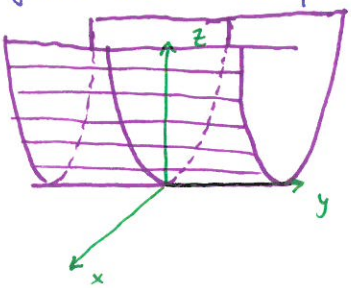
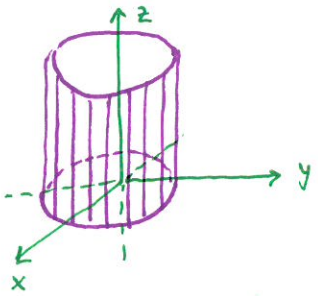
Cylinders: surface of all lines (called rulings) that are are parallel to a given line and pass through a given plane curve.

Ex. 1 $z = x^2$



Parabolic Cylinder

Ex. 2 $x^2 + y^2 = 1$



Circular Cylinder

Quadric Surfaces: graph of a second-degree equation in 3 variables. General Equation:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

where $A, B, \ldots, J$ are constants.

★ By translation and rotation brought into form

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or

$$Ax^2 + By^2 + Iz = 0$$

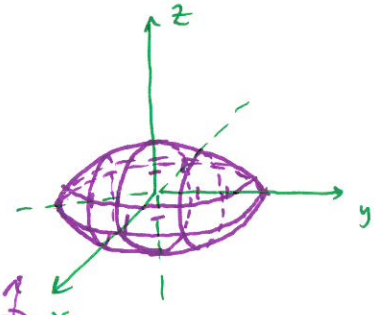
Trace: 2D graphs obtained by fixing one variable.

Ex. 3 Use traces to Sketch $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

traces in x: $x = 0$: Ellipse $\frac{y^2}{9} + \frac{z^2}{4} = 1$

traces in y: $y = 0$: Ellipse $x^2 + \frac{z^2}{4} = 1$

traces in z: $z = 0$: Ellipse $x^2 + y^2/9 = 1$



Ellipsoid

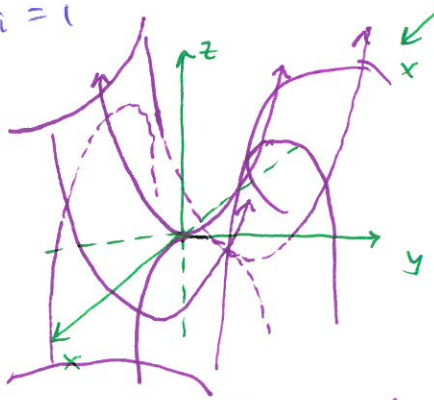
Ex. 5 Sketch $z = y^2 - x^2$

$x = K$: $z = y^2 - K^2$ parabolas

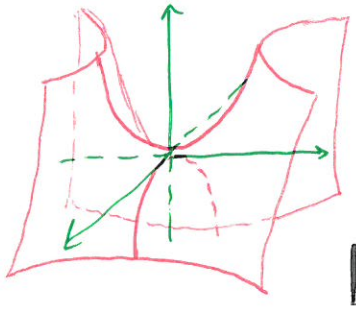
$y = K$: $z = K^2 - x^2$ parabolas

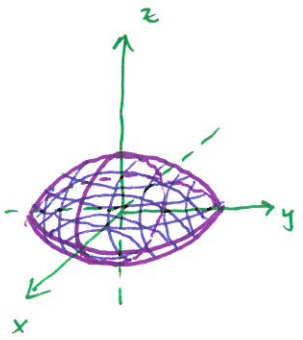
$z = K$: $y = \pm\sqrt{x^2 + K}$

$y^2 - x^2 = K$



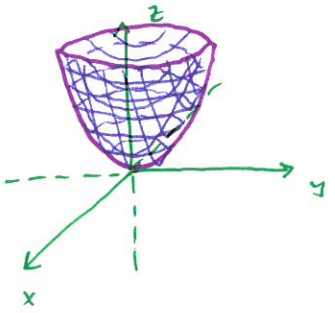
Hyperbolic Paraboloid





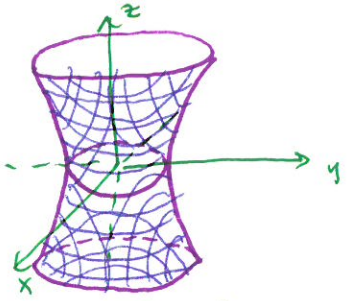
Ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Traces: All ellipses



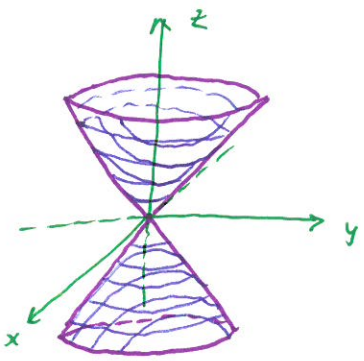
Elliptic Paraboloid: $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Traces: Horizontal ellipses
Vertical parabolas



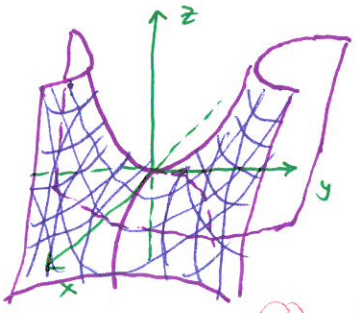
Hyperboloid of 1 Sheet: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Traces: Horizontal ellipses
Vertical hyperbolas



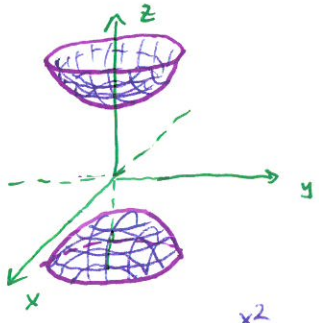
Cone: $\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Traces: Horizontal Ellipses
vertical line if x or y is 0
else hyperbolas



Hyperbolic Paraboloid: $\frac{z}{c}=-\frac{x^2}{a^2}-\frac{y^2}{b^2}$

Traces: Horizontal hyperbolas
Vertical parabolas

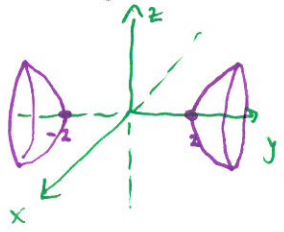


Hyperboloid of 2 Sheets: $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Traces: Horizontal Ellipses
Vertical hyperbolas

Ex. 7, 8 Identify and sketch $4x^2-y^2+2z^2+4=0$

$-x^2+\frac{y^2}{4}-\frac{z^2}{2}=1$



$x^2+2z^2-6x-y+10=0$

$y=(x^2-6x+9)+2(z^2)+10-9$

$(y-1)=(x-3)^2+2(z^2)$

Center: $(3,1,0)$