Section 16.1 - Vector Fields

Chapter 12 & 13:   3D, vectors, vector functions

Chapter 14:   Functions of more than 1 variable

Chapter 15:   Integrating functions of more than 1 variable

Chapter 16:   Integrating vector Fields - vector functions of more than 1 variable

- Vector Fields:

★ See the Wind map on website

- Visualizing vector fields:

[Example] A vector field on $\mathbb{R}^2$ is defined by $\vec{F}(x,y) = \langle -y, x \rangle$. Describe $\vec{F}$ by sketching some of the vectors. What can be said about the magnitude of the vectors as you move away from the origin? What can you say about the flow/direction of the vectors?

[Example] Sketch the vector field on $\mathbb{R}^3$ given by $\vec{F}(x,y,z) = \langle 0, 0, z \rangle$.

- Newton's Law of Gravitation:

  Magnitude of the gravitational force between two objects with masses m and M is $|\vec{F}| = \dfrac{mMG}{r^2}$ where r is the distance between the objects and G is the gravitational Constant.

  Taking M to be located at the origin with $\vec{r} = \langle x, y, z \rangle$ the position vector for m then Gravitation force exerted on the second object acts towards the origin, that is in the $\dfrac{-\vec{r}}{|\vec{r}|}$ direction. Thus the gravitational force (field) is $\vec{F}(\vec{r}) = \dfrac{-mMG}{|\vec{r}|^2} \vec{r}$.

  ★ Give a rough sketch of this vector field:

  ★ Demo 3D vector fields $\frac{1}{r^2}$ single
- Gradient Field:

  Example Find the gradient field of $f(x,y) = x^2 y - y^3$. Plot the gradient vector field together with a Contour map of f. How are they related?

  ★ Use 2D vector field plotter Scale = 0.01 Contours $K = 100, 50, 1, 0, -50, -200$

Section 16.1 - Vector fields

Example Find the gradient vector field of $f(x,y) = \sqrt{x^2 + y^2}$ and sketch it.

• Extra Examples

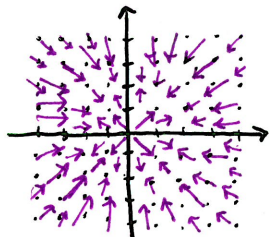# 29-32. Match the functions with the plots of their gradient vector fields I-IV

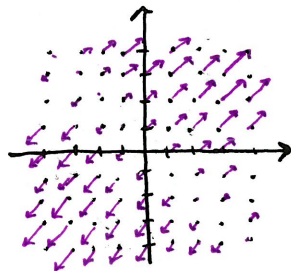29. $f(x,y) = x^2 + y^2$ 30. $f(x,y) = x(x+y)$ 31. $f(x,y) = (x+y)^2$ 32. $f(x,y) = \sin\sqrt{x^2 + y^2}$
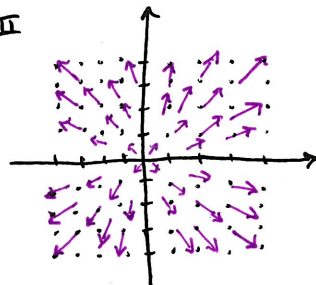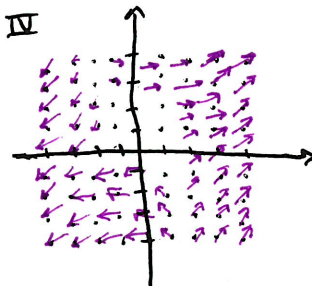
I

II

III

IV

28. Plot the gradient vector field of f together with a contour map of f using the online plotters, $f(x,y) = \cos(x) - 2\sin(y)$.