A numerical model of ultracold atoms and structured light in a driven optical cavity

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We study the behaviour of a Bose-Einstein Condensate (BEC) in a far red-detuned optical cavity driven by a Laguerre-Gaussian optical pump with orbital angular momentum (OAM).

In this document, we model the evolution of a BEC inside an optical cavity pumped with an optical field carrying OAM, which has a helical phase front and ring-shaped intensity distribution [1, 2].

Our model is based on the schematic set-up shown in Fig. 1. A spatial light modulator (SLM) displaying an m-forked diffraction grating is used to convert a Gaussian beam into a helical mode with OAM m [3]. This is used to pump a unidirectional ring cavity, of length $\mathcal{L}$ and indicatively built of four highly-reflective mirrors (of transmittivity T), containing a "pancake"-shaped BEC of length L, where $L << \mathcal{L}$. We consider a pump wavelength of 720nm which is far red-detuned from the atomic transition such that absorption can be neglected and the dipole interaction induces a focusing nonlinearity whereby atoms are attracted to intensity *maxima*, i.e. the atoms are "light-seeking" [4].

We base our model of the temporal evolution of the BEC and optical fields in the cavity on a description of co-propagating ultracold atomic and optical fields [4, 5]. By assuming an atomic medium with no mean group velocity, such that it remains static in the central cavity position, we adiabatically eliminate its excited state before performing a mean field procedure upon the optical field [6]. This gives the coupled equations:

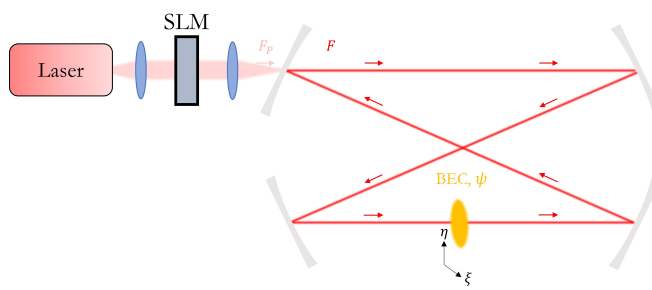

FIG. 1. Schematic set-up. A laser beam is incident on a SLM, which adds OAM m, before entering a unidirectional ring cavity, consisting of four highly reflective mirrors and interacting with a pancake-shaped BEC.

$$\partial_\tau \psi = \frac{\alpha_\psi}{\kappa}\left[i\nabla_\perp^2\psi - i\left(s|F|^2 - 2\beta_{dd}|F|^2|\psi|^2 + \beta_{col}|\psi|^2 - iL_3|\psi|^4\right)\psi\right], \tag{1}$$

$$\partial_\tau F = -\left(1+i\theta\right)F + i\alpha_F\nabla_\perp^2 F - i\frac{2L}{Tk_L w_s^2}\left(s|\psi|^2 - \beta_{dd}|\psi|^4\right)F + F_P. \tag{2}$$

Eq. (1) provides a Gross-Pitaevskii based description of the dynamics of the BEC field ψ, while Eq. (2) is a Lugiato-Lefever equation which extends the description of the optical field F to propagation in an optical cavity driven by an optical pump F_P, with θ the detuning between the input pump and the closest cavity resonance. We have introduced the parameters $\alpha_\psi = \hbar/(m_a w_s^2)$ where m_a is the atomic mass and w_s is an optical beam waist used for scaling, $\alpha_F = (2\mathcal{L})/(k_L w_s^2 T)$ where k_L is the optical wavenumber, and $\kappa = (cT)/(2\mathcal{L})$ where c is the speed of light, to leave all other parameters as defined in [4, 5].

Parameter $s = \pm 1$ represents the sign of the light-atom far-detuning: for $s = +1(-1)$, the fields are blue (red) detuned, and the BEC can be described as being 'dark (light) seeking' with respect to the optical field. β_{col} represents the interatomic scattering of the atoms within the BEC, providing an attractive or repulsive nonlinearity depending on the BEC's scattering length, a_s. We consider a BEC of Caesium atoms as this has a scattering length which may be tuned using magnetic fields around its Feshbach resonance [7]. Finally, the term in L_3 describes three-body loss ($L_3 \approx 10^{-4}$) in high-density regimes [8, 9]. Note that we neglect terms corresponding to dipole-dipole forces, as in Ref. [5], as well as any optical saturation, as their effects are marginal on the system dynamics studied here.

The transverse dimensions (ξ, η) have been scaled according to $(\xi, \eta) = \sqrt{2}(x, y)/w_s$, where w_s is a characteristic beam waist size that we set as 10μm for consistency with Ref. [5]. From (1), the temporal dynamics of the atomic field are effectively scaled by the factor α_ψ/κ, and we here consider $\kappa \approx 10^{-8}$, but note that a large range of κ values are accessible by altering the cavity parameters T and $\mathcal{L}$.

We assume that the BEC has a 'pancake' or disk form, i.e. with significantly larger transverse than longitudinal size, and that the initial transverse profile can be described by a Thomas-Fermi distribution:

$$\psi(r) = 1 - r^2 / \left(2w_\psi^2\right), \tag{3}$$

with $r = \sqrt{\xi^2 + \eta^2}$ and w_ψ the waist size of the BEC in terms of the transverse scaling w_s. Here we choose a 50μm radius BEC, s.t. $w_\psi = 5$.

Our optical pump takes the form of a Laguerre-Gaussian (LG) mode at the beam waist [2, 10]:

$$P = LG_0^m(r, \varphi) = r^{|m|} e^{-r^2/\left(2w_F^2\right)} e^{im\varphi}, \tag{4}$$

where m is an integer that represents the OAM-index of the beam and w_F is the optical beam waist in terms of the transverse scaling w_s. We initially select an optical pump with $m = 2$ and a beam waist of 40μm, such that $w_F = 4$, to ensure a maximal overlap between the pump field and the transverse domain of the BEC. The initial optical field, F, is comprised of noise at the 1% level of the maximal pump amplitude.

For both initial BEC and optical pump fields, we rescale the profiles to maximal amplitudes A_ψ and A_P, respectively. We select $A_\psi = 0.1, A_P = 4$, ensuring that the optical field dominates the dipole forces between fields. Finally, we consider a weakly repulsive interactions of $a_s \approx 12a_0$, with a_0 the Bohr radius, giving a value of $\beta_{col} \approx 2$, select a three body loss value $L_3 \approx 10^{-4}$, in accordance with prior estimates for Caesium [5], and consider no cavity detuning, $\theta = 0$.

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