THE UNIVERSITY OF AUCKLAND

SEMESTER TWO 2021
Campus: City

Software Engineering

Data Structures and Algorithms

(Time allowed: 2 hours and 30 min additional time)

NOTE: Answer all questions.
There are 21 marks in total for multiple choice questions in Section A
There are 24 marks in total in for short answer questions in Section B
Scan and upload to Inspera your answers to the short answer questions.

Section B: Short answer questions

1. Consider the following pseudo-code:

```
function meh(array a[0 ··· n − 1])
    if n > 2 then
        m ← ⌊(n−1)/2⌋
        for i ← 0 to m do
            if a[i] is even
                3 elementary operations e
            else
                2 elementary operations e
        recursive call meh(a[0, ··· , m])
return 0
```

   Assume the input array contains random 32-bits integers.

   (a) Find a recurrence formula $T(n)$ together with initial condition that calculates the number of elementary operation e the function performs in the average case. [4 marks]

   (b) What is the Big-Theta of $T(n)$? Briefly justify your answer (you do not need to show full working). [2 marks]

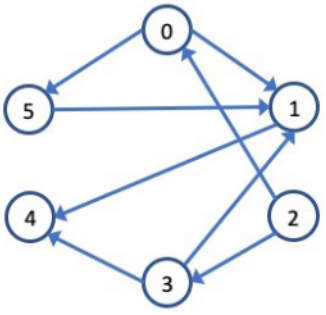
2. In the following, consider a hash table that uses chaining for collision resolution and has load factor λ. Assume the uniform hashing hypothesis.

   (a) What is the uniform hashing hypothesis? [1 mark]

   (b) Derive the probability of a collision in the hash table. [2 marks]

   (c) What is the expected running time for unsuccessful search in the hash table? Justify your answer. [2 marks]

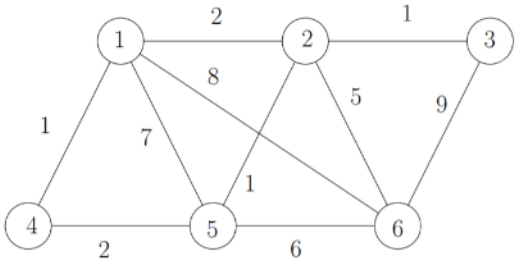
3. Consider the digraph G show below.



   (a) Find a topological order for G by performing DFS starting from 0 and choosing the node with the lowest index where there is a choice. Show your working. [2 marks]

   (b) Prove that a digraph H has a cycle if and only if there is a back arc when DFS is performed on H. [2 marks]

4. Find the weight of the minimum spanning tree by carrying out (by hand) Kruskal's algorithm on the following graph. When there is a choice, choose the node with the lowest index. Show your working step by step by showing the spanning forest at each iteration. [3 marks]



5. (a) Explain the main idea and key steps of the algorithm discussed in lectures that finds the strong components of a digraph $G = (V, E)$ in running time $\Theta(|V| + |E|)$. [2 marks]

   (b) Perform the algorithm on the digraph below by hand. Show your working. [4 marks]