Calc AB

$\boxed{\text{Lesson 51}}$

lessons 51
Integration by Guessing

1) Guess an antiderivative
2) Check by differentiating

Ex. $\int 5(x+2)^4\,dx = \boxed{(x+2)^5 + C}$

Guess: $(x+2)^5$ Check: $\frac{d}{dx}(x+2)^5 = 5(x+2)^4$ ✓

Ex. $\int 6\cos^5 t \sin t\,dt = \boxed{-\cos^6 t + C}$

Guess: $-\cos^6 t$ Check: $\frac{d}{dt}(-\cos^6 t) = +6\cos^5 t \cdot \sin t$ ✓

Fix

$\boxed{\text{Ex. 51.5}}$ $\int 8x(e^{4x^2})\,dx = \boxed{e^{4x^2} + C}$

Guess: e^{4x^2} Check: $\frac{d}{dx}(e^{4x^2}) = e^{4x^2}\cdot(8x)$ ✓

Ex. $\int 24x(2x^2+7)^5\,dx = \boxed{(2x^2+7)^6 + C}$

Guess: $(2x^2+7)^6$ Check: $\frac{d}{dx}(2x^2+7)^6 = 6(2x^2+7)^5\cdot(4x)$ ✓

$\boxed{\text{Ex. 51.4}}$ $\int \frac{3x^2}{2\sqrt{x^3+4}}\,dx = \boxed{(x^3+4)^{1/2} + C}$

Guess: $(x^3+4)^{1/2}$ Check: $\frac{1}{2}(x^3+4)^{-1/2}\cdot(3x^2)$ ✓

Calc AB

Lesson 52+53

10/19/15

<u>Agenda</u>: 10/19/15

HW leader:

lessons 52+53

Optimization Problems

Numerical integration

- Handout Calendar

★ Test 3 back after lesson

<u>Critical numbers</u>: local min, local max, inflection pts or where the derivative is undefined

When the derivative is 0 (horizontal tangent line to curve) or undefined

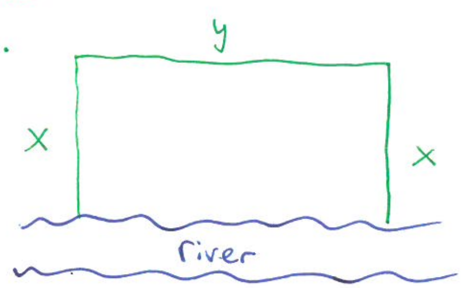
Optimization problems are applied problems that ask for the <u>absolute (global)</u> minimum or maximum of a function on an interval.

1. Finding the absolute max (or min) on an interval starts by finding all critical numbers of a function on the interval [This includes the endpoints]
2. Then find the function values at the critical numbers.
3. Choose the greatest (or least) value as answer.

<u>Ex. 52.1</u> A man with 100 yards of fence wants to form a rectangular field enclosed on 3 sides by fence and one side by a river. Find the <u>greatest area</u> that the fence can enclose.
to maximize

1. P = Perimeter A = Area

1.



2. $P = 100 = y + 2x$ so $y = 100 - 2x$

$A = x \cdot y$ or $A = x(100 - 2x)$ Domain: $(0, \infty)$

3. Find global max of A

$\frac{dA}{dx} = 100 - 4x$ $\frac{dA}{dx} = 0$ when $\boxed{x = 25}$ Only critical number

4. Check this is a max of A:

$\frac{d^2A}{dx^2} = -4$ so $\left.\frac{d^2A}{dx^2}\right|_{x=25} = -4 < 0 \Rightarrow \boxed{\text{Local max of } A}$

Since only critical number this is the global max.

Calc AB

$\boxed{\text{lesson } 52+53}$

10/19/15

Ex. A cylindrical can is to be made to hold $16 in^3$. If the material for the top and bottom costs \$0.03 per in^2 and the material for the side costs \$0.02 per in^2, find the dimensions which minimize the cost if the height must be between 1 and 5 inches.

1. Draw a picture label variables

V = volume $= 16 \text{ in}^3$

C = Cost of Can

2. $V = 16 = \pi r^2 \cdot h$ so $h = \frac{16}{\pi r^2}$

$C = 0.03(2\pi r^2) + 0.02(2\pi r h)$ so $C = 0.06\pi r^2 + 0.04\pi r\left(\frac{16}{\pi r^2}\right)$

3. Minimize $C(r)$ on $\left[\frac{16}{25\pi}, \frac{16}{\pi}\right]$

$C = .06\pi r^2 + \frac{.64}{r}$

Domain $h : [1,5]$

$r : \left[\frac{16}{25\pi}, \frac{16}{\pi}\right]$

Critical numbers: $\frac{dc}{dr} = .12\pi r - \frac{.64}{r^2} = 0$

$.12\pi r^3 = .64$

$r^3 = \frac{16}{3\pi}$ $\quad r = \left(\frac{16}{3\pi}\right)^{1/3}, \frac{16}{25\pi}, \frac{16}{\pi}$

4. Check for global minimum:

$C\left(\left(\frac{16}{3\pi}\right)^{1/3}\right) \approx \$0.804739$ ← Global minimum when

$$\boxed{\begin{aligned} r &= \left(\frac{16}{3\pi}\right)^{1/3} \text{ inches} \\ h &= \frac{16^{1/3} \cdot 3^{2/3}}{\pi^{1/3}} = \left(\frac{16 \cdot 9}{\pi}\right)^{1/3} \text{ inches} \end{aligned}}$$

$C\left(\frac{16}{25\pi}\right) \approx \$3.14942$

$C\left(\frac{16}{\pi}\right) \approx \$5.0149$

Lesson 53: Numerical Integration of Positive-Valued Functions on a Graphing Calc.

Ex. 53.2: Use a graphing calculator to approximate the area under the curve $y = \sin(x)$ between $x = 0$ and $x = \pi/6$.

$$\text{Area} = \int_0^{\pi/6} \sin(x)\,dx = -\cos(x)\Big|_0^{\pi/6} = -\cos\left(\tfrac{\pi}{6}\right) + \cos(0) = 1 - \tfrac{\sqrt{3}}{2} \approx 0.133975$$

$$\approx \underset{\substack{\uparrow \\ \text{Under } \boxed{\text{MATH}}}}{\text{fnInt}}(\overset{\text{function}}{\sin(x)}, \underset{\text{start}}{0}, \overset{\text{end}}{\pi/6}) \approx 0.133975$$

★ Calc in RADIAN Mode!

Calc AB

Lesson 54

10/20/15

Agenda: 10/20/15

Period 3

Period 4

HW leader:

lesson 54

Velocity and Acceleration

★ Quiz 6 on Friday

- Velocity is the instantaneous rate of change of position
- Acceleration is the instantaneous rate of change of velocity

★ we define positive velocities in the positive x-direction (right of 0) and call velocities in the opposite direction negative velocities (left of 0)

Ex. 54.2 The position of a particle moving along the x-axis at any time t (in seconds) is given by

$$x(t) = t^2 - 3t + 2$$

a) Find the times when the particle is at rest,

b) moving to the right

c) moving to the left

d) accelerating

e) decelerating

$v(t) = x'(t) = 2t - 3$

$a(t) = v'(t) = x''(t) = 2$

(a) when $v(t) = 0$ so $\boxed{t = 3/2 \text{ seconds}}$ the particle is at rest.

(b) when $v(t) > 0$ so $\boxed{t > 3/2 \text{ seconds}}$ the particle is moving right

(c) when $v(t) < 0$ so $\boxed{t < 3/2 \text{ seconds}}$ the particle is moving left

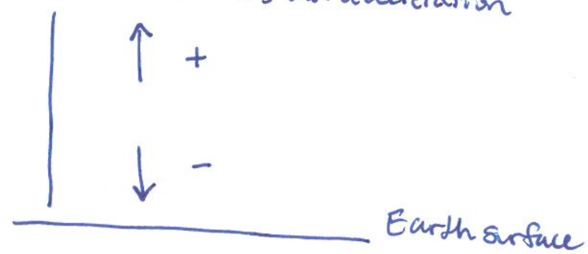
(d) when $a(t) > 0$ so $\boxed{\text{for all } t}$ the particle is accelerating

(e) when $a(t) < 0$ so never decelerating.

Ex. 54.3 A ball is thrown vertically into the air with an initial velocity of 20 m/s 2 m off the ground. It's height is then modeled by

$$h(t) = 2 + 20t - 4.9t^2$$

Vertical velocity and acceleration



(a) Find the height of the ball and velocity 1 second after it is thrown.

(b) At what time is the ball the greatest distance from the ground?

(c) How high will the ball go?

(d) What is the greatest value of the acceleration?

$$h(t) = 2 + 20t - 4.9t^2$$
$$v(t) = h'(t) = 20 - 9.8t$$
$$a(t) = -9.8$$

(a) $h(1) = 2 + 20 - 4.9 = \boxed{17.1\text{ m}}$

$v(1) = 20 - 9.8 = \boxed{10.2\text{ m/s}}$

(b) maximize $h(t)$ when $v(t) = 0$: $t = \frac{20}{9.8} \approx \boxed{2.0408\text{ seconds}}$

(c) $h(2.0408) \approx \boxed{22.4082\text{ meters}}$

(d) $\boxed{-9.8\text{ m/s}^2}$

Calc AB

$\boxed{\text{Lesson 56}}$

10/21/15

Agenda: 10/21/15

Period 3

Period 4

HW leader:

Lesson 56

More Integration by Guessing

★ Quiz 6 on Friday (lessons 46 - 56)

Ex. $\int \cos(3t)\,dt = \boxed{\frac{1}{3}\sin(3t) + C}$

Guess: $\sin(3t)$

Check: $\frac{d}{dt}(\sin(3t)) = \cos(3t)\cdot\underbrace{3}$

Guess: $\underbrace{\frac{1}{3}}\sin(3t)$

Check: $\frac{d}{dt}\left(\frac{1}{3}\sin(3t)\right) = \frac{1}{3}\frac{d}{dt}(\sin(3t))$

$= \cos(3t)$

Ex. $\int x^3(4x^4+5)^2\,dx = \boxed{\frac{1}{48}(4x^4+5)^3 + C}$

Guess: $(4x^4+5)^3$

Check: $\frac{d}{dx}(4x^4+5)^3 = \underbrace{3}(4x^4+5)^2\cdot\underbrace{16}x^3$

Guess: $\underbrace{\frac{1}{48}}(4x^4+5)^3$

Check: $\frac{d}{dx}\left(\frac{1}{48}(4x^4+5)^3\right) = \frac{3}{48}(4x^4+5)^2\cdot 16x^3$

Ex 56.3 $\int \frac{x^2\,dx}{\sqrt{x^3+1}} = \boxed{\underbrace{\frac{2}{3}}\left(\sqrt{x^3+1}\right) + C}$

Guess: $\sqrt{x^3+1}$

Check: $\frac{d}{dx}\left(\sqrt{x^3+1}\right) = \underbrace{\frac{1}{2}}(x^3+1)^{-1/2}\cdot\underbrace{3}x^2$

Ex. 56.6 $\int \frac{e^{\sqrt{x}}\,dx}{\sqrt{x}} = \boxed{\underbrace{2}e^{\sqrt{x}} + C}$

Guess: $e^{\sqrt{x}}$

Check: $\frac{d}{dx}\left(e^{\sqrt{x}}\right) = e^{\sqrt{x}}\cdot\underbrace{\frac{1}{2}}x^{-1/2}$

Ex. 56.8 $\int \frac{\cos(ax)\,dx}{\sqrt{b+\sin(ax)}} = \boxed{\frac{2}{a}\sqrt{b+\sin(ax)} + C}$

Guess: $\sqrt{b+\sin(ax)}$

Check: $\frac{d}{dx}\left(\sqrt{b+\sin(ax)}\right) = \underbrace{\frac{1}{2}}(b+\sin(ax))^{-1/2}\cdot\cos(ax)\cdot\underbrace{a}$

Calc AB

Lesson 57

10/22/15

Agenda: 10/22/15

Period 3

Period 4

HW leader:

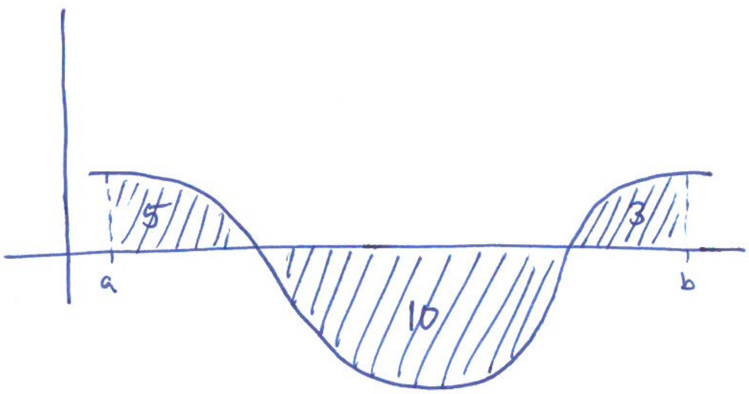
Lesson 57

Properties of the Definite Integral

★ Quiz 6 tomorrow

- The definite integral is a number that is the limit of a Rieman sum.

- The definite integral is the sum of areas above the x-axis below f and the negatives of the areas above the graph below the x-axis.



$$\int_a^b f(x)\,dx = 5 - 10 + 3 = \boxed{-2}$$

Properties of the Definite Integral

- $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_b^a f(x)\,dx = F(a) - F(b) = -(F(b)-F(a)) = -\int_a^b f(x)\,dx$
- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

  $\int_a^a f(x)\,dx = \int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^b f(x)\,dx - \int_a^b f(x)\,dx = 0$
- If $f(x) \geq 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \geq 0$
- If $f(x) = 0$ on $[a,b]$, then $\int_a^b f(x)\,dx = 0$
- If $f(x) \leq 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \leq 0$

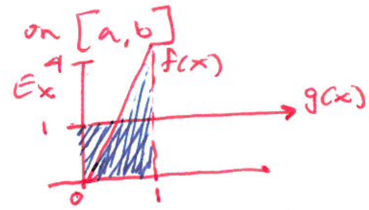
Calc AB

Lesson 57

10/22/15

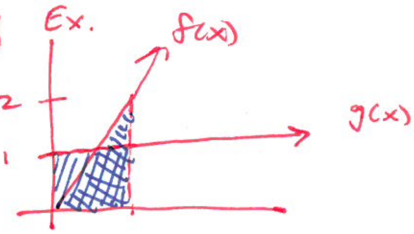
- If $g(x) < f(x)$ on $[a,b]$ then $\int_a^b g(x)\,dx < \int_a^b f(x)\,dx$

2 $\int_a^b g(x)\,dx < \int_a^b f(x)\,dx$ does not mean $g(x) < f(x)$ on $[a,b]$



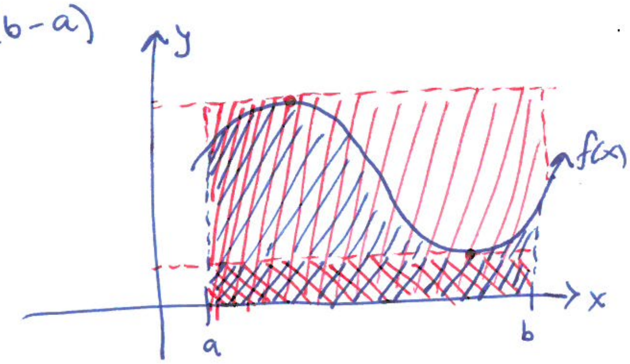
- If $g(x) = f(x)$ on $[a,b]$ then $\int_a^b g(x)\,dx = \int_a^b f(x)\,dx$

2 $\int_a^b g(x)\,dx = \int_a^b f(x)\,dx \not\Rightarrow g(x) = f(x)$ on $[a,b]$



Let M = max of f on $[a,b]$ m = min of f on $[a,b]$

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$



Ex

$\int_{-1}^{1} f(x)\,dx = 7$ $\int_{1}^{4} f(x)\,dx = 2$

Find $\int_{4}^{-1} f(x)\,dx$

$$\int_4^{-1} f(x)\,dx = -\int_{-1}^{4} f(x)\,dx = -\left[\int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx\right] = \boxed{-9}$$

Try Ex. 57.3 $\int_{-2}^{1} f(x)\,dx = 3$ $\int_{3}^{1} f(x)\,dx = 7$ Find $\int_{-2}^{3} f(x)\,dx$

$$= -7 + 3 = \boxed{-4}$$