PROBABILITY

1. <u>S is a simple space.</u>

   (i) $P(\phi) = 0$ (ii) $P(\bar{E}) = 1 - P(E)$ where $E \subseteq S$

   (iii) $E_1 \subseteq S$, $E_2 \subseteq S$ and $E_1 \subseteq E_2 \Rightarrow P(E_2 - E_1) = P(E_2) - P(E_1)$ and $P(E_1)$ and $P(E_1) \le P(E_2)$

   (iv) E_1, E_2 are events of S $\Rightarrow P(E_2 - E_1) = P(E_2) - P(E_1 \cap E_2)$

   (v) $E \subseteq S \Rightarrow 0 \le P(E) \le 1$

   (vi) Number of sample points favorable to E = n(E) = m
   (S is taken to contain equally likely simple events)

   Total number of sample points in S = n(S) = n. $P(E) = \frac{m}{n}$.

2. <u>Conditional event</u>

   If E_1, E_2 are events of a sample space S and if E_2 occurs after the occurance of E_1, then the even of occurance of E_2 after the even E_1 is called conditional event. It is denoted by $\frac{E_2}{E_1}$. Similarly we define $\frac{E_1}{E_2}$.

3. <u>Conditional Probability</u>

   $E_1 \subseteq S$, $E_2 \subseteq S$ and $P(E_1) \ne 0$. Then the probability of E_2 after the event E_1 has occurred is called the conditional probability of E_2 given E_1 and is denoted by $P\left(\frac{E_2}{E_1}\right)$.

   We define $P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)}$.

   Similarly we define $P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

   Note that $P\left(\frac{E_2}{E_1}\right) = \frac{n(E_1 \cap E_2)}{n(E_1)}, P\left(\frac{E_1}{E_2}\right) = \frac{n(E_1 \cap E_2)}{n(E_2)}$

   Also $P\left(\frac{E_2}{E_1}\right) + P\left(\frac{\overline{E_2}}{E_1}\right) = 1$.

4. If E is an even of a sample space S, then the odds in favour of E are defined as $P(E) : P(\bar{E})$ and the odds against E are defined as $P(\bar{E}) : P(E)$.

If P(E) : $P(\bar{E})$ = m : n, then P(E) = $\frac{m}{m+n}$ and $P(\bar{E}) = \frac{n}{m+n}$.

5. Independent and dependent events

If the occurrence of the even E_2 is not affected by the occurrence or non-occurrence of the event E_1, then the event E_2 is said to be independent of E_1 and $P\left(\frac{E_2}{E_1}\right) = P(E_2)$. If $P(E_1) \neq 0$, $P(E_2) \neq 0$ and E_2 is independent of E_1, then E_1 is independent of E_2. In this case we say that E_1, E_2 are mutually independent or simply independent.

If the occurrence of the event E_2 is effected by the occurrence of E_1 then the events E_1, E_2 are dependent and $P\left(\frac{E_2}{E_1}\right) \neq P(E_2)$.

e.g.: If a ball is drawn from a bag containing balls and replaced, the result of this drawing does not effect the outcome of the second and the two drawings are independent events.

If a ball is drawn from a bag containing balls and not replaced, the result of this drawing does effect the outcome of the second and the two drawing are dependent events.

6. Multiplication Theorem

$E_1 \subseteq S$, $E_2 \subseteq S$ and $P(E_1) \neq 0$, $P(E_2) \neq 0$.

Then $P(E_1 \cap E_2) = P(E_1).\ P\left(\frac{E_2}{E_1}\right)$ and $P(E_2 \cap E_1) = P(E_2)\ .\ P\left(\frac{E_1}{E_2}\right)$.

7. If E_1, E_2 are independent events, then

(i) $P(E_1 \cap E_2) = P(E_1).P(E_2)$

(ii) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1).P(E_2)$

- ❖ Note**:** If E_1, E_2 are mutually exclusive we write $P(E_1 \cup E_2)$ as $P(E_1 + E_2)$ and if E_1, E_2 are mutually independent we write $P(E_1 \cap E_2) = P(E_1\ E_2)$.

Mutually exclusive events in general are not independent.

If E_1, E_2 are mutually independent events such that $P(E_1) \neq 0$, $P(E_2) \neq 0$, then E_1, E_2 will have at least one common sample point.

($\because$ E_1, E_2 are independent $\Rightarrow P(E_1 \cap E_2) = P(E_1)\ P(E_2) \neq 0 \quad \Rightarrow E_1 \cap E_2 \neq \phi$).

8. (i) If E_1, E_2, E_3 are mutually independent events, then $P(E_1 \cap E_2 \cap E_3) = P(E_1).P(E_2).P(E_3)$ and conversely. Thus mutually independent events are pairwise independent.

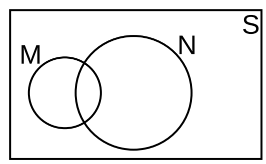
(ii) If E_1, E_2, E_3 are dependent events, then $P(E_1 \cap E_2 \cap E_3) = P(E_1 \cap E_2 \cap E_3) = P(E_1 \cap E_2) \cdot P\left(\frac{E_3}{E_1 \cap E_2}\right)$

$= P(E_1) \cdot P\left(\frac{E_2}{E_1}\right) . P\left(\frac{E_3}{E_1 \cap E_2}\right)$.

9. If A and B are independent events, then
(i) $\bar{A}$ and $\bar{B}$ (ii) A and $\bar{B}$ (iii) $\bar{A}$ and B are independent events

10. If A, B, C are any three events of a sample space S,
then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

11. If M and N are two events of a sample space S, the probability that exactly one of them occurs



$= P[(M \cap \bar{N}) \cup (\bar{M} \cap N)] = P(M \cap \bar{N}) + P(\bar{M} \cap N)$

$= P(M) - (M \cap N) + P(N) - P(M \cap N)$

$= P(M) + P(N) - 2P(M \cap N)$

$= 1 - P(\bar{M}) + 1 - P(\bar{N}) - 2[1 - P\left(\overline{M \cap N}\right)]$

$= -P(\bar{M}) - P(\bar{N}) + 2P\left(\bar{M} \cap \bar{N}\right)$

$= -P(k) - P(\bar{N}) + 2(P\left(\bar{M}\right)) + P(\bar{N}) - P\left(\bar{M} \cap \bar{N}\right)$

$= P(\bar{M}) + P(\bar{N}) - 2P\left(\bar{M} \cap \bar{N}\right)$

12. Description of normal park of cards (52)

Park (52)

Red (26) — Black (26)

Red (26): heart (13), diamond (13)

Black (26): club (13), spade (13)

heart (13): (A, K, Q, J, 10, 9, ..., 2

diamond (13): (A, K, Q, J, 10, 9, ..., 2

club (13): (A, K, Q, J, 10, 9, ..., 2

spade (13): (A, K, Q, J, 10, 9, ..., 2

Hearts, Diamonds, Clubs, Spades are four colours of a pack. King card, Queen card, Jack card are called face card. Thus there are 12 face cards in a normal pack.

PROBABILITY