HW5

2.6.7a, 3.1.9a,g, 3.1.22, 3.2.3a, 3.2.20c,e,k, 2.6.7

a. $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ 0 \end{bmatrix}$

it's not a linear transformation since

$$c\left(T\begin{bmatrix} x \\ y \end{bmatrix}\right) = c\begin{bmatrix} xy \\ 0 \end{bmatrix} = \begin{bmatrix} cxy \\ 0 \end{bmatrix}$$

$$T\left(c\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\begin{bmatrix} cx \\ cy \end{bmatrix} = \begin{bmatrix} c^2xy \\ 0 \end{bmatrix}$$

as the example above,
it doesn't preserve scalar multi-
plication.

3.1.9

a. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$\det(A+B) = 9$$

$$A+B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

But $\det A = 1$ $\det B = 4$

$$\det A + \det B = 5$$

So, $\det(A+B) \neq \det(A) + \det(B)$

9.

Theorem 3.1.3 says,

"If A is an $n \times n$ matrix, then

$\det(uA) = u^n \det A$ for any number u"

So since $u = -1$.

$\det(-1 \cdot A)$ certainly is $-1^n \det A$.

So it's true.

7.1.22

you can find the determinant by swaping the whole matrix upside down

So, for even number of rows, you need to swap $\frac{n}{2}$ times.

for odd number of rows, you need to swap $\frac{n-1}{2}$ times.

Every time you swap rows, you multiply -1 to the determinant.

So that's why we get $(-1)^k$ where $n=2k$ or $n=2k+1$

3.2.3

a. $\det(A^3 B C^T B^{-1})$

$= (-1)^3 (2)(3)\left(\frac{1}{2}\right)$

$= -1 \cdot \cancel{2} \cdot 3 \cdot \cancel{2} = -3$

3.2.20

C.

since $\det(AB) = \det(A) \cdot \det(B)$

and det. of a matrix is a scalar,

$\det(AB) = \det(BA)$.

Also consider $\det(B) = \det(B^T)$

then certainly

$\det(AB) = \det(B^T A)$ is true

e.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ since $A^T = -A$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -a & -c \\ -b & -d \end{bmatrix}$ which means

$a = 0$ and $d = 0$.

$b = -c$ and $c = -b$.

$\det \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix} = b^2$.

So NO $b^2 \neq -1$.

K.

certainly if $\det(AB) \neq 0$

then det(A) and det(B)

cannat be 0.

So yes if AB is invertable,

A and B are also invertable.