

#2 Let  $\vec{r}(t) = \langle \sqrt{2-t}, (e^t-1)/t, \ln(t+1) \rangle$

(a) Find the domain of  $\vec{r}$

(b) Find  $\lim_{t \rightarrow 0} \vec{r}(t)$

(c) Find  $\vec{r}'(t)$

(a)  $2 \geq t, t \neq 0, t > -1$   $D: [-1, 0) \cup (0, 2]$

(b)  $\lim_{t \rightarrow 0} \vec{r}(t) = \langle \sqrt{2}, 1, 0 \rangle$

(c)  $\vec{r}'(t) = \langle \frac{1}{2}(2-t)^{-1/2}(-1), \frac{e^t \cdot t - e^t}{t^2}, \frac{1}{t+1} \rangle$

Ex. Find a vector function that represents the curve of intersection of  
 $x^2 + y^2 + z = 4$  and  $x^2 + y^2 = 9$

$x = 3 \cos t$   $y = 3 \sin t$   $z = 4 - 9 = -5$

$\vec{r}(t) = \langle 3 \cos t, 3 \sin t, -5 \rangle$

Reparametrize  $\vec{r}(t) = \langle e^t, e^t, e^t \rangle$  with respect to arc length measured from  $(1, 0, 1)$  in the direction of increasing  $t$ .

$$s(t) = \int_0^t |\vec{r}'(u)| du = \int_0^t \sqrt{(e^{2u}) + e^{2u} + e^{2u}} du$$

$$= \int_0^t \sqrt{3} e^u du = \sqrt{3}(e^t - e^0) = \sqrt{3}e^t - \sqrt{3}$$

$t = \ln\left(\frac{s + \sqrt{3}}{\sqrt{3}}\right)$

$\vec{r}(s) = \left\langle \frac{\sqrt{3}}{3}s + 1, \left(\frac{\sqrt{3}}{3}s + 1\right), \left(\frac{\sqrt{3}}{3}s + 1\right) \right\rangle$

- #18 A particle starts at the origin with initial velocity  $\vec{i} - \vec{j} + 3\vec{k}$   
and its acceleration is  $\vec{a}(t) = 6t\vec{i} + 12t^2\vec{j} - 6t\vec{k}$ .  
Find its position function and its speed function.

$$\vec{v}(t) = \int \vec{a}(t) dt = \langle 3t^2, 4t^3, -3t^2 \rangle + \langle 1, -1, 3 \rangle$$

$$\vec{p}(t) = \int \vec{v}(t) dt = \langle t^3 + t, t^4 - t, -t^3 + 3t \rangle$$

$$\text{Speed} = |\vec{v}(t)| = \sqrt{(3t^2+1)^2 + (4t^3-1)^2 + (-3t^2+3)^2}$$