



# Automatic eye fixations identification based on analysis of variance and covariance

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## ABSTRACT

Eye movement is the simplest and repetitive movement that enables humans to interact with the environment. The common daily activities, such as reading a book or watching television, involve this natural activity, which consists of rapidly shifting our gaze from one region to another. In clinical application, the identification of the main components of eye movement during visual exploration, such as fixations and saccades, is the objective of the analysis of eye movements; however, in patients affected by motor control disorder the identification of fixation is not banal. This work presents a new fixation identification algorithm based on the analysis of variance and covariance: the main idea was to use bivariate statistical analysis to compare variance over  $x$  and  $y$  to identify fixation. We describe the new algorithm, and we compare it with the common fixations algorithm based on dispersion. To demonstrate the performance of our approach, we tested the algorithm in a group of healthy subjects and patients affected by motor control disorder.

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## 1. Introduction

Eye movements are an essential part of human vision as they drive the fovea and, consequently, visual attention toward a region of interest in the space. This enables visual system to process an image or its details with a high resolution power (Privitera and Stark, 2000).

The study of eye movements is an up-and-coming tool to study neurological disorders in clinical applications. Voluntary eye movements (saccades, smooth pursuit) are controlled by several structures in the central nervous system, which may enable easier distinction between peripheral and central lesions (Juhola et al., 2007). Brain's structures of the paramedian pontine reticular formation and the vestibulo-cerebellum are involved in the coordination of eye movements and in vestibular responses (Leigh, 2006). Some other neurological diseases, such as cerebellar ataxia, have an influence on saccade velocity, accuracy or latency.

Therefore, a correct analysis of eye movements can lead to distinguish patients from healthy subjects.

The fixations and saccades are the main features of eye movements; fixations are samples of points around a centre point (centroid) with long duration ( $\gg 50$  ms); these eye fixations are

intercalated by rapid eye jumps (saccade), which can be defined as rapid eye movement with velocities that may be higher than 500 deg/s and duration about 20–40 ms (Ramat et al., 2007); Fig. 1 shows a small portion of gaze sample during visual exploration on a psychological task: it is easy to identify three clusters of data points (fixations) and two saccades.

From a psychological point of view, the fixation is defined as the act of maintaining the visual gaze on a single location in order to make our environment visible (see Martinez-Conde et al. (2004) for a review of the role of fixations). From a technical point of view, fixation should be identified by a cluster of points around a centroid with a minimum duration; Irwin et al. (1990) found the theoretical minimum duration for a single fixation to be 150 ms, whereas Manor and Gordon (2003) argued that 100 ms can also be justified. Rayner (1998) indicated that the mean duration of a single fixation may depend on the nature of the task (225 ms on reading, 275 ms on visual search, 400 ms hand-eye coordination).

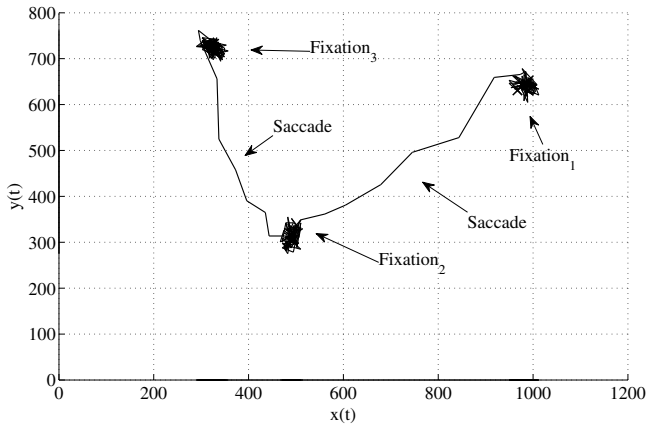
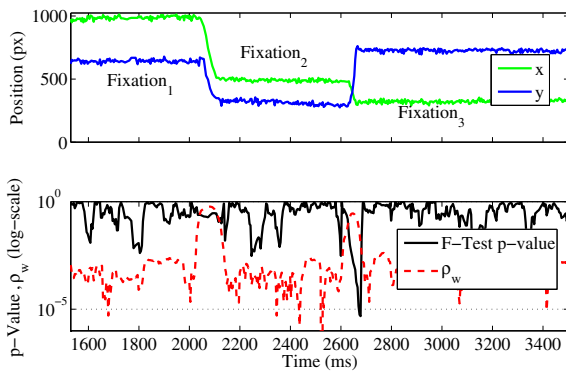
During fixation, the eye does not remain completely stable, but is affected by perturbations such as microsaccades, ocular drifts, and ocular microtremor, making it difficult to easily identify it by an algorithm.

### 1.1. Related works

In order to implement an efficient algorithm able to identify automatically fixations, the efforts have been concentrated on three parameters: fixations duration, dispersion and velocity.

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(a)  $x(t)$   $y(t)$ (b)  $x(t)$  and  $y(t)$ , distance covariance and p-value (bottom graph).

**Fig. 1.** A small portion of gaze of subject GV. For each sample  $(x(\tau), y(\tau))$  at time  $\tau$  on a time window  $(t - w/2, t + w/2)$ , we evaluated variance on  $x$  versus variance on  $y$  by bivariate F-test. When the  $p$ -value is larger than 5% the null hypothesis ( $H_0$ ) cannot be rejected, and variances are equal; then the maximum value should be interpreted as the centroid of fixations. The distance covariance  $\rho_w$  is maximum during saccades.

Generally speaking, the most common algorithms are based on clustering analysis (Urruty et al., 2007) or dispersion thresholding: Distance Dispersion Algorithm, Centroid-Distance Method (Anliker, 1976), Position-Variance Method and Salvucci I-DT Algorithm (Salvucci and Goldberg, 2000).

In Distance Dispersion Algorithm each point in that fixation must be no further than some threshold  $d_{max}$  from every other point. Position-Variance requires that  $M$  of  $N$  points have a standard deviation of distance from the centroid not exceeding  $\sigma_{max}$ . Centroid-Distance requires that  $M$  of  $N$  points be no further than some threshold from the centroid of the  $N$  points. Salvucci I-DT requires that the maximal horizontal distance and the maximal vertical distance is less than threshold.

Shic et al. (2008) accomplished a comparison of this algorithms: they found that choices in analysis can lead to very different interpretations of the same eye-tracking data. Blignaut (2009) found that the correct setting of dispersion threshold on I-DT is of utmost importance, especially if the subjects are not homogeneous.

## 1.2. Objective

We aimed to develop a robust algorithm able to identify fixations in clinical cases. The dispersion algorithm of I-DT Algorithm is accounted to be the most robust method (Shic et al., 2008), we implemented the new algorithm (Fixation Dispersion Algorithm based on Covariance – C-DT) based on the similar principle, but

we evaluated the bivariate variance around the centroid and the covariance to classify the data point. In Veneri et al. (2010b) we used the F-test of equal variance to identify fixations: the key principle of the proposed technique was based on supposing the variance along the  $x$  was not significantly different from that along the  $y$  during fixations: the algorithm was able to identify more fixations than I-DT, but in clinical cases, where eye movements should be affected by brain diseases, the assumptions such as the normality of distribution or the number of data points should affect the efficacy of the algorithm. For these reasons we used a mixed method based both on covariance and F-test for equal variance.

The first section of the paper describes the implemented algorithm, then a comparison between the C-DT and the I-DT Algorithm applied to simulated (artificial) fixations is proposed; the final section is devoted to a case study in normal subjects and patients.

## 2. Material and methods

### 2.1. Simulated fixations

In order to assess the proposed algorithm C-DT, we simulated 3500 fixations plus a small portion of two saccades at the endpoint of fixation. Fixation was defined as two dimensional sample vector data points  $x(t)$ ,  $y(t)$  around the fixation's centroid  $x_c(t)$ ,  $y_c(t)$ , distributed with random Gaussian variance ( $\sigma_x^2$ ,  $\sigma_y^2$ ) and duration  $D$   $\forall t \in (-D/2, D/2)$ . Saccades were defined by van Beers model (van Beers, 2008) with maximum velocity equal to 600 deg/s (Ramat et al., 2007).

The duration of fixation ranged from 50 to 550 ms. The variance over the  $x$  and  $y$  ranged from  $3px^2$  to  $50px^2$ , which corresponded to a max dispersion ranging from 10px to 300px.

### 2.2. Subjects' fixations

Fourteen healthy subjects and six patients with (diagnosed and well known) eye motor control disorder were enrolled aged 25–45. Subjects were seated at viewing distance of 78 cm from a 32" color monitor (51 cm  $\times$  31 cm). Eye position was recorded using ASL 6000 system, which consists of a remote-mounted camera sampling pupil location at 240 Hz. A 9-point calibration and 3-point validation procedure was repeated several times to ensure all recordings had a mean spatial error of less than 0.3 degree. Data was controlled by a Pentium4 dual core 3 GHz computer, which acquires signals by fast UART serial port. Head movement was restricted using a chin rest and bite. Subject was asked to fixate a centered red dot on the center; after 500 ms the centered red dot disappeared and subject could explore the scene. The displayed scenes were randomly chosen in a collection of real images (Privitera and Stark, 2000) or pop-up images. The fixations were manually identified by an expert operator.

## 3. Theory

In human visual search the source of variability should be due to the same system (Beers, 2007; Veneri et al., 2011); the key principle of the proposed technique is based on supposing  $x$  and  $y$  independent with the same variance during a fixation.

The method is based on this assumption, and evaluates the hypothesis by distance covariance coefficient and a statistical method such as the F-test for equal variance; the F-test is used to verify that two populations (with normal distribution) have the same mean or variance (this is the so called "null hypothesis"  $H_0$  to be tested against the alternative complementary hypothesis  $H_1$  that the two populations have heterogeneous variances) and is

a standard statistical procedure. In an informal way the F test calculates the F distribution

$$\frac{\text{between group variability}}{\text{within group variability}} = \frac{\text{variability over } x \text{ or } y}{\text{variability between } x \text{ y}} \quad (1)$$

and, then, evaluates the probability to get a result of the test less than the one actually observed when the null hypothesis  $H_0$  is true (in the C-DT algorithm there is no difference between the two variances); this probability is often indicated  $p$ -value or simply  $p$ . We must note that F-test was diagnosed as being extremely sensitive to non-normality (Markowski et al., 1990).

We assumed that  $x$  and  $y$  on fixations come from a normal distribution with same variance: then the highest relative  $p$ -values that did not accept the null hypothesis (greater than 5%) were classified as the centroid of the fixation.

The assumption of equal variance and the normality of the distribution, may be eligible for normal subjects, but in the case of patients affected by motor control disorders, the fixation is affected by nystagmus, drifts, square jerks (Leigh, 2006) and the method should fail. Covariance is a measure of how much the deviations of two or more variables or processes match and should be a good candidate to replace or integrate the F-test. Covariance is defined as:

$$\text{cov}(x, y) = \sum_{t=1}^n (x(t) - \bar{x})(y(t) - \bar{y}) \quad (2)$$

where  $\bar{x}$  is the mean of vector  $x$ . If  $x$  and  $y$  are independent, then their covariance is zero. The converse, however, is generally not true, for these reasons the correlation coefficient (Székely et al., 2007) (or a bivariate non-parametric test (Feuerverger, 1993)) is more suitable. Correlation coefficient is defined as

$$\rho(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y} \quad (3)$$

where  $\sigma_x$  and  $\sigma_y$  are the standard deviations of  $x$  and  $y$  respectively and the product provides the normalization factor to hold the Cauchy–Schwarz inequality  $0 \leq \rho(x, y) \leq 1$ .

Formally, the classification problem should be defined by: given a data point  $x(t)$ ,  $y(t)$  at time  $t^1$ .

$$x(\mathbf{t}), y(\mathbf{t}) \rightarrow \mathbb{F} \quad \text{if } |\rho(x(\tau), y(\tau))| \leq k \quad \forall \tau \in (\mathbf{t} - w/2, \mathbf{t} + w/2) \quad (4)$$

where  $(\mathbf{t} - w/2, \mathbf{t} + w/2)$  is a small time window,  $\mathbb{F}$  is the set of fixations and  $k$  is a constant (threshold)  $\in (0, 1)$ . From an intuitive point of view, when  $x$  and  $y$  are randomly distributed, they are independent and  $\rho(x, y) \approx 0$ .

#### 4. Calculation

The Eq. (4) is theoretically correct, but it was very sensitive to microsaccade, small changes and depended strictly from  $k$ . We used a windowed Pearson's Correlation Coefficient defined as:

$$\rho_w(x, y, \mathbf{t}) = \frac{|\text{cov}(x(\tau), y(\tau))|}{\sigma_x \sigma_y} \quad \forall \tau \in (\mathbf{t} - w/2, \mathbf{t} + w/2) \quad (5)$$

where  $w$  was set to the minimum “physiological wave” to take in consideration, which is, in our case, the saccade:  $w = 20 - 25 \text{ ms} \approx 6 \text{ sample}$  and  $\sigma_x$  is the variance of  $x$ . The Eq. (5) does not hold the Cauchy–Schwarz inequality, but keeps the independent condition for  $\rho_w = 0$ .

<sup>1</sup> A gaze's data point is identified by  $(x, y, t)$ , where  $x$  and  $y$  are the position of the eye and  $t$  the time when the sample  $(x, y)$  was taken. On the current paper, we use the notation  $x(t)$ ,  $y(t)$

To develop a more robust technique we integrated the proposed method with an empirical mixed method based on covariance distance and F-test:

$$\xi_w(x, y, \mathbf{t}) = \begin{cases} 0, & p_w > \alpha_1 \wedge \rho_w < \beta \\ 1, & p_w < \alpha_2 \wedge \rho_w > \beta \\ \rho_w(x, y, \mathbf{t}), & \text{otherwise} \end{cases} \quad (6)$$

where  $p_w = p_w(x, y, t)$  is the  $p$ -value of F-test for equal variance (Hogg and Ledolter, 1987) evaluated on  $(t - w/2, t + w/2)$ ;  $p$ -value can be calculated determining the degree of freedom and evaluating the F function  $= \sigma_x^2 / \sigma_y^2$  with  $n - 1$  and  $n - 1$  degree of freedom and, finally, finding the  $p$ -value on the F-distribution table.  $\beta$  is a very small empirical threshold  $\approx 0.01$ , and  $\alpha$  is the standard confidence interval of  $p$ -value:  $\alpha_1 = 0.05$  and  $\alpha_2 = 0.01$ . Finally we applied Eq. (4) for

$$k = \xi_w(x, y, \mathbf{t}) \quad \forall (x(t), y(t)) \quad \text{when } p_w(x, y, \mathbf{t}) > \alpha_1 \quad (7)$$

From an intuitive point of view,  $k$  is evaluated as mean of Eq. (6) only for the  $\xi_w(x, y, \mathbf{t})$  when  $p_w(x, y, \mathbf{t})$  is not significant.

Then for each  $x(t)$ ,  $y(t) \in \mathbb{F}$  we classified the contiguous points into the same fixation:

$$x(t), y(t) \rightarrow \phi_i \quad \text{if } x(t - \Delta t), y(t - \Delta t) \in \phi_i \quad (8)$$

where  $\phi_i \in \mathbb{F}$  is the fixation  $i$  and  $\Delta t$  is the sample time of eye-tracking machine (in our case  $1000/240 \approx 4.1667 \text{ ms}$ ). The C-DT does not take into consideration anything else than dispersion of fixations, avoiding any consideration on velocity or duration of fixation.

The fixation  $\phi_i$  is identified by its duration  $(t_0, t_1)$  and the centroid of fixation  $\bar{x}_i = |x(t)|$ ,  $\bar{y}_i = |y(t)|$  for all  $t \in (t_0, t_1)$ .

Starting from the centroid, the algorithm identifies the entire fixation extending the range  $(t_0, t_1)$  until

$$\sigma_x^2(\tau_0, \tau_1) \cdot \sigma_y^2(\tau_0, \tau_1) \leq \sigma_x^2(t_0, t_1) \cdot \sigma_y^2(t_0, t_1) \quad (9)$$

when  $\tau_0 \leq t_0$  and  $\tau_1 \geq t_1$ .

#### 4.1. Algorithm description

In summary:  $(x, y)$  where F-test for equal variance accepts the  $H_0$  is in fixation –  $(x, y)$  with high covariance is not in fixation – the remaining points are classified according to covariance and threshold  $k$  estimated through F-test for equal variance. The pseudo code A. 1 describes the algorithm:

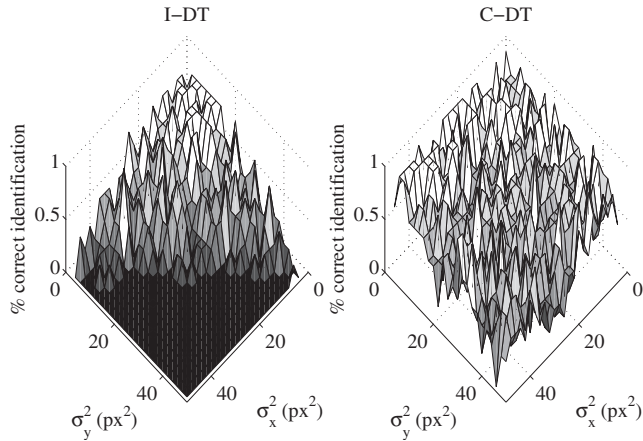
1. the covariance distance and equal variance are evaluated on a small time window (Eq. (6)),
2. threshold constant  $k$  is estimated through F-test for equal variance (Eq. (7)),
3. the points  $(x, y)$  having the same variance or small covariance ( $\leq k$ ) are assigned to fixations set (Eq. (4)),
4. the fixations are group of contiguous points (Eq. (8)),
5. the fixation is extended until variances product remains the same (Eq. (9)).

#### 5. Results

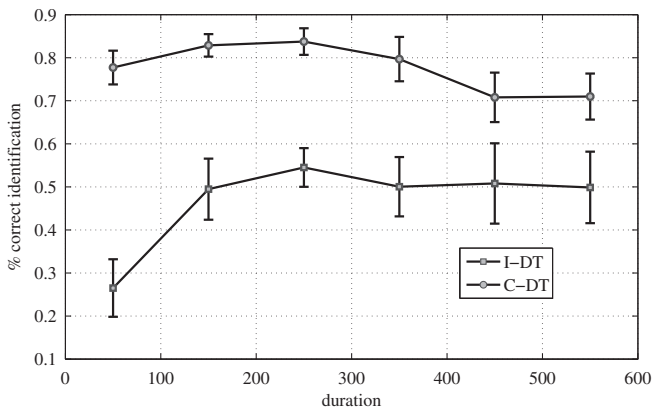
##### 5.1. Simulated fixations

Figs. 2 and 3 show C-DT compared to I-DT. I-DT failed when dispersion was greater than the maximum dispersion admitted by algorithm. We cannot consider this being an error, but it's the main characteristic of the I-DT algorithm; similar result was found by Shic et al. (2008). The C-DT does not consider maximum dispersion, but only the covariance between variance over  $x$  and  $y$ .

Generally speaking the C-DT identified simulated fixations with a precision of 87% and I-DT of 42%. The performance of I-DT, how-



**Fig. 2.** Percentage of correctly identified fixations by I-DT and C-DT varying variance of  $x$  axis,  $y$  axis and duration of fixation. Performance of I-DT and C-DT are similar for small values of  $\sigma_x$  and  $\sigma_y$ ; when dispersion of  $x$  and/or  $y$  increased (max dispersion varied 150px to 300px) I-DT was not able to identify fixation as expected, on the contrary, performance of C-DT remained stable. Performance of C-DT appeared to decrease when  $\sigma_x$  and  $\sigma_y$  were different.



**Fig. 3.** Percentage of correct fixation identified by I-DT and C-DT changing the fixation duration.

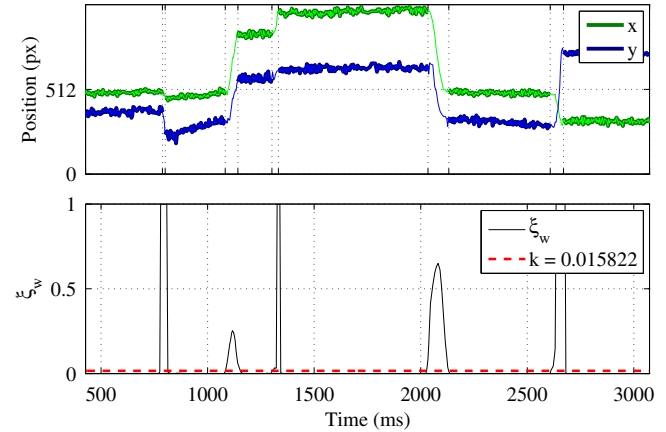
ever, strictly depends on the threshold dispersion set; when we chose a large threshold the I-DT identified more fixations, but in a real case should be affected by incorrect classification of saccade.

C-DT failed when it identified one fixation as two fixations (over segmentation problem).

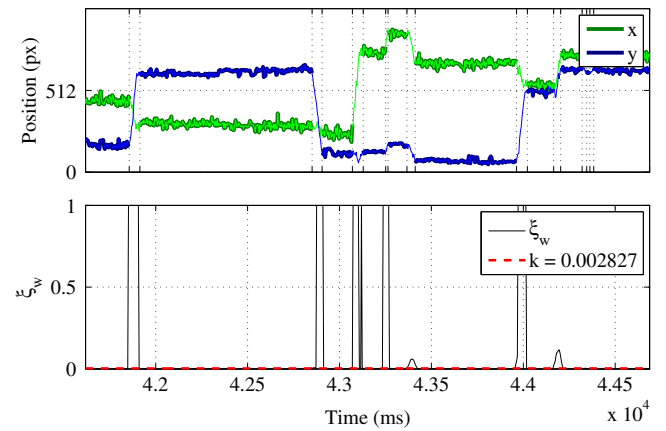
**Table 1**

Fixations identified by algorithms C-DT and I-DT on fourteen healthy subjects and six patients. C-DT identified 88,11% of fixations. I-DT identified 63,86% of fixations. C-DT provided also an estimation of correctness of fixation based on hypothesis of same variance over  $x$  and  $y$  during fixation.

Healthy subjects	AR	EU	FC	GV	TS	BO	CH
By human operator	31	26	52	43	46	40	27
C-DT	28	23	43	38	34	35	29
Rejected fixations by F-test	52%	60%	41%	67%	35%	60%	57%
I-DT	24	15	21	56	40	27	19
Healthy subjects	EPE	FF	GR	PP	PT	ANR	SL
By human operator	30	73	28	70	19	55	40
C-DT	26	71	22	59	18	53	34
Rejected fixations by F-test	57%	40%	46%	28%	86%	28%	48%
I-DT	23	56	23	57	35	61	16
Patients	GG	LB	SDN	XP	XV	CN	GF
By human operator	36	56	32	60	59	66	51
C-DT	40	48	24	52	55	65	48
Rejected fixations by F-test	61%	51%	66%	61%	50%	60%	80%
I-DT	31	30	11	49	42	36	16



(a) Subject GV



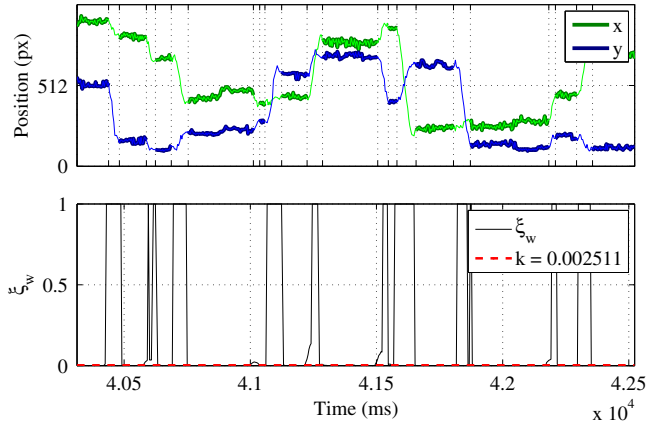
(b) Subject EU

**Fig. 4.** Small portion of exploration made by subjects: the tick lines are fixations identified by C-DT.

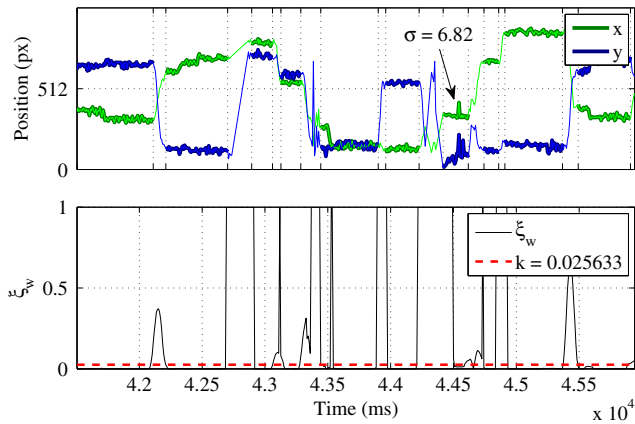
## 5.2. Case study

The fixations of fourteen healthy subjects and six patients with cerebellar disorders were analyzed: C-DT identified 88,11% of fixations. I-DT was set to max dispersion threshold ( $\approx 110px$ ) and identified 63,86% of fixations. The algorithms I-DT and C-DT were applied to raw data; when we applied a low-pass filter (Butterworth filter, 3rd order,  $f_c = 2.1$  Hz) the performances improved to  $\sim 93\%$ , but it depends from the type of filter and filter's parameters. Table 1 reports the results.

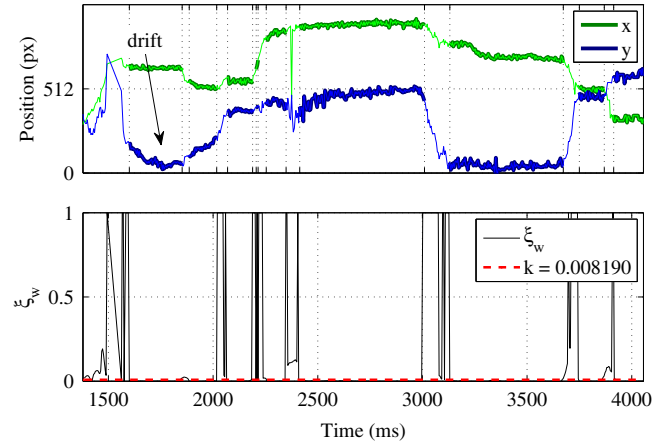




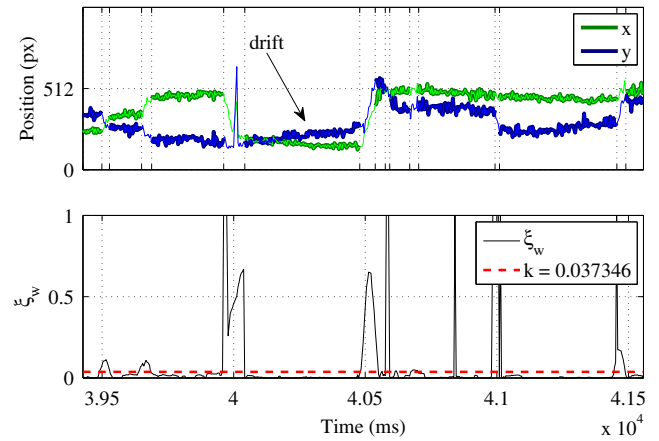
(a) Subject FC



(b) Subject AR



(a) Patient CN



(b) Patient GF

**Fig. 5.** Small portion of exploration made by subjects: C-DT identified the fixations (thick line) also on high variance cases.

Fig. 4 shows a small portion of gaze of subject GV and EU. C-DT identified  $\approx 90\%$  of fixations.

Fig. 5 shows a small portion of an exploration made by subject FC and AR: C-DT identified the fixations (thick line) also on high variance cases.

Fig. 6 shows the exploration made by patients: C-DT was insensitive to drifts, but failed for artifacts such as spikes.

An empirical analysis of lost fixations was performed; I-DT failed to identify correctly large fixations or very short fixations ( $\leq 60$  ms). The short fixations, however, are classified by some authors saccade interruptions (Findlay et al., 2001). C-DT failed on the identification a single fixation as two or more fixations (over segmentation problem) or on grouping two fixations separated by a very small saccade. In any case, a close comparison between I-DT and C-DT does not make sense because they work with different techniques and very often it is not trivial to define a “fixation”.

## 6. Conclusion

In the last decade a large effort has been made to identify fixations (Anliker, 1976, Salvucci and Goldberg, 2000, Urruty et al., 2007, Blignaut, 2009), however it is not yet easy to provide a formal mathematical definition of fixation: some authors have demonstrated that fixation's parameters depend strictly by the type of task (Rayner, 1998, Irwin et al., 1990, Manor and Gordon, 2003, Shic et al., 2008). We suggested a formal definition of fixations based on analysis of variance between  $x$  axis and  $y$  axis; the

**Fig. 6.** Small portion of exploration made by patients: C-DT was insensitive to drifts, but failed in the case of artifacts such as spikes.

implemented algorithm is based on the dispersion algorithm I-DT developed by Salvucci and Goldberg (2000) and integrates it with a statistical test (F-test) and covariance. The main advantage of the proposed technique is to provide a new definition of fixation which does not require the setting of any critical parameter or threshold, and provides a probability value of correctness.

Further work will be addressed to improve the extension mechanism described in the Algorithm 1 by a robust method such as the correntropy. Then, to avoid the normality assumption, the F-test should be replaced by a non parametric test, but it will require more points. Then, due to the high sensitivity of F-test, we will evaluate the opportunity to integrate it on a real-time application such as gaze-contingent (Veneri et al., 2010a).

Finally we will investigate towards improving the system of recognition of the whole establishment and the subsequent application to populations of patients with severe ocular motor disorders such as nystagmus or tremor (Federighi et al., 2011). In this kind of patients the source of variability of fixations should be different between  $x$  and  $y$  and the algorithm should fail; the difference, however, depends strictly on the subject and the ratio of variability between  $x$  and  $y$  should be calculated a priori.

## 7. Materials

Developed MATLAB algorithm should be downloaded at [THE JOURNAL URL].

## Acknowledgments

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## Appendix A. Algorithm

**Algorithm 1:** Operator “ $a:b$ ” means “all point between  $a$  and  $b$ ”. Operator “cov”, “var” or “ftest” are the functions which evaluates covariance, variance of a vector and ftest of two vectors. Operator “relativemax” is a function which calculates the relative maximum of contiguous points

```

1: {Calculate variance}
2:  $\sigma_x \leftarrow \text{var}(x)$ 
3:  $\sigma_y \leftarrow \text{var}(y)$ 
4: for all  $x(t), y(t)$  do
5:    $\tilde{x} \leftarrow x(t - w/2) : x(t + w/2)$ 
6:    $\tilde{y} \leftarrow y(t - w/2) : y(t + w/2)$ 
7:    $F(t) \leftarrow \text{ftest}(\tilde{x}, \tilde{y})$  {F-test on data  $x(t), y(t)$   $t \in (-w/2, w/2)$ }
8:    $R(t) \leftarrow |\text{cov}(\tilde{x}, \tilde{y})| / (\sigma_x \sigma_y)$ 
9:    $t \leftarrow t + \Delta t$  {Move to next point}
10:  if  $F(t) \geq 0.05$  AND  $R(t) \leq 0.01$  then
11:     $R(t) \leftarrow 0$ 
12:  end if
13:  if  $F(t) \leq 0.01$  AND  $R(t) \geq 0.01$  then
14:     $R(t) \leftarrow 1$ 
15:  end if
16: end for
17: {Evaluate threshold}
18:  $\chi \leftarrow \text{mean}(R(\tau)) \forall \tau: F(\tau) \geq 0.05$ 
19: {Identify fixations centroid}
20:  $R^* \leftarrow R \leq \chi$ 
21: {Look for centroids}
22:  $\bar{x}_i, \bar{y}_i \leftarrow \text{relativemax}(x(t), y(t), R^*)$ 
23: {Extend fixation}
24: for all  $\bar{x}_i, \bar{y}_i$  do
25:    $\tilde{x} \leftarrow x(t - w/2) : x(t + w/2)$ 
26:    $\tilde{y} \leftarrow y(t - w/2) : y(t + w/2)$ 
27:   {Calculate product of variances of x and y on data  $(x_t, y_t)$   $t \in (-W, W)$ }
28:    $s \leftarrow \text{var}(\tilde{x}) \cdot \text{var}(\tilde{y})$ 
29:    $ss \leftarrow s$ 
30:    $t_0 \leftarrow t - w/2$ 
31:    $t_1 \leftarrow t + w/2$ 
32:  while  $ss \leq s$  AND CONTINUE do
33:     $xx \leftarrow x(t_0 - \Delta t) : x(t_1)$ 
34:     $yy \leftarrow y(t_0 - \Delta t) : y(t_1)$ 
35:     $ss \leftarrow \text{var}(xx) \cdot \text{var}(yy)$ 
36:    if  $ss \leq s$  then
37:       $t_0 \leftarrow t_0 - \Delta t$ 
38:    CONTINUE
39:  end if
40:   $xx \leftarrow x(t_0) : x(t_1 + \Delta t)$ 
41:   $yy \leftarrow y(t_0) : y(t_1 + \Delta t)$ 
42:   $ss \leftarrow \text{var}(xx) \cdot \text{var}(yy)$ 
43:  if  $ss \leq s$  then
44:     $t_1 \leftarrow t_1 + \Delta t$ 
45:  CONTINUE
46:  end if
47: end while
48:  $\text{Fixations} \leftarrow \bar{x}_i, \bar{y}_i, t_0, t_1$  {Assign fixation}
49: end for

```

## Appendix B. Supplementary data

Supplementary data associated with this article can be found, in the online version, at [doi:10.1016/j.patrec.2011.06.012](https://doi.org/10.1016/j.patrec.2011.06.012).

## References

- Anliker, L., 1976. Eye movements: On-line measurement, analysis, and control. *Eye Move. Psych. Process.*, 185–199.
- Beers, R.J., 2007. The sources of variability in saccadic eye movements. *J. Neurosci.* 27, 8757–8770.
- Blignaut, P., 2009. Fixation identification: The optimum threshold for a dispersion algorithm. *Atten. Percept. Psychophys.* 71, 881–895.
- Federighi, P., Cevenini, G., Dotti, M.T., Rosini, F., Pretegianni, E., Federico, A., Rufa, A., 2011. Differences in saccade dynamics between spinocerebellar ataxia 2 and late-onset cerebellar ataxias. *Brain* 134, 879–891.
- Feuerverger, A., 1993. A consistent test for bivariate dependence. *Intern. Statist. Rev./Revue Internat. Statist.* 61, 419–433.
- Findlay, J.M., Brown, V., Gilchrist, I.D., 2001. Saccade target selection in visual search: The effect of information from the previous fixation. *Vision Res.* 41, 87–95.
- Hogg, R.V., Ledolter, J., 1987. *Engineering Statistics*. MacMillan.
- Irwin, D.E., Zacks, J.L., Brown, J.S., 1990. Visual memory and the perception of a stable visual environment. *Percept. Psychophys.* 47, 35–46.
- Juhola, M., Aalto, H., Hirvonen, T., 2007. Using results of eye movement signal analysis in the neural network recognition of otoneurological patients. *Comput. Methods Prog. Biomed.* 86, 216–226.
- Leigh, R.J.D., 2006. *The neurology of eye movements (Book/DVD)*, fourth ed. Oxford University Press, New York.
- Manor, B.R., Gordon, E., 2003. Defining the temporal threshold for ocular fixation in free-viewing visuocognitive tasks. *J. Neurosci. Methods* 128, 85–93.
- Markowski, Carol, A., Markowski, E.P., 1990. Conditions for the effectiveness of a preliminary test of variance. *Amer. Statist.* 44, 322v326.
- Martinez-Conde, S., Macknik, S.L., Hubel, D.H., 2004. The role of fixational eye movements in visual perception. *Nature Rev. Neurosci.* 5, 229–240.
- Privitera, C., Stark, L., 2000. Algorithms for defining visual regions-of-interest: Comparison with eye fixations. *IEEE Trans. Pattern Anal. Machine Intell.* 22, 970–982.
- Ramat, S., Leigh, R.J., Zee, D.S., Optican, L.M., 2007. What clinical disorders tell us about the neural control of saccadic eye movements. *Brain* 130, 10–35.
- Rayner, K., 1998. Eye movements in reading and information processing: 20 years of research. *Psychol. Bull.* 124, 372–422.
- Salvucci, D.D., Goldberg, J.H., 2000. Identifying fixations and saccades in eye-tracking protocols. In: *ETRA'00: Proc. 2000 Symp. on Eye Tracking Research & Applications*, New York, NY, USA. ACM, pp. 71–78.
- Shic, F., Scassellati, B., Chawarska, K., 2008. The incomplete fixation measure. In: *ETRA'08: Proc. 2008 Symp. on Eye Tracking Research and Applications*, New York, NY, USA. ACM, pp. 111–114.
- Székely, G.J., Bakirov, N.K., Rizzo, L.M., 2007. Measuring and testing independence by correlation of distances. *Ann. Statist.* 35, 2769–2794.
- Urruty, T., Lew, S., Djeraba, C., Simovici, D.A., 2007. Detecting eye fixations by projection clustering. In: *Proc. 14th Internat. Conf. on Image Analysis and Processing Workshops ICIAPW 2007*, pp. 45–50.
- van Beers, R.J., 2008. Saccadic eye movements minimize the consequences of motor noise. *PLoS One* 3, e2070.
- Veneri, G., Federighi, P., Rosini, F., Federico, A., Rufa, A., 2010a. Influences of data filtering on human-computer interaction by gaze-contingent display and eye-tracking applications. *Comput. Human Behavior* 26, 1555–1563.
- Veneri, G., Piu, P., Federighi, P., Rosini, F., Federico, A., Rufa, A., 2010b. Eye fixations identification based on statistical analysis – Case study. In: *2010 2nd International Workshop on Cognitive Information Processing (CIP)*. IEEE, pp. 446–451.
- Veneri, G., Federighi, P., Rosini, F., Federico, A., Rufa, A., 2011. Spike removal through multiscale wavelet and entropy analysis of ocular motor noise: A case study in patients with cerebellar disease. *J. Neurosci. Methods* 196, 318–326.