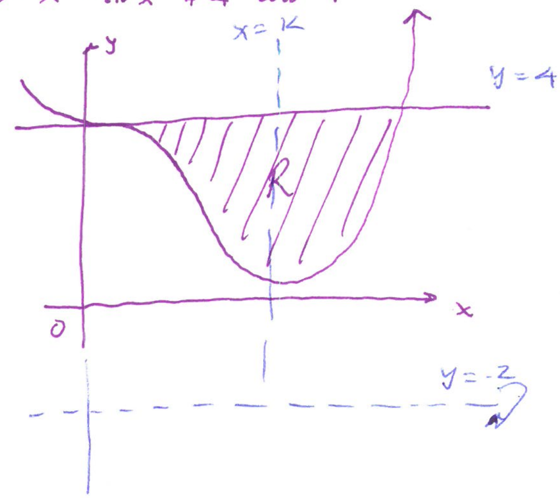
AB Calc

Review

2/1/16

#2 (2014) Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$ shown above.



(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$

$$\text{Volume} = \pi \int_0^{2.3} (R^2 - r^2)\,dx \qquad R = 4 - (-2) = 6$$

$$r = (x^4 - 2.3x^3 + 4) + 2$$

$$= \pi \int_0^{2.3} 36 - (x^4 - 2.3x^3 + 6)^2\,dx \approx \boxed{98.868}$$

$$= \text{fnInt}(\pi 36 - \pi(x^4 - 2.3x^3 + 6)^2, x, 0, 2.3)$$

Bounds:

$$x^3(x - 2.3) = 0$$

(a) Find the area of the region R.

$$\text{Area} = \int_0^{2.3} (4 - (x^4 - 2.3x^3 + 4))\,dx = \text{fnInt}(-x^4 + 2.3x^3, x, 0, 2.3) \approx \boxed{3.218}$$

) The vertical line $x = K$ divides R into 2 regions with equal areas. Write but do not solve, an equation involving integrals expressions whose solution gives the value of K.

$$\int_0^K (2.3x^3 - x^4)\,dx = \int_K^{2.3} (2.3x^3 - x^4)\,dx$$

or

$$\int_0^K (4 - f(x))\,dx = \int_K^{2.3} (4 - f(x))\,dx$$

#6 (2014) Consider the DE $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = 1$. f is defined for all real numbers.

(b) Write an equation for the line tangent to the solution curve at the point $(0,1)$. Use the equation to approximate $f(0.2)$.

$$\left.\frac{dy}{dx}\right|_{\substack{x=0\\y=1}} = 2\cos(0) = 2 \qquad \boxed{y = 2(x) + 1} \text{ tangent line to solution curve @ } (0,1)$$

$$f(0.2) \approx y(0.2) = \boxed{1.4}$$

(c) Find $y = f(x)$, the particular solution to the diff. equ with $f(0) = 1$.

$$\int \frac{1}{3-y}\,dy = \int \cos x\,dx \Rightarrow -\ln|3-y| = \sin x + C \Rightarrow -\ln 2 = C$$

$$|3 - y| = e^{-\sin x + \ln(2)} = 2e^{-\sin x} \Rightarrow$$

$y = 3 + 2e^{-\sin x}$ Doesn't satisfy $f(0) = 1$

or $\boxed{y = 3 - 2e^{-\sin x}}$