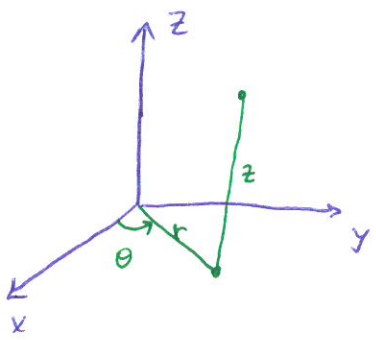
Section 15.8 - Cylindrical Coordinates

Vector Calc

3D equivalent to polar coordinates: (r, θ, z)

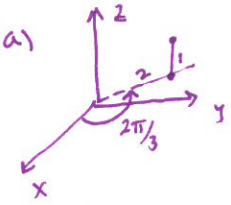


$$x = r\cos\theta \quad , \quad y = r\sin\theta \quad , \quad z = z$$

$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x} \quad , \quad z = z$$

Ex 1 (a) plot $(2, 2\pi/3, 1)$ find cartesian coords

(b) Find cylindrical coords for $(3, -3, -7)$

(a) 

$x^2 + y^2 = 4$, $\theta = \frac{2\pi}{3}$, $z = 1$

$x = 2\cos\left(\frac{2\pi}{3}\right) = -1$

$y = 2\sin\left(\frac{2\pi}{3}\right) = \sqrt{3}$

$\boxed{(-1, \sqrt{3}, 1)}$

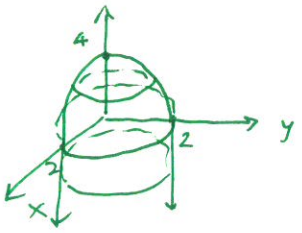
(b) $r^2 = (3)^2 + (-3)^2 = 18 \Rightarrow r = \sqrt{2}\cdot 3$ $\qquad z = -7$

$\tan\theta = -1$ in 4th Quadrant so $\theta = -\pi/4$ or $7\pi/4$

$\boxed{(3\sqrt{2}, 7\pi/4, -7)}$

Ex. Identify the surface $z = 4 - r^2$

$r^2 = x^2 + y^2$ so $z = 4 - x^2 - y^2$ paraboloid



Ex. 3 A solid E lies within $x^2 + y^2 = 1$, below $z = 4$, above $z = 1 - x^2 - y^2$.

The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

$$E = \{(r, \theta, z) \mid 0 \le r \le 1, \ 0 \le \theta \le 2\pi, \ 1 - r^2 \le z \le 4\} \qquad \rho(x, y, z) = k\sqrt{x^2 + y^2} = kr$$

$$\iiint_E \rho(x, y, z)\, dV = \int_0^{2\pi}\int_0^1\int_{1-r^2}^4 kr^2\, dz\, dr\, d\theta = 2\pi k\int_0^1 4r^2 - r^2 + r^4\, dr =$$

$$= 2\pi k\left[r^3 + \frac{r^5}{5}\right]_0^1 = 2\pi k\left[1 + \frac{1}{5}\right] = \boxed{\frac{12\pi k}{5}}$$

Ex 21 Evaluate $\iiint_E x^2\, dV$, where E is within $x^2 + y^2 = 1$, above $z = 0$, below $z^2 = 4x^2 + 4y^2$

$$= \int_0^{2\pi}\int_0^1\int_0^{2r} r^2\cos^2\theta\, dz\, dr\, d\theta = \int_0^{2\pi}\cos^2\theta\, d\theta \cdot \int_0^1 2r^4\, dr = \int_0^{2\pi}\cos^2\theta \cdot \frac{2}{5}$$

$$= \frac{2}{5}\int_0^{2\pi}\frac{1}{2}(1 + \cos 2\theta)\, d\theta = \frac{1}{5}\left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_0^{2\pi} = \boxed{\frac{2\pi}{5}}$$