Review Practice: Chapters 12 & 13

1. Find the equation of the plane through $A(0, 1, -1)$, $B(1, 0, -1)$, and $C(-1, 1, 0)$.

$\vec{AB} = \langle 1, -1, 0 \rangle$

$\vec{BC} = \langle -2, 1, 1 \rangle$

$\vec{AB} \times \vec{BC} = \langle -1, -1, - \rangle$

$\boxed{(x-0) + (y-1) + (z+1) = 0}$ or $\boxed{x + y + z = 0}$

2. Parametrize the line perpendicular to the plane with normal vector $\langle 0, 1, 1 \rangle$ through the point $(1, 2, 3)$.

$$\boxed{\begin{aligned} x(t) &= 1 + 0t \\ y(t) &= 2 + t \\ z(t) &= 3 + t \end{aligned}}$$

or $\boxed{\vec{r}(t) = \langle 1, 2+t, 3+t \rangle}$

3. Find the angle between the two planes from problem 1 and 2 above. Are they orthogonal?

$$\cos\theta = \frac{\langle 1,1,1\rangle \cdot \langle 0,1,1\rangle}{\sqrt{3} \cdot \sqrt{2}} \neq 0 \quad \text{so } \boxed{\text{not orthogonal}}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) \approx 35.264^\circ}$$

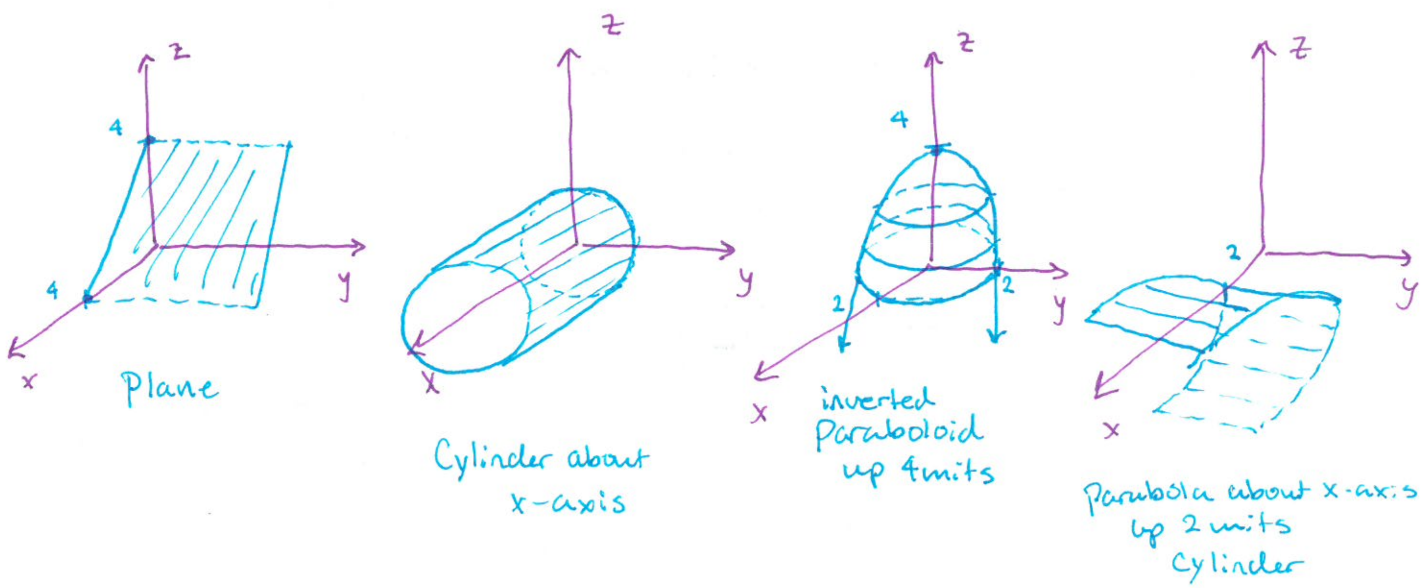
4. Sketch:

(a) $z = 4 - x$

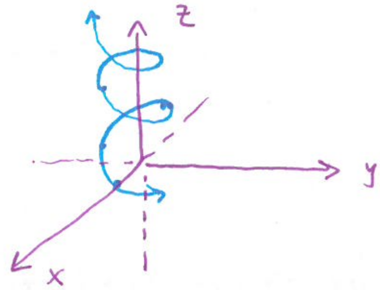
(b) $y^2 + z^2 = 9$

(c) $z = 4 - x^2 - y^2$

(d) $x = z^2 + 2$



5. Sketch $\mathbf{r}(t) = \langle \cos t, \sin t, -t \rangle$



Spiral up z-axis
Counter clockwise

Spiral down z-axis
Clockwise

6. Find $\lim_{t \to 0} \left\langle \frac{\sin t}{t}, t^2, \ln(t+1) \right\rangle$

$$= \left\langle \lim_{t \to 0} \frac{\cos t}{1},\ 0,\ \ln(1) \right\rangle = \boxed{\langle 1, 0, 0 \rangle}$$

used L'Hopital's rule

7. Find the derivative of $\mathbf{r}(t) = \langle \sin^2 t, e^{4t} + 1, 3t^4 + t^2 \rangle$

$$\boxed{\vec{r}\,'(t) = \langle 2\sin t \cdot \cos t,\ 4e^{4t},\ 12t^3 + 2t \rangle}$$

Chain Rule

8. Find the arc length of $\mathbf{r}(t) = \langle \sin(t^2), \cos(t^2), 2t^3 \rangle$ for $0 \le t \le 1$.

$$L = \int_0^1 \sqrt{(2t\cos(t^2))^2 + (-2t\sin(t^2))^2 + (6t^2)^2}\, dt$$

$$= \int_0^1 \sqrt{4t^2 + 36t^4}\, dt$$

$$= \int_0^1 2t\sqrt{1 + 9t^2}\, dt$$

$$= \frac{2}{3} \frac{(1+9t^2)^{3/2}}{9} \Big|_0^1$$

$$= \boxed{\frac{2}{27}\left(10^{3/2} - 1\right)}$$