Chapter 2
Sequences and Series

2.1 Introduction:

The INVENTOR of chess asked the King of the Kingdom that he may be rewarded in lieu of his INVENTION with one grain of wheat for the first square of the board, two grains for the second, four grains for the third, eight grains for the fourth, and so on for the sixty four squares. Fortunately, this apparently modest request was examined before it was granted. By the twentieth square, the reward would have amounted to more than a million grains of wheat; by the sixty-fourth square the number called for would have been astronomical and the bulk would have for exceeded all the grains in the kingdom.

The basis of this story a sequence of numbers that have a mathematical relationship --- has a great many important applications. Many of them are beyond the scope of this book, but we shall explore the means of dealing with a number of practical, and often entertaining, problems of this type.

2.2 Sequences:

A set of numbers arranged in order by some fixed rule is called as sequences.

For example

(i) 2, 4, 6, 8, 10, 12, 14 - - - - - - - - - - - -

(ii) 1, 3, 5, 7, 9, - - - - - - - - - - - -

(iii) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16},$ - - - - - - - - - - - -

In sequence $a_1, a_2, a_3, \ldots\ldots a_n, \ldots..$ a_1 is the first term, a_2 is the second term, a_3 is the third and so on.

A sequence is called **finite sequence** if it has finite terms e.g., 2, 4, 6, 8, 10, 12, 14, 16.

A sequence is called **infinite sequence** if it has infinite terms, e.g., , 4, 6, 8, 10, 12, 14, - - - - - - - -

2.3 Progression:

If a sequence of number is such that each term can be obtained from the preceding one by the operation of some law, the sequence is called a progression.

Note:- Each progression is a sequence but each sequence may or may not be a progression

2.4 Arithmetic Sequence:

A sequence in which each term after the first term is obtained from the preceding term by adding a fixed number, is called as an arithmetic sequence or Arithmetic Progression, it is denoted by A.P.

e.g., (i) 2, 4, 6, 8, 10, 12, - - - - - - - - -

(ii) 1, 3, 5, 7, 9, 11, - - - - - - - - - -

Common Difference:

The fixed number in above definition is called as common difference. It is denoted by d. it is obtained by subtracting the preceding terms from the next term i.e; $a_n - a_{n-1}$; n>1 .

For example

2, 4, 6, 8, 4, - - - - - - - -

d = Common difference = $a_2 - a_1 = 4 - 2 = 2$

Or d = Common difference = $a_3 - a_2 = 6 - 4 = 2$

The General Form of an Arithmetic Progression:

Let "a" be the first term and "d" be the common difference, then General form of an arithmetic progression is

a, a + d, a + 2d, - - - - - - - - - - a + (n – 1)d

2.5 nth term or General term(or, last term)of an Arithmetic Progression:

If "a" be the first term and "d" be the common difference then

a_1 = first term = a = a + (1 – 1)d

a_2 = 2nd term = a + d = a + (2 – 1)d

a_3 = 3rd term = a + 2d = a + (3 – 1)d

a_4 = 4th term = a + 3d = a + (4 – 1)d

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a_n = nth term = a + (n – 1)d

$\boxed{a_n = \text{nth term} = a + (n-1)d}$

in which a = 1st term

d = common difference

n = number of terms

Example 1:

Find the 7th term of A.P. in which the first term is 7 and the common difference is – 3.

Solution:

a_7 = 7th term = ?

$a_1 = 7$

d = 3

Putting these values in $a_n = a + (n - 1)d$

$a_7 = 7 + (7 - 1)(-3)$

$= 7 + 6 (-3)$
$= 7 - 18$
$\boxed{a_7 = -11}$

Example 2:

Find the 9th term of the A.P. $-\frac{5}{4}, -\frac{1}{4}, \frac{3}{4}, \ldots\ldots$

Solution:

$$a_1 = -\frac{5}{4}$$

$$d = -\frac{1}{4} - \left(-\frac{5}{4}\right) = \frac{-1}{4} + \frac{5}{4}$$

$$= \frac{-1+5}{4} = \frac{4}{4} = 1$$

9th term = $a_9 = ?$

$$a_n = a + (n-1)d$$

$$a_9 = -\frac{5}{4} + (9-1).1$$

$$= \frac{-5}{4} + 8$$

$$a_9 = \frac{-5+32}{4} = \frac{27}{4}$$

Example 3:

Find the sequence whose general term is $\frac{n(n-1)}{2}$

Solution:

Here $a_n = \frac{n(n-1)}{2}$

Put n = 1, $a_1 = \frac{1(1-1)}{2} = \frac{0}{2} = 0$

Put n = 2, $a_2 = \frac{2(2-1)}{2} = \frac{2(1)}{2} = 1$

Put n = 3, $a_3 = \frac{3(3-1)}{2} = \frac{3(2)}{2} = 3$

Put n = 4, $a_4 = \frac{4(4-1)}{2} = \frac{4(3)}{2} = 6$

Put n = 5, $a_5 = \frac{5(5-1)}{2} = \frac{5(4)}{2} = 10$

Therefore the required sequence is 0, 1, 3, 6, 10, - - - - - - - -

Exercise 2.1

Q.1 Find the terms indicated in each of the following A.P.

(i)	1, 4, 7, - - - - - - - -	7th term
(ii)	7, 17, 27, - - - - - - - -	13th term
(iii)	$\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$, - - - - - - - -	20th term

Q.2 Find the first four terms of A.P. in which first term is 7 and common difference is – 4.

Q.3 Find the number of terms in an A.P. in which $a_1 = 5$, d = 25 and $a_n = 130$.

Q.4 (i) Which term in the arithmetic progression 4, 1, -2 …. is – 77?
(ii) Which term in the arithmetic progression 17, 13, 9 …. is – 19?

Q.5 Find the 7th term of an A.P. whose 4th term is 5 and the common difference is –2.

Q.6 What is the first term of the eight term A.P. in which the common difference is 6 and the 8th term is 17.

Q.7 Find the 20th term of the A.P. whose 3rd term is 7 and 8th term is 17.

Q.8 If the 12th term of an A.P. is 19 and 17th term in 29, Find the first term and the common difference.

Q.9 The 9th term of an A.P. is 30 and the 17th term is 50. Find the first three terms.

Q.10 Find the sequence whose nth term is 4n + 5. Also prove that the sequence is in A.P.

Q.11 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P, show that $b = \frac{2ac}{a+c}$

Q.12 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P, show that the common difference is $\frac{a-c}{2ac}$

Answers 2.1

Q.1 (i) $a_7 = 19$ (ii) $a_{13} = 127$ (iii) $a_{20} = \frac{7}{4}$

Q.2 7, 3, –1, –5 Q.3 n = 6

Q.4 (i) n = 28 (ii) n = 10 Q.5 $a_{17} = -21$

Q.6 $a = -25$ Q.7 $a_{20} = 41$

Q.8 $a = -3, d = 2$ Q.9 $10, \frac{25}{2}, 15$

Q.10 9, 13, 17, - - - - - - and the difference between consecutive terms is equal. So the sequence is an A.P.

2.6 Arithmetic Means (A.Ms):

If a, A, b are three consecutive terms in an Arithmetic Progression, Then A is called the Arithmetic Mean (A.M) of a and b.

i.e. if a, A, b are in A.P. then

$$A - a = b - A$$
$$A + A = a + b$$
$$2A = a + b$$
$$A = \frac{a+b}{2}$$

The arithmetic mean of two numbers is equal to one half the sum of the two numbers.

Example 1:

Find the A.M. between $\sqrt{5} - 4$ and $\sqrt{5} + 4$

Solution:

Here $a = \sqrt{5} - 4$, $b = \sqrt{5} + 4$

$$\text{A.M.} = \quad A = \frac{a+b}{2}$$

$$A = \frac{\sqrt{5} - 4 + \sqrt{5} + 4}{2}$$

$$= \frac{2\sqrt{5}}{2} = \sqrt{5}$$

2.7 n Arithmetic Means between a and b:

The number A_1, A_2, A_3 - - - - - An are said to be n arithmetic means between a and b if a, A, A_2, A_3, - - - - - - An, b are in A.P. We may obtain the arithmetic means between two numbers by using $a_n = a + (n-1)d$ to find d, and the means can then be computed.

Example 2:

Insert three A.M's between – 18 and 4.

Solution:

Let A_1, A_2, A_3 be the required A.M's between –18 and 4, then

$-18, A_1, A_2, A_3, 4$ are in A.P.

Here $a = -18$, $n = 5$, $a_5 = 4$, $d = ?$

Using $a_n = a + (n-1)d$

$$a_5 = -18 + (5 - 1)d$$
$$4 = -18 + 4d$$
$$4d = 4 + 18$$
$$4d = 22$$
$$d = \frac{11}{2}$$

Therefore A_1 = 2nd term $= a + d$

$$= -18 + \frac{11}{2} = \frac{-25}{2}$$

A_2 = 3rd term = $A_1 + d$ $= \frac{-25}{2} + \frac{11}{2}$

$$= \frac{-25+11}{2}$$

$$A_2 = \frac{-14}{2} = -7$$

Thus the required A.M's are $\frac{-25}{2}, -7, \frac{-3}{2}$

Example 3:

Insert n A.M's between a and b.

Solution:

Let, $A_1, A_2, A_3, \ldots\ldots A_n$ be the n. A.M's between a and b.

Then $a, A_1, A_2, A_3, \ldots\ldots b$, are in A.P.

Let, d be the common difference

So, $a = a$, $n = n + 2$, $d = ?$ $a_n = b$

$$a_n = a + (n - 1)d$$
$$b = a + (n + 2 - 1)d$$
$$b = a + (n + 1)d$$
$$b - a = (n + 1)d$$
$$d = \frac{b - a}{n + 1}$$
$$A_1 = a + d = a + \frac{b - a}{n + 1} = \frac{a(n + 1) + b - a}{n + 1} = \frac{an + a + b - a}{n + 1}$$
$$A_1 = \frac{an + b}{n + 1}$$
$$A_2 = A_1 + d = \frac{an + b}{n + 1} + \frac{b - a}{n + 1} = \frac{an + a + b - a}{n + 1}$$

$$A_2 = \frac{(n-1)a + 2b}{n+1}$$

$$A_n = \frac{[n-(n-1)]a + nb}{n+1} = \frac{(n-n+1)a + nb}{n+1} \;\ldots\; \frac{a + nb}{n+1}$$

Thus n A.M's between a and b are:

$$\frac{an + b}{n+1}, \frac{(n-1)a + 2b}{n+1}, \frac{(n-2)a + 3b}{n+1} \;\ldots\; \frac{a + nb}{n+1}$$

Exercise 2.2

Q.1 Find the A.M. between
(i) 17 and –3 (ii) – 5 and 40
(iii) $2 + \sqrt{3}$ and $2 - \sqrt{3}$ (iv) x + b and x – b

Q.2 Insert two A.M's between – 5 and 40.

Q.3 Insert four A.M's between $\frac{\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$

Q.4 Insert five A.M's between 10 and 25.

Q.5 Insert six A.M's between 12 and –9.

Q.6 If 5 , 8 are two A.M's between a and b , find a and b

Q.7 Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the A.M's between a and b.

Q.8 Find the value of x if x + 1, 4x + 1 and 8x – 1 are the consecutive terms of an arithmetic progression.

Q.9 Show that the sum of n A.M's between a and b is equal to n times their single A.M.

Answer 2.2

Q.1 (i) 7 (ii) $\frac{35}{2}$ (iii) 2 (iv) x

Q.2 10, 25 Q.3 $\frac{7\sqrt{2}}{10}, \frac{9\sqrt{2}}{10}, \frac{11\sqrt{2}}{10}, \frac{13\sqrt{2}}{10}$

Q.4 $\frac{25}{2}, 15, \frac{35}{2}, 20, \frac{45}{2}$ Q.5 9, 6, 3, 0 –3, –6

Q.6 a= 2 , b = 11 Q.7 n = 0 Q.8 x = 2

Series:

The sum of the terms of a sequence is called as "series". For example: 1, 4, 9, 16, - - - - - - is a sequence.

Sum of the terms of sequence i.e., 1 + 4 + 9 + 16 - - - - - represent a series.

2.8 Arithmetic Series:

The sum of the terms of an Arithmetic sequence is called as Arithmetic series. For example:

7, 17, 27, 37, 47, - - - - - - - - - - is an A.P.

7 + 17 + 27 + 37 + 47 + - - - - - - - - - is Arithmetic series.

The sum of n terms of an Arithmetic Sequence:

The general form of an arithmetic sequence is a, a + d, a + 2d, - - - - - - - - a + (n – 1)d.

Let Sn denoted the sum of n terms of an Arithmetic sequence.

Then $Sn = a + (a + d)+(a + 2d) + - - - - + [a +(n - 1)d]$

Let nth term $[a + (n - 1)d] = \ell$

The above series can be written as

$Sn = a+(a + d) + (a + 2d) + - - - - - - + \ell$

Or, $Sn = a + (a + d) + (a + 2d) + - - - - - - + (\ell - 2d) + (\ell - d) + \ell$ -----(I)

Writing 1 in reverse order, we have

$Sn = \ell + (\ell - d) + (\ell - 2d) + - - -(a + 2d) + (a + d) + a$…… (II)

Adding I and II

$2Sn = (a + \ell) + (a + \ell) + (a + \ell) + - - - - + (a + \ell)$

$2Sn = n (a + \ell)$

$Sn = \frac{n}{2}(a + \ell)$ But $\ell = a + (n - 1)d$

$Sn = \frac{n}{2}[a + (a + (n - 1)d)]$

$= \frac{n}{2}[a + a + (n - 1)d]$

$$\boxed{Sn = \frac{n}{2}[2a + (n - 1)d]}$$

is the formula for the sum of n terms of an arithmetic sequence.

Example 1:

Find the sum of the series 3 + 11 + 19 + - - - - to 16 terms.

Solution:

Here $a = 3$, $d = 11 - 3 = 8$, $n = 16$

Using formula $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{16} = \frac{16}{2}[2(3) + (16-1)8]$$
$$= 8[6 + 15(8)]$$
$$= 8[6 + 120]$$
$$S_{16} = 8 \times 126 = 1008$$

Example 2:

Find the sum of all natural numbers from 1 to 500 which are divisible by 3.

Solution:

The sequence of numbers divisible by 3 is

3, 6, 9, 12, - - - - - 498 (which is in A.P.)

Here $a = 3$, $d = 6 - 3 = 3$, $n = ?$ $a_n = 498$

First we find n

For this using $a_n = a + (n-1)d$

$$498 = 3 + (n-1)(3)$$
$$498 = 3 + 3n - 3$$
$$3n = 498$$
$$n = 166$$

Now $a = 3$, $d = 3$, $n = 166$, $Sn = ?$

$$Sn = \frac{n}{2}[2a + (n-1)d]$$
$$S_{166} = \frac{166}{2}[2(3) + (166-1)3]$$
$$= 83[6 + 165(3)] = 83(6 + 495)$$
$$= 83 \times 501$$
$$S_{166} = 41583$$

Example 3:

` If the sum of n terms of an A.P. is $2n + 3n^2$. Find the nth term.

Solution:

We have $S_n = 2n + 3n^2$

$$S_{n-1} = 2(n-1) + 3(n-1)^2$$
$$= 2(n-1) + 3(n^2 - 2n + 1)$$
$$= 2n - 2 + 3n^2 - 6n + 3$$
$$S_{n-1} = 3n^2 - 4n + 1$$

nth term $= a_n = S_n - S_{n-1}$

$$= 2n + 3n^2 - (3n^2 - 4n + 1)$$
$$= 2n + 3n^2 - 3n^2 + 4n - 1$$
$$a_n = 6n - 1$$

Example 4:

The sum of three numbers in an A.P. is 12 and the sum of their cubes is 408. Find them.

Solution:

Let the required numbers be

$a - d, a, a + d$

According to 1st condition:

$$(a - d) + a + (a + d) = 12$$
$$a - d + a + a + d = 12$$
$$3a = 12$$
$$\boxed{a = 4}$$

According to 2nd given condition:

$$(a - d)^3 + a^3 + (a + d)^3 = 408$$
$$a^3 - d^3 - 3a^2d + 3ad^2 + a^3 + a^3 + d^3 + d^3 + 3a^2d + 3ad^2 = 408$$
$$3a^3 + 6ad^2 = 408$$
$$3(4)^3 + 6(4)\, d^2 = 408$$
$$24d^2 = 408 - 192$$
$$d^2 = 9$$
$$\Rightarrow \qquad d = \pm 3$$

When $a = 4$ $d = 3$ then number are $a - d, a, a + d$

i.e. $4 - 3, 4, 4 + 3$ i.e. $1, 4, 7$

when $a = 4$, $d = -3$ then numbers are $a - d, a, a + d$

$4 - (-3), 4, 4 + (-3)$

$4 + 3, 4, 4 - 3$ i.e., $7, 4, 1$

Hence the required numbers are 1, 4, 7 or 7, 4, 1

Note:

The problem containing three or more numbers in A.P. whose sum in given it is often to assume the number as follows.

If the required numbers in A.P are odd i.e. 3, 5, 7 etc. Then take ‘a’(first term) as the middle number and d as the common difference.

Thus three numbers are $a - d, a, a + d$. If the required numbers in A.P are even i.e. 2, 4, 6, etc. then take $a - d$, $a + d$ as the middle numbers and 2d as the common difference.

Thus four numbers are $a - 3d, a - d, a + d, a + 3d$ and six numbers are:

$a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

Example 5:

A man buys a used car for \$600 and agrees to pay \$100 down and \$100 per month plus interest at 6 percent on the outstanding indebtedness until the car paid for. How much will the car cost him?

Solution:

The rate of 6 percent per year is 0.5 percent per month.

Hence, when the purchaser makes his first payment, he will owe 1 month's interest.

The interest on \$500 = (500)(0.005) = \$2.50

The purchaser will pay in the second month = \$102.50

Since the purchaser pays \$100 on the principal, his interest from month to month is reduced by 0.5 percent of \$100, which is \$0.50 per month.

The final payment will be \$100 plus interest on 100 for 1 month, which is = \$100.50

Hence his payments on \$500 constitute an arithmetic progression

102.50 + - - - - - - + 100.50

Here a = 102.40, ℓ = 100.50 and n = 5

Therefore by the formula

$$S = \frac{n}{2}(a + \ell)$$

$$= \frac{5}{2}(102.50 + 100.50)$$

$$= \frac{5}{2}(203) = \$507.50$$

Thus, the total cost of the car will be \$ 607.50

Exercise 2.3

Q.1 Sum the series:

(i) 5 + 8 + 11 + 14 + ….. to n terms.

(ii) 51 + 50 + 49 + …… + 21.

(iii) 5 + 3 + 1 – 1 – …… to 10 terms.

(iv) $\frac{1}{1-\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1+\sqrt{x}}$ +-------- to n terms

Q.2 The nth term of a series in 4n + 1. Find the sum of its 1st n terms and also the sum of its first hundred terms.

Q.3 Find the sum of the first 200 odd positive integers.

Q.4 Find the sum of all the integral multiples of 3 between 4 and 97

Q.5 How many terms of the series:

(i) - 9 - 6 - 3 ……… amount to 66?

(ii) 5 + 7 + 9 + …… amount to 192?

Q.6 Obtain the sum of all the integers in the first 1000 positive integers which are neither divisible by 5 nor 2.

Q.7 The sum of n terms of a series is $7n^2 + 8n$. Show that it is an A.P and find its common difference.

Q.8 Sum the series
$1 + 3 - 5 + 7 + 9 - 11 + 13 + 15 - 17$ …… to 3n terms.

Q.9 If S_1, S_2, S_3 be sums to n, 2n, 3n terms of an arithmetic progression, Show that $S_3 = 3(S_2 - S_1)$.

Q.10 The sum of three numbers in A.P. is 24, and their product is 440. Find the numbers.

Q.11 Find four numbers in A.P. whose sum is 24 and the sum of whose square is 164.

Q.12 Find the five number in A.P. whose sum is 30 and the sum of whose square is 190.0

Q.13 How many bricks will be there in a pile if there are 27 bricks in the bottom row, 25 in the second row, etc., and one in the top row?

Q.14 A machine costs Rs. 3200, depreciates 25 percent the first year, 21 percent of the original value the second year, 17 percent of the original value of the third year, and so on for 6 years. What is its value at the end of 6 years.

Answers 2.3

Q.1 (i) $S_n = \frac{n}{2}[3n + 7]$ (ii) $n = 31$, $S_n = 1116$

(iii) $S_{10} = 40$ (iv) $\frac{n}{2}\left[\frac{2+(3-n)\sqrt{x}}{1-x}\right]$

Q.2 $S_n = n(2n + 3)$, $S_{100} = 20300$ Q.3 $S_{200} = 40,000$
Q.4 1581 Q.5 (i) $n = 11$ (ii) $n = 12$
Q.6 2000,000 Q.7 $d = 14$ Q.8 $S_{3n} = n(3n - 4)$
Q.10 5, 8, 11 or 11, 8, 5 Q.11 3, 5, 7, 9 or 9, 7, 5, 3
Q.12 4, 5, 6, 7, 8 or 8, 7, 6, 5, 4 Q.13 196 Q.14 Rs. 320.00

2.9 Geometric Sequence or Progression (G.P):

A geometric progression is a sequence of numbers each term of which after the first is obtained by multiplying the preceding term by a constant number called the common ratio. Common ratio is denoted by ‘r’.

Example:

(i) 2, 4, 8, 16, 32, …… is G.P
because each number is obtained by multiplying the preceding number by 2.

(ii) 2, 4, 8, ……

(iii) 4, 12, 36, ……

Note:- In geometric progression, the ratio between any two consecutive terms remains constant and is obtained by dividing the next term with the preceeding term, i.e., $r = \dfrac{a_n}{a_{n-1}}$, $n > 1$

2.10 nth term or General term(or, last term) of a Geometric Progression (G.P):

If a is the first term and r is the common ratio then the general form of G.P is a, ar, ar^2, ar^3, ……

If $a_1 = 1^{st}$ term $= a$

$a_2 = 2^{nd}$ term $= ar$

$a_3 = 3^{rd}$ term $= ar^2$

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a_n = nth term $= ar^{n-1}$

Which is the nth term of G.P in which:

a = 1st term

r = common ratio

n = number of terms

a_n = nth term = last term

Example 2:

Write the sequence in which

$a = 5, \ r = 3, \ n = 5$

$a_1 = a = 5$

$a_2 = ar = 5(3) = 15$

$a_3 = a_2r = 15(3) = 45$

$a_4 = a_3r = 45(3) = 135$

$a_5 = a_4r = 135(3) = 405$

Therefore, the required sequence is: 5, 15, 45, 135, 405

Example 3:

Find 4th term in the G.P. 5, 10, 20, ……

Solution:

$a = 5, \ r = \dfrac{10}{5} = 2, \ a_n = ?$

$a_n = ar^{n-1}$

$a_4 = a_4 = 5(2)^{4-1} = 5 \times 8 = 40$

Example 4:

Find n in the G.P. 4, –2, 1, …… if $a_n = \dfrac{1}{16}$

Solution: **Since** 4, –2, 1, ……

Here, $a = 4, r = -2, 4 = -\frac{1}{2}, a_n = \frac{1}{16}$

$a_n = ar^{n-1}$

$\frac{1}{16} = 4(-\frac{1}{2})^{n-1}$

let $\frac{1}{16} \text{ x } 4 = (-\frac{1}{2})^{n-1} = \frac{1}{64} = (-\frac{1}{2})^{n-1}$

$(-\frac{1}{2})^6 = (-\frac{1}{2})^{n-1}$

$\Rightarrow \quad n - 1 = 6 \quad \text{or} \quad n = 6 + 1 = 7$

Example 5:

Find the G.P. of which the third term is 4 and 6th is –32.

Solution:

Here $a_3 = 4, \; a_6 = -32$

$a_n = ar^{n-1}$

$a_3 = ar^{3-1}, a_6 = ar^{6-1}$

$4 = ar^2$ …… (i) $\qquad -32 = ar^5$ …… (ii)

Dividing (i) by (ii)

$\frac{ar^2}{ar^5} = \frac{4}{-32}$ or $\frac{1}{r^3} = \frac{-1}{8}$

$r^3 = -8 = (-2)^3$

$\Rightarrow r = -2$

Example 6:

The population of a town increases at the rate of 10% annually. Its present population is 2,00,000 what will be its population at the end of 5 years?

Solution:

Let, present population = a = 2,00,000 (given)

The increase of population at the end of 1st year

= a(10/100) = a(0.1)

Total population at the end of 1st year = a + a(0.1) = a(1.1)

Total population at the end of 2nd year = $a(1.1)(1.1) = a(1.1)^2$

The population at the end of 5years is the 6^{th} terms of G.P

$a , a(1.1) , a(1.1)^2$ ……

Here a = 2,00,000 , r = 1.1 , n =6 , a_6 = ?

Since, $a_n = ar^{n-1}$

$a_6 = 2,00,000\,(1.1)^5 = 2,00,000\,(1.61051) = 322102$

Example 7:

The value of an auto mobile depreciate at the rate of 15% per year. What will be the value of an automobile 3 years hence which is now purchased for Rs. 45,000?

Solution:

a = 45,000 = Purchased value of automobile

The amount depreciate at the end of 1st year = a (15/100) = 0.15a

The value of automobile at the end of 1st year = a – 0.15a

= a(1 – 0.15) = a(0.85)

The value of automobile at the end of 2nd year = a(0.85)(1 – 0.15)

= a(0.85)(0.85)

= $a(0.85)^2$

The value of automobile at the end of 3rd year = $a(0.85)^3$

= $45,000(0.85)^3$

= 45,000 (0.614125)

= 27635.63 rupees

Exercise 2.4

Q.1 Write the next five terms of the following G.Ps.

(i) 2, 10, ………. (ii) 27, 9, 3, ……….

(iii) $1, \frac{1}{2}, \frac{1}{4}, \ldots\ldots$

Q.2 Find the term indicated in each of the following G.Ps.

(i) $1, 3^3, 3^6$, ………., 6th term

(ii) i, –1, –i, 1, ………., 13th term

(iii) $\sqrt{2}, \sqrt{6}, 3\sqrt{2},$ ………., 15th term

(iv) $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}$ ………., 6th term

Q.3 Find the nth term of the G.P.

(i) $a = 8, \quad r = \frac{3}{2}, \; n = 5$

(ii) a = –1, r = –4, n = 6

(iii) a = 3, r = –2, n = 10

Q.4 Write down the finite geometric sequence which satisfies the given condition.

(i) a = 3, r = 5, n = 6

(ii) First term = 2, second term = – 6, n = 5

(iii) Third term = 9, sixth term = $\frac{1}{3}$, n = 8

(iv) Fifth term = 9, eight term = 243, n = 8

Q.5 If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are G.P ,show that the common ratio is $\pm\sqrt{\frac{a}{b}}$

Q.6 If the second term of a G.P is 2 and the 11th term is $\frac{1}{256}$, what is the first term? What is the nth term.

Q.7 Find the 10th term of a G.P if 2nd term 43 and 4th term is 9.

Q.8 What is the first term of a six term geometric progression in which the ratio is $\sqrt{3}$ and the sixth term is 27?

Q.9 A business concern pays profit at the rate of 15% compounded annually. If an amount of Rs. 2,00,000 is invested with the concern now, what total amount will become payable at the end of 5 years?

Q.10 A rubber bell is dropped from a height of 16dm, it continuously rebounds to $\frac{3}{4}$ of the distance of its previous fall. How high does it rebound its fourth time?

Q.11 Find three consecutive numbers in G.P whose sum is 26 and their product is 216.

Q.12 If the sum of the four numbers consecutive numbers of a G.P is 80 and A.M between second and fourth of them is 30.Find the terms.

Answers 2.4

1. (i) 50, 250, 1250, 6250, 31250 (ii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

   (iii) $\frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}$

2. (i) $(27)^5$ (ii) i (iii) $\sqrt{2}(3)^7$ (iv) $-\frac{1}{729}$

3. (i) $\frac{81}{2}$ (ii) 1024 (iii) –1536

4. (i) 3, 15, 75, 375, 1875, 9375

   (ii) 2, –6, 18, –54, 162

   (iii) $81, 27, 9, 3, 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

   (iv) $\frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, 81, 243$

6. $a = 4,\ a_n = 4\left(\frac{1}{2}\right)^{n-1}$ 7. 243

8. $a = \sqrt{3}$ 9. 402271.44 10. 5.06 dm
11. 2 , 6 , 18 or 18 , 6 , 2 12. 2 , 6 , 18 , 54

2.11 Geometric Mean:

When three quantities are in G.P., the middle one is called the Geometric Mean (G.M.) between the other two. Thus G will be the G.M. between a and b if a, G, b are in G.P.

To Find G.M between a and b:

Let, G be the G.M. between a and b

Then a. G. b are in G.P

$\therefore \quad \frac{G}{a} = \frac{b}{G} \Rightarrow G^2 = ab$

$G = \pm\sqrt{ab}$

Hence the G.M. between two quantities is equal to the square root of their product.

Example 1:

Find the G.M. between 8 and 72.

Solution:

$G = \pm\sqrt{ab}$

$G = \pm\sqrt{8 \text{ x } 72} = \pm\sqrt{8 \text{ x } 8 \text{ x } 9} = \pm 8 \text{x } 3$

$G = \pm 24$

2.12 n G.Ms Between a and b:

The numbers $G_1, G_2, G_3 \ldots\ldots G_n$ are said to be n G.Ms between a and b if $a, G_1, G_2, G_3 \ldots\ldots G_n, b$ are in G.P.

In order to obtain the G.M's between a and b, we use the formula $a_n = ar^{n-1}$ to find the value of r and then the G.M's can be computed.

To Insert n G.M's Between Two Numbers a and b

Let,$G_1, G_2, G_3 \ldots\ldots G_n$ be n G.Ms between a and b

Here $a = a$, $a_n = b$, $n = n + 2$, $r = ?$

$$a_n = ar^{n-1}$$
$$b = ar^{n-2-1} = ar^{n-1}$$
$$b/a = r^{n-1}$$
$$\Rightarrow r = (b/a)^{1/(n-1)}$$

So, $G_1 = ar = a(b/a)^{1/(n+1)}$

$$G_2 = G_1 r = a(b/a)^{1/(n+1)}(b/a)^{1/(n+1)} = a(b/a)^{2/(n+1)}$$
$$G_3 = G_2 r = a(b/a)^{2/(n+1)}(b/a)^{1/(n+1)} = a(b/a)^{3/(n+1)}$$
$$G_n = a(b/a)^{n/(n+1)}$$

Example 2:

Find three G.M's between 2 and 32.

Solution:

Let,G_1, G_2, G_3 …… G_n be n G.Ms between 2 and 32
Then 2, G_1, G_2, G_3 32 are in G.P.
Here a = 2, a_n = 32, r = ? n = 5

$$a_n = ar^{n-1}$$
$$32 = 2(r)^{5-1} = 2r^4$$
$$16 = r^4$$
$$2^4 = r^4$$
$$\Rightarrow r = 2$$

So, $G_1 = ar = 2(2) = 4$
$G_2 = G_1r = 4(2) = 8$
$G_3 = G_2r = 8(2) = 16$

Thus three G.M's between 2 and 32 are 4, 8, 16.

Example 3:

Insert 6 G.M's between 2 and 256.

Solution:

Let,G_1, G_2, G_3, G_4, G_5, G_6 be six G.M's between 2 and 256.
Then 2, G_1, G_2, G_3, G_4, G_5, G_6 256 are in G.P.
Here a = 2, a_n = 32, r = ? n = 5

$$a_n = ar^{n-1}$$
$$256 = 2(r)^{8-1} = 2r^7$$
$$128 = r^7$$
$$(2)^7 = r^2$$
$$\Rightarrow r = 2$$

So, $G_1 = ar = 2(2) = 4$
$G_2 = G_1r = 4(2) = 8$
$G_3 = G_2r = 8(2) = 16$
$G_4 = G_3r = 16(2) = 32$
$G_5 = G_4r = 32(2) = 64$
$G_6 = G_5r = 64(2) = 128$

Hence, required G.M's are 4, 8, 16, 32, 64, 128.

Example 4:

The A.M between two numbers is 10 and their G.M is 8.
Determine the numbers.

Solution: A.M $= \dfrac{a+b}{2} = 10$

$a + b = 20$ . . . . (1)

G.M. $= \sqrt{ab} = 8$

$\therefore$ $ab = 64$ . . . . (2)

from (2) $b = \dfrac{64}{a}$, Put in (1)

$a + \frac{64}{2} = 20$

$a^2 + 64 = 20a$

$(a - 16)(a - 4) = 0$

$\Rightarrow a = 16$ or $a = 4$

When, $a = 16$, $b = \frac{64}{16} = 4$

When, $a = 4$, $b = \frac{64}{16} = 16$

Hence the numbers are 4 and 16.

Exercise 2.5

Q1. Find G.M between

(i) 4, 64 (ii) $\frac{1}{3}$, 243 (iii) $\frac{8}{9}, \frac{8}{9}$

Q2. Insert two G.M's between $\sqrt{2}$ and 2.

Q3. Insert three G.M's between 256 and 1.

Q4. Insert four G.M's between 9 and $\frac{1}{27}$.

Q5. Show that A.M of two unequal positive quantities is greater than this G.M.

Q6. For what value of n is $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ the G.M between a and b, where a and b are not zero simultaneously.

Q7. Prove that the product of n G.M's between a and b is equal to the n power of the single G.M between them.

Q8. The A.M of two positive integral numbers exceeds their (positive)G.M by 2 and their sum is 20.Find the numbers.

Answers 2.5

Q1. (i) ± 16 (ii) ± 9 (iii) $\pm \frac{8}{9}$

Q2. $2^{2/3}, 2^{5/6}$ Q3. 64, 16, 4 Q4. 3, 1, $\frac{1}{3}, \frac{1}{9}$

Q6. $n = -\frac{1}{2}$ Q8. 16 , 4 or 4 , 16

2.13 Geometric Series

A geometric series is the sum of the terms of a geometric sequence.

If a, ar, ar^2, …… + ar^{n-1} is a geometric sequence.

Then $a + ar + ar^2 + \ldots\ldots + ar^{n-1}$ is a geometric series.

Sum of n Terms of a Geometric Series

Let, S_n be the sum of geometric series

i.e. $S_n = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$ ……………….. (1)

Multiplying by r on both sides

$rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n$……………….. (2)

Subtracting (2) from (1), we get

$S_n - r\,S_n = a - a\,r^n$

$(1 - r)S_n = a(1 - r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}\ ;\ r \neq 1$$

For convenience, we use :

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if } |r| < 1$$

and $$S_n = \frac{a(r^n-1)}{r-1} \quad \text{if } |r| > 1$$

Example 1:

Sum the series $\frac{2}{3}, -1, \frac{3}{2}, \ldots\ldots\ldots\ldots$ to 7 terms

Solution

Here $a = \frac{2}{3}$, $r = \frac{-1}{\frac{2}{3}} = \frac{-3}{2}$

$$S_n = \frac{a(1-r^n)}{1-r} \text{ (because r < 1)}$$

$$S_7 = \frac{\frac{2}{3}\left[1-\left(-\frac{3}{2}\right)^7\right]}{1-\left(-\frac{3}{2}\right)} = \frac{\frac{2}{3}\left[1+\frac{2187}{128}\right]}{\frac{5}{2}}$$

$$S_7 = \frac{2}{3}\left(\frac{2315}{128}\right)\frac{2}{5} = \frac{463}{96}$$

Example 2:

Sum to 5 terms the series 1 + 3 + 9 + ……..

Solution:

The given series is a G.P.

in which a = 1, r = $\frac{a_2}{a_1} = \frac{3}{1} = 3,\ n = 5$

$\therefore$ $S_n = \frac{a(r^4 - 1)}{r-1}$ (because r > 1)

$$S_n = \frac{1[(3)^5 - 1]}{3-1} = \frac{243-1}{2} = \frac{242}{2} = 121$$

Example 3:

Find S_n for the series 2 + 4 + 8 + ……….. + 2^n.

$\therefore$ Since r = r > 1

$\therefore$ $S_n = \frac{a(r^n - 1)}{r-1} = \frac{2(2^n - 1)}{2-1} = 2^{n+1} - 2$

Example 4:

How many terms of the series

$$\frac{2}{3} - \frac{1}{3} + \frac{1}{2} + \text{ . . . . amount to } \frac{55}{72}$$

Solution:

$$S_n = \frac{55}{72},\ n = ?\ \ a = \frac{2}{9},\ \ r = \frac{-\frac{1}{3}}{\frac{2}{9}} = \frac{-3}{2}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\frac{55}{72} = \frac{\frac{2}{9}\left[1-\left(-\frac{3}{2}\right)^n\right]}{1-\left(-\frac{3}{2}\right)} = \frac{\frac{2}{9}\left[1-\left(-\frac{3}{2}\right)^n\right]}{\frac{3}{2}}$$

$$\frac{55}{72} = \frac{4}{45}\left[1-\left(-\frac{3}{2}\right)^n\right]$$

$$\frac{45\text{x}55}{72\text{x}4} = 1-\left(-\frac{3}{2}\right)^n \qquad \Rightarrow \frac{275}{32} = 1-\left(-\frac{3}{2}\right)^n$$

$$\left(-\frac{3}{2}\right)^n = 1 - \frac{275}{32} = \frac{243}{32} = \left(-\frac{3}{2}\right)^5$$

$\Rightarrow$ n = 5

Example 5:

Sum the series:

(i) 0.2 + .22 + .222 + …… to n terms

(ii) $(x + y)(x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots$ to n terms.

Solutions:

(i) 0.2 + .22 + .222 + …… to n terms

Let, S_n = .2 + .22 + .222 + …… to n terms

= 2[.1 + .11 + .111 + …… to n terms]

Multiplying and dividing by 9

$$S_n = \frac{2}{9}[.9 + .99 + .999 + \ldots\ldots \text{ to n terms}]$$

$$= \frac{2}{9}[(1 - 1) + (1 - .01) + (0.1 - .001) + \ldots\ldots \text{ to n terms}]$$

$$= \frac{2}{9}[(1+1+1+\ldots\ldots \text{n terms})-(0.1+.01+.001+\ldots\ldots \text{to n terms})]$$

$$a = .1 \qquad r = \frac{.01}{.1} = 0.1 = \frac{1}{10}$$

$$a = \frac{1}{10}$$

We use $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_n = \frac{2}{9}\left[n - \frac{\frac{1}{10}\left\{1-\left(\frac{1}{10}\right)^n\right\}}{1-\frac{1}{10}}\right] = \frac{2}{9}\left[n - \frac{1}{9}\left\{1-\frac{1}{10^n}\right\}\right]$$

Solution (ii)

Let, $S_n = (x + y) + (x^2 + xy + y^2) + (x^3+x^2y+xy^2+y^3) + \ldots$ to n term.

Multiplying and dividing by (x – y)

$$S_n = \frac{1}{(x-y)}[(x+y)(x-y)+(x-y)(x^2+xy+y^2)+(x-y)(x^3+x^2y+xy^2+y^3)+\ldots$$

$$S_n = \frac{1}{(x-y)}[(x^2 - y^2)+(x^3 - y^3)+(x^4 - y^4)+\ldots \text{ to n term}]$$

$$S_n = \frac{1}{(x-y)}\,[(x^2 + x^3 + x^4 + \ldots \text{ to n term}) - (y^2 + x^3 + y^4 + \ldots \text{ to n term}]$$

We use $S_n = \dfrac{a(r^n - 1)}{r-1}$

$$S_n = \frac{1}{(x-y)}\left[\frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1}\right]$$

Example 6:

The sum of the first 10 terms of a G.P. is equal to 244 times the sum of first 5 terms. Find common ratio.

Solution:

Here, n = 10, n = 5, r = ?

So, $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_{10} = \frac{a(1-r^{10})}{1-r},\quad S_5 = \frac{a(1-r^5)}{1-r}$$

By the Given condition:

$S_{10} = 244S_5$

$$\frac{a(1-r^{10})}{1-r} = 244\left[\frac{a(1-r^5)}{1-r}\right]$$

$\Rightarrow 1 - r^{10} = 244(1 - r^5)$

$(1)^2 - (r^2)^5 = 244(1 - r^5)$

$(1 - r^5)(1 + r^5) = 244(1 - r^5) \quad \Rightarrow (1 - r^5)[1 + r^5 - 244] = 0$

$\Rightarrow 1 + r^5 - 244 = 0$ or $1 - r^5 = 0$

$1 + r^5 = 244 = 0$ $\qquad r^5 = 1$

$r^5 = 243 \quad \Rightarrow \boxed{r = 3}$ $\qquad r = 1$ which not possible

Example 7:

Given n = 6, $r = \dfrac{2}{3}$, $S_n = \dfrac{665}{144}$ find a.

Solution:

Formula $\quad S_n = \dfrac{a(r^n - 1)}{r-1} \qquad \because |r| > 1$

$$\frac{665}{144} = \frac{a\left[1-\left(\frac{2}{3}\right)^6\right]}{1-\frac{2}{3}}$$

$$= \frac{a\left[1-\frac{64}{729}\right]}{\frac{1}{3}}$$

$$\frac{665}{144} = a\left[\frac{665}{243}\right]$$

$$a = \frac{665}{144} \text{ x } \frac{243}{665}$$

$$\boxed{a = \frac{27}{16}}$$

Example 8:

If a man deposits \$ 200 at the beginning of each year in a bank that pays 4 percent compounded annually, how much will be to his credit at the end of 6 years?

Solution:

The man deposits \$ 200 at the beginning of each year.
The bank pays 4% compounded interest annually
At the end of first year the principle amount or credit becomes
= 200(1.04)
At the beginning of second year the principle amount or credit is
= 200 + 200 (1.04)
At the end of second year the principle amount or credit becomes

$$= 200(1.04) + 200(1.04)^2$$
$$= 200(1.04 + 1.04^2)$$

So at the end of 6 years the principle amount or credit becomes
$= 200\ (1.04 + 1.04^2 + \ldots$ sum upto 6 times)
Consider, $1.04 + 1.04^2 +$ - - - - - - - - 6 terms.

$a = 1.04$, $r = 1.04$, and $n = 6$

By the formula

$$S_n = \frac{a(r^n - 1)}{r - 1} \qquad \because |r| > 1$$

$$S_6 = \frac{1.04(1.04^6 - 1)}{1.04 - 1}$$

$$= \frac{1.04(1.2653 - 1)}{0.04}$$

$$= \frac{1.04 \text{ x } 0.2653}{0.04}$$

$$= 6.8983$$

Hence at the end of 6 years the credit is $= 200(6.8983)$
$= \$1379.66$

Exercise 2.6

Q1. Find the sum of each of the following series:

(i) $1+\frac{1}{3}+\frac{1}{9}+$ - - - - - - - to 6 terms

(ii) $x + x^2 + x^3$ - - - - - - - to 20 terms.

(iii) $\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+$ - - - - - - - $+ 64$

(iv) $3 + 3^2 + 3^3 +$ - - - - - - - - - - $+ 3^n$

Q2. How many terms of the series?

$\frac{2}{3}-1+\frac{3}{2}-\frac{9}{4}+$ - - - - - - - amount to $-\frac{133}{48}$

Q3. Sum the series.

(i) $.3 + .33 + .333 +$ - - - - - - - to n terms.

(ii) $3 + 33 + 333 +$ - - - - - - - to n terms.

(iii) $1 + (1 + x)r + (1+ x+x^2)r^2+(1+x+x^2+x^3)r^3 +$ - - - to n terms.

Q4. What is the sum of the geometric series for which $a = 2$, $n = 5$, $l = a_n = 32$?

Q5. A rubber ball is dropped from a height of 4.8 dm. It continuously rebounds, each time rebounding $\frac{3}{4}$ of the distance of the preceding fall. How much distance has it traveled when it strikes the ground for the sixth time?

Q6. The first term of geometric progression is $\frac{1}{2}$ and the 10th term is 256, using formula find sum of its 12 terms.

Q7. What is first term of a six term G.P. in which the common ratio is $\sqrt{3}$ and the sixth term is 27 find also the sum of the first three terms.

Answers 2.6

1. (i) $\frac{364}{243}$ (ii) $\frac{x(1-x^{20})}{1-x}$ (iii) 1023/8

(iv) $\dfrac{3(3^n-1)}{2}$

2. n = 6

3. (i) $\dfrac{1}{3}\left[n-\dfrac{1}{9}\left(1-\dfrac{1}{10^n}\right)\right]$ (ii) $\dfrac{1}{3}\left[\dfrac{10(10^n-1)}{9}-n\right]$

(iii) $\dfrac{1}{(1-x)}\left[\dfrac{1-r^n}{1-r}-\dfrac{x(1-r^n x^n}{1-rx}\right]$

4. 62 5. 26.76 6. $\dfrac{4085}{2}$ 7. $\sqrt{3};\dfrac{\sqrt{3}(3\sqrt{3}-1)}{\sqrt{3}-1}$

2.14 Infinite Geometric Sequence:

A geometric sequence in which the number of terms are infinite is called as infinite geometric sequence.

For example:

(i) $2, \dfrac{4}{3}, \dfrac{8}{9}, \dfrac{16}{27},$ - - - - - -

(ii) 2, 4, 8, 16, 32, - - - - - - -

Infinite Series:

Consider a geometric sequence a, ar, ar^2, - - - - - - - to n terms.

Let S_n denote the sum of n terms then $S_n = a + ar + ar^2 +$ - - - - - - to n terms.

Formula $S_n = \dfrac{a(1-r^n)}{1-r} \quad |r| < 1$

Taking limit as n → ∞ on both sides

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} a\frac{(1-r^n)}{1-r}$$

$$= \lim_{n\to\infty} a\left[\frac{1}{1-r}-\frac{r^n}{1-r}\right]$$

$$= \lim_{n\to\infty}\left(\frac{a}{1-r}\right)-\lim_{n\to\infty}\frac{ar^n}{1-r}$$

as $n \to \infty$, $r^n \to 0$

Therefore $S\infty = \dfrac{a}{1-r} - 0$

$$\boxed{S\infty = \frac{a}{1-r}}$$

$\therefore$ the formula for the sum of infinite terms of G.P.

Convergent Series:

An infinite series is said to be the convergent series when its sum tends to a finite and definite limit.

For example:

$\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+$ - - - - - - - is a series

Here $a = \frac{2}{3}$, $r = \frac{1}{3}+\frac{2}{3} = \frac{1}{2} < 1$

$$S\infty = \frac{a}{1-r}$$

$$= \frac{\frac{2}{3}}{1-\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{1}{2}}$$

$$= \frac{2}{3} \text{ x } \frac{1}{2} = \frac{4}{3}$$

Hence the series is convergent.

Divergent Series:

When the sum of an infinite series is infinite, it is said to be the Divergent series.

For example:

2 + 4 + 8 + 16 + 32 + - - - - - - - -

Here $a = 2$, $r = , 2 > 1$

Therefore we use formula

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{2(2^n-1)}{2-1}$$

$$S_n = 2^{n+1} - 2$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} (2^{n+1} - 2)$$

$$S\infty = 2^{\infty+1} - 2$$

$$= \infty \text{ as } n \to \infty,\ 2^{n+1} \to \infty$$

Hence the series is a divergent series.

2.14 Recurring Decimals:

When we attempt to express a common fraction such as $\frac{3}{8}$ or as $\frac{4}{11}$ as a decimal fraction, the decimal always either terminates or ultimately repeats.

Thus $\frac{3}{8}$ = 0.375 (Decimal terminate)

$\frac{4}{11}$ = 0.363636 (Decimal repeats)

We can express the recurring decimal fraction $0.\overline{36}$ (or $0.\dot{3}\dot{6}$) as a common fraction.

The bar ($0.\overline{36}$) means that the numbers appearing under it are repeated endlessly. i.e. $0.\overline{36}$ means 0.363636 - - - - - - -

Thus a non-terminating decimal fraction in which some digits are repeated again and again in the same order in its decimal parts is called a recurring decimal fraction.

Example 1:

Find the fraction equivalent to the recurring decimals $0.\overline{123}$.

Solution:

Let S $= 0.\overline{123}$

$= 0.123\ 123\ 123 - - - - - - - - - - - \infty$

$= 0.123 + 0.000123 + 0.000000123 - - - - - - \infty$

$= \frac{123}{1000} \frac{123}{1000\ 000} \frac{123}{1000\ 000\ 000} + - - - - - - \infty$

$= \frac{123}{10^3} + \frac{123}{10^6} + \frac{123}{10^9} + - - - - - - \infty$

Here $a = \frac{123}{10^3}$, $r = \frac{1}{10^3}$

$S = \frac{a}{1-r}$

$= \frac{\frac{123}{10^3}}{1-\frac{1}{10^3}} = \frac{\frac{123}{1000}}{1-\frac{1}{1000}} = \frac{\frac{123}{1000}}{\frac{1000-1}{1000}}$

$=$

$$= \frac{123}{1000} \text{x} \frac{1000}{999} = \frac{123}{999}$$

$$= \frac{41}{333}$$

Example 2:

Find the sum of infinite geometric series in which a = 128,

$r = -\frac{1}{2}$.

Solution:

$$a = 128, \ r = -\frac{1}{2}$$

Using $$S\infty = \frac{a}{1-r}$$

$$S\infty = \frac{128}{1-\left(\frac{1}{2}\right)} = \frac{128}{1+\frac{1}{2}}$$

$$= \frac{128}{\frac{3}{2}} = 128\text{x}\frac{2}{3}$$

$$S\infty = \frac{256}{3}$$

Exercise 2.7

Q.1 Find the sum of the following infinite geometric series

(i) $\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\text{-------}$

(ii) $2+\sqrt{2}+1+\text{-------}$

Q.2 Find the sum of the following infinite geometric series

(i) $a = 3, \ r = \frac{2}{3}$ (ii) $a = 3, \ r = \frac{3}{4}$

Q.3 Which of the following series are (i) divergent (ii) convergent

(i) 1 + 4 + 16 + 64 + ---------

(ii) $6+3+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\text{-------}$

(iii) $6 + 12 + 24 + 48 + \text{- - - - - - - - - -}$

Q.4 Find the fractions equivalent to the recurring decimals.

(i) $0.\overline{36}$ (ii) $2.\overline{43}$ (iii) $0.\overline{836}$

Q.5 Find the sum to infinity of the series $1 + (1 + k)r + 1 + k + k^2)r^2 + (1 + k + k^2 + k^3)r^3 + \text{- - - - - - - -}$ r and k being proper fraction.

Q.6 If $y = x + x^2 + x^3 + \text{- - - - - - - -} \infty$ and if x is positive and less than unity show that $x = \dfrac{y}{1+y}$

Q.7 What distance a ball travel before coming to rest if it is dropped from a height of 6 dm and after each fall it rebounds $\frac{2}{3}$ of the distance it fell.

Q.8 The sum of an infinite geometric series in 15 and the sum of the squares of its terms is 45. Find the series.

Answers 2.7

Q.1 (i) $S\infty = \dfrac{1}{4}$ (ii) $S\infty = \dfrac{2\sqrt{2}}{\sqrt{2}-1}$

Q.2 (i) 9 (ii) 12

Q.3 (i) Divergent (ii) Convergent (iii) Divergent

Q.4 (i) $\dfrac{4}{11}$ (ii) $\dfrac{241}{99}$ (iii) $\dfrac{5}{6}$

Q.5 $\dfrac{1}{(1-r)(1-Kr)}$

Q.7 30 dm.

Q.8 $5+\dfrac{10}{5}+\dfrac{20}{9}+ \text{- - - - - - -}$

Summary

1. nth term of General Term of an Arithmetic progression.
   $\boxed{a_n = a + (n-1)d}$
2. Arithmetic means between a and b
   $\boxed{A = \dfrac{a+b}{2}}$
3. Sum of the First n terms of an arithmetic series.
   (i) $S_n = \dfrac{n}{2}[2a + (n-1)d]$
   (ii) $S_n = \dfrac{n}{2}(a + l)$ when last term is given.
4. General or nth term of a G.P
   $a_n = ar^{n-1}$
5. Geometric means between a and b
   $G = \pm\sqrt{ab}$
6. Sum of n terms of a Geometric Series
   $S_n = \dfrac{a(1-r^n)}{1-r}$ if $r < 1$
   $S_n = \dfrac{a(r^n-1)}{r-1}$ if $r > 1$
7. Sum of an infinite Geometric Series
   $\boxed{S\infty = \dfrac{a}{1-r}}$

Short questions

Write the short answers of the following

Q.1: Define a sequence.

Q.2: Define finite sequence.

Q.3: Define infinite sequence.

Q.4: Define common difference.

Q.5: Write the nth term of arithmetic progression.

Q.6: Find the 7^{th} term of A.P. in which the first term is 7 and the common difference is –3.

Q.7: Find the 7^{th} term of an AP 1, 4, 7, ……………..

Q.8: Find the sequence whose general term in 4n +1.

Q.9: Define a series.

Q.10: Write the formula to find the sum of n term of an arithmetic sequence.

Q.11: Find the sum of the series 3 + 11 + 19 + ……to 16 terms.

Q.12: Find the sum of the series 5 + 8 + 11 + 14 + …….to n terms.

Q.13: Define arithmetic means (AMs).

Q.14: Find the A.M. between $\sqrt{5} - 4$ and $\sqrt{5} + 4$.

Q.15: Define a common ratio.

Q.16: Write the nth term of a geometric progressions .

Q.17: Find the term indicated in the following G.P. 1, 3^3, 3^6, ………… 6^{th} terms.

Q.18: write down the geometric sequence in which first term is 2 and the second term is – 6 and n = 5.

Q.19: Write the formula of sum of the first n terms of an geometric sequence for $|r| < 1$ and for $|r| > 1$

Q.20: Define geometric means.

Q.21: Find the G.M. between (i) 8 and 72 (ii) $\frac{4}{3}$, 243.

Q.22: Sum to 5 term the series 1 + 3 +9 + ……………

Q.23: Find the sum of the following series: $1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots$ to 6 terms.

Q.24: Find the sum of infinite geometric series in which a=128, $r = -\frac{1}{2}$

Q.25: Find the sum of following infinite geometric series $2+\sqrt{2}+1+\ldots\ldots\ldots\ldots$

Answers

Q6	$a_7 = -11$	Q7	$a_7 = 19$	Q8	5 , 9 , 13,………
Q11	1008	Q12	$\frac{n}{2}[7 + 3n]$	Q14	$\sqrt{5}$
Q17	$(27)^5$	Q18	2 , -6 , 18 , -54 , 162	Q21	(i) ± 24 (ii) ± 9
Q22	121	Q23	$\frac{364}{243}$	Q24	$\frac{256}{3}$
Q25	$\frac{4}{2-\sqrt{2}}$				

Objective Type Questions

Q.1 Each questions has four possible answers. Choose the correct answer and encircle it.

__1. The nth term of an A.P. whose 1st term is ‘a’ and common difference is ‘d’ is:

(a) $2a + (n + 1)d$ (b) $a + (n + 1)d$
(c) $a + (n - 1)d$ (d) $a + (d - 1)n$

__2. The nth term of an A.P. 1, 4, 7, …… is:

(a) 17 (b) 19 (c) 21 (d) 23

__3. If a, b, c are in A.P. then:

(a) $b - a = c - b$ (b) $\frac{b}{a} = \frac{c}{b}$
(c) $a + b = b + c$ (d) $\frac{a}{c} = \frac{b}{a}$

__4. The 10th term is 7, 17, 27, …… is:

(a) 97 (b) 98 (c) 99 (d) 100

__5. The sum of n terms of an A.P. with ‘a’ as 1st term and ‘d’ as common difference is:

(a) $\frac{n}{2}[a + (n - 1)d]$ (b) $\frac{n}{2}[2a + (n - 1)d]$
(c) $\frac{n}{2}[a + (n + 1)d]$ (d) $\frac{n}{2}[2a - (n - 1)d]$

__6. Arithmetic mean between $x - \sqrt{3}$ and $x + \sqrt{3}$ is:

(a) x (b) 2x (c) 3 (d) –3

__7. If $S_n = (n^2 + n + 1)$ then its 4th term will be:

(a) 21 (b) 40 (c) 41 (d) 101

__8. Arithmetic mean between – 7 and 7 is:

(a) $\frac{7}{2}$ (b) $-\frac{7}{2}$ (c) 0 (d) 14

__9. The sum of the series 1 + 2 + 3 + …… + 100 is:

(a) 100 (b) 5000 (c) 5050 (d) 500

__10. The nth term of a G.P a, ar, ar^2, …… is:

(a) ar^2 (b) ar^{n+1} (c) $\frac{1}{a}r^{n-1}$ (d) ar^{n-1}

__11. The 5th term of a G.P $1, \frac{1}{2}, \frac{1}{4}$, …… is:

(a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{32}$

__12. The 6th term of G.P $1, \sqrt{2}, \sqrt{4}, \ldots\ldots$ is:

(a) $4\sqrt{2}$ (b) 4 (c) $\sqrt{2}$ (d)

__13. The G.M. between a and b is:

(a) $\pm ab$ (b) ab (c) $\pm\sqrt{ab}$ (d) $\sqrt{ab}$

__14. If x, y, z are in G.P. then:

(a) $2y = x + z$ (b) $2y = xz$ (c) $y^2 = xz$ (d) $z^2 = xy$

__15. Geometric mean between 3 and 27 is:

(a) -9 (b) 12 (c) 15 (d) ± 9

__16. The sum of n terms of a geometric series:
$a + ar + ar^2 + \ldots\ldots; |r| < 1$

(a) $\frac{ar^{n-1}}{r-1}$ (b) $\frac{a(1-r^n)}{1-r}$

(c) $\frac{ar^{n+1}}{1-r}$ (d) $\frac{a(r^n-1)}{1-r}$

__17. The sum of 6 terms of the series $1 + 2 + 4 + \ldots\ldots$ is:

(a) 63 (b) 64 (c) 65 (d) 66

__18. The sum of 5 terms of the series $1 - 2 + 4 \ldots\ldots$:

(a) 16 (b) 11 (c) –11 (d) –16

__19. The sum of infinite terms of a G.P. a, ar_1, ar_1^2, …… if $|r| < 1$ is:

(a) $\frac{a}{1-r}$ (b) $\frac{a(1-r^n)}{1-r}$

(c) ar^{n-1} (d) None of these

__20. The sum of infinite geometric series $1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots$ is:

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

Answers

1.	c	2.	b	3.	a	4.	a	5.	b
6.	a	7.	a	8.	c	9.	c	10.	d
11.	c	12.	a	13.	c	14.	c	15.	d
16.	b	17.	a	18.	b	19.	a	20.	c