24-681 COMPUTER-AIDED DESIGN Spring 16

Carnegie Mellon University

PROBLEM SET 8

Due: **3/17/2016 (Thu) 3:00PM @ DH A302**
Issued: 3/1/2016 (Tue)
Weight: 3% of total grade
Note: * **Attach the last page of the problem set as the cover page of your paper.**

PS8-1 Cubic Hermite curves

The matrix form of Hermite curves is written as:

$$\mathbf{p}(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_0^u \\ \mathbf{p}_1^u \end{bmatrix}.$$

(1) What are the blending functions $F_1(u), F_2(u), F_3(u)$, and $F_4(u)$ of Hermite curves?

$$\mathbf{p}(u) = F_1(u)\,\mathbf{p}_0 + F_2(u)\,\mathbf{p}_1 + F_3(u)\,\mathbf{p}_0^u + F_4(u)\,\mathbf{p}_1^u$$

(2) What is the matrix form of the first derivative of Hermite curves, $\mathbf{p}^u(u)$? Show the derivation for full credit.

(3) What is the matrix form of the second derivative of Hermite curves, $\mathbf{p}^{uu}(u)$? Show the derivation for full credit.

PS8-2 C and G Continuity

Consider the following two parametric curve segments:

$$\mathbf{q}(u) = (\ u-1,\ u^2 - 2u + 1,\ 0\ ), \quad 0 \le u \le 1$$

$$\mathbf{p}(u) = (\ u,\ u^3,\ 0\ ), \quad 0 \le u \le 1$$

(1) These two curve segments share one end point. What is the coordinate value of the point where these curve segments connect? What continuity do the curve segments have at the joint, C^0, G^0, C^1, or G^1?

(2) The second curve segment $\mathbf{p}(u) = (\ u,\ u^3,\ 0\ ),\quad 0 \le u \le 1$ is a cubic curve. If you represent the curve as a cubic Hermite curve, what are the four geometry vectors, $\mathbf{p}_0,\ \mathbf{p}_1, \mathbf{p}_0^u,$ and $\mathbf{p}_1^u$?

PS8-3 Curve Continuity

Consider the following three cubic curve segments:

$$\mathbf{q}(u) = (\ u-1,\ u^2 - 2u + 1,\ 0\ ),\quad 0 \le u \le 1$$

$$\mathbf{p}(u) = (\ ?,\ ?,\ ?\ ),\ 0 \le u \le 1$$

$$\mathbf{r}(u) = (\ 1+u,\ 1+u^3,\ 1\ ),\ 0 \le u \le 1$$

(1) Find the second curve, $\mathbf{p}(u)$, so that it connects to the first curve, $\mathbf{q}(u)$, at $u = 0$ and the third curve, $\mathbf{r}(u)$, at $u = 1$ with the C^1 continuity.

(2) If you represent the second curve, $\mathbf{p}(u)$, with a cubic Hermite curve, what are the four geometry vectors, $\mathbf{p}_0,\ \mathbf{p}_1, \mathbf{p}_0^u,$ and $\mathbf{p}_1^u$?

(3) If you represent the second curve, $\mathbf{p}(u)$, with a cubic Bezier curve, what are the four geometry vectors, $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_4$?

PS8-4 Curve representation

Consider the following two curve segments:

$$\mathbf{q}(u) = (\ u-1,\ u^2 - 2u + 1,\ 0\ ),\quad 0 \le u \le 1$$
$$\mathbf{p}(t) = (\ ?,\ ?,\ ?\ ),\ 0 \le t \le 1$$

(1) The second curve, $\mathbf{p}(t)$, is a <u>quadratic</u> curve. Find the curve so that it connects to the first curve, $\mathbf{q}(u)$, at t = 0 with the C^1 continuity and that it goes through point (1, 1, 1) at t = 1.

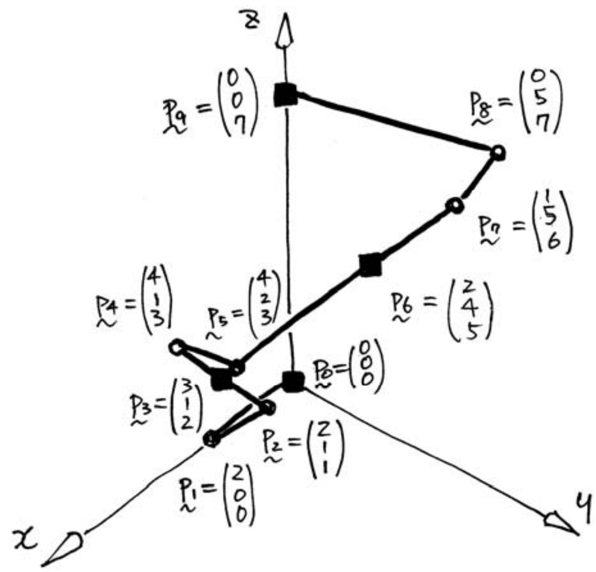
(2) What are the four geometry vectors, $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_4$, of the first curve, $\mathbf{q}(u)$, if we are to represent this curve as a Bezier curve?

(3) Now, take the first half of curve $\mathbf{q}(u)$, that is, $0 \le u \le 0.5$, and re-parameterize this curve so that the parametric range becomes $0 \le u \le 1.0$. What are the four Bezier geometry vectors, $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_4$, of this new piece of curve?

PS8-5 Drawing a Bezier curve

Write a program that draws Bezier curve segments and tangent vectors. Three sets of control points of cubic Bezier curve segments are given as follows:

first curve : P0 =(0,0,0), P1=(2,0,0), P2=(2,1,1), P3=(3,1,2)
second curve: P3=(3,1,2), P4=(4,1,3), P5=(4,2,3), P6=(2,4,5)
third curve: P6=(2,4,5), P7=(1,5,6), P8=(0,5,7), P9=(0,0,7)



(1) Write a subroutine that evaluates the location of a point and the tangent vector on a cubic Bezier curve.

```
void bezier (double b[4][3], double u, double p[3], double v[3])
 /* INPUT:  b[4][3]: four control points, b[0], b[1], b[2], b[3] */
 /*        u:       parameter value */
 /* OUTPUT p[3]:    the point location on curve */
 /*        v[3]:    the tangent vector on curve */
```

(2) Using your bezier subroutine, write a program that generates:

- bezier1.wrl: VRML file that shows the coordinate frame, control polygons and the curve
- bezier2.wrl: VRML file that shows the coordinate frame, the curve and the tangent vectors

  Approximate each curve by 20 line segments. Calculate tangent vectors at all the end points of the line segments. Use different colors for the coordinate frame, the curve, control polygons, and tangent vectors.

In your hand-in directory on AFS, make a new directory called ps8-5 (in lower case), and hand in:

- source codes, and
- bezier1.wrl and bezier2.wrl

Also hand in a printout of:

- source codes, and
- pictures of bezier1.wrl and bezier2.wrl

PS8

The first letter of your LAST name

__________________________ First Name

__________________________ Last Name

How many hours did you spend to complete this problem set?

___________ Hour(s)

How many no-penalty late days do you want to use for this problem set?

___________ Day(s)

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