AP Calc AB

Agenda: 11/18/15
Lesson 68
Even and Odd Functions

Lesson 68

A function $f(x)$ is **even** if $f(x) = f(-x)$ for all x in the domain of f. [Symmetric about y-axis]

A function $f(x)$ is **odd** if $f(-x) = -f(x)$ for all x in the domain of f. [Symmetric about origin]

★ If f is even then $\int_{-a}^{a} f(x) dx = 2 \int_{0}^{a} f(x) dx$

★ If f is odd then $\int_{-a}^{a} f(x) dx = 0$

★ A Poly is even if all exponents of x are even and odd if all exponents of x are odd.

   $-x^3 - (-x) = -(x^3 - x) = -f(x)$ [odd]

Ex. 68.2 Is $f(x) = x^3 - x$ an even, odd or neither function?

$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x)$ [odd]

So $\int_{-2}^{2} (x^3 - x) dx = 0$

Ex. 68.6 Are the following even, odd or neither functions?

(a) $f(x) = e^x$

(b) $g(x) = e^{-x^2}$

(a) $f(-x) = e^{-x} \neq -f(x)$ nor $f(x)$

   [Neither even nor Odd]

(b) $g(-x) = e^{-(-x)^2} = e^{-x^2} = g(x)$

   [Even]

Ex. 68.7 Is $K(x) = \dfrac{x^2 + \cos(x)}{\sin(x)}$ even, odd or neither?

$K(-x) = \dfrac{(-x)^2 + \cos(-x)}{\sin(-x)} = \dfrac{x^2 + \cos(x)}{-\sin(x)} = -K(x)$ [Odd]

★ Recall $\cos(x) = \cos(-x)$ because its even
and $\sin(-x) = -\sin(x)$ because its odd.