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Survey Paper

Detecting Changes in Signals and Systems—A Survey*†

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The problems of detection, estimation and diagnosis of changes in dynamical properties of signals or systems are addressed, with particular emphasis on statistical methods for detection, to provide a general framework for change detection in signals and systems.

Key Words—Time-varying signals and systems; signals segmentation; failure detection; diagnosis.

Abstract—The purpose of this paper is the presentation of a tentative general framework for change detection in signals and systems. It is based upon a non-exhaustive survey of available methods. The main emphasis is placed upon statistical (parametric) methods for detection, which are presented according to the increasing order of complexity of the change problem. Another noticeable feature is the joint presentation of two commonly disconnected aspects, namely the generation of the signals to be monitored and the design of statistical decision rules.

1. PROBLEMS STATEMENT AND APPLICATION EXAMPLES

1.1. *Introduction*

THE PROBLEM of detecting changes in dynamical properties of signals and systems has received growing attention during the last 15 years, as can be seen from the survey papers (Willsky, 1976; Mironovski, 1980; Basseville, 1982; Kligene and Telksnys, 1983; Isermann, 1984) and the books (Himmelblau, 1978; Pau, 1981; Nikiforov, 1983; Basseville and Benveniste (Eds), 1986; Telksnys (Ed.), 1986). Taking into account abrupt changes in statistical models appears as a natural complement of most of the adaptative techniques which track only slow variations of parameters. The analysis of the non-stationary behavior of many signals (see examples below) and the investigation of some exceptional phenomena in dynamical systems (e.g. sensors and actuators failures) show that a reasonable approach in such situations consists of using statistical parametric models in which one or several parameters may abruptly change. Actually, the problem of change detection arises in many areas of automatic control and signal processing, which may be classified as follows.

(1) Segmentation of signals and images for the purpose of recognition, and also for monitoring dynamical systems; in that case, the problems of interest are the detection of the changes and the estimation of the places (time or space) where the changes occur. This segmentation is a possible first step for recognition or monitoring.
(2) Failure detection in controlled systems; a short delay for detection is often of crucial importance, in view of the reconfiguration of the control law, for example.
(3) Gains updating in adaptive algorithms, for tracking quick variations of the parameters; the detection of abrupt changes in the characteristics of the analyzed system and the estimation of the change time and magnitude may allow a convenient updating of the gains of the algorithm.

Many applied fields have already been concerned with change detection: edge detection (Basseville *et al.*, 1981); continuous speech recognition (André-Obrecht, 1988); geophysical (Basseville and Benveniste, 1983a) and seismic (Nikiforov and Tikhonov, 1986) signals segmentation; biomedical signals processing (Gustavson *et al.*, 1978a,b; Sanderson and Segen, 1980; Ishii *et al.*, 1980; Appel and Brandt, 1983; Mottl *et al.*, 1983; Corge and Puech, 1986); aeronautics (Deckert *et al.*, 1977; Kerr, 1985); chemical

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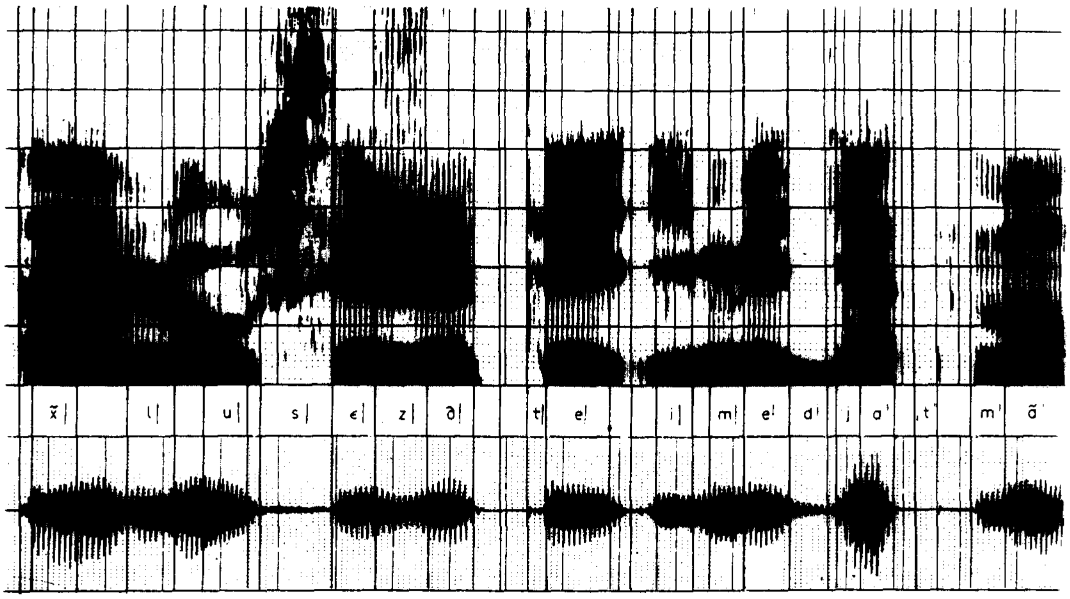

FIG. 1. Segmentation of continuous speech signal (the vertical lines indicate the detected jumps).

(Himmelblau, 1978) and nuclear (Desaï and Ray, 1984) industries; vibration monitoring (Basseville *et al.*, 1987); incidents detection on freeways (Willsky *et al.*, 1980); leak detection for pipelines (Isermann, 1984); control of air conditioning systems (Usoro *et al.*, 1985); prediction of municipal water demand (Sastri, 1987); and econometry (Shaban, 1980).

The interested reader is referred to Perriot-Mathonna (1984) and to Favier and Smolders (1984) for two examples of the use of change detection algorithms for solving a problem of type (3). Let us now investigate two typical problems of class (1) and one example of class (2).

1.1.1. *Example* 1: *segmentation of continuous speech signals.* A possible approach to automatic recognition of continuous speech consists of using an automatic segmentation of the signal as the first processing step (André-Obrecht, 1988). Without using any phonetic information, this segmentation results in a decomposition of the signal into units which are then labelled and processed at the second step called acoustic-phonetic decoding. An example of such an automatic segmentation can be seen on Fig. 1. The algorithm which is used will be discussed below. Let us just briefly mention that on-line detection of abrupt changes in the spectral characteristics of the signal (y_n) is performed via the comparison between a long term model M_0 identified in a growing window and a short term model M_1 identified in a sliding window of fixed length. The models which are used are autoregressive (AR) models excited by Gaussian white noises, namely:

$$y_n = \sum_{i=1}^{p} a_i y_{n-i} + \varepsilon_n \qquad (1)$$

where (ε_n) has variance σ^2.

The distance measure between the two estimated models is Kullback's divergence between the conditional probability laws of the signal with respect to these models. The key point which has to be kept in mind is that the vector parameter:

$$\theta \triangleq (a_1 \cdots a_p \sigma)^{\mathrm{T}} \qquad (2)$$

is monitored on-line, using a sophisticated function of the innovations of the two models M_0 and M_1.

1.1.2. *Example* 2: *vibration monitoring of structures under natural excitation.* Monitoring changes in the vibrating characteristics of a complex structure, such as an offshore platform subject to the swell, results in fatigue analysis. The main difficulties of such a monitoring lie in the highly non-stationary behavior of the unknown excitation, and furthermore finite elements model updating is impracticable when only a few measurements (from accelerometers) are available. A recent original solution consists of two steps (Basseville *et al.*, 1987), as follows.

(a) On site identification of a modal signature, namely of the vibrating frequencies and modes of the structure which is described by:

$$M\ddot{X} + C\dot{X} + KX = E \qquad (3)$$

where M, C, K are the matrices of mass, damping and stiffness coefficients and E is the unknown excitation. This signature is obtained with the aid of a parametric identification method based on ARMA modelling.

(b) Validation of this signature on subsequent records of signals. This test can monitor the structure globally, or focus the monitoring into particular subspaces and thus give information for diagnosis; i.e. identification and localization of the origin of the change.

The key point here is that we have to detect changes in the AR part of a vector auto regressive moving average (ARMA) process, with unknown and time-varying MA coefficients which thus have to be considered as nuisance parameters. The main feature of the solution is to transform this complex problem into the simpler problem of detecting a change in the mean value of a conveniently chosen random variable.

1.1.3. *Example* 3: *failure detection in air conditioning systems.* Fault detection and diagnosis in the heating, ventilation and air conditioning (HVAC) system of a building is of crucial importance for reducing energy use. A possible solution to that problem has been recently proposed by Usoro *et al.* (1985). It consists of using a continuous-time non-linear state space model for the air handler unit together with an extended Kalman filter for estimating its state. The various faults are detected by monitoring convenient functions of the innovations of this filter—sum of squares, likelihood functions, . . . , —and they are possibly estimated and located using a pre-established classification of the values of these functions for different failure types.

As in the previous example, the main feature of this application is the transformation of the complex initial problem into the problem of monitoring convenient residuals.

The sophisticated algorithms used in the examples above and the numerous fields of application which we mentioned show that a significant amount of methodological tools and experimental results is available now. The purpose of this paper is the presentation of some key solutions to the underlying detection, estimation and diagnosis problems.

1.2. *A twofold possible approach*

In designing change detection/estimation algorithms, it may be useful to distinguish two types of tasks generalizing the philosophy developed in Chow and Willisky (1980), as follows.

(i) Generation of "residuals" or change indicating signals: these artificial measurements are designed in order to reflect possible changes of interest in the analyzed signal or system. They are, for example, ideally close to zero when no change occurs, or, more generally, their mean value or their spectral properties change when the analyzed system is changing.

(ii) Design of decision rules based upon these residuals: this task consists of designing the convenient detector which monitors the changes as reflected by the "residuals".

Both deterministic and stochastic approaches have been used in the literature for solving these two tasks. In this paper, we will mainly concentrate on *parametric statistical methods,* especially for task (ii). Our experience and conviction are that a useful and powerful approach for solving change detection problems consists of the following.

- *First, transforming the possibly non-stochastic initial problem into a stochastic change detection problem such as the problem precisely stated below.* This step is the generalized task (i) mentioned above. A non-standard example of such a problem transformation may be found in Bouthemy (1987).
- *Second, using sophisticated statistical tools for solving the resulting stochastic problem, namely task* (ii).

We insist upon the fact that, as will be seen below, there exists a general statistical approach for change detection, namely the likelihood ratio approach, which leads most of the time to very powerful algorithms. Whenever such a solution can be used, i.e. when there are no constraints on algorithm complexity and no nuisance parameters, *this likelihood approach should be implemented directly on the initial system or signal without considering step* (i). The discussion about the detection of changes in spectral properties or eigenstructure will clarify this point. But we also emphasize that the solution of task (i) may be of key importance in complex systems, for example, in order to reduce the size and/or simplify the structure of the model to be monitored, or in order to get rid of nuisance parameters. In Section 8 will be found deterministic and stochastic solutions to this task.

1.3. *Problems statement*

According to the above discussion, from now on and until Section 7 included, we assume that our change detection problem has been transformed into the following stochastic problem. Let us consider a stochastic process (Y_t), with conditional distribution $p_\theta(y_t \mid y_{t-1}, \ldots, y_0)$. Given a record $(y_t)_{(0 \leq t \leq n)}$, decide between the two hypothesis:

$$\mathbb{H}_0 : \theta = \theta_0 \tag{4}$$

$\mathbb{H}_1$: there exists an instant $1 \leq r \leq n$ such that:

$$\begin{cases} \theta = \theta_0' & \text{for} \quad 0 \leq t \leq r-1 \\ \theta = \theta_1 & \text{for} \quad r \leq t \leq n. \end{cases} \tag{5}$$

As will be seen in the subsequent sections, the case $\theta_0' = \theta_0$ will often be considered for on-line approaches.

If $\mathbb{H}_1$ is decided, further questions are to estimate the change time r, possibly to estimate θ_0' and θ_1, and in some cases to diagnose which type of change actually occurs in the process. Of course, the relative importance of these subsequent questions depends upon the applications. Both off-line (n fixed) and on-line (n growing) procedures can be designed for solving such types of problems. We only recall that an off-line point of view may be useful to design an algorithm which will be implemented on-line and we refer the reader to Basseville and Benveniste (Eds, 1986, Chapter 4) for a complete discussion. We also refer to Benvensite *et al.* (1987) for the connection between change detection and model validation.

Finally, as is obvious in (5), we will consider only single change point alternatives. From an off-line point of view, multiple changes may be found by global search; from an on-line viewpoint, the changes are assumed to be detected one after the other.

1.4. *Choice of criteria*

The standard performance index for on-line change detection algorithms is the delay for detection, which has to be minimized for a fixed false alarm rate (Page, 1954; Shiryaev, 1963; Moustakides, 1986a). Lorden (1971) and Nikiforov (1983) use a slightly different definition of the delay. For other types of criteria used for deriving optimal stopping times for change detection, see Bojdecki and Hosza (1984) and Pelkowitz (1987). For off-line procedures, this question is more tricky, because change detection problems are multiple hypotheses testing problems for which there exists no optimum test, in the classical sense of test's power. Therefore asymptotic analyses have to be used, which may also be useful for designing tests, as we shall see later. Further discussions may be found in Basseville and Benveniste (Eds, 1986, Chapter 4).

Apart from the tradeoff between the mean time between false alarms and the delay for detection—both increasing when the sensitivity of the detector to high frequencies decreases—there exists another tradeoff to be kept in mind which is closely related to the first one: efficiency vs complexity. Actually, when the designed monitoring system involves at each alarm a complex time consuming processing and/or a reconfiguration of the control law, false alarms are more dramatic than in the simpler case of over-segmentation in signal recognition. Furthermore, it has to be noticed that the complexity of a change detection system is not only of a computational type but also of a technological nature: some failure detection procedures explicitly use the redundancy in the information given by several identical sensors, and reducing such a complexity without degrading the performance of the detector may be of interest. Other comments on these questions may be found in Willsky (1976) and in the discussion concerning open problems presented in the conclusion. Finally, model and sample sizes required by the designed change detection technique are of importance for application to real signals or systems.

1.5. *Organization of the paper*

As already mentioned, Sections 2–7 are mainly devoted to the design of parametric statistical solutions for task (ii), while both deterministic and stochastic solutions to task (i) are reported in Section 8. More precisely, we first investigate in Section 2 the simplest change detection problem—jump in the *mean*—for which we introduce the likelihood ratio and the cumulative sum tests. Section 3 is devoted to the extension of these tests to the detection of *additive* changes in *linear* systems. Then changes in *spectral* properties are considered in Section 4, where the so-called two-model approach is presented together with exact and approximate likelihood ratio tests for that situation. In Section 5, the *likelihood approach* is presented in a general framework. The (statistical) *local approach* is described in Section 6, where cumulative sum type algorithms are built for solving problems of changes in spectral properties or eigenstructure with a lower complexity cost than likelihood tests. In Section 7, we exhibit a counter-example for which none of the general solutions presented in Sections 5 and 6 can be used, because of *nuisance parameters*. We give another general solution for designing change detection algorithms in such cases; it still uses the local approach but no longer the likelihood function.

Afterwards, in Section 8, we investigate the problem of the choice of the signals to be monitored in order to perform change detection and diagnosis. Both redundancy approaches and filtering methods are described, and we discuss the question of problem transformation we mentioned above in Section 1.2. Then in Section 9 we discuss the *diagnosis* problem, namely the problem of deciding which type of change actually occurred and where—sometimes called the failure isolation problem—for which we give three types of solutions.

Finally some open problems and conclusions are given in Section 10.

Before going more deeply into technical details, two remarks have to be made. First, as far as the design of decision rules is concerned, we intentionally leave out the voting strategies which are often used for highly physically redundant systems, and we refer the interested reader to Willsky (1976), Kerr (1985) and to Desäi and Ray (1984) for an extension to degrees of consistency among residuals and/or measurements. Second, most of the parametric statistical detection rules presented below are based upon the likelihood ratio, with or without bayesian framework. It is of key importance to keep in mind that, for this type of decision rule, the independence hypothesis, which is often implicitly used for writing the likelihood function as a product, is only justified when the "residuals" or change indicating signals which are managed are the innovations of a Kalman filter, but generally not for the instantaneous or temporal redundancy relations. This point will be further investigated in Section 8.

2. DETECTING JUMPS IN THE MEAN

We begin this series of sections devoted to the design of statistical decision rules by investigating the simplest change detection problem, namely the problem of a change in the mean of independent identically distributed random variables. We introduce two basic tests—the likelihood ratio test and the Page–Hinkley stopping rule—and discuss their theoretical properties.

Let $(\varepsilon_n)_n$ be a white noise sequence with variance σ^2, and let $(y_n)_n$ be the sequence of observations (possibly the "residuals" of Section 8) such that:

$$y_n = \mu_n + \varepsilon_n$$

where:

$$\mu_n = \begin{cases} \mu_0 & \text{if} \quad n \leqslant r-1 \\ \mu_1 & \text{if} \quad n \geqslant r. \end{cases} \tag{6}$$

The problem is to detect the change in the mean μ_n, to estimate the change time r and possibly the mean values μ_0 and μ_1 before and after the jump. We first investigate the case where μ_0 and μ_1 are known, and then the case where only μ_0 is known—which is of interest in practice for on-line detection.

2.1. *Known means before and after the jump*

The detection problem consists of testing between the no change hypothesis:

$$\mathbb{H}_0 : r > n$$

and the change hypothesis:

$$\mathbb{H}_1 : r \leqslant n.$$

The likelihood ratio between these two hypotheses is:

$$\prod_{k=r}^{n} \frac{p_1(y_k)}{p_0(y_k)} \tag{7}$$

where p_i is the Gaussian probability density with mean μ_i $(i = 0, 1)$. Its logarithm is thus:

$$\Lambda_n(r) = \frac{\mu_1 - \mu_0}{\sigma^2} \sum_{k=r}^{n} \left(y_k - \frac{\mu_0 + \mu_1}{2} \right) = \frac{1}{\sigma^2} S_r^n(\mu_0, \nu)$$

where

$$S_i^j(\mu, \nu) = \nu \sum_{k=i}^{j} \left(y_k - \mu - \frac{\nu}{2} \right) \tag{8}$$

and

$$\nu = \mu_1 - \mu_0$$

is the magnitude of the jump.

Replacing the unknown jump time r by its maximum likelihood estimate (MLE) under $\mathbb{H}_1$, namely:

$$\hat{r}_n = \arg\max_{1 \leqslant r \leqslant n} \left[\prod_{k=0}^{r-1} p_0(y_k) \prod_{k=r}^{n} p_1(y_k) \right] = \arg\max_{1 \leqslant r \leqslant n} S_r^n(\mu_0, \nu) \tag{9}$$

we get the following change detector:

$$g_n \triangleq \Lambda_n(\hat{r}_n) = \max_r S_r^n(\mu_0, \nu) \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda \tag{10}$$

where λ is a threshold. In other words, decide $\mathbb{H}_1$ whenever g_n exceeds λ, and $\mathbb{H}_0$ otherwise. The detection is not very sensitive to the choice of the threshold λ; see further comments at the end of this section.

This detector may be described as follows: detect a jump in the mean at the first time n at

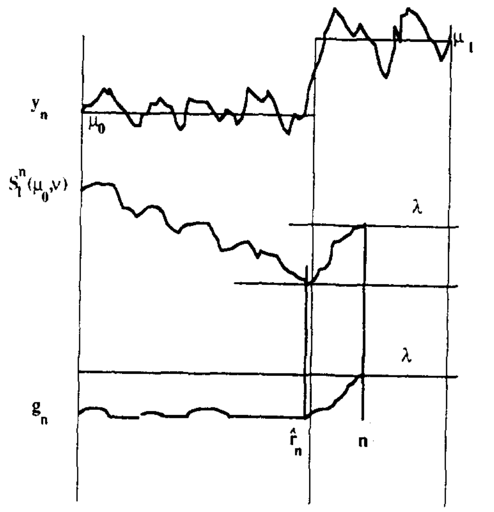

FIG. 2. Scheme for the Page–Hinkley stopping rule.

which:

$$g_n = S_1^n(\mu_0, \nu) - \min_{1 \le k \le n} S_1^k(\mu_0, \nu) > \lambda. \quad (11)$$

This is called the Page–Hinkley stopping rule (Page, 1954) or cumulative sum algorithm; and may be computed in the following recursive manner:

$$g_n = \left(g_{n-1} + y_n - \mu_0 - \frac{\nu}{2}\right)^+.$$

See Basseville and Benveniste (Eds, 1986, Chapter 1) for more details.

Its behavior is depicted on Fig. 2. It may be used more generally for detecting any change between two known probability laws p_{θ_0} and p_{θ_1}. In this case, compute:

$$S_i^j(p_{\theta_0}, p_{\theta_1}) = \sum_{k=i}^{j} \log \frac{p_{\theta_1}(y_k)}{p_{\theta_0}(y_k)}. \quad (12)$$

The theoretical properties of the test (11) have been investigated for a long time from both on-line and off-line points of view. The most significant works in that direction are Shiryaev (1963), Lorden, (1971), Hinkley (1971), Basseville (1981), and recently Moustakides (1986a). This last result is the only non-asymptotic one: the stopping rule (11) minimizes the mean delay for detection for a *fixed* (and not going to zero) false alarm rate. Further discussions may be found in Basseville and Benveniste (Eds, 1986, Chapter 1).

Another optimal stopping rule was obtained by Bojdecki and Hosza (1984) for a different criterion still in the general case of two known probability laws. Finally, recent results obtained by Yashchin (1985) should improve performance evaluation for the above-described cumulative sum algorithms.

2.2. *Unknown jump magnitude*

We now consider the more realistic case where the jump magnitude ν is unknown. From an on-line point of view, we may assume that μ_0 is known, but not μ_1. Two approaches may be used in such a case.

The first one consists of running two tests (11) in parallel, corresponding to an *a priori* chosen minimum jump magnitude ν_m and to two possible directions (increase or decrease in the mean). The corresponding stopping rules are as follows: for a decrease:

$$\begin{cases} T_0 = 0 \\ T_n = \sum_{k=1}^{n} \left(y_k - \mu_0 + \frac{\nu_m}{2}\right) \\ M_n = \max_{0 \le k \le n} T_k \\ \text{alarm} \quad \text{when } M_n - T_n > \lambda \end{cases} \quad (13)$$

and for an increase:

$$\begin{cases} U_0 = 0 \\ U_n = \sum_{k=1}^{n} \left(y_k - \mu_0 - \frac{\nu_m}{2}\right) \\ m_n = \min_{0 \le k \le n} U_k \\ \text{alarm} \quad \text{when } U_n - m_n > \lambda. \end{cases} \quad (14)$$

The decision which is taken corresponds to the rule which stops first, and the estimate of the jump time r is the last maximum (respectively minimum) time before detection. This approach was used in Basseville *et al.* (1981) for line-by-line edge detection in digital pictures.

The second approach in the case of unknown jump magnitude ν consists of replacing it by its MLE. The likelihood ratio test is then:

$$\max_{1 \le r \le n} \max_{\nu} S_r^n(\mu_0, \nu) \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \quad (15)$$

Because of (8), we have:

$$\hat{\nu}_n(r) \triangleq \arg\max_{\nu} S_r^n(\mu_0, \nu) = \frac{1}{n-r+1} \sum_{k=r}^{n} (y_k - \mu_0)$$

and thus the double maximization in (15) is actually only a single one.

We shall see in the next section that this property is still valid in a more general situation, and leads to an efficient change detection algorithm with reasonable computing cost.

Finally, let us mention that other algorithms—both on-line and off-line— for detecting changes in the mean are surveyed in Basseville (1982). The robustness and the superiority of the Page–Hinkley stopping rule (11) with respect to the classical "filtered derivatives" detectors are investigated in Basseville (1981) and Basseville and Benveniste (1983a).

3. ADDITIVE CHANGES IN LINEAR SYSTEMS

In this section we consider the problem of detecting *additive* changes in *linear* dynamical systems described in state space representation as follows:

$$\begin{cases} X_{n+1} = FX_n + GU_n + V_n(+\nu_x\delta_{r,n}) \\ Y_n \quad = HX_n + W_n \qquad (+\nu_y\delta_{r,n}) \end{cases} \tag{16}$$

where δ is the Kronecker symbol, $(V_n)_n$ and $(W_n)_n$ are two independent Gaussian white noises, and where the change of "magnitude" ν_x or ν_y may occur either on the state transition equation or on the observation equation.

3.1. *Generalized likelihood ratio test*

A rather old intuitive approach consists of monitoring the innovations $(\varepsilon_n)_n$ of a Kalman filter, for example (Mehra and Peschon, 1971). Actually, because of the linear property of the system and because of the additive effect of the change on the system, it may easily be shown (Willsky and Jones, 1976) that the effect of the change on the innovation ε_n is also additive. Moreover, the Gaussian characteristic of the state and observation noises in (16) ensures that the property of explicit solution in ν for the likelihood ratio test (15) is still valid in the present general situation of additive changes in linear systems. These points were exploited by Willsky and Jones (1976), who derived a *recursive* algorithm for the so-called *generalized likelihood ratio* (GLR) test (15) computed for the innovations ε_n of the Kalman filter designed under the no change hypothesis.

More precisely, because the distribution of these innovations is given by the *conditional* distribution of the observation with respect to its past values, the cumulative sum to be computed instead of (12) in the present case is:

$$S_i^j(p_{\theta_0}, p_{\theta_1}) = \sum_{k=i}^{j} \log \frac{p_{\theta_1}(Y_k \mid Y_{k-1}, \ldots, Y_0)}{p_{\theta_0}(Y_k \mid Y_{k-1}, \ldots, Y_0)} \tag{17}$$

where p_{θ_1} reflects the change of "magnitude" ν in (16). The GLR test is then:

$$\max_{1 \le r \le n} \max_{\theta_1} S_r^n(p_{\theta_0}, p_{\theta_1}) \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \tag{18}$$

As mentioned above, the maximization over θ_1 (or ν) is explicit, because of the assumptions of Gaussian white noises and additive changes.

Moreover, the computation of S_r^k and $\hat{\nu}_k$ are recursive. The only non-recursive computation is the discrete maximization over the change time r. In order to limit the computing time, Willsky and Jones (1976) decided to constrain the search in a window of fixed size M, namely to compute:

$$\max_{k-M+1 \le r \le k} S_r^k(p_{\theta_0}, p_{\hat{\theta}_1}) \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \tag{19}$$

The key point is that (19) is not a finite horizon technique because (17) is computed with the aid of *all* the past observations.

One interesting feature of this algorithm is the ability of updating the Kalman filter after change detection, with the aid of the estimate of the jump magnitude.

Another interesting property concerns the diagnosis problem, and will be discussed in Section 9. The theoretical properties of the GLR test will be reported in Section 5.

3.2. *A modified algorithm*

In practice, the main advantage of the GLR algorithm (19) is to give good estimates for the change time r and "magnitude" ν, even if the change actually occurs in more than one step in time.

However, a true drawback lies in the coupling effect between the window size M and the threshold λ in (19) and in the possibly high sensitivity with respect to the choice of λ.

For these reasons, a modified algorithm was derived by Basseville and Benveniste (1983a) and applied to geophysical signals. The decision is based upon the MLE $\hat{\nu}_k$ (and not on the likelihood ratio), and the resulting algorithm—filtering + detection + updating—works as a low-pass filter everywhere except at the change points.

An experimental comparison with the Page–Hinkley stopping rule (13)–(14) is done in Basseville and Benveniste (1983a), and also with a "mixed" algorithm involving Hinkley's stopping rule and Willsky's magnitude estimate.

Other ways of managing not necessarily additive changes in systems like (16) are reported in Section 9.

4. CHANGES IN SPECTRAL PROPERTIES OR EIGENSTRUCTURE

We now investigate the problem of changes in AR or ARMA models, or equivalently *in the state transition matrix F of model* (16). In this section, we are mainly interested in the problem of segmentation of *scalar* signals, while the vector case will be mainly investigated in Sections 5–8.

It may be of interest to keep in mind the following distinction between the two types of situations:

(i) either the analyzed system or signal is known to have the same behaviour as an AR or ARMA process, and then the model is descriptive enough for its parameters' behavior to be of interest;
(ii) or the model of the system or signal is not known, and the main issue is the detection of changes in its spectral characteristics; then the AR or ARMA model to be used is nothing but a tool for the detection of such changes. In this case, robustness properties of the detectors may be of key importance.

4.1. *Generalized likelihood ratio test*

In the present situation of detecting changes in (scalar) AR or ARMA models, the generalized likelihood ratio (GLR) test presented in the previous section may still be used. The log-likelihood is computed as in (17) using the *conditional* laws of the observations, and the *parameter* θ *defined in* (2) *for AR models.* In an on-line framework where $\theta_0' = \theta_0$ in (5) and the law p_{θ_0} is assumed to be known (possibly up to a convenient identification), the GLR test is exactly as in (18). But the maximization over θ_1 is no longer explicit, because the change is not additive on the observation. Moreover, in the ARMA case the cumulative sum (17) is no longer linear in the parameters. Therefore the test (17)–(18) is quite time-consuming.

Assume now that $\theta_0' \neq \theta_0$ in (5)—which is generally the case in off-line approaches. In that situation, the log-likelihood ratio for a sample of size n is;

$$S_1^n(p_{\theta_0}; r, p_{\theta_0'}, p_{\theta_1}) = \sum_{k=1}^{r-1} \log \frac{p_{\theta_0'}(Y_k|\cdots)}{p_{\theta_0}(Y_k|\cdots)} + \sum_{k=r}^{n} \log \frac{p_{\theta_1}(Y_k|\cdots)}{p_{\theta_0}(Y_k|\cdots)}. \quad (17')$$

If none of the parameters $\theta_0, \theta_0', \theta_1, r$ are known, the GLR test is as follows:

$$\max_{1\leq r\leq n} \min_{\theta_0} \max_{\theta_0'} \max_{\theta_1} S_1^n(p_{\theta_0}; r, p_{\theta_0'}, p_{\theta_1}) \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \quad (18')$$

In other words, the unknown parameters are again replaced by their MLE. Further details for the AR case will be given below. A complete theoretical investigation of this test is done in Deshayes and Picard (1986) with the aid of convenient statistical asymptotic analyses and will be reported in the next section.

Nevertheless, several design issues may be extracted from the GLR methodology, as will be shown below for detecting changes in AR models.

4.2. *The two-model approach for on-line change detection in AR models*

Let us consider an AR process:

$$y_n = \sum_{i=1}^{p} a_i^{(n)} y_{n-i} + \varepsilon_n \quad (20)$$

where (ε_n) is a Gaussian white noise with variance σ_n^2, and: for $1 \leq i \leq p$:

$$a_i^{(n)} = \begin{cases} a_i^0 & \text{for } n \leq r-1 \\ a_i^1 & \text{for } n \geq r \end{cases}$$

and

$$\sigma_n^2 = \begin{cases} \sigma_0^2 & \text{for } n \leq r-1 \\ \sigma_1^2 & \text{for } n \geq r. \end{cases} \quad (21)$$

Let us define:

$$\theta^j = (a_1^j, \ldots, a_p^j, \sigma_j^2) \quad (j = 0, 1)$$

and

$$Y^{n-1} = (Y_{n-1}, \ldots, Y_1)^{\mathrm{T}}.$$

The problem is to detect a change in θ and to estimate the change time r.

4.2.1. *Implementation of the GLR test.* An on-line implementation of the algorithm (17)–(18) is depicted in Fig. 3 and may be described as follows. If the AR model M_0 under the no change hypotheses (i.e. before the change) is not known, identify it with the aid of a recursive growing memory filter. On the other hand, for each possible change time r, use the data of the time window $\{r, r+1, \ldots, n-1, n\}$ for identifying the AR model M_1 after change, and compute the log-likelihood ratio S_r^n. Then maximize over r. Other distance measures between the two models M_0 and M_1 will be reported below.

If the full GLR algorithm (17')–(18') is implemented on-line, three models M_0, M_0', M_1 have to be identified: for fixed r, M_0 and M_0' with growing memory filters using the data of the time windows $\{1, \ldots, n\}$ and $\{1, \ldots, r-1\}$, respectively; M_1 using the time window $\{r, r+1,$

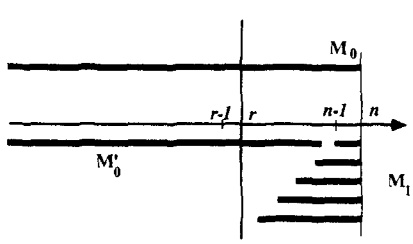

FIG. 3. Scheme for the GLR test.

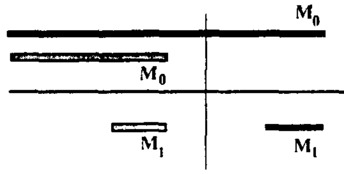

FIG. 4. Using two models, one "global" and one local.

$\ldots, n-1, n\}$. Then the maximization over r has to be performed. Actually it may easily be shown that, for ARMA processes, the GLR test (17′)–(18′) is:

$$\max_{1 \le r \le n} u_n(r) \gtrless \lambda$$

where:

$$u_n(r) = n \log \hat{\sigma}_0^2 - [(r-1)\log \hat{\sigma}_0'^2 + (n-r+1)\log \hat{\sigma}_1^2] \quad (22)$$

in which $\hat{\sigma}_0^2$, $\hat{\sigma}_0'^2$, $\hat{\sigma}_1^2$ are the variances of the innovations of the (estimated) models M_0, M_0', M_1, respectively.

A simplified and approximated implementation was proposed by Appel and Brandt (1983). The detection of the change is done first, using a fixed window length $N = n - r + 1$ for M_1, and the decision rule:

$$u_n(N) = n \log \hat{\sigma}_0^2 - [(n-N)\log \hat{\sigma}_0'^2 + N \log \hat{\sigma}_1^2] \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \quad (23)$$

The estimation of the change time is done in a second step. The resulting algorithm is very close to GLR.

4.2.2. *Other distance measurements between the two models*. By the same time, Basseville and Benveniste (1983b) also proposed to use the above-mentioned two-model approach for change detection (Fig. 4): compute a distance between a long term or global AR model M_0 and a short term or local AR model M_1. Several distance measurements may be used (Gray and Markel, 1976; Ishii *et al.*, 1979). The Euclidean distance between the AR parameters:

$$\sum_{i=1}^{p} (a_i^1 - a_i^0)^2$$

is bad, because of neither mathematical nor spectral theoretical meaning, but unfortunately still often used in practice. An efficient and theoretically sound distance between the spectral densities $S_j(e^{i\omega})(j = 0, 1)$ is:

$$\|\log S_1(e^{i\omega}) - \text{Log}\, S_0(e^{i\omega})\|_{L^2}$$

which may be well approximated by the cepstral distance, namely the Euclidean distance between the *cepstral* coefficients (Gray and Markel, 1976). Another distance was given in (22), which may be seen as the Chernoff distance between the *joint* distributions of the observations. Finally, Basseville and Benveniste (1983b) proposed the use of the Kullback divergence between the conditional distributions of the observations, which results in:

$$w_n = \frac{1}{2}\left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2} - 1\right) + \left(1 + \frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right)\frac{(e_n^0)^2}{2\hat{\sigma}_0^2} - \frac{e_n^0 e_n^1}{\hat{\sigma}_1^2} \quad (24)$$

where

$$e_n^j = y_n - \sum_{i=1}^{p} \hat{a}_i^j y_{n-i} \quad (j = 0, 1)$$

are the innovations of the growing memory and sliding filters, respectively.

Because the effect of a change in the AR model is reflected on w_n (24) by a change in the sign of its mean value, an accurate estimation of the change time may be obtained if we add the Page–Hinkley stopping rule (11) computed on the w_n.

The differences between the algorithms (22) of Appel and Brandt (1983) and (24) of Basseville and Benveniste (1983b) lie in the distance which is used between the two models M_0 and M_1 of Fig. 4, and in the estimation of the change time. But they are very similar in their spirit and have been compared by André-Obrecht (1988) for the segmentation of continuous speech signals. An example may be seen in Fig. 5. The same French sentence is shown in each part (a) and (b) of the figure, and processed by the algorithm (22) in (a) and the algorithm (24) in (b). In each case, the behavior of the statistics (22) or (24) is depicted below the signal, and the vertical lines indicate the estimated change times. It has to be noticed that the statistic (24) has a smoother behavior than (22), and thus leads to a more robust detector (lower sensitivity to the choice of the threshold). We refer the reader to Basseville and Benveniste (Eds, 1986, Chapter 6) for further discussions and comparisons between the two above-mentioned algorithms and also with the cepstral distance.

For other uses of distance measurements for segmentation, see Ishii *et al.* (1979).

4.2.3. *Comments on this approach*. Three key features have to be emphasized. First, this particular implementation of the two-model approach is more efficient than a previous one (Bodenstein and Praetorius, 1977) which is depicted in Fig. 6. It consists of the comparison of two local models M_0'' and M_1 identified inside two finite windows having the same length. Clearly the reference model M_0 (before change) is far more precisely identified than M_0'' and, if a

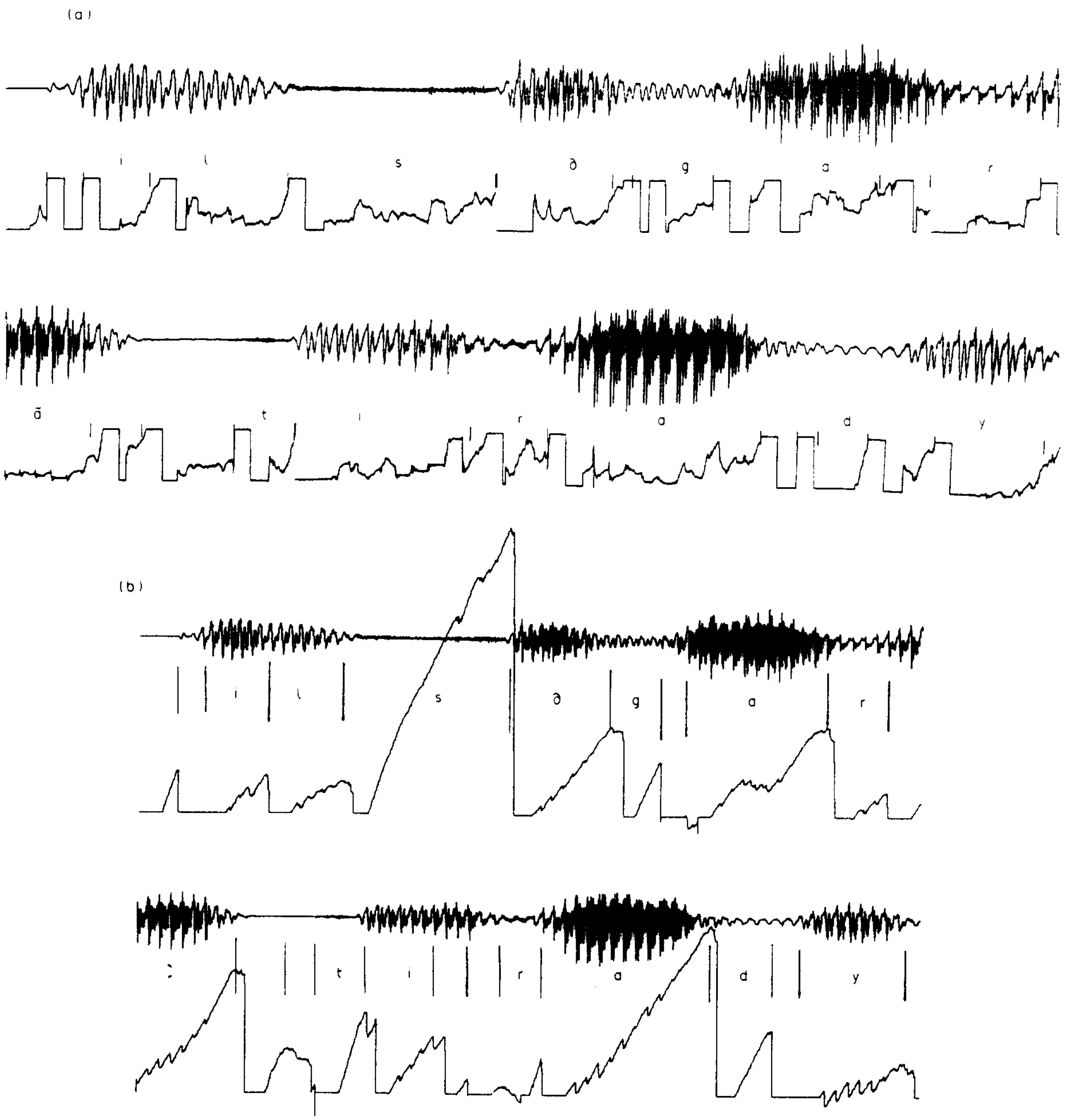

FIG. 5. Comparing the two algorithms [(22)—(a) and (24)—(b)] on continuous speech signal.

small forgetting ability is used, it will be slightly modified by the change, and thus the false alarm rate should be reduced.

Second, this two-model approach is more efficient than a one-model approach based on classical χ^2-type tests on the innovations of M_0, as:

$$u_n = \sum_{k=1}^{n} \left[\frac{(e_k^0)^2}{\hat{\sigma}_0^2} - 1 \right] \tag{25}$$

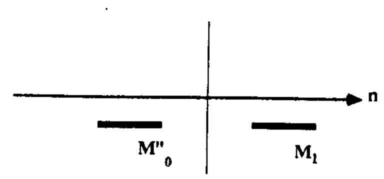

FIG. 6. Using two local models.

which was studied in Segen and Sanderson (1980), for example. This test is "blind", it is in fact designed to detect a deviation with respect to the noise behavior—thus the range of changes that can be detected is larger with the former approach (24) than with the latter (25)— and may have a high variance before change. See Basseville and Benveniste (1983b) for further discussions.

Finally, no theoretical result concerning performance evaluation of the two-model approach is available at this time, except in the case of exact implementation of the likelihood ratio test GLR which will be investigated in the next section, and for the local approach presented in Section 6.

A non-parametric treatment of this approach may be found in Kedem and Slud (1982) and will be commented on in the last section.

5. THE LIKELIHOOD APPROACH IN A GENERAL FRAMEWORK

As already mentioned in Section 1, the likelihood ratio approach is a fairly general tool for change detection which may be used in general situations encountered in automatic control and signal processing, such as changes in *multivariable* AR or ARMA models and changes in the state transition matrix of a state space model.

This approach again consists of computing the log-likelihood ratio (17) based upon the *conditional* distribution of the observations (conditioned by their past values), and in running the GLR test (18) which generally involves a double maximization of high computational cost. For this reason, several different approximations have been designed, even in the scalar case as described in the previous section. Another general tool for approximating the likelihood ratio will be presented in the next section. However, the GLR may at least be used as a benchmark for other algorithms, because its theoretical optimality has been completely investigated recently (Deshayes and Picard, 1986) from an off-line point of view.

Before giving the key result of this study, let us first briefly outline the main difficulty of the change detection problem.

Usually the criterion for performance evaluation of statistical tests between two hypotheses $\mathbb{H}_0$ and $\mathbb{H}_1$—here no change and change, respectively—is to maximize the power β (or probability of deciding $\mathbb{H}_1$ when $\mathbb{H}_1$ is true) for a fixed *level* α (or probability of deciding $\mathbb{H}_1$ when $\mathbb{H}_0$ is true). When $\mathbb{H}_1$ is not reduced to a single distribution, the best property for a test is to be uniformly most powerful (UMP) (for each distribution belonging to $\mathbb{H}_1$). Unfortunately, because the parameters of interest in change detection problems—namely the change time and the change magnitude—are such that the Neyman–Pearson lemma (Lehmann, 1959) is not valid, change detection problems are multiple hypotheses testing problems for which no UMP test exists. For this reason it is necessary to define an asymptotic framework in which UMP tests exist for change detection problems. Deshayes and Picard (1986) use the large deviations asymptotic analysis in which *exponential* error probabilities are introduced: α and $1-\beta$ are kept exponentially decreasing to zero while the number of observations goes to infinity. Considering only the corresponding exponential decay rates, and for specific families of densities, the GLR test is shown to be UMP. The asymptotic behavior of the test is derived together with the asymptotic distribution of the change time and magnitude estimates.

6. THE STATISTICAL LOCAL APPROACH

In this section we describe another approach for overcoming the main drawback of the GLR test (18), namely its computing time cost due to the double maximization. This approach is known by statisticians under the name of *local* approach and has been introduced in change detection problems by Nikiforov (1983, 1986) for on-line detection of changes in spectral characteristics. The use of the local approach in conjunction with functions other than the likelihood one has been proposed in Basseville *et al.* (1987) and Benveniste *et al.* (1987) and will be described in the next section.

6.1. *Local approach for changes in spectral properties*

The original idea of Nikiforov (1983, 1986) consists of looking for *small* changes in AR (or ARMA) models and using a special type of Taylor's *expansion* of the log-likelihood function which is called Le Cam's asymptotic expansion (Roussas, 1972). In other words, instead of monitoring the observations process $(y_n)_n$ or the innovation process, the local approach monitors;

$$z_n = \frac{\mathrm{d}}{\mathrm{d}\theta} \log p_\theta(y_n \mid y_{n-1}, \ldots,)|_{\theta=\theta_0}. \qquad (26)$$

The key theoretical point here (Deshayes and Picard, 1986) is that there exists a central limit theorem for z_n, the main consequence of which is as follows. Any change in θ (2) is reflected in a change in the *mean* of z_n for which the Page–Hinkley stopping rule (11) or the GLR (15) of Section 2 may be used.

Using two different kinds of *a priori* information about the changes to occur (changes along a known direction, changes outside an ellipsoïd centered at the reference model θ_0), Nikiforov developed two algorithms based upon the detection rule (11), which he called cumulative sum algorithms (CSA).

More precisely, recall that the considered detection rule was defined with the aid of:

$$g_n = \max_{1 \leq k \leq n} S_k^n(\theta_0, \nu). \qquad (27)$$

In the first case of *change along a known direction C*, the parameter θ evolves as:

$$\theta = \theta_0 + \eta C \qquad (28)$$

and the corresponding cumulative sum to be used in (27) is:

$$S_k^n(\theta_0, C) = \sum_{i=k}^{n} z_i^{\mathrm{T}} C \tag{29}$$

where z_k is defined in (26).

In the second case of *change outside an ellipsoïd* for which;

$$(\theta - \theta_0)^{\mathrm{T}} \mathcal{F} (\theta - \theta_0) \gtrless \eta \tag{30}$$

the corresponding function in (27) is:

$$S_k^n = \left(\sum_{i=k}^{n} z_i\right)^{\mathrm{T}} \mathcal{F}^{-1} \left(\sum_{i=k}^{n} z_i\right). \tag{31}$$

We refer the reader to Nikiforov (1986) for further details. Let us emphasize that these cumulative sum algorithms may be designed for any change in a *multivariable* ARMA process. The application of this methodology to seismic signal processing is described in Nikiforov and Tikhonov (1986).

6.2. *A tool for performance evaluation of CSA*

Another interesting part of the above-mentioned work is the derivation of a convenient tool for performance evaluation of cumulative sum algorithms. This tool, known as the *average run length,* permits the computation of both the false alarm rate and the delay for detection with the aid of a single function which is the expectation of the detection time under the convenient probability law. We refer to Nikiforov (1986) for more details. With respect to the criteria discussed in Section 1.4, the delay for detection which is evaluated is defined in the same manner as in Lorden (1971).

7. A COUNTER-EXAMPLE AND ANOTHER USE OF THE LOCAL APPROACH

In spite of the wide applicability and the good properties of the statistical change detection/estimation algorithms described above—namely the likelihood ratio and the local approaches—there exist situations where none of these two approaches can be used because of *coupling effects* in the likelihood function between the parameters to be monitored and unknown nuisance parameters.

The purpose of this section is the presentation of such a counter-example and of the corresponding solution which has been recently derived (Basseville *et al.*, 1987). This solution may be extended to more general situations as will be shown below: to any recursive parameter estimation algorithm may be associated a change detection and a model validation scheme (Benveniste *et al.*, 1987), using the local approach for conveniently chosen statistics.

7.1. *An example of change detection in the presence of nuisance parameters*

The example 2 of vibration monitoring of structures under natural excitation, which was described in 1.1.2., may be easily stated as a problem of detecting changes in the AR part of a multivariable ARMA process having unknown and time-varying MA coefficients to be considered as nuisance parameters. Because the Fisher information matrix of an ARMA process is not block-diagonal with respect to the AR and MA parameters, neither the likelihood function nor its "Taylor's" expansion (local approach) are of any help for solving this particular detection problem (see the discussion on nuisance parameters in Deshayes and Picard, 1986). The solution presented in Basseville *et al.* (1987) uses two basic tools. Let:

$$Y_n = \sum_{i=1}^{p} A_i Y_{n-i} + \sum_{j=0}^{p-1} B_j(n) E_{n-j} \tag{32}$$

be the considered ARMA process, where $(E_n)_n$ is a standard Guassian white noise sequence. The first tool to be used is what we call the *instrumental statistics*:

$$U_n = \sum_{k=p+N-1}^{n} Z_k W_k^{\mathrm{T}} \tag{33}$$

where

$$Z_k^{\mathrm{T}} = (Y_{k-p}^{\mathrm{T}} \cdots Y_{k-p-N+1}^{\mathrm{T}})$$

is the vector of past observations and:

$$W_k = Y_k - A_1 Y_{k-1} - \cdots - A_p Y_{k-p}$$

is the MA part.

The second tool to be used is the *local* approach described in Section 6, however not connected to the likelihood function but to the above instrumental statistics U_n (33). In other words, we again look for *small* changes in the AR coefficients $(A_i)_{1 \leq i \leq p}$ of (32). It turns out that, because of the non-stationary central limit theorem of Moustakides and Benveniste (1986), these changes are reflected in changes in the *mean* of the instrumental statistics U_n which is furthermore asymptotically Gaussian distributed with covariance matrix Σ_n. As we followed for that application an off-line model validation approach (validation of a "signature" on a new record of measurements), the convenient test for detecting a change in the mean of U_n is simply the χ^2 test:

$$U_n^{\mathrm{T}} \Sigma_n^{-1} U_n \underset{\mathbb{H}_0}{\overset{\mathbb{H}_1}{\gtrless}} \lambda. \tag{34}$$

As shown in Basseville *et al.* (1987), this test is very powerful in practice. Its theoretical properties are investigated in Rougée *et al.* (1987). We will see in Section 9 how it can be used for solving the diagnosis problem. Finally, an on-line implementation is reported in André-Obrecht (1988) for another application.

7.2. *Extended use of the local approach for change detection and model validation*

The detection solution presented above may be extended to more general situations than ARMA models. Actually the key idea in the previous example was to use, for the detection problem, the same starting point as in the *identification* problem. It was known that the instrumental variable identification method for estimating the AR part of an ARMA process was theoretically (Benveniste and Fuchs, 1985) and experimentally (Prevosto *et al.*, 1983) robust with respect to the unknown and time-varying MA part. Thus the "instrumental statistics" U_n (33) was defined and theoretically studied for deriving the detector (34).

In the same manner, starting from any general recursive parameter identification algorithm:

$$\theta_n = \theta_{n-1} + \gamma_n H(\theta_{n-1}, X_n) \tag{35}$$

and applying the local approach to the statistics $H(\theta_0, X_n)$ where θ_0 is a nominal model, it is possible to prove a central limit theorem which transforms the problem of detecting changes in the parameter vector θ into the problem of detecting changes in the mean value of an asymptotically Gaussian distributed process which is a convenient cumulative sum of the function H. We refer to Benveniste *et al.* (1987) for more details. No theoretical result concerning the performances of these tests is available now, except for (34).

We shall see in Section 9 how this general approach may be used for solving the diagnosis problem.

8. GENERATING THE SIGNALS TO BE MONITORED

This section is devoted to the presentation of different types of algorithms for solving the task (i) described in 1.2, namely the problem of generating the signals to be monitored in order to achieve change detection. For this purpose, both deterministic and stochastic algorithms may be used, and we distinguish two classes of methods which operate the compression of information in different ways: redundancy and filtering operations. The generation of such "residuals" and "change indicating signals" Δ may be generally summarized as in the diagram of Fig. 7 (Mironovski, 1980), where P is the studied process or system having inputs U and outputs Y, and where H and F will be defined below.

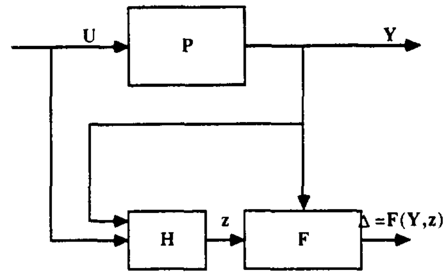

FIG. 7. Diagram for residuals generation.

8.1. *Redundancy methods*

These techniques, which are well known in the automatic control community, are basically deterministic. They exploit either the direct physical redundancy of the system, namely the identical measurement and/or control units present in duplicate, triplicate and even quadruplicate in the system; or the analytical redundancy, i.e. the deterministic instantaneous or temporal relations existing between various measurements, for example kinematic. If the redundancy is high enough, a diagnosis of the change can be obtained as will be shown below.

8.1.1. *Direct or physical redundancy.* If several identical sensors measuring the same quantities are available, the differences betwen the two signals contained in each possible pair may of course reflect a failure. In the diagram of Fig. 7, for a duplicate system H is the second system, and the operator $F(Y, Z)$ producing the "residual" Δ is a simple difference. These "residuals" are generally processed with the aid of voting methods (Willsky, 1976). But another possible processing consists of searching, given an error bound for each sensor, for subsets of measurements with different degrees of consistency. The most consistent subset is used for estimating the measured quantity, and the less consistent one—if it exists—for isolating the failure. This has been done in Desaï and Ray (1984) for multidimensional measurements, for example speed and acceleration in a three-dimensional space. It has to be noticed that this method processes simultaneously real measurements and artificial measurements resulting from the investigation of the analytical redundancy of the system. Furthermore it is possible to solve the problem of calibration between measurements from identical sensors having different biases for example (Ray and Desaï, 1983).

8.1.2. *Indirect or analytical redundancy.* Analytical redundancy is the set of all existing instantaneous or temporal relationships between the inputs of the actuators and the outputs of the sensors of the system which are identically zero when no change occurs. These techniques seem to have been developed independently in the United States (Deckert *et al.*, 1977; Chow and Willsky, 1980, 1984) and in the Soviet Union (Mironovski, 1980). We present here some key points when deriving redundancy relations in the two situations where the system is represented by a state space model—first deterministic and then noisy—or by a block-diagram connecting elements with known transfer functions.

8.1.2.1. *Deterministic case*. Consider a deterministic system described by:

$$\begin{cases} \dot{X} = AX + BU & X \in \mathbb{R}^n, \quad U \in \mathbb{R}^r \\ Y = CX & Y \in \mathbb{R}^s. \end{cases} \tag{36}$$

Let us introduce a so-called redundant variable z such that:

$$\Delta = F(Y, Z) = M(y_1, \ldots, y_s) + z = 0 \tag{37}$$

where M is an instantaneous relation which may be linear in the y_i. Several methods are available for designing z, namely for designing the block H in Fig. 7.

For an observable system, a possible solution, when M is linear, consists of using as H a Luenberger observer (Kailath, 1980) designed for estimating the instantaneous linear combination of the states of the system: $-MY = -MCX = LX$. Recall that for a noisy system the best observer of X, in the mean square sense, is the Kalman filter, which is of order n and thus is equivalent to a system duplication. Luenberger derived a method which reduces the order to $n - s$, where s is the dimension of the observation Y. The order may be further reduced if LX is estimated and not X.

Applying this theory modifies the diagram of Fig. 7 with a Luenberger observer as H and the linear combination M as F. This results in the diagram of Fig. 8. Consider now the problem of designing such a device of minimum order k. It may be simply shown (Mironosvski, 1980) that,

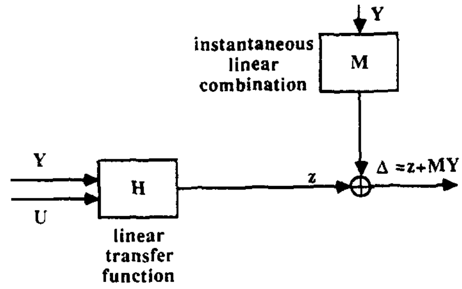

FIG. 8. Diagram for residuals generation with analytical redundancy.

because H is a linear stationary block with scalar output, the diagram of Fig. 8 may be described by the equation:

$$p^k \Delta = p^k MY + \sum_{j=0}^{k-1} p^j(\alpha_j Y + \beta_j U - \gamma_j z) \tag{38}$$

where p^i is the i-th differential operator.

Because of (36) and (37), this relation is satisfied for any input U if and only if:

$$(\alpha_0 + \gamma_0 M; \alpha_1 + \gamma_1 M; \ldots; \alpha_{k-1} + \gamma_{k-1} M; M) \begin{pmatrix} C \\ CA \\ \vdots \\ CA^k \end{pmatrix} = 0 \tag{39}$$

and for j from 0 to $k - 1$:

$$\beta_j = -\sum_{i=j+1}^{k} (\alpha_i + \gamma_i M) CA^{i-1-j} B$$

where $\alpha_k = 0$ and $\gamma_k = 1$.

Mironovski (1979) has shown that the minimum order k of a device such as Fig. 8 is always between the smallest and the largest Kronecker invariant index, and that it is possible to choose M in order to reach these bounds in k. From (38) and (39), it can be seen that this approach leads to the concept of "parity check" studied by Chow and Willsky (1980, 1984). They use an ARMA model (38) and look for the orthogonal space of the range of the observability matrix (39).

8.1.2.2. *Extension to noisy systems*. If the model (36) is perturbed by noises on the state X or the observation Y, the relation (37) no longer holds even when no change occurs. Therefore redundancy relations which are robust with respect to the noise have to be defined. One possible solution consists of searching for conditions similar to (39) but related to an extended observability space taking noise into account (Chow *et al.*, 1986). Using the same approach, it is possible to take into account uncertainties on the parameters A, B, C. One can also avoid such a modelling and work directly with the empirical covariances of the observations (Pattipati *et al.*, 1984).

8.1.2.3. *Systems defined with transfer functions*. The use of redundant variables has been also studied for linear systems described by any block diagram connecting elements with known transfer functions $W_1(p), \ldots, W_n(p)$ (Mironovski, 1980).

It can be shown (Britov and Mironovski, 1972) that a redundancy relation z of the form:

$$z = \sum_{i=1}^{n} (\alpha_{0i}u_i + \beta_{0i}y_i) + \sum_{k=1}^{m} W^k(p) \sum_{i=1}^{n} (\alpha_{ki}u_i + \beta_{ki}y_i) \quad (40)$$

can be obtained if and only if the transfer functions $W_i(p)$ are rational; then the coefficients αs and βs in (40) can be written in closed form.

Consequently, for any system defined with components having rational transfer functions, it is possible to design a detection device such as Fig. 8, of order equal to the largest order of the elements belonging to the system, and thus generally far lower than the system order.

8.2. *Filtering methods*

8.2.1. *Kalman filters and state observers.* A rather old solution (Willsky, 1976) to the change or failure detection problem consists of monitoring the innovations or prediction errors or some conveniently chosen estimation filter(s) or parameter identification algorithm(s) which fills the block H of the diagram of Fig. 7. This idea has been developed along two main axes. The first viewpoint led to design filters especially sensitive to the changes under study: this is precisely what was described above for the various observers; a geometric framework for this approach is presented in Massoumnia (1986). The other approach consists of using the optimal state estimate, namely the Kalman filter, designed according to the model of the system (or signal) in its normal operating mode. As will be shown in the next section, if diagnosis is desired in addition to detection, a possible solution consists of using a bank of Kalman filters designed according to all the available possible models of the system (or signal) under all the change hypotheses. The corresponding decision rules will be discussed in the next section.

But, as far as the design of decision rules is concerned, we insist upon the fact that the Kalman filter is the only one which produces zero-mean and independent residuals—under the no-change hypothesis—when state and/or measurement noises are present. This is generally not the case for the instantaneous or temporal redundancy relations described above. Therefore the assumption of *independence* in statistical decision rules is valid *only* for the innovations of Kalman filters. For monitoring other types of "residuals", it may be of interest to use the algorithms for detecting changes in spectral characteristics presented in Section 4.

8.2.2. *Generalization to extended or decentralized Kalman filters.* The extension of the above approach to non-linear dynamical systems may be achieved with the use of extended Kalman filters (Himmelblau, 1978; Usoro *et al.*, 1985). On the other hand, in order to reduce the implementation cost of Kalman filters, and also to introduce protection against some subsystem failures, the use of decentralized filters is currently under investigation in the aeronautic domain (Kerr, 1985).

8.2.3. *Extension to other identification methods.* The detection strategy which is commonly chosen in connection with filtering methods for change detection, consists of testing how much the sequence of *innovations* has deviated from the "white noise" hypothesis. See Mehra and Peschon (1971) and Willsky *et al.*, (1975), for example. The tests which are used are then classical tests for zero-mean, independence, unit variance, ... But, in some practical problems, it may be useful and even necessary to monitor some more complex function of the innovations than the innovations themselves. This is the case in the Example 2 of Section 1, and we described in Section 7 a systematic approach transforming a possibly complex change detection problem into the simple problem of change in the mean of a Gaussian process with known covariance matrix.

9. THE DIAGNOSIS PROBLEM

In this section we investigate the diagnosis problem, namely the problem of estimating the origin of the change and possibly its location in the system. Two types of situation have to be distinguished, of unequal degree of difficulty.

(i) *Diagnosis of changes on identifiable model sets*
In this case, there is a one-to-one correspondence between the parameters used for detecting model changes and the parameters for which diagnosis is desired. For example, one monitors an AR or ARMA model, and one wishes to know, when a change is detected, which poles actually moved.

(ii) *Diagnosis of changes on non-identifiable model sets*
This situation is much more difficult to solve because the convenient parameterization for diagnosis is in terms of not identifiable parameters. An example of such a situation is described in Basseville *et al.* (1987) and Moustakides (1986b) (see Example 2 of Section 1): it is desired to diagnose the changes in terms of the mechanical characteristics of the structure

which are not identifiable partly because of a model reduction performed in practice for monitoring.

To our knowledge, three types of methods have been developed for answering such questions. The multiple model (MM) approach is probably the older one and has been investigated in several directions. The generalized likelihood ratio (GLR) methodology using several possible models is another solution. Finally, the local approach of Sections 6 and 7 may be used for solving the diagnosis problem even in situation (ii) described above.

9.1. *The multiple model approach*

The use of several possible models of the system under consideration is of common practice especially in automatic control for different purposes. An overview of such an approach for state estimation can be found in Pattipati and Sandell (1983). Adaptive identification is discussed in Tugnait (1982b), and the design of adaptive gains in recursive identification is reported in Andersson (1985). The use of the MM approach in change detection is reviewed in Willsky (1976). Generally speaking, in an MM environment change detection is based upon the monitoring of the *a posteriori* probabilities of the different models, and, in order to avoid the exponential growth of the size of the filters bank to be used, several suboptimal strategies have been proposed. See for example Willsky *et al.* (1975, 1980) and Tugnait (1982a).

Of course, this methodology brings information for diagnosis: each model corresponds to a different change situation and the maximum *a posteriori* probability indicates what is the most likely change. This approach involves implicitly Bayesian techniques, for which an extensive study may be found in Peterka (1981).

9.2. *The GLR methodology*

If a list of N possible (additive) failure directions f_i is known for the system (16), then activating in parallel N GLR tests corresponding to these directions leads to a diagnosis of the change, according to the largest of these tests.

9.3. *Two uses of the local approach*

The local approach presented in Section 6 may be used for solving the diagnosis problem in the following manner. If several possible directions of changes C_j are known in advance, running the corresponding cumulative sum algorithms (27)–(29) in parallel leads to diagnosis of the change.

A second possible use of the local approach for diagnosis is related to the Example 2 of Section 1 and reported in Basseville *et al.* (1987) and Moustakides (1986b). It consists of focussing the instrumental test (34) on some pre-specified subspaces of the parameter space using convenient Jacobians. In situation (ii) described above, the computation of the relevant Jacobians is much more complicated, because of the necessary model reduction, but a solution does exist and provides satisfactory results for diagnosis in terms of the mechanical parameters M and K of (3).

This approach may be generalized in order to associate to *any* recursive parameter identification algorithm a procedure for diagnosing changes even in terms of non-identifiable model sets. We refer to Benveniste *et al.* (1987) for further details.

10. OPEN PROBLEMS AND CONCLUSIONS

We have described what we think to be the state-of-the-art about change detection, estimation and diagnosis in signals and dynamical systems. We have mainly investigated parametric statistical approaches, especially for the design of convenient decision rules, with special attention to the general likelihood ratio methodology.

One important issue we have not addressed for change detection algorithms is the problem of robustness with respect to unmodelled phenomena. Some discussions about this point may be found in Chow *et al.* (1986) and Basseville and Benveniste (1983a,b), for example. One basic conclusion of these works is that it is possible to obtain accurate change detection and estimation using both a simplified model of the monitored signal or process and a convenient detector. For example, the two-model approach with AR models and Kullback's divergence (see 4.2.2.) leads to a sensible segmentation of continuous speech signals (André-Obrecht, 1988), even though it is well known that AR models are not convenient for modelling speech.

A radical way of getting rid of this robustness problem consists of using non-parametric techniques (Kassam, 1980). For example, Kedem and Slud (1982) use zero-crossings statistics in order to characterize the global and the local models of Fig. 3 as well as a distance measurement between them. Deshayes and Picard (1986) report some theoretical results about off-line Kolmogorov–Smirnov's tests applied to the empirical cumulative spectral distribution function (integral of the periodogram). Finally, Darkhovskii (1985) gives a consistency result for an estimate of the change time based upon an extension of the Mann–Whitney's statistics.

Apart from this important robustness problem, the open problems are of three different types. First, as far as on-line signal segmentation is concerned, the two-model approach we described is not convenient when the segments to be found are short: other types of parametric models than classical AR or ARMA models should probably be used, as in Friedlander and Porat (1987). Second, still in an on-line framework, other types of criteria than delay for detection should be optimized, especially for the problems of change or failure detection in controlled industrial processes: convenient criteria should include inspection and repairing costs, for example. Finally, much remains to be done in the field of diagnosis, especially when the parametric models which are used for monitoring are of far lower dimension than the monitored system. The general solution proposed in Benveniste *et al.* (1987) should be used in experiments on different types of systems, and such algorithms should be included in general preventive maintenance strategies.

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