Q2. $0.5x^2 + 3.2x + 4.5y^2 - 6y + 7$ = objective fn

$$Z = 0.5\left[x^2 + 0.9y^2\right] + 3.2x - 6y + 7$$

↳ const ⇒ irrelevant

$$Z = 0.5 \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} x & y \end{Bmatrix} \begin{Bmatrix} 3.2 \\ -6 \end{Bmatrix}$$

$$Z = \frac{1}{2} \begin{bmatrix} x & y \end{bmatrix} \begin{Bmatrix} 1 & 0 \\ 0 & 0.9 \end{Bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} 3.2 & -6 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

Thus we have, $H = \begin{bmatrix} 1 & 0 \\ 0 & 0.9 \end{bmatrix}$ $\quad f = \begin{bmatrix} 3.2 \\ -6 \end{bmatrix}$

Subject to $x, y \geq 0$

$x + 3y \geq 15$

$2x + 5y \leq 100$

$3x + 4y \leq 80$

$$\Rightarrow \underbrace{\begin{bmatrix} -1 & -3 \\ 2 & 5 \\ 3 & 4 \end{bmatrix}}_{A} \begin{Bmatrix} x \\ y \end{Bmatrix} \leq \underbrace{\begin{Bmatrix} -15 \\ 100 \\ 80 \end{Bmatrix}}_{b}$$

$-x - 3y \leq -15$

$2x + 5y \leq 100$

$3x + 4y \leq 80$

$A_{eq} = [\,]$, $b_{eq} = [\,]$

$lb = [\,]$, $ub = [\,]$

$$A = \begin{bmatrix} -1 & -3 \\ 2 & 5 \\ 3 & 4 \end{bmatrix} \qquad b = \begin{Bmatrix} -15 \\ 100 \\ 80 \end{Bmatrix}$$

Q3. Cost = $C_1 W + C_2 d$

$W = \rho V$ (Taking weight = mass)

$= \rho \frac{\pi H}{4} [d_2^2 - d_1^2]$

$= \frac{\pi \rho H}{4} [d_2 + d_1][d_2 - d_1]$

$= \frac{\pi \rho H}{4} [d + d + 2t](2t)$

$= \pi \rho H [d + t]t = \pi \rho H [dt + t^2]$

Thus cost = $C_1 \pi \rho H [dt + t^2] + C_2 d$

$= 4 \pi \rho H [dt + t^2] + 2d$

Given an approximation of 'small t', we have t^2=0

Thus Cost=4*pi*ro*H*d*t + 2d

Column must not buckle $\Rightarrow \sigma_b < \sigma_c$

$\frac{P}{\pi dt} < \frac{\pi E I}{H^2 dt}$ (As $d \neq 0$ $t \neq 0$ & $d > 0$ $t > 0$, we can cancel)

$\frac{P}{\pi} - \frac{\pi E I}{H^2} < 0 \Rightarrow \frac{P}{\pi} - \frac{\pi E}{H^2} \times \frac{\pi}{8} dt (d^2 + t^2) < 0$

$\Rightarrow \frac{P}{\pi} - \frac{\pi^2 E}{8 H^2} dt (d^2 + t^2) < 0$ ——— (1)

Must not yield under compression

$$\Rightarrow \sigma_c < \sigma_y$$

$$\Rightarrow \frac{P}{\pi dt} < \sigma_y \Rightarrow \boxed{\frac{P}{\pi dt} - \sigma_y < 0} \quad - (2)$$

Must not yield under buckling

$$\sigma_b < \sigma_y \Rightarrow \frac{\pi EI}{H^2 dt} - \sigma_y < 0$$

$$\Rightarrow \frac{\pi E}{H^2 dt} \times \frac{\pi}{8} dt \, (d^2 + t^2) - \sigma_y < 0$$

$$\boxed{\frac{\pi^2 E}{8H^2} (d^2 + t^2) - \sigma_y < 0} \quad - (3)$$

$$\boxed{d_1 \leq d \leq d_2} \quad - (4), (5)$$

$$\boxed{t_1 \leq t \leq t_2} \quad - (6) (7)$$