Section 14.6 - Directional Derivatives and the gradient Vector

Summary:

1. $D_{\vec{u}} f(x,y) = \nabla f \cdot \vec{u}$
2. $\nabla f = \langle f_x, f_y \rangle$
3. Max of $D_{\vec{u}} f$ is $|\nabla f|$
4. Direction of max $D_{\vec{u}} f$ is ∇f
5. $\nabla f \perp$ any tangent vector
6. ∇f is a normal vector for the tangent plane
7. ∇f direction of normal line
8. ∇f makes a $90°$ angle with the level curves, points in direction of increasing z

$\boxed{\#39}$ Second Directional Derivative

$$D_{\vec{u}}^2 f(x,y) = D_{\vec{u}}(D_{\vec{u}} f(x,y))$$

$f(x,y) = x^3 + 5x^2y + y^3$ $\vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$ find $D_u^2 f(x,y)$.

$$D_{\vec{u}} f(x,y) = \frac{3}{5}(3x^2 + 10xy) + \frac{4}{5}(5x^2 + 3y^2)$$

$$D_{\vec{u}}^2 f(x,y) = \frac{3}{5}\left(\frac{3}{5}(6x+10y) + \frac{4}{5}(10x)\right) + \frac{4}{5}\left(\frac{3}{5}(10x) + \frac{4}{5}(6y)\right)$$

$\boxed{\#55}$ Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parrallel to the plane $x + y = z$?

$$\langle 1, 1, -1 \rangle = \lambda \nabla (x^2 - y^2 - z^2) = \lambda \langle 2x, -2y, -2z \rangle$$

point $\lambda \langle 1, -1, 1 \rangle$

On hyperboloid? $(\lambda)^2 - \lambda^2 - \lambda^2 = -\lambda^2 \neq 1$ for any λ

Thus there are no points.