2.2 - 7, 9, 11ac, 15, 17

2.2.7

$A(\vec{x}_0 + t\vec{u} + s\vec{v}) \leftarrow \vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \vec{v} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$

$= A\vec{x}_0 + tA\vec{u} + sA\vec{v}$

$= b + t(0) + s(0) = b$

$$\boxed{\text{so, } \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + s\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \text{ is two-parameter family of solutions to } Ax = b}$$

2.2.9

$$a_1 X a_2 = \begin{bmatrix} 0-1 \\ 1-0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = b$$

to check b is really not a linear combination,

$$b \cdot a_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = 0 - 1 + 1 = 0$$

so, $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ is in fact not a linear combination of a_1, a_2, a_3.

2.2.11

a.

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} -x \\ y \end{bmatrix}, \text{ so } \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

c.

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} -y \\ -x \end{bmatrix} \text{ so } \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

2.2.15

$$x = \begin{bmatrix} x_1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} + \begin{bmatrix} 0 \\ x_2 \\ 0 \\ \vdots \end{bmatrix} + \cdots$$

$$A = \begin{bmatrix} a_1 & a_2 & \cdots \end{bmatrix}$$

$$Ax = a_1 \begin{bmatrix} x_1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ x_2 \\ 0 \\ \vdots \end{bmatrix} + \cdots$$

So if A had a 0 row at some j

Ax will have a zero entry at row j.

2.2.17

Assume b is a linear combination of A.

There should be coefficients $c_1, c_2, \ldots c_n$

Then there

such that $B = cA$

or $B = C_1 A_1, C_2 A_2 \; \ldots \; C_n A_n$

So if $Ax = b$ has no solution,

it contradicts that b is a linear combination of A.

therefore, if $Ax = b$ is inconsistent,

it implies that b is not a linear combination of A