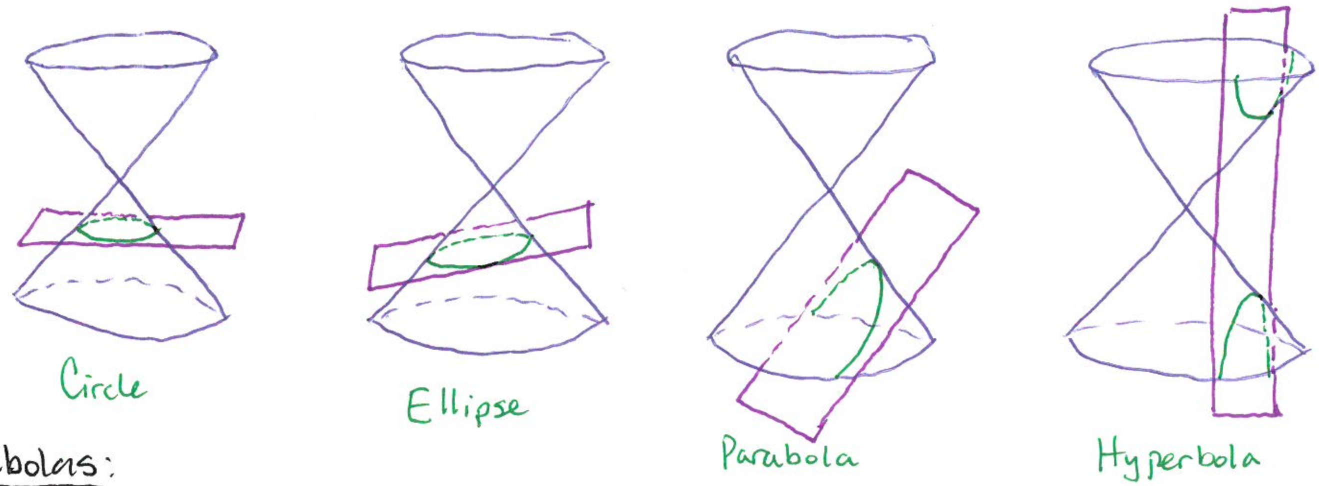
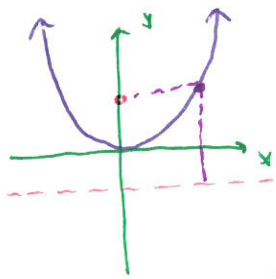
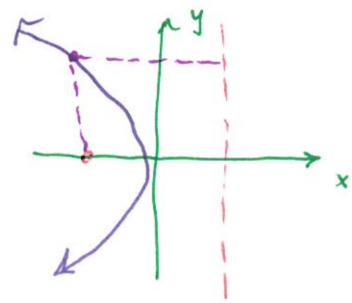


Parabolas:

Set of all points in a plane that are equidistant from a point (focus) and a line (Directrix).





$y = \frac{1}{4p}x^2$ Focus: $(0, p)$ Directrix: $y = -p$

$x = \frac{1}{4p}y^2$ Focus: $(p, 0)$ Directrix: $x = -p$

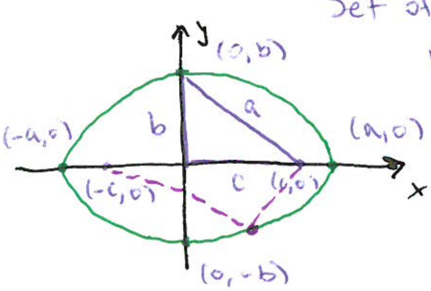
Standard Form: $y = a(x-h)^2 + k$

Vertex: (h, k)

General Form: $ax^2 + bx + cy + d = 0$

Ellipses:

Set of all points in a plane, sum of their distances from two fixed points (foci) is constant.



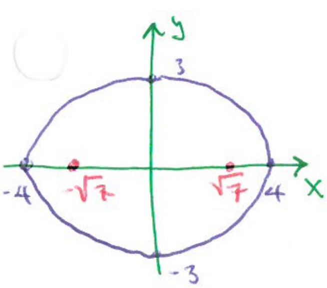
Centered at the origin

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a \geq b > 0$

  Major axis: x-axis foci $(\pm c, 0)$ $c^2 = a^2 - b^2$

- $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ $a \geq b > 0$

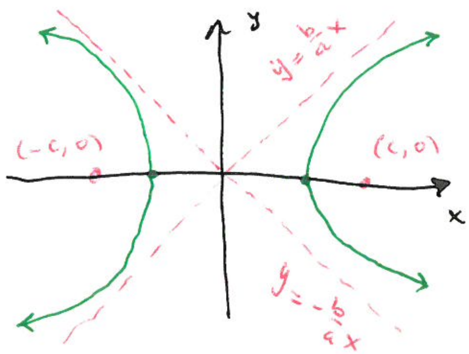
  Major axis: y-axis foci $(0, \pm c)$ $c^2 = a^2 - b^2$

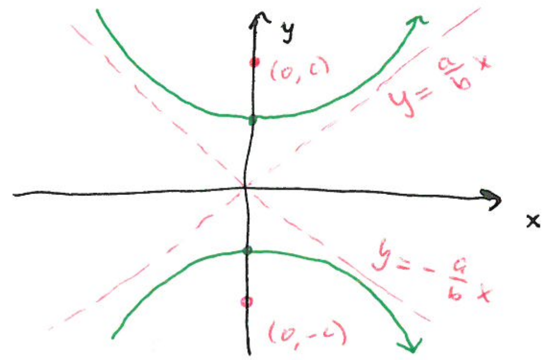


Example 2 Sketch $9x^2 + 16y^2 = 144$ and locate foci.

$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ $c = \pm\sqrt{16 - 9} = \pm\sqrt{7}$

Hyperbolas: Set of all points in a plane, the difference of their distances from two fixed points (foci) is constant.





Opens on x-axis:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

foci: $(\pm c, 0)$ $\quad c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{b}{a} x$

Opens on y-axis:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

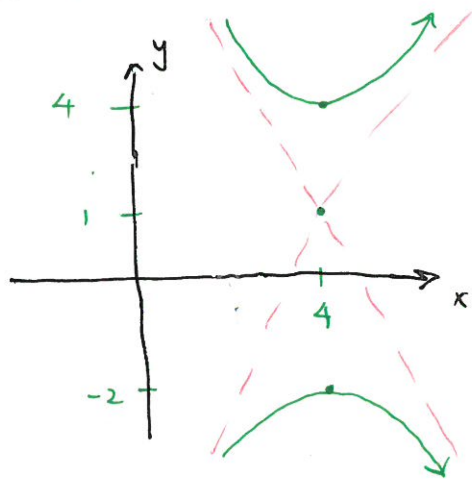
foci: $(0, \pm c)$ $\quad c^2 = a^2 + b^2$

Asymptotes: $y = \pm \frac{a}{b} x$

Shifted Conics: Move center to (h, k) from $(0,0)$

replace (x) with $(x-h)$ and (y) with $(y-k)$

Example 7 Sketch $9x^2 - 4y^2 - 72x + 8y + 176 = 0$ and find its foci.



$$9(x^2 - 8x + 16) - 4(y^2 - 2y + 1) = -176 + 9(16) - 4$$

$$9(x-4)^2 - 4(y-1)^2 = -36$$

$$\frac{(y-1)^2}{9} - \frac{(x-4)^2}{4} = 1$$

$(y-1) = \pm \frac{9}{4}(x-4)$ $\quad$ foci $(4, 1 \pm \sqrt{13})$