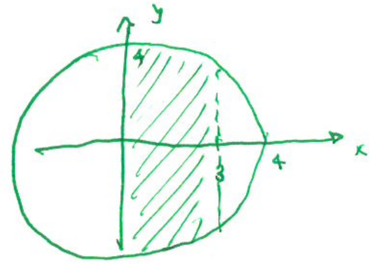
Test 15 - Review

Vector Calc

① Sketch the region of integration

$$\int_0^3 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} f(x,y)\,dy\,dx$$



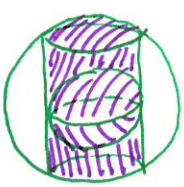
② Use polar coords to evaluate

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sec^2(x^2+y^2)\,dy\,dx = \int_{-\pi/2}^{\pi/2} \int_0^4 \sec^2(r^2)\,r\,dr\,d\theta = \pi\left[\frac{\tan(r^2)}{2}\right]\Big|_0^4$$

$$= \boxed{\frac{\pi}{2}\tan(16)}$$

③ Set up the integral to find the volume between the spheres

$x^2+y^2+z^2=4$ and $x^2+y^2+z^2=1$ inside $x^2+y^2=1$



$$\boxed{2\int_0^{2\pi}\int_0^1\int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} dz \cdot r\,dr\,d\theta}$$

④ Find $\iiint_E z\,dV$ where V is described in ③.

$$2\cdot 2\pi \cdot \int_0^1 \left((4-r^2)-(1-r^2)\right)\cdot r\,dr = 4\pi \cdot 3\left(\tfrac{1}{2}\right) = \boxed{6\pi}$$

⑤ use spherical coords to evaluate $\iiint_E y\,dV$ where E is the region above the xy-plane between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=1$.

$$\int_0^{\pi/2}\int_0^{2\pi}\int_1^2 \rho\sin\varphi\sin\theta \cdot \rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi = \int_1^2 \rho^3\,d\rho \int_0^{2\pi} \sin\theta\,d\theta \int_0^{\pi/2} \sin^2\varphi\,d\varphi$$

$$= \frac{1}{4}\left[16-1\right]\cdot\left[-\cos 2\pi + \cancel{\cos(0)}\right]\cdots$$

$$= \boxed{0}$$