1. Suppose we have eight seats, three boys and five girls. In how many different ways can these people sit such that no two boys are sitting next to each other?

We note that each boy will need to occupy one seat each, and there will need to be at least two seats separating the three boys. This leaves us to consider the three remaining seats. We choose to interpret this scenario as a stars and bars problem: ie we need to position the three seats around the three boys. There are four placements for each seat to go (before the first boy, in between any of the three boys, or after the third boy), and using the stars and bars formula with $n = 3$ objects and $k = 4$ we note that there are $\binom{n+(k-1)}{(k-1)} = \binom{6}{3}$ such possible arrangements. Further, there are $3!$ possible orderings of the boys, and $5!$ possible orderings of the girls, hence the final answer is $\binom{6}{3} \cdot 3! \cdot 5! = 14400$.

2. What is the explicit form of the following recurrence relation $T(n) = 3T(n/3) + 1; T(1) = 1$.

We start by assuming $n = 3^k$ for some positive integer k, then we observe the following:
$T(3^k) = 3T(3^{k-1}) + 3^0$
$T(3^k) = 3^2T(3^{k-2}) + 3^0 + 3^1$
$T(3^k) = 3^3T(3^{k-3}) + 3^0 + 3^1 + 3^2$
$T(3^k) = 3^kT(3^{k-k}) + 3^0 + 3^1 + 3^2 + \cdots + 3^{k-1}$
$T(3^k) = 3^k + 3^0 + 3^1 + 3^2 + \cdots + 3^{k-1}$
$T(3^k) = \sum_{i=0}^{k} 3^k = (3^{k+1} - 1)/2 = (3n - 1)/2$

3. What is the explicit form of the following recurrence relation $T(n) = T(n-1) + \log_2 n; T(0) = 0$. Hint $n!$ is approximately $\sqrt{2\pi n} n^n e^{-n}$.

$T(n) = T(n-1) + \log_2 n$
$T(n) = T(n-2) + \log_2(n) + \log_2(n-1)$
$T(n) = T(n-3) + \log_2(n) + \log_2(n-1) + \log_2(n-2)$
$T(n) = T(n-n) + \log_2(n) + \log_2(n-1) + \log_2(n-2) + \cdots + \log_2(1)$
$T(n) = \log_2(n!) \approx \log_2(\sqrt{2\pi n} n^n e^{-n})$
$T(n) \approx \log_2(\sqrt{2\pi n}) + \log_2(n^n) + \log_2(e^{-n})$
$T(n) \approx \log_2(\sqrt{2\pi n}) + n \log_2(n) - n * \log_2(e)$

4. Consider mergesort.

   (a) Give two sorted arrays of size 4 whose merging requires the maximum number of comparisons.

   (b) What is the minimum and maximum number of comparisons needed when merging two nonempty sorted lists of size n into a single list?

For (a) we consider the arrays $(1, 3, 5, 7)$ and $(2, 4, 6, 8)$. For (b), let's call the two lists A_1 and A_2. The minimum number of comparisons happens when all the elements in one of the list are smaller than all the elements in the other list. So let's say we are trying to sort everything in ascending order and all the items in A_1 are smaller than A_2. Then we would only need to compare each element of A_1 with the first element in A_2 once, making the total number of comparisons n. The maximum number of comparison occurs when $A_1[i] < A_2[i] < A_1[i+1]$ or $A_2[i] < A_1[i] < A_2[i+1]$ for all $i < n$. The total number of comparisons in this case is $2n - 1$ (the algorithm will have to zig-zag through the arrays).