Calc AB 8/24/15

Agenda: 8/24/15

HW Leader: None

Lesson 18
  Function Composition
    Review
★ Test 1 on Friday
   Lessons 1-24
★ Handout Calendar part 2

Warm-up:
① A company produces bolts of fabric with a fixed width. The cost of producing x yards is $C = f(x)$ dollars.
  (a) What does $f'(x)$ mean? What are the units?
  (b) In practical terms what does $f'(1000) = 9$ mean?
  (c) Which is greater $f'(50)$, $f'(500)$ or $f'(5000)$?
② Find $\frac{d}{dx}(\sqrt{x})$.

Let f and g be two functions.

- $(f+g)(x) = f(x) + g(x)$
- $(fg)(x) = f(x) \cdot g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Function Composition   [pluging in a function to a function]

$$f \circ g\,(x) = f(g(x)) \qquad g \circ f\,(x) = g(f(x))$$

Ex. $f(x) = \sin(x)$ and $g(x) = x^2 - 1$

$$f \circ g(x) = f(g(x)) = \sin(g(x)) = \boxed{\sin(x^2 - 1)} \quad \begin{array}{l} D = \mathbb{R} \\ R = [-1, 1] \end{array}$$

$$g \circ f(x) = g(f(x)) = (f(x))^2 - 1 = \boxed{\sin^2(x) - 1} \quad \begin{array}{l} D = \mathbb{R} \\ R = [-2, 0] \end{array}$$

Domain of a Composition:
  All real numbers that produce an output from the first that are also acceptable to the second.

Ex. 18.4　Write $h(x) = e^{-2x+1}$ as the composition of two functions.

$$f(x) = e^x \qquad g(x) = -2x+1$$

$$h(x) = f(g(x)) = e^{g(x)} = e^{-2x+1} \quad \checkmark$$

Ex. 18.6　Let $f(x) = \sqrt{x}$ and $g(x) = 2x+3$. Find the domain and range of $f \circ g$ and $g \circ f$.

$$f \circ g(x) = \sqrt{2x+3}$$

Need output of $g(x) \geq 0$

so　$x \geq -\frac{3}{2}$ ⟶

Domain of f: $[0, \infty)$

Domain of g: $\mathbb{R}$

Domain of $f \circ g$: $[-\frac{3}{2}, \infty)$

Range of $f \circ g$: $[0, \infty)$ ↵

$$g \circ f(x) = 3\sqrt{x} + 3$$

Range: $[3, \infty)$ ⟵

Domain of g: $\mathbb{R}$

Domain of f: $[0, \infty)$

Domain of $g \circ f$: $[0, \infty)$