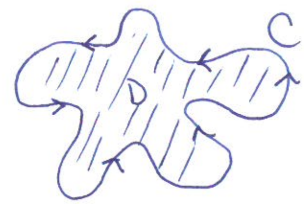
Section 16.4 - 16.5 Review Vector Calc

- **Section 16.4 - Green's Theorem**

  - Positive orientation - Counter clockwise

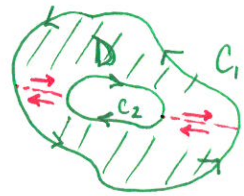
Green's Theorem: C - positively oriented, piecewise smooth, simple closed
D - region bounded by C
P, Q have continuous partials on D

$$\boxed{\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA}$$



★ notation: $\oint_C P\,dx + Q\,dy = ∲_C P\,dx + Q\,dy$

Green's Theorem Extends to regions with holes



Ex. Compute $\int_C \vec{F}\cdot d\vec{r}$ where:

$\vec{F} = \langle y - \cos y,\ x\sin y \rangle$ C: circle $(x-3)^2 + (y+4)^2 = 4$ clockwise

$x = 2\cos t + 3 \quad y = 2\sin t - 4 \quad 0 \le t \le 2\pi$

$$\int_C \vec{F}\cdot d\vec{r} = -\int_{-C} \vec{F}\cdot d\vec{r}$$

Green's Thm

$$= -\int_0^{2\pi}\int_0^2 (\sin y - (1+\sin y))\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 r\,dr\,d\theta = \boxed{4\pi}$$

- **Section 16.5 - Curl and Divergence**

$\vec{F} = \langle P, Q, R\rangle$

$$\text{Curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial x}\right)\vec{k}$$

$$\text{div}\,\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = \nabla\cdot\vec{F}$$

Theorems:
(3) f has continuous second partials $\Rightarrow \text{Curl}(\nabla f) = \vec{0}$
(4) $\vec{F}$ defined on all $\mathbb{R}^3$, components have continuous partials,
$\text{Curl}\,\vec{F} = 0 \Rightarrow \vec{F}$ Conservative
(11) $\vec{F}$ components have continuous second partials $\Rightarrow \text{div}(\text{curl}(\vec{F})) = 0$

$F = \langle P, Q\rangle$ then $\boxed{\oint_C \vec{F}\cdot d\vec{r} = \iint_D \text{Curl}\,\vec{F}\cdot\vec{k}\,dA}$ vector form of Green's Theorem.

Ex. Find the curl and divergence of $\vec{F} = \langle xye^z, 0, yze^x\rangle$

$$\text{Curl}\,\vec{F} = \boxed{(ze^x)\vec{i} - (yze^x - xye^z)\vec{j} + (-ye^z)\vec{k}}$$

$$\text{div}\,\vec{F} = ye^z + 0 + ye^x = \boxed{y(e^z + e^x)}$$