

# Applying Radial Basis Networks and Markov Chains for on-line detection of concept drift in non-stationary environments

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## Abstract

The amount of information produced by computer systems has grown sharply in recent decades. Most of this information is presented in the form of uninterrupted and potentially infinite sequences known as data streams. These streams are usually produced in non-stationary environments, in which the data distribution can change over time, possibly deteriorating the system performance. In the literature, this phenomenon is named concept drift. Nevertheless, most drift detection methods are unsuited for non-stationary environments with data streams. These algorithms or require the correct labeling of data - infeasible in these settings, or do not match the strict response time and resource usage restrictions inherent to data streams. In an attempt to mitigate the aforementioned problem, this paper proposes a novel proactive method for on-line drift detection, called RBFChain. The proposed method relies on Radial Basis Networks implicit clustering property and uses Markov Chains to model the drifts transitions. To assess the proposed method as a viable concept drift detector, an analysis of sensitivity, accuracy, and noise tolerance was performed using synthetic datasets, and results were compared to the most established algorithms in the literature. Furthermore, the algorithm was applied to the real-world problem of eye-tracking. A problem with impact in different areas of knowledge, since many behavioral experiments use eye-tracking information as a relevant analysis factor. Performed experiments reveal that RBFChain can classify fixations and saccades in real-time, with high accuracy, noise tolerance, and using limited resources.

*Keywords:* Concept Drift, Drift Detection, Eye tracking

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## 1. Introduction

In recent years, the volume of data produced by computer systems has grown dramatically. Technological advances favored this growth, such as the pervasiveness of mobile devices, the popularization of social networks and the expansion of the internet of things [1].

A significant portion of this information is produced in the form of uninterrupted and potentially infinite sequences [2]. In literature, sequences with these characteristics are called data streams and are present in various fields of application such as financial market monitoring [3], road traffic monitoring [4], telecom network management [5], real-time sentiment analysis [6] and intruder prevention and identification systems [7].

Most of the environments that produce data streams are non-stationary. That is, the joint probability distribution changes arbitrarily over time, such as a switch in the conditional probability distribution on a classification problem, or a change of some moment (such as mean and variance) on a time series forecasting problem [8]. Systems applied to these environments may be unable to adapt to the new information, hence dramatically deteriorating their performance. This phenomenon is known as concept drift [9].

Still, most drift detection methods are improper for non-stationary environments with data streams. These methods or require the correct labeling of data - impracticable in these contexts, or do not meet the severe response time and resource usage restrictions intrinsic to data streams.

This paper proposes a novel proactive method for on-line drift detection, called RBFChain. The proposed algorithm is based on Radial Basis Networks implicit clustering property and employs Markov Chains to model the drifts transitions. To validate the proposed method as a viable concept drift detector, an examination of sensitivity, accuracy, and noise tolerance was performed using synthetic datasets, and results were compared to the most established algorithms in the literature.

Moreover, the algorithm was also applied to the real-world problem of eye-tracking. A problem with impact in different areas of knowledge, since many behavioral experiments use eye-tracking information as a relevant analysis factor. Performed experiments reveal that RBFChain can classify fixations and saccades in real-time, with high accuracy, noise tolerance, and using limited resources.

The rest of the paper is organized as follows: Section 2 describes the concept drift phenomenon and the main detection techniques; Section 3 presents

the eye-tracking problem, with a focus on the identification of fixations and saccades; Section 4 describes the RBFChain algorithm and its pseudo-code; Section 5 shows the experiment configuration for synthetic datasets and the obtained results; Section 6 presents the examination setup with a real-world eye-tracking problem and observed outcomes; and, finally, Section 7 provides conclusions and discusses future work.

## 2. Concept Drift

Many relevant real-world problems can be considered as non-stationary environments. Examples include financial market monitoring, telecom networks, intruder detection, spam filtering, among others [9].

In the literature, Bayesian Theory is commonly used as a background to define the concept drift phenomenon formally [10]: consider the posterior probability of a sample  $x$  belonging to a class  $y$ , a concept drift happens when this probability changes over time, that is,  $P_{t+1}(y|x) \neq P_t(y|x)$ . In a supervised learning scenario, this can be interpreted as when the relationship between the input data and the target variable change over time.

According to [8, 9], concept drifts can occur in four main patterns:

- **Abrupt:** occurs when a concept A switches abruptly to another concept B.
- **Gradual:** occurs when a concept A is being exchanged for the B concept gradually. In this case, while there is no definitive change from concept A to concept B, occurrences of B become more frequent, while fewer events of A are observed.
- **Incremental:** occurs when a concept A is being exchanged for B through intermediate concepts. These concepts differ little from its predecessor and successor. So changes are noticeable only in the long run.
- **Recurrent:** occurs when a previously active concept reappears after a certain period. However, this can not be understood as a periodic seasonality.

Figure 1 demonstrates these patterns:

Algorithms for detecting concept drift characterize and quantify concept drifts through the delimitation of the moments or time intervals in which

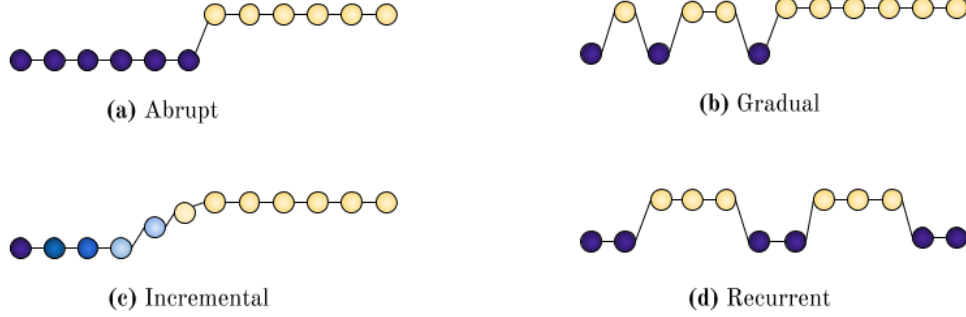


Figure 1: Concept Drift Patterns

changes occur [11]. These algorithms fall into two categories, according to the need for data labeling [12]:

- **Explicit Algorithms/Supervised:** These methods adopt a passive approach, as they depend on the correct labeling of the data to act. The model performance is monitored continuously, and drifts are detected when its performance starts to deteriorate, reaching a threshold.
- **Implicit Algorithms/Unsupervised:** These algorithms take a proactive approach and are independent of correct data labeling. Concept drifts are detected through the analysis of incoming data or indicators produced by the applied learning techniques. Although they are more prone to false alarms, they are an alternative to scenarios where obtaining labels is expensive, time-consuming or unviable. Also, this approach can lead to better results, since it is possible to refit the model or adjust the data, before the deterioration of the predictions.

The algorithm proposed in this paper classifies itself as an unsupervised algorithm and adopts a proactive approach. Briefly, its operation can be described: The Radial Basis Function Networks continuously cluster all incoming data. Changes in the generated cluster (a different center is activated) reflect in a Markov Chain, which keeps an online model of the possible system transitions and its probabilities. Drifts are triggered when the transition probability reaches a parametric threshold.

To assess the proposed method as a viable concept drift detector, an analysis of sensitivity, accuracy, and noise tolerance was performed using synthetic datasets. Moreover, results were compared to the most established algorithms in the literature, demonstrating the competitiveness of the method.

### 3. Fixations and Saccades detection

Visual perception involves six types of eye movements [13], among which fixations and saccades are the most relevant. During fixation, the eye is kept relatively stable on an area of interest (AOI). In contrast, saccades are fast eye movements enabling the fovea to fixate different regions of the scene [14]. Thus, the process of looking at a scene can be represented by a sequence of fixations and saccades, the so-called visual scan path. Research on scan path analysis and visual perception has benefited from the recent development of eye trackers. Today's eye-tracking systems allow a precise recording of eye movements at high sampling rates, thus enabling a detailed analysis of the viewing behavior.

Despite recent advances, reliable automated clustering of eye movements is still challenging, even more so in dynamic scenarios. In many applications, e.g., human-computer gaze-based interaction, driving assistance systems, on-line adaptation of digital content based on gaze analysis, the identification of fixations and saccades has to occur in an online fashion. There is a wide variety of methods for the online analysis of eye-tracking data and the recognition of fixations and saccades. However, only a few of them are suited for online applicability to dynamic scenes. Such methods have to quickly adapt not only to the individual viewing behavior but also to the changes occurring in the viewing scene. This small group of highly promising methods is based on probabilistic formalizations, e.g., as Markov Models [15, 16], Bayesian Mixture Models [17], etc.

Prior techniques for the automated recognition of different types of eye movements from eye-tracking data fall into two main categories: (i) threshold-based methods, where the distinction of fixations from saccades is based on dispersion, velocity, or acceleration thresholds, and (ii) probabilistic methods. These groups of techniques will be briefly discussed in the following.

Threshold-based methods distinguish between fixations and saccades based on the assumption that the distances, velocities, or accelerations occurring between subsequent fixations differ from those occurring between saccades.

127 The goal then is to identify a threshold based on which saccades can be  
128 reliably distinguished from fixations.

129 When distance thresholds are used, fixation clusters are usually identi-  
130 fied by searching for data points that are close enough to each other (i.e.,  
131 below the established threshold) within a predefined time window [18]. A  
132 representative of this group, is the Dispersion Threshold Identification (I-  
133 DT) algorithm [15]. Other similar approaches differ mainly in the way the  
134 threshold is calculated [19, 20].

135 Other algorithms in this realm are based on the computation of Minimum  
136 Spanning Trees (MST). In [15] an MST is built on the eye-tracking points  
137 within a temporal window of predefined length. An edge (i.e., representing  
138 the distance between two points) is classified as a saccade if its length is  
139 significantly larger than the lengths of neighboring edges, which have been  
140 previously classified as distances between fixations. Yet other methods em-  
141 ploy smart clustering algorithms, e.g., [21, 22] but have serious limitations  
142 concerning their applicability to dynamic online scenarios, since, in such sce-  
143 narios, the cluster properties for fixations and saccades show high variability.

144 Methods that are based on velocity or acceleration thresholds work simi-  
145 larly. A representative of this group is the Velocity-Threshold Identification  
146 (I-VT) algorithm, where a point is identified as a saccade point, if the im-  
147 plicit velocity along the distance from the previous data point to that point  
148 exceeds a predefined threshold. Otherwise the data point is assigned to a  
149 fixation cluster [15].

150 In summary, the major drawback of threshold-based methods is that they  
151 rely on thresholds that have to be empirically adjusted to the individual  
152 viewing behavior, the viewing area, and the specific task. Each of these pa-  
153 rameters can have significant influence on the classification result [16, 15].  
154 For this reason and because of the fact that the viewing behavior is strongly  
155 physically and physiologically-dependent, such methods are not reliable, es-  
156 pecially when real-time analysis of eye-tracking data is needed.

157 Probabilistic methods are built on soft decision rules, which are formalized  
158 as probabilities, e.g., the probability of a data point being a saccade given  
159 the previous observations. The probabilities and thus, the decisions are  
160 adjusted to the observations.

161 One of the most prominent probabilistic methods applied to the identifica-  
162 tion of fixations and saccades is the Hidden Markov Model (HMM). An HMM  
163 is a simple dynamic Bayesian network with variables representing values from  
164 a discrete state and observation space. The state of a variable represents the

165 class of the current observation. It is only dependent on the state (i.e., class  
 166 of the previous observation). Because of this sequential nature, such mod-  
 167 els are a popular choice for the analysis of successively arising data points  
 168 (i.e., observations). For the detection of fixations and saccades from eye  
 169 data, HMMs have been used with velocity observations between successive  
 170 data points, thus allowing the adaptation of the model to the physiological  
 171 viewing behavior [15]. In the model of [15] (coined I-HMM), the two states  
 172 used represent discretized velocity distributions over fixations and saccades.  
 173 Transition probabilities between the states represent the probability of the  
 174 current sample belonging to a fixation cluster or a saccade, given the previ-  
 175 ous state [18]. Due to the above probabilistic representation, no thresholds  
 176 are needed. The I-HMM is reported to outperform fixed-threshold methods,  
 177 such as I-VT [15]. In summary, the sequential, dynamic, and probabilistic  
 178 nature of HMMs makes them an adequate choice for data arising in an online  
 179 fashion and containing variability in its features.

180 Probabilistic mixture models, such as the Bayesian Mixture Model (BMM)  
 181 presented in [17], build on the assumption that the observed data is generated  
 182 from a mixture of unknown density distributions. The goal is to estimate  
 183 the parameters of these distributions based on observed data points and to  
 184 derive the most probable distribution that might have generated a given data  
 185 point.

186 The algorithm presented in [17] could distinguish between fixations and  
 187 saccades in an online fashion, only by considering the Euclidean distances  
 188 between subsequent data points. The underlying model is based on the as-  
 189 sumption that distances between subsequent fixation points will, in general,  
 190 be shorter than distances between subsequent saccade points; that is, dis-  
 191 tances between subsequent fixation points would be generated from a specific  
 192 Gaussian distribution and those between subsequent saccade points from an-  
 193 other. This intuition was modeled by a Bayesian Online Mixture Model. The  
 194 benefit of the Bayesian formalization of the mixture model is that the param-  
 195 eters of the two distributions are updated and learned in an online fashion as  
 196 more and more data is observed. For every new data point, the prior prob-  
 197 abilities are replaced by the latest estimates. For practical purposes, this  
 198 means that for every new user the algorithm needs a relatively small num-  
 199 ber of data points to adjust to that user and learn user- or scene-dependent  
 200 parameters.

201 In summary, probabilistic methods come with three main advantages over  
 202 threshold-based ones:

- 203 1. No fixed thresholds are needed. Instead, the parameters of the model  
204 (e.g., state transition probabilities, label emission probabilities, and  
205 other settings) are learned from labeled data.
- 206 2. Both HMMs and BMMs can adapt to the individual (i.e., physiological)  
207 viewing behavior of a subject and the specific task.
- 208 3. Given the dynamic nature of the underlying models, the methods are  
209 naturally suited for data arising in an online fashion, such as eye-  
210 tracking data.

## 211 4. RBFChain algorithm

212 This section details the RBFChain implementation. However, before de-  
213 scribing the proposed method, it is significant to present the main applied  
214 concepts of Radial Base Function Networks and Markov Chains.

### 215 4.1. Radial Basis Function Networks (RBFN)

216 Radial Basis Function Networks (RBFN) are used in various disciplines  
217 with a reasonable degree of success. The broad applicability is a result of  
218 their excellent ability to make function approximation, especially when the  
219 relationships among the variables of interest are nonlinear [23].

220 A radial basis function network is a type of artificial neural network  
221 (ANN), and most neural networks are known to be useful in modeling com-  
222 plex and nonlinear relationships. An RBFN has advantages in specific appli-  
223 cations in that for a given parameter set, RBFN networks do not require an  
224 iterative procedure to learn the model. Iterative learning for most ANN types  
225 is computationally expensive and vulnerable to the local minima problem.

226 The topology of an RBFN is given in Fig. 2 as a multiple input sin-  
227 gle output feedforward network. Assume that there are  $n$  input variables  
228 labeled from  $x_1$  to  $x_n$ . The network receives input samples as vectors  $x =$   
229  $(x_1, x_2, \dots, x_n)$  of size  $1 \times n$ . The initial layer is only a buffer that feeds the in-  
230 put values to the intermediate layer, which is called the hidden layer. There  
231 are  $n_h$  processing elements in the hidden layer. Each processing element  
232 in the hidden layer processes the input vector and produces a single value  
233 output. This processing is performed through a basis function  $\phi$ . Finally,  
234 the output layer weights the results of the intermediate layer by weights,  
235 aggregating them linearly to compose the final network response.

236 Among many candidates for basis functions, Gaussian radial basis func-  
237 tion (RBF), presented in Eq. 1, is used in this study. The main reason



for this choice is that it can be shown that an RBFN with Gaussian RBF can sufficiently approximate any given function for a large enough number of hidden layer elements [24].

Probabilistic methods are built on soft decision rules, which are formalized as probabilities, e.g., the probability of a data point being a saccade given the previous observations. The probabilities and thus, the decisions are adjusted to the observations.

$$\varphi(v_i) = e^{-(\sigma r)^2} \quad (1)$$

In the hidden layer, each processing element has a separate vector called the center, which has the same dimensions as the input vector. For  $n_h$  hidden layer elements we have  $n_h$  center vectors as  $(c_1; c_2; \dots; c_{n_h})$ . Then each processing element looks at the distance between the input vector and its center and uses this distance to create its output (activation phase).

This work uses only the initial and intermediary layers of the presented architecture. The initial layer channels the incoming data to the middle layer, which implicitly forms clusters during the activation phase. The formed grouping has an active center that changes according to the processed value. Changes in the active center are interpreted as possible concept drifts.

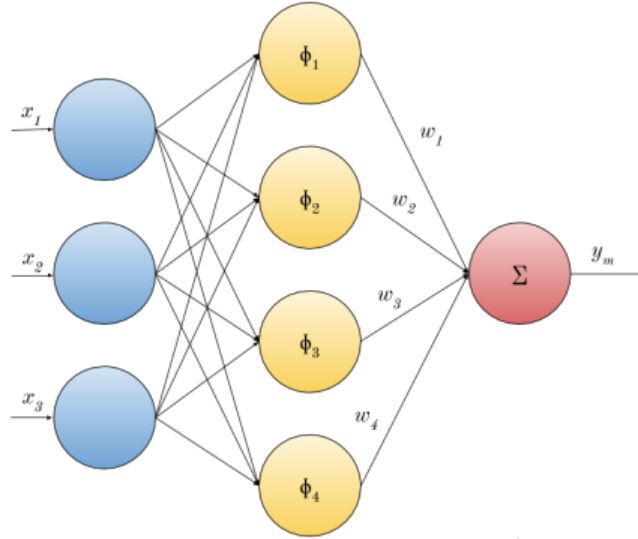


Figure 2: Topology of a RBFN

#### 255 4.2. Markov Chains

256 A Markov chain model can be defined by the tuple  $(S; A; \lambda)$ .  $S$  corre-  
257 sponds to the state space,  $A$  is a matrix representing transition probabilities  
258 from one state to another, and  $\lambda$  is the initial probability distribution of the  
259 states in  $S$ . If there are  $n$  states in our Markov chain, then the matrix of  
260 transition probabilities  $A$  is of size  $n \times n$ .

261 The fundamental property of the Markov model is the dependency on the  
262 previous state. If the vector  $s(t)$  denotes the probability vector for all the  
263 states at time  $t$ , then:

$$\hat{s}(t) = \hat{s}(t-1)A \quad (2)$$

264 In this proposal, Markov chains are used to model the transitions (ac-  
265 tivations) between centers in the Radial Basis Function Network. For this  
266 formulation, a Markov state corresponds to one of the centers.

267 When the RBFN identifies a different center, a new state is registered in  
268 the Markov Chain. Initially, all possible transitions from this center have  
269 a zero value. If another center is activated, this change produces an incre-  
270 ment in the probability of the correspondent transition. In paralell, all other  
271 transitions probabilities are decreased proportionally to the total number of  
272 possible transitions.

273 The use of a Markov Chain allows the proposed algorithm to keep an  
274 online model of the transitions. The probabilities sustained in this model  
275 are compared to parametric thresholds, to indicate when a warning zone is  
276 triggered, or a concept drift happens.

#### 277 4.3. RBFChain

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## 279 5. Analyses on Synthetic Datasets with Concept Drift

### 280 5.1. Experimental Setup

### 281 5.2. Results

## 282 6. Detection of Saccade and Fixation

### 283 6.1. Experimental Setup

### 284 6.2. Results

## 285 7. Concluding Remarks

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