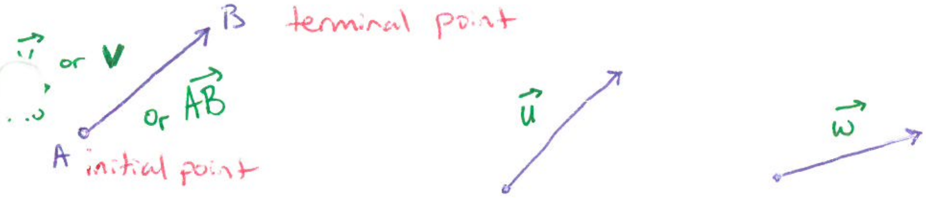
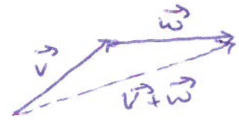


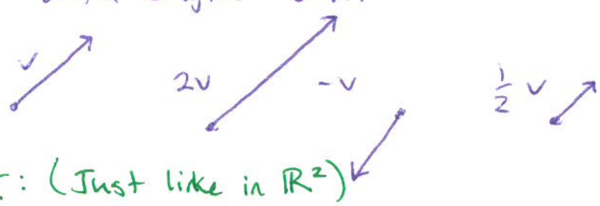

$|\vec{v}|$ : length or magnitude

Equivalent : if both vectors have the same direction and magnitude ($\vec{u}$ and $\vec{v}$)

Addition : Sum components $\vec{v}+\vec{w}$ :




Scalar multiplication : c a scalar, $\vec{v}$ a vector $c\vec{v}$ is a vector in the direction of $\vec{v}$ with length $c \cdot |v|$




Position of a vector Does not affect its notation

Components of a vector : (Just like in $\mathbb{R}^2$)

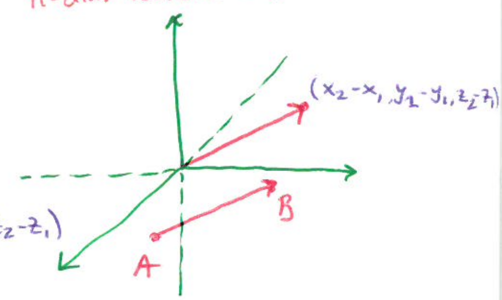
$\mathbb{R}^2$: $\vec{v} = \langle x_1, y_1 \rangle$ $\mathbb{R}^3$: $\vec{v} = \langle x, y, z \rangle$

Vectors in $\mathbb{R}^2$ : $V^2$ Vectors in $\mathbb{R}^3$ : $V^3$ n-dim vectors : $V^n$

Given $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$

$$\vec{v} = \vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Called the position vector of point $(x_2-x_1, y_2-y_1, z_2-z_1)$




Magnitude :

$\mathbb{R}^2$: $|\vec{v}| = |\langle x, y \rangle| = \sqrt{x^2+y^2}$ $\mathbb{R}^3$: $|\vec{v}| = |\langle x, y, z \rangle| = \sqrt{x^2+y^2+z^2}$

Operations of vectors

$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle$
$= \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle$
$= \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$

$c\langle a_1, a_2, a_3 \rangle = \langle c \cdot a_1, c \cdot a_2, c \cdot a_3 \rangle$

Properties of vectors

$\vec{v}, \vec{u}, \vec{w}$ in $V^n$ c, d scalars

1. $\vec{v} + \vec{u} = \vec{u} + \vec{v}$ Commutative
2. $\vec{v} + (\vec{u} + \vec{w}) = (\vec{v} + \vec{u}) + \vec{w}$ associative
3. $\vec{v} + \vec{0} = \vec{v}$ additive identity
4. $\vec{v} + (-\vec{v}) = \vec{0}$ additive inverse
5. $c(\vec{v} + \vec{u}) = c\vec{v} + c\vec{u}$ Distributive Laws
6. $(c+d)\vec{v} = c\vec{v} + d\vec{v}$ Distributive Laws
7. $(cd)\vec{v} = c(d\vec{v}) = d(c\vec{v})$ Distributive Laws
8. $1 \cdot \vec{v} = \vec{v}$ Scalar identity

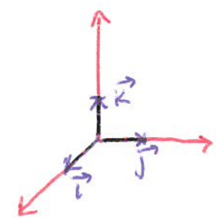
**Example 4** $\vec{a} = \langle 4, 0, 3 \rangle$ $\vec{b} = \langle -2, 1, 5 \rangle$ Find $|\vec{a}|$ and $2\vec{a} + 5\vec{b}$

$$|\vec{a}| = |\langle 4,0,3\rangle| = \sqrt{4^2+0^2+3^2} = \sqrt{25} = \boxed{5}$$

$$2\vec{a} + 5\vec{b} = 2\langle 4,0,3\rangle + 5\langle -2,1,5\rangle = \langle 8,0,6\rangle + \langle -10,5,25\rangle = \boxed{\langle -2,5,31\rangle}$$

Standard Basis vectors

$$\vec{i} = \langle 1,0,0\rangle \qquad \vec{j} = \langle 0,1,0\rangle \qquad \vec{k} = \langle 0,0,1\rangle$$




**Example 6** Express $\vec{v} = \langle 2, -1, -2\rangle$ interms of $\vec{i}, \vec{j}, \vec{k}$ and find a unit vector in the direction of $\vec{v}$.

Unit vector: vector whose length is 1; in direction of a nonzero vector $\vec{v}$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{|\vec{v}|} \cdot \vec{v}$$

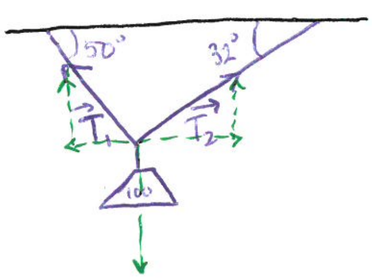
$$\vec{v} = 2\vec{i} + (-1)\vec{j} + (-2)\vec{k} = \boxed{2\vec{i} - \vec{j} - 2\vec{k}}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 2,-1,-2\rangle}{|\langle 2,-1,-2\rangle|} = \frac{\langle 2,-1,-2\rangle}{\sqrt{4+1+4}} = \boxed{\left\langle \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right\rangle}$$

Applications: Velocity and acceleration in space, forces

★ Resultant Force: sum of forces acting on an object

**Example 7** 100 lb weigh hangs from two wires. Find the tension forces $\vec{T_1}$ and $\vec{T_2}$.




Vertical Components:

① $|T_1| \sin 50° + |T_2| \sin 32° = 100$

Horizontal Components:

② $|T_1| \cos 50° = |T_2| \cos 32° \Rightarrow |T_2| = \dfrac{|T_1| \cos 50°}{\cos 32°}$

① $|T_1| = \dfrac{100}{\sin 50° + \tan 32° \cos 50°} \approx \boxed{85.64 \text{ lb}}$

② $|T_2| \approx \boxed{64.91 \text{ lb}}$

$$\vec{T_1} = |\vec{T_1}| \cos 50° \vec{i} + |\vec{T_1}| \sin 50° \vec{j} \approx \boxed{-55.05\vec{i} + 65.60\vec{j}}$$

$$\vec{T_2} = |\vec{T_2}| \cos 32° \vec{i} + |\vec{T_2}| \sin 32° \vec{j} \approx \boxed{55.05\vec{i} + 34.40\vec{j}}$$