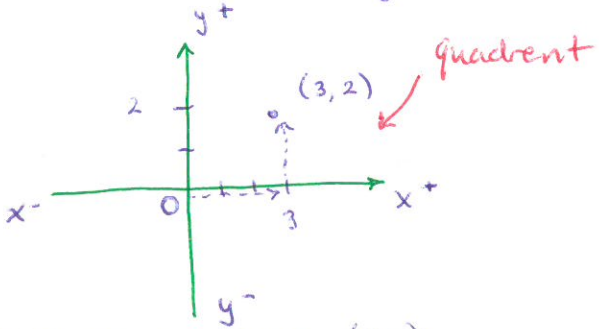
Section 12.1 - 3D Coordinate System

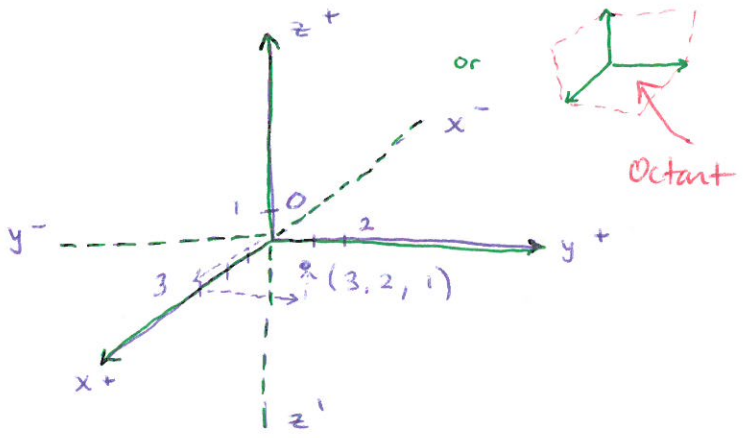
Vector Calc

Cartesian Coordinate System



Point on a plane (x, y)

3D Coordinate System



Point in space (x, y, z)

★ Visualize the corner of a room

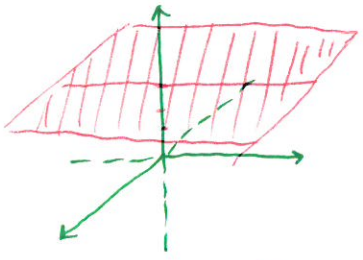
Right Hand Rule - fingers on x^+ axis, curl to y^+ axis, thumb up to z^+ axis

Cartesian	3D
$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y) \mid x, y \in \mathbb{R}\}$	$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$
equation x, y is a <u>curve</u>	equation x, y, z is a <u>surface</u>

Example 1 What surfaces in $\mathbb{R}^3$ are represented by the equations:

(a) $z = 3$

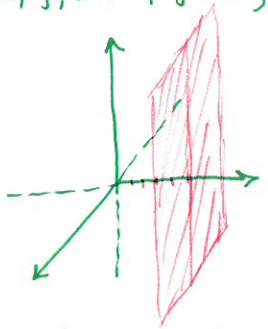
$\{(x, y, z) \mid z = 3\}$



Horizontal plane at $z = 3$
Parallel to xy-plane

(b) $y = 5$

$\{(x, y, z) \mid y = 5\}$



Vertical plane at $y = 5$
Parallel to xz-plane

Distance formula between 2 points:

$P_1(x_1, y_1)$ $P_2(x_2, y_2)$

$\mathbb{R}^2$: $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

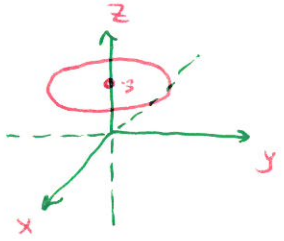
$P_1(x_1, y_1, z_1)$ $P_2(x_2, y_2, z_2)$

$\mathbb{R}^3$: $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

Example 2 (a) Which points (x, y, z) satisfy $x^2+y^2=1$ and $z=3$?
(b) What does the equation $x^2+y^2=1$ represent in $\mathbb{R}^3$?

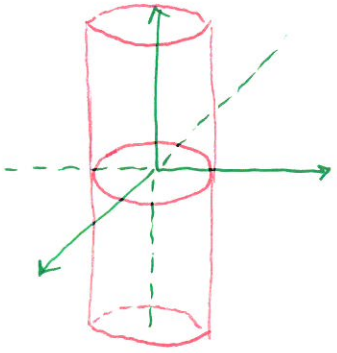
(a) $\{(x, y, z) \mid x^2+y^2=1 \text{ and } z=3\}$

- points on the plane $z=3$ w/ $x^2+y^2=1$
- Circle of points, radius 1, centered on z-axis, on plane $z=3$.



(b) Circle of points, radius 1, centered on z-axis for all values of z

Cylinder of radius 1, centered on z-axis



Equation of a Sphere:

★Recall: a circle is the set of all points in $\mathbb{R}^2$ equadistant from the center.
Hence a sphere is the set of all points in $\mathbb{R}^3$ equadistant from the center.

Radius r, center $C(h,k,l)$: $\boxed{(x-h)^2+(y-k)^2+(z-l)^2=r^2}$

Radius r, center O: $\boxed{x^2+y^2+z^2=r^2}$

Example Show $x^2+y^2+z^2+4x=0$ is the equation of a sphere.

Complete the square: $(x^2+4x+\underline{4})+y^2+z^2=0+\underline{4}$

$\boxed{(x+2)^2+y^2+z^2=4}$ $\boxed{\text{Center: } (-2,0,0) \quad \text{Radius: } 2}$

Example 7 What region in $\mathbb{R}^3$ is represented by $1 \le x^2+y^2+z^2 \le 4$ and $z \le 0$?

All points outside circle of radius 1: $1^2 = x^2+y^2+z^2$

But inside circle of radius 2: $x^2+y^2+z^2=2^2$

Below xy-plane (semi-spheres): $z \le 0$