Section 16.6 - Parametric Surfaces and their Areas

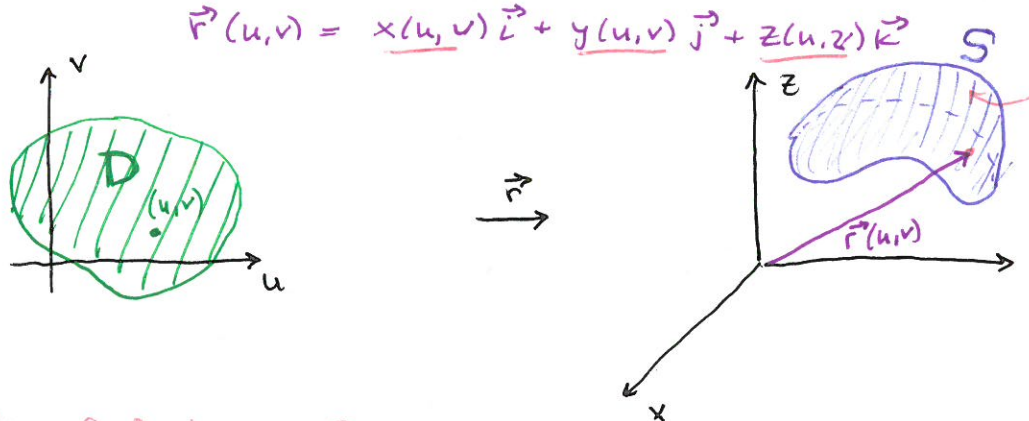
Chapter 12 - Special surfaces from functions of two variables or level surfaces of functions of three variables

Now use vector functions to describe general surfaces - Parametric Surfaces

Space curve C described by a vector function $\vec{r}(t)$

parametric surface S described by a vector function $\vec{r}(u,v)$ of two parameters u, v

$$\vec{r}(u,v) = \underline{x(u,v)}\,\vec{i} + \underline{y(u,v)}\,\vec{j} + \underline{z(u,v)}\,\vec{k}$$



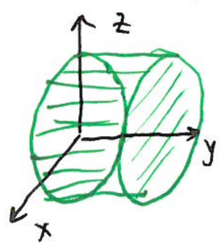
Called the parametric equations of S

* tip of $\vec{r}$ traces S as (u,v) move throughout D

Ex 1 Identify and sketch the surface with vector equation

$$\vec{r}(u,v) = 2\cos u\,\vec{i} + v\,\vec{j} + 2\sin u\,\vec{k}$$

Parametric equations: $x = 2\cos u \quad y = v \quad z = 2\sin u$

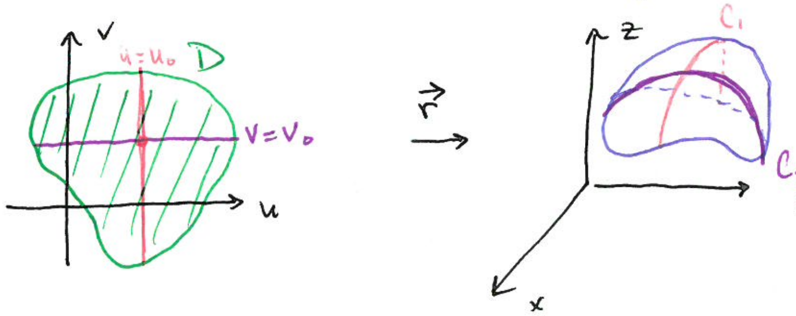


$x^2 + z^2 = 4$ - Cross sections ∥ xz plane are all circles of radius 2

No restrictions on u, v gives a cylinder of radius 2.

Useful Family of Curves: Called grid curves where u is constant and then v is constant

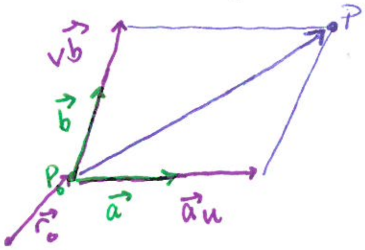
$\vec{r}(u_0, v)$ - space curve $\qquad \vec{r}(u, v_0)$ - space curve



This is what Computers use when graphing Parametric Surfaces

Section 16.6 - Parametric Surfaces and their Areas   Vector Calc

Ex 3 Find a vector function that represents the plane through the point P_0, position vector $\vec{r_0}$, containing two nonparallel vectors $\vec{a}$ and $\vec{b}$.



P any point in plane

$\vec{P_0P} = u\vec{a} + v\vec{b}$ so $\vec{r} = \vec{r_0} + \vec{P_0P} = \vec{r_0} + u\vec{a} + v\vec{b}$

$$\boxed{\vec{r}(u,v) = \vec{r_0} + u\vec{a} + v\vec{b}}$$

Parametric Equations:

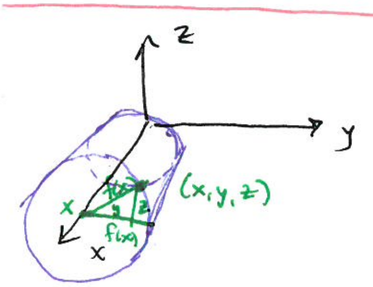
$x = x_0 + ua_1 + vb_1 \quad y = y_0 + ua_2 + vb_2 \quad z = z_0 + ua_3 + vb_3$

Ex 7 Find a parametric representation for the surface $z = 2\sqrt{x^2+y^2}$, that is, the top half of the cone $z^2 = 4x^2 + 4y^2$.

(1) $x = x \;,\; y = y \;,\; z = 2\sqrt{x^2+y^2}$ x, y are the parameters

(2) $x = r\cos\theta \;,\; y = r\sin\theta \;,\; z = 2r$ $r \geq 0 \;,\; 0 \leq \theta \leq 2\pi$ r, θ parameters

Surfaces of Revolution: Obtaining surfaces by rotating the curve $y = f(x)$ about x-axis, $a \leq x \leq b$, $f(x) \geq 0$

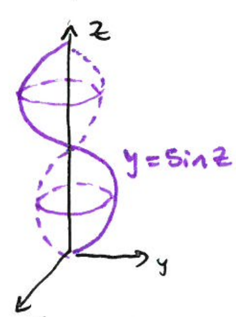


Point on Surface: (x, y, z)

parametrization:

$$\boxed{\begin{array}{lll} x = x & y = f(x)\cdot\cos\theta & z = f(x)\cdot\sin\theta \\ a \leq x \leq b, & 0 \leq \theta \leq 2\pi & \end{array}}$$

Ex Find a parametrization for the surface obtained by rotating one period of $y = \sin z$ about the z-axis



$z = z \quad y = \sin z \cdot \cos\theta \quad x = \sin z \sin\theta$

$0 \leq z \leq 2\pi$ and $0 \leq \theta \leq 2\pi$