

HW 1

$$1.1 - 6, 11, 14$$

$$1.2 - 1, 2, 12(a, b)$$

$$\underline{1.1.6}$$

No solution:

$$0x + 0y + 1 = 0$$

one solution:

$$2x + 3 = 0$$

∞ solution:

$$x - y = 0$$

$$1.1.11$$

$$a. \quad 3x - 2y = 5 \xrightarrow{\times 4} 12x - 8y = 20$$

$$-12x + 8y = -20$$

$0 = 0$, So ∞ solutions

b. $3x - 2y = 5 \xrightarrow{\times 4} 12x - 8y = 20$
 $-12x + 8y = 6$

$0 = 26$, So No solutions

1.1.14

a.  $\begin{matrix} & \overbrace{\hspace{2cm}}^n \\ \underbrace{\hspace{1cm}}_m & \end{matrix}$ false. m rows

b. either  intersect or  same line, so false

c. yes.

Elementary operations including Row operation do not effect the result.

d. yes. Same reason as c.

1.2.1

None of them are in RREF.

a, c, d, e are in REF.

1.2.2

a.

$$I \rightarrow \begin{bmatrix} 0 & -1 & 2 & 1 & 2 & 1 & -1 \\ 0 & 1 & -2 & 2 & 7 & 2 & 4 \\ 0 & -2 & 4 & 3 & 7 & 1 & 0 \\ 0 & 3 & -6 & 1 & 6 & 4 & 1 \end{bmatrix}$$

$$b = b + a$$

$$c = c + 2a$$

$$d = d - 3a$$

$$\rightarrow \begin{bmatrix} 0 & 1 & -2 & 2 & 7 & 2 & 4 \\ 0 & 0 & 0 & 3 & 9 & 3 & 3 \\ 0 & 0 & 0 & 7 & 21 & 5 & 8 \\ 0 & 0 & 0 & -5 & -15 & -2 & -11 \end{bmatrix}$$

$$b = \frac{1}{3}b$$

$$a = a - 2b$$

$$\rightarrow \begin{bmatrix} 0 & 1 & -2 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 & 1 & 1 \end{bmatrix}$$

$$\begin{aligned} c &= c - 7b \\ d &= d + 5b \end{aligned} \left[\begin{array}{cccccc|c} 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 & 4 \end{array} \right]$$

$$\begin{aligned} c &= -\frac{1}{2}c \\ b &= b - c \\ d &= d - 3c \end{aligned} \left[\begin{array}{cccccc|c} 0 & 1 & -2 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{2} \end{array} \right]$$

$$\begin{aligned} d &= \frac{2}{5}d \\ a &= a - 2d \\ b &= b - \frac{3}{2}d \\ c &= b + \frac{1}{2} \end{aligned} \left[\begin{array}{cccccc|c} 0 & 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$b. \left[\begin{array}{cccccc|c} 0 & -1 & 3 & 1 & 3 & 2 & 1 \\ 0 & -2 & 6 & 1 & -5 & 0 & -1 \\ 0 & 3 & -9 & 2 & 4 & 1 & -1 \\ 0 & 1 & -3 & -1 & 3 & 0 & 1 \end{array} \right]$$

$$\begin{aligned} \text{I} \\ b &= b + 2a \\ c &= c - 3a \\ d &= d + a \end{aligned} \left[\begin{array}{cccccc|c} 0 & 1 & -3 & -1 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 5 & -5 & 1 & -4 \\ 0 & 0 & 0 & 0 & 6 & 2 & 2 \end{array} \right]$$

$$\begin{array}{l}
 b = -b \\
 a = a + b \\
 c = c - 5b
 \end{array}
 \rightarrow
 \begin{bmatrix}
 0 & 1 & -3 & 0 & 2 & 0 & 0 \\
 0 & 0 & 0 & 1 & -1 & 0 & -1 \\
 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 6 & 2 & 2
 \end{bmatrix}$$

$$\begin{array}{l}
 I \\
 c = \frac{1}{6}c \\
 a = d - 2c \\
 b = b + c
 \end{array}
 \rightarrow
 \begin{bmatrix}
 0 & 1 & -3 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & \frac{1}{3} & -\frac{2}{3} \\
 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} \\
 0 & 0 & 0 & 0 & 0 & 1 & 1
 \end{bmatrix}$$

$$\begin{array}{l}
 b = b - \frac{1}{3}d \\
 c = c - \frac{1}{3}d
 \end{array}
 \rightarrow
 \begin{bmatrix}
 0 & 1 & -3 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 1
 \end{bmatrix}$$

1.2.12

← the equations are dependent

a. $\left| \begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 2 & 4 \end{array} \right|$, false

b. $\left| \begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right|$, true

