Topic: Rational Functions
- Definition
- Zeros, holes, vertical Asymptotes
- horizontal and slant Asymptotes

☆ Handout WS 1

★ Handout last test Study Guide

Definition - A rational function $R(x)$ is of the form

$$R(x) = \frac{P(x)}{q(x)}$$

where $P(x)$ and $q(x)$ are polynomials, with $q(x) \neq 0$.

Examples:

rational

$$y = \frac{x-1}{x+3}$$

$$y = \frac{3}{(x+1)^2}$$

$$y = x^2 - 3$$

Not rational

$$y = \frac{1}{\sqrt{x}}$$

$$y = \frac{1}{3 + e^x}$$

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holes, zeros and Vertical Asymptotes: $R(x) = \frac{P(x)}{q(x)}$

- **holes** happen at values of x that make $P(x) = 0$ __and__ $q(x) = 0$
  (and cancel so no zero in the denominator)

- **Zeros** happen at values of x where $P(x) = 0$ and $\underline{q(x) \neq 0}$

- **Vertical Asymptotes** happen at values of x where $q(x) = 0$ and $\underline{P(x) \neq 0}$

ex. find any holes, zeros, and vertical Asymptotes:

$$y = \frac{3(x-2)}{(x-2)(x+3)}$$

$$y = \frac{(x-1)(x+1)}{(x+3)(x-2)}$$

$$y = \frac{-2(x+5)^2}{(x+5)}$$

$$y = \frac{-2(x+5)}{(x+5)^2}$$

holes: $\left(2, \frac{3}{5}\right)$ None $(-5, 0)$ None

zeros: None $x = 1, -1$ None   None

VA: $x = -3$ $x = -3, \ x = 2$ None $x = -5$

Horizontal and Slant Asymptotes $R(x) = \frac{P(x)}{q(x)}$ (Only One!)

- If degree P < degree q then Horizontal asymptote at $y = 0$

    (as $x \to \infty$, $y \to 0$ and as $x \to -\infty$, $y \to 0$)

- If degree P = degree q then Horizontal asymptote at

$$y = \frac{\text{leading coef } P}{\text{leading coef } q}$$

- If degree P = degree q + 1 then Slant Asymptote at

$$y = \text{quotient of } P(x)/q(x)$$

Examples: Find any slant or horizontal Asymptote for

	$y = \frac{3(x-2)}{(x-2)(x+3)}$	$y = \frac{(4x-1)(x+1)}{(x+3)(7x-2)}$	$y = \frac{-2(x+5)^2}{(x+5)}$	$y = \frac{(x+3)^2(x-1)}{(x+2)}$
degree P:	1	2	2	3
degree q:	2	2	1	1
HA/SA:	HA $y = 0$	HA $y = \frac{4}{7}$	SA $y = -2(x+5)$	None

Topic : Rational Functions
- End behavior
- Graphs with unique zeros and asymptotes

☆ Handout WS2

<u>End behavior</u> $R(x) = \dfrac{P(x)}{q(x)}$

- Horizontal Asymptote - clear
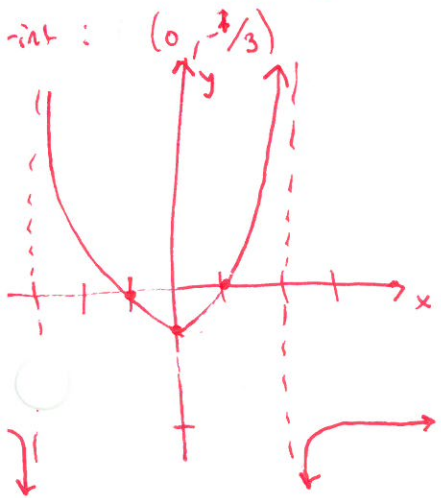
- No horizontal Asymptote look at

$$\lim_{x \to \infty} R(x) \qquad \text{and} \qquad \lim_{x \to -\infty} R(x)$$

Ex.

$$'' = \frac{-2(x-1)(x+1)}{(x+3)(x-2)}$$

holes :

zeros: $x = 1, -1$

'A: $x = -3 \quad x = 2$

A/SA : $y = -2$

end behavior: $y = -2$

-int : $(0, -\tfrac{2}{3})$

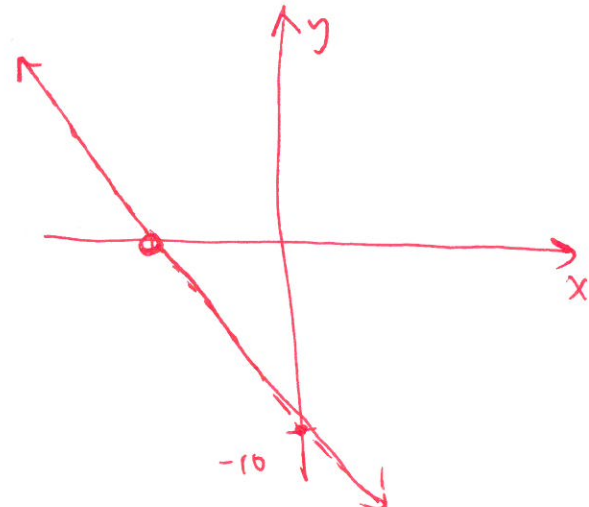$$y = \frac{3(x-2)}{(x-2)(x+3)}$$

$(2, \tfrac{3}{5})$

None

$x = -3$

$y = 0$

$y = 0$

$(0, 1)$
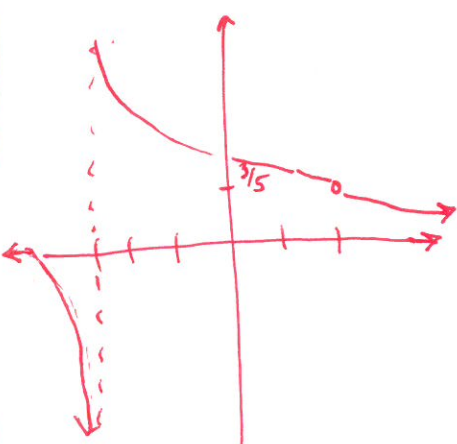
$$y = \frac{-2(x+5)^2}{x+5}$$

$(-5, 0)$

None

None

$y = -2x - 10$

$\lim_{x \to \infty} y = -\infty \qquad \lim_{x \to -\infty} y = \infty$

$(0, -10)$

Topic: Rational Functions
- Graphs with zeros/asymptotes powers > 1   ★ Handout ws 3

Example:

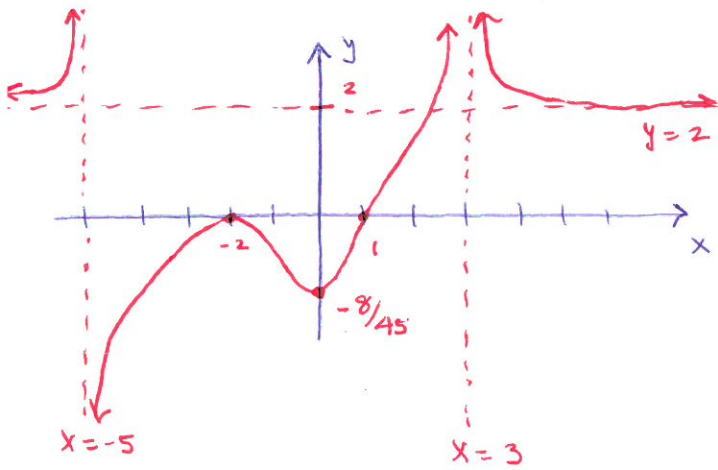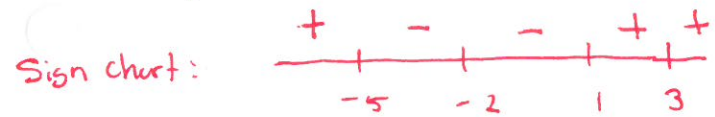$$y = \frac{2(x-1)(x+2)^2}{(x-3)^2(x+5)}$$

y-int: $\left(0, -\frac{8}{45}\right)$

Holes: None

Zeros: mult

1	-2	
1	2	

VA: $x = 3$ $x = -5$

HA/SA: (End behavior) $y = 2$

Sign chart:

$$y = \frac{(1-x)^3(x+4)(x+1)}{(6-x)^2(x+3)}$$

$\left(0, \frac{4}{108}\right)$

None

1	-4	-1
3	1	1

$x = 6$ $x = -3$

$\lim\limits_{x \to \infty} y = -\infty$ $\lim\limits_{x \to -\infty} y = -\infty$
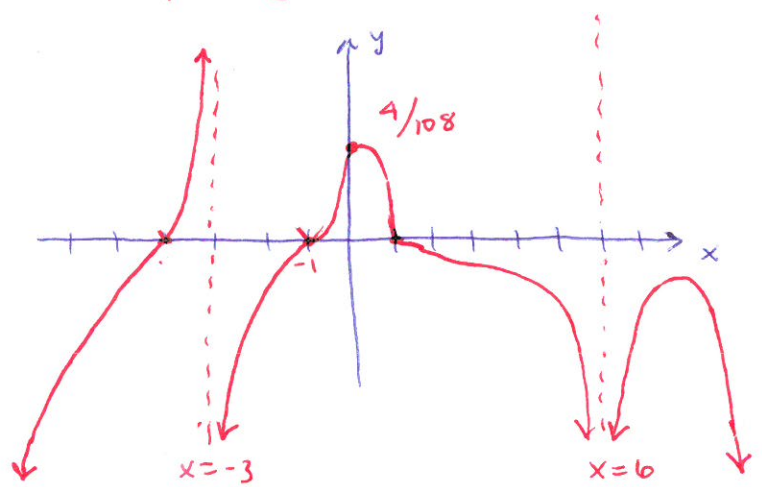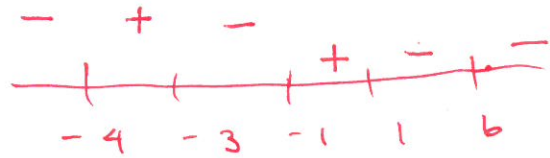
$$y = \frac{x^2+1}{x-2}$$

y-int: $\left(0, -\frac{1}{2}\right)$

zeros: none

holes: None

VA: $x = 2$

HA/SA = $x - 2 \overline{)\begin{array}{l} x+2 \\ x^2+1 \\ \underline{x^2-2x} \\ 2x+1 \end{array}}$

$\boxed{y = x + 2}$
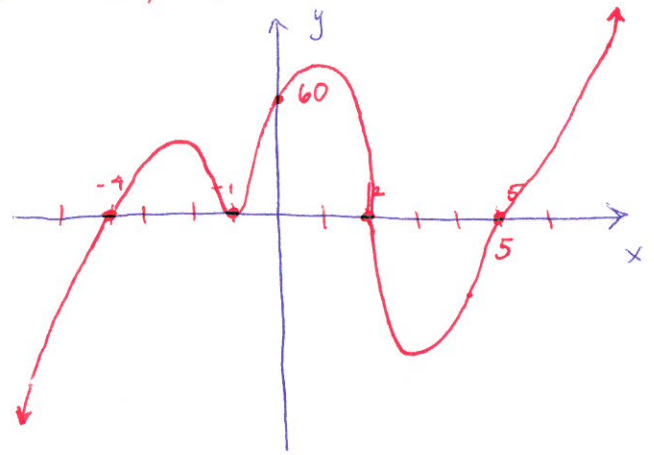
Sign Chart

$y = x + 2$
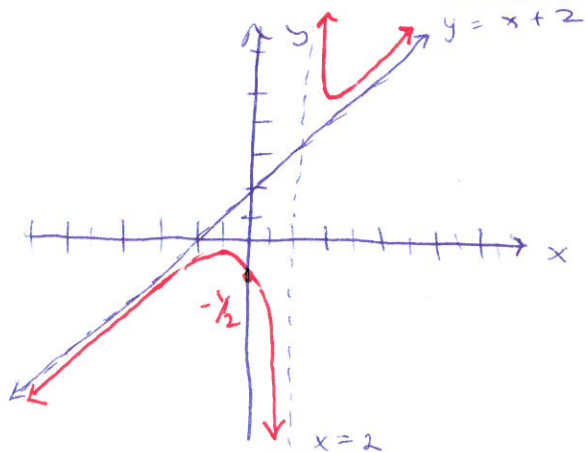
Topic: Polynomial and Rational function Review

Ex. $f(x) = \dfrac{-3}{8}(2-x)^3(x+1)^2(x+4)(x-5)$

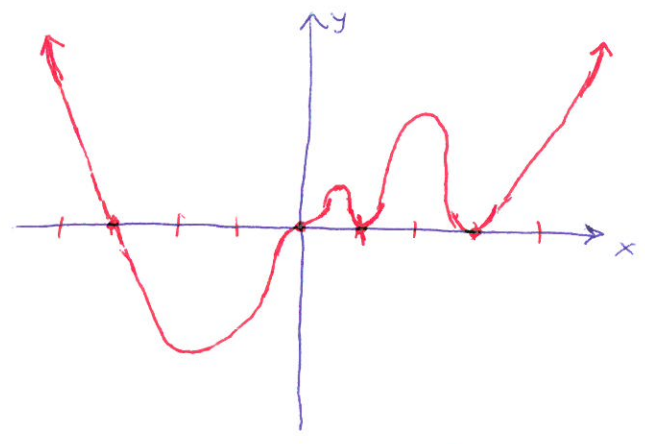Lead Term: $\dfrac{3}{8}x^7$ End Behavior:

Zeros	-4	-1	2	5
mult	1	2	3	1

y-int (0, 60)

$f(x) = (3-x)^2(x)^5(1-2x)^2(x+3)$

Lead Term: $4x^{10}$ End behavior:

Zeros	-3	0	1	3
mult	1	5	2	2

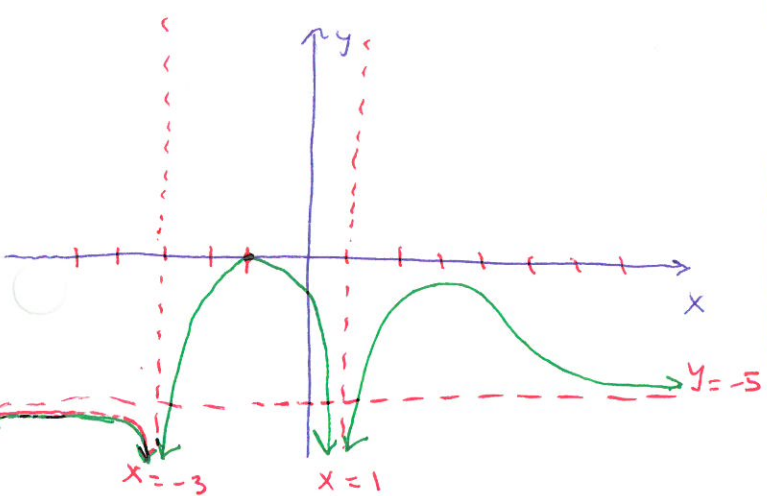y-int: (0, 0)

$R(x) = \dfrac{-5(x-2)^3(x+1)^2}{(x-2)(x+3)^2(x-1)^2}$

y-int: (0, -20/9)   hole: (2, 0)

Zeros: $x = -1$

VA: $x = -3$ $x = 1$

HA/SA/End behavior: $y = -5$

Sign Chart:
```
 -    -    -    -    -
+----+----+----+----+
    -3   -1    1    2
```

$R(x) = \dfrac{(3-x)(4+x)(x+1)^2}{(x-1)^3(x+2)^2(x-5)}$

y-int: (0, 12/-20)   hole: None

Zeros: $x = 3, x = -4, -1$

VA: $x = 1, -2, 5$

HA/SA/End behavior: $y = 0$

Sign Chart:
```
 +   -   -   -   +   -   +
+---+---+---+---+---+---+
 -4  -2  -1   1   3   5
```

$Y = -5$
$X = -3$ $X = 1$

$X = -2$ $X = 1$ $X = 5$