Review Practice: Chapters 14 & 15

1. Consider $f(x, y) = x^2 + 2x - y$

   (a) Find all first and second partials

   $\boxed{f_x = 2x+2, \quad f_y = -1; \quad f_{xx} = 2, \quad f_{yy} = 0, \quad f_{xy} = f_{yx} = 0}$

   (b) Find the gradient

   $\boxed{\nabla f = \langle 2x+2, -1 \rangle}$

   (c) What types of graphs are the level curves of f?

   $K = x^2 + 2x - y$

   $y = x^2 + 2x + K \rightarrow$ upward parabolas

   $y = (x+1)^2 + (K-1)$ vertex: $(-1, K-1)$

2. Find all critical points of $f(x, y) = x^3 - 12x + y^2$ and classify them using the second derivative test.

   $\vec{0} = \nabla f = \langle 3x^2 - 12, 2y \rangle$

   $x = \pm 2, \quad y = 0$

   $D = f_{xx} \cdot f_{yy} - f_{xy}^2 = (6x)(2) - 0 = 12x$

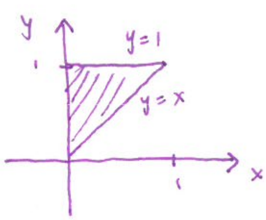
   - $D(2,0) > 0$, $f_{xx}(2,0) > 0 \Rightarrow$ local min at $(2,0)$
   - $D(-2,0) < 0 \Rightarrow$ Saddle point at $(-2,0)$

3. If $x = f(x, y)$ and $x = g(r, s)$ and $y = h(r, s)$ use chain rule to find $\frac{\partial z}{\partial s}$

   $$\frac{\partial z}{\partial s} = f_x \frac{\partial x}{\partial s} + f_y \frac{\partial y}{\partial s}$$

   $$\boxed{\frac{\partial z}{\partial s} = f_x(g,h)\,g_s + f_y(g,h)\,h_s}$$

4. Compute by changing the order of integration: $\int_0^1 \int_x^1 y^2 \sin(xy)\, dy\, dx$



$$= \int_0^1 \int_0^y y^2 \sin(xy)\, dx\, dy$$

$$= \int_0^1 -\frac{y^2 \cos(xy)}{y} \Big|_0^y \, dy$$

$$= \int_0^1 -y\cos(y^2) + y \, dy$$

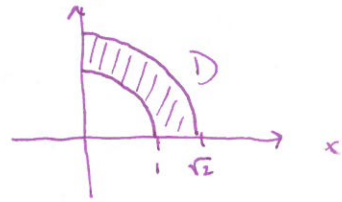
$$= -\frac{\sin(y^2)}{2} + \frac{y^2}{2} \Big|_0^1 = \boxed{\frac{1}{2}\left(-\sin(1) + 1\right)}$$

5. Compute $\int\int_D x\, dA$ where D is the region in the first quadrant between $x^2+y^2=1$ and $x^2+y^2=2$.

$$\iint_D x\,dA = \int_0^{\pi/2}\int_1^{\sqrt{2}} r\cos\theta \cdot r\,dr\,d\theta = \int_0^{\pi/2}\cos\theta\,d\theta \cdot \int_1^{\sqrt{2}} r^2\,dr$$

$$= \sin\theta\Big|_0^{\pi/2} \cdot \frac{r^3}{3}\Big|_1^{\sqrt{2}}$$

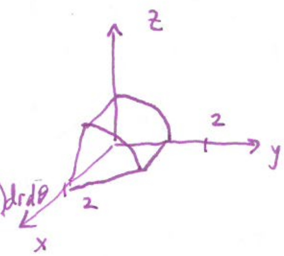
$$= \boxed{\frac{1}{3}\left(2^{3/2}-1\right)}$$



6. Compute $\int\int\int_E z\, dV$ where E is the region in the first octant between $y^2+z^2=1$ and $x+y=2$.

$$\iiint_E z\,dV = \iint_D \int_0^{2-y} z\,dx\,dA$$

$$= \iint_D 2z - yz\,dA = \int_0^{\pi/2}\int_0^1 r^2(2\sin\theta - r\sin\theta\cos\theta)\,dr\,d\theta$$



$$= \int_0^{\pi/2} \frac{2}{3}\sin\theta - \frac{1}{4}\sin\theta\cos\theta\,d\theta = -\frac{2}{3}\cos\theta - \frac{1}{4}\frac{\sin^2\theta}{2}\Big|_0^{\pi/2} = \boxed{-\frac{1}{8}+\frac{2}{3}}$$

7. If the cylinderical coords of a point are $(2\sqrt{3}, 3\pi/4, 2)$ find the sphereical coords of the point.

$$(r,\theta,z) = (2\sqrt{3}, \underline{3\pi/4}, 2) \longrightarrow (\rho, \underline{\theta}, \varphi) = \boxed{(4, 3\pi/4, \pi/3)}$$

$x = 2\sqrt{3}\cos\frac{3\pi}{4} = -\sqrt{6}$

$y = 2\sqrt{3}\sin\frac{3\pi}{4} = \sqrt{6}$

$z = 2$

$\rho^2 = (-\sqrt{6})^2 + (\sqrt{6})^2 + (2)^2$

$\rho^2 = 16 \Rightarrow \rho = 4$

$2 = 4\cos\varphi$

$\cos\varphi = \frac{1}{2} \Rightarrow \varphi = \pi/3$