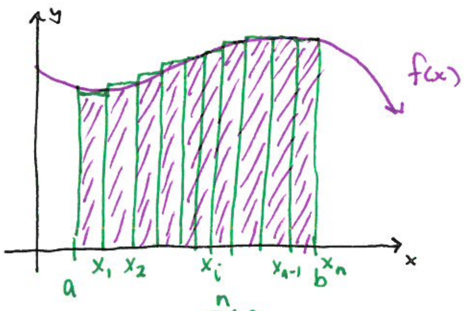
Section 15.1 - Double Integral over Rectangles

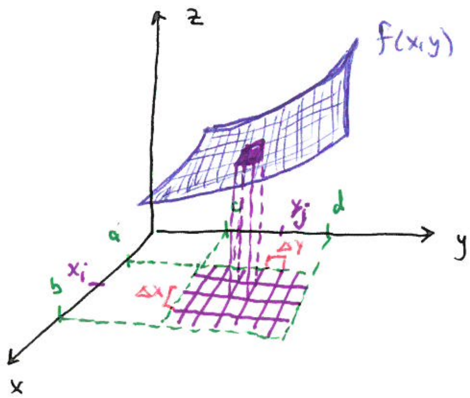
Review Definite Integral:

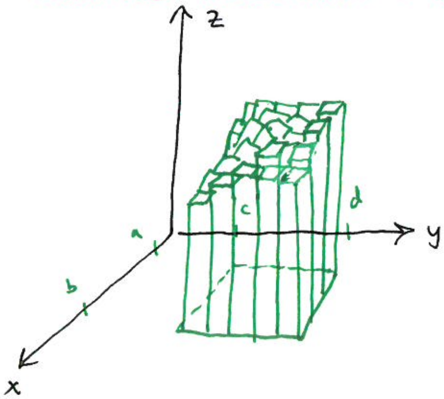


$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$S_R(n) = \sum_{i=1}^{n} f(x_i) \cdot \Delta X$ Sum of areas of rectangles

Volumes and double Integrals: Compute volume under a surface in a similar manner as a sum of volumes of rectangular prisms





Volume of one rectangle: Area of base × height

$$= (\Delta x \cdot \Delta y) \cdot f(x_i, y_j)$$
$$= \Delta A \cdot f(x_i\, y_j)$$

Sum of volues to approximate volume under f:

$$V \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\,\Delta A$$

Thus $V = \lim_{m,n \to \infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\,\Delta A$ gives the volume under f above xy-plane

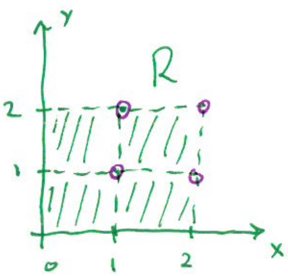
Definition - the double integral of f over the rectangle $R = \{(x,y) \mid a \le x \le b,\ c \le y \le d\}$

$$\boxed{\iint_R f(x,y)\,dA = \lim_{m,n \to \infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\,\Delta A}$$

★ Note when $f(x,y) \ge 0$ on R then the volume under f is the double integral.

Section 15.1 - Double Integral over Rectangles

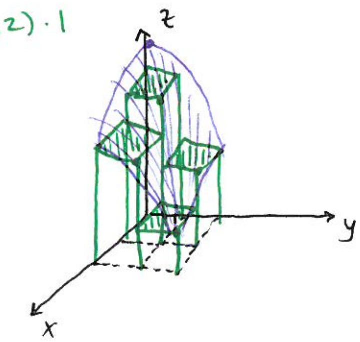
Ex 1 Estimate the volume of the solid that lies above the square $R=[0,2]\times[0,2]$ and below $z=16-x^2-2y^2$, by dividing R into 4 equal squares and choosing the upper right corner of each square for taking the height of the rectangular prism.



$$V \approx \sum_{i=1}^{2}\sum_{j=1}^{2} f(x_i, y_j)\Delta A \qquad \Delta A = \Delta x \cdot \Delta y = 1\cdot 1 = 1$$

$$= f(1,1)\cdot 1 + f(1,2)\cdot 1 + f(2,1)\cdot 1 + f(2,2)\cdot 1$$

$$= 13 + 7 + 10 + 4 = \boxed{34 \text{ units}^3}$$



★ Can take any point (x_i, y_j) in the rectangle R_{ij} with area ΔA for the definition of $\iint_R f(x,y)\,dA$, such as midpoint.

$$\iint_R f(x,y)\,dA \approx \sum_{i=1}^{n}\sum_{j=1}^{m} f(\bar{x}_i, \bar{y}_j)\Delta A$$

where $\bar{x}_i$ is the midpoint of $[x_{i-1}, x_i]$ and $\bar{y}_j$ is the midpoint of $[y_{j-1}, y_j]$.

Average Value:

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Similar, the average value of $f(x,y)$ over a rectangle R is

$$\boxed{f_{ave} = \frac{1}{\text{Area}(R)}\iint_R f(x,y)\,dA}$$

Properties of Double Integrals:

① $\iint_R [f(x,y)+g(x,y)]\,dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$

② $\iint_R c\cdot f(x,y)\,dA = c\iint_R f(x,y)\,dA$

③ If $f(x,y) \geq g(x,y)$ for all (x,y) in R then

$$\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$$