

# Applying Radial Basis Networks and Markov Chains to detect concept drift in non-stationary environments

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## Abstract

Most real-world problems experience a phenomenon known as concept drift, which is a change in data distribution that can affect the system performance. However, the majority of drift detection methods work reactively. These algorithms continuously monitor the system performance and detect concept drifts when the performance drops past a threshold. Nevertheless, some environments can not afford this deterioration in performance. In an attempt to mitigate the aforementioned problem, this paper proposes a novel proactive method for on-line drift detection, called RBFChain. The proposed method relies on Radial Basis Networks implicit clustering property, besides using Markov Chains to model the transitions and for better noise tolerance. To assess the proposed method as a viable concept drift detector, an analysis of sensitivity, accuracy, and noise tolerance was performed using synthetic datasets. Moreover, results were compared to the most established algorithms in the literature, demonstrating the competitiveness of the method. Furthermore, the algorithm was also tested with the real-world problem of eye-tracking. This problem is relevant because many behavioral experiments use eye-tracking information (fixations and saccades) as a relevant analysis factor. Performed experiments reveal that RBFChain can classify fixations and saccades in real-time, with high accuracy, noise tolerance, and using limited resources.

*Keywords:* Concept Drift, Drift Detection, Eye tracking

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## 1. Introduction

Add concept drift paragraph

3 Eye movements are one of the most natural and repetitive motions of the  
4 human being. Worldly activities, such as reading a book or watching televi-  
5 sion, are only possible because of these. This regular movement, known as  
6 saccadic movement, is required by the retina to obtain a clean image. Dur-  
7 ing the analysis of a scene, the eye tends to fixate during a few milliseconds  
8 on the most significant areas. After this fixation, it moves towards a new  
9 area of interest. Therefore, it is possible to describe the trajectory of the eye  
10 as a sequence of fixation periods interrupted by saccades. Fixation periods  
11 allow the brain to process information, given that during saccades, the brain  
12 does not have time to process the images transmitted by the visual system.  
13 Hence, the brain assigns semantics to the image only during fixation periods.  
14 Thus, the study of fixations and saccades is essential for the interpretation  
15 and understanding of the eye movements.

16 Most commercial eye-tracking systems provide horizontal and vertical  
17 coordinates of the point of regard (POR) relative to the screen of a monitor.  
18 The results presented are composed of sequences of points corresponding to  
19 positions of the eyes of an individual. The generated eye path consists of eye  
20 fixations (with a high density of points) separated by vast spaces where only  
21 a few isolated points (saccades) are present.

22 Automated identification of fixation and saccades is a necessary aspect  
23 in the analysis of visual behavior. These classifications serve as underlying  
24 knowledge for the metrics used for interpreting eye movements (number of  
25 saccades, the average amplitude of saccades, number of fixations, duration  
26 of the first fixation, etc). Unfortunately, the majority of the algorithms still  
27 employ a velocity, acceleration, or dispersion threshold to detect potential  
28 saccades [1]. Some techniques differ by applying additional procedures, such  
29 as principal component analysis, to distinguish between smooth search, sac-  
30 cades, and noise [2].

31 However, advances in the area of behavioral analysis have made measured  
32 eye movements more variable and complex, thus making the main methods  
33 found in the literature less effective [3].

34 This paper proposes a novel algorithm, called RBFChain, which applies  
35 Radial Basis Function Networks and Markov Chains to classify scan paths  
36 into fixations and saccades in real-time, with high accuracy, noise tolerance,  
37 and using limited resources. Besides that, experiments conducted in this  
38 work show that the true detection rate obtained by RBFChain is comparable  
39 to that obtained by state of the art.

40 The rest of the paper is organized as follows: Section 2 describes the

41 concept drift phenomenon and the main detection techniques; Section 3  
 42 presents the fixations and saccades detection problem; Section 4 describes  
 43 the RBFChain algorithm and its pseudo-code; Section 5 shows the exper-  
 44 iment configuration for synthetic datasets and results obtained; Section 6  
 45 presents the examination setup with a real-world eye-tracking problem and  
 46 observed outcomes; and, finally, Section 7 provides conclusions and discusses  
 47 future work.

## 48 2. Concept Drift

49 Most real-world problems experience a phenomenon known as concept  
 50 drift [4]. This situation happens in datasets where the joint probability dis-  
 51 tribution changes arbitrarily over time, such as a switch in the conditional  
 52 probability distribution on a classification problem, or a change of some mo-  
 53 ment (such as mean and variance) on a time series forecasting problem [5].

54 Environments that generate this kind of data are considered non-stationary.  
 55 In these environments, concepts tend to evolve frequently, and the system  
 56 may be unable to adapt to the new information, hence dramatically deterio-  
 57 rating its performance. Many relevant real-world problems can be considered  
 58 as non-stationary environments. Examples include financial market monitor-  
 59 ing, telecom networks, intruder detection, spam filtering, among others [4].

60 In the literature, Bayesian Theory is commonly used as a background  
 61 to define the concept drift phenomenon formally [6]: consider the posterior  
 62 probability of a sample  $x$  belonging to a class  $y$ , a concept drift happens  
 63 when this probability changes over time, that is,  $P_{t+1}(y|x) \neq P_t(y|x)$ . In a  
 64 supervised learning scenario, this can be interpreted as when the relationship  
 65 between the input data and the target variable change over time.

66 According to [5, 4], concept drifts can occur in four main patterns:

- 67 • **Abrupt:** occurs when a concept A switches abruptly to another con-  
 68 cept B.
- 69 • **Gradual:** occurs when a concept A is being exchanged for the B con-  
 70 cept gradually. In this case, while there is no definitive change from  
 71 concept A to concept B, occurrences of B become more frequent, while  
 72 fewer events of A are observed.
- 73 • **Incremental:** occurs when a concept A is being exchanged for B  
 74 through intermediate concepts. These concepts differ little from its

75 predecessor and successor. So changes are noticeable only in the long  
 76 run.

- 77 • **Recurrent:** occurs when a previously active concept reappears after  
 78 a certain period. However, this can not be understood as a periodic  
 79 seasonality.

80 Figure 1 demonstrates these patterns:

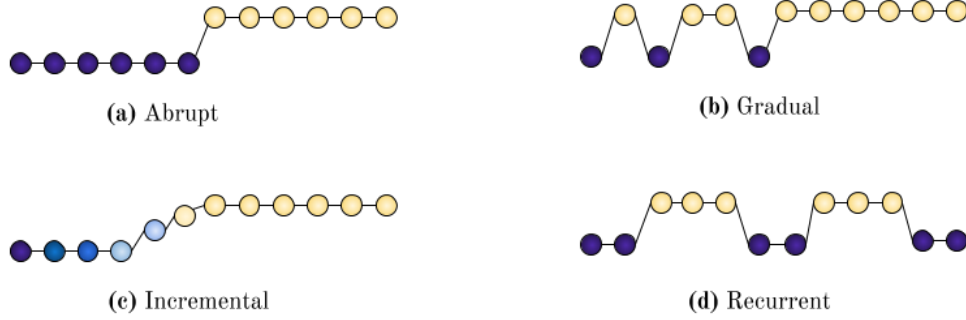


Figure 1: Concept Drift Patterns

81 Algorithms for detecting concept drift characterize and quantify concept  
 82 drifts through the delimitation of the moments or time intervals in which  
 83 changes occur [7]. These algorithms fall into two categories, according to the  
 84 need for data labeling [8]:

- 85 • **Explicit Algorithms/Supervised:** These methods adopt a passive  
 86 approach, as they depend on the correct labeling of the data to act. The  
 87 model performance is monitored continuously, and drifts are detected  
 88 when its performance starts to deteriorate, reaching a threshold.
- 89 • **Implicit Algorithms/Unsupervised:** These algorithms take a proac-  
 90 tive approach and are independent of correct data labeling. Concept  
 91 drifts are detected through the analysis of incoming data or indica-  
 92 tors produced by the applied learning techniques. Although they are  
 93 more prone to false alarms, they are an alternative to scenarios where  
 94 obtaining labels is expensive, time-consuming or unviable. Also, this  
 95 approach can lead to better results, since it is possible to refit the model  
 96 or adjust the data, before the deterioration of the predictions.

97 The algorithm proposed in this paper classifies itself as an unsupervised  
 98 algorithm and adopts a proactive approach. Briefly, its operation can be  
 99 described: The Radial Basis Function Networks continuously cluster all in-  
 100 coming data. Changes in the generated cluster (a different center is activated)  
 101 reflect in a Markov Chain, which keeps an online model of the possible sys-  
 102 tem transitions and its probabilities. Drifts are triggered when the transition  
 103 probability reaches a parametric threshold.

104 To assess the proposed method as a viable concept drift detector, an  
 105 analysis of sensitivity, accuracy, and noise tolerance was performed using  
 106 synthetic datasets. Moreover, results were compared to the most established  
 107 algorithms in the literature, demonstrating the competitiveness of the method.

### 108 **3. Fixations and Saccades detection**

109 Visual perception involves six types of eye movements [9], among which  
 110 fixations and saccades are the most relevant. During fixation, the eye is kept  
 111 relatively stable on an area of interest (AOI). In contrast, saccades are fast  
 112 eye movements enabling the fovea to fixate different regions of the scene [10].  
 113 Thus, the process of looking at a scene can be represented by a sequence of  
 114 fixations and saccades, the so-called visual scan path. Research on scan path  
 115 analysis and visual perception has benefited from the recent development of  
 116 eye trackers. Today's eye-tracking systems allow a precise recording of eye  
 117 movements at high sampling rates, thus enabling a detailed analysis of the  
 118 viewing behavior.

119 Despite recent advances, reliable automated clustering of eye movements  
 120 is still challenging, even more so in dynamic scenarios. In many applications,  
 121 e.g., human-computer gaze-based interaction, driving assistance systems, on-  
 122 line adaptation of digital content based on gaze analysis, the identification  
 123 of fixations and saccades has to occur in an online fashion. There is a wide  
 124 variety of methods for the online analysis of eye-tracking data and the recog-  
 125 nition of fixations and saccades. However, only a few of them are suited for  
 126 online applicability to dynamic scenes. Such methods have to quickly adapt  
 127 not only to the individual viewing behavior but also to the changes occurring  
 128 in the viewing scene. This small group of highly promising methods is based  
 129 on probabilistic formalizations, e.g., as Markov Models [11, 12], Bayesian  
 130 Mixture Models [13], etc.

131 Prior techniques for the automated recognition of different types of eye  
 132 movements from eye-tracking data fall into two main categories: (i) threshold-

133 based methods, where the distinction of fixations from saccades is based on  
134 dispersion, velocity, or acceleration thresholds, and (ii) probabilistic meth-  
135 ods. These groups of techniques will be briefly discussed in the following.

136 Threshold-based methods distinguish between fixations and saccades based  
137 on the assumption that the distances, velocities, or accelerations occurring  
138 between subsequent fixations differ from those occurring between saccades.  
139 The goal then is to identify a threshold based on which saccades can be  
140 reliably distinguished from fixations.

141 When distance thresholds are used, fixation clusters are usually identi-  
142 fied by searching for data points that are close enough to each other (i.e.,  
143 below the established threshold) within a predefined time window [14]. A  
144 representative of this group, is the Dispersion Threshold Identification (I-  
145 DT) algorithm [11]. Other similar approaches differ mainly in the way the  
146 threshold is calculated [15, 16].

147 Other algorithms in this realm are based on the computation of Minimum  
148 Spanning Trees (MST). In [11] an MST is built on the eye-tracking points  
149 within a temporal window of predefined length. An edge (i.e., representing  
150 the distance between two points) is classified as a saccade if its length is  
151 significantly larger than the lengths of neighboring edges, which have been  
152 previously classified as distances between fixations. Yet other methods em-  
153 ploy smart clustering algorithms, e.g., [17, 18] but have serious limitations  
154 concerning their applicability to dynamic online scenarios, since, in such sce-  
155 narios, the cluster properties for fixations and saccades show high variability.

156 Methods that are based on velocity or acceleration thresholds work simi-  
157 larly. A representative of this group is the Velocity-Threshold Identification  
158 (I-VT) algorithm, where a point is identified as a saccade point, if the im-  
159 plicit velocity along the distance from the previous data point to that point  
160 exceeds a predefined threshold. Otherwise the data point is assigned to a  
161 fixation cluster [11].

162 In summary, the major drawback of threshold-based methods is that they  
163 rely on thresholds that have to be empirically adjusted to the individual  
164 viewing behavior, the viewing area, and the specific task. Each of these pa-  
165 rameters can have significant influence on the classification result [12, 11].  
166 For this reason and because of the fact that the viewing behavior is strongly  
167 physically and physiologically-dependent, such methods are not reliable, es-  
168 pecially when real-time analysis of eye-tracking data is needed.

169 Probabilistic methods are built on soft decision rules, which are formalized  
170 as probabilities, e.g., the probability of a data point being a saccade given

171 the previous observations. The probabilities and thus, the decisions are  
172 adjusted to the observations.

173 One of the most prominent probabilistic methods applied to the identifica-  
174 tion of fixations and saccades is the Hidden Markov Model (HMM). An HMM  
175 is a simple dynamic Bayesian network with variables representing values from  
176 a discrete state and observation space. The state of a variable represents the  
177 class of the current observation. It is only dependent on the state (i.e., class  
178 of the previous observation). Because of this sequential nature, such mod-  
179 els are a popular choice for the analysis of successively arising data points  
180 (i.e., observations). For the detection of fixations and saccades from eye  
181 data, HMMs have been used with velocity observations between successive  
182 data points, thus allowing the adaptation of the model to the physiological  
183 viewing behavior [11]. In the model of [11] (coined I-HMM), the two states  
184 used represent discretized velocity distributions over fixations and saccades.  
185 Transition probabilities between the states represent the probability of the  
186 current sample belonging to a fixation cluster or a saccade, given the previ-  
187 ous state [14]. Due to the above probabilistic representation, no thresholds  
188 are needed. The I-HMM is reported to outperform fixed-threshold methods,  
189 such as I-VT [11]. In summary, the sequential, dynamic, and probabilistic  
190 nature of HMMs makes them an adequate choice for data arising in an online  
191 fashion and containing variability in its features.

192 Probabilistic mixture models, such as the Bayesian Mixture Model (BMM)  
193 presented in [13], build on the assumption that the observed data is generated  
194 from a mixture of unknown density distributions. The goal is to estimate  
195 the parameters of these distributions based on observed data points and to  
196 derive the most probable distribution that might have generated a given data  
197 point.

198 The algorithm presented in [13] could distinguish between fixations and  
199 saccades in an online fashion, only by considering the Euclidean distances  
200 between subsequent data points. The underlying model is based on the as-  
201 sumption that distances between subsequent fixation points will, in general,  
202 be shorter than distances between subsequent saccade points; that is, dis-  
203 tances between subsequent fixation points would be generated from a specific  
204 Gaussian distribution and those between subsequent saccade points from an-  
205 other. This intuition was modeled by a Bayesian Online Mixture Model. The  
206 benefit of the Bayesian formalization of the mixture model is that the param-  
207 eters of the two distributions are updated and learned in an online fashion as  
208 more and more data is observed. For every new data point, the prior prob-

abilities are replaced by the latest estimates. For practical purposes, this means that for every new user the algorithm needs a relatively small number of data points to adjust to that user and learn user- or scene-dependent parameters.

In summary, probabilistic methods come with three main advantages over threshold-based ones:

1. No fixed thresholds are needed. Instead, the parameters of the model (e.g., state transition probabilities, label emission probabilities, and other settings) are learned from labeled data.
2. Both HMMs and BMMs can adapt to the individual (i.e., physiological) viewing behavior of a subject and the specific task.
3. Given the dynamic nature of the underlying models, the methods are naturally suited for data arising in an online fashion, such as eye-tracking data.

## 4. RBFChain algorithm

This section details the RBFChain implementation. However, before describing the proposed method, it is significant to present the main applied concepts of Radial Base Function Networks and Markov Chains.

### 4.1. Radial Basis Function Networks (RBFN)

Radial Basis Function Networks (RBFN) are used in various disciplines with a reasonable degree of success. The broad applicability is a result of their excellent ability to make function approximation, especially when the relationships among the variables of interest are nonlinear [19].

A radial basis function network is a type of artificial neural network (ANN), and most neural networks are known to be useful in modeling complex and nonlinear relationships. An RBFN has advantages in specific applications in that for a given parameter set, RBFN networks do not require an iterative procedure to learn the model. Iterative learning for most ANN types is computationally expensive and vulnerable to the local minima problem.

The topology of an RBFN is given in Fig. 2 as a multiple input single output feedforward network. Assume that there are  $n$  input variables labeled from  $x_1$  to  $x_n$ . The network receives input samples as vectors  $x = (x_1, x_2, \dots, x_n)$  of size  $1 \times n$ . The initial layer is only a buffer that feeds the input values to the intermediate layer, which is called the hidden layer. There



are  $n_h$  processing elements in the hidden layer. Each processing element in the hidden layer processes the input vector and produces a single value output. This processing is performed through a basis function  $\phi$ . Finally, the output layer weights the results of the intermediate layer by weights, aggregating them linearly to compose the final network response.

Among many candidates for basis functions, Gaussian radial basis function (RBF), presented in Eq. 1, is used in this study. The main reason for this choice is that it can be shown that an RBFN with Gaussian RBF can sufficiently approximate any given function for a large enough number of hidden layer elements [20].

Probabilistic methods are built on soft decision rules, which are formalized as probabilities, e.g., the probability of a data point being a saccade given the previous observations. The probabilities and thus, the decisions are adjusted to the observations.

$$\varphi(v_i) = e^{-(\sigma r)^2} \quad (1)$$

In the hidden layer, each processing element has a separate vector called the center, which has the same dimensions as the input vector. For  $n_h$  hidden layer elements we have  $n_h$  center vectors as  $(c_1; c_2; \dots; c_{n_h})$ . Then each processing element looks at the distance between the input vector and its center and uses this distance to create its output (activation phase).

This work uses only the initial and intermediary layers of the presented architecture. The initial layer channels the incoming data to the middle layer, which implicitly forms clusters during the activation phase. The formed grouping has an active center that changes according to the processed value. Changes in the active center are interpreted as possible concept drifts.

#### 4.2. Markov Chains

A Markov chain model can be defined by the tuple  $(S; A; \lambda)$ .  $S$  corresponds to the state space,  $A$  is a matrix representing transition probabilities from one state to another, and  $\lambda$  is the initial probability distribution of the states in  $S$ . If there are  $n$  states in our Markov chain, then the matrix of transition probabilities  $A$  is of size  $n \times n$ .

The fundamental property of the Markov model is the dependency on the previous state. If the vector  $s(t)$  denotes the probability vector for all the states at time  $t$ , then:

$$\hat{s}(t) = \hat{s}(t-1)A \quad (2)$$

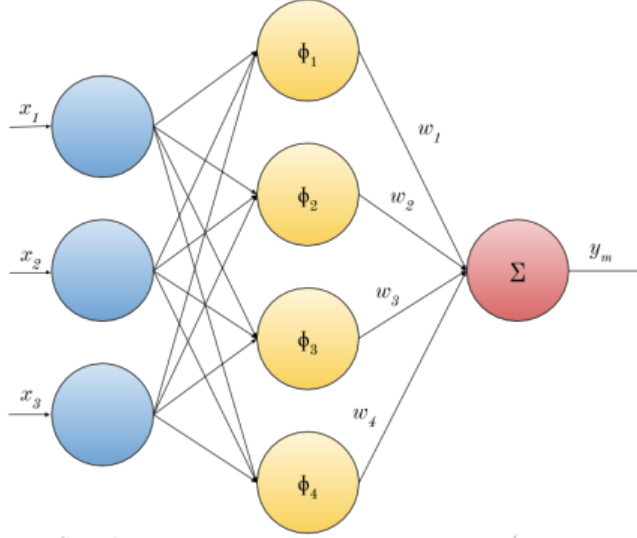


Figure 2: Topology of a RBFN

276 In this proposal, Markov chains are used to model the transitions (ac-  
 277 tivations) between centers in the Radial Basis Function Network. For this  
 278 formulation, a Markov state corresponds to one of the centers.

279 When the RBFN identifies a different center, a new state is registered in  
 280 the Markov Chain. Initially, all possible transitions from this center have  
 281 a zero value. If another center is activated, this change produces an incre-  
 282 ment in the probability of the correspondent transition. In paralell, all other  
 283 transitions probabilités are decreased proportionally to the total number of  
 284 possible transitions.

285 The use of a Markov Chain allows the proposed algorithm to keep an  
 286 online model of the transitions. The probabilities sustained in this model  
 287 are compared to parametric thresholds, to indicate when a warning zone is  
 288 triggered, or a concept drift happens.

#### 289 4.3. *RBFChain*

290 . . .

## 291 **5. Analyses on Synthetic Datasets with Concept Drift**

### 292 *5.1. Experimental Setup*

### 293 *5.2. Results*

## 294 **6. Detection of Saccade and Fixation**

### 295 *6.1. Experimental Setup*

### 296 *6.2. Results*

## 297 **7. Concluding Remarks**

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