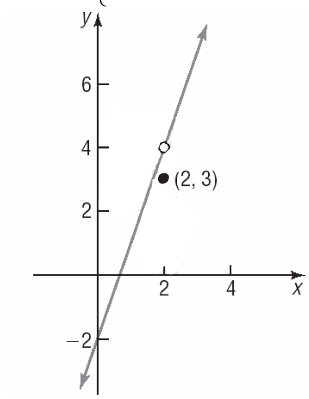
Chapter 14
A Preview of Calculus:
The Limit, Derivative, and Integral of a Function

Section 14.1

1. $f(x)=\begin{cases}3x-2 & \text{if } x\neq 2\\ 3 & \text{if } x=2\end{cases}$



3. $\lim\limits_{x\to c} f(x)$

5. True

7. $\lim\limits_{x\to 2}\left(4x^3\right)$

X	Y1
1.99	31.522
1.999	31.952
1.9999	31.995
2.0001	32.005
2.001	32.048
2.01	32.482

Y1=4X^3

$\lim\limits_{x\to 2}\left(4x^3\right)=32$

9. $\lim\limits_{x\to 0}\left(\dfrac{x+1}{x^2+1}\right)$

X	Y1
-.01	.9899
-.001	.999
-1E-4	.9999
1E-4	1.0001
.001	1.001
.01	1.0099

Y1=(X+1)/(X²+1)

$\lim\limits_{x\to 0}\left(\dfrac{x+1}{x^2+1}\right)=1$

11. $\lim\limits_{x\to 4}\left(\dfrac{x^2-4x}{x-4}\right)$

X	Y1
3.99	3.99
3.999	3.999
3.9999	3.9999
4.0001	4.0001
4.001	4.001
4.01	4.01

Y1=(X²-4X)/(X-4)

$\lim\limits_{x\to 4}\left(\dfrac{x^2-4x}{x-4}\right)=4$

13. $\lim\limits_{x\to 0}\left(e^x+1\right)$

X	Y1
-.01	1.99
-.001	1.999
-1E-4	1.9999
1E-4	2.0001
.001	2.001
.01	2.0101

Y1=e^(X)+1

$\lim\limits_{x\to 0}\left(e^x+1\right)=2$

15. $\lim\limits_{x\to 0}\left(\dfrac{\cos x-1}{x}\right)$

X	Y1
-.01	.005
-.001	5E-4
-1E-4	5E-5
1E-4	-5E-5
.001	-5E-4
.01	-.005

Y1=(cos(X)-1)/X

$\lim\limits_{x\to 0}\left(\dfrac{\cos x-1}{x}\right)=0$

17. $\lim\limits_{x\to 2} f(x)=3$

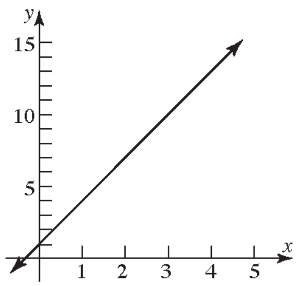
The value of the function gets close to 3 as x gets close to 2.

19. $\lim\limits_{x\to 2} f(x)=4$

The value of the function gets close to 4 as x gets close to 2.

21. $\lim\limits_{x\to 3} f(x)$ does not exist because as x gets close to 3, but is less than 3, $f(x)$ gets close to 3. However, as x gets close to 3, but is greater than 3, $f(x)$ gets close to 6.

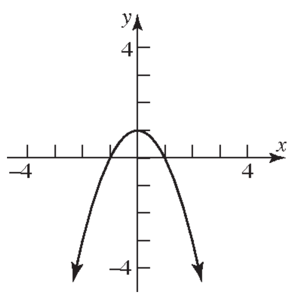
23. $f(x) = 3x + 1$



$\lim_{x \to 4} (3x + 1) = 13$

The value of the function gets close to 13 as x gets close to 4.

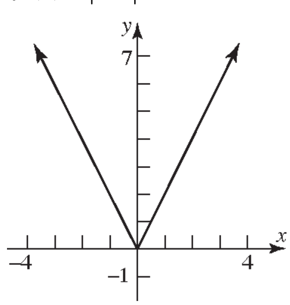
25. $f(x) = 1 - x^2$



$\lim_{x \to 2} \left(1 - x^2\right) = -3$

The value of the function gets close to -3 as x gets close to 2.

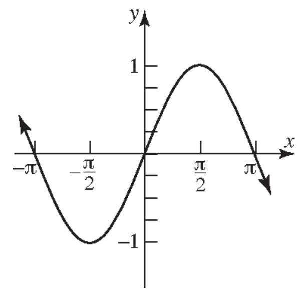
27. $f(x) = |2x|$



$\lim_{x \to -3} |2x| = 6$

The value of the function gets close to 6 as x gets close to -3.

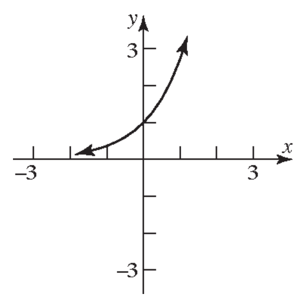
29. $f(x) = \sin x$



$\lim_{x \to \frac{\pi}{2}} (\sin x) = 1$

The value of the function gets close to 1 as x gets close to $\frac{\pi}{2}$.

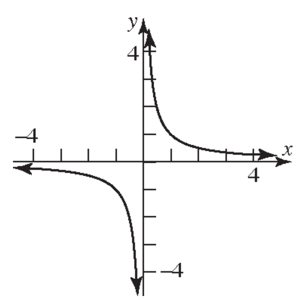
31. $f(x) = e^x$



$\lim_{x \to 0} \left(e^x\right) = 1$

The value of the function gets close to 1 as x gets close to 0.

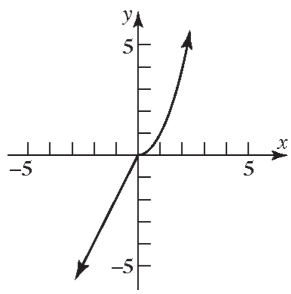
33. $f(x) = \dfrac{1}{x}$



$\lim_{x \to -1} \left(\frac{1}{x}\right) = -1$

The value of the function gets close to -1 as x gets close to -1.

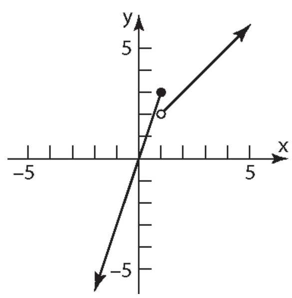
35. $f(x)=\begin{cases} x^2 & x\ge 0 \\ 2x & x<0 \end{cases}$



$\lim_{x\to 0} f(x)=0$

The value of the function gets close to 0 as x gets close to 0.

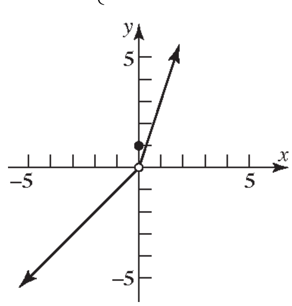
37. $f(x)=\begin{cases} 3x & x\le 1 \\ x+1 & x>1 \end{cases}$



$\lim_{x\to 1} f(x)$ does not exist

The value of the function does not approach a single value as x approaches 1. For $x<1$, the function approaches the value 3, while for $x>1$ the function approaches the value 2.

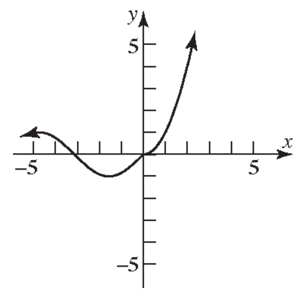
39. $f(x)=\begin{cases} x & x<0 \\ 1 & x=0 \\ 3x & x>0 \end{cases}$



$\lim_{x\to 0} f(x)=0$

The value of the function gets close to 0 as x gets close to 0.

41. $f(x)=\begin{cases} \sin x & x\le 0 \\ x^2 & x>0 \end{cases}$



$\lim_{x\to 0} f(x)=0$

The value of the function gets close to 0 as x gets close to 0.

43. $\lim_{x\to 1}\left(\dfrac{x^3-x^2+x-1}{x^4-x^3+2x-2}\right)$

X	Y1
.99	.66663
.999	.66667
.9999	.66667
1	ERROR
1.0001	.66667
1.001	.66667
1.01	.66663

Y1=(X^3−X²+X−1)…

$\lim_{x\to 1}\left(\dfrac{x^3-x^2+x-1}{x^4-x^3+2x-2}\right)\approx 0.67$

45. $\lim_{x\to 2}\left(\dfrac{x^3-2x^2+4x-8}{x^2+x-6}\right)$

X	Y1
1.99	1.5952
1.999	1.5995
1.9999	1.6
2	ERROR
2.0001	1.6
2.001	1.6005
2.01	1.6048

Y1=(X^3−2X²+4X−…

$\lim_{x\to 2}\left(\dfrac{x^3-2x^2+4x-8}{x^2+x-6}\right)=1.60$

47. $\lim_{x\to -1}\left(\dfrac{x^3+2x^2+x}{x^4+x^3+2x+2}\right)$

X	Y1
-1.01	.01042
-1.001	.001
-1	ERROR
-.9999	-1E-4
-.999	-1E-3
-.99	-.0096

Y1=(X^3+2X²+X)/…

$\lim_{x\to -1}\left(\dfrac{x^3+2x^2+x}{x^4+x^3+2x+2}\right)=0.00$

49. $d(P_1,P_2)=\sqrt{(6-2)^2+\left(-11-(-3)\right)^2}$
$=\sqrt{4^2+(-8)^2}$
$=\sqrt{16+64}=\sqrt{80}=4\sqrt{5}$

The coordinates of the midpoint are:
$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$
$=\left(\frac{6+2}{2},\frac{-11+(-3)}{2}\right)$
$=\left(\frac{8}{2},\frac{-14}{2}\right)=(4,-7)$

51. $A=Pe^{rt}$
$P=4000, r=0.06, t=10$
$A=4000e^{(0.06)(10)}$
$=\$7288.48$

Section 14.2

1. $\frac{(x)^2-(2)^2}{x-2}=\frac{x^2-4}{x-2}=\frac{(x-2)(x+2)}{x-2}$

3. product

5. c

7. False; the function may not be defined at 5.

9. $\lim\limits_{x\to 1}(5)=5$

11. $\lim\limits_{x\to 4}(x)=4$

13. $\lim\limits_{x\to -2}(5x)=5(-2)=-10$

15. $\lim\limits_{x\to 2}(3x+2)=3(2)+2=8$

17. $\lim\limits_{x\to -1}(3x^2-5x)=3(-1)^2-5(-1)=8$

19. $\lim\limits_{x\to 1}(5x^4-3x^2+6x-9)=5(1)^4-3(1)^2+6(1)-9$
$=5-3+6-9$
$=-1$

21. $\lim\limits_{x\to 1}(x^2+1)^3=\left(\lim\limits_{x\to 1}(x^2+1)\right)^3=\left(1^2+1\right)^3=2^3=8$

23. $\lim\limits_{x\to 1}\sqrt{5x+4}=\sqrt{\lim\limits_{x\to 1}(5x+4)}$
$=\sqrt{5(1)+4}$
$=\sqrt{9}=3$

25. $\lim\limits_{x\to 0}\left(\frac{x^2-4}{x^2+4}\right)=\frac{\lim\limits_{x\to 0}(x^2-4)}{\lim\limits_{x\to 0}(x^2+4)}=\frac{0^2-4}{0^2+4}=\frac{-4}{4}=-1$

27. $\lim\limits_{x\to 2}(3x-2)^{5/2}=\left(\lim\limits_{x\to 2}(3x-2)\right)^{5/2}$
$=\left(3(2)-2\right)^{5/2}$
$=4^{5/2}=32$

29. $\lim\limits_{x\to 2}\left(\frac{x^2-4}{x^2-2x}\right)=\lim\limits_{x\to 2}\left(\frac{(x-2)(x+2)}{x(x-2)}\right)$
$=\lim\limits_{x\to 2}\left(\frac{x+2}{x}\right)$
$=\frac{2+2}{2}=\frac{4}{2}=2$

31. $\lim\limits_{x\to -3}\left(\frac{x^2-x-12}{x^2-9}\right)=\lim\limits_{x\to -3}\left(\frac{(x-4)(x+3)}{(x-3)(x+3)}\right)$
$=\lim\limits_{x\to -3}\left(\frac{x-4}{x-3}\right)$
$=\frac{-3-4}{-3-3}=\frac{-7}{-6}=\frac{7}{6}$

33. $\lim\limits_{x\to 1}\left(\frac{x^3-1}{x-1}\right)=\lim\limits_{x\to 1}\left(\frac{(x-1)(x^2+x+1)}{x-1}\right)$
$=\lim\limits_{x\to 1}\left(x^2+x+1\right)$
$=1^2+1+1=3$

35. $\lim\limits_{x\to -1}\left(\frac{(x+1)^2}{x^2-1}\right)=\lim\limits_{x\to -1}\left(\frac{(x+1)^2}{(x-1)(x+1)}\right)$
$=\lim\limits_{x\to -1}\left(\frac{x+1}{x-1}\right)$
$=\frac{-1+1}{-1-1}=\frac{0}{-2}=0$

37. $\lim_{x\to 2}\left(\frac{x^3-2x^2+4x-8}{x^2+x-6}\right)$

$=\lim_{x\to 2}\left(\frac{x^2(x-2)+4(x-2)}{(x+3)(x-2)}\right)$

$=\lim_{x\to 2}\left(\frac{(x-2)(x^2+4)}{(x+3)(x-2)}\right)=\lim_{x\to 2}\left(\frac{x^2+4}{x+3}\right)$

$=\frac{2^2+4}{2+3}=\frac{8}{5}$

39. $\lim_{x\to -1}\left(\frac{x^3+2x^2+x}{x^4+x^3+2x+2}\right)$

$=\lim_{x\to -1}\left(\frac{x(x^2+2x+1)}{x^3(x+1)+2(x+1)}\right)$

$=\lim_{x\to -1}\left(\frac{x(x+1)^2}{(x+1)(x^3+2)}\right)$

$=\lim_{x\to -1}\left(\frac{x(x+1)}{x^3+2}\right)=\frac{-1(-1+1)}{(-1)^3+2}$

$=\frac{-1(0)}{-1+2}=\frac{0}{1}=0$

41. $\lim_{x\to 2}\left(\frac{\sqrt{x}-\sqrt{2}}{x-2}\right)=\lim_{x\to 2}\left(\frac{\sqrt{x}-\sqrt{2}}{x-2}\cdot\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right)$

$=\lim_{x\to 2}\left(\frac{x-2}{(x-2)(\sqrt{x}+\sqrt{2})}\right)$

$=\lim_{x\to 2}\left(\frac{1}{\sqrt{x}+\sqrt{2}}\right)=\left(\frac{1}{\sqrt{2}+\sqrt{2}}\right)$

$=\frac{1}{2\sqrt{2}}=\frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{4}$

43. $\lim_{x\to 2}\left(\frac{f(x)-f(2)}{x-2}\right)=\lim_{x\to 2}\left(\frac{(5x-3)-7}{x-2}\right)$

$=\lim_{x\to 2}\left(\frac{5x-10}{x-2}\right)$

$=\lim_{x\to 2}\left(\frac{5(x-2)}{x-2}\right)$

$=\lim_{x\to 2}(5)=5$

45. $\lim_{x\to 3}\left(\frac{f(x)-f(3)}{x-3}\right)=\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$

$=\lim_{x\to 3}\left(\frac{(x-3)(x+3)}{x-3}\right)$

$=\lim_{x\to 3}(x+3)$

$=3+3=6$

47. $\lim_{x\to -1}\left(\frac{f(x)-f(-1)}{x-(-1)}\right)=\lim_{x\to -1}\left(\frac{x^2+2x-(-1)}{x+1}\right)$

$=\lim_{x\to -1}\left(\frac{x^2+2x+1}{x+1}\right)$

$=\lim_{x\to -1}\left(\frac{(x+1)^2}{x+1}\right)$

$=\lim_{x\to -1}(x+1)$

$=-1+1=0$

49. $\lim_{x\to 0}\left(\frac{f(x)-f(0)}{x-0}\right)=\lim_{x\to 0}\left(\frac{3x^3-2x^2+4-4}{x}\right)$

$=\lim_{x\to 0}\left(\frac{3x^3-2x^2}{x}\right)$

$=\lim_{x\to 0}(3x^2-2x)=0$

51. $\lim_{x\to 1}\left(\frac{f(x)-f(1)}{x-1}\right)=\lim_{x\to 1}\left(\frac{\frac{1}{x}-1}{x-1}\right)=\lim_{x\to 1}\left(\frac{\frac{1-x}{x}}{x-1}\right)$

$=\lim_{x\to 1}\left(\frac{-1(x-1)}{x(x-1)}\right)=\lim_{x\to 1}\left(\frac{-1}{x}\right)$

$=\frac{-1}{1}=-1$

53. $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=\lim_{x\to 0}\left(\frac{\frac{\sin x}{\cos x}}{x}\right)=\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)$

$=\left(\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\right)\cdot\left(\lim_{x\to 0}\left(\frac{1}{\cos x}\right)\right)$

$=1\cdot\left(\frac{\lim_{x\to 0}1}{\lim_{x\to 0}(\cos x)}\right)=1\cdot\frac{1}{1}=1$

55. $\lim_{x\to 0}\left(\frac{3\sin x+\cos x-1}{4x}\right)$

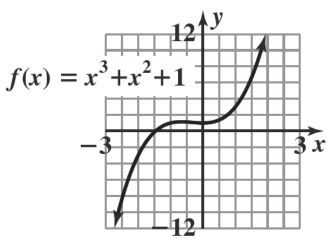
$=\lim_{x\to 0}\left(\frac{3\sin x}{4x}+\frac{\cos x-1}{4x}\right)$

$=\lim_{x\to 0}\left(\frac{3\sin x}{4x}\right)+\lim_{x\to 0}\left(\frac{\cos x-1}{4x}\right)$

$=\frac{3}{4}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)+\frac{1}{4}\lim_{x\to 0}\left(\frac{\cos x-1}{x}\right)$

$=\frac{3}{4}\cdot 1+\frac{1}{4}\cdot 0=\frac{3}{4}$

57.



59. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

We are finding the angle θ, $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$, whose sine equals $\frac{\sqrt{3}}{2}$.

$$\sin\theta=\frac{\sqrt{3}}{2},\qquad -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$$

$$\theta=\frac{\pi}{3}$$

$$\arcsin\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$$

Section 14.3

1. $f(0)=0^2=0$, $f(2)=5-2=3$

3. True

5. True

7. one-sided

9. continuous, c

11. True

13. Domain: $[-8,-6)\cup(-6,4)\cup(4,6]$ or $\{x\mid -8\le x<-6 \text{ or } -6<x<4 \text{ or } 4<x\le 6\}$

15. x-intercepts: –8, –5, –3

17. $f(-8)=0$; $f(-4)=2$

19. $\lim_{x\to -6^-} f(x)=\infty$

21. $\lim_{x\to -4^-} f(x)=2$

23. $\lim_{x\to 2^-} f(x)=1$

25. $\lim_{x\to 4} f(x)$ does exist.
$\lim_{x\to 4} f(x)=0$ since $\lim_{x\to 4^-} f(x)=\lim_{x\to 4^+} f(x)=0$

27. f is not continuous at -6 because $f(-6)$ does not exist, nor does the $\lim_{x\to -6} f(x)$.

29. f is continuous at 0 because
$f(0)=\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=3$

31. f is not continuous at 4 because $f(4)$ does not exist.

33. $\lim_{x\to 1^+}(2x+3)=2(1)+3=5$

35. $\lim_{x\to 1^-}\left(2x^3+5x\right)=2(1)^3+5(1)=2+5=7$

37. $\lim_{x\to \frac{\pi}{2}^+}\left(\sin x\right)=\sin\frac{\pi}{2}=1$

39. $\lim_{x\to 2^+}\left(\frac{x^2-4}{x-2}\right)=\lim_{x\to 2^+}\left(\frac{(x+2)(x-2)}{x-2}\right)$

$=\lim_{x\to 2^+}(x+2)$

$=2+2=4$

41. $$\lim_{x\to -1^-}\left(\frac{x^2-1}{x^3+1}\right)=\lim_{x\to -1^-}\left(\frac{(x+1)(x-1)}{(x+1)(x^2-x+1)}\right)$$
$$=\lim_{x\to -1^-}\left(\frac{x-1}{x^2-x+1}\right)$$
$$=\frac{-1-1}{(-1)^2-(-1)+1}=-\frac{2}{3}$$

43. $$\lim_{x\to -2^+}\left(\frac{x^2+x-2}{x^2+2x}\right)=\lim_{x\to -2^+}\left(\frac{(x+2)(x-1)}{x(x+2)}\right)$$
$$=\lim_{x\to -2^+}\left(\frac{x-1}{x}\right)$$
$$=\frac{-2-1}{-2}=\frac{-3}{-2}=\frac{3}{2}$$

45. $f(x)=x^3-3x^2+2x-6$; $c=2$

1. $f(2)=2^3-3\cdot 2^2+2\cdot 2-6=-6$
2. $\lim\limits_{x\to 2^-} f(x)=2^3-3\cdot 2^2+2\cdot 2-6=-6$
3. $\lim\limits_{x\to 2^+} f(x)=2^3-3\cdot 2^2+2\cdot 2-6=-6$

Thus, $f(x)$ is continuous at $c=2$.

47. $f(x)=\dfrac{x^2+5}{x-6}$; $c=3$

1. $f(3)=\dfrac{3^2+5}{3-6}=\dfrac{14}{-3}=-\dfrac{14}{3}$
2. $\lim\limits_{x\to 3^-} f(x)=\dfrac{3^2+5}{3-6}=\dfrac{14}{-3}=-\dfrac{14}{3}$
3. $\lim\limits_{x\to 3^+} f(x)=\dfrac{3^2+5}{3-6}=\dfrac{14}{-3}=-\dfrac{14}{3}$

Thus, $f(x)$ is continuous at $c=3$.

49. $f(x)=\dfrac{x+3}{x-3}$; $c=3$

Since $f(x)$ is not defined at $c=3$, the function is not continuous at $c=3$.

51. $f(x)=\dfrac{x^3+3x}{x^2-3x}$; $c=0$

Since $f(x)$ is not defined at $c=0$, the function is not continuous at $c=0$.

53. $f(x)=\begin{cases}\dfrac{x^3+3x}{x^2-3x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$; $c=0$

1. $f(0)=1$
2. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\left(\dfrac{x^3+3x}{x^2-3x}\right)$

$$=\lim_{x\to 0^-}\left(\frac{x(x^2+3)}{x(x-3)}\right)$$
$$=\lim_{x\to 0^-}\left(\frac{x^2+3}{x-3}\right)=\frac{3}{-3}=-1$$

Since $\lim\limits_{x\to 0^-} f(x)\neq f(0)$, the function is not continuous at $c=0$.

55. $f(x)=\begin{cases}\dfrac{x^3+3x}{x^2-3x} & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$; $c=0$

1. $f(0)=-1$
2. $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\left(\dfrac{x^3+3x}{x^2-3x}\right)$

$$=\lim_{x\to 0^-}\left(\frac{x(x^2+3)}{x(x-3)}\right)$$
$$=\lim_{x\to 0^-}\left(\frac{x^2+3}{x-3}\right)=\frac{3}{-3}=-1$$

3. $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\left(\dfrac{x^3+3x}{x^2-3x}\right)$

$$=\lim_{x\to 0^+}\left(\frac{x(x^2+3)}{x(x-3)}\right)$$
$$=\lim_{x\to 0^+}\left(\frac{x^2+3}{x-3}\right)=\frac{3}{-3}=-1$$

The function is continuous at $c=0$.

57. $f(x)=\begin{cases}\dfrac{x^3-1}{x^2-1} & \text{if } x<1\\ 2 & \text{if } x=1\\ \dfrac{3}{x+1} & \text{if } x>1\end{cases}$; $c=1$

1. $f(1)=2$

2. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(\frac{x^3-1}{x^2-1}\right)$

$= \lim_{x\to 1^-}\left(\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}\right)$

$= \lim_{x\to 1^-}\left(\frac{x^2+x+1}{x+1}\right) = \frac{3}{2}$

Since $\lim_{x\to 1^-} f(x) \ne f(1)$, the function is not continuous at $c = 1$.

59. $f(x) = \begin{cases} 2e^x & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \dfrac{x^3+2x^2}{x^2} & \text{if } x > 0 \end{cases}$; $c = 0$

1. $f(0) = 2$
2. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(2e^x\right) = 2e^0 = 2\cdot 1 = 2$
3. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{x^3+2x^2}{x^2}\right)$

$= \lim_{x\to 0^+}\left(\frac{x^2(x+2)}{x^2}\right)$

$= \lim_{x\to 0^+}(x+2) = 0+2 = 2$

The function is continuous at $c = 0$.

61. The domain of $f(x) = 2x+3$ is all real numbers, and $f(x)$ is a polynomial function. Therefore, $f(x)$ is continuous everywhere.

63. The domain of $f(x) = 3x^2 + x$ is all real numbers, and $f(x)$ is a polynomial function. Therefore, $f(x)$ is continuous everywhere.

65. The domain of $f(x) = 4\sin x$ is all real numbers, and trigonometric functions are continuous at every point in their domains. Therefore, $f(x)$ is continuous everywhere.

67. The domain of $f(x) = 2\tan x$ is all real numbers except odd integer multiples of $\frac{\pi}{2}$, and trigonometric functions are continuous at every point in their domains. Therefore, $f(x)$ is continuous everywhere except where $x = \frac{k\pi}{2}$ where k is an odd integer. $f(x)$ is discontinuous at $x = \frac{k\pi}{2}$ where k is an odd integer.

69. $f(x) = \frac{2x+5}{x^2-4} = \frac{2x+5}{(x-2)(x+2)}$. The domain of $f(x)$ is all real numbers except $x = 2$ and $x = -2$, and $f(x)$ is a rational function. Therefore, $f(x)$ is continuous everywhere except at $x = 2$ and $x = -2$. $f(x)$ is discontinuous at $x = 2$ and $x = -2$.

71. $f(x) = \frac{x-3}{\ln x}$. The domain of $f(x)$ is $(0,1)$ or $(1,\infty)$. Thus, $f(x)$ is continuous on the interval $(0,\infty)$ except at $x = 1$. $f(x)$ is discontinuous at $x = 1$.

73. $R(x) = \frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)}$. The domain of R is $\{x \mid x \ne -1, x \ne 1\}$. Thus R is discontinuous at both –1 and 1.

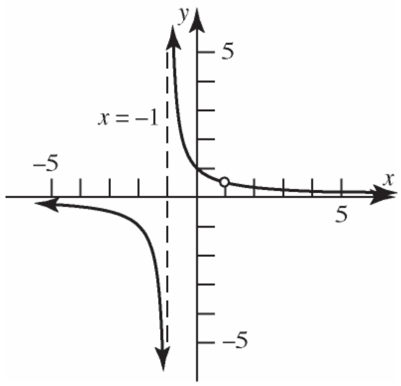
$\lim_{x\to -1^-} R(x) = \lim_{x\to -1^-}\left(\frac{x-1}{(x-1)(x+1)}\right)$

$= \lim_{x\to -1^-}\left(\frac{1}{x+1}\right) = -\infty$

since when $x < -1$, $\frac{1}{x+1} < 0$, and as x approaches -1, $\frac{1}{x+1}$ becomes unbounded.

$\lim_{x\to -1^+} R(x) = \lim_{x\to -1^+}\left(\frac{1}{x+1}\right) = \infty$ since when $x > -1$, $\frac{1}{x+1} > 0$, and as x approaches -1, $\frac{1}{x+1}$ becomes unbounded.

$\lim_{x\to 1} R(x) = \lim_{x\to 1}\left(\frac{1}{x+1}\right) = \frac{1}{2}$. Note there is a hole in the graph at $\left(1, \dfrac{1}{2}\right)$.



75. $R(x) = \dfrac{x^2 + x}{x^2 - 1} = \dfrac{x(x+1)}{(x-1)(x+1)}$. The domain of R is $\{x \mid x \neq -1, x \neq 1\}$. Thus R is discontinuous at both -1 and 1.

$\lim\limits_{x \to 1^-} R(x) = \lim\limits_{x \to 1^-} \left(\dfrac{x(x+1)}{(x-1)(x+1)} \right) = \lim\limits_{x \to 1^-} \left(\dfrac{x}{x-1} \right) = -\infty$

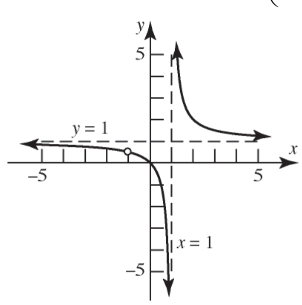
since when

$0 < x < 1, \dfrac{x}{x-1} < 0$, and as x approaches 1, $\dfrac{x}{x-1}$ becomes unbounded. $\lim\limits_{x \to 1^+} R(x) = \lim\limits_{x \to 1^+} \dfrac{x}{x-1} = \infty$

since when

$x > 1, \dfrac{x}{x-1} > 0$, and as x approaches 1, $\dfrac{x}{x-1}$ becomes unbounded.

$\lim\limits_{x \to -1} R(x) = \lim\limits_{x \to -1} \left(\dfrac{x}{x-1} \right) = \dfrac{-1}{-2} = \dfrac{1}{2}$. Note there is a hole in the graph at $\left(-1, \dfrac{1}{2}\right)$.



77. $R(x) = \dfrac{x^3 - x^2 + x - 1}{x^4 - x^3 + 2x - 2} = \dfrac{x^2(x-1) + 1(x-1)}{x^3(x-1) + 2(x-1)}$

$= \dfrac{(x-1)(x^2+1)}{(x-1)(x^3+2)} = \dfrac{x^2+1}{x^3+2}, x \neq 1$

There is a vertical asymptote where $x^3 + 2 = 0$. $x = -\sqrt[3]{2}$ is a vertical asymptote. There is a hole in the graph at $x = 1$ (at the point $\left(1, \frac{2}{3}\right)$).

79. $R(x) = \dfrac{x^3 - 2x^2 + 4x - 8}{x^2 + x - 6} = \dfrac{x^2(x-2) + 4(x-2)}{(x+3)(x-2)}$

$= \dfrac{(x-2)(x^2+4)}{(x+3)(x-2)} = \dfrac{x^2+4}{x+3}, x \neq 2$

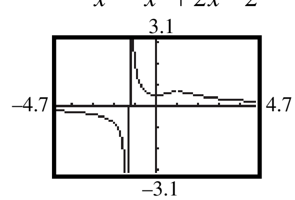
There is a vertical asymptote where $x + 3 = 0$. $x = -3$ is a vertical asymptote. There is a hole in the graph at $x = 2$ (at the point $\left(2, \frac{8}{5}\right)$).

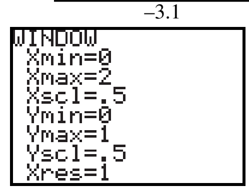
81. $R(x) = \dfrac{x^3 + 2x^2 + x}{x^4 + x^3 + 2x + 2} = \dfrac{x(x^2 + 2x + 1)}{x^3(x+1) + 2(x+1)}$

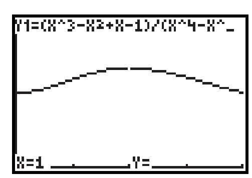
$= \dfrac{x(x+1)^2}{(x+1)(x^3+2)} = \dfrac{x(x+1)}{x^3+2}, x \neq -1$

There is a vertical asymptote where $x^3 + 2 = 0$. $x = -\sqrt[3]{2}$ is a vertical asymptote. There is a hole in the graph at $x = -1$ (at the point $(-1, 0)$).

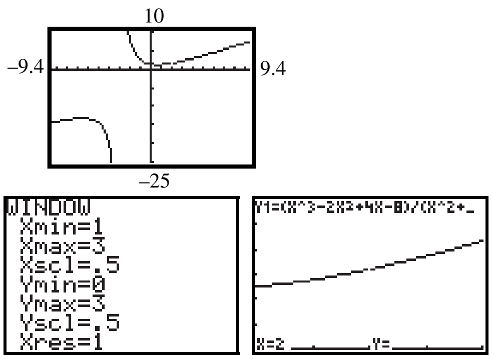
83. $R(x) = \dfrac{x^3 - x^2 + x - 1}{x^4 - x^3 + 2x - 2}$



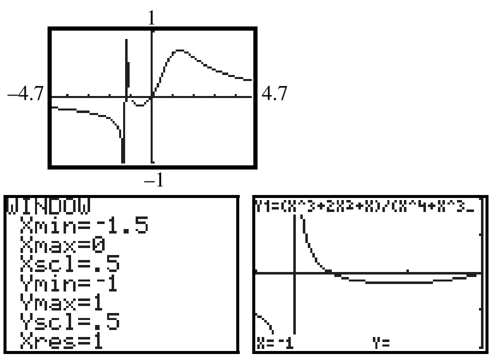




85. $R(x)=\dfrac{x^3-2x^2+4x-8}{x^2+x-6}$



87. $R(x)=\dfrac{x^3+2x^2+x}{x^4+x^3+2x+2}$



89. Answers will vary. Three possible functions are:
$f\left(x\right)=x^2;\; g\left(x\right)=\sin\left(x\right);\; h\left(x\right)=e^x$.

91. $f(x)=\dfrac{3x-4}{x-4}$ is in lowest terms so the vertical asymptote is the zero of the denominator. The vertical asymptote is: $x=4$.
Since the degree of the numerator and denominator are equal the horizontal asymptote is the quotient of the leading coefficients. The horizontal asymptote is: $y=3$.

93.
$$\begin{aligned}5\ln x+2\ln y-4\ln z &= \ln x^5+\ln y^2-\ln z^4\\ &= \ln(x^5y^2)-\ln z^4\\ &= \ln\left(\frac{x^5y^2}{z^4}\right)\end{aligned}$$

Section 14.4

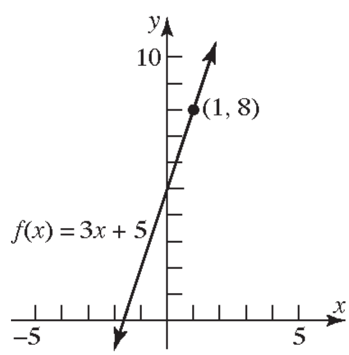
1. Slope 5; containing point $\left(2,-4\right)$
$$\begin{aligned}y-y_1&=m\left(x-x_1\right)\\ y-\left(-4\right)&=5\left(x-2\right)\\ y+4&=5x-10\\ y&=5x-14\end{aligned}$$

3. tangent line

5. velocity

7. True

9. $f(x)=3x+5$ at $(1,8)$
$$\begin{aligned}m_{\tan}&=\lim_{x\to1}\left(\frac{f(x)-f(1)}{x-1}\right)=\lim_{x\to1}\left(\frac{3x+5-8}{x-1}\right)\\ &=\lim_{x\to1}\left(\frac{3x-3}{x-1}\right)=\lim_{x\to1}\left(\frac{3(x-1)}{x-1}\right)\\ &=\lim_{x\to1}\left(3\right)=3\end{aligned}$$
Tangent Line: $y-8=3(x-1)$
$$\begin{aligned}y-8&=3x-3\\ y&=3x+5\end{aligned}$$

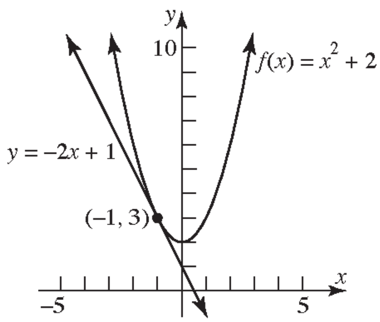


11. $f(x)=x^2+2$ at $(-1,3)$
$$\begin{aligned}m_{\tan}&=\lim_{x\to-1}\left(\frac{f(x)-f(-1)}{x+1}\right)=\lim_{x\to-1}\left(\frac{x^2+2-3}{x+1}\right)\\ &=\lim_{x\to-1}\left(\frac{x^2-1}{x+1}\right)=\lim_{x\to-1}\left(\frac{(x+1)(x-1)}{x+1}\right)\\ &=\lim_{x\to-1}(x-1)=-1-1=-2\end{aligned}$$

Tangent Line: $y-3=-2(x-(-1))$

$$y-3=-2x-2$$

$$y=-2x+1$$



13. $f(x)=3x^2$ at $(2,12)$

$$m_{\tan}=\lim_{x\to 2}\left(\frac{f(x)-f(2)}{x-2}\right)=\lim_{x\to 2}\left(\frac{3x^2-12}{x-2}\right)$$

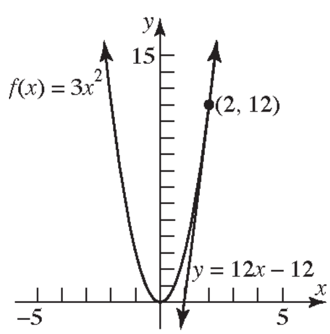
$$=\lim_{x\to 2}\frac{3\left(x^2-4\right)}{x-2}=\lim_{x\to 2}\left(\frac{3(x+2)(x-2)}{x-2}\right)$$

$$=\lim_{x\to 2}\left(3(x+2)\right)=3(2+2)=12$$

Tangent Line: $y-12=12(x-2)$

$$y-12=12x-24$$

$$y=12x-12$$



15. $f(x)=2x^2+x$ at $(1,3)$

$$m_{\tan}=\lim_{x\to 1}\left(\frac{f(x)-f(1)}{x-1}\right)=\lim_{x\to 1}\left(\frac{2x^2+x-3}{x-1}\right)$$

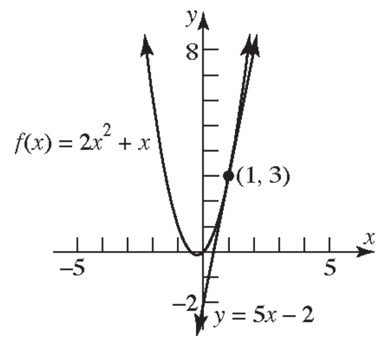
$$=\lim_{x\to 1}\left(\frac{(2x+3)(x-1)}{x-1}\right)=\lim_{x\to 1}(2x+3)$$

$$=2(1)+3=5$$

Tangent Line: $y-3=5(x-1)$

$$y-3=5x-5$$

$$y=5x-2$$



17. $f(x)=x^2-2x+3$ at $(-1,6)$

$$m_{\tan}=\lim_{x\to -1}\frac{f(x)-f(-1)}{x+1}=\lim_{x\to -1}\frac{x^2-2x+3-6}{x+1}$$

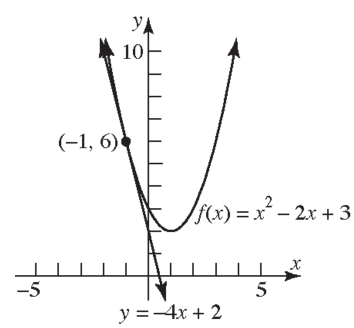
$$=\lim_{x\to -1}\frac{x^2-2x-3}{x+1}=\lim_{x\to -1}\frac{(x+1)(x-3)}{x+1}$$

$$=\lim_{x\to -1}(x-3)=-1-3=-4$$

Tangent Line: $y-6=-4(x-(-1))$

$$y-6=-4x-4$$

$$y=-4x+2$$



19. $f(x)=x^3+x$ at $(2,10)$

$$m_{\tan}=\lim_{x\to 2}\left(\frac{f(x)-f(2)}{x-2}\right)=\lim_{x\to 2}\left(\frac{x^3+x-10}{x-2}\right)$$

$$=\lim_{x\to 2}\left(\frac{x^3-8+x-2}{x-2}\right)$$

$$=\lim_{x\to 2}\left(\frac{(x-2)(x^2+2x+4)+\left(x-2\right)\cdot 1}{x-2}\right)$$

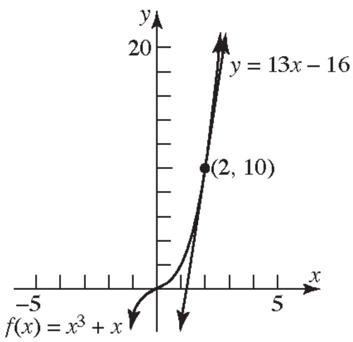
$$=\lim_{x\to 2}\left(\frac{(x-2)(x^2+2x+4+1)}{x-2}\right)$$

$$=\lim_{x\to 2}(x^2+2x+5)=4+4+5=13$$

Tangent Line: $y-10=13(x-2)$

$y-10=13x-26$

$y=13x-16$



21. $f(x)=-4x+5$ at 3

$$\begin{aligned}f'(3)&=\lim_{x\to 3}\left(\frac{f(x)-f(3)}{x-3}\right)\\&=\lim_{x\to 3}\left(\frac{-4x+5-(-7)}{x-3}\right)\\&=\lim_{x\to 3}\left(\frac{-4x+12}{x-3}\right)\\&=\lim_{x\to 3}\left(\frac{-4(x-3)}{x-3}\right)\\&=\lim_{x\to 3}(-4)=-4\end{aligned}$$

23. $f(x)=x^2-3$ at 0

$$\begin{aligned}f'(0)&=\lim_{x\to 0}\left(\frac{f(x)-f(0)}{x-0}\right)\\&=\lim_{x\to 0}\left(\frac{x^2-3-(-3)}{x}\right)\\&=\lim_{x\to 0}\left(\frac{x^2}{x}\right)\\&=\lim_{x\to 0}(x)=0\end{aligned}$$

25. $f(x)=2x^2+3x$ at 1

$$\begin{aligned}f'(1)&=\lim_{x\to 1}\left(\frac{f(x)-f(1)}{x-1}\right)\\&=\lim_{x\to 1}\left(\frac{2x^2+3x-5}{x-1}\right)\\&=\lim_{x\to 1}\left(\frac{(2x+5)(x-1)}{x-1}\right)\\&=\lim_{x\to 1}(2x+5)=7\end{aligned}$$

27. $f(x)=x^3+4x$ at -1

$$\begin{aligned}f'(-1)&=\lim_{x\to -1}\left(\frac{f(x)-f(-1)}{x-(-1)}\right)\\&=\lim_{x\to -1}\left(\frac{x^3+4x-(-5)}{x+1}\right)\\&=\lim_{x\to -1}\left(\frac{x^3+1+4x+4}{x+1}\right)\\&=\lim_{x\to -1}\left(\frac{(x+1)(x^2-x+1)+4(x+1)}{x+1}\right)\\&=\lim_{x\to -1}\left(\frac{(x+1)(x^2-x+1+4)}{x+1}\right)\\&=\lim_{x\to -1}(x^2-x+5)\\&=(-1)^2-(-1)+5=7\end{aligned}$$

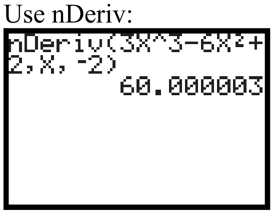
29. $f(x)=x^3+x^2-2x$ at 1

$$\begin{aligned}f'(1)&=\lim_{x\to 1}\left(\frac{f(x)-f(1)}{x-1}\right)\\&=\lim_{x\to 1}\left(\frac{x^3+x^2-2x-0}{x-1}\right)\\&=\lim_{x\to 1}\left(\frac{x(x^2+x-2)}{x-1}\right)\\&=\lim_{x\to 1}\left(\frac{x(x+2)(x-1)}{x-1}\right)\\&=\lim_{x\to 1}\left(x(x+2)\right)\\&=1(1+2)=3\end{aligned}$$

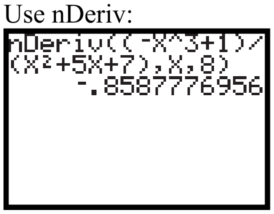
31. $f(x)=\sin x$ at 0

$$f'(0)=\lim_{x\to 0}\left(\frac{f(x)-f(0)}{x-0}\right)=\lim_{x\to 0}\left(\frac{\sin x-0}{x-0}\right)=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$$

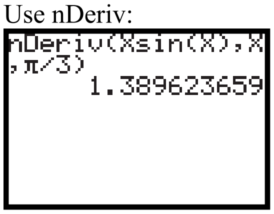
33. Use nDeriv:



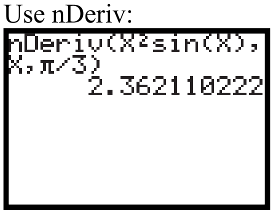
35. Use nDeriv:



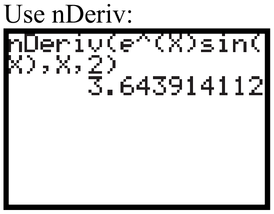
37. Use nDeriv:



39. Use nDeriv:



41. Use nDeriv:



43. $V(r)=3\pi r^2$ at $r=3$

$$V'(3)=\lim_{r\to 3}\left(\frac{V(r)-V(3)}{r-3}\right)=\lim_{r\to 3}\left(\frac{3\pi r^2-27\pi}{r-3}\right)=\lim_{r\to 3}\left(\frac{3\pi(r^2-9)}{r-3}\right)=\lim_{r\to 3}\left(\frac{3\pi(r-3)(r+3)}{r-3}\right)=\lim_{r\to 3}\left(3\pi(r+3)\right)=3\pi(3+3)=18\pi$$

At the instant $r=3$ feet, the volume of the cylinder is increasing at a rate of 18π cubic feet per foot.

45. $V(r)=\frac{4}{3}\pi r^3$ at $r=2$

$$V'(2)=\lim_{r\to 2}\left(\frac{V(r)-V(2)}{r-2}\right)=\lim_{r\to 2}\left(\frac{\frac{4}{3}\pi r^3-\frac{32}{3}\pi}{r-2}\right)=\lim_{r\to 2}\left(\frac{\left(\frac{4}{3}\pi(r^3-8)\right)}{r-2}\right)=\lim_{r\to 2}\left(\frac{\left(\frac{4}{3}\pi(r-2)(r^2+2r+4)\right)}{r-2}\right)=\lim_{r\to 2}\left(\frac{4}{3}\pi(r^2+2r+4)\right)=\frac{4}{3}\pi(4+4+4)=16\pi$$

At the instant $r=2$ feet, the volume of the sphere is increasing at a rate of 16π cubic feet per foot.

47. a. $-16t^2 + 96t = 0$
$-16t(t-6) = 0$
$t = 0 \text{ or } t = 6$
The ball strikes the ground after 6 seconds.

b. $$\frac{\Delta s}{\Delta t} = \frac{s(2)-s(0)}{2-0} = \frac{-16(2)^2 + 96(2) - 0}{2} = \frac{128}{2} = 64 \text{ feet/sec}$$

c.
$$\begin{aligned}
s'(t_0) &= \lim_{t\to t_0}\left(\frac{s(t)-s(t_0)}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(\frac{-16t^2+96t-\left(-16t_0{}^2+96t_0\right)}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(\frac{-16t^2+16t_0{}^2+96t-96t_0}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(\frac{-16\left(t^2-t_0{}^2\right)+96\left(t-t_0\right)}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(\frac{-16\left(t-t_0\right)\left(t+t_0\right)+96\left(t-t_0\right)}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(\frac{\left(t-t_0\right)\left(-16\left(t+t_0\right)+96\right)}{t-t_0}\right)\\
&= \lim_{t\to t_0}\left(-16\left(t+t_0\right)+96\right)\\
&= \left(-16\left(t_0+t_0\right)+96\right)\\
&= -32t_0+96 \quad \text{ft/sec}
\end{aligned}$$
The instantaneous speed at time t is $-32t+96$ feet per second.

d. $s'(2) = -32(2) + 96 = -64 + 96 = 32$ feet/sec

e. $s'(t) = 0$
$-32t + 96 = 0$
$-32t = -96$
$t = 3$ seconds

f. $$\begin{aligned} s(3) &= -16(3)^2 + 96(3)\\ &= -144 + 288\\ &= 144 \text{ feet}\end{aligned}$$

g. $$\begin{aligned} s'(6) &= -32(6) + 96\\ &= -192 + 96\\ &= -96 \text{ feet/sec}\end{aligned}$$

49. a. $$\frac{\Delta s}{\Delta t} = \frac{s(4)-s(1)}{4-1} = \frac{917-987}{3} = \frac{-70}{3} = -23\frac{1}{3} \text{ feet/sec}$$

b. $$\frac{\Delta s}{\Delta t} = \frac{s(3)-s(1)}{3-1} = \frac{945-987}{2} = \frac{-42}{2} = -21 \text{ feet/sec}$$

c. $$\frac{\Delta s}{\Delta t} = \frac{s(2)-s(1)}{2-1} = \frac{969-987}{1} = \frac{-18}{1} = -18 \text{ feet/sec}$$

d. $s(t) = -2.631t^2 - 10.269t + 999.933$

e.
$$\begin{aligned}
s'(1) &= \lim_{t\to 1}\left(\frac{s(t)-s(1)}{t-1}\right)\\
&= \lim_{t\to 1}\left(\frac{-2.631t^2-10.269t+12.9}{t-1}\right)\\
&= \lim_{t\to 1}\left(\frac{-2.631t^2+2.631t-12.9t+12.9}{t-1}\right)\\
&= \lim_{t\to 1}\left(\frac{-2.631t(t-1)-12.9(t-1)}{t-1}\right)\\
&= \lim_{t\to 1}\left(\frac{(-2.631t-12.9)(t-1)}{t-1}\right)\\
&= \lim_{t\to 1}(-2.631t-12.9)\\
&= -2.631(1)-12.9 = -15.531 \text{ feet/sec}
\end{aligned}$$
At the instant when $t = 1$, the instantaneous speed of the ball is –15.531 feet / sec.

51. Complete the square to put in standard form:

$$x^2 - 2x - 2y + 7 = 0$$
$$x^2 - 2x + 1 = 2y - 7 + 1$$
$$(x-1)^2 = 2y - 6$$
$$(x-1)^2 = 2(y-3)$$

The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 2$ or $a = \frac{1}{2}$, $h = 1$, and $k = 3$. Thus, we have:

Vertex: (1, 3); Focus: $\left(1, \frac{7}{2}\right)$

Directrix: $y = 3$

53. $${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot2\cdot1}$$
$$= \frac{5\cdot4}{2} = 10$$

Section 14.5

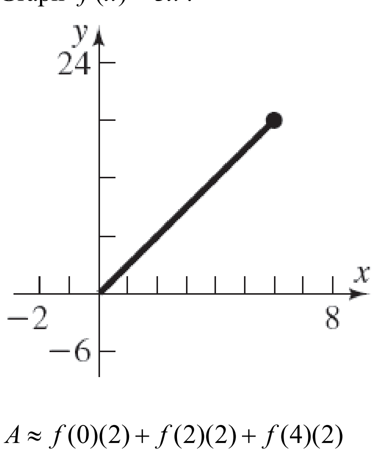
1. $A = lw$

3. $\int_a^b f(x)\,dx$

5. $A \approx f(1)\cdot1 + f(2)\cdot1 = 1\cdot1 + 2\cdot1 = 1 + 2 = 3$

7. $$A \approx f(0)\cdot2 + f(2)\cdot2 + f(4)\cdot2 + f(6)\cdot2$$
$$= 10\cdot2 + 6\cdot2 + 7\cdot2 + 5\cdot2$$
$$= 20 + 12 + 14 + 10 = 56$$

9. a. Graph $f(x) = 3x$:



b. $$A \approx f(0)(2) + f(2)(2) + f(4)(2)$$
$$= 0(2) + 6(2) + 12(2)$$
$$= 0 + 12 + 24 = 36$$

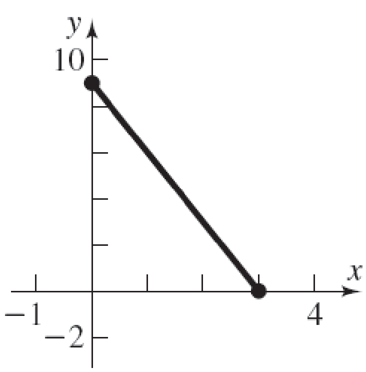
c. $A \approx f(2)(2) + f(4)(2) + f(6)(2)$
$= 6(2) + 12(2) + 18(2)$
$= 12 + 24 + 36 = 72$

d. $A \approx f(0)(1) + f(1)(1) + f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1)$
$= 0(1) + 3(1) + 6(1) + 9(1) + 12(1) + 15(1)$
$= 0 + 3 + 6 + 9 + 12 + 15 = 45$

e. $A \approx f(1)(1) + f(2)(1) + f(3)(1) + f(4)(1) + f(5)(1) + f(6)(1)$
$= 3(1) + 6(1) + 9(1) + 12(1) + 15(1) + 18(1)$
$= 3 + 6 + 9 + 12 + 15 + 18$
$= 63$

f. The actual area is the area of a triangle: $A = \frac{1}{2}(6)(18) = 54$

11. a. Graph $f(x) = -3x + 9$:



b. $A \approx f(0)(1) + f(1)(1) + f(2)(1)$
$= 9(1) + 6(1) + 3(1)$
$= 9 + 6 + 3 = 18$

c. $A \approx f(1)(1) + f(2)(1) + f(3)(1)$
$= 6(1) + 3(1) + 0(1)$
$= 6 + 3 + 0 = 9$

d. $A \approx f(0)\left(\frac{1}{2}\right) + f\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + f(1)\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) + f(2)\left(\frac{1}{2}\right) + f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)$
$= 9\left(\frac{1}{2}\right) + \frac{15}{2}\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right) + \frac{9}{2}\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) + \frac{3}{2}\left(\frac{1}{2}\right)$
$= \frac{9}{2} + \frac{15}{4} + 3 + \frac{9}{4} + \frac{3}{2} + \frac{3}{4} = \frac{63}{4} = 15.75$

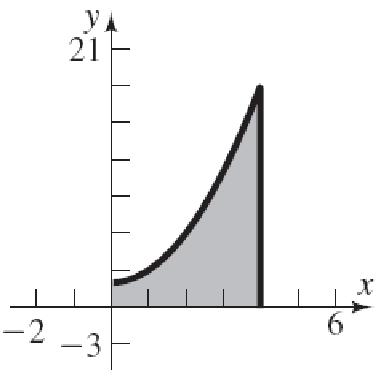
e. $A \approx f\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+f(1)\left(\frac{1}{2}\right)+f\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)+f(2)\left(\frac{1}{2}\right)+f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)+f(3)\left(\frac{1}{2}\right)$

$=\frac{15}{2}\left(\frac{1}{2}\right)+6\left(\frac{1}{2}\right)+\frac{9}{2}\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)+\frac{3}{2}\left(\frac{1}{2}\right)+0\left(\frac{1}{2}\right)$

$=\frac{15}{4}+3+\frac{9}{4}+\frac{3}{2}+\frac{3}{4}+0=\frac{45}{4}=11.25$

f. The actual area is the area of a triangle: $A=\frac{1}{2}(3)(9)=\frac{27}{2}=13.5$

13. a. Graph $f(x)=x^2+2,\quad [0, 4]$:



b. $A \approx f(0)(1)+f(1)(1)+f(2)(1)+f(3)(1)=2(1)+3(1)+6(1)+11(1)\ =2+3+6+11=22$

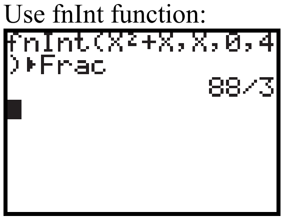
c. $A \approx f(0)\left(\frac{1}{2}\right)+f\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+f(1)\left(\frac{1}{2}\right)+f\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)+f(2)\left(\frac{1}{2}\right)+f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)+f(3)\left(\frac{1}{2}\right)+f\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)$

$=2\left(\frac{1}{2}\right)+\frac{9}{4}\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)+\frac{17}{4}\left(\frac{1}{2}\right)+6\left(\frac{1}{2}\right)+\frac{33}{4}\left(\frac{1}{2}\right)+11\left(\frac{1}{2}\right)+\frac{57}{4}\left(\frac{1}{2}\right)$

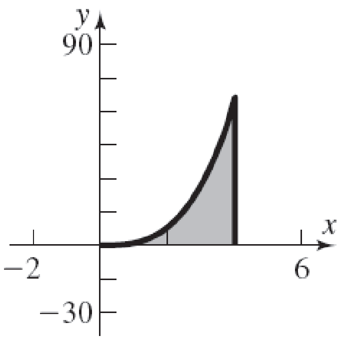
$=1+\frac{9}{8}+\frac{3}{2}+\frac{17}{8}+3+\frac{33}{8}+\frac{11}{2}+\frac{57}{8}=\frac{51}{2}=25.5$

d. $A=\int_0^4 (x^2+2)dx$

e. Use fnInt function:



15. **a.** Graph $f(x)=x^3,\quad [0,4]$:



b. $A \approx f(0)(1)+f(1)(1)+f(2)(1)+f(3)(1)=0(1)+1(1)+8(1)+27(1)\ =0+1+8+27=36$

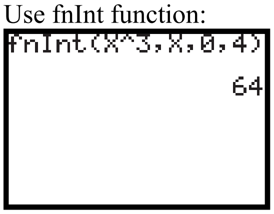
c. $A \approx f(0)\left(\frac{1}{2}\right)+f\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+f(1)\left(\frac{1}{2}\right)+f\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)+f(2)\left(\frac{1}{2}\right)+f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)+f(3)\left(\frac{1}{2}\right)+f\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)$

$=0\left(\frac{1}{2}\right)+\frac{1}{8}\left(\frac{1}{2}\right)+1\left(\frac{1}{2}\right)+\frac{27}{8}\left(\frac{1}{2}\right)+8\left(\frac{1}{2}\right)+\frac{125}{8}\left(\frac{1}{2}\right)+27\left(\frac{1}{2}\right)+\frac{343}{8}\left(\frac{1}{2}\right)$

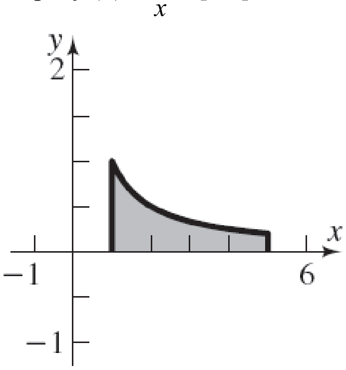
$=0+\frac{1}{16}+\frac{1}{2}+\frac{27}{16}+4+\frac{125}{16}+\frac{27}{2}+\frac{343}{16}=49$

d. $A=\int_0^4 x^3\,dx$

e. Use fnInt function:



17. **a.** Graph $f(x)=\frac{1}{x},\quad [1,5]$:



b. $A \approx f(1)(1)+f(2)(1)+f(3)(1)+f(4)(1)=1(1)+\frac{1}{2}(1)+\frac{1}{3}(1)+\frac{1}{4}(1)\ =1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{25}{12}$

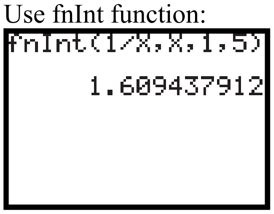
c. $A \approx f(1)\left(\frac{1}{2}\right)+f\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)+f(2)\left(\frac{1}{2}\right)+f\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)+f(3)\left(\frac{1}{2}\right)+f\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)+f(4)\left(\frac{1}{2}\right)+f\left(\frac{9}{2}\right)\left(\frac{1}{2}\right)$

$=1\left(\frac{1}{2}\right)+\frac{2}{3}\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\right)+\frac{2}{5}\left(\frac{1}{2}\right)+\frac{1}{3}\left(\frac{1}{2}\right)+\frac{2}{7}\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)+\frac{2}{9}\left(\frac{1}{2}\right)$

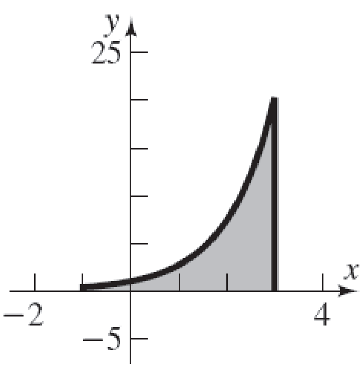
$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}=\frac{4609}{2520}\approx 1.829$

d. $A=\int_1^5 \frac{1}{x}\,dx$

e. Use fnInt function:



19. a. Graph $f(x)=e^x$, $[-1, 3]$:



b. $A \approx \left(f(-1)+f(0)+f(1)+f(2)\right)(1) \approx (0.3679+1+2.7183+7.3891)(1) \approx 11.475$

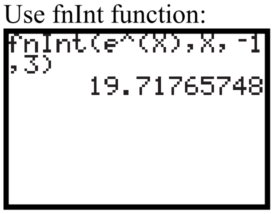
c. $A \approx \left(f(-1)+f\left(-\frac{1}{2}\right)+f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)+f\left(\frac{5}{2}\right)\right)\cdot\left(\frac{1}{2}\right)$

$\approx \left(0.3679+0.6065+1+1.6487+2.7183+4.4817+7.3891+12.1825\right)\left(0.5\right)$

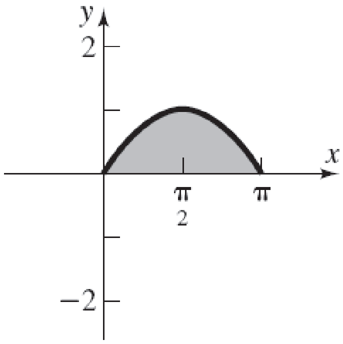
$=30.3947\left(0.5\right)\approx 15.197$

d. $A=\int_{-1}^{3} e^x\,dx$

e. Use fnInt function:



21. a. Graph $f(x)=\sin x, \quad [0, \pi]$:



b. $A \approx \left(f(0)+f\left(\frac{\pi}{4}\right)+f\left(\frac{\pi}{2}\right)+f\left(\frac{3\pi}{4}\right)\right)\left(\frac{\pi}{4}\right)=\left(0+\frac{\sqrt{2}}{2}+1+\frac{\sqrt{2}}{2}\right)\left(\frac{\pi}{4}\right)$

$=\left(1+\sqrt{2}\right)\left(\frac{\pi}{4}\right)\approx 1.896$

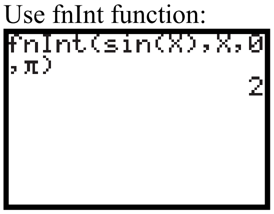
c. $A \approx \left(f(0)+f\left(\frac{\pi}{8}\right)+f\left(\frac{\pi}{4}\right)+f\left(\frac{3\pi}{8}\right)+f\left(\frac{\pi}{2}\right)+f\left(\frac{5\pi}{8}\right)+f\left(\frac{3\pi}{4}\right)+f\left(\frac{7\pi}{8}\right)\right)\left(\frac{\pi}{8}\right)$

$\approx \left(0+0.3827+0.7071+0.9239+1+0.9239+0.7071+0.3827\right)\left(\frac{\pi}{8}\right)$

$=5.0274\left(\frac{\pi}{8}\right)\approx 1.974$

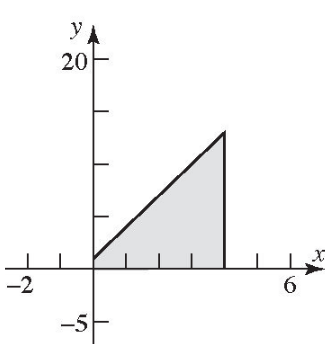
d. $A=\int_0^{\pi}\sin x dx$

e. Use fnInt function:

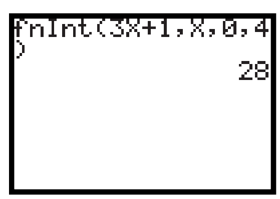


23. a. The integral represents the area under the graph of $f(x)=3x+1$ from $x=0$ to $x=4$.

b.

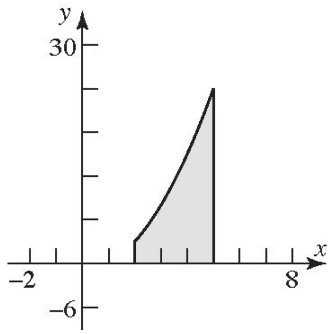


c.

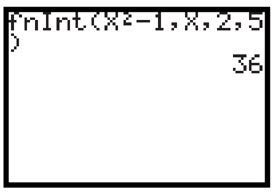


25. **a.** The integral represents the area under the graph of $f(x) = x^2 - 1$ from $x = 2$ to $x = 5$.

b.

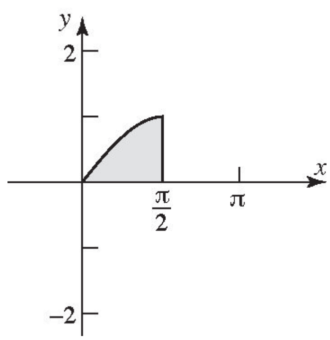


c.

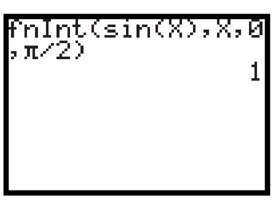


27. **a.** The integral represents the area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$.

b.

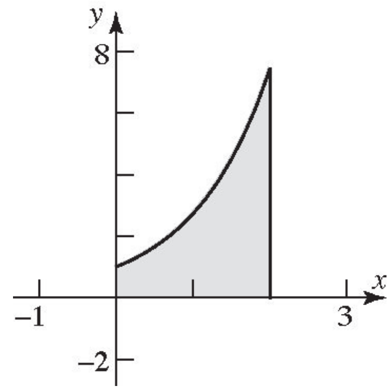


c.



29. **a.** The integral represents the area under the graph of $f(x) = e^x$ from $x = 0$ to $x = 2$.

b.



c.

31. Using left endpoints:

$n = 2: \quad 0 + 0.5 = 0.5$

$n = 4: \quad 0 + 0.125 + 0.25 + 0.375 = 0.75$

$n = 10: \quad 0 + 0.02 + 0.04 + 0.06 + \cdots + 0.18 = \frac{10}{2}(0 + 0.18) = 0.9$

$n = 100: \quad 0 + 0.0002 + 0.0004 + 0.0006 + \cdots + 0.0198 = \frac{100}{2}(0 + 0.0198) = 0.99$

Using right endpoints:

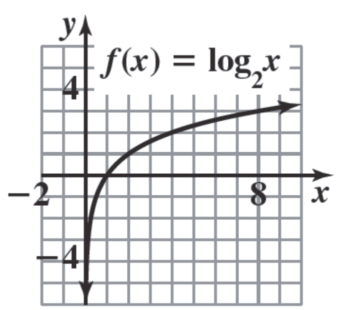
$n = 2: \quad 0.5 + 1 = 1.5$

$n = 4: \quad 0.125 + 0.25 + 0.375 + 0.5 = 1.25$

$n = 10: \quad 0.02 + 0.04 + 0.06 + \cdots + 0.20 = \frac{10}{2}(0.02 + 0.20) = 1.1$

$n = 100: \quad 0.0002 + 0.0004 + 0.0006 + \cdots + 0.02 = \frac{100}{2}(0.0002 + 0.02) = 1.01$

33.



35. $f(x) = 2x^2 + 3x + 1$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{\left[2(x+h)^2 + 3(x+h) + 1\right] - \left[2x^2 + 3x + 1\right]}{h}$$
$$= \frac{2\left(x^2 + 2xh + h^2\right) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$
$$= \frac{2x^2 + 4xh + 2h^2 - 3h - 2x^2}{h} = \frac{4xh + 2h^2 - 3h}{h}$$
$$= 4x + 2h - 3$$

Chapter 14 Review Exercises

1. $\lim_{x \to 2} \left(3x^2 - 2x + 1\right) = 3(2)^2 - 2(2) + 1$
$= 12 - 4 + 1 = 9$

2. $\lim_{x \to -2} \left(x^2 + 1\right)^2 = \left(\lim_{x \to -2} \left(x^2 + 1\right)\right)^2$
$= \left((-2)^2 + 1\right)^2 = 5^2 = 25$

3. $\lim_{x \to 3} \sqrt{x^2 + 7} = \sqrt{\lim_{x \to 3} (x^2 + 7)}$
$= \sqrt{3^2 + 7}$
$= \sqrt{16} = 4$

4. $\lim_{x \to 1^-} \sqrt{1 - x^2} = \sqrt{\lim_{x \to 1^-} (1 - x^2)}$
$= \sqrt{1 - 1^2}$
$= \sqrt{0} = 0$

5. $\lim_{x \to 2} (5x + 6)^{3/2} = \left(\lim_{x \to 2} (5x + 6)\right)^{3/2}$
$= \left(5(2) + 6\right)^{3/2}$
$= 16^{3/2} = 64$

6. $\lim_{x\to -1}\left(\frac{x^2+x+2}{x^2-9}\right)=\frac{\lim_{x\to -1}(x^2+x+2)}{\lim_{x\to -1}(x^2-9)}$

$=\frac{(-1)^2+(-1)+2}{(-1)^2-9}$

$=\frac{2}{-8}=-\frac{1}{4}$

7. $\lim_{x\to 1}\left(\frac{x-1}{x^3-1}\right)=\lim_{x\to 1}\left(\frac{x-1}{(x-1)(x^2+x+1)}\right)$

$=\lim_{x\to 1}\left(\frac{1}{x^2+x+1}\right)$

$=\frac{1}{1^2+1+1}=\frac{1}{3}$

8. $\lim_{x\to -3}\left(\frac{x^2-9}{x^2-x-12}\right)=\lim_{x\to -3}\left(\frac{(x-3)(x+3)}{(x-4)(x+3)}\right)$

$=\lim_{x\to -3}\left(\frac{x-3}{x-4}\right)=\frac{-3-3}{-3-4}$

$=\frac{-6}{-7}=\frac{6}{7}$

9. $\lim_{x\to -1^-}\left(\frac{x^2-1}{x^3-1}\right)=\lim_{x\to -1^-}\left(\frac{(x+1)(x-1)}{(x-1)(x^2+x+1)}\right)$

$=\lim_{x\to -1^-}\left(\frac{x+1}{x^2+x+1}\right)$

$=\frac{-1+1}{(-1)^2+(-1)+1}$

$=\frac{0}{1}=0$

10. $\lim_{x\to 2}\left(\frac{x^3-8}{x^3-2x^2+4x-8}\right)$

$=\lim_{x\to 2}\left(\frac{(x-2)(x^2+2x+4)}{x^2(x-2)+4(x-2)}\right)$

$=\lim_{x\to 2}\left(\frac{(x-2)(x^2+2x+4)}{(x-2)(x^2+4)}\right)$

$=\lim_{x\to 2}\left(\frac{x^2+2x+4}{x^2+4}\right)=\left(\frac{2^2+2(2)+4}{2^2+4}\right)$

$=\frac{12}{8}=\frac{3}{2}$

11. $\lim_{x\to 3}\left(\frac{x^4-3x^3+x-3}{x^3-3x^2+2x-6}\right)$

$=\lim_{x\to 3}\left(\frac{x^3(x-3)+1(x-3)}{x^2(x-3)+2(x-3)}\right)$

$=\lim_{x\to 3}\left(\frac{(x-3)(x^3+1)}{(x-3)(x^2+2)}\right)$

$=\lim_{x\to 3}\left(\frac{x^3+1}{x^2+2}\right)$

$=\frac{3^3+1}{3^2+2}=\frac{28}{11}$

12. $f(x)=3x^4-x^2+2;\ c=5$

1. $f(5)=3(5)^4-5^2+2=1852$
2. $\lim_{x\to 5^-} f(x)=3(5)^4-5^2+2=1852$
3. $\lim_{x\to 5^+} f(x)=3(5)^4-5^2+2=1852$

Thus, $f(x)$ is continuous at $c=5$.

13. $f(x)=\frac{x^2-4}{x+2};\ c=-2$

Since $f(x)$ is not defined at $c=-2$, the function is not continuous at $c=-2$.

14. $f(x)=\begin{cases}\frac{x^2-4}{x+2} & \text{if } x\neq -2\\ 4 & \text{if } x=-2\end{cases};\quad c=-2$

1. $f(-2)=4$
2. $\lim_{x\to -2^-} f(x)=\lim_{x\to -2^-}\left(\frac{x^2-4}{x+2}\right)$

$=\lim_{x\to -2^-}\left(\frac{(x-2)(x+2)}{x+2}\right)$

$=\lim_{x\to -2^-}(x-2)=-4$

Since $\lim_{x\to -2^-} f(x)\neq f(-2)$, the function is not continuous at $c=-2$.

15. $f(x)=\begin{cases}\frac{x^2-4}{x+2} & \text{if } x\neq -2\\ -4 & \text{if } x=-2\end{cases};\quad c=-2$

1. $f(-2)=-4$

2. $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-}\left(\dfrac{x^2-4}{x+2}\right)$
$= \lim\limits_{x\to -2^-}\left(\dfrac{(x-2)(x+2)}{x+2}\right)$
$= \lim\limits_{x\to -2^-}(x-2) = -4$

3. $\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+}\left(\dfrac{x^2-4}{x+2}\right)$
$= \lim\limits_{x\to -2^+}\left(\dfrac{(x-2)(x+2)}{x+2}\right)$
$= \lim\limits_{x\to -2^+}(x-2) = -4$

The function is continuous at $c = -2$.

16. Domain:
$\{x \mid -6 \le x < 2 \text{ or } 2 < x < 5 \text{ or } 5 < x \le 6\}$

17. Range: $(-\infty, \infty)$, or all real numbers

18. x-intercepts: 1, 6

19. y-intercept: 4

20. $f(-6) = 2$; $f(-4) = 1$

21. $\lim\limits_{x\to -4^-} f(x) = 4$

22. $\lim\limits_{x\to -4^+} f(x) = -2$

23. $\lim\limits_{x\to 2^-} f(x) = -\infty$

24. $\lim\limits_{x\to 2^+} f(x) = \infty$

25. $\lim\limits_{x\to 0} f(x)$ does not exist because
$\lim\limits_{x\to 0^-} f(x) = 4 \ne \lim\limits_{x\to 0^+} f(x) = 1$

26. f is not continuous at 0 because
$\lim\limits_{x\to 0^-} f(x) = 4 \ne \lim\limits_{x\to 0^+} f(x) = 1$

27. f is continuous at 4 because
$f(4) = \lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^+} f(x)$

28. $R(x) = \dfrac{x+4}{x^2-16} = \dfrac{x+4}{(x-4)(x+4)}$. The domain of R is $\{x \mid x \ne -4, x \ne 4\}$. Thus R is discontinuous at both –4 and 4.

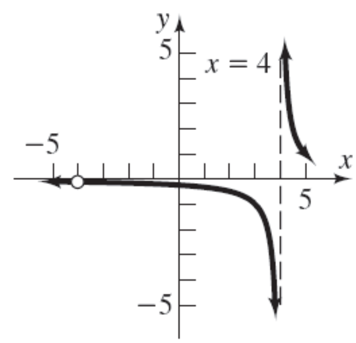
$\lim\limits_{x\to 4^-} R(x) = \lim\limits_{x\to 4^-}\left(\dfrac{x+4}{(x-4)(x+4)}\right)$
$= \lim\limits_{x\to 4^-}\left(\frac{1}{x-4}\right) = -\infty$

since when $x < 4$, $\dfrac{1}{x-4} < 0$, and as x approaches 4, $\frac{1}{x-4}$ becomes unbounded.

$\lim\limits_{x\to 4^+} R(x) = \lim\limits_{x\to 4^+}\left(\frac{1}{x-4}\right) = \infty$ since when $x > 4$, $\frac{1}{x-4} > 0$, and as x approaches 4, $\frac{1}{x-4}$ becomes unbounded. Thus, there is a vertical asymptote at $x = 4$.

$\lim\limits_{x\to -4} R(x) = \lim\limits_{x\to -4}\left(\frac{1}{x-4}\right) = -\dfrac{1}{8}$.

Thus, there is a hole in the graph at $\left(-4, -\frac{1}{8}\right)$.



29. $R(x) = \dfrac{x^3-2x^2+4x-8}{x^2-11x+18}$
$= \dfrac{x^2(x-2)+4(x-2)}{(x-9)(x-2)}$
$= \dfrac{(x-2)(x^2+4)}{(x-9)(x-2)}$
$= \dfrac{x^2+4}{x-9}, x \ne 2$

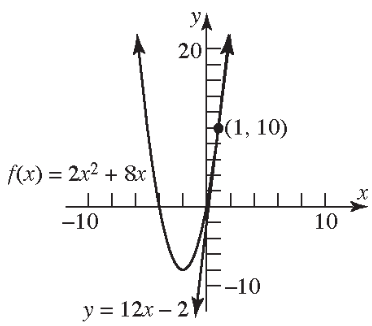
Undefined at $x = 2$ and $x = 9$. There is a vertical asymptote where $x - 9 = 0$. $x = 9$ is a vertical asymptote. There is a hole in the graph at $x = 2$, the point $\left(2, -\dfrac{8}{7}\right)$.

30. $f(x)=2x^2+8x$ at $(1,10)$

$$m_{\tan}=\lim_{x\to1}\left(\frac{f(x)-f(1)}{x-1}\right)=\lim_{x\to1}\left(\frac{2x^2+8x-10}{x-1}\right)$$
$$=\lim_{x\to1}\left(\frac{2(x+5)(x-1)}{x-1}\right)=\lim_{x\to1}\left(2(x+5)\right)$$
$$=2(1+5)=12$$

Tangent Line: $y-10=12(x-1)$

$$y-10=12x-12$$
$$y=12x-2$$

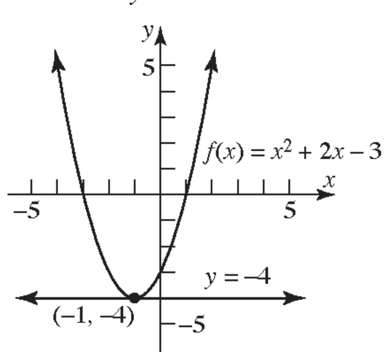


31. $f(x)=x^2+2x-3$ at $(-1,-4)$

$$m_{\tan}=\lim_{x\to-1}\left(\frac{f(x)-f(-1)}{x+1}\right)$$
$$=\lim_{x\to-1}\left(\frac{x^2+2x-3-(-4)}{x+1}\right)$$
$$=\lim_{x\to-1}\left(\frac{x^2+2x+1}{x+1}\right)$$
$$=\lim_{x\to-1}\left(\frac{(x+1)^2}{x+1}\right)$$
$$=\lim_{x\to-1}(x+1)$$
$$=-1+1=0$$

Tangent Line: $y-(-4)=0(x-(-1))$

$$y+4=0$$
$$y=-4$$

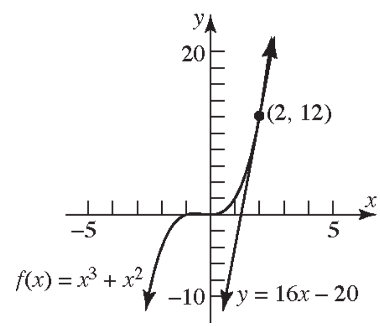


32. $f(x)=x^3+x^2$ at $(2,12)$

$$m_{\tan}=\lim_{x\to2}\left(\frac{f(x)-f(2)}{x-2}\right)=\lim_{x\to2}\left(\frac{x^3+x^2-12}{x-2}\right)$$
$$=\lim_{x\to2}\left(\frac{x^3-2x^2+3x^2-12}{x-2}\right)$$
$$=\lim_{x\to2}\left(\frac{x^2(x-2)+3(x-2)(x+2)}{x-2}\right)$$
$$=\lim_{x\to2}\left(\frac{(x-2)(x^2+3x+6)}{x-2}\right)$$
$$=\lim_{x\to2}(x^2+3x+6)=4+6+6=16$$

Tangent Line: $y-12=16(x-2)$

$$y-12=16x-32$$
$$y=16x-20$$



33. $f(x)=-4x^2+5$ at 3

$$f'(3)=\lim_{x\to3}\left(\frac{f(x)-f(3)}{x-3}\right)$$
$$=\lim_{x\to3}\left(\frac{-4x^2+5-(-31)}{x-3}\right)$$
$$=\lim_{x\to3}\left(\frac{-4x^2+36}{x-3}\right)$$
$$=\lim_{x\to3}\left(\frac{-4(x^2-9)}{x-3}\right)$$
$$=\lim_{x\to3}\left(\frac{-4(x-3)(x+3)}{x-3}\right)$$
$$=\lim_{x\to3}\left((-4)(x+3)\right)$$
$$=-4(6)=-24$$

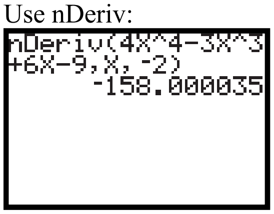
34. $f(x) = x^2 - 3x$ at 0

$$f'(0) = \lim_{x\to 0}\left(\frac{f(x)-f(0)}{x-0}\right) = \lim_{x\to 0}\left(\frac{x^2-3x-0}{x}\right) = \lim_{x\to 0}\left(\frac{x(x-3)}{x}\right) = \lim_{x\to 0}(x-3) = -3$$

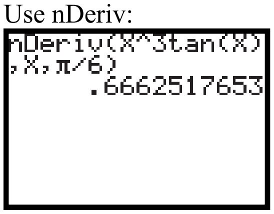
35. $f(x) = 2x^2 + 3x + 2$ at 1

$$f'(1) = \lim_{x\to 1}\left(\frac{f(x)-f(1)}{x-1}\right) = \lim_{x\to 1}\left(\frac{2x^2+3x+2-7}{x-1}\right) = \lim_{x\to 1}\left(\frac{2x^2+3x-5}{x-1}\right) = \lim_{x\to 1}\left(\frac{(2x+5)(x-1)}{x-1}\right) = \lim_{x\to 1}(2x+5) = 7$$

36. Use nDeriv:



37. Use nDeriv:



38. a.
$$-16t^2 + 96t + 112 = 0$$
$$-16(t^2 - 6t - 7) = 0$$
$$-16(t+1)(t-7) = 0$$
$$t = -1 \text{ or } t = 7$$
The ball strikes the ground after 7 seconds in the air.

b.
$$-16t^2 + 96t + 112 = 112$$
$$-16t^2 + 96t = 0$$
$$-16t(t-6) = 0$$
$$t = 0 \text{ or } t = 6$$
The ball passes the rooftop after 6 seconds.

c.
$$\frac{\Delta s}{\Delta t} = \frac{s(2)-s(0)}{2-0} = \frac{-16(2)^2+96(2)+112-112}{2} = \frac{128}{2} = 64 \text{ feet/sec}$$

d.
$$s'(t_0) = \lim_{t\to t_0}\left(\frac{s(t)-s(t_0)}{t-t_0}\right)$$
$$= \lim_{t\to t_0}\left(\frac{-16t^2+16t_0^{\,2}+96t-96t_0}{t-t_0}\right)$$
$$= \lim_{t\to t_0}\left(\frac{-16\left(t^2-t_0^{\,2}\right)+96\left(t-t_0\right)}{t-t_0}\right)$$
$$= \lim_{t\to t_0}\left(\frac{-16\left(t-t_0\right)\left(t+t_0\right)+96\left(t-t_0\right)}{t-t_0}\right)$$
$$= \lim_{t\to t_0}\left(\frac{\left(t-t_0\right)\left(-16\left(t+t_0\right)+96\right)}{t-t_0}\right)$$
$$= \lim_{t\to t_0}\left(-16\left(t+t_0\right)+96\right)$$
$$= \left(-16\left(t_0+t_0\right)+96\right)$$
$$= -32t_0 + 96 \quad \text{ft/sec}$$
The instantaneous speed at time t is $-32t + 96$ feet per second.

e. $s'(2) = -32(2) + 96 = -64 + 96 = 32$ feet/sec

f.
$$s'(t) = 0$$
$$-32t + 96 = 0$$
$$-32t = -96$$
$$t = 3 \text{ seconds}$$

g.
$$s'(6) = -32(6) + 96 = -192 + 96 = -96 \text{ feet/sec}$$

h. $s'(7) = -32(7) + 96$
$= -224 + 96$
$= -128$ feet/sec

39. a. $\dfrac{\Delta R}{\Delta x} = \dfrac{8775 - 2340}{130 - 25} = \dfrac{6435}{105} \approx \$61.29\text{/watch}$

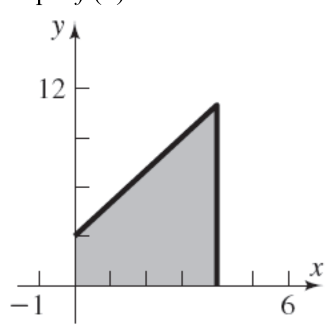
b. $\dfrac{\Delta R}{\Delta x} = \dfrac{6975 - 2340}{90 - 25} = \dfrac{4635}{65} \approx \$71.31\text{/watch}$

c. $\dfrac{\Delta R}{\Delta x} = \dfrac{4375 - 2340}{50 - 25} = \dfrac{2035}{25} \approx \$81.40\text{/watch}$

d. $R(x) = -0.25x^2 + 100.01x - 1.24$

e. $R'(25) = \lim\limits_{x \to 25} \left(\dfrac{R(x) - R(25)}{x - 25} \right)$
$= \lim\limits_{x \to 25} \left(\dfrac{-0.25x^2 + 100.014x - 2344}{x - 25} \right)$
$= \lim\limits_{x \to 25} \left(\dfrac{(x-25)(-0.25x + 93.76)}{x - 25} \right)$
$= \lim\limits_{x \to 25} (-0.25x + 93.76)$
$= -0.25(25) + 93.76 = \$87.51$ /watch

40. a. Graph $f(x) = 2x + 3$:



b. $A \approx (f(0) + f(1) + f(2) + f(3))(1)$
$= (3 + 5 + 7 + 9)(1)$
$= 24(1) = 24$

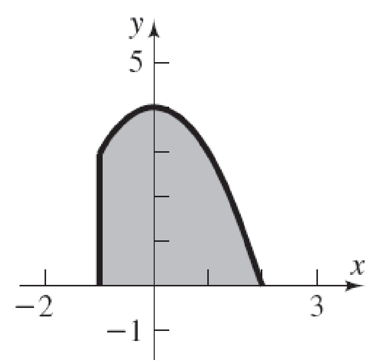
c. $A \approx (f(1) + f(2) + f(3) + f(4))(1)$
$= (5 + 7 + 9 + 11)(1)$
$= 32(1) = 32$

d. $A \approx \left(f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) + f\left(\frac{7}{2}\right) \right) \cdot \left(\frac{1}{2}\right)$
$= (3 + 4 + 5 + 6 + 7 + 8 + 9 + 10)\left(\frac{1}{2}\right)$
$= 52\left(\frac{1}{2}\right) = 26$

e. $A \approx \left(f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) + f\left(\frac{7}{2}\right) + f(4) \right) \cdot \left(\frac{1}{2}\right)$
$= (4 + 5 + 6 + 7 + 8 + 9 + 10 + 11)\left(\frac{1}{2}\right)$
$= 60\left(\frac{1}{2}\right) = 30$

f. The actual area is the area of a trapezoid:
$A = \frac{1}{2}(3 + 11)(4) = \frac{56}{2} = 28$

41. a. Graph $f(x) = 4 - x^2$, $[-1, 2]$:

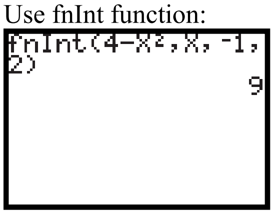


b. $A \approx (f(-1) + f(0) + f(1))(1)$
$= (3 + 4 + 3)(1)$
$= 10(1) = 10$

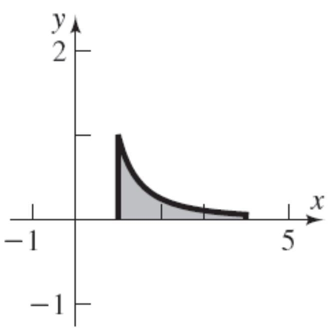
c. $$A \approx \left(f(-1) + f\left(-\frac{1}{2}\right) + f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) \right) \cdot \left(\frac{1}{2}\right)$$
$$= \left(3 + \frac{15}{4} + 4 + \frac{15}{4} + 3 + \frac{7}{4}\right)\left(\frac{1}{2}\right)$$
$$= \frac{77}{4}\left(\frac{1}{2}\right) = \frac{77}{8} = 9.625$$

d. $A = \int_{-1}^{2} \left(4 - x^2\right) dx$

e. Use fnInt function:



42. a. Graph $f(x) = \frac{1}{x^2}$, $[1, 4]$:

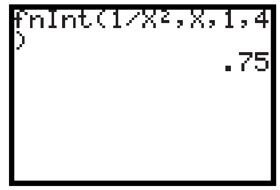


b. $$A \approx \left(f(1) + f(2) + f(3) \right)(1)$$
$$= \left(1 + \frac{1}{4} + \frac{1}{9}\right)(1)$$
$$= \frac{49}{36}(1) = \frac{49}{36} \approx 1.36$$

c. $$A \approx \left(f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) + f\left(\frac{7}{2}\right) \right) \cdot \left(\frac{1}{2}\right)$$
$$= \left(1 + \frac{4}{9} + \frac{1}{4} + \frac{4}{25} + \frac{1}{9} + \frac{4}{49}\right)\left(\frac{1}{2}\right)$$
$$= 1.02$$

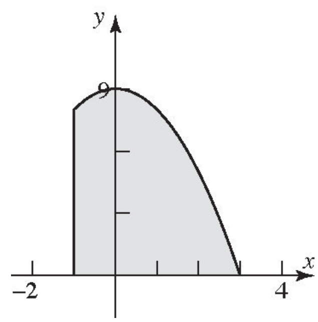
d. $A = \int_{1}^{4} \left(\frac{1}{x^2}\right) dx$

e. Use fnInt function:

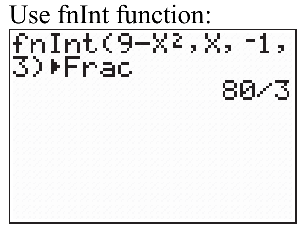


43. a. The integral represents the area under the graph of $f(x) = 9 - x^2$ from $x = -1$ to $x = 3$.

b.

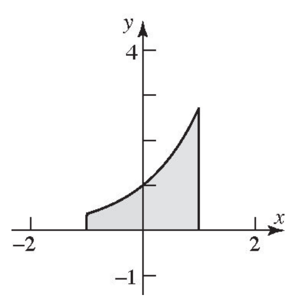


c. Use fnInt function:

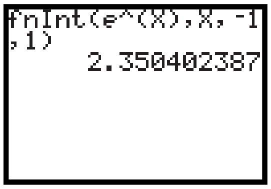


44. a. The integral represents the area under the graph of $f(x) = e^x$ from $x = -1$ to $x = 1$.

b.



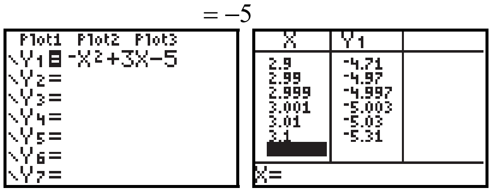
c. Use fnInt function:



Chapter 14 Test

1. Here we are taking the limit of a polynomial. Therefore, we evaluate the polynomial expression for the given value.

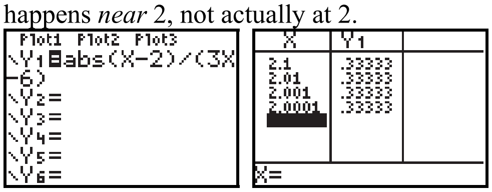
$$\lim_{x\to 3}\left(-x^2+3x-5\right)=-(3)^2+3(3)-5$$
$$=-9+9-5$$
$$=-5$$



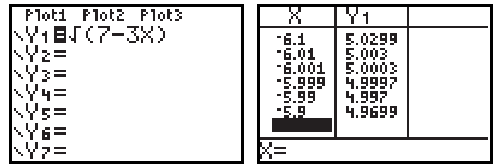
2. For this problem, direct substitution does not work because it would yield the indeterminate form $\frac{0}{0}$. However, notice that this is a one-sided limit from the right. As we approach 2 from the right, we will have x values such that $x>2$. Therefore, we have $|x-2|=x-2$ and get the following:

$$\lim_{x\to 2+}\frac{|x-2|}{3x-6}=\lim_{x\to 2+}\frac{x-2}{3x-6}$$
$$=\lim_{x\to 2+}\frac{x-2}{3(x-2)}$$
$$=\lim_{x\to 2+}\frac{1}{3}$$
$$=\frac{1}{3}$$

Remember that we can cancel the common factor $(x-2)$ because we are interested in what happens *near* 2, not actually at 2.

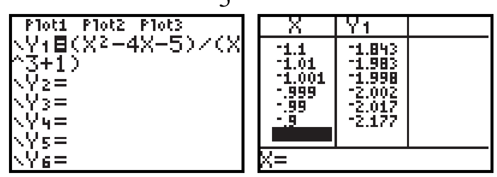


3. $$\lim_{x\to -6}\sqrt{7-3x}=\sqrt{7-3(-6)}$$
$$=\sqrt{7+18}$$
$$=\sqrt{25}=5$$

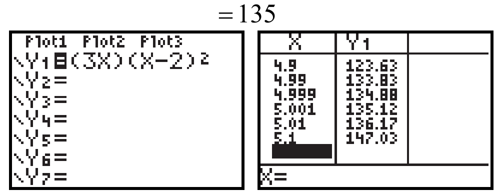


4. Note that direct substitution will yield the indeterminate form $\frac{0}{0}$. For rational functions, this means that there is a common factor that can be cancelled before taking the limit.

$$\lim_{x\to -1}\frac{x^2-4x-5}{x^3+1}=\lim_{x\to -1}\frac{(x-5)(x+1)}{(x+1)\left(x^2-x+1\right)}$$
$$=\lim_{x\to -1}\frac{x-5}{x^2-x+1}$$
$$=\frac{\lim_{x\to -1}(x-5)}{\lim_{x\to -1}\left(x^2-x+1\right)}$$
$$=\frac{-1-5}{(-1)^2-(-1)+1}$$
$$=\frac{-6}{3}=-2$$



5. $$\lim_{x\to 5}\left[(3x)(x-2)^2\right]=\lim_{x\to 5}(3x)\cdot\lim_{x\to 5}(x-2)^2$$
$$=\lim_{x\to 5}3\cdot\lim_{x\to 5}x\cdot\left[\lim_{x\to 5}(x-2)\right]^2$$
$$=3\cdot 5\cdot(5-2)^2$$
$$=15(3)^2$$
$$=135$$

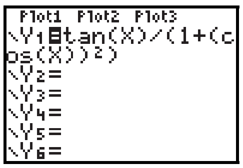


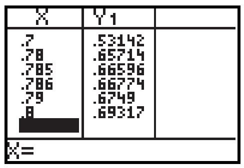
6. $$\lim_{x\to\frac{\pi}{4}} \frac{\tan x}{1+\cos^2 x} = \frac{\lim_{x\to\frac{\pi}{4}} \tan x}{\lim_{x\to\frac{\pi}{4}} 1+\cos^2 x}$$

$$= \frac{\tan\frac{\pi}{4}}{1+\cos^2\frac{\pi}{4}} = \frac{1}{1+\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}}$$

$$= \frac{2}{3}$$





7. To be continuous at a point $x = c$, we need to show that $\lim_{x\to c-} f(x) = \lim_{x\to c+} f(x) = f(c)$.

$$\lim_{x\to 4-} f(x) = \lim_{x\to 4-} \frac{x^2-9}{x+3} = \frac{4^2-9}{4+3} = 1$$
$$\lim_{x\to 4+} f(x) = \lim_{x\to 4+} (kx+5) = 4k+5$$
$$f(4) = \frac{4^2-9}{4+3} = 1$$

Therefore, we need to solve

$$4k+5=1$$
$$4k=-4$$
$$k=-1$$

8. To find the limit, we look at at the values of f when x is close to 3, but more than 3. From the graph, we conclude that $\lim_{x\to 3+} f(x) = -3$.

9. To find the limit, we look at at the values of f when x is close to 3, but less than 3. From the graph, we conclude that $\lim_{x\to 3-} f(x) = 5$.

10. To find the limit, we look at at the values of f when x is close to -2 on either side. From the graph, we see that the limits from the left and right are the same and conclude that $\lim_{x\to -2} f(x) = 2$.

11. For a limit to exist, the limit from the left and the limit from the right must both exist and be equal. From the graph we see that the limit exists since $\lim_{x\to 1-} f(x) = \lim_{x\to 1+} f(x) = 2 = \lim_{x\to 1} f(x)$

Note that the limit need not be equal to the function value at the point c. In fact, the function does not even need to be defined at c. We simply need the left and right limits to exist and be the same.

12. **a.** The graph has a hole at $x=-2$ and the function is undefined. Thus**,** the function is not continuous at $x=-2$.

b. The function is defined at $x=1$ and $\lim_{x\to 1} f(x)$ exists, but $\lim_{x\to 1} f(x) \neq f(1)$ so the function is not continuous at $x=1$.

c. The function is defined at $x=3$, but there is a gap in the graph. That is, $\lim_{x\to 3-} f(x) \neq \lim_{x\to 3+} f(x)$ so the two-sided limit as $x\to 3$ does not exist. Thus, the function is not continuous at $x=3$.

d. From the graph we can see that $\lim_{x\to 4-} f(x) = \lim_{x\to 4+} f(x) = f(4)$. Therefore, the function is continuous at $x=4$.

13. $R(x) = \dfrac{x^3+6x^2-4x-24}{x^2+5x-14}$

Begin by factoring the numerator and denominator, but do not cancel any common factors yet.

$$R(x) = \frac{x^3+6x^2-4x-24}{x^2+5x-14}$$
$$= \frac{x^2(x+6)-4(x+6)}{(x+7)(x-2)}$$
$$= \frac{(x+6)(x^2-4)}{(x+7)(x-2)}$$
$$= \frac{(x+6)(x+2)(x-2)}{(x+7)(x-2)}$$

From the denominator, we can see that the function is undefined at the values $x=-7$ and $x=2$ because these values make the denominator equal 0.

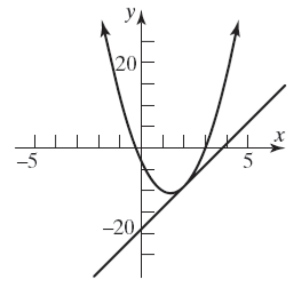
To determine whether an asymptote or hole occur at these restricted values, we need to write the function in lowest terms by canceling common factors.

$$R(x)=\frac{(x+6)(x+2)(x-2)}{(x+7)(x-2)}=\frac{(x+6)(x+2)}{(x+7)}$$

(where $x \neq 2$)

Since $x=-7$ still makes the denominator equal to 0, there will be a vertical asymptote at $x=-7$. Since $x=2$ no longer makes the denominator equal to 0 when the expression is in lowest terms, there will be a hole in the graph at $x=2$.

c.



14. a. $f'(2)$

$$\begin{aligned}
&= \lim_{x\to 2}\frac{f(x)-f(2)}{x-2}\\
&= \lim_{x\to 2}\frac{(4x^2-11x-3)-(4(2)^2-11(2)-3)}{x-2}\\
&= \lim_{x\to 2}\frac{4x^2-11x-3-(-9)}{x-2}\\
&= \lim_{x\to 2}\frac{4x^2-11x+6}{x-2}\\
&= \lim_{x\to 2}\frac{(x-2)(4x-3)}{x-2}\\
&= \lim_{x\to 2}(4x-3)\\
&= 4(2)-3=5
\end{aligned}$$

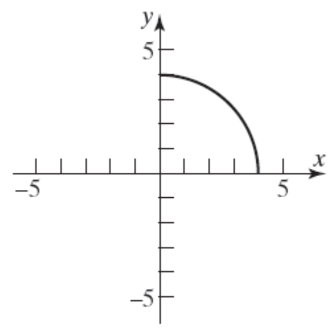
b. The derivative evaluated at $x=2$ is the slope of the tangent line to the graph of *f* at $x=2$. From part (a), we have $m_{\tan}=5$. Using the slope and the given point, $(2,-9)$, we can find the equation of the tangent line.

$$\begin{aligned}
y-y_1 &= m(x-x_1)\\
y-(-9) &= 5(x-2)\\
y+9 &= 5x-10\\
y &= 5x-19
\end{aligned}$$

Therefore, the equation of the tangent line to the graph of *f* at $x=2$ is $y=5x-19$.

15. a.

x	$y=f(x)$	(x,y)
0	$y=\sqrt{16-0^2}=4$	$(0,4)$
2	$y=\sqrt{16-2^2}=2\sqrt{3}$	$(2,2\sqrt{3})$
3	$y=\sqrt{16-3^2}=\sqrt{7}$	$(3,\sqrt{7})$
4	$y=\sqrt{16-4^2}=0$	$(4,0)$



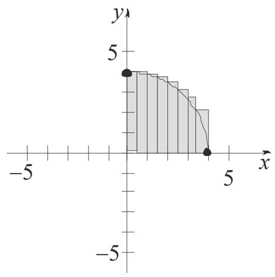
b. Each subinterval will have length

$$\frac{b-a}{8}=\frac{4-0}{8}=\frac{1}{2}$$

Since $u=$ left endpoint , we have

$$a=u_0=0,\ u_1=\frac{1}{2},\ u_2=1,\ u_3=\frac{3}{2},$$

$$u_4=2,\ u_5=\frac{5}{2},\ u_6=3,\ u_7=\frac{7}{2}$$



$$A \approx \frac{1}{2}[f(0)+f\left(\tfrac{1}{2}\right)+f(1)+f\left(\tfrac{3}{2}\right)+f(2)$$
$$+f\left(\tfrac{5}{2}\right)+f(3)+f\left(\tfrac{7}{2}\right)]$$
$$=\frac{1}{2}[4+3.969+3.873+3.708+3.464$$
$$+3.123+2.646+1.937]$$
$$=\frac{1}{2}(26.72)$$
$$=13.36 \text{ square units}$$

c. The desired region is one quarter of a circle with radius $r=4$. The area of this region is
$$A=\frac{1}{4}\left[\pi(4)^2\right]$$
$$=\frac{1}{4}(16\pi)=4\pi\approx 12.566 \text{ square units}$$
The estimate in part (b) is slightly larger than the actual area. Since the function is decreasing over the entire interval, the largest value of the function on a subinterval always occurs at the left endpoint. Therefore, we would expect our estimate to be larger than the actual value.

16. Area $=\int_1^4\left(-x^2+5x+3\right)dx$

17. The average rate of change is given by
$$a.r.c.=\frac{s(6)-s(3)}{6-3}=\frac{137-31}{6-3}$$
$$=\frac{106}{3}=35\frac{1}{3}\approx 35.33 \text{ ft. per sec.}$$

Chapter 14 Projects

Project I – Internet-based Project

Answers may will vary passed on the year that is used.

Total Midyear Population for the World: 1950-2050

t	Year	Population
0	1950	2,555,360,972
1	1951	2,593,139,857
2	1952	2,635,192,901
3	1953	2,680,522,529
4	1954	2,728,486,476
5	1955	2,779,929,940
6	1956	2,832,880,780
7	1957	2,888,699,042
8	1958	2,945,196,478
9	1959	2,997,522,100
10	1960	3,039,585,530
11	1961	3,080,367,474
12	1962	3,136,451,432
13	1963	3,205,956,565
14	1964	3,277,024,728
15	1965	3,346,002,675
16	1966	3,416,184,968
17	1967	3,485,881,292
18	1968	3,557,690,668
19	1969	3,632,294,522
20	1970	3,707,475,887
21	1971	3,784,957,162
22	1972	3,861,537,222
23	1973	3,937,599,035
24	1974	4,013,016,398
25	1975	4,086,150,193
26	1976	4,157,827,615
27	1977	4,229,922,943
28	1978	4,301,953,661
29	1979	4,376,897,872

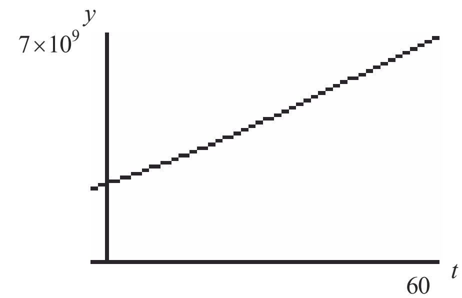
t	Year	Population
30	1980	4,452,584,592
31	1981	4,528,511,458
32	1982	4,608,410,617
33	1983	4,689,840,421
34	1984	4,769,886,824
35	1985	4,851,592,622
36	1986	4,934,892,988
37	1987	5,020,809,215
38	1988	5,107,404,183
39	1989	5,194,105,912
40	1990	5,281,653,820
41	1991	5,365,480,276
42	1992	5,449,369,636
43	1993	5,531,014,635
44	1994	5,611,269,983
45	1995	5,691,759,210
46	1996	5,770,701,020
47	1997	5,849,885,301
48	1998	5,927,556,529
49	1999	6,004,170,056
50	2000	6,079,603,571
51	2001	6,153,801,961
52	2002	6,226,933,918
53	2003	6,299,763,405
54	2004	6,372,797,742
55	2005	6,446,131,400

Source: U.S. Bureau of the Census, International Database.
Note: Data updated 9-30-2004
http://www.census.gov/ipc/www/worldpop.html

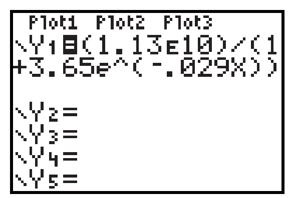
1.

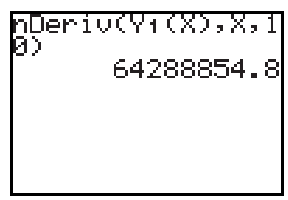
$$y = \frac{1.13\times 10^{10}}{1+3.65e^{-0.029t}}$$

2.



3.



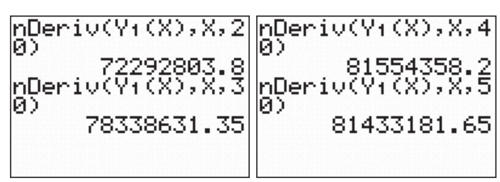


4. $y(1961) \approx 3{,}039{,}585{,}530 + 64{,}288{,}855$

$= 3{,}103{,}874{,}385$

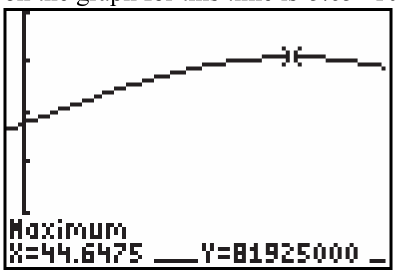
The actual population in 1961 was 3,080,367,474.

5.



The instantaneous growth is slowing down. Thus, Malthus' contention is not true.

6. The growth rate is largest in 1994. Then, the growth rate begins to decrease. The y value on the graph for this time is 5.65×10^9.



7. $\lim_{t\to\infty} f(t) = \lim_{t\to\infty}\left(\frac{1.13\times 10^{10}}{1+3.65e^{-0.029t}}\right)$

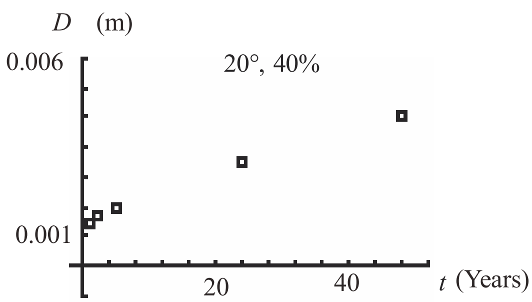
$= \frac{1.13\times 10^{10}}{1+0} = 1.13\times 10^{10}$

The carrying capacity of the Earth is 1.13×10^{10} people.

8. If the population exceeds the carrying capacity, the population will begin to die off very quickly due to hunger and disease in particular. There will not be enough agricultural growth to keep up with the increase in population. Urban sprawl will cause agricultural growth to diminish since land will be taken away.

Project II

1.



2. $t = 1$ to $t = 2$:

$\frac{\Delta D}{\Delta t} = \frac{0.0014 - 0.0011}{2-1} = 0.003$ m/hr

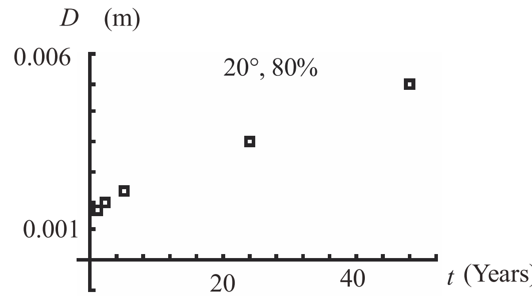
$t = 2$ to $t = 5$:

$\frac{\Delta D}{\Delta t} = \frac{0.0017 - 0.0014}{5-2} = 0.001$ m/hr

$t = 24$ to $t = 48$:

$\frac{\Delta D}{\Delta t} = \frac{0.004 - 0.003}{48-24} = 0.000042$ m/hr

3.



4. t = 1 to t = 2:

$\frac{\Delta D}{\Delta t} = \frac{0.002 - 0.0017}{2-1} = 0.0003 m/hr$

t = 2 to t = 5:

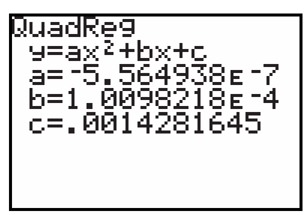
$\frac{\Delta D}{\Delta t} = \frac{0.0023 - 0.002}{5-2} = 0.0001 m/hr$

t = 24 to t = 48:

$\frac{\Delta D}{\Delta t} = \frac{0.006 - 0.004}{48-24} = 0.000083 m/hr$

5. There are no differences until the time span from t = 24 to t = 48. The highest rate is for 80 % RH from t = 24 to t = 48. The lowest rate is for 20% for the time span t = 24 to t = 48.

6.



$D(t) = -0.0000006t^2 + 0.0001t + 0.0014$

7. $D'(t) = \lim_{h\to 0} \dfrac{-0.0000006(t+h)^2 + 0.0001(t+h) + 0.0014 - (-0.0000006t^2 + 0.0001t + 0.0014)}{h}$

$= \lim_{h\to 0} \dfrac{-0.0000006(t^2 + 2th + h^2) + 0.0001(t+h) + 0.0014 + 0.0000006t^2 - 0.0001t - 0.0014}{h}$

$= \lim_{h\to 0} \dfrac{-0.0000012th - 0.0000006h^2 + 0.0001h}{h}$

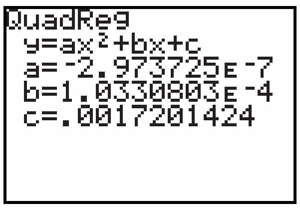
$= \lim_{h\to 0}(-0.0000012t - 0.000006h + 0.0001)$

$= -0.0000012t + 0.0001$

$D'(2) = 0.0000976$ m/hr ≈ 0.0001 m/hr

$D'(24) = 0.0000712$ m/hr

8.



$D(t) = -0.0000003t^2 + 0.0001t + 0.0017$

$D'(t) = \lim_{h\to 0} \dfrac{-0.0000003(t+h)^2 + 0.0001(t+h) + 0.0017 - (-0.0000003t^2 + 0.0001t + 0.0017)}{h}$

$= \lim_{h\to 0} \dfrac{-0.0000003(t^2 + 2th + h^2) + 0.0001(t+h) + 0.0017 + 0.0000003t^2 - 0.0001t - 0.0017}{h}$

$= \lim_{h\to 0} \dfrac{-0.0000006th - 0.0000003h^2 + 0.0001h}{h}$

$= \lim_{h\to 0}(-0.0000006t - 0.000003h + 0.0001)$

$= -0.0000006t + 0.0001$

$D'(2) = 0.0000988 m/hr \approx 0.0001 m/hr$

$D'(24) = 0.0000856 m/hr \approx 0.00009 m/hr$

The instantaneous rate of change is the same at t = 2, but the rates are different for t = 24.

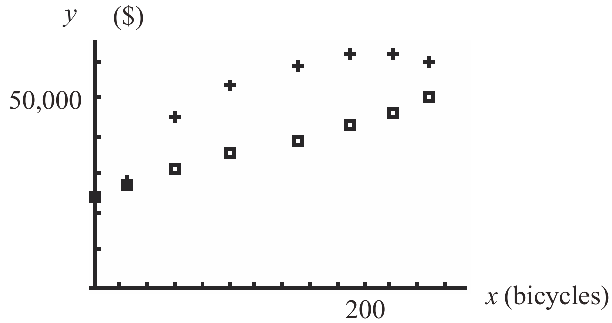
Project III

1.

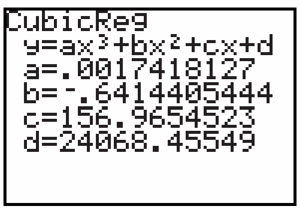
X	P (total profit)
0	-24000
25	250
60	8500
102	18150
150	20160
190	19610
223	14985
249	9625

The profit-maximizing level is 150 bicycles.

2.

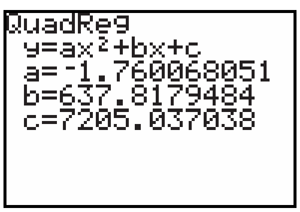


3.



$C(x) = 0.002x^3 - 0.6x^2 + 157x + 24068$

4.



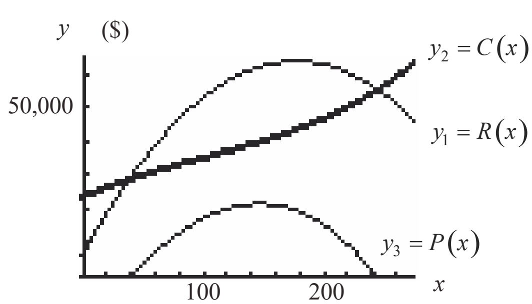
$R(x) = -1.8x^2 + 638x + 7205$

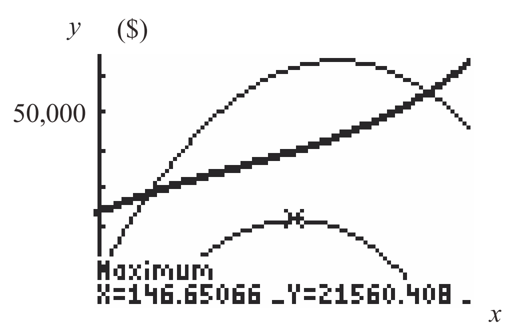
5. $P(x) = R(x) - C(x)$

$$= (-1.8x^2 + 638x + 7205) - (0.002x^3 - 0.6x^2 + 157x + 24068)$$

$$= -0.002x^3 - 2.4x^2 + 795x + 31273$$

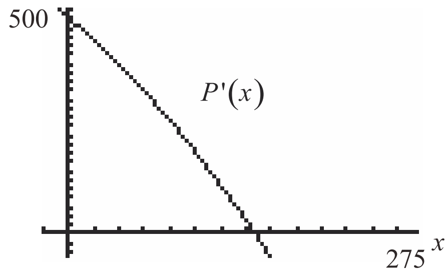
6.



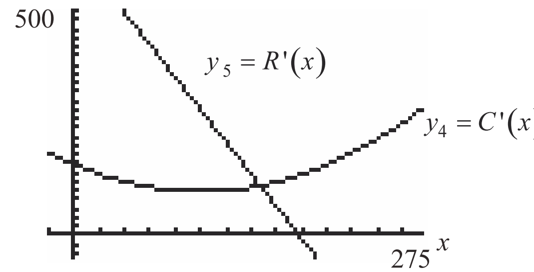


The answer in (a) was 150 and this one is 147. Those are very close.

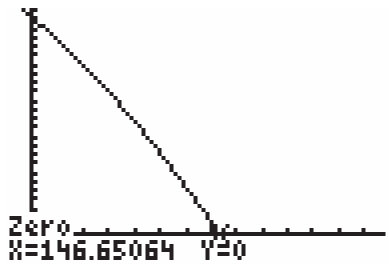
7.



9.

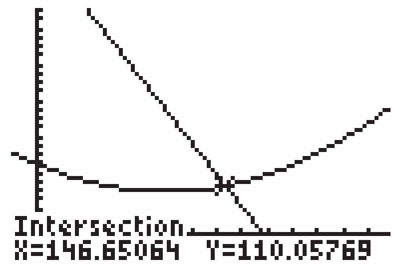


8.



This is the same result as in part (f).

10.



11. Marginal revenue is the rate of change of the revenue as another bicycle is produced. Marginal cost gives the change in cost that making the next bicycle will cause. When the rate of change of the revenue is the same as the rate of change of the cost, the two changes offset each other, thus maximizing the cost.