$$y+\frac{2}{3}(4)=\frac{10}{3}$$
$$y+\frac{8}{3}=\frac{10}{3}$$
$$y=\frac{2}{3}$$

Substitute 4 for z and $\frac{2}{3}$ for y in equation (1).

$$x+\frac{3}{2}\left(\frac{2}{3}\right)-(4)=\frac{1}{2}$$
$$x+1-4=\frac{1}{2}$$
$$x=\frac{1}{2}+3=\frac{7}{2}$$

The solution is the ordered triple $\left(\frac{7}{2}, \frac{2}{3}, 4\right)$.

15. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} 2 & 5 & -6 \\ -6 & -15 & 18 \end{array}\right] \qquad \left(R_1=\frac{1}{2}r_1\right)$$
$$=\left[\begin{array}{cc|c} 1 & \frac{5}{2} & -3 \\ -6 & -15 & 18 \end{array}\right] \qquad \left(R_2=6r_1+r_2\right)$$
$$=\left[\begin{array}{cc|c} 1 & \frac{5}{2} & -3 \\ 0 & 0 & 0 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x+\frac{5}{2}y=-3 & (1) \\ 0=0 & (2) \end{cases}$$

The statement 0 = 0 in equation (2) indicates that the system is dependent and has an infinite number of solutions.
The solution to the system is $\{(x, y) \mid 2x+5y=-6\}$.

16. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 8 \\ 2 & 3 & -10 & 19 \\ -1 & -2 & 7 & -11 \end{array}\right] \qquad \left(\begin{array}{l} R_2=-2r_1+r_2 \\ R_3=r_1+r_3 \end{array}\right)$$
$$=\left[\begin{array}{ccc|c} 1 & 1 & -3 & 8 \\ 0 & 1 & -4 & 3 \\ 0 & -1 & 4 & -3 \end{array}\right] \qquad \left(R_3=r_2+r_3\right)$$
$$=\left[\begin{array}{ccc|c} 1 & 1 & -3 & 8 \\ 0 & 1 & -4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x+y-3z=8 & (1) \\ y-4z=3 & (2) \\ 0=0 & (3) \end{cases}$$

The statement $0=0$ in equation (3) indicates that the system is dependent and has an infinite number of solutions.

Solve equation (2) for y.

$$y-4z=3$$
$$y=4z+3$$

Substitute $4z+3$ for y in equation (1).

$$x+(4z+3)-3z=8$$
$$x+z+3=8$$
$$x=-z+5$$

The solution to the system is
$\{(x, y, z) | x = -z + 5, y + 4z + 3, z$ is any real number$\}$

17. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} -2 & 3 & 4 \\ 10 & -15 & 2 \end{array}\right] \qquad \left(R_1=-\frac{1}{2}r_1\right)$$
$$=\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -2 \\ 10 & -15 & 2 \end{array}\right] \qquad \left(R_2=-10r_1+r_2\right)$$
$$=\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -2 \\ 0 & 0 & 22 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x-\frac{3}{2}y=-2 & (1) \\ 0=22 & (2) \end{cases}$$

Substitute $\frac{11}{34}$ for z in equation (2).

$$y-\frac{3}{10}\left(\frac{11}{34}\right)=-\frac{13}{20}$$
$$y-\frac{33}{340}=-\frac{13}{20}$$
$$y=-\frac{47}{85}$$

Substitute $\frac{11}{34}$ for z and $-\frac{47}{85}$ for y in equation (1).

$$4x+5\left(-\frac{47}{85}\right)-2\left(\frac{11}{34}\right)=0$$
$$4x-\frac{47}{17}-\frac{11}{17}=0$$
$$4x-\frac{58}{17}=0$$
$$4x=\frac{58}{17}$$
$$x=\frac{29}{34}$$

The solution is the ordered triple $\left(\frac{29}{34}, -\frac{47}{85}, \frac{11}{34}\right)$.

45. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} 1 & -3 & 3 \\ -2 & 6 & -6 \end{array}\right] \quad (R_2 = 2r_1 + r_2)$$
$$=\left[\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x-3y=3 & (1) \\ 0=0 & (2) \end{cases}$$

The statement 0 = 0 in equation (2) indicates that the system is dependent and has an infinite number of solutions.
The solution to the system is $\{(x, y) \mid x-3y=3\}$.

47. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ -2 & 4 & -1 & -5 \\ -8 & 16 & 1 & -21 \end{array}\right] \quad \begin{pmatrix} R_2 = 2r_1 + r_2 \\ R_3 = 8r_1 + r_3 \end{pmatrix}$$
$$=\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 9 & 3 \end{array}\right] \quad (R_3 = -9r_2 + r_3)$$
$$=\left[\begin{array}{ccc|c} 1 & -2 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -6 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x-2y+z=3 & (1) \\ z=1 & (2) \\ 0=-6 & (3) \end{cases}$$

The statement 0 = –6 in equation (3) indicates that the system is inconsistent. The system has no solution. The solution set is ∅ or { }.

49. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ -4 & 0 & 3 & -4 \\ -2 & 2 & -1 & 4 \end{array}\right] \quad \begin{pmatrix} R_2 = 4r_1 + r_2 \\ R_3 = 2r_1 + r_3 \end{pmatrix}$$
$$=\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 4 & -5 & 12 \\ 0 & 4 & -5 & 12 \end{array}\right] \quad \left(R_2 = \frac{1}{4}r_2\right)$$
$$=\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 4 & -5 & 12 \end{array}\right] \quad (R_3 = -4r_2 + r_3)$$
$$=\left[\begin{array}{ccc|c} 1 & 1 & -2 & 4 \\ 0 & 1 & -\frac{5}{4} & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix.

$$\begin{cases} x+y-2z=4 & (1) \\ y-\frac{4}{5}z=3 & (2) \\ 0=0 & (3) \end{cases}$$

The statement 0 = 0 in equation (3) indicates that the system is dependent and has an infinite number of solutions.
Solve equation (2) for y.

$$y=\frac{5}{4}z+3$$

Substitute $\frac{5}{4}z+3$ for y in equation (1).

$$x+\left(\frac{5}{4}z+3\right)-2z=4$$
$$x+\frac{5}{4}z-\frac{8}{4}z=1$$
$$x=\frac{3}{4}z+1$$

The solution is the ordered triple

$$\left\{(x,\ y,\ z)\middle|\, x=\frac{3}{4}z+1,\ y=\frac{5}{4}z+3,\ z \text{ is any real number}\right\}.$$

51. $\begin{cases} x+4y=-5 & (1) \\ y=-2 & (2) \end{cases}$

This system is consistent and independent.
Substitute -2 for y in equation (1).

$$x+4(-2)=-5$$
$$x-8=-5$$
$$x=3$$

The solution is the ordered pair (3, −2).

53. $\begin{cases} x+3y-2z=6 & (1) \\ y+5z=-2 & (2) \\ 0=4 & (3) \end{cases}$

This system is inconsistent. The solution is $\varnothing$ or $\{\ \}$.

55. $\begin{cases} x-2y-z=3 & (1) \\ y-2z=-8 & (2) \\ z=5 & (3) \end{cases}$

This system is consistent and independent.
Substitute 5 for z in equation (2).

$$y-2(5)=-8$$
$$y-10=-8$$
$$y=2$$

Substitute 5 for z and 2 for y in equation (1).

$$x-2(2)-5=3$$
$$x-4-5=3$$
$$x-9=3$$
$$x=12$$

The solution is the ordered triple $(12,\ 2,\ 5)$.

57. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} 1 & -3 & 18 \\ 2 & 1 & 1 \end{array}\right] \quad (R_2=-2r_1+r_2)$$
$$=\left[\begin{array}{cc|c} 1 & -3 & 18 \\ 0 & 7 & -35 \end{array}\right] \quad \left(R_2=\frac{1}{7}r_2\right)$$
$$=\left[\begin{array}{cc|c} 1 & -3 & 18 \\ 0 & 1 & -5 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix

$\begin{cases} x-3y=18 & (1) \\ y=-5 & (2) \end{cases}$

This system is consistent and independent.
Substitute –5 for y in equation (1).

$$x-3(-5)=18$$
$$x+15=18$$
$$x=3$$

The solution is the ordered pair (3, –5).

59. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} 2 & 4 & 10 \\ 1 & 2 & 3 \end{array}\right] \quad (\text{Interchange } r_1 \text{ and } r_2)$$
$$=\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 4 & 10 \end{array}\right] \quad (R_2=-2r_1+r_2)$$
$$=\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 4 \end{array}\right]$$

The system is inconsistent. The system has no solution. The solution set is $\varnothing$ or $\{\ \}$.

61. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{cc|c} 1 & -6 & 8 \\ 2 & 8 & -9 \end{array}\right] \quad (R_2=-2r_1+r_2)$$
$$=\left[\begin{array}{cc|c} 1 & -6 & 8 \\ 0 & 20 & -25 \end{array}\right] \quad \left(R_2=\frac{1}{20}r_2\right)$$
$$=\left[\begin{array}{cc|c} 1 & -6 & 8 \\ 0 & 1 & -\frac{5}{4} \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix

$\begin{cases} x-6y=8 & (1) \\ y=-\frac{5}{4} & (2) \end{cases}$

This system is consistent and independent.

Substitute $-\frac{5}{4}$ for y in equation (1).

Substitute –5 for z in equation (2).

$$y-\frac{3}{5}(-5)=3$$
$$y+3=3$$
$$y=0$$

Substitute –5 for z and 0 for y in equation (1).

$$x+3(0)-2(-5)=14$$
$$x+10=14$$
$$x=4$$

The solution is the ordered triple $(4,\ 0,-5)$.

69. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ -1 & 3 & 1 & -4 \\ 3 & 1 & 7 & -2 \end{array}\right] \quad (\text{Interchange } r_1 \text{ and } r_2)$$

$$=\left[\begin{array}{ccc|c} -1 & 3 & 1 & -4 \\ 2 & -1 & 3 & 1 \\ 3 & 1 & 7 & -2 \end{array}\right] \quad (R_1=-1\cdot r_1)$$

$$=\left[\begin{array}{ccc|c} 1 & -3 & -1 & 4 \\ 2 & -1 & 3 & 1 \\ 3 & 1 & 7 & -2 \end{array}\right] \quad \begin{pmatrix} R_2=-2r_1+r_2 \\ R_3=-3r_1+r_3 \end{pmatrix}$$

$$=\left[\begin{array}{ccc|c} 1 & -3 & -1 & 4 \\ 0 & 5 & 5 & -7 \\ 0 & 10 & 10 & -14 \end{array}\right] \quad (R_3=-2r_2+r_3)$$

$$=\left[\begin{array}{ccc|c} 1 & -3 & -1 & 4 \\ 0 & 5 & 5 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system is dependent and has an infinite number of solutions.

Write the system of equations that corresponds to the row-echelon matrix

$$\begin{cases} x-3y-z=4 & (1) \\ 5y+5z=-7 & (2) \\ 0=0 & (3) \end{cases}$$

Solve equation (2) for y.

$$5y+5z=-7$$
$$5y=-5z-7$$
$$y=-z-\frac{7}{5}=-z-1.4$$

Substitute $-z-1.4$ for y in equation (1).

$$x-3(-z-1.4)-z=4$$
$$x+3z+4.2-z=4$$
$$x+2z+4.2=4$$
$$x=-2z-0.2$$

The solution to the system is

$\{(x,y,z)|x=-2z-0.2,$
$y=-z-1.4,\ z \text{ is any real number}\}$.

71. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 3 & 1 & -4 & 0 \\ -2 & -3 & 1 & 5 \\ -1 & -5 & -2 & 3 \end{array}\right] \quad (\text{Interchange } r_1 \text{ and } r_3)$$

$$=\left[\begin{array}{ccc|c} -1 & -5 & -2 & 3 \\ -2 & -3 & 1 & 5 \\ 3 & 1 & -4 & 0 \end{array}\right] \quad (R_1=-1\cdot r_1)$$

$$=\left[\begin{array}{ccc|c} 1 & 5 & 2 & -3 \\ -2 & -3 & 1 & 5 \\ 3 & 1 & -4 & 0 \end{array}\right] \quad \begin{pmatrix} R_2=2r_1+r_2 \\ R_3=-3r_1+r_3 \end{pmatrix}$$

$$=\left[\begin{array}{ccc|c} 1 & 5 & 2 & -3 \\ 0 & 7 & 5 & -1 \\ 0 & -14 & -10 & 9 \end{array}\right] \quad (R_3=2r_2+r_3)$$

$$=\left[\begin{array}{ccc|c} 1 & 5 & 2 & -3 \\ 0 & 7 & 5 & -1 \\ 0 & 0 & 0 & 7 \end{array}\right]$$

The system is inconsistent. The system has no solution. The solution set is $\varnothing$ or $\{\ \}$.

73. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & -1 \\ 3 & 1 & -4 & 3 \\ 1 & 7 & -2 & 2 \end{array}\right] \quad (\text{Interchange } r_1 \text{ and } r_3)$$

$$=\left[\begin{array}{ccc|c} 1 & 7 & -2 & 2 \\ 3 & 1 & -4 & 3 \\ 2 & -1 & 3 & -1 \end{array}\right] \quad \begin{pmatrix} R_2=-3r_1+r_2 \\ R_3=-2r_1+r_3 \end{pmatrix}$$

$$=\left[\begin{array}{ccc|c} 1 & 7 & -2 & 2 \\ 0 & -20 & 2 & -3 \\ 0 & -15 & 7 & -5 \end{array}\right] \quad \left(R_2=-\frac{1}{20}r_2\right)$$

$$=\left[\begin{array}{ccc|c} 1 & 7 & -2 & 2 \\ 0 & 1 & -\frac{1}{10} & \frac{3}{20} \\ 0 & -15 & 7 & -5 \end{array}\right] \quad (R_3=15r_2+r_3)$$

$$=\left[\begin{array}{ccc|c} 1 & 7 & -2 & 2 \\ 0 & 1 & -\frac{1}{10} & \frac{3}{20} \\ 0 & 0 & \frac{11}{2} & -\frac{11}{4} \end{array}\right] \quad \left(R_3 = \frac{2}{11}r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & 7 & -2 & 2 \\ 0 & 1 & -\frac{1}{10} & \frac{3}{20} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix

$$\begin{cases} x+7y-2z=2 & (1) \\ y-\frac{1}{10}z=\frac{3}{20} & (2) \\ z=-\frac{1}{2} & (3) \end{cases}$$

This system is consistent and independent.

Substitute $-\frac{1}{2}$ for z in equation (2).

$$y-\frac{1}{10}\left(-\frac{1}{2}\right)=\frac{3}{20}$$
$$y+\frac{1}{20}=\frac{3}{20}$$
$$y=\frac{2}{20}=\frac{1}{10}$$

Substitute $-\frac{1}{2}$ for z and $\frac{1}{10}$ for y in equation (1).

$$x+7\left(\frac{1}{10}\right)-2\left(-\frac{1}{2}\right)=2$$
$$x+\frac{7}{10}+1=2$$
$$x+\frac{17}{10}=2$$
$$x=\frac{3}{10}$$

The solution is the ordered triple $\left(\frac{3}{10}, \frac{1}{10}, -\frac{1}{2}\right)$.

75. Write the augmented matrix of the system and then put it in row echelon form.

$$\left[\begin{array}{ccc|c} 3 & 5 & 2 & 6 \\ 0 & 10 & -2 & 5 \\ 6 & 0 & 4 & 8 \end{array}\right] \quad \left(R_1 = \frac{1}{3}r_1\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 10 & -2 & 5 \\ 6 & 0 & 4 & 8 \end{array}\right] \quad \left(R_3 = -6r_1 + r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 10 & -2 & 5 \\ 0 & -10 & 0 & -4 \end{array}\right] \quad \left(R_3 = -\frac{1}{10}r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 10 & -2 & 5 \\ 0 & 1 & 0 & \frac{2}{5} \end{array}\right] \quad \left(\text{Interchange } r_2 \text{ and } r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 10 & -2 & 5 \end{array}\right] \quad \left(R_3 = -10r_2 + r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & -2 & 1 \end{array}\right] \quad \left(R_3 = -\frac{1}{2}r_3\right)$$

$$=\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{2}{3} & 2 \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix

$$\begin{cases} x+\frac{5}{3}y+\frac{2}{3}z=2 & (1) \\ y=\frac{2}{5} & (2) \\ z=-\frac{1}{2} & (3) \end{cases}$$

This system is consistent and independent.

Substitute $\frac{2}{5}$ for y and $-\frac{1}{2}$ for z in equation (1).

$$x+\frac{5}{3}\left(\frac{2}{5}\right)+\frac{2}{3}\left(-\frac{1}{2}\right)=2$$
$$x+\frac{2}{3}-\frac{1}{3}=2$$
$$x+\frac{1}{3}=2$$
$$x=\frac{5}{3}$$

The solution is the ordered triple $\left(\frac{5}{3}, \frac{2}{5}, -\frac{1}{2}\right)$.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 20{,}000 \\ 0.03 & 0.04 & 0.06 & 820 \\ 1 & 0 & -1 & 3000 \end{array}\right] \begin{pmatrix} R_2 = -0.03r + r_2 \\ R_3 = -1r_1 + r_3 \end{pmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20{,}000 \\ 0 & 0.01 & 0.03 & 220 \\ 0 & -1 & -2 & -17{,}000 \end{array}\right] (R_2 = 100r_2)$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20{,}000 \\ 0 & 1 & 3 & 22{,}000 \\ 0 & -1 & -2 & -17{,}000 \end{array}\right] (R_3 = r_2 + r_3)$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 20{,}000 \\ 0 & 1 & 3 & 22{,}000 \\ 0 & 0 & 1 & 5000 \end{array}\right]$$

Write the system of equations that corresponds to the row-echelon matrix

$$\begin{cases} t + m + c = 20{,}000 & (1) \\ m + 3c = 22{,}000 & (2) \\ c = 5000 & (3) \end{cases}$$

Substitute 5000 for c in equation (2).

$$m + 3(5000) = 22{,}000$$
$$m + 15{,}000 = 22{,}000$$
$$m = 7000$$

Substitute 5000 for c and 7000 for m in equation (1).

$$t + 7000 + 5000 = 20{,}000$$
$$t + 12{,}000 = 20{,}000$$
$$t = 8000$$

Therefore, Carissa should invest \$8000 in Treasury bills, \$7000 in municipal bonds, and \$5000 in corporate bonds.

83. Write the augmented matrix of the system and then put it in reduced row echelon form.

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ -3 & -2 & -5 \end{array}\right] \quad (R_1 = 2r_1 + r_2)$$

$$= \left[\begin{array}{cc|c} 1 & 0 & -3 \\ -3 & -2 & -5 \end{array}\right] \quad (R_2 = 3r_1 + r_2)$$

$$= \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & -2 & -14 \end{array}\right] \quad \left(R_2 = -\frac{1}{2}r_2\right)$$

$$= \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 7 \end{array}\right]$$

Write the system of equations that corresponds to the reduced row echelon matrix

$$\begin{cases} x = -3 \\ y = 7 \end{cases}$$

The solution is the ordered pair $(-3,\ 7)$.

85. Write the augmented matrix of the system and then put it in reduced row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 1 & -4 & 25 \\ -3 & 2 & 1 & 0 \end{array}\right] \quad \begin{pmatrix} R_2 = -2r_1 + r_2 \\ R_3 = 3r_1 + r_3 \end{pmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -6 & 19 \\ 0 & 5 & 4 & 9 \end{array}\right] \quad \begin{pmatrix} R_1 = r_1 + r_2 \\ R_3 = 5r_2 + r_3 \end{pmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -5 & 22 \\ 0 & -1 & -6 & 19 \\ 0 & 0 & -26 & 104 \end{array}\right] \quad \left(R_3 = -\frac{1}{26}r_3\right)$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -5 & 22 \\ 0 & -1 & -6 & 19 \\ 0 & 0 & 1 & -4 \end{array}\right] \quad \begin{pmatrix} R_1 = 5r_3 + r_1 \\ R_2 = 6r_3 + r_2 \end{pmatrix}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & -5 \\ 0 & 0 & 1 & -4 \end{array}\right] \quad (R_2 = -1 \cdot r_2)$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

Write the system of equations that corresponds to the reduced row echelon matrix

$$\begin{cases} x = 2 \\ y = 5 \\ z = -4 \end{cases}$$

The solution is the ordered triple $(2,\ 5,\ -4)$.

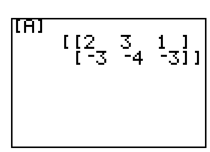
87. Answers will vary.

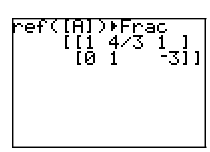
89. Multiply each entry in row 2 by $\frac{1}{5}$ (or divide each entry of row 2 by 5). That is, use the row operation $R_2 = \frac{1}{5}r_2$.

91. Write the augmented matrix of the system.

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ -3 & -4 & -3 \end{array}\right]$$

Enter the system into a 2 by 3 matrix, [A]. Then, use the **ref(** command along with the ▸**frac** command to write the matrix in row echelon form with the entries in fractional form.





5. $D = \begin{vmatrix} 3 & 2 \\ -2 & -1 \end{vmatrix} = 3(-1) - (-2)(2) = -3 + 4 = 1$

$D_x = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = 1(-1) - 1(2) = -1 - 2 = -3$

$D_y = \begin{vmatrix} 3 & 1 \\ -2 & 1 \end{vmatrix} = 3(1) - (-2)(1) = 3 + 2 = 5$

$x = \dfrac{D_x}{D} = \dfrac{-3}{1} = -3$; $y = \dfrac{D_y}{D} = \dfrac{5}{1} = 5$

Thus, the solution is the ordered pair $(-3,\ 5)$.

6. $D = \begin{vmatrix} 4 & -2 \\ -6 & 3 \end{vmatrix} = 4(3) - (-6)(-2) = 12 - 12 = 0$

Since $D = 0$, Cramer's Rule does not apply.

7. $\begin{vmatrix} 2 & -3 & 5 \\ 0 & 4 & -1 \\ 3 & 8 & -7 \end{vmatrix} = 2\begin{vmatrix} 4 & -1 \\ 8 & -7 \end{vmatrix} - (-3)\begin{vmatrix} 0 & -1 \\ 3 & -7 \end{vmatrix} + 5\begin{vmatrix} 0 & 4 \\ 3 & 8 \end{vmatrix}$

$= 2\left[4(-7) - 8(-1)\right] + 3\left[0(-7) - 3(-1)\right] + 5\left[0(8) - 3(4)\right]$
$= 2(-28 + 8) + 3(0 + 3) + 5(0 - 12)$
$= 2(-20) + 3(3) + 5(-12)$
$= -40 + 9 - 60$
$= -91$

8. $\begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & -3 \\ -5 & -1 & -5 \end{vmatrix} = 3\begin{vmatrix} 1 & -3 \\ -1 & -5 \end{vmatrix} - 2\begin{vmatrix} 1 & -3 \\ -5 & -5 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -5 & -1 \end{vmatrix}$

$= 3[1(-5) - (-1)(-3)] - 2[1(-5) - (-5)(-3)] + 1[1(-1) - (-5)(1)]$
$= 3(-5 - 3) - 2(-5 - 15) + 1(-1 + 5)$
$= 3(-8) - 2(-20) + 1(4)$
$= -24 + 40 + 4$
$= 20$

9. $x = \dfrac{D_x}{D} = \dfrac{2}{4} = \dfrac{1}{2}$

$y = \dfrac{D_y}{D} = \dfrac{-8}{4} = -2$

$z = \dfrac{D_z}{D} = \dfrac{-4}{4} = -1$

Thus, the solution is the ordered triple $\left(\dfrac{1}{2}, -2, -1\right)$.

C.4 Exercises

11. $\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = 4(3) - 1(2) = 12 - 2 = 10$

13. $\begin{vmatrix} -2 & -4 \\ 1 & 3 \end{vmatrix} = -2(3) - 1(-4) = -6 + 4 = -2$

15. $D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1) - 1(1) = -1 - 1 = -2$

$D_x = \begin{vmatrix} -4 & 1 \\ -12 & -1 \end{vmatrix}$
$= -4(-1) - (-12)(1) = 4 + 12 = 16$

$D_y = \begin{vmatrix} 1 & -4 \\ 1 & -12 \end{vmatrix} = 1(-12) - 1(-4) = -12 + 4 = -8$

$x = \dfrac{D_x}{D} = \dfrac{16}{-2} = -8$; $y = \dfrac{D_y}{D} = \dfrac{-8}{-2} = 4$

Thus, the solution is the ordered pair $(-8, 4)$.

17. $D = \begin{vmatrix} 2 & 3 \\ -3 & 1 \end{vmatrix} = 2(1) - (-3)(3) = 2 + 9 = 11$

$D_x = \begin{vmatrix} 3 & 3 \\ -10 & 1 \end{vmatrix} = 3(1) - (-10)(3) = 3 + 30 = 33$

$D_y = \begin{vmatrix} 2 & 3 \\ -3 & -10 \end{vmatrix}$
$= 2(-10) - (-3)(3) = -20 + 9 = -11$

$x = \dfrac{D_x}{D} = \dfrac{33}{11} = 3$; $y = \dfrac{D_y}{D} = \dfrac{-11}{11} = -1$

Thus, the solution is the ordered pair (3, −1).

19. $D = \begin{vmatrix} 3 & 4 \\ -6 & 8 \end{vmatrix} = 3(8) - (-6)(4) = 24 + 24 = 48$

$D_x = \begin{vmatrix} 1 & 4 \\ 4 & 8 \end{vmatrix} = 1(8) - 4(4) = 8 - 16 = -8$

$D_y = \begin{vmatrix} 3 & 1 \\ -6 & 4 \end{vmatrix} = 3(4) - (-6)(1) = 12 + 6 = 18$

$x = \dfrac{D_x}{D} = \dfrac{-8}{48} = -\dfrac{1}{6}$; $y = \dfrac{D_y}{D} = \dfrac{18}{48} = \dfrac{3}{8}$

Thus, the solution is the ordered pair $\left(-\dfrac{1}{6}, \dfrac{3}{8}\right)$.

21. The system in standard form is: $\begin{cases} 2x - 6y = 12 \\ 3x - 5y = 11 \end{cases}$

$D = \begin{vmatrix} 2 & -6 \\ 3 & -5 \end{vmatrix} = 2(-5) - 3(-6) = -10 + 18 = 8$

$D_x = \begin{vmatrix} 12 & -6 \\ 11 & -5 \end{vmatrix}$
$= 12(-5) - 11(-6)$
$= -60 + 66$
$= 6$

$D_y = \begin{vmatrix} 2 & 12 \\ 3 & 11 \end{vmatrix} = 2(11) - 3(12) = 22 - 36 = -14$

$x = \dfrac{D_x}{D} = \dfrac{6}{8} = \dfrac{3}{4}$; $y = \dfrac{D_y}{D} = \dfrac{-14}{8} = -\dfrac{7}{4}$

Thus, the solution is the ordered pair $\left(\dfrac{3}{4}, -\dfrac{7}{4}\right)$.

23. $\begin{vmatrix} 2 & 0 & -1 \\ 3 & 8 & -3 \\ 1 & 5 & -2 \end{vmatrix}$

$= 2\begin{vmatrix} 8 & -3 \\ 5 & -2 \end{vmatrix} - 0\begin{vmatrix} 3 & -3 \\ 1 & -2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 8 \\ 1 & 5 \end{vmatrix}$
$= 2[8(-2) - 5(-3)] - 0[3(-2) - 1(-3)]$
$\quad + (-1)[3(5) - 1(8)]$
$= 2(-16 + 15) - 0(-6 + 3) + (-1)(15 - 8)$
$= 2(-1) - 0(-3) + (-1)(7)$
$= -2 + 0 - 7$
$= -9$

25. $\begin{vmatrix} -3 & 2 & 3 \\ 0 & 5 & -2 \\ 1 & 4 & 8 \end{vmatrix}$

$= (-3)\begin{vmatrix} 5 & -2 \\ 4 & 8 \end{vmatrix} - 2\begin{vmatrix} 0 & -2 \\ 1 & 8 \end{vmatrix} + 3\begin{vmatrix} 0 & 5 \\ 1 & 4 \end{vmatrix}$
$= -3[5(8) - 4(-2)] - 2[0(8) - 1(-2)]$
$\quad + 3[0(4) - 1(5)]$
$= -3(40 + 8) - 2(0 + 2) + 3(0 - 5)$
$= -3(48) - 2(2) + 3(-5)$
$= -144 - 4 - 15$
$= -163$