

# Size Reduction

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# Introduction

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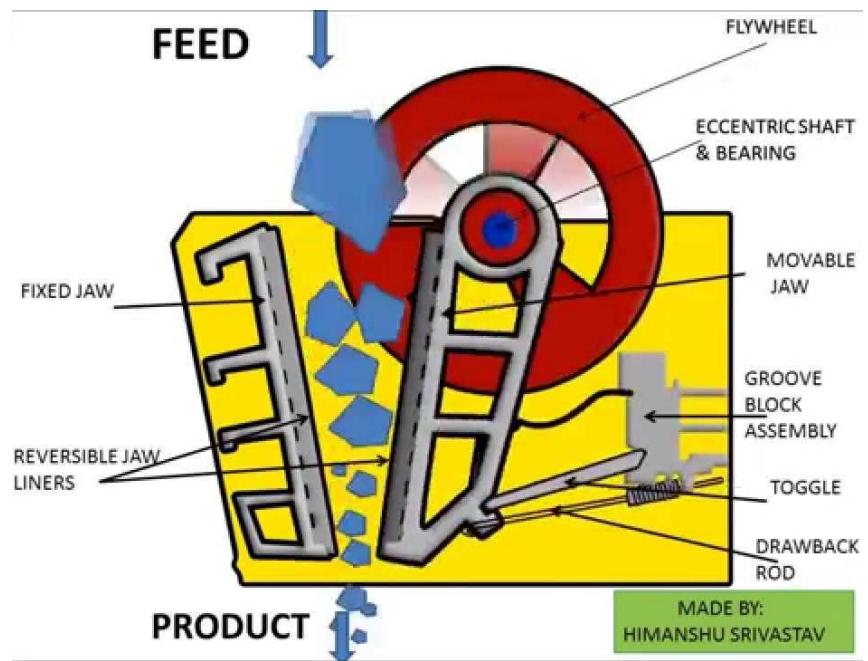
- Size reduction or comminution is an important step in the processing of many solid materials
- It may be used to create particles of a certain size and shape, to increase surface area available for chemical reaction
- Size reduction of solids is an energy intensive and highly inefficient process
- Design and scale-up of comminution processes is usually based on experience and testing

# Equipment for Particle Size Reduction

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- Jaw Crushers
- Gyratory Crushers
- Roll Crushers
- Hammer Mill Grinders
- Revolving Grinding Mills

# Jaw Crushers

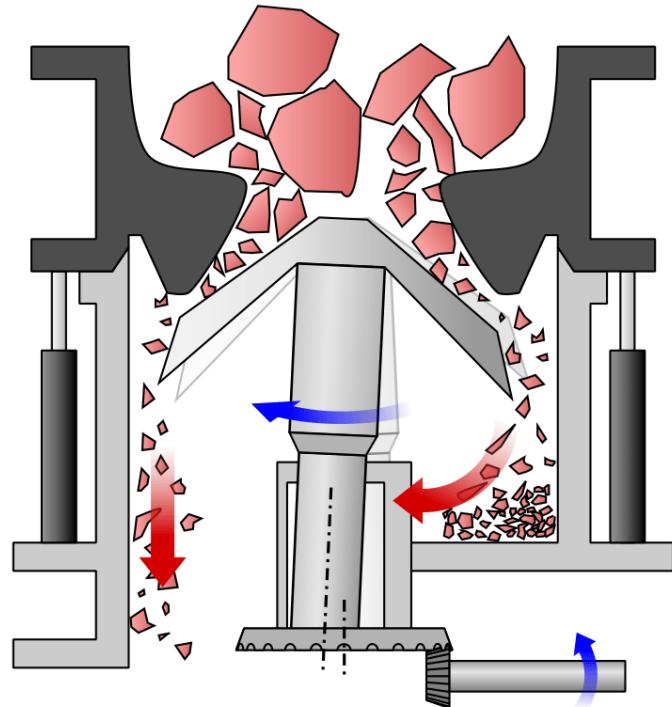


<https://zenithdream.en.made-in-china.com/product/CvjxzYsgrZVT/China-Factory-Price-Primary-Jaw-Crusher-Jaw-Crushers-for-Sale.html>

<https://www.youtube.com/watch?v=yTlePG1E-kY>

# Gyratory Crushers

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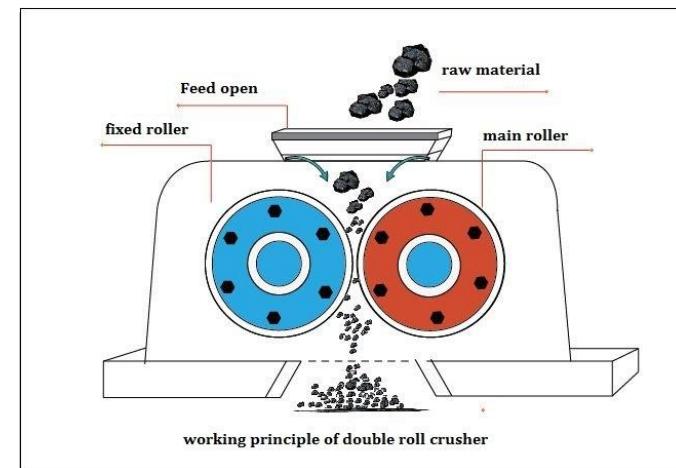
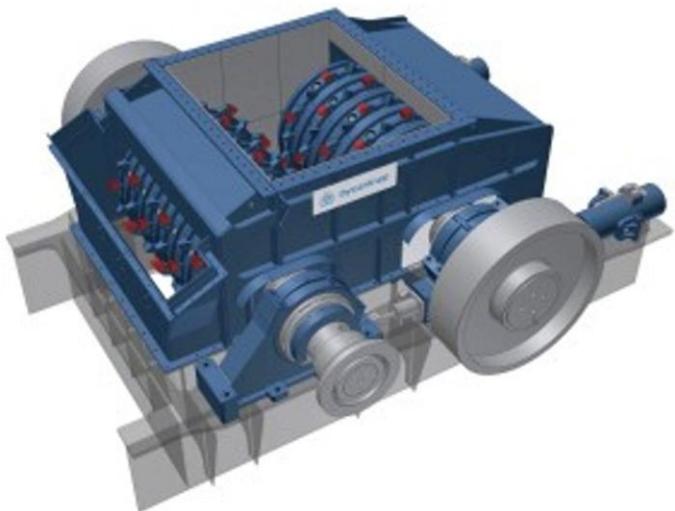


<https://savree.com/en/encyclopedia/gyratory-crusher>

<https://www.sbmchina.com/equipments/hgt.html>

# Roll Crushers

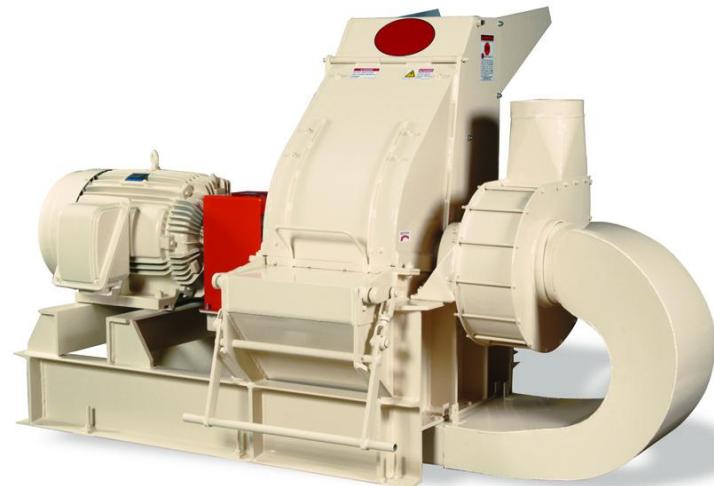
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<https://www.thyssenkrupp-polysius.com/en/products/crushing-technologies/double-roll-crusher>

<https://citicic.en.made-in-china.com/product/TKBJECuxOLUj/China-Large-Capacity-Double-Roll-Crusher-with-ISO-Certificate.html>

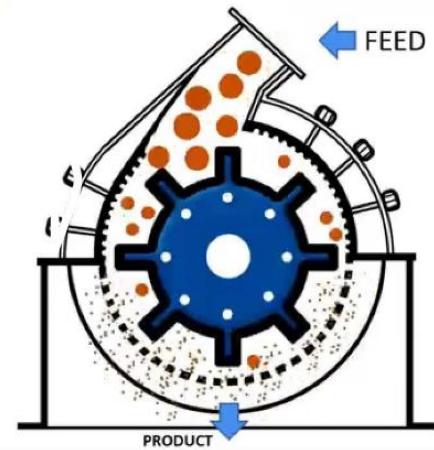
# Hammer Mill Grinders



<https://www.hammermills.com/equipment/13-series-industrial-grinding-hammer-mill/>



[https://www.thomassci.com/Equipment/Mills/\\_/Wiley-Mill-4-12-Horse-Power-Unit-Accessories](https://www.thomassci.com/Equipment/Mills/_/Wiley-Mill-4-12-Horse-Power-Unit-Accessories)

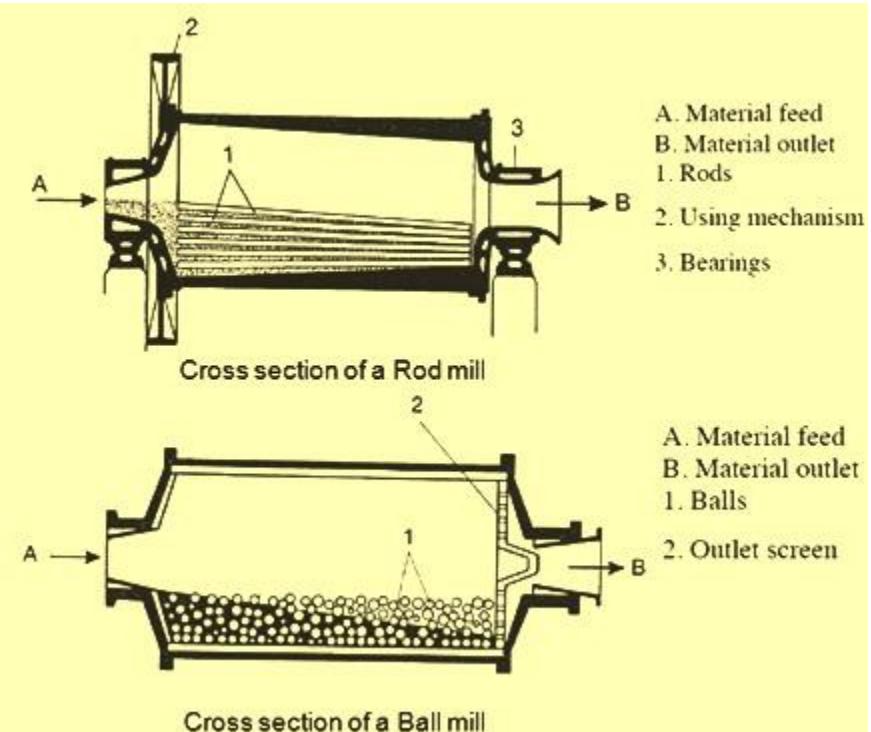


<https://www.saintytec.com/working-principle-hammer-mills/>

# Revolving Grinding Mills



<https://www.quarrymagazine.com/2022/08/09/websterbscs-quick-turn-around-on-coupling-upgrades/>



<https://www.ispatguru.com/grinding-mills-and-their-types/>

# Grinding and Cutting

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- Grinding and cutting reduce the size of solid materials by mechanical action, dividing them into smaller particles.
- In the grinding process, materials are reduced in size by fracturing them.
- The force applied may be compression, impact, or shear, and both the magnitude of the force and the time of application affect the extent of grinding achieved.
- The important factors to be studied in the grinding process are the amount of energy used and the amount of new surface formed by grinding.

# Energy Used in Grinding

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Assumption: the energy (E) required to produce a change  $dX$  in a particle of a typical size dimension  $X$  is a simple power function of  $X$ :

$$\frac{dE}{dX} = -\frac{C}{X^n}$$

where:

$X$  = size or diameter of the particle in mm,

$n$  and  $C$  = constants depending on the type and size of material and the type of machine used.

# Rittinger's Law

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Peter Ritter Von Rittinger (1867) proposed a law that states that the work in crushing is proportional to the new surface created. Since area is proportional to the length squared:

$$\frac{dE}{dX} = -\frac{C}{X^2}$$

$$E = K_R \left( \frac{1}{X_2} - \frac{1}{X_1} \right)$$

where E is the work required to reduce a unit mass feed from X<sub>1</sub> to X<sub>2</sub>, and K<sub>R</sub> is a constant.

Used for grinding *fine powders*.



# Kick's Law

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Friedrich Kick (1885) assumed that the energy required to reduce a material in size was directly proportional to the size reduction ratio ( $n=1$ ):

$$\frac{dE}{dX} = -\frac{C}{X} \quad E = C \ln\left(\frac{X_1}{X_2}\right)$$

$$E = K_R \log\left(\frac{X_1}{X_2}\right)$$

where  $K_R$  is a constant.

# Bond's Law



Fred C. Bond (1952) suggest that the work required using a **large-size feed** ( $X_1 \approx 0$ ) is proportional to the square root of the surface/volume ratio of the product ( $n=1.5$ ):

$$\frac{dE}{dX} = -\frac{C}{X^{1.5}}$$

$$E = K_B \frac{1}{\sqrt{X_2}}$$

where  $K_B$  is a constant.

# Bond's Law



Bond proposed a work index  $E_i$  as the work in kW(h/ton) required to reduce a unit weight from very large size to 80% passing a 100- $\mu\text{m}$  screen. Bond's final equation in terms of English units:

$$\frac{P}{T} = 1.46E_i \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

where P is in hp, T is the feed rate in tons/min,  $D_F$  is the size of the feed in ft, and  $D_P$  is the product size in ft.

Typical values of  $E_i$  can be found in our handbook (Table 21-15, page 21-70 9<sup>th</sup> Ed)

TABLE 21-15 Average Work Indices for Various Materials\*

Material	No. of tests	Specific gravity	Work index <sup>†</sup>	Material	No. of tests	Specific gravity	Work index <sup>†</sup>
All materials tested	2088	—	13.81	Taconite	66	3.52	14.87
Andesite	6	2.84	22.13	Kyanite	4	3.23	18.87
Barite	11	4.28	6.24	Lead ore	22	3.44	11.40
Basalt	10	2.89	20.41	Lead-zinc ore	27	3.37	11.35
Bauxite	11	2.38	9.45	Limestone	119	2.69	11.61
Cement clinker	60	3.09	13.49	Limestone for cement	62	2.68	10.18
Cement raw material	87	2.67	10.57	Manganese ore	15	3.74	12.46
Chrome ore	4	4.06	9.60	Magnesite, dead burned	1	5.22	16.80
Clay	9	2.23	7.10	Mica	2	2.89	134.50
Clay, calcined	7	2.32	1.43	Molybdenum	6	2.70	12.97
Coal	10	1.63	11.37	Nickel ore	11	3.32	11.88
Coke	12	1.51	20.70	Oil shale	9	1.76	18.10
Coke, fluid petroleum	2	1.63	38.60	Phosphate fertilizer	3	2.65	13.03
Coke, petroleum	2	1.78	73.80	Phosphate rock	27	2.66	10.13
Copper ore	308	3.02	13.13	Potash ore	8	2.37	8.88
Coral	5	2.70	10.16	Potash salt	3	2.18	8.23
Diorite	6	2.78	19.40	Pumice	4	1.96	11.93
Dolomite	18	2.82	11.31	Pyrite ore	4	3.48	8.90
Emery	4	3.48	58.18	Pyrrhotite ore	3	4.04	9.57
Feldspar	8	2.59	11.67	Quartzite	16	2.71	12.18
Ferrochrome	18	6.75	8.87	Quartz	17	2.64	12.77
Ferromanganese	10	5.91	7.77	Rutile ore	5	2.84	12.12
Ferrosilicon	15	4.91	12.83	Sandstone	8	2.68	11.53
Flint	5	2.65	26.16	Shale	13	2.58	16.40
Fluorspar	8	2.98	9.76	Silica	7	2.71	13.53
Gabbro	4	2.83	18.45	Silica sand	17	2.65	16.46
Galena	7	5.39	10.19	Silicon carbide	7	2.73	26.17
Garnet	3	3.30	12.37	Silver ore	6	2.72	17.30
Glass	5	2.58	3.08	Sinter	9	3.00	8.77
Gneiss	3	2.71	20.13	Slag	12	2.93	15.76
Gold ore	209	2.86	14.83	Slag, iron blast furnace	6	2.39	12.16
Granite	74	2.68	14.39	Slate	5	2.48	13.83
Graphite	6	1.75	45.03	Sodium silicate	3	2.10	13.00
Gravel	42	2.70	25.17	Spodumene ore	7	2.75	13.70
Gypsum rock	5	2.69	8.16	Syenite	3	2.73	14.90
Ilmenite	7	4.27	13.11	Tile	3	2.59	15.53
Iron ore	8	3.96	15.44	Tin ore	9	3.94	10.81
Hematite	79	3.76	12.68	Titanium ore	16	4.23	11.88
Hematite—specular	74	3.29	15.40	Trap rock	49	2.86	21.10
Oolitic	6	3.32	11.33	Uranium ore	20	2.70	17.93
Limanite	2	2.53	8.45	Zinc ore	10	3.68	12.42
Magnetite	83	3.88	10.21				

\*Allis-Chalmers Corporation.

†Caution should be used in applying the average work index values listed here to specific installations since individual variations between materials in any classification may be quite large.

# Example 1:

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It is desired to crush 10 ton/h of iron ore hematite. The size of the feed is such that 80% passes a 3-in. (76.2-mm) screen and 80% of the product is to pass a 1/8-in. (3.175-mm) screen. Calculate the gross power required.

Iron ore Hematite		8 79		3.96 3.76		15.44 12.68
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Solution:

$$E_i = 12.68 \text{ (9}^{\text{th}} \text{ Ed)}$$

$$D_F = 3 \text{ in} = 0.250 \text{ ft}$$

$$D_P = 1/8 \text{ in} = 0.0104 \text{ ft}$$

# Example 1 Solution

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Solution:

$$E_i = 12.68 \text{ (9<sup>th</sup> Ed)}$$

$$D_F = 3 \text{ in} = 0.250 \text{ ft}$$

$$D_p = 1/8 \text{ in} = 0.0104 \text{ ft}$$

$$T = 10 \text{ tons/h} = 0.167 \text{ tons/min}$$

$$\frac{P}{0.167} = 1.46(12.68) \left( \frac{1}{\sqrt{0.0104}} - \frac{1}{\sqrt{0.250}} \right)$$

$$P = 24.1$$

$P = 24.1 \text{ hp}$

$$P = 17.96 \text{ kW}$$

# Example 2 (Problem 29.3-1)

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In crushing a certain ore, the feed is such that 80% is less than 50.8 mm in size, and the product size is such that 80% is less than 6.35 mm. The power required is 89.5 kW. What will be the power required using the same feed so that 80% is less than 3.18 mm? Use the Bond equation.

Solution:

$$\frac{P}{T} = 1.46 E_i \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

$$P = 1.46 E_i T \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

$$P = K \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

# Example 2 (Problem 29.3-1)

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Solution (cont.):

$$P = K \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

For  $D_F = 50.8$ ,  $D_P = 6.35$ ,  $P = 89.5$  kW:

$$89.5 = K \left( \frac{1}{\sqrt{6.35}} - \frac{1}{\sqrt{50.8}} \right)$$

$$K = 348.88$$

# Example 2 (Problem 29.3-1)

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Solution (cont.):

$$P = 348.88 \left( \frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

For  $D_F = 50.8$ ,  $D_P = 3.18$ ,  $P = ?$  :

$$P = 348.88 \left( \frac{1}{\sqrt{3.18}} - \frac{1}{\sqrt{50.8}} \right)$$

$$P = 146.69$$

**P = 146.69 kW**