

## PS4 SOLUTIONS

### PS4-1 2D transformation of a point and a parabola

(1)

$$\text{Translation matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Scale matrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rotate matrix} \begin{bmatrix} c45 & -s45 & 0 \\ s45 & c45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By multiplying these transformation matrices from left to right in the order, we can get the result.

$$\begin{bmatrix} c45 & -s45 & 0 \\ s45 & c45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} -3\sqrt{2} \\ 2 \\ \frac{11\sqrt{2}}{2} \\ 1 \end{bmatrix}$$

(2)

$$\text{Translation matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv T$$

$$\text{Rotate matrix} \begin{bmatrix} c(-30) & -s(-30) & 0 \\ s(-30) & c(-30) & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv R(-30)$$

$$\text{Scale matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv S$$

By multiplying these transformation matrices from left to right in the order, we can get the result.

$$SR(-30)T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Take the inverse to represent x, y by using x' and y'.

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = (SR(-30)T)^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T^{-1}R(-30)^{-1}S^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Here,

$$T^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R(-30)^{-1} = R(30) = \begin{bmatrix} c(30) & -s(30) & 0 \\ s(30) & c(30) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The result transformation matrix is,

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{4} & -1 \\ \frac{1}{2} & \frac{\sqrt{3}}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

$$x = \frac{\sqrt{3}}{2}x' - \frac{1}{4}y' - 1, \quad y = \frac{1}{2}x' + \frac{\sqrt{3}}{4}y'$$

By plugging these into the given equation  $y = x^2$ , we can get the solution.

$$\frac{1}{2}x' + \frac{\sqrt{3}}{4}y' = \left( \frac{\sqrt{3}}{2}x' - \frac{1}{4}y' - 1 \right)^2$$

#### PS4-2 Homogeneous geometric/coordinate transformation

(1)

$$R = \begin{bmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} & 0 & 0 \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = TR \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

By using these, we can express x, y, z by using x', y', z' as follows.

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = R^{-1}T^{-1} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \frac{\pi}{6} & \sin \frac{\pi}{6} & 0 & 0 \\ -\sin \frac{\pi}{6} & \cos \frac{\pi}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

So,

$$x = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' - \frac{\sqrt{3}}{2}$$

$$y = -\frac{1}{2}x' + \frac{\sqrt{3}}{2}y' + \frac{1}{2}$$

$$z = z'$$

Plug these into the original plane equation.

$$x + y + z + 1 = \frac{\sqrt{3}-1}{2}x' + \frac{1+\sqrt{3}}{2}y' + \frac{1-\sqrt{3}}{2}z' + 1 = 0$$

$$\boxed{\frac{\sqrt{3}-1}{2}x' + \frac{1+\sqrt{3}}{2}y' + \frac{1-\sqrt{3}}{2}z' + 1 = 0}$$

(2)

Transformation matrices

$${}^W A = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^W B = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^W C = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from Cube A to Cube B:  ${}^A B$

$${}^A B = {}^A W {}^W B = [{}^W A]^{-1} {}^W B = \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 2 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & -4 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from Cube B to Cube C:  ${}^B C$

$${}^B C = {}^B W {}^W C = [{}^W B]^{-1} {}^W C = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation from Cube C to Cube A:  ${}^C A$

$${}^C A = {}^C W {}^W A = [{}^W C]^{-1} {}^W A = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 & 3 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 6 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & -1 & 4 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{5\sqrt{2}}{2} \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### PS4-3 Converting a polygon file to a VRML file

Next problem solution includes reading .dat file and writing the data into vrml file. Please refer to the solution for PS4-4.

### PS4-4 Rotation and scaling of an object

#### MATLAB Code:

```
%%  
  
% ps3-1  
  
clear all  
  
close all  
  
clc  
  
str = 'triceratops.dat';  
  
%str = 'shape.dat';  
  
%str = 'cube.dat';  
  
fid = fopen(str, 'r'); % for triceratops.data  
  
Vertex=[]; % initialize vertex container matrix  
Order=[]; % initialize vertex order container matrix  
while (1)  
    Line = fgetl(fid); % read a line from the file  
    if (Line == -1) break; end % if the line is end, terminate this loop  
    Line = strread(Line,'%s'); % read the first character  
    if( strcmp(Line{1}, 'v')) % if the character is v, read the remaining information as a number and  
store it to the vertex matrix  
        Vertex = [Vertex ; str2num(Line{2}), str2num(Line{3}), str2num(Line{4})];  
    elseif( strcmp(Line{1}, 'f')) % if the character is f,  
        Numbers = { Line{2:end} }; % change the string cell matrix to string array.  
        Data = []; % initialize the Data array  
        for i=1:size(Numbers,2) % iterate i to the number of remaining ordering numbers, read remaining  
information as a number  
            Data=[Data,str2num(Numbers{i})];  
        end  
        %Data = Data+1; % the index of facet starts from 0. So, modify the indices by adding one  
        Order = [Order ; {Data}]; % make the vertex order  
    end  
end  
  
fclose(fid); % reading file ends. close the file.
```

```

fid = fopen('triceratops.wrl', 'w');

fprintf(fid, '#VRML V2.0 utf8\n');

fprintf(fid, 'Background{ skyColor 1 1 1 }\n');

fprintf(fid, 'DEF Axes Shape{\n');

fprintf(fid, '    geometry Extrusion\n');

fprintf(fid, '    {\n');

fprintf(fid, '        spine[0 0 0, 10 0 0, 10 0 0, 11 0 0]\n');

fprintf(fid, '        crossSection[0.5 0.5, 0.5 -0.5, -0.5 -0.5, -0.5 0.5, 0.5 0.5]\n');

fprintf(fid, '        scale[0.1 0.1, 0.1 0.1, 0.3 0.3, 0 0]\n');

fprintf(fid, '    }\n');

fprintf(fid, '    appearance Appearance{material Material{diffuseColor 0 0 0}}\n');

fprintf(fid, '}\n');

fprintf(fid, 'Transform{\n');

fprintf(fid, '    rotation 0 0 1 1.570796327\n');

fprintf(fid, '    children[ USE Axes]\n');

fprintf(fid, '}\n');

fprintf(fid, 'Transform{\n');

fprintf(fid, '    rotation 0 1 0 -1.570796327\n');

fprintf(fid, '    children[ USE Axes]\n');

fprintf(fid, '}\n');

fprintf(fid, 'Transform{\n');

fprintf(fid, '    translation 5 10 0\n');

fprintf(fid, '    children[Shape{geometry Text{\n');

fprintf(fid, '        string["SooHo Park"]\n');

fprintf(fid, '        fontStyle DEF Fonts \n');

fprintf(fid, '        FontStyle{\n');

fprintf(fid, '            family "SANS"\n');

fprintf(fid, '            justify "MIDDLE"\n');

fprintf(fid, '            style "BOLD"\n');

fprintf(fid, '            size 1\n');

fprintf(fid, '        }\n');

fprintf(fid, '    }\n');

fprintf(fid, '    appearance DEF colors Appearance{material Material{diffuseColor 0 0 0}}\n');

fprintf(fid, '}\n');

fprintf(fid, ']\n');

```

```

fprintf(fid, '      }\n');
%%
fprintf(fid, 'Shape{ \n');
fprintf(fid, '\t geometry IndexedFaceSet{ \n');
fprintf(fid, '\t\t coord Coordinate{ point[\n');

for i=1:size(Vertex,1)
    fprintf(fid, '\t\t\t %f %f %f,\n', Vertex(i,1), Vertex(i,2), Vertex(i,3));
end

fprintf(fid, '\t\t] }\n');
fprintf(fid, 'coordIndex[\n');
for i = 1:size(Order,1) % check how many the point is. iterate this loop
    Polygon = Order(i); % read a cell of a polygon
    VertexOrder = Polygon{1}(1:size(Polygon{1},2)); % read the vertex order
    fprintf(fid, '\t\t\t');
    for j=1:size(VertexOrder,2)
        fprintf(fid, '%d, ', VertexOrder(j));
    end
    fprintf(fid, '-1, ');
    fprintf(fid, '\n');
end

fprintf(fid, '\t\t]\n');
fprintf(fid, '\t}\n');
fprintf(fid, 'appearance Appearance{ material Material{ diffuseColor 0 0 1}}\n');
fprintf(fid, '}\n');

%%
fprintf(fid, 'Shape{ \n');
fprintf(fid, '\t geometry IndexedFaceSet{ \n');
fprintf(fid, '\t\t coord Coordinate{ point[\n');

theta = pi/4;
c = cos(theta);
s = sin(theta);
t = 1-c;
axis = [1,1,1];

```

```

x = axis(1)/norm(axis);
y = axis(2)/norm(axis);
z = axis(3)/norm(axis);

RotMatrix = [ t*x*x+c,t*x*y-z*s,t*x*z+y*s,0;
              t*x*y+z*s,t*y*y+c,t*y*z-x*s,0;
              t*x*z-y*s,t*y*z+x*s,t*z*z+c,0;
              0,0,0,1
            ];

ScaleMatrix = [2,0,0,0;
               0,1,0,0;
               0,0,1,0;
               0,0,0,1
            ];

for i=1:size(Vertex,1)
    Vertex2 = ScaleMatrix * RotMatrix * [Vertex(i,:),1]';
    fprintf(fid, '\t\t\t %f %f %f,\n', Vertex2(1), Vertex2(2), Vertex2(3));
end

fprintf(fid, '\t\t\t ]\n');
fprintf(fid, 'coordIndex[\n');

for i = 1:size(Order,1) % check how many the point is. iterate this loop
    Polygon = Order(i); % read a cell of a polygon
    VertexOrder = Polygon{1}(1:size(Polygon{1},2)); % read the vertex order
    fprintf(fid, '\t\t\t\t');
    for j=1:size(VertexOrder,2)
        fprintf(fid, '%d, ', VertexOrder(j));
    end
    fprintf(fid, '-1, ');
    fprintf(fid, '\n');
end

fprintf(fid, '\t\t\t]\n');
fprintf(fid, '\t]\n');

fprintf(fid, 'appearance Appearance{ material Material{ diffuseColor 1 0 1}}\n');
fprintf(fid, ')\n');
fclose(fid);

```



## C++ Code:

```
// ps3.cpp by Arbtip Dheeravongkit
```

```
#include <iostream.h>
#include <fstream.h>
#include <iomanip.h>
#include <stdlib.h>
#include <string.h>
#include "ps3.h"

int main()
{
    //Open and read data from file
    ifstream dataFile("triceratops.dat");
    if ( !dataFile ) {
        cerr << "File could not be opened\n";
        exit (1);
    }

    //Open output files
    ofstream outFile;
    outFile.open ("triceratops.wrl", ios::out);
    if (!outFile) {
        cerr << "Cannot open this file for output.\n";
        exit (-1);
    }

    ofstream outFile2;
    outFile2.open ("triceratops2.wrl", ios::out);
    if (!outFile2) {
        cerr << "Cannot open this file for output.\n";
        exit (-1);
    }

    //Draw axes on vrml output files
    vrml_axes (outFile, MY_NAME);
    vrml_axes (outFile2, MY_NAME);

    //Declare and initialize variables
    double dataVertex[2900][4]; //Matrix contains coordinates of vertices
    int dataFace [2900][15]; //Matrix contains coordIndex of faces

    for (int x=0; x<2900; x++)
        for(int y=0; y<14; y++)
            dataFace[x][y] = 0;

    char buffer[100];
    char *tokenPtr;
    double test;
    int vIndex;
    int j=0;
    int b=0;
    double U[3]; //Unit vector to rotate about
    double R[4][4]; //Rotation matrix
    double S[4][4]; //Scale matrix
    double RS[4][4]; //Rotation then Scale matrix

    //Assign values to matrices and vector
    U[0] = U[1] = U[2] = 1.0/(sqrt(3.0));

    R[0][0] = cos(angle) + (U[0]*U[0]*(1-cos(angle)));
    R[0][1] = (-U[2]*sin(angle)) + (U[0]*U[1]*(1-cos(angle)));
    R[0][2] = (U[1]*sin(angle)) + (U[2]*U[0]*(1-cos(angle)));
    R[0][3] = 0.0;
    R[1][0] = (U[2]*sin(angle)) + (U[0]*U[1]*(1-cos(angle)));
    R[1][1] = cos(angle) + (U[1]*U[1]*(1-cos(angle)));
    R[1][2] = (-U[0]*sin(angle)) + (U[1]*U[2]*(1-cos(angle)));
    R[1][3] = 0.0;
    R[2][0] = (-U[1]*sin(angle)) + (U[2]*U[0]*(1-cos(angle)));
    R[2][1] = (U[0]*sin(angle)) + (U[1]*U[2]*(1-cos(angle)));
    R[2][2] = cos(angle) + (U[2]*U[2]*(1-cos(angle)));
    R[2][3] = 0.0;
```

```

R[3][0] = R[3][1] = R[3][2] = 0.0;
R[3][3] = 1.0;

for(int s=0;s<4;s++) {
    for(int t=0;t<4;t++)
        S[s][t] = 0.0;
}
S[0][0] = 2.0;
S[3][3] = S[1][1] = S[2][2] = 1.0;

//Rotate-then-Scale matrix
for(int l=0; l< 4; l++) {
    for(int m=0; m<4; m++) {
        RS[l][m]=0;
        for (int n=0; n<4; n++)
            RS[l][m] += R[l][n]*S[n][m];
    }
}

//Read values from input file
while (!dataFile.eof())
{
    dataFile.getline(buffer,100);
    tokenPtr = strtok(buffer," ");

    int i=0;
    if (tokenPtr[0] != 'f') {
        while (tokenPtr != NULL) {
            if (tokenPtr[0]!='v') {
                test = atof(tokenPtr);
                dataVertex[j][i] = test;
                i++;
            }
            tokenPtr = strtok(NULL," ");
        }
        dataVertex[j][3] = 1.0;
        j++;
    }
    else {
        //Read face data
        //and store values in a matrix dataFace
        int a=0;
        while (tokenPtr!=NULL) {
            if (tokenPtr[0]!='f') {
                vIndex = atoi(tokenPtr);
                dataFace[b][a+1] = vIndex;
                a++;
            }
            tokenPtr = strtok(NULL," ");
        }
        dataFace[b][0] = a;
        b++;
    }
    //Read the next line from input file
}

int numVertex = j;
int numFace = b;

//Total number of vertices
//Total number of faces

//Transform coordinates
double transf[2900][4];
for(int f=0; f< numVertex; f++) {
    for(int g=0; g<4; g++) {
        transf[f][g]=0;
        for (int h=0; h<4; h++)
            transf[f][g] += dataVertex[f][h]*RS[h][g];
    }
}

//Draw Faces on output vrmf files
vrmf_drawFace (outFile, dataFace, dataVertex, numFace, numVertex, 1, 0); //Original on first output
vrmf_drawFace (outFile2, dataFace, dataVertex, numFace, numVertex, 1, 0); //Original on second output
vrmf_drawFace (outFile2, dataFace, transf, numFace, numVertex, 0, 1); //Transformed on second out

```

```

R[3][0] = R[3][1] = R[3][2] = 0.0;
R[3][3] = 1.0;

for(int s=0;s<4;s++) {
    for(int t=0;t<4;t++)
        S[s][t] = 0.0;
}
S[0][0] = 2.0;

```

```
//vrml.cpp by Arbtip Dheeravongkit
```

```
#include "ps3.h"

void vrml_axes (ofstream &outFile, char* my_name) {
    Draw in the output vrml file, background, heading, and axis
    // outFile: pointer to the output vrml file
    // my_name: my name to be shown in the heading

    // Create Header
    outFile << "#VRML V2.0 utf8\n";
    outFile << "#-----\n";
    outFile << "# 24-786: Geometric Modeling\n";
    outFile << "# <<MY_NAME<<\n";
    outFile << "# Transforming Triceratops\n";
    outFile << "#-----\n\n";

    // Set up background
    outFile << "# Specify the background color\n";
    outFile << "#-----\n";
    outFile << "Background {\n\tskyColor 1 1 1\n}\n\n"; // White background

    // Put text heading on the VRML screen
    outFile << "# Draw texts\n";
    outFile << "#-----\n";
    outFile << "Transform {\n";
    outFile << "\ttranslation 0 13 0\n";
    outFile << "\tchildren Shape {\n";
    outFile << "\t\tappearance Appearance {\n";
    outFile << "\t\t\tmaterial Material {\n";
    outFile << "\t\t\t\tdiffuseColor 0 0 0\n";
    outFile << "\t\t\t}\n\t\t}\n";
    outFile << "\t\tgeometry Text {\n";
    outFile << "\t\t\tstring [\"24-786: Geometric Modeling\", \n";
    outFile << "\t\t\t\"Transforming Triceratops\", \n";
    outFile << "\t\t\t\"<<MY_NAME<<\" ]\n";
    outFile << "\t\t\tfontStyle FontStyle {\n";
    outFile << "\t\t\t\tfamily \"SANS\"\n";
    outFile << "\t\t\t\tjustify \"MIDDLE\"\n";
    outFile << "\t\t\t\tstyle \"BOLD\"\n";
    outFile << "\t\t\t\tsize 1\n\t\t\t}\n\t\t}\n\n";

    // Add the x-axis
    outFile << "# Define the x axis\n";
    outFile << "#-----\n";
    outFile << "Transform {\n";
    outFile << "\ttranslation 0 0 0\n";
    outFile << "\tchildren DEF AXIS Shape {\n";
    outFile << "\t\tappearance Appearance {\n";
    outFile << "\t\t\tmaterial Material {\n";
    outFile << "\t\t\t\tdiffuseColor 0.0 0.0 0.0\n\t\t\t}\n\t\t}\n";
    outFile << "\t\tgeometry Extrusion {\n";
    outFile << "\t\t\tcrossSection [2 0, 0 -2, -2 0, 0 2, 2 0]\n";
    outFile << "\t\t\tscale [0.035 0.035, 0.035 0.035, 0.15 0.15, 0 0]\n";
    outFile << "\t\t\tspine [-10 0 0, 8 0 0, 8 0 0, 10.5 0 0]\n\t\t\t}\n\t\t}\n\n";

    // Add the y-axis
    outFile << "# Draw the y axis\n";
    outFile << "#-----\n";
    outFile << "Transform {\n";
    outFile << "\trotation 0 0 1 1.5707\n";
    outFile << "\tchildren USE AXIS\n}\n\n";

    // Add the z-axis
    outFile << "# Draw the z axis\n";
    outFile << "#-----\n";
    outFile << "Transform {\n";
    outFile << "\trotation 0 1 0 -1.5707\n";
    outFile << "\tchildren USE AXIS\n}\n\n";
}
```

```
void vrml_drawFace (ofstream &outFile2, int M[][15], double N[][4], int numFace, int numVertex, int R, int C)
//Draw a faces on output vrml file
```

