CONTENTS

	ord	
Снарте	r 1. Introduction	. 1
1–1	Notation	. 1
1–2	Instruction Set and Execution Time Model	
Снарте	r 2. Basics	11
2-1	Manipulating Rightmost Bits	11
2-2	Addition Combined with Logical Operations	16
2-3	Inequalities among Logical and Arithmetic Expressions	17
2–4	Absolute Value Function	18
2-5	Average of Two Integers	19
2-6	Sign Extension	19
2-7	Shift Right Signed from Unsigned	20
2-8	Sign Function	20
2–9	Three-Valued Compare Function	21
2-10	Transfer of Sign Function	22
2-11	Decoding a "Zero Means 2**n" Field	22
2-12	Comparison Predicates	23
2-13	Overflow Detection	28
2-14	Condition Code Result of Add, Subtract, and Multiply	36
2-15	Rotate Shifts	37
2-16	Double-Length Add/Subtract	38
2-17	Double-Length Shifts	39
2-18	Multibyte Add, Subtract, Absolute Value	40
2-19	Doz, Max, Min	41
2-20	Exchanging Registers	45
2-21	Alternating among Two or More Values	48
2-22	A Boolean Decomposition Formula	51
2-23	Implementing instructions for all 16 Binary Boolean	
	Operations	53
Снарте	R 3. POWER-OF-2 BOUNDARIES	59
3–1	Rounding Up/Down to a Multiple of a Known Power of 2	59
3–1	Rounding Up/Down to the Next Power of 2	60
3–3	Detecting a Power-of-2 Boundary Crossing	63
5 5	Detecting a rower or 2 Doundary Crossing	55

viii CONTENTS

Снарты	R 4. ARITHMETIC BOUNDS	67
4–1	Checking Bounds of Integers	67
4–2	Propagating Bounds through Add's and Subtract's	70
4–3	Propagating Bounds through Logical Operations	73
Снарты	R 5. COUNTING BITS	81
5–1	Counting 1-Bits	81
5–2	Parity	96
5–3	Counting Leading 0's	99
5–4	Counting Trailing 0's	107
Снарты	R 6. SEARCHING WORDS	117
6–1	Find First 0-Byte	117
6–2	Find First String of 1-Bits of a Given Length	123
6–3	Find Longest String of 1-Bits	125
6–4	Find Shortest String of 1-Bits	126
Снарте	R 7. REARRANGING BITS AND BYTES	129
7–1	Reversing Bits and Bytes	129
7–2	Shuffling Bits	139
7–3	Transposing a Bit Matrix	141
7–4	Compress, or Generalized Extract	150
7–5	Expand, or Generalized Insert	156
7–6	Hardware Algorithms for Compress and Expand	157
7–7	General Permutations, Sheep and Goats Operation	161
7–8	Rearrangements and Index Transformations	165
7–7	An LRU Algorithm	166
Снарты	R 8. MULTIPLICATION	171
8-1	Multiword Multiplication	171
8-2	High-Order Half of 64-Bit Product	173
8–3	High-Order Product Signed from/to Unsigned	174
8–4	Multiplication by Constants	175
Снарты	R 9. INTEGER DIVISION	181
9–1	Preliminaries	181
9–2	Multiword Division	184
9–3	Unsigned Short Division from Signed Division	189

CONTENTS ix

9–4 9–5	Unsigned Long Division	192 197
9–3	Doubleword Division from Long Division	197
CHAPTER	R 10. INTEGER DIVISION BY CONSTANTS	205
10-1	Signed Division by a Known Power of 2	205
10-2	Signed Remainder from Division by a Known Power of 2	206
10–3	Signed Division and Remainder by Non-Powers of 2	207
10–4	Signed Division by Divisors ≥ 2	210
10-5	Signed Division by Divisors ≤ -2	218
10–6	Incorporation into a Compiler	220
10-7	Miscellaneous Topics	223
10-8	Unsigned Division	227
10–9	Unsigned Division by Divisors ≥ 1	230
10-10	Incorporation into a Compiler (Unsigned)	232
10-11	Miscellaneous Topics (Unsigned)	234
10-12	Applicability to Modulus and Floor Division	237
10–13	Similar Methods	237
10-14	Sample Magic Numbers	238
10-15	Simple Code in Python	240
10–16	Exact Division by Constants	240
10-17	Test for Zero Remainder after Division by a Constant	248
10–18	Methods Not Using Multiply High	251
	Remainder by Summing Digits	262
10-20	Remainder by Multiplication and Shifting Right	268
10-21	Converting to Exact Division	274
10-22	A Timing Test	276
10–23	A Circuit for Dividing by 3	276
CHAPTER	R 11. SOME ELEMENTARY FUNCTIONS	279
11–1	Integer Square Root	279
11–2	Integer Cube Root	287
11–3	Integer Exponentiation	288
11–4	Integer Logarithm	291
Снартег	R 12. UNUSUAL BASES FOR NUMBER SYSTEMS	299
12-1	Base –2	299
12–2	Base -1 + <i>i</i>	306
12–3	Other Bases	308
12–4	What Is the Most Efficient Base?	

x CONTENTS

CHAPTER	13. Gray Code	311
13–2 13–3	Gray Code Incrementing a Gray-Coded Integer Negabinary Gray Code Brief History and Applications	311 313 315 315
CHAPTER	14. CYCLIC REDUNDANCY CHECK	319
14–2	Introduction Theory Practice	319 320 323
CHAPTER	15. Error correcting codes	331
15–2 15–3	Introduction The Hamming Code Software for SEC-DED on 32 Information Bits Error Correction Considered More Generally	331 332 337 342
CHAPTER	16. Hilbert's Curve	355
16–2 16–3 16–4 16–5 16–6	A Recursive Algorithm for Generating the Hilbert Curve Coordinates from Distance along the Hilbert Curve Distance from Coordinates on the Hilbert Curve Incrementing the Coordinates on the Hilbert Curve Non-Recursive Generating Algorithms Other Space-Filling Curves Applications	356 358 366 368 371 371 372
CHAPTER	17. FLOATING-POINT	375
17–2 17–3 17–4 17–5	IEEE Format Floating-Point To/From Integer Conversions Comparing Floating-Point Numbers Using Integer Operations An Approximate Reciprocal Square Root Routine The Distribution of Leading Digits Table of Miscellaneous Values	375 377 381 383 385 387
CHAPTER	18. FORMULAS FOR PRIMES	391
18–2 18–3	Introduction Willans's Formulas Wormell's Formula Formulas for Other Difficult Functions	391 393 397

CONTENTS xi

Answers to Exercises	405
APPENDIX A. ARITHMETIC TABLES FOR A 4-BIT MACHINE	453
APPENDIX B. NEWTON'S METHOD	457
APPENDIX C. A GALLERY OF GRAPHS OF DISCRETE FUNCTIONS	459
C-1 Plots of Logical Operations on Integers	459
C–2 Plots of Addition, Subtraction, and Multiplication	
C-3 Plots of Functions Involving Division	463
C–4 Plots of the Compress, SAG, and Rotate Left Functions	464
C–5 2D Plots of some Unary Functions	466
Bibliography	. 471
Index	481