

Q2.1

$$I_{t+1} = I_t + \sum w_c B_c$$

Expanding, we have,

$$I_{t+1} = I_t + w_1 B_1 + w_2 B_2 + \dots w_n B_n$$

Taking dot product with B_j

$$(I_{t+1}) B_j = I_t B_j + \sum_{i=1}^n w_i B_i \cdot B_j$$

Since bases are orthogonal, the dot product $B_i \cdot B_j = 0 \quad \forall i \neq j$ & $B_i \cdot B_j = |B_j|^2 \quad \forall i=j$

$$\Rightarrow I_{t+1} B_j = I_t B_j + w_j |B_j|^2$$

$$(I_{t+1} - I_t) B_j = w_j |B_j|^2$$

$$w_j = \frac{(I_{t+1} - I_t) \cdot B_j}{|B_j|^2}$$