HW10

6.1.2aeg, 6.2.1ace, 6.2.7ac, 6.3.6ac, 6.3.12acm

Exercise 6.1.2 Are the following sets vector spaces with the indicated operations? If not, why not?

a. The set V of nonnegative real numbers; ordinary addition and scalar multiplication.

No, because if you multiply any negative c, then it wouldn't be in V.

e. The set V of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$; operations of $\mathbf{M}_{22}$.

if $a=b=c=0$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} + \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ 0+0 & c+c \end{bmatrix}$$ closed in +

$$u \cdot \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} ua & ub \\ u\cdot 0 & uc \end{bmatrix}$$ Closed in *

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} + \begin{bmatrix} -a & -b \\ 0 & -c \end{bmatrix} = 0$$

So the set $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a Vector space

g. The set V of 2×2 matrices with zero determinant; usual matrix operations.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det\left(k\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$

$$= k^2 \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 0$$

$$\det(A+B) \neq \det(A) + \det(B)$$

So Not a Vector space

Exercise 6.2.1 Which of the following are subspaces of $\mathbf{P}_3$? Support your answer.

a. $U = \{f(x) \mid f(x) \in \mathbf{P}_3,\ f(2) = 1\}$

$f(2) = 1 \neq 0$

so V doesn't have the zero poly of P_3

is Not a subspace

c. $U = \{xg(x) \mid g(x) \in \mathbf{P}_3\}$

$xg(x)$ is P_4

so it's certainly not subspace

e. $U =$ The set of all polynomials in $\mathbf{P}_3$ with constant term 0

$\underline{ax^3 + bx^2 + cx + 0}$

certainly contains 0

$(ax^3 + bx^2 + cx + 0) + (a_1 x_1^3 + bx^2 + cx + 0)$

still has 0 constant term

$a(ax^3 + bx^2 + cx + 0)$

still has 0 constant term

is subspace.

Exercise 6.2.7 Determine whether $\mathbf{v}$ lies in span $\{\mathbf{u}, \mathbf{w}\}$ in each case.

a. $\mathbf{v} = 3x^2 - 2x - 1$; $\mathbf{u} = x^2 + 1$, $\mathbf{w} = x + 2$

$3u = 3x^2 + 3$

$-2w = -2x - 4$

$3u - 2w = 3x^2 - 2x - 1$

sa tes

c. $\mathbf{v} = \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}; \mathbf{u} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

$V = au + bw$

$V - au = bw$

$V - u = \begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}$

there isn't any scalar b

that $b \cdot \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}$

So, NO

Exercise 6.3.6 Exhibit a basis and calculate the dimension of each of the following subspaces of $\mathbf{P}_2$.

a. $\{a(1+x)+b(x+x^2) \mid a \text{ and } b \text{ in } \mathbb{R}\}$

basis : $\{(1+x), (x+x^2)\}$

dim : 2

c. $\{p(x) \mid p(1) = 0\}$

basis: $\{(x-1), (x-1)^2\}$

dim : 2

Exercise 6.3.12 In each case, either prove the assertion or give an example showing that it is false.

a. Every set of four nonzero polynomials in $\mathbf{P}_3$ is a basis.

Not every.

needs to contain each $(1), (x), (x^2), (x^3)$

to make all possible P_3

c. $\mathbf{P}_2$ has a basis of polynomials $f(x)$ such that $f(0) = 1$.

basis: $\{1, (1+x), (1+x)^2\}$

to prove that

$$a \cdot 1 + b \cdot (1+x) + c \cdot (1+x)^2 = 0$$

if and only if $a = b = c = 0$

$$a + b + bx + c(1 + 2x + x^2) = 0$$

$$(a+b+c) + (b+2c)x + cx^2 = 0$$

$$a+b+c = 0$$

$$b + 2c = 0$$

$$c = 0$$

so $a = b = c = 0$

or

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

0 0 1

which shows that it's [I

m. If $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{0}$ then $\{\mathbf{u},\ \mathbf{v}\}$ is dependent if and only if one is a scalar multiple of the other.

$au + bv = 0$

at least one of them not equal to 0

would make it dependent

forward direction

$au + bv = 0$ for some $a, b \neq 0$

then $au = -bv$

$\therefore$ $u = \frac{-b}{a} v$

backward direction

for $u = av$ for some a

a can't be 0, since $u, v \neq 0$

so $u - av = 0$

for $a \neq 0$

then it is linearly dependent.