PS3-1 Calculation of the area a 2D polygon

1) **Pseudo Code**

```
1.  Open text file
2.  Read all points into structure
3.  Close text file

4.  Initialize Area,
5.  Repeat i
6.      Calculate area of triangle segment
7.      PolygonArea <- PolygonArea + area of triangle
8.  Until all vertices are visited

9.  Display
    Problem Set 3-1
    Kenji Shimada
    File Name
    # of vertices
    Area
```

2) **Source code**

```
clear all
close all
clc
%% read the .txt file
fname = 'bird';
%fname = 'mouse';
fid = fopen([fname,'.txt']);     % open file to read
C = textscan(fid,'%f %f');       % read the data
fclose(fid);                     % close the file

V = [C{1,1}, C{1,2}];            % vertex list
nVertice = size(V,1);            % number of vertices

%% calculate the area
```

```
V = [V; V(1,:)];                    % add the first vertex again
area = 0;                           % initialize area
for i=1:nVertice
    area = area + 0.5*(V(i,1)*V(i+1,2) - V(i,2)*V(i+1,1));
end

%% display the results
disp('Problem Set 3-1');
disp('Kenji Shimada');
disp(sprintf('File Name: %s.txt', fname));
disp(sprintf('# of vertices: %d', nVertice));
disp(sprintf('Area: %f', area));
```

3) **Output**

```
Problem Set 3-1
Kenji Shimada
File Name: bird.txt
# of vertices: 172
Area: 34911.023

Problem Set 3-1
Kenji Shimada
File Name: mouse.txt
# of vertices: 180
Area: 157764.93
```

4) **Readme**

Programming language: MATLAB

Operating system: MAC OS X Yosemite

Complier & interpreter: MATLAB

PS3-2 Calculation of the surface area and the volume of a polyhedron

5) Pseudo Code

```
1.  Open data file
2.  Initialize Vertex, Order
3.  While forever
4.      Read a line from the file
5.      If file ends
6.          break While loop
7.      Else
8.          continue
9.      Endif
10.     Read first character of the line
11.     If the character is 'v'
12.         Store remaining numbers into Vertex
13.     Else If first character of the line is 'f'
14.         Store all remaining numbers into Order
15.     EndIf
16. Endwhile
17. Close data file

18. Initialize Area, Volume
19. Repeat i
20.     Read vertex information of polygon from Order(i)
21.     Define the first point as a pivot point to calculate Area
22.     Define the normal vector of first triangle as a plus sign
vector of Area

23.     Initialize PolygonArea, PolygonVolume
24.     Repeat j
25.         Define j-th triangle of the polygon
26.         Calculate area of triangle
27.         Calculate sign of area
28.         Multiply sign of area and area of triangle
29.         PolygonArea <- PolygonArea + area of triangle
```

```
30.         Calculate volume of tetrahedron
31.         PolygonVolume <- PolygonVolume + volume of tetrahedron
32.     Until all the triangles are visited

33.     Area <- Area + area of polygons
34.     Volume <- Volume + volumes of tetrahedrons
35. Until all polygons are visited

36. Set the sing of Volume to positive.

37. Display
    Problem Set 3-2
    Sooho Park
    File Name
    # of vertices
    # of faces
    Area
    Volume
```

6) Source code

```
clear all
close all
clc
str = ‘triceratops.dat’;
% str = ‘shape.dat’;
% str = ‘cube.dat’;
fid = fopen(str, 'r'); % for triceratops.data

Vertex=[]; % initialize vertex container matrix
Order=[]; % initialize vertex order container matrix
while (1)
   Line = fgetl(fid); % read a line from the file
   if (Line == -1) break;  end % if the line is end, terminate this loop
   Line = strread(Line,'%s'); % read the first character
   if( strcmp(Line{1}, 'v'))
      Vertex = [Vertex ; str2num(Line{2}), str2num(Line{3}), str2num(Line{4})];
```

```
    elseif( strcmp(Line{1}, 'f')) % if the character is f,
        Numbers = { Line{2:end} }; % change the string cell matrix to string array.
        Data = []; % initialize the Data array
        for i=1:size(Numbers,2)
            Data=[Data,str2num(Numbers{i})];
        end
        Data = Data+1;
        Order = [Order ; {Data}]; % make the vertex order
    end
end
fclose(fid); % reading file ends. close the file.

Orig = [0,0,0]; % origin point to calculate volume. this can be whatever.
Area = 0; % initialize area
Volume = 0; % initialize volume
for i = 1:size(Order,1) % check how many the point is. iterate this loop
    Polygon = Order(i); % read a cell of a polygon
    VertexOrder = Polygon{1}(1:size(Polygon{1},2)); % read the vertex order
    p0 = Vertex(VertexOrder(1),:);
    p1 = Vertex(VertexOrder(2),:);
    p2 = Vertex(VertexOrder(3),:);
    SignVector = cross((p1-p0),(p2-p0));
    PolygonArea = 0; % initialize polygon area
    PolygonVolume = 0; % initialize polygon volume
    for j = 2:(size(VertexOrder,2)-1)
        p1 = Vertex(VertexOrder(j),:);
        p2 = Vertex(VertexOrder(j+1),:);
        PolygonAreaSign = sign(dot ( cross((p1-p0),(p2-p0)),  SignVector));
        PolygonArea = PolygonArea + PolygonAreaSign*(1/2)*norm(cross((p1-p0),(p2-p0)));
        PolygonVolume = PolygonVolume + (1/6)*dot( (Orig-p0), cross((p1-p0),(p2-p0)) );
    end
    Area = Area + abs(PolygonArea);
    Volume = Volume + PolygonVolume;
end
Volume = abs(Volume);
```

```
disp('Problem Set 3-2');
disp('Sooho Park');
disp(sprintf('File Name: %s', str));
disp(sprintf('# of vertices: %d', size(Vertex,1)));
disp(sprintf('# of faces: %d', size(Order,1)));
disp(sprintf('Area: %f', Area));
disp(sprintf('Volume: %f', Volume));
```

7) Output

Problem Set 3-2

Sooho Park

File Name: shape.dat

# of vertices: 33

# of faces: 66

Area: 0.016716

Volume: 0.00014955

Problem Set 3-2

Sooho Park

File Name: triceratops.dat

# of vertices: 2832

# of faces: 2834

Area: 219.499905

Volume: 136.741966

8) Readme

Programming language: Matlab

Operating system: Windows XP

Complier & interpreter: Matlab

PS3-3 homogeneous 2D geometric transformation

1) The transformation is pure skewing in the x direction:

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

or

$$x + ay = x'$$
$$y = y'$$

This transformation moves one of the corner points, (0, 5), to a new position, (5, 5):

$$\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$$

Using this condition, we find a:

$$0 + 5a = 5$$
$$5 = 5$$

$$a = 1$$

The skewing transformation is:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

By applying this transformation to the four corner points of the original square, we can confirm that all the points are moved to the correct new locations.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix}$$

2) Please update the solution as we discussed on the phone.

Let's assume the homogeneous transformation matrix which moves the all corner points as in the figure.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

This transformation moves one of the corner point (0,0) to (1,2)

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},$$

Using this condition, we find,

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$c = 1, \quad f = 2$$

This transformation moves one of the corner point (0,5) to (4,11)

$$\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 4 \\ 11 \\ 1 \end{bmatrix}$$

Using this condition, we find,

$$\begin{bmatrix} a & b & 1 \\ d & e & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 11 \\ 1 \end{bmatrix}$$

$$5b + 1 = 4$$
$$5e + 2 = 11$$

$$b = \frac{3}{5}$$

$$e = \frac{9}{5}$$

This transformation moves one of the corner point (5,0) to (8,4)

$$\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 8 \\ 4 \\ 1 \end{bmatrix}$$

Using this condition, we find,

$$\begin{bmatrix} a & \frac{3}{5} & 1 \\ d & \frac{9}{5} & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 1 \end{bmatrix}$$

$$5a + 1 = 8$$

$$5d + 2 = 4$$

$$a = \frac{7}{5}$$

$$d = \frac{2}{5}$$

As the result,

$$\begin{bmatrix} \frac{7}{5} & \frac{3}{5} & 1 \\ \frac{2}{5} & \frac{9}{5} & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

By applying this transformation to the four corner points of the original square, we can confirm that all the points are moved to the correct new locations.

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 4 \\ 11 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} 8 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix} \to \begin{bmatrix} 11 \\ 13 \\ 1 \end{bmatrix}$$