Review Practice: Chapter 14

Formulas:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$L(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

$$\mathbf{D_u} f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}$$

$$\text{Second Derivative Test:} \qquad D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - (f_{xy})^2$$

$$\text{Lagrange Multipliers:} \qquad \nabla f(x, y, z) = \lambda \nabla g(x_0, y_0, z_0), \quad g(x, y, z) = k$$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y}, \qquad \frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$$

1. True or False: If $(2, 1)$ is a critical point of f and $f_{xx}(2, 1) f_{yy}(2, 1) < f_{xy}(2, 1)$ then f has a saddle point at $(2, 1)$.

2. Find the partial derivatives of $G(x, y, z) = e^{xy} \sin(y/z)$.

3. If $v = x^2 \sin y + y e^{xy}$ where $x = s + 2t$ and $y = st$, use the chain rule to find $\frac{\partial v}{\partial t}$ and $\frac{\partial v}{\partial s}$ when $s = 0$ and $t = 1$.

4. Find the maximum rate of change of $f(x, y) = x^2 y + \sqrt{x}$ at the point $(2, 1)$. In what direction does it occur?

5. Use Lagrange Multipliers to find the max/min values of $f(x, y) = \dfrac{1}{x} + \dfrac{1}{y}$ subject to the constraint $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$. [Note I will only ask you for the set up of these problems]

6. Find the absolute max and min values of f on the set D given:

$$f(x,y) = e^{-x^2-y^2}(x^2+y^2) \quad \text{and} \quad D = \{(x,y) \mid x^2+y^2 \le 4\}$$