

Coulomb interaction revised in the presence of material with negative permittivity

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Coulomb law is one of the fundamental laws in Physics. It describes the magnitude of the electrostatic force between two electric charges. Counterintuitively the repulsion force between two equal electric charges in a vacuum, stated by the Coulomb law, turn into the attraction force between the same electric charges when they are placed next to a material with negative permittivity and the distance between them is larger than some critical distance. As a result the equally charged particles “crystallize” occupying equilibrium positions. We prove this claim with the method of images for two charged particles placed next to a material with negative permittivity.

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I. INTRODUCTION

Most materials have positive permittivity. If such a material is placed in an electric field, then the direction of the field induced inside the material will have the same orientation as the applied field. Whereas the field inside a material with negative permittivity would be oriented in the opposite direction to the applied field. The most popular substances with negative permittivity are gaseous plasmas and solid-state plasmas [1, 2]. In a plasma with no magnetic field, the permittivity $\varepsilon/\varepsilon_0$ (ε_0 -vacuum permittivity) is given by

$$\varepsilon/\varepsilon_0 = 1 - \omega_0^2/\omega^2, \quad (1)$$

where $\omega_0 = \sqrt{4\pi Ne^2/m}$ is the plasma frequency, N is the concentration of the carriers, e is their charge, and m is their mass. It is not hard to check that when the frequency ω is smaller compared to the plasma frequency ω_0 , then ε is negative.

It has been shown that wire structures with lattice spacing of the order of a few millimeters behave like a plasma with a resonant frequency, ω_0 , in the GHz regions. These metamaterials gain their properties from the structure rather than the composition [3, 4]. Such materials attract growing interest for the last decade from theoretical and experimental perspectives. The presence of simultaneously negative permeability and permittivity in these materials [4, 5] lead to many unusual effects which find various applications in areas such as perfect and hyper lenses [6–12], suppression of spontaneous emission of an atom in front of a mirror made by metamaterial [13], magnification of objects that are smaller than the wavelength [14] and creation of second-harmonic generation [15].

Veselago made a theoretical study of materials with negative electrical permittivity, ε and negative magnetic permeability, μ in his seminal paper [1]. The conclusion of his article is that this type of materials support electromagnetic waves description, but the energy flow, directed by the Poynting vector, is in the opposite direction to the wave vector. This means that rays travel in the opposite direction to waves. The most important

result of Veselago’s work is that when both ε and μ are negative, the refraction index, defined by the equation $n^2 = \varepsilon\mu$, is negative

$$n = -\sqrt{\varepsilon\mu}. \quad (2)$$

The key experimental consequence is the rather unusual manifestation of Snell’s law. In the passage of a ray of light from one medium with positive refraction index $n_1 > 0$ into another, with $n_2 < 0$ the Snell’s law is satisfied but n_1/n_2 is negative. In materials with negative refraction index the Cherenkov effect is reversed, just like the Doppler effect [16].

The Veselago theory is focused on the optical properties of materials with negative permittivity and permeability. In the present paper we consider the electric forces in the presence of material with negative permittivity. We explore theoretically the force acting on a single charged particle and the interaction between two charged particles placed next to a material with negative permittivity. Our examination relies on the Pendry’s criterion for validity of electrostatic limit in the present problem [17]: the wavelength, corresponding to the frequency ω in Eq.1, should be longer compare with the distance of the charged particles to the material with negative permittivity. We show, using the method of images [18–20], that for negative permittivity of the material, the force between the material and the charged particle can be attractive or repulsive and can even have a bigger value compared to the conventional materials. Furthermore, the repulsion force between two equally charged electric particles in a vacuum, stated by the Coulomb law, turn into the attraction force between the same electric charges when they are placed next to a material with negative permittivity and the distance between them is larger than some critical distance. As a result the two equally charged particles “crystallize” occupying equilibrium states.

II. METHOD OF IMAGES FOR MATERIAL WITH NEGATIVE PERMITTIVITY

A. A point charge above dielectric plane

Initially we start our examination with the simplest possible case of a single point charge q , which is embedded in a semi-infinite dielectric with permittivity ε_1 at a distance d away from a plane interface that separates the first medium from another semi-infinite dielectric with permittivity ε_2 (Fig.1). We want to find what the force acting on the charge q is. From the point of view of Mathematics our problem is to solve Poisson's equation in a region with permittivity ε_1 , and a point charge q , subject to the boundary conditions at the plane interface that separates the first medium from the second. The solution of this problem is easily found by the method of images [18–20]. This method is a powerful and easy way to handle solutions of differential equations, in which the domain of the sought function is extended by the addition of its mirror image with respect to a symmetry hyperplane. In our case the electrical potential will be fully reconstructed in the space occupied by the dielectric ε_1 if we place an image of q with a charge magnitude

$$q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} q \quad (3)$$

at a distance d away from a plane in the space occupied by the dielectric ε_2 [18–20] (Fig.1). Therefore the net force acting on the charge q is

$$F = \frac{1}{4\pi\varepsilon_1} \frac{qq'}{(2d)^2} = \frac{q^2}{16\pi\varepsilon_1 d^2} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}, \quad (4)$$

which force could be attractive or repulsive depending on the ratio $(\varepsilon_1 - \varepsilon_2) / (\varepsilon_1 + \varepsilon_2)$. The maximal attraction force for conventional materials ($\varepsilon_1 > 0$, $\varepsilon_2 > 0$) is [18–20]

$$F = -\frac{q^2}{16\pi\varepsilon_0 d^2}. \quad (5)$$

We are interested in the case when the first media is a vacuum, but the second media is a material with negative permittivity ($\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 < 0$). In this case the net force acting on the charge q is

$$F = \frac{q^2}{16\pi\varepsilon_0 d^2} \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}. \quad (6)$$

If the permittivity ε_2 of the material satisfies

$$|\varepsilon_2| < \varepsilon_0, \quad (7)$$

then the force from Eq.(6) has a positive sign, therefore the charge is repelled from this material. The force from Eq.(6) is attractive if

$$|\varepsilon_2| > \varepsilon_0 \quad (8)$$

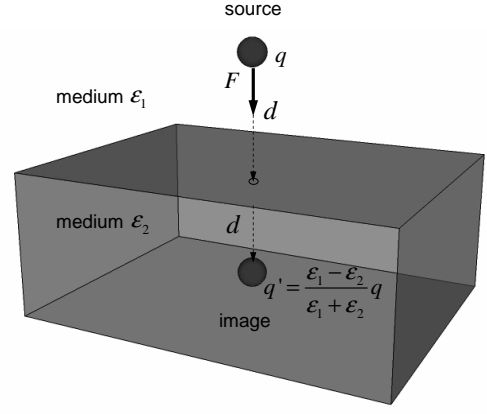


FIG. 1: A point charge q embedded in a semi-infinite dielectric with permittivity ε_1 at a distance d away from a plane interface that separates the two mediums and its image charged symmetrically placed in a semi-infinite dielectric with permittivity ε_2 .

Obvious singularity happens in Eq.(6) when $\varepsilon_2 = -\varepsilon_0$. One might make a mistake thinking that this singularity leads to an infinite force, but one should note that dispersionless material is an abstraction and a finite positive imaginary component of the permittivity always exists [17]. The imaginary component of the permittivity can no longer be neglected when the difference between the real parts of the two dielectric permittivities is a small number, which resolve the paradox with the infinite force.

An important case is when we can neglect the imaginary part of the permittivity ε_2 . Then the force from Eq.(6) could be repulsive and bigger compared to the module of the force from Eq.(5). For example if $\varepsilon_2 = -\varepsilon_0/2$, then the net force acting on the charge q is

$$F = \frac{3q^2}{16\pi\varepsilon_0 d^2}, \quad (9)$$

which is a repulsive force and three times stronger than the module of the force that the same charge q , placed at the same distance d , feels from the plane interface of a metal (see Eq.(5)).

B. Two point charges above a dielectric plane

Even a more interesting situation with a counterintuitive result is the case when two charged particles are placed in a vacuum next to a material with negative permittivity ε (Fig.2). To examine the situation in more details, we consider the case when the two charged particles q_1 and q_2 are in a vacuum at a distance d away from the material with negative permittivity ε . The method of images states that the system is described equivalently by two images for each charge q_1 and q_2 placed at a distance d away from the surface of the material ε (Fig.2)

with charges

$$q'_1 = \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} q_1, \quad (10)$$

$$q'_2 = \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} q_2. \quad (11)$$

For convenience let us represent the force in two components:

$$F_{\parallel} = \frac{q_1 q_2}{4\pi\varepsilon_0 L^2} \left(1 + \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \frac{L^3}{(L^2 + 4d^2)^{3/2}} \right), \quad (12)$$

$$F_{\alpha\perp} = \frac{q_\alpha^2}{16\pi\varepsilon_0 d^2} \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \left(1 + \frac{q_1 q_2}{q_\alpha^2} \frac{8d^3}{(L^2 + 4d^2)^{3/2}} \right), \quad (13)$$

where $\alpha = 1$ or 2 and L is the distance between the charged particles. The perpendicular to the material component of the force can be thought of as an effective force between the charged particles and the material, while the parallel component as an effective interaction between the particles.

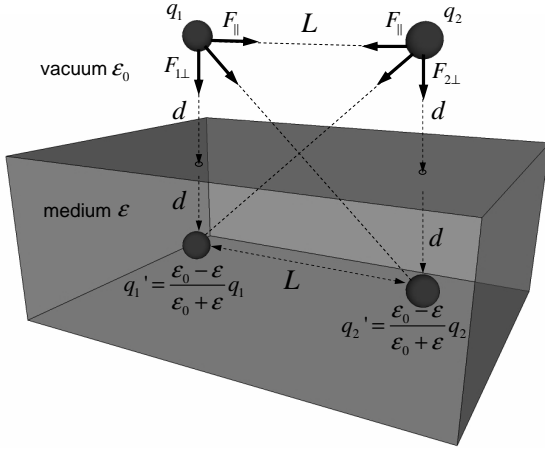


FIG. 2: Two point charges q_1 and q_2 placed in a vacuum at a distance d away from a plane interface that separates the vacuum with a semi-infinite dielectric with permittivity ε and their images q'_1 and q'_2 .

The expression in the brackets in Eq.(12) is positive when $\varepsilon > 0$ for arbitrary values of d and L . This means that the character of the interaction (attractive or repulsive) is the same as that stated by the Coulomb law.

When $\varepsilon < 0$ and $|\varepsilon| > \varepsilon_0$ there is a critical distance between the particles L_{cr} at which the interaction is zero ($F_{\parallel}(L_{cr}) = 0$)

$$L_{cr} = \frac{2d}{\sqrt{\left(\frac{|\varepsilon| + \varepsilon_0}{|\varepsilon| - \varepsilon_0}\right)^{2/3} - 1}}. \quad (14)$$

From Eq.(12) it follows that: when the two particles are closer ($L < L_{cr}$), the second term in the brackets is

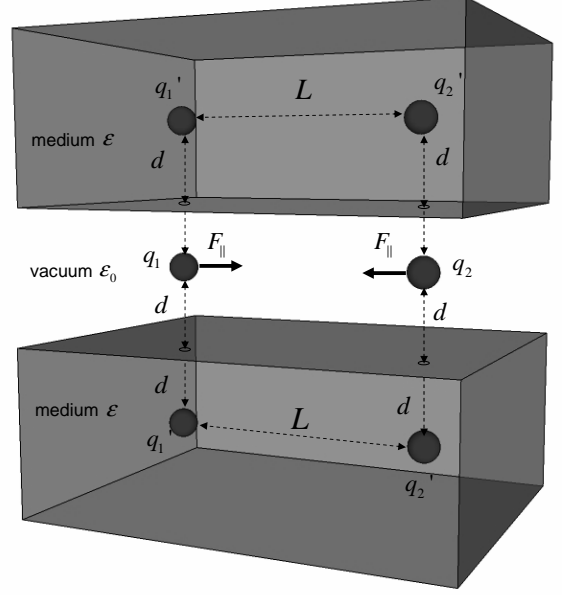


FIG. 3: The symmetric geometry case of two charged particles q_1 and q_2 , placed in a middle way between two identical semi-infinite metamaterials with permittivity ε and their images q'_1 and q'_2 .

smaller than the first one and the interaction between the charges is the same as stated by the Coulomb law. But when the distance between the particles is larger than the critical one ($L_{cr} < L$), the expression in the brackets of Eq.(12) is negative and the force has an opposite sign compared to the force given by the Coulomb law.

The total perpendicular component of the force can be zero ($L_{\alpha\perp} = 0$) if a symmetric geometry is realized. For example this is the case when the charged particles are placed in a vacuum sandwich with two identical semi-infinite materials with negative permittivity ε (Fig.3). In the particular case when the two particles have equal charges $q_1 = q_2 = q$ and $|\varepsilon| > \varepsilon_0$, the interaction between them is

$$F_{\parallel} = \frac{q^2}{4\pi\varepsilon_0 L^2} \left(1 - 2 \frac{|\varepsilon| + \varepsilon_0}{|\varepsilon| - \varepsilon_0} \frac{L^3}{(L^2 + 4d^2)^{3/2}} \right), \quad (15)$$

The factor 2 in the brackets in Eq (15) is owing to the geometry of the system. It changes when the geometry of the system is altered. The critical distance at which the force between particles Eqs (15) is zero ($F_{\parallel}(L'_{cr}) = 0$) depends on the geometry too

$$L'_{cr} = \frac{2d}{\sqrt{\left(2 \frac{|\varepsilon| + \varepsilon_0}{|\varepsilon| - \varepsilon_0}\right)^{2/3} - 1}}. \quad (16)$$

The force, Eq (15), between two equal charged particles is repulsive when they are closer than the critical distance L'_{cr} , and it is attractive when the distance is larger

than the critical one. As a result the particles “crystallize” occupying equilibrium positions at a distance equal to the critical distance L'_{cr} .

The scheme could be extended to many particles. By increasing the number of the particles, the critical distance decreases. In this way we can make an array of equally charged particles situated at a very small distance from each other. This permits us to construct a new type of equally charged particles trap.

By making the permittivity ε to vary from negative to positive, we release the particles and they will have a kinetic energy depending on the critical distance between them. This enables us to accelerate the charges in a new fashion.

III. CONCLUSIONS

The examined force between a charged particle and a material with negative permittivity has the potential to

be not only a curious and intriguing example of what the artificial material can do, but also a useful technique to levitate small objects with charges and therefore to make frictionless devices.

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