Vector Function - function with domain a set of real numbers and whose range is a set of vectors.

✱ Most interested in vector functions whose range is 3D vectors.

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\vec{i} + g(t)\vec{j} + h(t)\vec{k}$$

- $\lim\limits_{t \to a} \vec{r}(t) = \left\langle \lim\limits_{t \to a} f(t), \lim\limits_{t \to a} g(t), \lim\limits_{t \to a} h(t) \right\rangle$

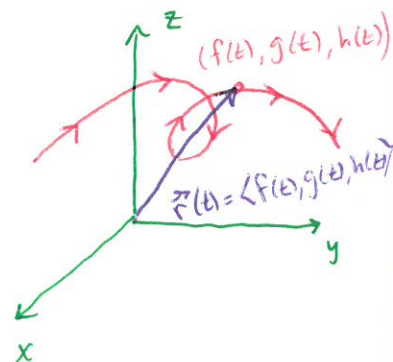 Provided the limits of the components exist.

- A vector function is <u>continuous at a</u> if $\lim\limits_{t \to a} \vec{r}(t) = \vec{r}(a)$.

Example 2 Find $\lim\limits_{t \to 0} \vec{r}(t)$ where $\vec{r}(t) = (1+t^3)\vec{i} + te^{-t}\vec{j} + \frac{\sin t}{t}\vec{k}$

$$\lim\limits_{t \to 0} \vec{r}(t) = \left[\lim\limits_{t \to 0}(1+t^3)\right]\vec{i} + \left[\lim\limits_{t \to 0} te^{-t}\right]\vec{j} + \left[\lim\limits_{t \to 0} \frac{\sin t}{t}\right]\vec{k}$$

$$= \vec{i} + 0\vec{j} + \vec{k}$$

✱ All limit rules for functions in one variable still apply
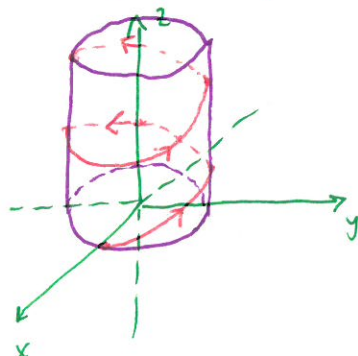
- A <u>Space Curve</u> C is the set of points (x,y,z) where

  (Parametric Equations) $x = f(t)$ $y = g(t)$ and $z = h(t)$

  t varies throughout an interval I.

Example 4 Sketch the curve whose vector equation is

$$\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\vec{k}$$

$x = \cos t$ $y = \sin t$ $z = t$ $x^2 + y^2 = 1$ ⟹ Curve lies on a cylinder

As we go around the unit circle in x and y z is increasing so we move up in a counter clockwise fashion.

This curve is called a <u>helix</u>.

Double helix model for our DNA.

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Example 5 Find a vector equation and parametric equations for the line segment that joins the point $P(1, 3, -2)$ and $Q(2, -1, 3)$.

$$(12.5) \qquad \vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1} \qquad \text{or} \qquad \vec{r}(t) = \vec{r_0} + t(\vec{r_1} - \vec{r_0}) \qquad 0 \le t \le 1$$
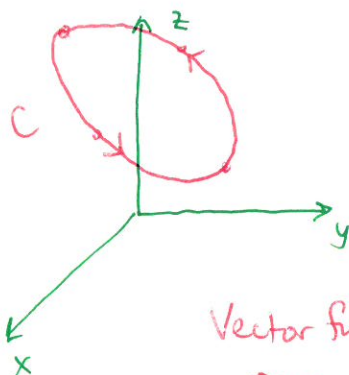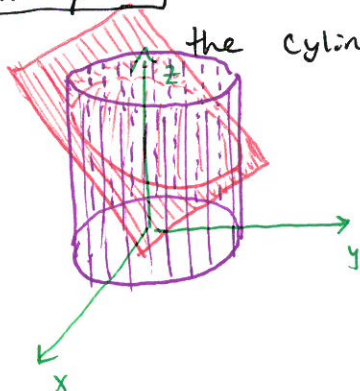
$$\vec{r_0} = \langle 1, 3, -2 \rangle \qquad\qquad \vec{r_1} = \langle 2, -1, 3 \rangle$$

$$\boxed{\begin{array}{l} x = 1+t \\ y = 3-4t \qquad 0 \le t \le 1 \\ z = -2+5t \end{array}}$$

$$\vec{r}(t) = \langle 1, 3, -2 \rangle + t\langle 1, -4, 5 \rangle \qquad 0 \le t \le 1$$

$$\boxed{\vec{r}(t) = \langle 1+t, \ 3-4t, \ -2+5t \rangle \qquad 0 \le t \le 1}$$

Example 6 Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$.

$$\boxed{\begin{array}{l} x = \cos t \qquad y = \sin t \\ \\ z = 2 - \sin t \\ \\ 0 \le t \le 2\pi \end{array}}$$

Vector function:

$$\vec{r}(t) = \langle \cos t, \ \sin t, \ 2 - \sin t \rangle \qquad 0 \le t \le 2\pi$$

Example Use a computer to graph the toroidal spiral:

$$x = (2 + \sin 20t)\cos t \qquad y = (4 + \sin 20t)\sin t \qquad z = \cos 20t$$

★ Pull up Wolframalpha.com