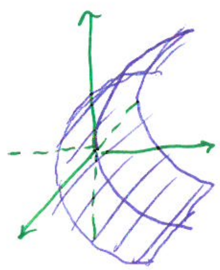
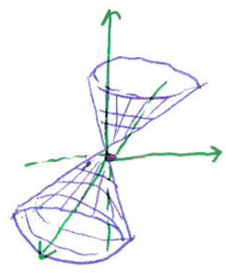
(1) Sketch the following

a) $y = z^2$    b) $x^2 = y^2 + 4z^2$    c) $-4x^2 + y^2 - 4z^2 = 4$

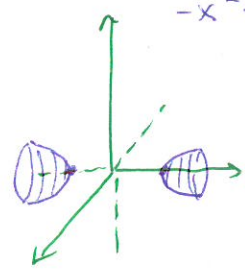
$-x^2 + \frac{y^2}{4} - z^2 = 1$



Cylindrical parabola



Cone



Hyperboloid of 2 sheets

(2) Find parametric equations for the lines:

a) line through $(4,-1,2)$ and $(1,1,5)$

$\vec{r_0} = \langle 4,-1,2 \rangle$

$\vec{v} = \langle -3, 2, 3 \rangle$

$\vec{r_1} = \langle 1, 1, 5 \rangle$

$\vec{r}(t) = \vec{r_0} + t\vec{v} = \langle 4,-1,2 \rangle + \langle -3t, 2t, 3t \rangle$

$= \boxed{\langle 4-3t, -1+2t, 2+3t \rangle}$

$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1}$

$= \langle 4-4t+1t, -1+t+t \ldots \rangle$

(3) Find the plane through $(2,-1,-1)$ parallel to $x + 4y - 3z = 1$

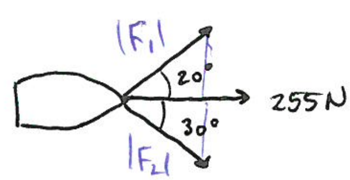
$\vec{n} = \langle 1, 4, -3 \rangle$

$1x + 4y - 3z = d$

$2 - 4 + 3 = d = 1$

$\boxed{x + 4y - 3z = 1}$

(4) A boat is pulled onto shore using 2 ropes. If a force of 255N is needed, find the magnitude of the force in each rope.




$F_1 \cos 20° + F_2 \cos 30° = 255$

$F_1 \sin 20° + F_2 \sin 30° = 0$

27, 33, 53,

(7) Find a vector for the tangent line to the curve of intersection of the cylinders $x^2+y^2=25$ and $y^2+z^2=20$ at the point $(3,4,2)$

$x^2+20-z^2=25$

$x^2-z^2=5$

$x=5\cos t$

$y=5\sin t$

$z=\pm\sqrt{20-25\sin^2 t}$

$\frac{dx}{dt}=-5\sin t$ $\qquad \left.\frac{dx}{dt}\right|_{(3,4,2)}=-4$ $\qquad \cos t=\frac{3}{5}$

$\frac{dy}{dt}=5\cos t$ $\qquad =3$ $\qquad \sin t=\frac{4}{5}$

$\frac{dz}{dt}=\frac{1}{2}(20-25\sin^2 t)^{-1/2}\cdot(-50\sin t\cdot\cos t) = \frac{1}{2\sqrt{2}}\cdot{-2\cdot3\cdot4}=-6$

$$\boxed{\vec{r}(t)=(3-4t)i+(4+3t)j+(2-6t)k}$$

(33) $\vec{r}_1(t)=\langle t,t^2,t^3\rangle$ $\quad \vec{r}_2(t)=\langle \sin t, \sin 2t, t\rangle$ both intersect the origin.
Find their angele of intersection.

$\vec{r}_1'(t)=\langle 1,2t,3t^2\rangle$ $\qquad \vec{r}_2'(t)=\langle \cos t, 2\cos(2t), 1\rangle$

$\vec{r}_1'(0)=\langle 1,0,0\rangle$ $\qquad \vec{r}_2'(0)=\langle 1,2,1\rangle$

$\frac{|\vec{r}_1'\cdot\vec{r}_2'(0)|}{|\vec{r}_1'|\cdot|\vec{r}_2'|}=\cos\theta \Rightarrow \cos\theta=\frac{1}{\sqrt{6}} \Rightarrow \theta\approx\boxed{65.91^\circ}$

- If $|r(t)|=c$ is a constant, then show $r'(t)$ is orthogonal to $r(t)$.

Show $r'(t)\cdot r(t)=0$ $\qquad$ recall $r(t)\cdot r(t)=|r(t)|^2=c^2$

so $0=\frac{d}{dt}(r(t)\cdot r(t))=2r'(t)\cdot r(t) \Rightarrow r'(t)\cdot r(t)=0.$