Section 12.6 – Cylinders & Quadric Surfaces
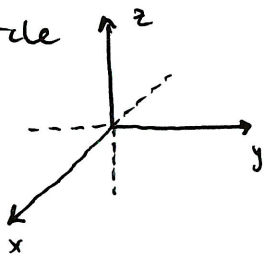
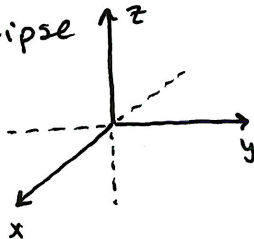★ Conic Sections are 2D curves what about 3D surfaces?

- **Cylinders**: surface of all lines (called rulings) parallel to one another, passing through a given plane curve perpendicular to the plane containing the plane curve.
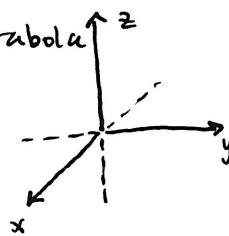
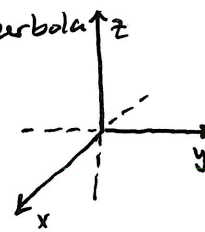[Example] Sketch a cylinder for each conic section:

① Circle $\uparrow z$

② Ellipse $\uparrow z$

③ Parabola $\uparrow z$

④ Hyperbola $\uparrow z$

- **Quadric Surfaces**: graph of a second-degree equation in 3-variables.

General Equation: $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$

By translation and/or rotation all can be rewritten as:

- **Method of sketching**: layer <u>traces</u> –

[Example 3] use traces to sketch
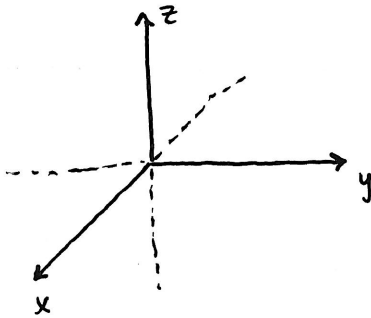$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

[Example 5] Sketch using traces
$$z = y^2 - x^2$$

- 6 Types of Quadric Surfaces: meaningful ways to Combine 3 conic sections
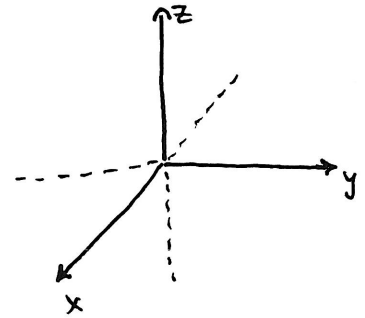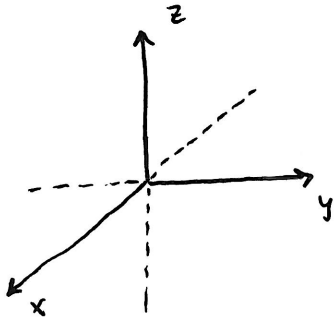
Ellipsoid:

Traces:

Hyperboloid of 1 Sheet:

Traces:
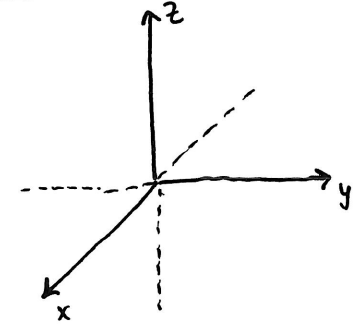
Cone:

Traces:
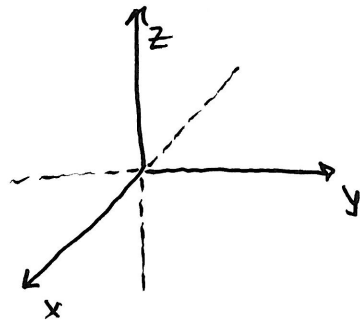
Hyperboloid of 2 sheets:

Traces:
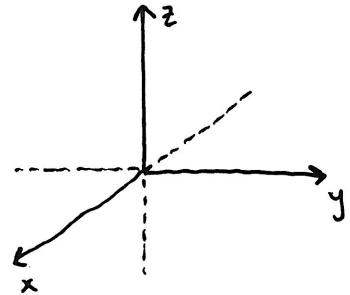
Elliptic Paraboloid:

Traces:

Hyperbolic Paraboloid:

Traces:

Example 7 | Identify & sketch
$$4x^2 - y^2 + 2z^2 + 4 = 0$$

Example 8 | Identify & sketch
$$x^2 + 2z^2 - 6x - y + 10 = 0$$

- Quadric Surfaces 3D applet: www.geogebra.org/m/VunKpsBA

- Extra Examples:

#37-40 use the 3D applet to sketch the following by identifying the quadric surface and the values of a, b, c.

#37 $-4x^2 - y^2 + z^2 = 1$

#38 $x^2 - y^2 - z = 0$

#39 $-4x^2 - y^2 + z^2 = 0$

#40 $x^2 - 6x + 4y^2 - z = 0$

#47 Traditionally, the earth's surface has been modeled as a sphere, but the World Geodetic System uses an ellipsoid as a more accurate model. It places the earth's center at the origin and north pole on the z-axis. The distance from the center to the poles is 6356.523 km and the distance to the equator is 6378.137 km.

(a) Find the model

(b) Curves of equal latitude are traces in the planes $z = k$. What are the curves?

(c) Meridians (curves of equal longitude) are traces in planes $y = mx$. What are they?

$\boxed{3/3}$