24-681 COMPUTER-AIDED DESIGN Spring 2016

Carnegie Mellon University

PROBLEM SET 2

Due: **1/28/2016 (Thu) 3:00PM @ DH A302**
Issued: 1/19/2016 (Tue)
Weight: 3% of total grade
Note: *** Attach the last page of the problem set as the cover page of your paper.**
*** Use the mathematical notation scheme and the representations of lines and planes discussed in the lectures on 1/14 – 1/26.**

PS2-1 Intersection of a line and a plane

Find the intersection of a line, Line 1, and a plane, Plane 1. Line 1 goes through two points, (1, 2, 3) and (8, 3, 5). Plane 1 goes through three points, (0, 1, 0), (1, 2, 4), and (3, 3, 3).

PS2-2 Minimum distance from a point to a line

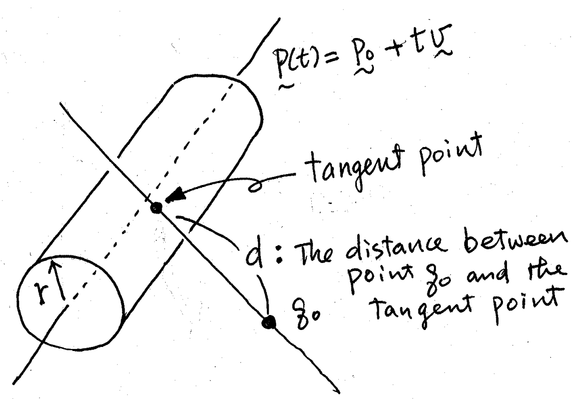
Find the minimum distance from a point (0,0,1) to the intersection line of two planes, $x+y+z=1$ and $2y-z=2$. Also find the foot of the perpendicular from the point to the line (the point where the perpendicular from (0,0,1) to the line intersects the line.)

PS2-3 Minimum distance between two lines

Given two lines, Line 1 and Line2, where
Line 1 goes through point $\mathbf{p}_1$ and its direction vector is $\mathbf{v}_1$
Line 2 goes through point $\mathbf{p}_2$ and its direction vector is $\mathbf{v}_2$
find the minimum distance between the two lines.

PS2-4 Tangent distance from a point to a cylinder

There exists a family of lines that go through a point, $\mathbf{q}_0$, and are tangent to a cylinder whose axis is $\mathbf{p}(t)=\mathbf{p}_0+t\,\mathbf{v}$ and whose radius is *r*. Among such tangent lines, consider the two lines that minimize the distance from point $\mathbf{q}_0$ to the tangent point on the cylinder. How do you find the minimum of the distance, *d*?



PS2-5 Solving a vector equation

Given a plane, $\mathbf{p}(u,v) = \mathbf{p} + u\mathbf{a} + v\mathbf{b}$, and a line, $\mathbf{p}(t) = \mathbf{q} + t\mathbf{v}$, find the intersection point by solving vector equation $\mathbf{p} + u\mathbf{a} + v\mathbf{b} = \mathbf{q} + t\mathbf{v}$.

PS2-6 Intersection of three planes

Show that the intersection of three planes is given by

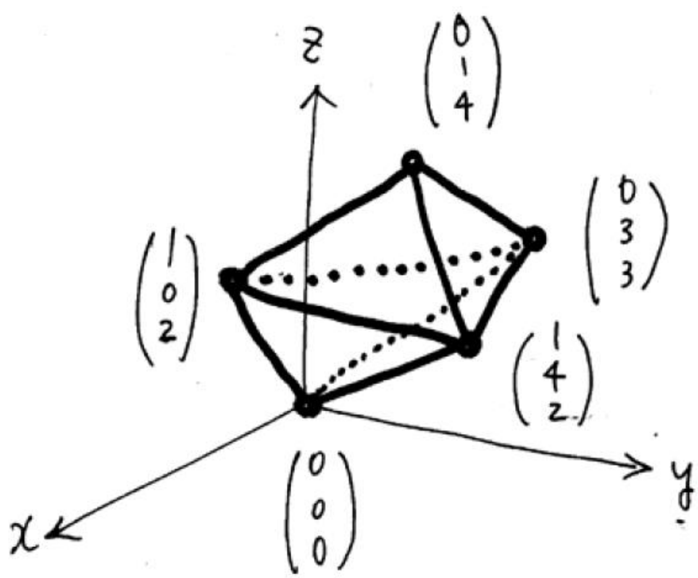
$$\mathbf{p_c} = \frac{d_1(\mathbf{n_2} \times \mathbf{n_3}) + d_2(\mathbf{n_3} \times \mathbf{n_1}) + d_3(\mathbf{n_1} \times \mathbf{n_2})}{\mathbf{n_1} \cdot (\mathbf{n_2} \times \mathbf{n_3})}$$

where $\mathbf{n_1}$, $\mathbf{n}_2$, and $\mathbf{n}_3$ are unit normal vectors of the three planes, and d_1, d_2 , and d_3 are the perpendicular distances to each plane from the origin. Thus the equations of the planes are $\mathbf{n}_1 \cdot \mathbf{p} = d_1$, $\mathbf{n}_2 \cdot \mathbf{p} = d_2$, and $\mathbf{n}_3 \cdot \mathbf{p} = d_3$. You do not have to derive the above equation. Just show that the point $\mathbf{p}_c$ lies on the three planes.
Hint: Use the "triple scalar product" formula:

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

PS2-7 Volume of a polyhedron

Find the volume of the polyhedron shown in the figure below.



PS2

The first letter of your LAST name ____________________ ____________________

First Name Last Name

How many hours did you spend to complete this problem set?

__________ Hour(s)

How many no-penalty late days do you want to use for PS2?

__________ Day(s)

PS2-1	PS2-2	PS2-3	PS2-4	PS2-5	PS2-6	PS2-7	Following Instructions

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