Alignment Completeness for Relational Hoare Logics

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LICS 29 June 2021

This work was partially supported by NSF award CNS 1718713
and ONR N00014-17-1-2787.

Relational Properties of Programs

$$c \quad \widehat{=} \quad z := x; \textsf{if } x \geq 10 \textsf{ then } z := 2 * z \textsf{ else } z := 3 * x$$
$$c' \quad \widehat{=} \quad z := x; \textsf{if } z \geq 10 \textsf{ then } z := z + z \textsf{ else } z := z + z + z$$

Equiv: from states with the same value for x, get same value for z

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- given $x = x'$, after $z := x \mid z := x$ have $x = x' = z = z'$
- at which point $x \geq 10 \iff z' \geq 10$
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Lockstep aligned conditional rule

$$\frac{\mathcal{R} \Rightarrow e = e' \qquad c|c' : \mathcal{R} \wedge e \approx\!> \mathcal{S} \qquad d|d' : \mathcal{R} \wedge \neg e \approx\!> \mathcal{S}}{\mathsf{if}\ e\ \mathsf{then}\ c\ \mathsf{else}\ d\ \mathsf{fi} \mid \mathsf{if}\ e'\ \mathsf{then}\ c'\ \mathsf{else}\ d'\ \mathsf{fi} : \mathcal{R} \approx\!> \mathcal{S}}$$

One-rule Cook complete RHL

Consider the following RHL rule[1]:

$$\frac{c; d' : \mathcal{R} \rightsquigarrow \mathcal{S} \qquad d' \text{ is renamed copy of } d}{c \mid d : \mathcal{R} \approx\!> \mathcal{S}} \ \textsc{SeqProd}$$

If $c|d : \mathcal{R} \approx\!> \mathcal{S}$ is true then it is provable in HL + SeqProd.

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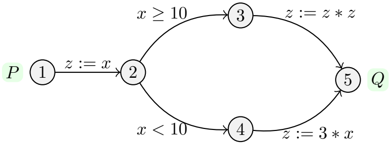
Floyd completeness of Hoare logic



$z := x$; if $x \geq 10$ then $z := 2{*}z$ else $z := 3{*}x$

Given a valid annotation, an, for $c : P \rightsquigarrow Q$, there is a HL proof using only judgements derived from an.

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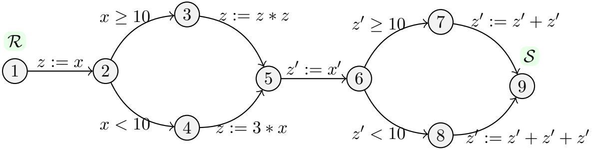

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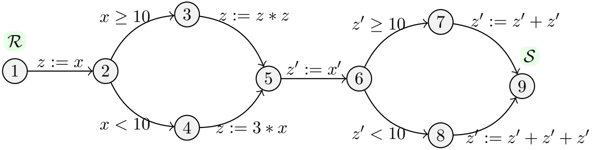

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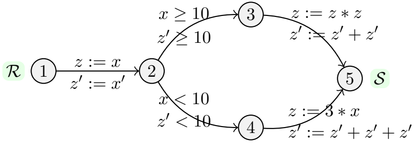

$$\frac{\mathcal{R} \Rightarrow e = e' \qquad c|c' : \mathcal{R} \wedge e \approx\!> \mathcal{S} \qquad d|d' : \mathcal{R} \wedge \neg e \approx\!> \mathcal{S}}{\text{if } e \ldots c \ldots d \mid \text{if } e' \ldots c' \ldots d' : \mathcal{R} \approx\!> \mathcal{S}}$$

Thm: Lockstep rules are alignment complete for lockstep products.