

CSE-103

DISCRETE MATHEMATICS

ASSIGNMENT NO : 05
ASSIGNED TO : PROF. DR. M. KAYKOBAD

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SUBMITTED BY

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SECTION : B

LEVEL-1 TERM-2

(Out of the text assignment #3)

Problem Statement :

10 prisoners were told that they would be given an opportunity if being free. Next day in the morning they will be arranged as per desire of the jailor in a line so that anybody can see heads of all people in the front. Then they would be given a cap on each one's head from large collection of white and black hats. Then the person standing last in the line would be asked the color of the hat he was wearing. If the answer was correct he would be set free. Then the next person would be asked the same question, and finally the person in the front of the line. Prisoners after reaching cell devised a way in which more than half of the prisoners would be set free. What is the strategy? What about if there were m prisoners and each of them was given hat of any of the known m colours ?

Answer :

In this problem, 9 prisoners can be saved with 100 % certainty. But for the last one the chance is 50%.

The strategy to be free :

For 2 Color :

The strategy is if the present prisoner sees the number of black hats in front of him is even then he will say "Black", otherwise he will say "White". Then in front of him the next

prisoner will count the number of white and black hats in front of him and will say "Black" if the number of black hat in front of him is odd (according to the previous prisoners statement) , otherwise will say "White".

Thus the other all will count the number of white and black hat in front of him and will answer "Black" or "White" depending on the previous one's answer .

By taking this strategy the $(N-1)$ can be saved with 100% certainty if there is N prisoners but the last one can't answer with 100% certainty. The chance for the last one is 50% to be free.

For m color :

If there is n prisoners and m colors labeling $\{ 1 , 2 , 3 , \dots, m \}$ then the last one will sum of other $(n - 1)$ hats in front of him and mod them with k . After this the second last one calculate for his own hat's color depending on the last one's answer.

Now the third prisoner, according to the last two person's answer, can deduce the sum of the last $(n - 2)$ prisoners can deduce the sum of $(n - 3)$ prisoners and can answer correctly of the color of his hat . And so on for the next, next and next one until the end.

But again, as in previous question and it's answer, the last prisoner can't answer correctly of the color of his hat. There is a possibility of $(100 / m)\%$ of being correct that he answered .