Review Practice: Chapters 16

1. Find the equation and parametric equations of the tangent plane at $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -2\right)$ to the parametric surface S: $\mathbf{r}(u,v) = \langle v\cos u, v\sin u, 2v\rangle$ for $-2 < v < 2$ and $0 < u < \pi$.

$\vec{r}_u = \langle -v\sin u, v\cos u, 0\rangle$ $\quad \vec{r}_u(\frac{3\pi}{4}, -1) = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\rangle = \frac{\sqrt{2}}{2}\langle 1,1,0\rangle$

$\vec{r}_v = \langle \cos u, \sin u, 2\rangle$ $\quad \vec{r}_v = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2\rangle = \frac{\sqrt{2}}{2}\langle -1, 1, 2\sqrt{2}\rangle$

Point (u,v):

$2v = -2 \Rightarrow v = -1$

$v\cos u = \frac{\sqrt{2}}{2}$

$\cos u = -\frac{\sqrt{2}}{2} \Rightarrow u = \frac{3\pi}{4}$

$\vec{n} = \vec{r}_u \times \vec{r}_v = \frac{1}{2}\langle 1,1,0\rangle \times \langle -1,1,2\sqrt{2}\rangle$

$= \langle \sqrt{2}, -\sqrt{2}, 1\rangle$

$\boxed{x = \frac{\sqrt{2}}{2} + \sqrt{2}t, \quad y = -\frac{\sqrt{2}}{2} - \sqrt{2}t, \quad z = -2 + t \\ \sqrt{2}(x - \sqrt{2}/2) - \sqrt{2}(y + \sqrt{2}/2) + (z+2) = 0}$

2. Find a parametrization of the surface given by:

$$3x + x^2 + 2y^2 - z^2 = 3 \quad \text{for} \quad z \le 0$$

$z^2 = 3x + x^2 + 2y^2 - 3$

$\boxed{z = \sqrt{3x + x^2 + 2y - 3}, \quad x = x, \quad y = y \quad \text{with} \quad 3x + x^2 + 2y \ge 3}$

3. Consider $\mathbf{F} = \langle xye^z, yze^x, xze^y\rangle$

   (a) Compute Div $\mathbf{F}$
   (b) Compute Curl $\mathbf{F}$
   (c) Is $\mathbf{F}$ conservative? Why or why not.

$\boxed{\text{Div } \vec{F} = ye^z + ze^x + xe^y}$

$\boxed{\text{Curl } \vec{F} = \langle xze^y - ye^x, ze^y - xye^z, yze^x - xe^z\rangle}$

$\boxed{\text{Since Curl } \vec{F} \neq \vec{0} \Rightarrow \vec{F} \text{ is not conservative}}$

4. $\mathbf{F} = \langle \overset{y\cos z}{y}, x\cos z, -xy\sin z\rangle$ find $\int_C \mathbf{F}\cdot d\mathbf{r}$ for any curve with initial point $(0,0,0)$ and terminal point $(1,1,0)$.

$\Downarrow$

$\vec{F}$ is conservative

$f = \int f_x\,dx = xy\cos z + C(y,z)$

$f = \int f_y\,dy = xy\cos z + C(x,z)$

$f = \int f_z\,dz = xy\cos z + C(x,y)$

$f = xy\cos z$

$\int_C \vec{F}\cdot d\vec{r} = \int_C \nabla f\cdot d\vec{r} \overset{\text{FTC}}{=} f(1,1,0) - f(0,0,0)$

$= 1 - 0$

$= \boxed{1}$

5. Set up only: $\int\int_S xy\, dS$ over D, where S is part of the graph of $z^2 = 4x^2 + 4y^2$ between the planes $z = -2$ and $z = -4$ and D is the region for your parameters.

D: $z = -\sqrt{4x^2+4y^2}$ $\quad -4 < z < -2$

or $1 \le x^2+y^2 \le 4$

or

$x = r\cos\theta \quad y = r\sin\theta \quad z = -2r$

$0 \le \theta \le 2\pi \qquad 1 \le r \le 2$

$\vec{r}_r = \langle \cos\theta, \sin\theta, -2\rangle \quad \vec{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$

$\vec{r}_r \times \vec{r}_\theta = \langle 2r\cos\theta, -2r\sin\theta, r\rangle$

$$\iint_S xy\,dS = \iint_D xy \cdot |\langle 2r\cos\theta, -2r\sin\theta, r\rangle|\,dr\,d\theta$$

$$= \boxed{\int_0^{2\pi}\int_1^2 r\cos\theta \cdot r\sin\theta\,\sqrt{5}\,r\,dr\,d\theta}$$

6. Use Stoke's Theorem to compute: $\int\int_S \text{Curl } \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \langle y, -x, z^2\rangle$ and S is part of $z = -x^2 - y^2$ above $z = -4$.

C: $-4 = -x^2 - y^2$ or $x^2 + y^2 = 4$

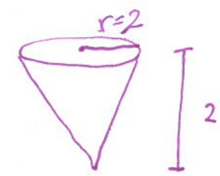
$\vec{r}(\theta) = \langle 2\cos\theta, 2\sin\theta, -4\rangle$

$$\iint_S \text{Curl}\,\vec{F}\cdot d\vec{S} = \int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle y, -x, z^2\rangle \cdot \vec{r}\,'(\theta)\,d\theta$$

$$= \int_0^{2\pi} \langle 2\sin\theta, -2\cos\theta, 16\rangle \cdot \langle -2\sin\theta, 2\cos\theta, 0\rangle\,d\theta$$

$$= \int_0^{2\pi} -4\,d\theta = \boxed{-8\pi}$$

7. Use the divergence theorem to compute: $\int\int_S \mathbf{F}\cdot d\mathbf{S}$ where $\mathbf{F} = \langle zy, 2y, 3z\rangle$ and S is the surface of the solid right cone $z^2 = x^2 + y^2$ for $0 \le z \le 2$.



$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \text{Div}\,\vec{F}\,dV = \iiint_E 0+2+3\,dV$$

$$= 5 \cdot V(\text{cone})$$

$$= 5 \cdot \frac{1}{3}\pi(2)^2\cdot(2)$$

$$= \boxed{\frac{40}{3}\pi}$$