

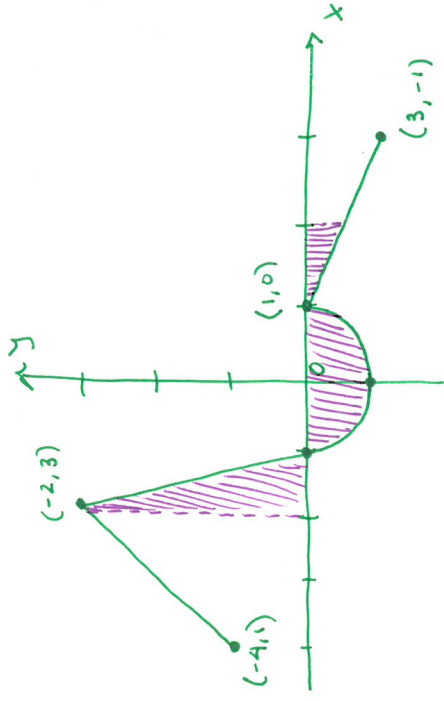
Agenda: 11/19/15

★ Handout AP Packets

AP Problems - Tagged Line Problems

- Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of 3 line segments and a semi circle centered at the origin, is given to the right. Let g be the function given by

$$g(x) = \int_1^x f(t) dt$$

(a) Find $g(2)$ and $g(-2)$

$$g(2) = \int_1^2 f(t) dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = \boxed{\frac{1}{4} \text{ units}^2}$$

$$g(-2) = \int_1^{-2} f(t) dt = -\int_{-2}^1 f(t) dt = -\left[\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1)^2\right] = -\frac{3}{2} + \frac{\pi}{2} = \boxed{\frac{\pi-3}{2} \text{ units}^2}$$

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

$$g'(x) = \frac{d}{dx} \int_1^x f(t) dt = f(x) \quad \text{so } g'(-3) = f(-3) = \boxed{2}$$

$$g''(-3) = f'(-3) = \boxed{1} \quad \text{slope of the tangent line to } f(x) \text{ at } x = -3$$

(c) Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points determine whether g has a relative minimum, relative maximum, or neither. Justify your answers.

g has a horizontal tangent line when $g'(x) = f(x) = 0$ so at $\boxed{x = -1, \text{ and } x = 1}$

Sign of f

$$\begin{array}{c} + & - & + & - \\ -1 & & 1 & \end{array}$$

So g has a local max at $x = -1$

Because f going from positive to negative at $x = -1$ means g goes from increasing to decreasing at $x = -1$.

At $x = 1$ it is neither a maximum nor a minimum as f does not change sign (stays negative) meaning g is decreasing before and after at $x = 1$.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning. When $f'(x) = 0$ or undefined

$$\begin{array}{c} + & - & - & + & - \\ -2 & -1 & 0 & 1 & \end{array}$$

Inflection points at $x = -2, 0$ and 1 because this is where $f(x)$ goes from increasing/decreasing meaning $g(x)$ is changing concavity at these points.