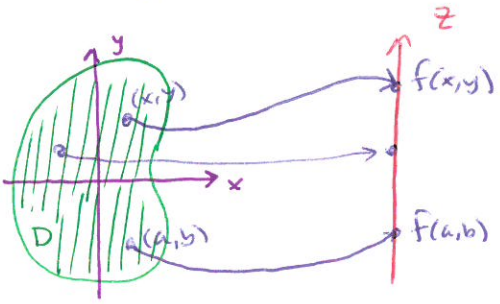
Section 14.1 – Functions of Several Variables

Vector Calc

A function of two variables – is a rule that assigns to each ordered pair (x,y) in D domain a real number that is unique $f(x,y)=z$. $D=\{f(x,y)\mid (x,y)\in D\}$



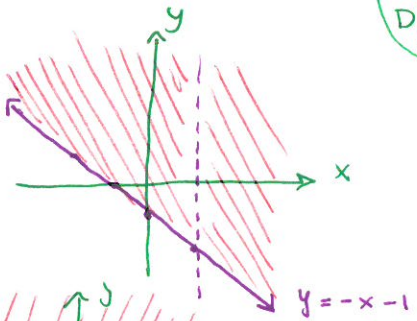
Example 1 Evaluate $f(3,2)$ and sketch the domain

(a) $f(x,y)=\dfrac{\sqrt{x+y+1}}{x-1}$ (b) $f(x,y)=x\ln(y^2-x)$

(a) $f(3,2)=\dfrac{\sqrt{6}}{2}$

$x+y+1\geq 0 \quad y\geq -x-1$

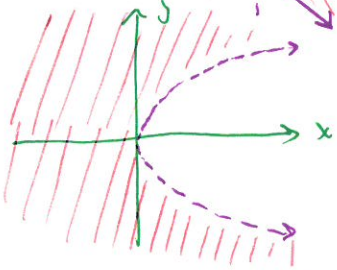
$x\neq 1$



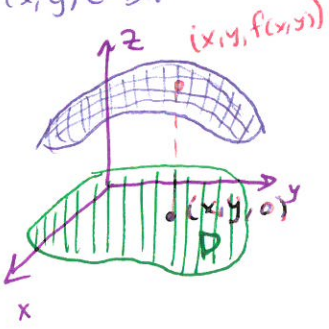
(b) $f(3,2)=3\ln(1)=0$

$y^2-x>0$

$y^2>x$



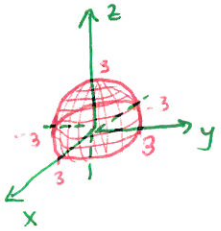
- If f is a function of 2 variables with domain D, then the graph of f is the set of all points $(x,y,z)\in\mathbb{R}^3$ such that $f(x,y)=z$ and $(x,y)\in D$.



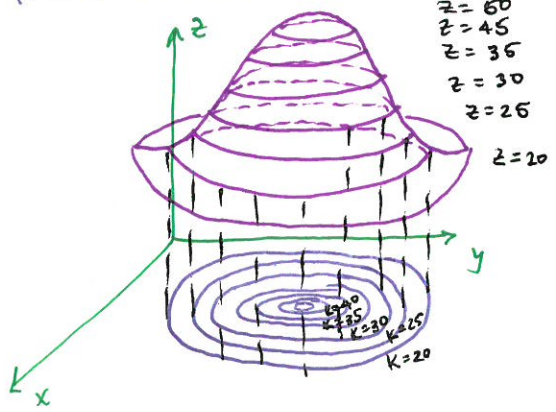
Example 6 Sketch the graph of $g(x,y)=\sqrt{9-x^2-y^2}$

$z^2=g(x,y)^2=9-x^2-y^2, \quad z\geq 0$

$x^2+y^2+z^2=3^2$ with $z\geq 0$



- The **level curves** of a function f are the curves with equations $f(x,y)=k$ where k is a constant in the range of f. (Think horizontal slices of your 3D graph)



- Level curves can be easier to draw than 3D graph
- easier to read info from level curves
- easier visualization with level curves

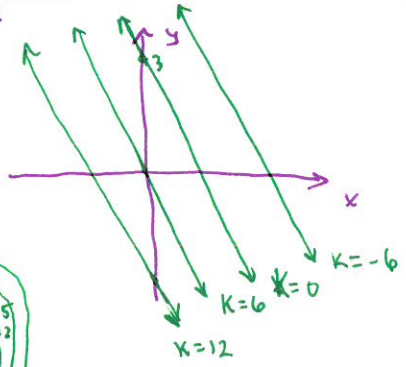
Ex. Topographic maps – Figure 12

Weather maps for Temp – Figure 13

level curves called isothermals

Ex 10 Sketch the level curves of the function $f(x,y) = 6 - 3x - 2y$ for the values $K = -6, 0, 6, 12$.

level curves: $K = 6 - 3x - 2y$ or $y = \frac{-3x + 6 - K}{2}$ lines with slope $-\frac{3}{2}$

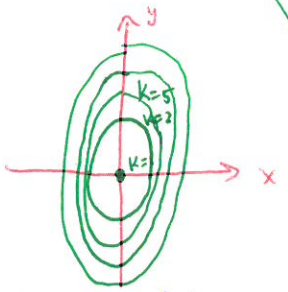


Ex 12 Sketch some level curves of $h(x,y) = 4x^2 + y^2 + 1$

Elliptic paraboloid up!

level curves: $K = 4x^2 + y^2 + 1$ or

$$\frac{x^2}{\frac{1}{4}(K-1)} + \frac{y^2}{K-1} = 1 \quad \text{(ellipses)}$$



Functions of 3 or More Variables:

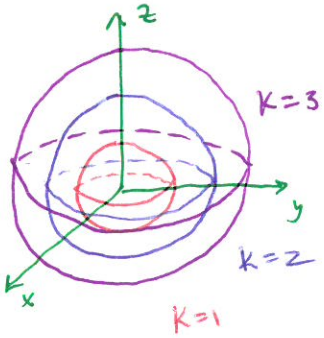
- A function of 3 variables is a rule that assigns to each ordered triple $(x,y,z) \in D \subseteq \mathbb{R}^3$ a unique real number $f(x,y,z)$.

Ex. 14 Find the domain of $f(x,y,z) = \ln(z - y) + xy \sin z$

Need $z - y > 0$ so $z > y$ $\quad D = \{(x,y,z) \in \mathbb{R}^3 \mid z > y\}$

Ex 15 Find the level surfaces of the function $f(x,y,z) = x^2 + y^2 + z^2$

level surfaces: $x^2 + y^2 + z^2 = K$ Spheres



★ Can't visualize in 4D can only visualize their shadows in 3D

★ 3D movie of life watched all at once – outside of time

- A function of n variables is a rule that assigns to each n-tuple $(x_1, x_2, \ldots, x_n)$ a unique number $z = f(x_1, x_2, \ldots, x_n)$.

Ex. Company uses n items to make a product, cost function

$$C = f(x_1, \ldots, x_n) = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$\vec{c} = \langle c_1, c_2, \ldots, c_n \rangle$ $\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$ then $C = f(\vec{x}) = \vec{c} \cdot \vec{x}$. (Vector function)