Q1.1 We translate each image coordinate system so that origins coincide with the principal point by translations, $\tilde{u} \simeq S^{-1}u \quad \tilde{u}' = S^{-1}u'$

Let the principal point be (u_0, v_0)

The new fundamental matrix is

$$\tilde{F} = S^{T}FS = \begin{bmatrix} F_{2\times2} & F_{2\times3}\, u_0 \\ u_0^T F_{3\times2} & \boxed{u_0^T F u_0} \end{bmatrix}.$$

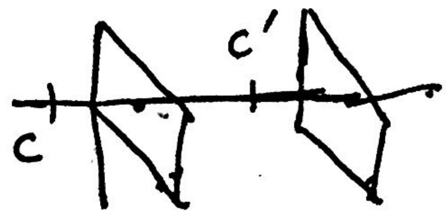
Since there is a case of fixation of cameras therefore u_0 in the left & right images satisfy epipolar constraint $\Rightarrow$ $u_0^T \underline{F u_0} = 0$

Thus new $\tilde{F}$ can be expressed as

$$\tilde{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & 0 \end{bmatrix}$$

Q1.2. Consider the 2^{nd} camera differing from the first by pure translation.

Thus we have $\underbrace{C = C'}_{\text{translation}}$ & $\underbrace{R = I}_{\text{zero rotation}}$

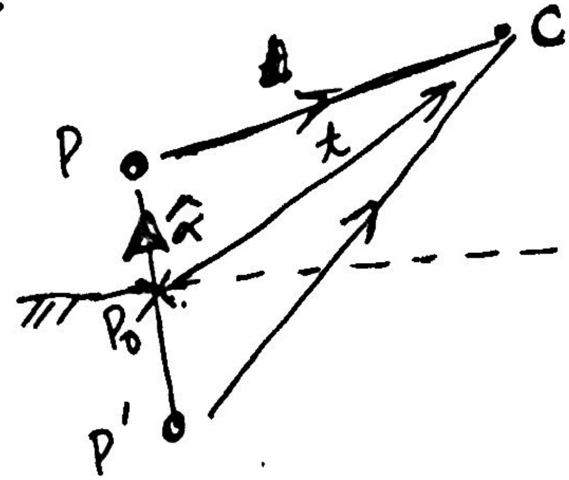


$$F = [C']^{-T} [T_x] R [C^{-1}]$$
$$= [C^{-T}] [T_x] [C^{-1}]$$
$$= [CT]_x = [e']_x = [e]_x$$

Thus ~~identical~~ equal epipolar lines imply a ~~scaling &~~ parallel nature

Thus epipolar lines of the 2 images are parallel

Q3.



In a global frame,

$$P_0 = \frac{P + P'}{2}$$

$$(2P_0 - P) = P'$$

$$P' = \underline{2P_0 - P}$$

Thus we have,
$$\begin{Bmatrix} P' \\ P \end{Bmatrix} = \begin{bmatrix} \cancel{I}-2\hat{\alpha}\alpha' & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} P \\ 1 \end{Bmatrix}$$

On transforming this system to camera

$$C = [I \mid t] \quad \text{(Pure translation)}$$

Thus
$$p = CP = [I \mid t] \begin{Bmatrix} P \\ 1 \end{Bmatrix}$$

$$p' = CP' = [I \mid t] \begin{Bmatrix} P' \\ 1 \end{Bmatrix}$$

$$= [I \mid t] \begin{Bmatrix} I-2\hat{\alpha}\alpha' & 0 \\ 0 & 1 \end{Bmatrix} \begin{Bmatrix} P_w \\ 1 \end{Bmatrix}$$

$$= [Ex] \begin{Bmatrix} P_w \\ 1 \end{Bmatrix}$$

The dual of this problem can be

seen as 2 cameras P & P' observing an object c. But the transformation b/w these cameras is a reflective one.

ie $[2\alpha\alpha^T \; t]$.

We can now note that the essential matrix

$$E = [T_x] R = [2\alpha\alpha^T t]_\times \underbrace{R}_{I}$$

skew symm

$$\Rightarrow \boxed{E^T = -E}$$

$$F = K^{-T} E K^{-1}$$

$$F^T = (K^{-T} E K^{-1})^T = K^{-T} E^T K^{-1}$$

$$= -K^{-T} E K^{-1} = -F$$

$$\boxed{F^T = -F}$$

Thus it is a skew symmetric fundamental matrix.