The Economic Role of Non-GAAP Earnings

January 9, 2026

Abstract

This study provides a theoretical framework to explain the economic role of non-GAAP earnings as an efficient, equilibrium response to conflicting demands from capital providers. I model a firm serving creditors who require conservative earnings for contracting, and equity investors who require informative signals for valuation. In a GAAP-only regime, the model formalizes the fundamental trade-off: raising the recognition threshold toward fair value improves equity informativeness but degrades debt contracting efficiency; maintaining conservatism protects creditors but creates information gaps for equity investors. As a result, any uniform mandatory standard is inherently second-best. Conservative accounting creates residual uncertainty that pools firms and increases the cost of equity, creating demand for a second, valuation-relevant signal: non-GAAP earnings. Managers resolve this pooling by voluntarily disclosing a non-GAAP adjustment that reveals unrecognized gains in the GAAP earnings, enabling signal specialization and reducing the cost of equity. However, such disclosure simultaneously increases the cost of debt by exposing asset volatility. This trade-off yields a leverage threshold that partitions firms: intangible-intensive firms with low leverage adopt dual reporting to minimize WACC, whereas highly leveraged firms rely on GAAP-only reporting. Policy efforts to force convergence would destroy this efficient sorting mechanism. My analysis provides a structural explanation for why dual reporting emerges endogenously and why regulatory attempts to suppress non-GAAP reporting are most value-destroying for high-growth, intangible-intensive firms.

Keywords: Non-GAAP Earnings, Conditional Conservatism, Debt Contracting, Information Asymmetry, Intangible Assets.

JEL Classifications: M41, G14, G32, D82.

1 Introduction

The divergence between Generally Accepted Accounting Principles (GAAP) and the alternative performance metrics disclosed by management represents one of the most persistent phenomena in modern capital markets. Despite two decades of regulatory scrutiny, including the implementation of Regulation G in 2003 and subsequent guidance by the Securities and Exchange Commission (SEC), the prevalence of Non-GAAP reporting has not abated; rather, it has become institutionalized (Heflin and Hsu, 2008; Black *et al.*, 2017). For the majority of firms in the S&P 500, the street earnings number used by analysts and investors now systematically deviates from the audited financial statements (Bentley *et al.*, 2018; Dechow *et al.*, 2025).

This persistent decoupling of the primary accounting signal has generated a polarized debate. One stream of research views the divergence through the lens of *managerial opportunism*, arguing that managers utilize discretion to exclude recurring expenses and obfuscate performance, representing a breakdown in reporting quality that requires regulatory containment (Doyle *et al.*, 2003; Black and Christensen, 2009; Doyle *et al.*, 2013). Conversely, a second stream emphasizes *informativeness*, arguing that the rigidity of accounting standards renders GAAP earnings an increasingly noisy proxy for fundamental value, prompting managers to provide non-GAAP adjustments that strip out transitory noise and convey more precise signals of core earnings to equity investors (Bradshaw and Sloan, 2002; Leung and Veenman, 2018).

While both perspectives offer empirical support, neither fully explains the structural persistence of the phenomenon across diverse firm life-cycles and regulatory regimes. If Non-GAAP reporting were purely opportunistic, the disciplining forces of efficient markets and regulation should have curtailed its use. If it were purely informative, standard setters should have arguably converged GAAP toward these 'better' metrics to reduce information asymmetry. The survival of two conflicting signals suggests that the prevailing binary choice, either truth-telling or opportunism, is a false dichotomy.

In this paper,[1] I develop a theoretical framework to explain why the divergence between GAAP and Non-GAAP earnings is an efficient, WACC-minimizing response to the conflicting demands of capital providers. I model a firm that must communicate with two audiences:

[1] An online appendix with complete formal proofs for all mathematical results in this paper is available at `https://github.com/rngpaper/onlineappendix`.

creditors who require a conservative, verifiable signal for debt contracting, and equity investors who demand a timely signal to value intangible assets.

The intuition for this framework relates to the fundamental conflict between the two primary roles of financial reporting. The first role is *stewardship and contracting*. Following the positive accounting theory of Watts and Zimmerman (1986), financial statements serve as the verifiable metric upon which the firm's nexus of contracts, including debt covenants, compensation agreements, and supplier terms, is written. To function effectively as a contracting technology, accounting signals must minimize the moral hazard of asset substitution and ex-post settling up. This requires a reporting regime characterized by rigidity, verifiability, and, most crucially, conditional conservatism (Basu, 1997). By requiring the timely recognition of losses and delaying the recognition of gains, GAAP creates a 'hard' lower bound on firm value that protects creditors and monitors against managerial over-optimism.

The second role is *equity valuation*. In the linear information dynamics framework of (Ohlson, 1995) and Feltham and Ohlson (1995), the accounting signal serves as the input for pricing the firm's intrinsic value. To function effectively as a pricing mechanism, the signal must be an unbiased estimator of future cash flows. Equity investors, as residual claimants, require a metric that capitalizes the expected payoff of risky growth options, regardless of their verifiability.

For firms with low asset volatility, such as mature manufacturers with tangible capital, these two objectives largely overlap. The liquidation value (contracting income) approximates the going-concern value (valuation income). However, as the economy shifts toward high-volatility, intangible-intensive firms, the mathematical objectives of these two masters diverge. The very features that make GAAP efficient for contracting, such as the immediate expensing of R&D to prevent collateral inflation, render it increasingly biased and noisy for valuation.

This creates a fundamental impossibility: a single mandatory accounting signal cannot simultaneously optimize both stewardship and valuation because these objectives impose conflicting demands on the signal's statistical properties. A signal that is sufficiently conservative to satisfy the debt market is structurally too pessimistic for the equity market. Conversely, a signal aggressive enough to capture the value of intangible growth options is too soft to support efficient debt contracting. Following Gjesdal (1981) and the economic logic of the Tinbergen Principle (Tinbergen, 1952)—which establishes that achieving two independent policy targets requires two independent policy instruments—I demonstrate that this impossibility dilemma necessitates a *structural decoupling*, where the firm optimizes its cost of capital by providing two distinct signals: a conservative GAAP number for stewardship and an adjusted Non-GAAP number for valuation.

My study provides a parsimonious formal framework for evaluating the economic function of the dual-reporting environment. In doing so, this study responds to the recent call by Breuer, Labro, Sapra and Zakolyukina (2024) for richer analytical theories that explicitly bridge the gap between reporting institutions and empirical phenomena. By endogenizing both the mandatory GAAP signal and the discretionary Non-GAAP signal, I characterize the modern reporting environment not as a failure of standardization, but as a sophisticated, WACC-minimizing architecture.

This framework yields several insights regarding the economic function of Non-GAAP reporting. First, the model demonstrates that the demand for Non-GAAP earnings is structurally rooted in the asymmetric (un)informativeness of conservative accounting. Because GAAP recognizes losses immediately but delays the recognition of gains, the mandatory signal functions effectively as a 'hard' lower bound on firm value, a property that satisfies the verifiability requirements of debt contracting. However, for firms with high intangible intensity, this conservative bias renders GAAP earnings increasingly uninformative for equity valuation. I characterize the unrecognized economic gain as a call option on the firm's intangible productivity. As firms shift toward high-volatility, innovation-driven business models, the variance of this unrecognized component accelerates, creating a structural wedge between the GAAP signal and the firm's fundamental value. This informational friction generates the demand for a second, discretionary signal to resolve the residual uncertainty that GAAP, by design, ignores. By modeling conservatism as *structural censoring* (truncation at a verification threshold) rather than distributional noise, the framework shows that voluntary disclosure emerges endogenously as the equilibrium response. This result reverses the finding of Gigler and Hemmer (2001) that conservatism and disclosure are substitutes; instead, when conservatism takes the form of structural censoring, it creates complementarity between conservative GAAP and voluntary Non-GAAP disclosure.

Second, the analysis reveals that the provision of this second signal creates a fundamental tension between the cost of equity and the cost of debt. While Non-GAAP disclosure reduces the cost of equity by lowering the liquidity discount associated with information asymmetry, it simultaneously increases the cost of debt. The mechanism driving this result is the creditor's pricing of volatility. Because debt is a concave claim on firm value, creditors view the revelation of significant 'unrecognized upside' not merely as good news, but as evidence of higher asset volatility and tail risk. By explicitly modeling debt pricing via convex credit risk functions, I show that the cost of debt is strictly increasing and convex in the magnitude of the Non-GAAP adjustment. Consequently, voluntary disclosure is not a costless transfer of information; it exerts real effects on the firm's cash flows by triggering a repricing of the firm's liabilities.

These opposing forces yield a sorting equilibrium where firms endogenously partition into reporting regimes based on their capital structure. The model identifies a critical leverage threshold, defined as the firm's Informational Debt Capacity. For firms with leverage below this threshold, typically high-growth, intangible-intensive firms, the liquidity benefit of resolving equity uncertainty outweighs the marginal increase in borrowing costs. These firms optimally engage in Structural Decoupling, providing a conservative GAAP number to anchor debt contracts and an aggressive Non-GAAP number to guide equity pricing. In contrast, for firms with leverage above this threshold, the real cost of revealing volatility to the credit market is prohibitive. These firms optimally revert to a 'GAAP-only' reporting strategy, relying on the coarseness of the conservative signal to shield their cost of debt from the pricing of tail risk. This mechanism explains why highly levered firms engage in more conservative Non-GAAP reporting, addressing the concern that the voluntary channel lacks credibility by demonstrating that capital market discipline, rather than regulatory enforcement, provides the endogenous limit on reporting aggressiveness.

Third, I demonstrate that the standard setter faces an intractable optimization problem when constrained to a single mandatory signal. The recognition threshold governing GAAP conservatism has opposing effects: raising the threshold toward fair value reduces the cost of equity by making GAAP more informative, but simultaneously increases the cost of debt by introducing volatility into covenant triggers. Because the optimal weights on these objectives vary dramatically across firms, mature utilities versus growth startups, any uniform standard represents an unsatisfying compromise. This analysis formalizes the intuition that a single accounting signal cannot serve two masters with incompatible objective functions. The dual-reporting regime resolves this impossibility by allowing signal specialization: GAAP optimizes for contracting efficiency while voluntary Non-GAAP disclosure serves equity valuation. This separation constitutes a market-based application of the Tinbergen principle—two instruments for two targets. Critically, the optimal accounting standard in a dual-reporting regime is a corner solution: maximal GAAP conservatism (immediate expensing of intangibles). This corner solution would be infeasible in a GAAP-only world, where the standard setter must balance contracting and valuation objectives. However, when voluntary Non-GAAP disclosure is available, GAAP can specialize entirely in contracting, providing a theoretical justification for current standards that require immediate expensing of intangibles, which appear inefficient from a pure valuation perspective but are efficient when understood as part of a dual-signal architecture.

Fourth, and most critically, I show that the information environment has real effects on capital allocation through the adverse selection channel identified by Myers and Majluf (1984). Under GAAP-only reporting, conservative accounting creates pooling: high-productivity firms cannot credibly separate from low-productivity firms because the censored GAAP signal renders them observationally equivalent. This mispricing creates an adverse selection wedge that raises the effective cost of capital, causing systematic underinvestment in intangible assets relative to the first-best level. Non-GAAP disclosure mitigates this distortion by allowing high-quality firms to signal their type, reducing the cost of capital and restoring efficient investment. The welfare gain from dual reporting decomposes into two components: reduced mispricing of existing assets and reduced underinvestment in new projects. This investment efficiency result provides the normative foundation for evaluating disclosure regulation: restrictions on Non-GAAP reporting effectively function as a tax on innovation, with welfare losses concentrated among low-leverage, high-growth firms where adverse selection is most severe.

This framework makes four primary contributions. First, I establish that disclosure regulation has real effects on investment efficiency and aggregate welfare. By explicitly modeling the firm's intangible investment decision, I demonstrate that the information environment directly impacts capital allocation through adverse selection. Under GAAP-only reporting, conservative accounting creates pooling equilibria where high-productivity firms are systematically underpriced, raising their effective cost of capital and causing underinvestment relative to first-best. Non-GAAP disclosure mitigates this distortion by enabling separation, lowering the cost of capital and restoring efficient investment in innovation. This result provides the normative foundation for evaluating disclosure policy: a regulatory ban on Non-GAAP reporting would destroy this separation mechanism, forcing high-growth firms back into the pooling equilibrium and creating deadweight losses through foregone investment in intangible assets. The welfare cost is concentrated precisely where it is most economically damaging, among low-leverage, R&D-intensive firms that drive innovation. This contribution extends the Myers-

Majluf (1984) underinvestment framework to the disclosure setting and provides a counterpoint to Chen, Hemmer and Zhang (2007), who emphasize the benefits of uniform reporting when strategic complementarities are high. My analysis demonstrates that in the presence of extreme accounting conservatism, uniform regimes destroy allocative efficiency.

Second, I formalize the standard setter's dilemma and characterize the dual-reporting regime as the solution to the impossibility dilemma. A standard setter constrained to a single mandatory signal faces an intractable optimization: the recognition threshold that minimizes the cost of equity (high $\overline{R}_C$, approaching fair value) is fundamentally incompatible with the threshold that minimizes the cost of debt (low $\overline{R}_C$, maintaining conservatism). Because these objectives have opposing gradients and the optimal weights vary across heterogeneous firms, any uniform standard is inherently second-best. The dual-reporting regime eliminates this trade-off through signal specialization: GAAP optimizes for debt contracting while voluntary Non-GAAP serves equity valuation. This market-based solution instantiates the Tinbergen principle, two instruments for two incompatible targets, and demonstrates that the observed divergence between GAAP and Non-GAAP is not a regulatory failure but an efficient equilibrium outcome. This result provides a theoretical foundation for principles-based regulation that permits reporting flexibility rather than mandating convergence.

Third, the analysis contributes to the literature on the confirmatory role of mandatory reporting. Standard theory posits that the credibility of 'soft' voluntary disclosure is sustained by its subsequent verification against 'hard' mandatory reports (e.g., Gigler and Hemmer, 1998). However, I identify a structural failure in this mechanism driven by conditional conservatism. Because conservative accounting systematically censors upside realizations through structural truncation (rather than distributional noise), GAAP effectively loses the capacity to confirm the 'good news' inherent in Non-GAAP adjustments. To bridge this verification gap, the model demonstrates that the cost of debt assumes the disciplining role typically assigned to the auditor. This substitution effect provides a structural rationalization for the 'loss reversal' phenomenon. Rather than *prima facie* evidence of opportunism, the analysis reveals that firms report Non-GAAP profits amidst GAAP losses precisely when the mandatory signal becomes locally uninformative, with the creditor's pricing schedule serving as the credibility bond (Leung and Veenman, 2018). This result demonstrates that when conservatism is modeled as structural censoring, voluntary disclosure emerges endogenously as the equilibrium response, reversing the finding of Gigler and Hemmer (2001) that conservatism and disclosure are substitutes.

Fourth, this paper contributes to the nascent literature on the interaction between debt and voluntary disclosure. Building on Beyer and Dye (2021), who examine how leverage impacts disclosure when managers are compensated on enterprise value, I show that the nature of the accounting signal (conservative vs. non-conservative) creates a real effect on capital structure. I introduce the concept of Informational Debt Capacity, demonstrating that the revelation of tail risk through Non-GAAP metrics imposes a "volatility tax" in the credit market. This characterizes the divergence between GAAP and Non-GAAP as a sophisticated adaptation of institutional decoupling (Meyer and Rowan, 1977). While sociologists view decoupling as ceremonial, my model proves it is economically functional: GAAP serves as a binding 'hard' constraint for creditors, while Non-GAAP resolves the liquidity discounts for equity investors. Critically, the policy implication is that optimal regulation should focus on strengthening the credibility of Non-GAAP metrics, through enhanced governance, audit committee oversight, and ex post penalties for manipulation, rather than prescriptive restrictions on content. This shifts the regulatory focus from suppressing the disclosure channel to disciplining its use.

Ultimately, this study suggests that the 'two-signal' structure of modern financial reporting is not an accident of regulatory drift, but an endogenous market necessity (Einhorn, 2005). As Chen, Hemmer and Zhang (2007) and Bertomeu, Darrough and Xue (2017) have noted, conservatism is essential for contract efficiency, yet it inevitably creates informational frictions for equity valuation. My model shows that Non-GAAP reporting is the market's endogenous solution to this friction, a dual architecture that allows signal specialization when a single mandatory standard cannot simultaneously optimize contracting and valuation objectives. The normative implication is that regulatory policy should abandon the quest for convergence and instead focus on the mechanisms that ensure the credibility of voluntary disclosure, thereby preserving the investment efficiency gains while mitigating the agency costs of managerial discretion.

2 Background and Motivation

The central premise of this study is that financial reporting serves two distinct economic functions with mathematically incompatible informational requirements: contracting (stewardship) and valuation (decision-usefulness). While the analytical accounting literature has long recognized this tension, prior work largely treats it as an unsolvable dilemma for standard setters—a binding constraint that forces regulators to accept second-best compromises. This paper reframes the tension by demonstrating that the market endogenously resolves the conflict through dual reporting: GAAP specializes in contracting via conservative recognition rules, while voluntary non-GAAP disclosure specializes in valuation by revealing censored information about intangible productivity.

This section proceeds in three parts. First, it establishes the theoretical impossibility of a single accounting signal simultaneously optimizing both stewardship and valuation objectives, drawing on the seminal work of Gjesdal (1981) and the economic logic of the Tinbergen Principle (Tinbergen, 1952). Second, it situates the modeling approach within the analytical tradition of conservatism, tracing the theoretical development from unconditional bias (Feltham and Ohlson, 1995) through option-based frameworks (Zhang, 2000) to verification-constrained systems (Gao, 2013). Third, it demonstrates how voluntary non-GAAP reporting emerges not as an evasion of standards but as the efficient market response to this foundational impossibility.

2.1 The Foundational Impossibility: One Signal Cannot Serve Two Masters

The starting point for this analysis is a result established rigorously by Gjesdal (1981) in his seminal paper 'Accounting for Stewardship.' Gjesdal proved analytically that the information system optimal for stewardship (monitoring managerial performance to determine compensation or control rights) is not necessarily the same information system optimal for valuation (helping investors predict future cash flows). The core of his result is that these two objectives impose conflicting demands on the statistical properties of the accounting signal.

The Stewardship Demand: Hardness and Downward Bias

From a stewardship perspective, the primary function of accounting is to provide an enforceable measure of managerial performance that can be contracted upon (Watts and Zimmerman, 1986). For debt contracts specifically, the relevant stakeholder is the creditor, who holds a concave claim: receiving a fixed payment in solvency states but bearing full downside risk in default. This asymmetry generates the classic agency conflict of asset substitution (Jensen and Meckling, 1976)—managers acting on behalf of shareholders have an incentive to take excessive risk because equity holders capture the upside while creditors bear the downside.

To mitigate this moral hazard, creditors demand an accounting signal characterized by *con-*

ditional conservatism: the asymmetric recognition of economic gains and losses (Basu, 1997; Watts, 2003). Recent analytical work establishes the micro-foundations for this demand. Gao (2013) demonstrates that because managers possess private information and incentives to inflate reported performance, the optimal ex-ante accounting rule must impose stricter verification requirements for recognizing gains than losses. This asymmetry prevents managers from distributing unrealized gains as dividends or using 'paper profits' to artificially satisfy debt covenants. Similarly, Bertomeu *et al.* (2017) show that as agency frictions intensify, the optimal reporting system becomes increasingly conservative to maintain incentive compatibility.

Importantly, this contracting demand necessitates that the accounting signal be *hard*—meaning verifiable by third parties and based on realized transactions rather than subjective valuations (Holthausen and Watts, 2001). For intangible investments such as R&D or brand development, which lack separable collateral value and exhibit highly skewed payoffs, efficient contracting requires immediate expensing to preserve the integrity of debt covenants. The resulting mandatory signal functions as a 'lower bound' on firm value: it reliably captures downside risk but systematically understates the firm's going-concern value.

The Valuation Demand: Matching and Unbiasedness

In contrast to creditors, equity investors act as residual claimants with a convex payoff structure. Their objective is not maximizing the probability of solvency but rather maximizing the expected present value of future cash flows. This creates a fundamentally different informational demand. Following Ohlson (1995), equity value is a function of current book value and the present value of expected future residual earnings. For accounting earnings to serve as an efficient summary measure in this valuation mapping, they must satisfy the clean surplus relation and, critically, must reflect the *matching principle*: expenses should be recognized in the same period as the revenues they generate.

However, the contracting constraint described above compels a systematic violation of this principle for intangible-intensive firms. When R&D expenditures are expensed immediately under conservative GAAP, despite generating benefits in future periods, a temporal mismatch is introduced that degrades value relevance (Lev and Zarowin, 1999). As documented empirically by Barth *et al.* (2001), this accounting treatment creates a structural wedge between reported performance and future cash flow generation. During high-investment phases, the conservative signal is mechanically depressed, severing the link between current earnings and future value creation.

Moreover, as Gigler *et al.* (2009) demonstrate analytically, the 'false alarms' generated by conservative signals degrade informational efficiency for equity valuation. While useful for creditor monitoring, these false alarms prevent investors from distinguishing economic distress from aggressive investment. Investors cannot simply apply a mechanical adjustment to reverse the GAAP bias because the bias itself embeds proprietary, unobservable information about investment success.

The Tinbergen Principle and the Impossibility Result

The conflict between these two informational demands can be understood through the lens of the *Tinbergen Principle* (Tinbergen, 1952), a foundational concept in the theory of economic policy. The principle asserts a mathematical constraint on optimization: to achieve N independent policy targets, a policymaker requires at least N independent policy instruments. When the number of instruments is less than the number of targets, the system is over-determined, and the policymaker must accept a second-best trade-off.

Applied to accounting standard setting, the Tinbergen Principle formalizes Gjesdal's impossibility result. The standard setter controls a single instrument—the mandatory accounting signal (GAAP earnings)—but faces two distinct targets: optimizing debt contracting efficiency (which requires conservatism) and maximizing equity informativeness (which requires matching). As established by Paul (1992) and Bushman and Indjejikian (1993), the properties that optimize monitoring—hardness, conservatism, asymmetric timeliness—systematically degrade the informativeness of stock prices for equity valuation.

Consider a hypothetical standard setter attempting to choose the recognition threshold $\overline{R}_C$ (the maximum return that GAAP recognizes; returns above this threshold are censored) to minimize the firm's weighted average cost of capital in a GAAP-only regime:

$$\overline{R}_C^* = \arg\min_{\overline{R}_C} \left[w_D \cdot r_L^{GAAP}(\overline{R}_C) + w_E \cdot r_E^{GAAP}(\overline{R}_C) \right]$$

where $r_L^{GAAP}(\overline{R}_C)$ denotes the cost of debt and $r_E^{GAAP}(\overline{R}_C)$ denotes the cost of equity, both as functions of the recognition threshold $\overline{R}_C$.

This optimization yields an inherently second-best solution. Raising $\overline{R}_C$ toward fair value reduces residual uncertainty for equity investors ($\partial r_E^{GAAP}/\partial \overline{R}_C < 0$) but degrades GAAP's utility as a contracting benchmark for creditors ($\partial r_L^{GAAP}/\partial \overline{R}_C > 0$). The weights w_D and w_E vary dramatically across firms, and any interior solution represents an unsatisfying compromise that serves neither constituency optimally. As Holthausen and Watts (2001) emphasize, ignoring contracting needs to focus solely on 'value relevance' destroys efficient contracting, while maintaining strict conservatism creates information gaps that penalize equity holders.

From Dilemma to Solution

The literature reviewed above establishes the *problem*—that a single accounting signal cannot simultaneously optimize stewardship and valuation—but largely treats this as an intractable constraint on standard setting. When voluntary disclosure is permitted, this impossibility dilemma has a natural market-based solution. By allowing firms to supplement the mandatory conservative signal with voluntary non-GAAP adjustments, the economy effectively implements the Tinbergen prescription: two instruments (GAAP and non-GAAP) are deployed to hit two targets (contracting and valuation). The key insight is that the market, not the regulator, determines the equilibrium information environment through firms' endogenous disclosure choices, which are disciplined by the convex pricing of debt.

2.2 Modeling Conservatism: A Evolution of Analytical Approaches

Having established the economic conflict that motivates dual reporting, this section situates the modeling approach within the analytical tradition of conservatism, clarifying how my framework builds on prior work while introducing a distinct modeling choice: treating conservatism as *structural censoring* (truncation of the signal at a verification threshold) rather than as distributional noise or deterministic bias. This distinction is critical for understanding why the results differ from earlier studies, particularly the influential work of Gigler and Hemmer (2001).

Baseline: Feltham and Ohlson (1995) — Unconditional Conservatism in Valuation Dynamics

The modern analytical treatment of conservatism begins with Feltham and Ohlson (1995), who model accounting bias within the Linear Information Dynamics framework. In their specification, conservatism enters as a persistent downward bias in book value: expected book value is systematically less than expected market value because the accounting system expenses investments (like R&D) that generate future growth. This unconditional conservatism is operationalized as a parameter governing the depreciation rate of intangible capital, creating a deterministic wedge between accounting measures and economic value.

My framework extends the Feltham-Ohlson baseline in two directions. First, rather than treating the bias as a deterministic depreciation parameter, it models conservatism as *stochastic*: the conservative bias depends on the realization of the intangible return, which is privately observed by the manager. Second, this stochastic bias creates *local information loss* in the 'gain state' that cannot be mechanically undone by a linear valuation model. While Feltham and Ohlson demonstrate how conservatism affects valuation dynamics in a rational expectations equilibrium, the focus here is on how the *revelation* of this bias through voluntary disclosure affects the firm's cost of capital across both equity and debt markets.

Convexity and Options: Zhang (2000) — Conservatism and Growth Options

A critical refinement of the conservatism literature is provided by Zhang (2000), who was among the first to rigorously link accounting conservatism to option-pricing mechanics. Zhang models conservatism not merely as a static bias but as a property that affects the firm's growth options—specifically, the value of expansion and abandonment decisions. His key insight is that conservative accounting creates *convexity* in the relationship between the bias and the firm's intangible intensity: as the firm scales up risky investments, the variance of the unrecognized component accelerates.

This paper builds directly on Zhang's approach. Following his insight, the conservative bias is modeled as having the exact payoff structure of a *call option* on intangible returns. This convexity property is inherited directly from the option-like structure Zhang identified. However, this result is extended by characterizing how *revealing* the value of this option through non-GAAP disclosure penalizes the firm via the debt market's pricing of volatility. The creditor, who holds a short put position on the firm's assets, responds to revealed tail risk by demanding higher yields, creating a convex cost function that disciplines aggressive reporting. This extension connects Zhang's valuation insight to the contracting friction, closing the loop between conservatism and capital structure.

The Critical Counterpoint: Gigler and Hemmer (2001) — Probabilistic Conservatism

A particularly important paper for positioning this contribution is Gigler and Hemmer (2001), who study conservatism's effect on voluntary disclosure incentives in a setting with probabilistic signals. In their model, conservatism is operationalized as an increase in the likelihood of a ‘bad’ report conditional on a bad state—effectively, conservatism introduces distributional noise that increases ‘false alarms.’ Their central result is that conservatism *reduces* the incentive for voluntary disclosure because it lowers the informativeness of silence (the market becomes less pessimistic about non-disclosers).

Their conclusion diverges from my findings in this paper that conservatism *increases* disclosure incentives. The resolution lies in the distinct modeling approaches, which represent fundamentally different characterizations of how conservatism affects information. Gigler and Hemmer treat conservatism as a parameter shift in the conditional probability distribution: the probability of a low report given a low state exceeds the probability of a high report given a high state. In my framework, conservatism is not distributional noise but rather *structural censoring*: the GAAP signal is truncated at a verification threshold, creating a region (the ‘gain state’) where the mandatory signal becomes locally uninformative about the magnitude of value creation. Under censoring, multiple types are pooled at the truncation point, generating severe residual uncertainty and a large liquidity discount. This pooling *necessitates* non-GAAP disclosure to separate high-productivity firms from the pool, whereas in Gigler and Hemmer’s model, the signal remains informative (albeit noisy) across all states.

The contrast is subtle but economically critical. Probabilistic conservatism (as in Gigler and Hemmer) is a smoothing friction: the signal is less precise but covers the full state space. Structural censoring (as in this study) is a truncation friction: the signal is precise where it reports but silent where it does not. The former reduces the stigma of silence; the latter amplifies it, reversing the disclosure incentives. This distinction is the single most important modeling choice in the framework, as it determines whether conservatism and voluntary disclosure function as substitutes (Gigler and Hemmer) or complements (this study).

The Interaction of Mandatory and Voluntary Signals: Einhorn (2005) and the Lundholm Effect

A foundational paper for understanding dual-reporting environments is Einhorn (2005), who establishes that the value relevance of a voluntary signal depends entirely on its correlation with the mandatory signal. In settings where multiple correlated signals measure the same underlying fundamental, Lundholm (1988) demonstrates that a voluntary disclosure can exhibit an *inverse price-signal relationship*: a high voluntary signal, conditional on an observed mandatory signal, can reveal that the mandatory signal contained a positive error, leading to a downward revision of expectations. Einhorn (2005) formalizes Lundholm's intuition, showing that the coefficient on a voluntary signal can be negative when the correlation between signal errors is sufficiently high.

The Einhorn-Lundholm result might suggest a concern for the current framework: if GAAP and non-GAAP are correlated signals of firm value, could non-GAAP adjustments exhibit negative price coefficients? The concern does not apply to the current setting for a fundamental structural reason. The Lundholm effect requires that the two signals have correlated errors such that learning about one signal's error reveals information about the other signal's error. In the current model, the GAAP measurement error is explicitly assumed to be independent of all fundamental variables and therefore independent of the non-GAAP adjustment.

The independence of measurement errors implies that observing the non-GAAP adjustment provides no information about the GAAP measurement error. The two signals are not correlated estimates of the same object (as in Lundholm's setting) but rather complementary components that sum to total earnings: GAAP captures the recognized component (minus measurement error), while non-GAAP reveals the unrecognized component that was censored by conservative accounting. The additive structure of GAAP and non-GAAP, combined with error independence, ensures a positive coefficient on non-GAAP adjustments, consistent with the empirical evidence in Bradshaw and Sloan (2002) and Leung and Veenman (2018).

Moreover, in the censoring region where non-GAAP disclosure occurs, GAAP earnings become locally uninformative about true economic earnings because they are constant at the recognition threshold (except for measurement error). The informational structure in the censoring region makes the Lundholm inverse effect mathematically impossible: for an inverse relationship to occur, GAAP must be informative about true earnings, but in the censoring region, GAAP provides no information about the magnitude of value creation above the threshold, while non-GAAP directly reveals the censored component. Therefore, non-GAAP adjustments cannot exhibit negative price coefficients in the current framework.

Verification Micro-Foundations: Gao (2013) — Asymmetric Thresholds

The question of *why* GAAP is conservative—rather than simply taking conservatism as an exogenous parameter—is addressed by Gao (2013). Gao provides the micro-foundations for asymmetric recognition by modeling the accounting system as a two-stage verification process.

In his framework, reported transactions must satisfy a verifiability constraint, and conservatism is defined as requiring a higher verification threshold for gains than for losses. Crucially, Gao shows that this asymmetry is the optimal ex-ante response to managers' ex-post manipulation incentives: by forcing stricter verification for 'good news,' the accounting system deters opportunistic inflation while preserving the credibility of 'bad news.'

This micro-foundation justifies the use of the recognition threshold as the primitive that generates conservatism. Consistent with Gao's logic, GAAP is modeled as constrained by an asymmetric verification requirement: economic gains are recognized only if the return exceeds a threshold, whereas losses are recognized immediately. This constraint is what *forces* unverified ('soft') information about intangible productivity into the voluntary non-GAAP channel. In this paper, the recognition threshold is a policy instrument controlled by the standard setter—analogous to Gao's verification threshold—and the central result is that the optimal threshold in a dual-reporting regime differs fundamentally from the optimal threshold in a GAAP-only regime. By permitting voluntary disclosure to handle soft information, the regulator can set the threshold low (maximizing conservatism and contracting value) without sacrificing equity informativeness.

Political Economy Foundations: Bertomeu and Magee (2015) — Why GAAP Is Conservative

While Gao (2013) provides the micro-foundations for *how* conservatism operates (asymmetric verification), Bertomeu and Magee (2015) explain *why* GAAP takes this form through a political economy lens. They model the standard-setting process as a political game where different constituencies (creditors, equity investors, managers) lobby for accounting rules that favor their interests. Their key result is that efficient mandatory disclosure policy focuses on 'bad news' (a lower threshold for recognizing losses) because creditors, who hold concave claims, have stronger incentives to lobby for conservative standards that protect their downside. This provides the structural justification for why GAAP is conservative: it is not an arbitrary design choice but the equilibrium outcome of a political process where debt holders' preferences dominate.

Bertomeu and Magee (2015)'s political economy foundation complements the verification-based micro-foundations of Gao (2013). Together, these papers establish that conservatism is both *technically optimal* (Gao's verification argument) and *politically inevitable* (Bertomeu and

Magee's lobbying argument). This paper takes the politically determined 'bad news' focus of GAAP as a given constraint and solves for the manager's equilibrium response (non-GAAP disclosure) to fill the valuation void. The key insight is that by allowing voluntary disclosure to handle 'good news,' the standard setter can set the mandatory threshold at the corner solution (maximal conservatism) without sacrificing equity informativeness, a result that would be infeasible in a GAAP-only world.

Optimal Conservatism with Manipulation: Bertomeu, Darrough, and Xue (2017)

A recent refinement of the conservatism literature is provided by Bertomeu *et al.* (2017), who model a regulator choosing the optimal degree of conservatism while anticipating that the manager will manipulate the signal. They find an interior optimum: conservatism should be neither maximal (which would eliminate informativeness) nor minimal (which would permit excessive manipulation). Their result balances the costs and benefits of conservatism within a single-signal framework.

The current analysis extends this logic by showing that the introduction of a *second* signal fundamentally alters the optimal conservatism choice. While Bertomeu et al. find an interior solution for the recognition threshold in a GAAP-only regime, when voluntary non-GAAP disclosure is available, the regulator optimally chooses a *corner solution*: maximal conservatism (immediate expensing) for the mandatory signal. The intuition is that with dual reporting, GAAP can specialize entirely in contracting (where conservatism is valuable) while the voluntary channel handles valuation (where matching is valuable). This result provides a normative justification for current accounting standards that require immediate expensing of intangibles, which appear suboptimal from a pure valuation perspective but are efficient when understood as part of a dual-reporting architecture.

Summary: The Modeling Approach

The review above clarifies the position within the analytical tradition. This paper builds on Feltham-Ohlson's treatment of conservatism in valuation dynamics, Zhang's insight linking conservatism to convexity and options, Gao's verification-based micro-foundations, and Bertomeu et al.'s framework for optimal standard setting. The distinct modeling choice is to treat conservatism as *structural censoring* (truncation at a verification threshold) rather than as distributional noise or deterministic bias. This specification, combined with the explicit modeling of debt pricing via Merton's structural credit risk framework, allows the framework to characterize the equilibrium information architecture where GAAP and non-GAAP serve complementary rather than competing roles. The censoring structure is what differentiates these results from Gigler and Hemmer (2001) and generates the prediction that conservatism and non-GAAP disclosure are complements, not substitutes.

2.3 non-GAAP Reporting: From Anomaly to Equilibrium Solution

Having established both the theoretical impossibility facing standard setters (Gjesdal's result) and the analytical tradition for modeling conservatism (the FO-Zhang-Gao lineage), this section reframes the role of non-GAAP reporting. The dominant view in both the regulatory debate and much of the empirical literature treats non-GAAP disclosure as an inherently problematic deviation from standardized reporting—evidence of either opportunistic manipulation or, at best, a necessary evil to correct for GAAP's deficiencies. My analysis challenges this framing by demonstrating that non-GAAP reporting is not an anomaly requiring regulatory suppression but rather the *efficient market response* to the foundational impossibility dilemma established in Section 2.1.

The Received View: Opportunism versus Informativeness

The empirical literature on non-GAAP reporting has largely organized itself around a dichotomy between two competing hypotheses. The *opportunism hypothesis* posits that managers use non-GAAP adjustments to mislead investors by selectively excluding legitimate expenses or inflating performance metrics to meet benchmarks or boost compensation. The *informativeness hypothesis* counters that non-GAAP metrics provide value-relevant information by filtering out transitory items or correcting for accounting mismatches, thereby improving investors' ability to forecast future earnings.

While this debate has generated valuable empirical insights, it implicitly treats non-GAAP disclosure as a deviation from a hypothetical ideal where GAAP alone would suffice. The question posed is: 'Is non-GAAP reporting opportunistic or informative?' This paper suggests a different question is more appropriate: 'Given that GAAP *must* be conservative for contracting purposes (as established in Section 2.1), what is the efficient equilibrium information architecture?' Reframed this way, non-GAAP disclosure is not a deviation from an ideal single-signal system but rather an *endogenous adaptation* that implements the Tinbergen solution to the impossibility dilemma.

Modern Parallel: Bertomeu, Vaysman, and Xue (2021) — Mandatory Disclosure of Bad News

A recent contribution that provides a modern parallel to this paper is **?**, who study the efficiency of mandatory disclosure policy in a setting with both mandatory and voluntary channels. Their key insight is that efficient mandatory policy focuses on 'bad news' (a lower threshold for mandatory disclosure) to prevent excessive voluntary disclosure and reduce unraveling costs. This mirrors the result here where GAAP handles the downside (mandatory, conservative) and non-GAAP handles the upside (voluntary, revealing censored gains), though their focus is on liquidation/unraveling logic rather than debt pricing.

The conceptual parallel is that both frameworks recognize the efficiency of asymmetric mandatory disclosure: mandatory rules should focus on bad news to serve contracting needs, while voluntary disclosure handles good news to serve valuation needs. The key difference is the disciplining mechanism: Bertomeu et al. rely on disclosure costs and unraveling logic to constrain voluntary disclosure, whereas this paper introduces the cost of debt as a real effect that disciplines the voluntary disclosure of good news. This extension is particularly relevant for firms with significant leverage, where the debt channel provides a more immediate and observable constraint than unmodeled disclosure costs.

Existing Analytical Foundations: McClure and Zakolyukina (2024)

Recent analytical work has begun to formalize the efficiency role of non-GAAP reporting, though primarily in equity-only settings. McClure and Zakolyukina (2024) develop a dynamic structural model where managers choose investment and non-GAAP bias to maximize a combination of current stock price and long-term value. Their key insight is that non-GAAP reporting creates a tradeoff: it removes transitory noise from GAAP (improving investment efficiency) but enables opportunistic manipulation (distorting investment). Through structural estimation using Simulated Method of Moments, they demonstrate that non-GAAP adoption increases investment intensity and, under their calibrated parameters, appears to improve firm value relative to a GAAP-only regime.

However, their analysis cannot determine whether non-GAAP is welfare-improving without calibration. As they explicitly acknowledge: 'Although a theoretical model can lay out the trade-offs involved, it cannot quantify the extent to which non-GAAP earnings are value enhancing or destroying' (p. 343). Their structural estimation suggests non-GAAP is beneficial under their calibrated parameters, but this finding is sensitive to unobservable quantities like the manager's personal cost of manipulation and degree of myopia. This limitation creates ambiguity about the cross-sectional heterogeneity in welfare effects: which firms benefit from non-GAAP, and which are harmed?

This paper addresses this ambiguity by introducing an observable friction—the firm's capital structure—that generates parameter-free predictions about when non-GAAP improves welfare. The differences between the approaches extend beyond the debt channel to fundamental modeling choices.

The Conservatism Modeling Distinction A critical difference concerns how conservatism is operationalized. McClure and Zakolyukina model GAAP earnings as containing a transitory shock with systematic negative bias. This specification treats conservatism as *additive noise with non-zero mean*—effectively, measurement error that is known to investors (since the bias parameter is part of their rational expectations equilibrium). In a rational expectations equilibrium, if investors know the systematic bias, they can mechanically correct it, and the stock price should reflect this adjustment.

In contrast, this study models conservatism as *structural censoring*: GAAP earnings are a truncated version of economic earnings, where gains above a verification threshold are not recognized. This creates genuine information loss because the unrecognized component depends on the realization of the intangible return, which only the manager observes. Even if investors know the recognition rule (the threshold), they cannot recover the censored gains without the manager's private signal. This creates persistent information asymmetry that survives rational expectations, justifying the need for non-GAAP disclosure. While McClure and Zakolyukina model conservatism as a known bias parameter, this study models it as structural censoring dependent on the state realization. This distinction allows the current paper to preserve information asymmetry even under rational expectations.

The Investment Channel Distinction A second fundamental difference concerns the mechanism through which non-GAAP affects investment. In McClure and Zakolyukina's model, non-GAAP influences investment through *managerial myopia*: the manager's utility includes a weighted stock price term, creating an incentive to boost current price even though it's based on imperfect information. This generates overinvestment when non-GAAP allows the manager to inflate earnings while offsetting the price-depressing effect of high investment.

This study sidesteps the contracting puzzle by focusing on the *cost of capital channel* rather than behavioral distortions. non-GAAP disclosure affects investment not through managerial myopia but through its real effect on financing costs: disclosure reduces information asymmetry, lowering the cost of equity, which makes investment objectively more attractive regardless of managerial incentives. To the extent managers care about stock price (through equity compensation), this arises from optimal contracting to align effort incentives, not from an unexplained behavioral bias. The investment response is therefore robust to governance improvements.

The Debt Channel and Welfare Predictions The key innovation is introducing the *debt contracting channel*, which McClure and Zakolyukina abstract from entirely. By modeling the cost of debt as convex in the magnitude of non-GAAP adjustments, creditors penalize the revelation of tail risk through higher yields. This creates a leverage-dependent tradeoff: low-leverage firms benefit from disclosure (equity effect dominates), while high-leverage firms are harmed (debt effect dominates).

This extension generates *signed welfare predictions* that do not require calibration. Specifically, non-GAAP disclosure is welfare-improving if and only if firm leverage is below a critical threshold. This threshold depends only on observables (leverage, intangible intensity, GAAP informativeness) rather than unobservable parameters (cost of manipulation, degree of myopia). This resolves the ambiguity in McClure and Zakolyukina's framework: rather than 'non-GAAP might be good or bad depending on unobservables,' the conclusion is 'non-GAAP is welfare-improving for firms below the threshold and should be permitted; for firms above the threshold it may destroy value and warrants scrutiny.'

The two papers are *complementary* rather than competing. McClure and Zakolyukina characterize the dynamic tradeoff between information and manipulation within a single firm over time, using structural estimation to quantify magnitudes. This study characterizes the cross-sectional tradeoff between equity information benefits and debt contracting costs across firms with different capital structures, using analytical methods to derive parameter-free conditions. Together, the papers suggest that non-GAAP reporting's welfare effects depend on both time-series factors (persistence of shocks, adjustment costs) and cross-sectional factors (leverage, intangible intensity).

Signal Specialization and the Tinbergen Solution

The central insight of this analysis is that dual reporting allows for *signal specialization* in a way that resolves the Tinbergen impossibility. Rather than forcing GAAP to serve two masters—an optimization that Gjesdal proved is generically infeasible—the economy deploys two instruments. First, GAAP specializes in providing a conservative, verifiable lower bound on firm value. By setting the recognition threshold low (aggressive conservatism), the standard setter ensures that GAAP reliably captures downside risk and serves as a 'hard' trigger for debt covenants. This satisfies the creditor's primary informational demand without requiring the signal to also provide unbiased forecasts of future cash flows. Second, non-GAAP specializes in revealing soft information about intangible productivity. By voluntarily disclosing an adjustment, the manager communicates private information about the censored component of value creation, allowing equity investors to form refined valuations that incorporate growth option value.

Critically, the market—not the regulator—determines which firms engage in dual reporting through an endogenous sorting mechanism. Firms with low leverage and high intangible intensity optimally disclose non-GAAP metrics because the equity valuation benefit (reduced liquidity discount) exceeds the debt cost penalty (increased yield). Firms with high leverage or low intangibles optimally remain silent, relying on the coarseness of GAAP to minimize creditor-perceived volatility. This heterogeneity is efficient: it allows the informational architecture to adapt to firm-specific trade-offs without requiring the standard setter to implement firm-specific rules.

Disciplining the Voluntary Channel: The Role of Creditors

A potential critique of this signal specialization story is that it appears to give managers unconstrained discretion over the 'soft' signal (non-GAAP), raising concerns about opportunistic manipulation. I directly addresses this concern by characterizing the endogenous disciplining mechanism provided by creditors. Because creditors hold a concave claim and are particularly sensitive to tail risk, they rationally price the *magnitude* of the non-GAAP adjustment, not merely its mean. Specifically, the cost of debt is strictly increasing and convex in the absolute size of the adjustment. Such convexity creates an accelerating penalty for aggressive reporting: small adjustments have minimal impact on yield, but large deviations from GAAP trigger disproportionate increases in borrowing costs.

Importantly, this disciplining mechanism operates *without* requiring GAAP-level verification of the non-GAAP signal. While the adjustment is not contractible (consistent with the 'soft information' constraint emphasized by Hart and Moore (1998)), creditors incorporate it into pricing decisions, creating a pecuniary penalty that the manager internalizes. Empirically, this channel aligns with Christensen *et al.* (2019), who document that following debt covenant violations—periods of heightened creditor scrutiny—firms systematically reduce the aggressiveness of non-GAAP reporting and improve the quality of reconciliations. This dynamic supports the theoretical prediction that creditor discipline provides a market-based brake on manipulation that substitutes for regulatory restrictions.

This mechanism contrasts with alternative disciplining approaches in the literature. Beyer and Dye (2021) examine how leverage impacts disclosure when managers are compensated on enterprise value, but they focus on a 'reputation' mechanism where managers disclose to build credibility over time. In contrast, this paper emphasizes a 'real effect' mechanism where debt repricing creates an immediate, pecuniary penalty that the manager internalizes through equity ownership. The debt pricing channel provides a harder, more immediate disciplining mechanism than reputational effects, as it operates through observable capital market prices rather than unmodeled future reputation concerns. This distinction is particularly relevant for understanding why highly levered firms engage in more conservative non-GAAP reporting: the debt channel provides a binding constraint that reputation alone cannot replicate.

Empirical Manifestations: Loss Reversals and Audience-Specific Reporting

The dual-reporting architecture modeled here generates empirical predictions that align with documented patterns in non-GAAP disclosure. First, consider the 'loss reversal' phenomenon, where firms report GAAP losses but non-GAAP profits. Rather than interpreting this as prima facie evidence of opportunism, this paper suggests it is a structural inevitability for intangible-intensive firms. When conservative accounting forces immediate expensing of R&D or brand investments, a firm generating positive economic returns on these outlays will mechanically report a GAAP loss. The non-GAAP adjustment serves to reverse this accounting treatment, aligning reported income with economic performance. Consistent with this interpretation, Leung and Veenman (2018) provide compelling evidence that for loss-making firms, non-GAAP exclusions are highly predictive of future operating performance and are valued by investors as informative signals of core operations.

Second, this paper rationalizes the empirical regularity documented by Black *et al.* (2022) that firms systematically vary non-GAAP definitions across audiences. Specifically, firms report different metrics in earnings announcements (targeted at equity investors) than in proxy statements (targeted at compensation committees and, indirectly, creditors). This behavior is precisely what the model predicts: the firm is not choosing between truth-telling and deception but rather engaging in *signal specialization*, tailoring the disclosure to the informational demands of each constituency. The earnings announcement emphasizes valuation-relevant adjustments (adding back intangible investments), while the proxy statement emphasizes performance metrics aligned with verifiable contracting benchmarks.

In the sections that follow, these insights are formalized within a parsimonious model that strips the problem down to its economic core. By deliberately abstracting away from other frictions—such as proprietary costs of disclosure or heterogeneity in investor sophistication—the analysis isolates the specific mechanism by which conservatism and voluntary disclosure interact to implement the Tinbergen solution to the standard setter's impossibility dilemma.

3 GAAP and Non-GAAP Earnings

3.1 Setting up the Model

I consider a single-period economy ($t = 0, t$) featuring a representative firm, a manager, and a competitive market of equity investors. To focus the analysis on the information frictions driving disclosure choice, I normalize the risk-free rate to zero.

The sequence of events, depicted in Figure 1, highlights the strategic interaction between corporate reporting and capital pricing. At $t = 0$, the firm invests capital to fund a portfolio of tangible and intangible assets. Nature determines the productivity of these assets, which is observed privately by the manager. The accounting system generates a mandatory GAAP report. Subsequently, the manager decides whether to voluntarily disclose a supplemental Non-GAAP signal. Finally, at $t = t$, the terminal value is realized, and all claims are settled.

I formalize the firm's asset structure and the stochastic nature of its payoffs as follows.[2]

[2]See Appendix A for the complete list of model symbols and definitions. All mathematical results in this paper have been formally verified using the Lean 4 proof assistant. The online appendix (`https://github.`

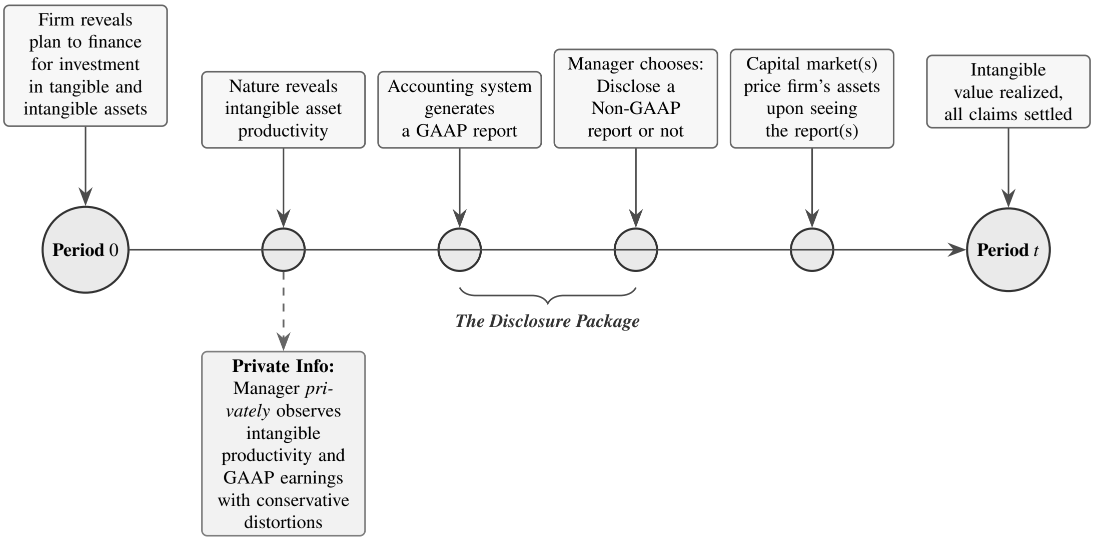

Figure 1: Timeline

Note: This figure depicts the model's timeline. At $t = 0$, the firm plants investments. Nature reveals intangible productivity, observed privately by the manager along with accounting distortions. A mandatory GAAP report is generated, followed by the manager's choice whether to disclose a supplemental Non-GAAP signal. Markets then price the firm based on the disclosure package. At $t = T$, firm value is realized, regulatory penalties assessed, and all claims settled.

Assumption 1 (Asset Structure and Payoffs)**.** *The firm's terminal liquidating value, $\widetilde{T}_t$, is the sum of the initial investment and the net economic earnings generated by intangible assets:*

$$\widetilde{T}_t = K + I_0 + \widetilde{e}_t \tag{3.1}$$

where $\widetilde{e}_t = I_0 \cdot \widetilde{R}_I$ represents the stochastic net earnings. The variable $\widetilde{R}_I$ is the net return on intangible investment, composed of a fundamental productivity component and idiosyncratic noise:

$$\tilde{R}_I = \tilde{\theta} + \tilde{\nu}, \quad \textit{where} \quad \tilde{\theta} \sim N(\mu_\theta, \sigma_\theta^2) \quad \textit{and} \quad \tilde{\nu} \sim N(0, \sigma_\nu^2) \tag{3.2}$$

This structure captures the essential economics of modern firms. The term $\widetilde{e}_t$ represents the economic profit or loss relative to the initial capital base. When $\widetilde{e}_t > 0$, the intangible assets generate a return exceeding the cost of investment. Crucially, I_0 acts as a scaling factor for risk. The variance of the intangible payoff is $\sigma_I^2 = I_0^2(\sigma_\theta^2 + \sigma_\nu^2)$, implying that total fundamental volatility is strictly increasing in the initial intangible investment I_0. This formulation allows me to define *Intangible Intensity* directly via the magnitude of I_0, enabling analytical comparisons between mature, capital-light firms (low I_0) and high-growth, innovation-driven firms (high I_0).

The core information friction arises from the asymmetry regarding $\tilde{\theta}$. While the market

`com/rngpaper/onlineappendix`) contains a condensed mathematical summary and complete formal proofs for Sections 3–5, including Propositions 1–6, Lemmas 3.1–4.2, and all corollaries.

knows only the prior distribution, the manager privately observes the realization of $\tilde{\theta}$ at $t = 0$. The residual term $\tilde{v}$ represents outcome uncertainty—such as macroeconomic shocks—that remains unknown to all parties until the terminal date.

3.2 The Informativeness of GAAP Earnings

The accounting system generates a mandatory public report, y_G. To capture the economic impact of accounting standards, Following Basu (1997); Watts (2003), I model this signal as conditional conservatism applied to the firm's net economic earnings, which is consistent with typical accounting practices such as the *Lower of Cost or Market* principle: losses are recognized immediately, while gains are deferred until realized.[3]

Assumption 2 (GAAP Reporting and Conservative Bias)**.** *GAAP earnings, y_G, are a censored version of the net economic earnings $\widetilde{e}_t = I_0 \cdot \widetilde{R}_I$. Let $\overline{R}_C$ denote the verification threshold for recognizing gains. The GAAP signal is defined as:*

$$y_G = I_0 \min(\widetilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}, \quad \textit{where} \quad \tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2) \tag{3.3}$$

The Conservative Bias, $\tilde{g}$, is defined as the economic earnings unrecognized by the accounting system:

$$\tilde{g} \equiv \tilde{e}_t - (y_G - \tilde{\varepsilon}) = I_0 \max(\widetilde{R}_I - \overline{R}_C, 0) \tag{3.4}$$

This formulation reveals that the conservative accounting bias has the exact payoff structure of a *Call Option* (the unrecognized upside) on the intangible return, which follows the analytical frameworks of Zhang (2000) who similarly model conservative accounting as providing a 'floor' value while treating the upside potential as an option-like claim. In 'bad states' where

[3]My approach differs fundamentally from McClure and Zakolyukina (2024), who model conservatism as GAAP earnings containing transitory noise with a systematic *negative mean*. In their specification, investors do not observe the specific realization of this noise in any given period, which creates a logical tension: in their rational expectations equilibrium, investors should be able to learn the systematic bias from repeated observations and mechanically correct for it. Once the average bias is learned, investors should be able to mechanically adjust their expectations upward, eliminating the information asymmetry that motivates Non-GAAP disclosure in their model.

In contrast, I model this conservative signal not as an arbitrary noise process, but a censored version of the net economic earnings, which creates *structural information loss* that cannot be mechanically undone. The unrecognized component $\tilde{g} = I_0 \max(\widetilde{R}_I - \overline{R}_C, 0)$ depends on the realization of $\widetilde{R}_I$, which only the manager has private information about. Even if investors know the recognition rule (threshold $\overline{R}_C$), they cannot recover the censored gains without the manager's private signal. This creates genuine information asymmetry that persists under rational expectations. This modeling choice is consistent with rational expectations equilibrium (investors cannot mechanically undo structural censoring) and allows for deriving the *optimal degree of conservatism* (the threshold $\overline{R}_C$) as a policy choice that can be optimized by standard-setters, whereas modeling conservatism as a fixed bias parameter precludes such normative analysis.

returns fall below the threshold $\overline{R}_C$, the bias is zero because GAAP fully recognizes the economic loss. In 'good states,' the bias is positive and increasing, representing the *unrecognized upside* filtered out by the verifiability constraint.[4]

Because GAAP writes down losses but censors gains, the unrecognized component $\tilde{g}$ functions like a call option on intangible returns. As shown in the Appendix (Corollary B.1), the variance of this bias, $\sigma_g^2(I_0) = I_0^2 \cdot V(\overline{R}_C)$, is strictly increasing and strictly convex in the firm's intangible intensity I_0. This convexity implies that for high-growth firms, the volatility of the accounting distortion accelerates dramatically, progressively degrading the quality of the mandatory report as intangible investment scales up.

Market Valuation with GAAP Earnings Only

Consider a benchmark scenario where the only public signal is the mandatory GAAP report ($\Omega = \{y_G\}$). The market's valuation problem is to infer the true economic earnings from the censored signal.

Proposition 1 (Market's Non-GAAP Adjustment). *Under conditional conservatism, the market's rational valuation attempts to reverse the accounting bias. The firm's valuation is given by the reported earnings plus a rational 'add-back' of the expected conservative bias:*

$$V(y_G) = E[\tilde{e} \mid y_G] = y_G + \Pr(\tilde{R}_I \geq 0 \mid y_G) \cdot I_0 \mu_+ \tag{3.5}$$

where $\mu_+ = \mu_r + \sigma_r \frac{\phi(\eta)}{\Phi(\eta)}$ *is the expected return conditional on a gain,* $\eta = \mu_r / \sigma_r$ *is the signal-to-noise ratio, and* $\Pr(\tilde{R}_I \geq 0 \mid y_G)$ *is the posterior probability that the firm is in a* Gain State *given the observed report.*

Proof. See Appendix B.2 □

This result highlights the structural limitation of the GAAP signal. While the market rationally adds back the *expected* bias, it cannot observe the *realized* bias. In Gain States, earnings are censored, rendering the signal locally uninformative about the magnitude of value creation. This leads to significant residual uncertainty.

[4]This approach captures the full spectrum of conditional conservatism: The threshold $\overline{R}_C$ can be interpreted as the degree of verifiability required for the recognition of economic earnings (Watts, 2003). Returns below the threshold (losses) are fully recognized, while returns above the threshold (gains) are censored at $\overline{R}_C$. (1) if $\overline{R}_C = -1$, even the recovery of the initial intangible investment is treated as an unrecognized gain until realization. (2) if $\overline{R}_C = 0$, the standard impairment regime where any negative return is recognized but all positive returns are censored. (3) if $-1 < \overline{R}_C < 0$, meaning only severe losses are recognized as impairments, while moderate losses are censored. (4) if $\overline{R}_C > 0$, allowing for the recognition of some unrealized gains.

A direct implication of Proposition 1 concerns the market's response to GAAP earnings. Because conditional conservatism compresses earnings variance by censoring gains, the market must rationally amplify the GAAP signal to recover the variance of true economic earnings. This creates *signal amplification*: the GAAP Earnings Response Coefficient exceeds unity ($\beta_G > 1$), as the market scales up the reported earnings to compensate for the compression. However, this amplification mechanism creates a vulnerability—simultaneously amplifying the measurement error $\tilde{\varepsilon}$. Consequently, for high-intangible firms, the mandatory report becomes an increasingly noisy basis for valuation. The formal derivation of the GAAP-ERC and its properties are provided in Appendix B.

However, this amplification mechanism creates a vulnerability: scaling up the signal ($\beta_G > 1$) simultaneously amplifies the measurement error $\tilde{\varepsilon}$. Consequently, for high-intangible firms, the mandatory report becomes an increasingly noisy basis for valuation. In other words, a conservative GAAP earnings signal cannot fully reveal true earnings. Formally, this insight is described by the following corollary:

Corollary 1 (Residual Uncertainty under GAAP)**.** *Let* $\Sigma_{ND} \equiv \mathrm{Var}(\tilde{e} \mid y_G)$ *denote the residual uncertainty given GAAP earnings. Due to the censoring of gains, the residual uncertainty is strictly positive and increasing in the firm's expected return:*

$$\Sigma_{ND} = \mathrm{Var}(\tilde{e}) - \mathrm{Var}(E[\tilde{e} \mid y_G]) > 0 \tag{3.6}$$

Proof. See Appendix B.2 □

This residual uncertainty Σ_{ND} establishes the 'Demand Side' for Non-GAAP reporting. For high-growth firms, the probability of censoring is high, forcing investors to rely on imprecise priors. This results in a noisy valuation, creating a strategic imperative for the manager to provide a voluntary disclosure to reveal their private knowledge regarding the conservative bias.

3.3 The Non-GAAP Report as a Managerial Response

Faced with a censored GAAP signal, the manager can voluntarily issue a Non-GAAP metric to provide a more relevant signal of value. I model this disclosure as a discretionary adjustment, $\mathscr{A}$, applied to the mandatory report:

$$y_{NG} = y_G + \mathscr{A} \tag{3.7}$$

I assume the manager observes the information set $\mathscr{I}_M = \{\tilde{\theta}, y_G\}$. While the manager has an informational advantage, they do not possess perfect foresight regarding the terminal noise $\tilde{v}$. Consequently, the manager forms a private expectation of the unrecognized bias, $\hat{g}_M \equiv E[\tilde{g} \mid \mathscr{I}_M]$, which serves as the basis for the adjustment.

Because the underlying primitives are normally distributed, the manager's private estimate of the bias behaves predictably. Under the distributional assumptions above, the manager's private expectation $\hat{g}_M \equiv E[\tilde{g} \mid \mathscr{I}_M = \{\tilde{\theta}, y_G\}]$ satisfies standard regularity conditions: its support forms a connected interval $[\underline{g}, \bar{g}]$ with $\underline{g} \geq 0$, and its probability density function is log-concave, ensuring a non-decreasing hazard rate. These properties, formalized in Appendix B (Lemma B.1), guarantee the existence and uniqueness of the disclosure equilibrium.

3.3.1 Managerial Objectives and Reporting Incentives

The manager selects the adjustment $\mathscr{A}$ to maximize a utility function that reflects the agency conflicts inherent in financial reporting. Unlike a sole proprietor, a manager in an IPO setting retains only a fraction of the equity and faces distinct career concerns.

Assumption 3 (Manager's Utility Function)**.** *The manager chooses $\mathscr{A}$ to maximize U_M, conditional on their private information set $\mathscr{I}_M$:*

$$U_M(\mathscr{A} \mid \mathscr{I}_M) = \phi_1 P(\Omega) + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2 \tag{3.8}$$

where $\phi_1 \in [0,1]$, $\phi_2 \geq 0$, and $\psi_P \geq 0$.

This specification captures the tension between valuation, private benefits, and governance. The term $\phi_1 P(\Omega)$ aligns the manager with shareholders. The term ϕ_2 captures the 'hype' incentive, the manager's direct marginal benefit for every dollar of overstatement relative to their private estimate. These are counterbalanced by the third term, where ψ_P governs the severity of the penalty for falsification. I interpret ψ_P as the cost of regulatory enforcement or audit committee scrutiny that the manager incurs when the reported adjustment deviates from the latent truth $\hat{g}_M$.[5]

[5]This implies that the enforcing body has the same information acess as the manager, which is the case for board of directors or audit committees.

3.3.2 Equilibrium Market Pricing

I assume the capital market is competitive but imperfectly liquid. Following the standard microstructure framework of Diamond and Verrecchia (1991), I model the equilibrium share price as the expected terminal value conditional on public information, discounted by the residual risk borne by investors.

Assumption 4 (Equity Market Pricing with Liquidity Discount)**.** *Let* Ω *denote the public information set available at time* t*. The market prices the firm based on expected terminal value minus a liquidity discount for residual uncertainty:*

$$P(\Omega) = (K + I_0) + E[\tilde{e} \mid \Omega] - \lambda \cdot \mathrm{Var}(\tilde{e} \mid \Omega) \tag{3.9}$$

where $\lambda > 0$ *represents the illiquidity parameter or the aggregate risk aversion of market makers.*

The manager's disclosure strategy determines the content of Ω, creating two distinct pricing regimes.

No Disclosure Regime In the No Disclosure regime ($\Omega = \{y_G\}$), investors rely on imprecise priors to correct the GAAP bias. The price P^{ND} is discounted by the substantial residual uncertainty Σ_{ND}:

$$P^{ND} = (K + I_0) + (y_G + E[\tilde{g} \mid y_G]) - \lambda \Sigma_{ND} \tag{3.10}$$

where $\Sigma_{ND} \equiv Var(\tilde{e} \mid y_G)$ denotes the posterior variance when investors rely solely on the mandatory report. For intangible-intensive firms, Σ_{ND} is substantial because the conservative censoring of upside potential degrades the precision of the signal.

Disclosure Regime When the manager voluntarily discloses Non-GAAP earnings, the market gains access to a second signal ($\Omega = \{y_G, \mathscr{A}\}$). The manager reports $\mathscr{A} = \hat{g}_M + B^*$, where $\hat{g}_M = E[\tilde{g} \mid \tilde{\theta}, y_G]$ is the manager's private expectation and B^* is the equilibrium bias. Investors rationally filter out the equilibrium bias B^* and combine the corrected Non-GAAP signal with the GAAP report.

$$\hat{g}_M = E[\tilde{g} \mid y_G, \mathscr{A}] = \mathscr{A} - B^* \tag{3.11}$$

The equilibrium price under disclosure is therefore:

$$P^D = (K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda \Sigma_D \tag{3.12}$$

where $\Sigma_D \equiv Var(\tilde{e} \mid y_G, \mathscr{A})$ represents the reduced posterior variance under disclosure.

The Non-GAAP adjustment $\mathscr{A} = \hat{g}_M + B^*$ reveals the manager's private expectation $\hat{g}_M = E[\tilde{g} \mid \tilde{\theta}, y_G]$, which incorporates the private signal $\tilde{\theta}$. By standard properties of Bayesian updating, this additional signal strictly reduces posterior variance: learning $\mathscr{A}$ is equivalent to learning $\tilde{\theta}$, which resolves part of the uncertainty embedded in Σ_{ND}. By the law of total variance, the posterior variance under disclosure satisfies $\Sigma_D < \Sigma_{ND}$, with the gap reflecting the informativeness of the manager's private signal (see Appendix B.1, for the formal proof).

The Pricing Benefit of Disclosure

Define the uncertainty reduction ratio:

$$\omega \equiv \frac{\Sigma_D}{\Sigma_{ND}} \in (0,1) \tag{3.13}$$

The term $(1-\omega)$ represents the fraction of uncertainty resolved by the manager's voluntary disclosure.

Corollary 2 (Pricing Benefit of Disclosure)**.** *The price difference between disclosure and non-disclosure decomposes as:*

$$P^D - P^{ND} = (\hat{g}_M - \bar{g}^{ND}) + \Delta_{\text{Liquidity}} \tag{3.14}$$

where $\bar{g}^{ND} \equiv E[\tilde{g} \mid y_G, \textit{No Disclosure}]$ is the market's expectation of the conservative bias conditional on the manager remaining silent, and the Liquidity Benefit *is defined as:*

$$\Delta_{\text{Liquidity}} \equiv \lambda(\Sigma_{ND} - \Sigma_D) = \lambda(1-\omega)\Sigma_{ND} > 0 \tag{3.15}$$

Disclosure increases the stock price ($P^D > P^{ND}$) if and only if:

$$\hat{g}_M > \bar{g}^{ND} - \Delta_{\text{Liquidity}} \tag{3.16}$$

That is, when the manager's private signal exceeds the market's expectation under silence, adjusted for the liquidity benefit. The liquidity effect creates a structural bias favoring disclosure: even moderately pessimistic signals can increase price through uncertainty reduction.

Proof. See Appendix B.2. □

The first term, $(\hat{g}_M - \bar{g}^{ND})$, captures the *information effect*: the price adjustment from updating beliefs about the true bias. This term can be positive (good news: $\hat{g}_M > \bar{g}^{ND}$), negative (bad news: $\hat{g}_M < \bar{g}^{ND}$), or zero (news confirms prior). The second term, $\Delta_{\text{Liquidity}}$, captures the *liquidity effect*: disclosure reduces uncertainty, shrinking the liquidity discount by

$\lambda(\Sigma_{ND} - \Sigma_D) = \lambda(1-\omega)\Sigma_{ND} > 0$. This term is always positive.

Critically, even when disclosure conveys bad news ($\hat{g}_M < \bar{g}^{ND}$), the stock price can still increase if the liquidity benefit is sufficiently large. This creates a structural bias favoring disclosure: managers can increase price through transparency alone, even with moderately pessimistic signals. The liquidity benefit is increasing in the illiquidity parameter λ, the baseline uncertainty Σ_{ND}, and the informativeness of the manager's signal (which determines $1-\omega$). For intangible-intensive firms where Σ_{ND} is large, this benefit provides a powerful incentive for voluntary disclosure.

Price Effects vs. Managerial Incentives However, the condition for price increase ($P^D > P^{ND}$) is *not* the same as the manager's disclosure decision. Recall managers utility function in Equation (3.8), while the manager internalizes the price effect through their equity stake (ϕ_1), they also face personal costs and benefits from disclosure—including private benefits from 'hype' (ϕ_2) and penalties for aggressive reporting (ψ_P). These additional frictions create a wedge between the price-maximizing disclosure strategy and the manager's equilibrium choice. The next subsection characterizes the manager's optimal disclosure threshold, accounting for these personal incentives.

3.3.3 Manager's Decision for Non-GAAP Disclosure

I now characterize the manager's decision to voluntarily disclose the Non-GAAP signal. The manager faces a standard trade-off: disclosure reduces the firm's cost of capital by resolving uncertainty, but it requires the manager to incur the personal and regulatory costs associated with issuing a non-standardized report.

Intuitively, the manager will only voluntarily disclose when the 'unrecognized upside' censored by GAAP is sufficiently large. If the manager's private estimate of the conservative bias, $\hat{g}_M$, is low, the marginal benefit of boosting the stock price is outweighed by the scrutiny and potential penalties of the disclosure. Consequently, the optimal strategy must take the form of a threshold rule.

Lemma 3.1 (Optimal Disclosure Threshold)**.** *The manager discloses Non-GAAP earnings if and only if their private expectation of the conservative bias exceeds a critical threshold:*

$$\text{Disclose} \iff \hat{g}_M \geq g^* = \bar{g}^{ND} + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}} \tag{3.17}$$

where $\bar{g}^{ND} \equiv E[\tilde{g} \mid y_G, \text{No Disclosure}]$ *is the market's expectation of the bias conditional on silence. The threshold is determined by two opposing wedges:*

$$\Delta_{\text{Personal}} = \frac{(\phi_1 + \phi_2)(\phi_1 - \phi_2)}{2\phi_1 \psi_P}; \tag{3.18}$$

$$\Delta_{\text{Liquidity}} = \lambda(\Sigma_{ND} - \Sigma_D) \tag{3.19}$$

Proof. See Appendix B.2 □

The threshold g^* encapsulates the tension between governance and valuation. The term Δ_{Personal} represents the monitoring effect by governance mechanisms such as the audit committee. This term represents the net disutility of reporting, scaled by the severity of the penalty ψ_P. It raises the bar for disclosure. The manager remains silent unless the underlying economic reality is sufficiently positive to justify the personal risk of 'sticking their neck out.'

Counteracting this is the $\Delta_{\text{Liquidity}}$ term (defined in Corollary 2). Because the manager holds an equity stake ϕ_1, they internalize the market's valuation reward for reduced uncertainty. This benefit lowers the threshold. Thus, the model predicts that managers of firms with opaque GAAP earnings (high $\Delta_{\text{Liquidity}}$) will be more willing to disclose Non-GAAP metrics, even when the news is only marginally positive, effectively "buying" a higher stock price with increased transparency.

3.3.4 The Optimal Reporting Strategy

Conditional on the decision to disclose, the manager must determine the magnitude of the Non-GAAP adjustment, $\mathscr{A}$. This choice involves a trade-off: inflating the adjustment increases the stock price and yields private benefits, but simultaneously increases the expected penalty for falsification.

I solve for the optimal reporting strategy by substituting the pricing function $P^D = (K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda\Sigma_D$ (Equation 3.12) into the manager's utility. The manager chooses $\mathscr{A}$ to maximize:

$$U_M^D(\mathscr{A}) = \phi_1\left[(K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda\Sigma_D\right] + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2 \tag{3.20}$$

The intuition for the solution is best understood through the first-order condition, which equates the marginal benefit of inflation to the marginal cost.

$$\text{FOC:} \quad \frac{\partial U_M^D}{\partial \mathscr{A}} = \phi_1 + \phi_2 - \psi_P(\mathscr{A} - \hat{g}_M) = 0 \tag{3.21}$$

The marginal benefit consists of two components: the sensitivity of the stock price to the disclosure, weighted by the manager's equity stake (ϕ_1), and the direct private benefit of 'hype' (ϕ_2). The marginal cost is the increasing marginal penalty of regulatory scrutiny ($\psi_P(\mathscr{A} - \hat{g}_M)$).

Solving reveals that the optimal report $\mathscr{A}^*$ equals the private expectation $\hat{g}_M$ plus a systematic bias.

Lemma 3.2 (Equilibrium Reporting Bias)**.** *In any disclosure equilibrium, the manager systematically overstates the Non-GAAP adjustment. The optimal bias, $B^* \equiv \mathscr{A}^* - \hat{g}_M$, is:*

$$B^* = \frac{\phi_1 + \phi_2}{\psi_P} \tag{3.22}$$

Proof. See Appendix B.2 □

The bias is determined by the ratio of marginal benefits to marginal costs. The numerator $\phi_1 + \phi_2$ captures the manager's total incentive to inflate: ϕ_1 reflects the price benefit from a higher reported adjustment, while ϕ_2 captures private benefits such as reputation or career advancement. The denominator ψ_P reflects the personal cost of falsification—regulatory scrutiny, litigation risk, and reputational damage. A stricter enforcement regime (higher ψ_P) directly suppresses manipulation.

It is important to emphasize that in a rational expectations equilibrium, this bias does not deceive the market. Investors anticipate the manager's incentives and rationally filter out B^* when pricing the firm. Consequently, the bias B^* represents a deadweight loss: the manager is forced to incur the 'deadweight cost of lying' (risk of penalty) merely to meet the market's expectations, without successfully altering the equilibrium price on average.

3.3.5 Equilibrium Characterization

I now assemble the partial equilibrium results derived in the previous lemmas to characterize the general equilibrium. The following proposition establishes that the interplay between the manager's incentive to influence the stock price and the market's demand for liquidity results in a unique, stable disclosure outcome.

Proposition 2 (Equilibrium Existence and Uniqueness)**.** *Under the model primitives (Assumptions 1-4), there exists a unique Bayesian disclosure equilibrium characterized by:*

1. Disclosure threshold (from Lemma 3.1): *The manager discloses if and only if the private expectation of the bias exceeds a critical cutoff, $\hat{g}_M \geq g^*$, where:*

$$g^* = \bar{g}^{ND} + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}} \tag{3.23}$$

2. Equilibrium bias (from Lemma 3.2): *Conditional on disclosure, the reported adjustment is $\mathscr{A}^* = \hat{g}_M + B^*$, where the systematic bias is:*

$$B^* = \frac{\phi_1 + \phi_2}{\psi_P} \tag{3.24}$$

3. Market pricing: *The market rationally prices the firm based on the available information set (From Equations 3.10 and 3.12):*

$$P^{ND} = (K + I_0) + y_G + E[\tilde{g} \mid y_G, \text{ND}] - \lambda \Sigma_{ND}$$
$$P^{D} = (K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda \Sigma_D$$

Proof. See Appendix B.2. The proof establishes existence via the Intermediate Value Theorem and uniqueness via the log-concavity property of Lemma B.1 □

With the equilibrium established, I next examine its implications for the firm's cost of capital. I define the cost of equity, $r_E(\Omega) = \lambda \cdot \text{Var}(\tilde{e} \mid \Omega)/P(\Omega)$, as the required return per unit of price, driven by the residual risk investors must bear.

A direct implication of the equilibrium pricing is that Non-GAAP disclosure strictly reduces the cost of equity. The reduction in the cost of equity is $\Delta r_E = r_E^{ND} - r_E^{D} = \lambda(1-\omega)\Sigma_{ND}/P > 0$, where $(1-\omega)$ captures the fraction of uncertainty resolved by disclosure. This highlights the mechanism by which disclosure creates value: it is not merely the transmission of good news, but the resolution of uncertainty that lowers the discount rate. However, this reduction is contingent upon the credibility of the signal. As the penalty parameter $\psi_P \to 0$, the equilibrium bias $B^* \to \infty$, causing the market to rationally disregard the signal. In this limit, the disclosure provides no reduction in uncertainty or cost of capital, underscoring the necessity of external disciplining mechanisms.

Remark 3.1 (Personal Costs as Credibility Mechanism)**.** *It is important to emphasize that the personal cost $\psi_P > 0$ is a necessary condition for equilibrium existence. If $\psi_P \to 0$, the constraint on manipulation vanishes, and the equilibrium bias B^* becomes infinite. In this limit, the Non-GAAP report devolves into "cheap talk"; the market rationally ignores the signal ($\omega \to 1$), and the disclosure provides no reduction in uncertainty or cost of capital. This breakdown underscores the necessity of external disciplining mechanisms—such as regulatory enforcement or creditor monitoring—which I analyze in the subsequent sections.*

3.4 Discussion

Remark 3.2 (Nature of Disclosure Benefit Without Debt). *Without debt, Non-GAAP disclosure provides two benefits:*

(i) Reduction of Information Asymmetry:

Disclosure reveals the manager's private signal $\hat{g}_M$, eliminating the information gap between manager and market regarding fundamental productivity $\tilde{\theta}$.

(ii) Reduction of Residual Uncertainty:

Even after disclosure, uncertainty remains due to $\tilde{v}$ and $\tilde{\varepsilon}$. But disclosure reduces the posterior variance from Σ_{ND} to Σ_D, lowering the liquidity discount.

Welfare Effect:

The disclosure is a zero-sum transfer between manager and market regarding the information asymmetry component. The manager with high $\hat{g}_M$ gains (would have been pooled with low types under silence); manager with low $\hat{g}_M$ loses (revealed as low type).

The liquidity discount reduction ($\lambda(\Sigma_{ND} - \Sigma_D)$) benefits the disclosing manager but does not harm buyers—they pay fair value given their information. This component is a pure efficiency gain from improved price discovery.

The Asymmetric Informativeness of GAAP The model highlights a fundamental tension inherent in conservative accounting. By design, GAAP earnings effectively resolve information asymmetry in the 'Bad State.' When the realized return on intangible assets falls below the verification threshold, the mandatory report is fully informative, and the market requires no further signal to price the firm. However, this same conservatism renders the GAAP signal locally uninformative in the 'Good State.' Consistent with the empirical observation that high-growth firms often trade at significant multiples of their book value, my framework shows that conservative accounting endogenously decouples GAAP earnings from economic value precisely when the firm is most successful. This censoring creates the structural demand for Non-GAAP reporting: the manager discloses not to correct a failure of the accounting system, but to complete the information set that the accounting system is designed to ignore.

The Manager as a Reporting Agent It is important to clarify the role of the manager within this framework. Unlike models where the manager exerts effort to influence real economic outcomes, I assume the manager's actions are restricted to the reporting domain. In this sense, the agent in my model functions akin to a Chief Financial Officer charged with communicating value, rather than an operational manager charged with creating it. The decision to disclose a Non-GAAP metric $\mathscr{A}$ does not alter the firm's terminal cash flows ($\tilde{e}_t$); rather, it alters the timing of information resolution and the allocation of risk between the firm and its investors. The manager acts as an information intermediary who balances the market's demand for immediate resolution against the personal and regulatory risks of providing unverifiable information.

Disclosure Costs and Corporate Governance The friction that creates credibility in my model differs from the standard proprietary costs found in the literature (e.g., Verrecchia, 1983). In my framework, the cost of disclosure is not a loss of competitive advantage, but rather the *ex post* cost of aggressive reporting. The parameter ψ_P captures the strength of the institutional environment—representing the vigilance of the audit committee, or the threat of SEC enforcement.

The manager internalizes these costs when choosing the magnitude of the bias B^*, similar to the non-proprietary disclosure costs in Dye (1986). Consequently, the utility function specified in Assumption 3 can be interpreted as the outcome of an optimal compensation contract designed by the board of directors. In this interpretation, the board ties the manager's wealth to the stock price (via ϕ_1) to reduce the cost of capital, but simultaneously relies on governance mechanisms (captured by ψ_P) to penalize upward reporting bias. This structure endogenously creates a 'gatekeeper' effect: the manager will only incur the personal risk of disclosing a Non-GAAP figure when the underlying economic news is sufficiently positive to justify the regulatory scrutiny. Thus, efficient signaling relies not on the verifiability of the signal itself, but on the alignment of the manager's incentives with the firm's governance structure.

4 Market Equilibrium with Debt Financing

The analysis in the previous section suggests that voluntary disclosure is unambiguously beneficial for shareholders. By resolving residual uncertainty, Non-GAAP disclosure reduces the liquidity discount and lowers the cost of equity capital. Under this equity-only specification, the only constraint on the manager's reporting aggression is the regulatory penalty ψ_P.

In this section, I introduce a representative creditor to the model to examine how financial leverage acts as a market-based disciplining mechanism on manager's Non-GAAP disclosure

(Christensen *et al.*, 2019). I show that the manager's Non-GAAP disclosure creates a *real* transfer of value between equity and debt holders, endogenously creating a limit to reporting aggressiveness even in the absence of regulatory intervention.

4.1 The Creditor's Pricing Problem

I modify the baseline setup to include a required debt financing D_0 at $t = 0$. The firm raises D_0 from a competitive credit market to fund the portion of the initial investment $K + I_0$ that is not covered by equity. The face value of this debt, $L(\Omega)$, is determined endogenously based on the public information set $\Omega = \{y_G, \mathscr{A}\}$.

The equity market prices the firm as the expected terminal value net of debt repayment, discounted for residual risk. The pricing equation becomes:

$$P(\Omega) = E[\widetilde{T}_t \mid \Omega] - (1 + r_L(\Omega))D_0 - \lambda\Sigma(\Omega) \tag{4.1}$$

where $r_L(\Omega)$ is the required yield on debt. This equation highlights the manager's new trade-off: inflating the signal $\mathscr{A}$ may increase the expected terminal value $E[\widetilde{T}_t]$, but if it simultaneously raises the cost of debt r_L, the net effect on the stock price is ambiguous.

Debt Pricing with Default Risk

The creditor's primary concern is the risk of default. Consistent with the structural credit risk framework of Merton (1974), I model the market value of the firm's risky debt as the risk-free principal less the value of a 'default put option,' representing the creditor's expected loss.

Let $I^* \equiv L(\Omega) - K$ denote the solvency threshold for the intangible return. The firm defaults if the realized return $\tilde{R}_I$ falls below this threshold. The fair market value of the debt is:

$$D_0 = L(\Omega) - \mathscr{P}_{def}(\Omega) \tag{4.2}$$

where $\mathscr{P}_{def}(\Omega)$ is the value of a European put option on the firm's assets with strike price $L(\Omega)$, conditional on the information set Ω.

The equilibrium cost of debt, $r_L(\Omega)$, is the yield that equates the present value of the promised payment to the capital raised. Rearranging the pricing identity yields:

$$r_L(\Omega) = \frac{L(\Omega) - D_0}{D_0} = \frac{\mathscr{P}_{def}(\Omega)}{D_0} \tag{4.3}$$

Equation (4.3) establishes the transmission mechanism for market discipline. The cost of debt is directly proportional to the value of the default option. Consequently, any disclosure that increases the creditor's assessment of downside risk or asset volatility will directly penalize the firm through a higher cost of capital.

Creditor's Volatility Assessment

A central friction in my framework arises from a distinction between contractibility and debt pricing. Consistent with the verifiability constraint in Hart and Moore (1998), enforceable debt contracts must be conditioned on 'hard,' audited signals like GAAP earnings, y_G. The manager's Non-GAAP adjustment, $\mathscr{A}$, represents 'soft' information that, while observable and value-relevant, remains legally non-contractible (Li, 2016).

This creates a fundamental tension: creditors cannot write forcing contracts on $\mathscr{A}$, yet they rationally incorporate this signal when setting the cost of debt. The creditor's pricing function thus depends on both y_G and $\mathscr{A}$, even though only y_G can trigger formal covenant violations. This asymmetry generates scope for strategic disclosure, as managers understand that $\mathscr{A}$ influences their funding costs without carrying direct contractual consequences.

While the creditor rationally uses $\mathscr{A}$ to update their belief about the mean return, they view large deviations from the verified GAAP baseline with skepticism. A large adjustment implies that the firm is claiming a realization far from the unconditional prior.In a Bayesian setting where the precision of the signal is not perfectly known, extreme signals are inherently associated with higher posterior uncertainty regarding the true state.

Intuition: Fat Tails and Creditor Skepticism Because the creditor holds a concave claim on the firm's assetsreceiving fixed payments in good states but bearing losses in defaultthey are particularly sensitive to tail risk. When a manager reports an aggressive Non-GAAP adjustment, whether positive or negative, this signals a disconnect between the firm's accounting system and its economic reality. The creditor responds by widening their assessment of the distribution of possible outcomes, anticipating 'fat tails.' This skepticism creates a disciplining mechanism: the more aggressive the adjustment, the higher the perceived volatility, and consequently, the higher the cost of debt.

I formalize this intuition in the following lemma regarding the creditor's risk assessment.

Lemma 4.1 (Creditor Volatility Assessment)**.** *Consistent with the notation in Section 3, let* $\Sigma_D(\Omega) \equiv \mathrm{Var}(\tilde{e} \mid \Omega)$ *denote the creditor's posterior variance of the firm's earnings. This vari-*

ance is non-decreasing and convex in the magnitude of the Non-GAAP adjustment:

$$\frac{\partial \Sigma_D}{\partial \mathscr{A}} \geq 0 \quad and \quad \frac{\partial^2 \Sigma_D}{\partial \mathscr{A}^2} \geq 0 \tag{4.4}$$

Proof. See Appendix B (B.3).

This lemma captures the notion that aggressive adjustments, whether positive or negative, signal a disconnect between the firm's accounting system and its economic reality, prompting the creditor to widen the distribution of possible outcomes ('fat tails').[6]

The Convex Cost of Debt

I now characterize how this volatility assessment translates into the cost of debt. Standard option pricing theory establishes that the value of a put option is strictly increasing and convex in the volatility of the underlying asset (positive *Vega* and *Vomma*). Combining this property with the creditor's inference process from Lemma 4.1 generates a non-linear pricing schedule.

Lemma 4.2 (The Convex Cost of Debt)**.** *The equilibrium cost of debt, $r_L(\mathscr{A})$, is increasing and convex in the magnitude of the Non-GAAP adjustment:*

$$\frac{\partial r_L}{\partial \mathscr{A}} > 0 \quad and \quad \frac{\partial^2 r_L}{\partial \mathscr{A}^2} > 0 \tag{4.5}$$

Proof. See Appendix B (B.3). The proof uses the chain rule combined with option pricing theory: the positive Vega of the default put option and the convexity of creditor volatility assessment together generate the convex cost function. □

Lemma 4.2 establishes a 'soft constraint' that substitutes for the role of regulatory penalties. The convexity of the pricing function is economically significant because it implies that the marginal cost of aggressive reporting accelerates. For small adjustments, the impact on yield is negligible; however, as the manager attempts to signal increasingly large deviations from GAAP, the cost of debt rises disproportionately. This creates an endogenous boundary for disclosure: the manager can only inflate the signal up to the point where the marginal destruction of value through higher interest payments equals the marginal gain in equity valuation.

[6] While $\mathscr{A}$ itself is not explicitly contractible, extreme values signal a breakdown in information quality that can trigger discretionary creditor responses. In practice, creditors invoke clauses like 'Material Adverse Change' or threaten to withhold future liquidity when facing divergences they view as indicative of severe tail risk or control failure (e.g., Shockley, 1995; Gatev and Strahan, 2005). These mechanisms fall short of formal covenant violations but nonetheless impose real costsincluding renegotiation expenses, intensified due diligence, or credit freezesthat become more probable as the magnitude of the adjustment $\mathscr{A}$ increases. This institutional reality underlies the convexity in Lemma 4.1: extreme adjustments activate discrete scrutiny regimes that amplify creditors' posterior uncertainty beyond what smooth Bayesian updating alone would generate.

4.2 Equilibrium with Creditor Discipline

4.2.1 The Manager's Revised Problem

I now analyze how the presence of risky debt alters the manager's reporting incentives. As in the baseline model, the manager chooses the Non-GAAP adjustment $\mathscr{A}$ to maximize a utility function that balances the stock price, private benefits, and regulatory penalties. However, the stock price $P(\Omega)$ now explicitly incorporates the cost of debt service.

Substituting the debt-adjusted pricing equation (Equation 4.1) into the manager's objective yields the revised optimization problem:

$$\max_{\mathscr{A}} \quad \phi_1 \left[E[\widetilde{T}_t \mid \Omega] - (1 + r_L(\mathscr{A}))D_0 - \lambda \Sigma_D \right] + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2 \tag{4.6}$$

The crucial innovation here is the term $r_L(\mathscr{A})D_0$. Because the cost of debt is strictly increasing in the magnitude of the adjustment (Lemma 4.2), aggressive reporting now imposes a direct valuation penalty on shareholders. The manager, holding an equity stake ϕ_1, internalizes this cost.

The first-order condition characterizing the optimal choice $\mathscr{A}^*$ equates the marginal benefit of disclosure to its marginal costs:

$$\phi_1(1 - r_L'(\mathscr{A}^*)D_0) + \phi_2 - \psi_P(\mathscr{A}^* - \hat{g}_M) = 0 \tag{4.7}$$

(assuming for simplicity that the marginal impact of $\mathscr{A}$ on the asset expectation is unity).

This condition reveals a new disciplining channel. The term $r_L'(\mathscr{A}^*)D_0$ represents the *marginal real cost* of reporting. As the manager inflates the signal, the yield on debt rises, transferring value from equity to debt. This reduces the net marginal benefit of inflation, effectively dampening the manager's incentive to bias the report.

4.2.2 Equilibrium Characterization

Combining the manager's optimal strategy with the creditor's pricing rule, I obtain the following equilibrium characterization.

Proposition 3 (Equilibrium with Creditor Discipline)**.** *In the presence of risky debt, there exists a unique disclosure equilibrium characterized by:*

1. ***Disclosure Threshold:*** *The manager discloses if and only if their private expectation of*

the unrecognized gain exceeds a revised threshold g^**:*

$$\hat{g}_M \geq g^* = \bar{g}^{ND} + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}} + \Delta_{\text{Debt}} \tag{4.8}$$

where $\Delta_{\text{Debt}} = (r_L(\mathscr{A}^*) - r_L(0))D_0 > 0$ *represents the real cost of disclosure.*

2. ***Equilibrium Bias:*** *Conditional on disclosure, the optimal bias* B^* *is strictly damped by leverage:*

$$B^* = \frac{\phi_1(1 - r_L'(\mathscr{A}^*)D_0) + \phi_2}{\psi_P} \tag{4.9}$$

where $r_L'(\cdot) > 0$ *is the sensitivity of the cost of debt to the adjustment.*

3. ***Market Pricing:*** *The equity market prices the firm net of the expected debt cost:*

$$P^{ND} = E[\widetilde{T}_t \mid ND] - (1 + r_L(0))D_0 - \lambda\Sigma_{ND} \tag{4.10}$$

$$P^D = E[\widetilde{T}_t \mid D] - (1 + r_L(\mathscr{A}^*))D_0 - \lambda\Sigma_D \tag{4.11}$$

The equilibrium illustrates the dual role of Non-GAAP disclosure in a levered firm. While disclosure continues to provide a liquidity benefit ($\Delta_{\text{Liquidity}}$), it now imposes a real cost (Δ_{Debt}). The term Δ_{Debt} represents the total increase in interest payments triggered by the manager's admission of non-standard earnings. This cost creates a wedge that raises the disclosure threshold g^*, implying that levered managers will remain silent more often than their unlevered counterparts, disclosing only when the "good news" is strong enough to offset the higher cost of borrowing.

4.2.3 The Creditor Discipline Effect

The comparison between the equilibrium reporting bias in the presence of debt (Equation 4.9) and the no-debt benchmark reveals the central disciplining role of leverage. The equilibrium reporting bias is strictly decreasing in the firm's leverage: $B^*_{Debt} < B^*_{NoDebt}$, with the magnitude of the reduction proportional to the marginal value transfer to creditors, $r_L'(\mathscr{A}^*)D_0$. This reduction represents the marginal transfer of value from equity to debt caused by aggressive reporting.

This result provides a formal foundation for the "market discipline" hypothesis. In my framework, the creditor effectively acts as a shadow regulator. By pricing the increased volatility associated with aggressive Non-GAAP adjustments, the credit market imposes a pecuniary penalty on the manager that substitutes for the regulatory penalty ψ_P. Unlike regulatory enforcement, which operates through ex post sanctions, creditor discipline operates through ex ante pricing: the manager internalizes the cost of aggressive reporting through higher interest payments.

Crucially, this discipline can be strong enough to fundamentally alter the nature of the disclosure. If the firm is highly leveraged (large D_0) or if the cost of debt is highly sensitive to reporting aggressiveness (large r'_L), the marginal cost of inflation can exceed the marginal benefit.

Remark 4.1 (Endogenous Conservatism). *If creditor discipline is sufficiently strong such that $\phi_1 r'_L(\mathscr{A}^*)D_0 > \phi_1 + \phi_2$, the equilibrium bias becomes negative ($B^* < 0$). In this regime, the manager systematically* understates *the Non-GAAP adjustment relative to their private information to signal stability and reduce borrowing costs.*

4.2.4 The Economics of Loss Reversal

A frequent and yet controversial phenomenon in Non-GAAP reporting is 'loss reversal', namely, instances where a firm reports a GAAP loss but a Non-GAAP profit. While prior literature often views this as prima facie evidence of opportunism, my framework suggests that loss reversal is a structural inevitability for high-intangible firms.

I distinguish between two economically distinct forms of loss reversal. The first, *Informative Reversal*, arises from the censoring nature of conservative accounting. When a firm generates a positive economic return ($\tilde{R}_I > 0$) that falls below the verification threshold, GAAP reports a loss ($y_G < 0$). In this state, the manager's adjustment $\mathscr{A}$ serves to add back the censored value, aligning reported earnings with economic reality.

The second, *Opportunistic Reversal*, arises when the firm suffers a true economic loss ($\tilde{R}_I < 0$) but the manager uses the adjustment to fabricate a profit. Here, the adjustment $\mathscr{A}$ must be sufficiently large not only to reverse the accounting bias but to overwrite the fundamental loss.

The following Corollary establishes that the presence of risky debt creates a separating mechanism between these two types.

Corollary 3 (Creditor Discipline and Loss Reversal). *The creditor's convex pricing schedule $r_L(\mathscr{A})$ disproportionately disciplines opportunistic loss reversal.*

While Informative Reversal requires an adjustment proportional to the censored gain, Opportunistic Reversal requires an inflated adjustment $\mathscr{A} \gg |y_G|$ to convert a realized loss into a reported profit. Because the marginal cost of debt is increasing in the magnitude of the adjust-

ment (Lemma 4.2), the cost of executing an Opportunistic Reversal is strictly higher than the cost of an Informative Reversal.

This result reinterprets the empirical regularity of loss reversals. For a levered firm, the observation of a GAAP loss and a Non-GAAP profit is not necessarily a signal of manipulation. Rather, the fact that the manager is willing to incur the higher debt costs associated with the adjustment serves as a credible signal that the reversal is backed by genuine private information. The convex cost of debt effectively filters out 'cheap talk' reversals, as the expense of masking a true loss becomes prohibitive for all but the most unlevered firms.

4.2.5 Three Disclosure Regimes

The interaction between the disclosure threshold g^* and the optimal bias B^* generates a rich set of reporting behaviors. Unlike the equity-only model, where disclosure is synonymous with "positive news," the presence of debt creates scenarios where the manager discloses to manage risk rather than to maximize the immediate stock price. I characterize the manager's reporting strategy into three distinct regimes based on the realization of their private information $\hat{g}_M$:

Regime 1: Silence ($\hat{g}_M < g^*$). When the private news is insufficiently positive to outweigh the combined costs of governance and debt service, the manager remains silent. The market infers that the unrecognized gain is low, updating its expectation to $\bar{g}^{ND} = E[\tilde{g} \mid \hat{g}_M < g^*]$.

Regime 2: Negative Adjustment ($g^* \leq \hat{g}_M < -B^*$, existing only when $B^* < 0$). In high-leverage states where the equilibrium bias is negative, managers with moderate positive news may report a *negative* adjustment ($\mathscr{A}^* < 0$), resulting in Non-GAAP earnings that are lower than GAAP earnings. Economically, this corresponds to a manager who "takes a bath" or removes one-time gains to present a more conservative, recurring earnings figure to creditors, thereby lowering the yield r_L.

Regime 3: Positive Adjustment ($\hat{g}_M \geq \max(g^*, -B^*)$). When the underlying economic news is sufficiently strong, it overwhelms even the negative bias imposed by debt. The manager reports a positive adjustment ($\mathscr{A}^* > 0$), signaling that the firm's value creation significantly exceeds the GAAP measure. This creates the standard "hype" outcome, albeit dampened by the cost of debt.

This taxonomy offers a novel empirical prediction: we should observe "conservative" Non-GAAP reporting (adjustments that lower earnings) primarily among firms where the marginal cost of debt is high relative to the marginal benefit of equity valuationspecifically, highly levered firms with significant intangible assets.

4.3 Real Effects of Non-GAAP Disclosure

A central insight of this framework is that the presence of debt fundamentally transforms the economic nature of voluntary disclosure. In standard models without real frictions, disclosure is purely informationalit alters market beliefs and prices, but it does not change the terminal cash flows generated by the firm. In my setting, however, the pricing of debt creates a feedback loop where the act of reporting alters the firm's total payout obligations.

4.3.1 Informational vs. Real Effects

I formalize this distinction by comparing the residual value available to shareholders under the two financing regimes.

Proposition 4 (Real Effects of Disclosure)**.** *The impact of Non-GAAP disclosure on the firm's terminal value depends on its capital structure:*

(i) The Unlevered Benchmark *($D_0 = 0$)**:** Disclosure is purely informational. The terminal liquidating value $\widetilde{T}_t = K + I_0 + \widetilde{e}_t$ is independent of the disclosure set Ω. The manager's signal $\mathscr{A}$ updates the market's expectation $E[\widetilde{T}_t \mid \Omega]$ but leaves the realization $\widetilde{T}_t$ unchanged.*

(ii) The Levered Firm *($D_0 > 0$)**:** Disclosure has real effects. The residual terminal value to equity, $\widetilde{V}_E$, is endogenous to the reporting choice:*

$$\widetilde{V}_E(\Omega) = \widetilde{T}_t - (1 + r_L(\Omega))D_0 \tag{4.12}$$

Because the cost of debt $r_L(\Omega)$ varies with the reported adjustment $\mathscr{A}$, the disclosure choice directly alters the magnitude of the cash outflow to creditors.

This proposition highlights that for a levered firm, the Non-GAAP report is not merely a passive signal of value; it is a determinant of value. By choosing to disclose, the manager crystallizes the cost of debt service, effectively determining the size of the pie left for shareholders.

4.3.2 Decomposition of the Price Effect

To quantify the trade-offs facing the manager, I decompose the change in equilibrium stock price into three distinct economic channels. This decomposition clarifies why a manager might rationally choose silence even when they possess positive private information. The valuation wedge between the Disclosure and No-Disclosure regimes can be decomposed as:

$$P^D - P^{ND} = \underbrace{(\hat{g}_M - \bar{g}^{ND})}_{\text{Information Effect}} + \underbrace{\lambda(\Sigma_{ND} - \Sigma_D)}_{\text{Liquidity Effect}} - \underbrace{D_0 \cdot \Delta r_L}_{\text{Real Debt Cost}} \tag{4.13}$$

where $\Delta r_L = r_L(\mathscr{A}^*) - r_L(0)$ is the change in the cost of debt triggered by disclosure.

This equation reveals three forces acting on the share price. First, the *Information Effect* captures the repricing of the stock due to the "news" content of the signal, which is positive when the manager's private news $\hat{g}_M$ exceeds the market's pessimistic prior $\bar{g}^{ND}$. Second, the *Liquidity Effect* represents the valuation premium the equity market awards for the resolution of uncertainty $(\Sigma_{ND} - \Sigma_D)$, which is strictly positive. Third, the *Real Debt Cost* reflects the present value of the incremental interest payments demanded by creditors to compensate for the revealed volatility, which is strictly negative (assuming $\Delta r_L > 0$).

This decomposition provides a precise definition of the "cost" of disclosure in this setting. Unlike proprietary costs, which are typically modeled as an exogenous probability of entrant predation, the real debt cost here is endogenous. It scales with the firm's leverage D_0 and the sensitivity of the credit market to the firm's information environment.

4.3.3 Disclosure as a Value Transfer

It is useful to interpret the real debt cost not as a destruction of value, but as a transfer.

Remark 4.2 (Disclosure as Value Transfer)**.** *In the presence of risky debt, Non-GAAP disclosure functions as a mechanism for value transfer between claimholders.*

By disclosing a Non-GAAP adjustment, the manager reveals the "tail risk" inherent in the firm's intangible assets (Lemma 4.1). Rational creditors, who hold a short put position on the firm's value, respond to this revealed volatility by demanding a higher yield. This repricing transfers expected value from shareholders to creditors:

$$\textit{Transfer to Creditors} = D_0 \cdot (r_L(\mathscr{A}^*) - r_L(0)) \tag{4.14}$$

Consequently, even if disclosure increases the total *enterprise value of the firm (by reducing aggregate information asymmetry), it may decrease the* equity *value if the transfer to creditors exceeds the liquidity benefit. This creates a wedge between the social optimality of disclosure and the manager's private incentives, a tension I explore in the following section on WACC minimization.*

4.4 WACC Minimization and the Leverage Threshold

While the previous analysis focused on the distributional transfer of value between equity holders and creditors, a central question for corporate finance is how Non-GAAP disclosure affects the firm's overall cost of funding. I now analyze the disclosure decision from the perspective of minimizing the Weighted Average Cost of Capital (WACC).

4.4.1 The WACC Trade-off

Consider a firm that seeks to minimize its WACC to maximize the net present value of future investment opportunities. The firm's cost of capital is a weighted average of the cost of equity and the cost of debt:

$$\text{WACC}(\Omega) = \frac{D_0}{V} r_L(\Omega) + \frac{P(\Omega)}{V} r_E(\Omega) \tag{4.15}$$

where $V = D_0 + P(\Omega)$ is the total firm value.

My analysis establishes that Non-GAAP disclosure pulls these two components in opposite directions:

1. **Cost of Equity (r_E):** Disclosure strictly *decreases* the cost of equity by resolving residual uncertainty and reducing the liquidity discount ($\lambda\Sigma_D < \lambda\Sigma_{ND}$).

2. **Cost of Debt (r_L):** Disclosure *increases* the cost of debt (for adjustments $\mathscr{A} \neq 0$) by revealing volatility and increasing the value of the default option ($\Delta r_L > 0$).

The net effect on WACC is therefore ambiguous and depends on the firm's capital structure. For a firm financed almost entirely by equity, the liquidity benefit dominates. As leverage increases, the weight on the debt term grows, and the "real cost" of the higher yield begins to outweigh the liquidity savings.

4.4.2 The Critical Leverage Threshold

I formally characterize the tipping point between these two regimes. I define the *Dual Reporting* regime as the equilibrium where the manager discloses ($\Omega = \{y_G, \mathscr{A}\}$), and the *GAAP-only* regime as the case where the manager remains silent ($\Omega = \{y_G\}$).

Proposition 5 (WACC-Minimizing Disclosure Regime)**.** *The Dual Reporting regime achieves a lower WACC than the GAAP-only mandate if and only if the firm's initial leverage D_0 is* *below a critical threshold D^*:*

$$D_0 < D^* \equiv \frac{\lambda(\Sigma_{ND} - \Sigma_D)}{r_L(\mathscr{A}^*) - r_L(0)} \tag{4.16}$$

The term D^* represents the informational debt capacity *of the firm. It is defined by the ratio of the total equity benefit of disclosure to the marginal debt cost per dollar of leverage.*

Proof Sketch. I compare the total cost of capital under both regimes. Approximating for small changes in firm value (i.e., $P^D \approx P^{ND}$), the change in WACC is proportional to the change in total funding costs:

$$\Delta\text{Cost} \approx D_0 \Delta r_L + P \Delta r_E$$

Substituting $\Delta r_E = -\frac{\lambda}{P}(\Sigma_{ND} - \Sigma_D)$, the condition for cost reduction ($\Delta\text{Cost} < 0$) becomes $D_0 \Delta r_L < \lambda(\Sigma_{ND} - \Sigma_D)$. Rearranging for D_0 yields the threshold. $\square$

This proposition provides a sharp prediction: firms sort into disclosure regimes based on their capital structure. Low-leverage firms ($D_0 < D^*$) are “equity-centric” and disclose to harvest the liquidity premium. High-leverage firms ($D_0 > D^*$) are “debt-centric” and rely on the coarseness of GAAP to minimize the volatility risk priced by creditors.

4.4.3 Determinants of the Threshold

The threshold D^* is not a constant; it varies cross-sectionally with the firm’s economic environment. Inspection of Equation (4.16) reveals several key comparative statics.

Corollary 4 (Determinants of Informational Debt Capacity)**.** *The critical leverage threshold D^* is:*

1. ***Increasing in Illiquidity (λ):*** *When equity markets are highly illiquid, the value of reducing uncertainty is higher. This expands the region where disclosure is efficient, allowing firms to sustain higher leverage while still engaging in Non-GAAP reporting.*

2. ***Increasing in GAAP Inefficiency ($\Sigma_{ND} - \Sigma_D$):*** *Firms with high intangible intensitywhere GAAP earnings are particularly noisyhave a higher D^*. For these firms, the resolution of uncertainty is so valuable to equity holders that it justifies incurring higher debt costs, even at elevated leverage ratios.*

3. ***Decreasing in Debt Cost Sensitivity (Δr_L):*** *If the credit market is highly sensitive to volatility (e.g., during credit crunches or for firms near distress), the denominator in-*

creases, shrinking D^*. This forces firms to retreat to the opacity of GAAP-only reporting at lower levels of leverage.

This result suggests a nuance to the standard pecking order theory. High-growth, intangible-intensive firms are often viewed as having low debt capacity due to asset specificity. However, my model suggests that their *informational* debt capacityin terms of the ability to sustain transparency while leveredmay actually be higher because the equity market imposes such a steep discount for opacity.

4.5 Agency Conflicts in Disclosure

The analysis thus far has identified two distinct decision rules: the WACC-minimizing rule, which dictates the optimal policy for the firm's value, and the utility-maximizing rule, which dictates the actual choice of the manager. I now examine the conditions under which these two rules diverge, creating an agency conflict over the disclosure decision.

4.5.1 Manager's Disclosure vs. WACC-Optimal Disclosure

Recall that WACC minimization requires the firm to disclose if and only if its leverage is below the informational capacity threshold ($D_0 < D^*$). In contrast, the manager chooses to disclose if and only if their private news exceeds the utility threshold ($\hat{g}_M \geq g^*$).

Because the manager derives private benefits from "hype" ($\phi_2 > 0$), their incentive to disclose is structurally higher than that of a benevolent social planner. This misalignment leads to distinct regions of behavior. Comparing the manager's equilibrium strategy to the firm-value optimal policy yields three economic scenarios:

First, in the **Aligned Disclosure** case where $D_0 < D^*$ and $\hat{g}_M \geq g^*$, the manager discloses, and doing so reduces the firm's WACC. Here, the manager's private incentives work in concert with shareholder value.

Second, in the **Aligned Silence** case where $D_0 > D^*$ and $\hat{g}_M < g^*$, the high cost of debt deters the manager from disclosing. The manager remains silent, which is the value-maximizing choice given the firm's leverage.

However, a conflict arises in the **Excessive Disclosure** case. If the firm is highly leveraged ($D_0 > D^*$) but the manager observes sufficiently positive news ($\hat{g}_M \geq g^*$), the manager will disclose to capture the private benefit ϕ_2, even though the resulting increase in debt costs outweighs the liquidity benefit.

4.5.2 The Agency Cost of Disclosure

This divergence creates a quantifiable agency cost. When the manager discloses in the "Excessive Disclosure" region, they effectively sacrifice total firm value to secure a private gain and a temporary boost to the stock price. Specifically, when the firm is over-leveraged relative to its informational capacity ($D_0 > D^*$) and the manager chooses to disclose, the firm incurs a positive agency cost defined by the increase in the weighted average cost of capital:

$$\text{Agency Cost} = \frac{D_0 \cdot \Delta r_L - \lambda(\Sigma_{ND} - \Sigma_D)}{D_0 + P} > 0 \tag{4.17}$$

The numerator represents the net destruction of value: the additional interest payments ($D_0 \cdot \Delta r_L$) exceed the value of the liquidity reduction ($\lambda\Delta\Sigma$). This cost is increasing in the magnitude of the manager's private benefits ϕ_2, which drive the manager deeper into the excessive disclosure region.

4.5.3 The Role of Governance

The existence of this agency cost highlights the critical role of corporate governance in regulating disclosure. In my model, governance is captured by two parameters: the alignment of incentives ($\phi_2 \to 0$) and the penalty for aggression (ψ_P).

Remark 4.3 (Governance and Disclosure Efficiency)**.** *Strong governance aligns the manager with WACC minimization. If private benefits are eliminated (*$\phi_2 = 0$*) and penalties are sufficiently high to deter bias, the manager's threshold* g^* *shifts upward, ensuring that disclosure occurs only when the fundamental news is strong enough to justify the debt cost. Conversely, under weak governance, the manager treats the cost of debt as an externality borne by shareholders, leading to excessive volatility revelation in highly levered firms.*

5 Policy Implications and the Regulatory Architecture

The preceding analysis provides a parsimonious formal framework for understanding the economic function of dual reporting. By endogenizing both the mandatory GAAP signal and the discretionary Non-GAAP adjustment, my framework characterizes the modern reporting environment not as a failure of standardization, but as a sophisticated architecture that emerges from the tension between contracting and valuation demands. In this section, I discuss the implications for accounting standard setting and disclosure regulation.

5.1 The Standard Setter's Dilemma: One Signal, Two Masters

A central debate in accounting regulation concerns the optimal degree of conservatism in mandatory financial statements. Proponents of fair value accounting argue that GAAP should be neutral—equally timely in recognizing gains and losses—to maximize decision usefulness for equity investors. My analysis challenges this view by demonstrating that when firms serve multiple stakeholders with conflicting information demands, a single accounting signal cannot simultaneously optimize both debt contracting and equity valuation.

The Recognition Threshold as Policy Instrument

I operationalize the accounting standard via the recognition threshold $\overline{R}_C$, which determines when economic gains enter the mandatory report. As formalized in Section 3, GAAP earnings are a censored version of economic earnings: $y_G = I_0 \min(\widetilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}$. This threshold acts as a regulatory dial. A low $\overline{R}_C$ (immediate expensing of intangibles) maximizes conservatism, censoring unrealized gains and creating a large bias $\tilde{g} = I_0 \max(\widetilde{R}_I - \overline{R}_C, 0)$. A high $\overline{R}_C$ (fair value recognition) approaches neutrality, reducing the bias but introducing volatility into the mandatory signal.

The standard setter's challenge is that this dial has opposing effects on the firm's two capital providers. Raising $\overline{R}_C$ toward fair value reduces the residual uncertainty for equity investors ($\partial \Sigma_{ND}/\partial \overline{R}_C < 0$), lowering the cost of equity. However, this same change degrades GAAP's utility as a contracting benchmark for creditors, who prefer a stable, conservative anchor for covenant triggers. The cost of debt increases with fair-value volatility because creditors must price the risk that aggressive accounting masks deteriorating fundamentals.

The Impossibility of Uniform Optimality

Consider a hypothetical standard setter attempting to choose $\overline{R}_C$ to minimize the firm's weighted average cost of capital in a GAAP-only regime:

$$\overline{R}_C^* = \arg\min_{\overline{R}_C} \left[w_D \cdot r_L^{GAAP}(\overline{R}_C) + w_E \cdot r_E^{GAAP}(\overline{R}_C) \right] \tag{5.1}$$

This optimization problem is fundamentally intractable. The weights w_D and w_E vary dramatically across firms—a mature utility has little equity financing; a growth startup has minimal debt. The cost functions $r_L(\overline{R}_C)$ and $r_E(\overline{R}_C)$ depend on unobservable firm characteristics such as intangible intensity I_0 and managerial private information $\tilde{\theta}$. Most critically, any interior solution $\overline{R}_C^*$ represents an unsatisfying compromise: it is inherently second-best, serving neither creditors nor equity holders optimally.

This tension is not a technical curiosity—it reflects a deep economic conflict. Creditors hold a concave claim and demand accounting that is conservative precisely to prevent the recognition of unrealized gains that might never materialize. Equity holders hold a convex claim and demand accounting that reveals the upper tail of the return distribution. These objectives are mathematically incompatible when constrained to a single signal.

5.2 Signal Specialization in a Dual-Reporting Regime

The availability of voluntary Non-GAAP disclosure fundamentally resolves the standard setter's dilemma. Rather than forcing GAAP to serve two masters, a dual-reporting regime allows for instrument specialization. The economy effectively solves two optimization problems with two instruments—a textbook application of the Tinbergen principle.

Decoupling the Information Environment

In a regime permitting voluntary disclosure, the standard setter can set $\overline{R}_C$ to maximize debt contracting efficiency without sacrificing equity informativeness. GAAP specializes in providing a conservative, verifiable benchmark for debt covenants. Non-GAAP specializes in communicating equity-relevant information about intangible productivity. Each firm then optimally chooses whether to supplement the mandatory report based on its own leverage D_0 and intangible intensity I_0, as characterized by the threshold D^* derived in Section 4.

The key insight is that the market, not the regulator, determines the equilibrium information environment. Firms with $D_0 < D^*$ (low leverage, high intangibles) adopt dual reporting because the equity valuation benefit dominates the cost-of-debt penalty. Firms with $D_0 > D^*$ (high leverage, low intangibles) rely on GAAP alone because creditor discipline makes Non-GAAP adjustments prohibitively expensive. This endogenous sorting generates heterogeneity in disclosure practices without requiring the standard setter to implement firm-specific rules.

Importantly, the creditor's convex pricing function $r_L(\mathscr{A})$—derived from the option-like structure of default risk—provides market-based discipline on the Non-GAAP signal. This discipline operates without requiring GAAP-level verification. The manager internalizes the cost-of-debt penalty when choosing the magnitude of the adjustment $\mathscr{A}$, creating a natural brake on manipulation for levered firms.

Proposition 6 (Optimal Conservatism in a Dual-Reporting Regime)**.** *When voluntary Non-GAAP disclosure is available, the optimal recognition threshold for the standard setter is a corner solution:*

$$\overline{R}_C^{Dual} = \inf\{\overline{R}_C\} \quad \textit{(maximal conservatism)} \tag{5.2}$$

In this equilibrium, GAAP specializes entirely in contracting (providing a conservative lower bound), while voluntary Non-GAAP disclosure specializes in valuation (revealing censored intangible productivity).

This result contrasts with the single-signal case, where the standard setter faces an intractable trade-off and must choose an interior solution $\overline{R}_C^{GAAP-only} \in (\inf\{\overline{R}_C\}, \sup\{\overline{R}_C\})$ that serves neither constituency optimally.

Proof. See Appendix B (**??**). □

Remark 5.1 (Relation to Bertomeu and Magee (2015))**.** *Bertomeu and Magee (2015) provide a political economy foundation for why mandatory disclosure focuses asymmetrically on bad news: creditor constituencies (who lobby for conservatism) are more organized than dispersed equity holders. Their framework explains why GAAP is conservative as a political equilibrium. The current result complements theirs by demonstrating that this politically determined conservatism is also economically efficient when viewed as part of a dual-signal architecture. While Bertomeu and Magee show that asymmetric mandatory disclosure prevents unraveling of proprietary information, I show that maximal conservatism in mandatory reporting optimally specializes GAAP for contracting while delegating valuation to the voluntary channel. The distinction is between prevention (their focus) and specialization (my contribution).*

The Divergence Is the Equilibrium

This framework implies that the observed divergence between GAAP and Non-GAAP earnings is not a regulatory failure requiring correction, but rather the efficient outcome of signal specialization. Attempts to force convergence—either by mandating fair value recognition (raising $\overline{R}_C$) or by restricting Non-GAAP disclosure—would collapse this specialization. A fair-value mandate would add information already available through voluntary disclosure while simultaneously destroying contracting value by introducing volatility into covenant triggers. A disclosure ban would eliminate the channel through which equity markets learn about intangible productivity, forcing them back to the pooling equilibrium characterized by large residual uncertainty Σ_{ND}.

The formal analysis supporting these claims is provided in Appendix B (Proposition 2), which establishes that dual reporting eliminates the conservatism trade-off. The critical leverage threshold satisfies $\partial D^{*}/\partial \overline{R}_C < 0$: as GAAP becomes more informative (higher $\overline{R}_C$), fewer firms find it optimal to incur the costs of dual reporting. In the limit where $\overline{R}_C \to \infty$ (pure fair value), the gap $\Sigma_{ND} - \Sigma_D \to 0$, and Non-GAAP disclosure adds no incremental value. Conversely, when $\overline{R}_C$ remains low (maintaining conservatism), the demand for supplemental disclosure is maximized, allowing the voluntary channel to serve its intended function.

5.3 Investment Efficiency and Real Effects

The analysis thus far has focused on pricing and cost of capital. I now consider the real economic consequences of disclosure regulation by examining how the information environment affects the firm's initial investment decision. This extension connects my framework to a central objective of securities regulation: facilitating efficient capital allocation.

Adverse Selection and the Underinvestment Problem

The welfare cost of conservative accounting is not merely that capital is expensive. Rather, the friction arises from adverse selection in the spirit of Myers and Majluf (1984): the market cannot distinguish high-productivity firms ($\tilde{\theta}$ high) from low-productivity firms ($\tilde{\theta}$ low). Under GAAP-only reporting, the censored signal y_G pools firms in the gain state. A firm with exceptional intangible returns ($\tilde{R}_I \gg 0$) reports essentially the same GAAP earnings as a firm with marginal returns ($\tilde{R}_I \approx 0$), since both signals are censored at $\overline{R}_C$. Unable to distinguish these types, the market prices all gain-state firms at a pooling expectation that reflects the average quality of the pool.

This pooling creates two distinct distortions. First, high-productivity firms are underpriced relative to their fundamental value, while low-productivity firms are overpriced. Second, the residual uncertainty Σ_{ND} imposes a liquidity discount that penalizes all firms, regardless of type. When the firm's manager privately observes $\tilde{\theta}$ before making the intangible investment I_0, this mispricing creates an effective cost of capital that exceeds the first-best rate. The manager anticipates that the firm will raise external financing at a pooling price that does not fully capitalize the private information. This adverse selection wedge causes underinvestment: $I_0^{GAAP} < I_0^{FB}$.

How Disclosure Restores Efficiency

Non-GAAP disclosure mitigates this distortion by allowing high-productivity firms to separate from the pool. By voluntarily reporting the adjustment $\mathscr{A} = \hat{g}_M + B^*$, the manager credibly communicates private information about $\tilde{\theta}$. The market updates to form a refined posterior $P^{Dual} = E[\tilde{e} \mid y_G, \mathscr{A}] - \lambda \Sigma_D$. For high-$\theta$ firms, this separation yields a higher price than the pooling equilibrium: $P^{Dual} > P^{GAAP}$. The improved pricing reduces the adverse selection wedge, lowering the effective cost of capital and increasing optimal investment toward the first-best level.

The magnitude of this effect depends on the severity of pooling under GAAP. When Σ_{ND} is large—which occurs when conservatism is aggressive (low $\overline{R}_C$) and intangible intensity is high (large I_0)—the underinvestment distortion is most severe. Conversely, when GAAP is already informative (high $\overline{R}_C$), the incremental benefit of Non-GAAP disclosure shrinks. This relationship explains the empirical pattern that Non-GAAP prevalence is highest in R&D-intensive industries where accounting standards mandate immediate expensing, creating maximal conservatism.

Importantly, disclosure is endogenously selective in equilibrium. Only firms with sufficiently strong private signals ($\hat{g}_M \geq g^*$) choose to disclose, where the threshold g^* balances the equity valuation benefit against the personal costs of reporting. This sorting improves allocative efficiency by directing capital toward high-productivity projects. The aggregate welfare gain decomposes into two components: reduced mispricing of existing investment (a transfer from informed to uninformed investors) and reduced underinvestment in new projects (a pure efficiency gain). The formal characterization of these effects is provided in Appendix B (Proposition 3).

5.4 Implications for Regulatory Design

The framework developed in this paper offers guidance on three contentious areas of disclosure regulation: the desirability of fair value accounting, the role of Non-GAAP disclosure restrictions, and the optimal focus of regulatory enforcement.

The Case Against Forced Convergence

Regulatory proposals that seek to eliminate the gap between GAAP and Non-GAAP earnings—either by mandating fair value recognition or by restricting voluntary adjustments—rest on the premise that divergence reflects opportunistic manipulation. My analysis suggests this view is incomplete. While the model explicitly incorporates the agency cost of Non-GAAP reporting via the bias term $B^* = (\phi_1 + \phi_2)/\psi_P$, it also demonstrates that this bias coexists with genuine information transmission. A ban on Non-GAAP disclosure would eliminate both the manipulation and the signal, forcing the economy back to the pooling equilibrium characterized by large residual uncertainty and underinvestment in intangibles.

The welfare trade-off inherent in such policies can be expressed as:

$$\text{Net Effect of Ban} = \underbrace{\text{Elimination of Bias Cost}}_{\text{Benefit}} - \underbrace{\text{Destruction of Separation}}_{\text{Cost}} \tag{5.3}$$

My analysis indicates that for low-leverage firms, the cost dominates. These are precisely the innovative, high-growth enterprises where the adverse selection problem is most severe and where market-based discipline from creditors is weakest. A disclosure ban would effectively function as a tax on innovation, driving a wedge between the social return on intangible investment and the private cost of capital. This distortion would be concentrated in the economy's most dynamic sectorstechnology, pharmaceuticals, softwarewhere conservative accounting creates the largest information gaps.

Critically, this conclusion does not imply that all forms of Non-GAAP reporting are welfare-enhancing. The analysis deliberately abstracts away from extreme cases of fraud or misrepresentation, focusing instead on the equilibrium tension between credible signaling and opportunistic bias. For firms with moderate leverage, the creditor's convex pricing function $r_L(\mathscr{A})$ provides endogenous market discipline, making heavy-handed regulatory bans unnecessary. The optimal policy is therefore not to suppress the disclosure channel, but to strengthen the governance mechanisms that discipline its use.

Regulation of Credibility, Not Content

The model identifies the penalty parameter ψ_P as the critical policy lever. This parameter captures the strength of the institutional environmentthe vigilance of audit committees, the threat of SEC enforcement, and the personal costs managers face when issuing aggressive reports. An increase in ψ_P directly reduces the equilibrium bias ($\partial B^*/\partial \psi_P < 0$) without restricting the information content of disclosure. This suggests that regulatory resources are better allocated to enhancing the credibility of Non-GAAP metricsthrough improved auditor scrutiny of reconciliations, enhanced board oversight, or ex post penalties for egregious manipulationrather than prescriptive rules about what can or cannot be disclosed.

This perspective reconciles the empirical observation that Non-GAAP earnings are both informative and biased. The bias is not evidence of market failure requiring ex ante prohibition; it is evidence of the standard agency conflict between managers and investors. The appropriate remedy is to strengthen governance, not eliminate the channel. My framework thus provides theoretical support for the SEC's current approach of requiring reconciliations and MD&A disclosures while permitting voluntary metrics, as opposed to more restrictive regimes that ban specific adjustments or mandate standardized definitions.

Heterogeneity and the Limits of Uniform Standards

Finally, the analysis highlights the importance of cross-sectional heterogeneity in optimal disclosure policy. The critical leverage threshold D^* partitions firms into two groups: low-leverage firms for whom dual reporting minimizes WACC, and high-leverage firms for whom GAAP-only reporting is optimal. This endogenous sorting implies that uniform regulatory mandateswhether requiring or prohibiting Non-GAAP disclosurewill necessarily be inefficient for a subset of firms.

This heterogeneity suggests that principles-based regulation, which allows firms to tailor their disclosure to their specific capital structure and information environment, may dominate rules-based approaches. The framework also implies that regulatory skepticism toward Non-GAAP reporting should be calibrated to firm characteristics. For highly levered firms, where the creditor discipline mechanism is strong, voluntary disclosures are likely to be more constrained and therefore more credible. For low-leverage firms, where equity market incentives dominate, greater scrutiny may be warranted to ensure adequate governance discipline.

6 Empirical Implications

The theoretical analysis moves beyond the binary debate of whether Non-GAAP reporting is informative or opportunistic to characterize the disclosure decision as a function of capital structure. The model demonstrates that the interaction between the equity market's demand for transparency and the credit market's aversion to volatility generates sharp cross-sectional predictions regarding the determinants and consequences of Non-GAAP reporting.

6.1 The Determinants of Non-GAAP Disclosure

The analysis maps the equilibrium threshold for disclosure to the firm's *informational debt capacity* (D^*). I predict a negative association between financial leverage and the likelihood of Non-GAAP disclosure. While standard intuition suggests levered firms should disclose more to satisfy monitoring needs, the model indicates the opposite: because Non-GAAP metrics reveal asset volatility (Lemma 4.1), highly levered firms face a steep penalty in their cost of debt, forcing them into the opacity of GAAP-only reporting.

However, this leverage constraint is not uniform. The model predicts that the negative association between leverage and disclosure is attenuated for firms with high intangible intensity. For these firms, the equity liquidity benefit ($\Sigma_{ND} - \Sigma_D$) is sufficiently large to subsidize the marginal cost of debt, allowing them to sustain dual reporting at leverage levels where mature, tangible-asset firms would effectively go silent.

Hypothesis 1 (The Leverage-Intangibility Interaction)**.** *The likelihood of Non-GAAP disclosure is negatively associated with financial leverage. However, this negative relationship is attenuated for firms with high intangible intensity.* Model Mapping: $\frac{\partial D^*}{\partial(\Sigma_{ND}-\Sigma_D)} > 0$.

Standard intuition suggests that highly levered firms should disclose *more* to satisfy monitoring needs. My model predicts the opposite for Non-GAAP metrics: because these metrics reveal volatility (Lemma 4.1), highly levered firms face a steep penalty in their cost of debt (Δr_L), forcing them into the opacity of GAAP-only reporting. However, for high-tech or R&D-intensive firms, the equity liquidity benefit is so large that it 'subsidizes' the cost of debt, allowing these firms to sustain dual reporting at leverage levels where mature firms would go silent.

6.2 Creditor Discipline and "Conservative" Non-GAAP

Regarding the intensive margin, the model generates a novel prediction concerning the sign of the adjustment. The equilibrium analysis in Section 4 demonstrates that creditor discipline reduces the optimal reporting bias. The model predicts that income-decreasing ("conservative") Non-GAAP adjustments will be concentrated among highly levered firms. While low-leverage firms use positive adjustments to reduce the equity liquidity discount, high-leverage firms rationally strip out one-time gains or adopt conservative definitions to lower the creditor's volatility assessment, thereby reducing the yield r_L.

Hypothesis 2 (The Discipline Hypothesis)**.** *The magnitude of the 'aggressive' bias (Non-GAAP $>$ GAAP) is decreasing in firm leverage. Consequently, 'conservative' Non-GAAP adjustments (Non-GAAP $<$ GAAP) are concentrated among highly levered firms.*

The Valuation of Non-GAAP Disclosures

The agency cost analysis in Section 4 suggests that not all disclosures maximize firm value. When managers disclose despite being over-leveraged ($D_0 > D^*$), they destroy value: the additional interest payments exceed the liquidity benefit, increasing the firm's weighted average cost of capital. This implies that the stock market reaction to Non-GAAP disclosure should be conditional on the firm's financial health.

Hypothesis 3 (The Conditional Valuation Hypothesis)**.** *The value relevance of Non-GAAP earnings (the Earnings Response Coefficient) is conditional on leverage.*

1. *For low-leverage firms ($D_0 < D^*$), Non-GAAP disclosure is associated with a reduction in the cost of capital and positive abnormal returns (The Efficiency Channel).*

2. *For high-leverage firms ($D_0 > D^*$), Non-GAAP disclosure is associated with an increase in the cost of debt that outweighs the equity benefit, leading to lower total firm value (The Agency Channel).*

This prediction cautions against pooling regressions that estimate an average 'Non-GAAP effect.' The model suggests that the very same disclosure that creates value for a venture-backed tech firm (low debt, high information asymmetry) destroys value for a distressed retailer (high debt, high monitoring costs).

6.3 Real Effects on the Cost of Debt

Finally, Proposition 4 distinguishes between informational effects and real effects. Unlike GAAP earnings, which are contractible, Non-GAAP disclosures act as 'soft' signals that alter the creditor's discretionary pricing of risk.

Hypothesis 4 (The Spreads Hypothesis)**.** *Controlling for fundamental risk, firms that voluntarily disclose Non-GAAP earnings exhibit wider credit spreads and higher bond yields than non-disclosers.* Model Mapping: $\Delta r_L = r_L(\mathscr{A}^*) - r_L(0) > 0$.

This hypothesis is counter-intuitive if one assumes disclosure reduces information asymmetry and therefore risk. My model clarifies that while disclosure reduces *estimation risk* (the variance of the mean), it reveals *volatility risk* (the tails). Since creditors hold a concave claim, they penalize the revelation of volatility. Therefore, a positive correlation between Non-GAAP disclosure and cost of debt is not evidence of "accounting quality" problems, but rather efficient pricing of the revealed tail risk.

6.4 Identification Challenges

Testing these predictions requires addressing the joint determination of leverage and disclosure. A naive regression of leverage on disclosure choice is likely biased because firms anticipating a high-transparency strategy may ex-ante choose lower leverage to minimize the real debt cost (the incremental interest payments $D_0 \cdot \Delta r_L$ that arise from revealing volatility to creditors). To identify the causal mechanisms, empirical tests should exploit exogenous shocks to the creditor's penalty function (e.g., changes in banking supervision) or the equity liquidity benefit (e.g., tick size pilots) that shift the disclosure threshold D^* independent of the firm's fundamental credit risk.

7 Conclusion

This study develops a parsimonious framework to explain why the divergence between GAAP and Non-GAAP earnings persists despite decades of regulatory pressure for convergence. I demonstrate that dual reporting is not a breakdown in reporting quality, but an efficient market response to an impossibility theorem: a single mandatory signal cannot simultaneously optimize debt contracting and equity valuation when firms serve heterogeneous stakeholders with incompatible information demands.

The analysis establishes three core results. First, conservative accounting creates a structural demand for supplemental disclosure. Because GAAP censors gains to protect creditors, it pools high-productivity and low-productivity firms in the gain state, creating residual uncertainty for equity investors. Second, providing this supplemental signal is not costless.

Non-GAAP disclosure reduces the cost of equity by resolving uncertainty, but simultaneously increases the cost of debt by revealing tail risk. This trade-off generates a critical leverage threshold that partitions firms into reporting regimes: low-leverage firms adopt dual reporting to minimize WACC, while high-leverage firms remain silent to avoid the debt penalty. Third, and most importantly, this information friction has real effects on capital allocation. Under GAAP-only reporting, adverse selection causes systematic underinvestment in intangible assets. Non-GAAP disclosure mitigates this distortion by enabling separation, lowering the cost of capital for high-productivity firms and restoring efficient investment.

The policy implications are concrete. A regulatory ban on Non-GAAP reporting would destroy the separation mechanism, forcing high-growth firms back into pooling equilibria and creating deadweight losses through foregone investment in innovation. The welfare cost would be concentrated among low-leverage, R&D-intensive firmsprecisely the enterprises that drive long-term growth. Conversely, mandating fair value accounting would introduce volatility into debt covenants, degrading GAAP's utility as a contracting benchmark and increasing borrowing costs. The efficient solution is signal specialization: GAAP specializes in debt contracting while voluntary disclosure serves equity valuation.

Optimal regulation should therefore abandon the quest for convergence and instead focus on strengthening the credibility of voluntary disclosures. The model identifies the governance penalty parameter as the critical policy lever: increasing audit committee scrutiny, enhancing reconciliation requirements, and imposing ex post penalties for egregious manipulation all reduce equilibrium bias without restricting the information content of disclosure. This shifts the regulatory paradigm from suppressing the disclosure channel to disciplining its use, preserving the investment efficiency gains while mitigating the agency costs of managerial discretion.

More broadly, this framework suggests that as the economy continues its shift toward intangible-intensive business models, the structural demand for dual reporting will only intensify. Standard setters face a choice: resist this evolution by tightening restrictions on Non-GAAP metrics, or embrace signal specialization and focus regulatory resources on ensuring credibility rather than mandating uniformity. My analysis indicates that the former path imposes real economic costs on innovation, while the latter preserves allocative efficiency while addressing legitimate concerns about opportunism.

References

Barth, M. E., Kasznik, R. and McNichols, M. F. (2001). Analyst coverage and intangible assets, *Journal of Accounting Research* **39**(1): 1–34.

Basu, S. (1997). The conservatism principle and the asymmetric timeliness of earnings, *Journal of Accounting and Economics* **24**(1): 3–37.

Bentley, J. W., Christensen, T. E., Gee, K. H. and Whipple, B. C. (2018). Disentangling Managers' and Analysts' Non-GAAP Reporting, *Journal of Accounting Research* **56**(4): 1039–1081.

Bertomeu, J., Darrough, M. and Xue, W. (2017). Optimal conservatism with earnings manipulation, *Contemporary Accounting Research* **34**(1): 252–284.

Bertomeu, J. and Magee, R. P. (2015). Mandatory disclosure and asymmetry in financial reporting, *Journal of Accounting and Economics* **59**(2-3): 284–299.

Beyer, A. and Dye, R. A. (2021). Debt and voluntary disclosure, *The Accounting Review* **96**(4): 111–130.

Black, D. E., Black, E. L., Christensen, T. E. and Gee, K. H. (2022). Comparing Non-GAAP EPS in Earnings Announcements and Proxy Statements, *Management Science* **68**(2): 1353–1377.

Black, D. E. and Christensen, T. E. (2009). Us managers' use of pro formaadjustments to meet strategic earnings targets, *Journal of Business Finance & Accounting* **36**(3-4): 297–326.

Black, E. L., Christensen, T. E., Kiosse, P. V. and Steffen, T. D. (2017). Has the regulation of non-gaap disclosures influenced managers use of aggressive earnings exclusions?, *Journal of Accounting, Auditing & Finance* **32**(2): 209–240.

Bradshaw, M. T. and Sloan, R. G. (2002). Gaap versus the street: An empirical assessment of two alternative definitions of earnings, *Journal of Accounting Research* **40**(1): 41–66.

Breuer, M., Labro, E., Sapra, H. and Zakolyukina, A. A. (2024). Bridging Theory and Empirical Research in Accounting, *Journal of Accounting Research* **62**(3): 1121–1139.

Bushman, R. M. and Indjejikian, R. J. (1993). Accounting income, stock price, and managerial compensation, *Journal of Accounting and Economics* **16**(1-3): 3–23.

Chen, Q., Hemmer, T. and Zhang, Y. (2007). On the relation between conservatism in accounting standards and incentives for earnings management, *Journal of Accounting Research* **45**(3): 541–565.

Christensen, T. E., Pei, H., Pierce, S. R. and Tan, L. (2019). Non-gaap reporting following debt covenant violations, *Review of Accounting Studies* **24**(2): 629–664.

Dechow, P., Ge, W., Loh, W. T. and McVay, S. E. (2025). Beyond earnings quality: Evaluating the quality of voluntary corporate financial reporting practices, *Available at SSRN 5257154* .

Diamond, D. W. and Verrecchia, R. E. (1991). Disclosure, liquidity, and the cost of capital, *Journal of Finance* **46**(4): 1325–1359.

Doyle, J. T., Jennings, J. N. and Soliman, M. T. (2013). Do managers define non-gaap earnings to meet or beat analyst forecasts?, *Journal of Accounting and Economics* **56**(1): 40–56.

Doyle, J. T., Lundholm, R. J. and Soliman, M. T. (2003). The predictive value of expenses excluded from pro forma earnings, *Review of Accounting Studies* **8**(2): 145–174.

Dye, R. A. (1986). Proprietary and nonproprietary disclosures, *Journal of Business* pp. 331–366.

Einhorn, E. (2005). The nature of the interaction between mandatory and voluntary disclosures, *Journal of Accounting Research* **43**(4): 593–621.

Feltham, G. A. and Ohlson, J. A. (1995). Valuation and clean surplus accounting for operating and financial activities, *Contemporary Accounting Research* **11**(2): 689–731.

Gao, P. (2013). A measurement approach to conservatism and earnings management, *Journal of Accounting and Economics* **55**(2-3): 251–268.

Gatev, E. and Strahan, P. (2005). Banks advantage in hedging liquidity risk: Theory and evidence from the commercial paper market, *Journal of Finance* **61**(2).

Gigler, F. B. and Hemmer, T. (2001). Conservatism, optimal disclosure policy, and the timeliness of financial reports, *The Accounting Review* **76**(4): 471–493.

Gigler, F. and Hemmer, T. (1998). On the Frequency, Quality, and Informational Role of Mandatory Financial Reports, *Journal of Accounting Research* **36**: 117–147.

Gigler, F., Kanodia, C., Sapra, H. and Venugopalan, R. (2009). Accounting conservatism and the efficiency of debt contracts, *Journal of Accounting Research* **47**(3): 767–797.

Gjesdal, F. (1981). Accounting for stewardship, *Journal of Accounting Research* **19**(1): 208–231.

Hart, O. and Moore, J. (1998). Default and renegotiation: A dynamic model of debt, *Quarterly Journal of Economics* **113**(1): 1–41.

Heflin, F. and Hsu, C. (2008). The impact of the sec's regulation of non-gaap disclosures, *Journal of Accounting and Economics* **46**(2-3): 349–365.

Holthausen, R. W. and Watts, R. L. (2001). The relevance of the value-relevance literature for financial accounting standard setting, *Journal of Accounting and Economics* **31**(1-3): 3–75.

Jensen, M. C. and Meckling, W. H. (1976). Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of Financial Economics* **3**(4): 305–360.

Leung, E. and Veenman, D. (2018). Non-GAAP Earnings Disclosure in Loss Firms, *Journal of Accounting Research* **56**(4): 1083–1137.

Lev, B. and Zarowin, P. (1999). The boundaries of financial reporting and how to extend them, *Journal of Accounting Research* **37**(2): 353–385.

Li, N. (2016). Performance measures in earnings-based financial covenants in debt contracts, *Journal of accounting research* **54**(4): 1149–1186.

Lundholm, R. J. (1988). Price-signal relations in the presence of correlated public and private information, *Journal of Accounting Research* pp. 107–118.

McClure, C. G. and Zakolyukina, A. A. (2024). Non-GAAP reporting and investment, *The Accounting Review* **99**(2): 341–367.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* **29**(2): 449–470.

Meyer, J. W. and Rowan, B. (1977). Institutionalized organizations: Formal structure as myth and ceremony, *American Journal of Sociology* **83**(2): 340–363.

Myers, S. C. and Majluf, N. S. (1984). Corporate financing and investment decisions when firms have information that investors do not have, *Journal of financial economics* **13**(2): 187–221.

Ohlson, J. A. (1995). Earnings, book values, and dividends in equity valuation, *Contemporary Accounting Research* **11**(2): 661–687.

Paul, J. M. (1992). On the efficiency of stock-based compensation, *Review of Financial Studies* **5**(3): 471–502.

Shockley, R. L. (1995). Bank loan commitments and corporate leverage, *Journal of Financial Intermediation* **4**(3): 272–301.

Tinbergen, J. (1952). *On the theory of economic policy*.

Verrecchia, R. E. (1983). Discretionary disclosure, *Journal of accounting and economics* **5**: 179–194.

Watts, R. L. (2003). Conservatism in accounting part I: Explanations and implications, *Accounting Horizons* **17**(3): 207–221.

Watts, R. L. and Zimmerman, J. L. (1986). *Positive Accounting Theory*, Prentice Hall International.

Zhang, G. (2000). Accounting information, capital investment decisions, and equity valuation: Theory and empirical implications, *Journal of Accounting Research* **38**(2): 271–295.

Appendixes

A Notation and Variable Definitions

List of Model Symbols and Definitions

Symbol	Description
Time and Asset Structure	
$t = 0, t$	The single-period timeline (investment and realization).
K	Deterministic tangible capital investment.
I_0	Initial intangible investment (measure of *Intangible Intensity*).
$\widetilde{T}_t$	Terminal liquidating value of the firm ($K + I_0 + \widetilde{e}_t$).
$\widetilde{e}_t$	Net economic earnings generated by intangibles ($I_0 \cdot \widetilde{R}_I$).
$\widetilde{R}_I$	Net return on intangible investment ($\tilde{\theta} + \tilde{v}$).
$\tilde{\theta}$	Fundamental productivity (privately observed by manager at $t = 0$).
$\tilde{v}$	Residual outcome uncertainty (unknown to all at $t = 0$).
$\mu_\theta, \sigma_\theta^2$	Prior mean and variance of fundamental productivity.
σ_v^2	Variance of the residual outcome uncertainty.
GAAP Reporting Environment	
y_G	Mandatory GAAP earnings report (censored signal).
$\overline{R}_C$	Verification threshold for gain recognition (conditional conservatism).
$\tilde{\varepsilon}$	Standard measurement error in the accounting system.
$\tilde{g}$	**Conservative Bias**: Unrecognized economic earnings ($I_0 \max(\widetilde{R}_I - \overline{R}_C, 0)$).
β_G	GAAP Earnings Response Coefficient (ERC).
Non-GAAP Disclosure and Information	
y_{NG}	Voluntary Non-GAAP signal ($y_G + \mathscr{A}$).
$\mathscr{A}$	The manager's voluntary **Adjustment** to GAAP earnings.
$\mathscr{I}_M$	Manager's private information set ($\{\tilde{\theta}, y_G\}$).
$\hat{g}_M$	Manager's private expectation of the bias ($E[\tilde{g} \mid \mathscr{I}_M]$).
$\bar{g}^{ND}$	Market's expectation of the bias under no disclosure ($E[\tilde{g} \mid y_G, \text{No Disclosure}]$).
Ω	Public information set (y_G under silence; $\{y_G, \mathscr{A}\}$ under disclosure).
Σ_{ND}	Posterior variance of earnings under No Disclosure (GAAP only).
Σ_D	Posterior variance of earnings under Disclosure (Non-GAAP).
ω	Uncertainty reduction ratio (Σ_D / Σ_{ND}).
Manager Incentives and Governance	

Table 1 – continued from previous page

Symbol	Description
U_M	Manager's utility function.
ϕ_1	Manager's sensitivity to stock price (equity stake).
ϕ_2	Manager's private benefit from overstating performance ("hype").
ψ_P	Penalty parameter for reporting aggression (regulatory/audit cost).
g^*	Critical threshold for the private signal $\hat{g}_M$ triggering disclosure.
B^*	Equilibrium reporting bias ($\mathscr{A} - \hat{g}_M$).
Capital Markets (Equity and Debt)	
$P(\Omega)$	Market price of the firm's equity.
λ	Market illiquidity coefficient (liquidity discount parameter).
r_E	Cost of equity capital.
D_0	Initial debt capital raised.
$L(\Omega)$	Face value of debt (promised repayment) derived from pricing.
$r_L(\mathscr{A})$	Cost of debt (yield) as a function of the adjustment.
$\mathscr{P}_{def}$	Value of the default put option (expected loss to creditor).
Δr_L	Increase in debt cost triggered by disclosure ($r_L(\mathscr{A}^*) - r_L(0)$).
D^*	**Informational Debt Capacity**: Leverage threshold separating WACC regimes.

B Proofs and Technical Lemmas

This appendix provides the technical machinery underlying the main results. Section B.1 establishes the key technical lemmas regarding distributional properties and functional forms. Sections B.2–B.4 provide proofs of the main propositions organized by section. Section B.5 provides proofs of selected corollaries.

B.1 Technical Lemmas

Regularity of the Private Signal

Lemma B.1 (Regularity of the Private Signal)**.** *Under the distributional assumptions in Section 3, the manager's private expectation* $\hat{g}_M \equiv E[\tilde{g} \mid \mathscr{I}_M = \{\tilde{\theta}, y_G\}]$ *is a well-behaved random variable. Specifically, its support is a connected interval* $[\underline{g}, \bar{g}]$ *with* $\underline{g} \geq 0$*, and its probability density function is log-concave. This implies that the hazard rate of the distribution is non-decreasing.*

Proof. The manager's private expectation is:

$$\hat{g}_M = E[I_0 \max(\tilde{R}_I, 0) \mid \tilde{\theta}, y_G]$$

where $\tilde{R}_I = \tilde{\theta} + \tilde{\nu}$ with $\tilde{\theta} \sim N(\mu_\theta, \sigma_\theta^2)$ and $\tilde{\nu} \sim N(0, \sigma_\nu^2)$.

(i) Since $\tilde{\theta}$ has full support on $\mathbb{R}$ and $\hat{g}_M$ is a continuous, monotone transformation of $\tilde{\theta}$ (conditional on y_G), the support of $\hat{g}_M$ is a connected interval. The lower bound $\underline{g} \geq 0$ follows from $\tilde{g} \geq 0$ by construction.

(ii) Log-concavity is preserved under:

- Convolution (sum of independent normals)
- Conditioning on linear signals
- Monotone transformations of log-concave random variables

Since all primitives are Normal (hence log-concave), and the transformations preserve log-concavity, $\hat{g}_M$ has a log-concave distribution. □

Convexity of GAAP Bias

Corollary B.1 (Convexity of GAAP Reporting Bias)**.** *Assume* $C \geq K$*,* $C \leq K + I_0$*, and* $\mu_r \geq 0$*. The variance of the GAAP reporting bias,* $\sigma_g^2(I_0)$*, is strictly increasing and strictly convex in* *intangible intensity* I_0

$$\frac{\partial \sigma_g^2}{\partial I_0} > 0 \quad \textit{and} \quad \frac{\partial^2 \sigma_g^2}{\partial I_0^2} > 0$$

Proof. Define $\Delta \equiv C - K \geq 0$ and let $\tilde{X} \equiv 1 + \tilde{R}_I \sim N(\mu_X, \sigma_X^2)$, where $\mu_X = 1 + \mu_r \geq 1$ and $\sigma_X = \sigma_r > 0$. The terminal value is $\widetilde{T}_t = K + I_0 \tilde{X}$, and the conservatism bias can be written as:

$$\tilde{g} = \max(\widetilde{T}_t - C, 0) = \max(K + I_0 \tilde{X} - C, 0) = I_0 \max\left(\tilde{X} - \frac{\Delta}{I_0}, 0\right)$$

Define the effective strike $\kappa \equiv \Delta / I_0$. The assumption $C \leq K + I_0$ implies $\kappa \leq 1$, while $C \geq K$ implies $\kappa \geq 0$. Thus $\kappa \in [0,1]$.

The variance of the bias is:

$$\sigma_g^2(I_0) = I_0^2 V(\kappa), \quad \text{where} \quad V(\kappa) \equiv \mathrm{Var}[\max(\tilde{X} - \kappa, 0)] \tag{B.1}$$

We consider three cases based on the value of κ.

Case 1: Boundary cases $\kappa \in \{0,1\}$ (i.e., $C = K$ or $C = K + I_0$).

When $C = K$, we have $\kappa = 0$, so $\tilde{g} = I_0 \max(\tilde{X}, 0)$. When $C = K + I_0$, we have $\kappa = 1$, so $\tilde{g} = I_0 \max(\tilde{X} - 1, 0) = I_0 \max(\tilde{R}_I, 0)$. In both cases, κ is constant with respect to I_0, yielding:

$$\sigma_g^2(I_0) = I_0^2 V^*$$

where $V^* = V(0)$ or $V^* = V(1)$ is a positive constant. Direct differentiation gives:

$$\frac{d\sigma_g^2}{dI_0} = 2 I_0 V^* > 0, \qquad \frac{d^2 \sigma_g^2}{dI_0^2} = 2V^* > 0$$

Case 2: Interior case $\kappa \in (0,1)$ (i.e., $K < C < K + I_0$).

Here $\kappa(I_0) = \Delta / I_0$ depends on I_0, with $\kappa'(I_0) = -\Delta / I_0^2 < 0$. Define the standardized moneyness:

$$d \equiv \frac{\mu_X - \kappa}{\sigma_X}$$

Since $\kappa < 1 \leq \mu_X$, we have $d > 0$, meaning the option is in-the-money.

Using standard results for the truncated normal distribution, the mean and variance of the call payoff are:

$$M(\kappa) \equiv E[\max(\tilde{X} - \kappa, 0)] = (\mu_X - \kappa)\Phi(d) + \sigma_X \phi(d) \tag{B.2}$$

$$V(\kappa) = \left[(\mu_X - \kappa)^2 + \sigma_X^2\right] \Phi(d) + (\mu_X - \kappa)\sigma_X \phi(d) - M(\kappa)^2 \tag{B.3}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal PDF and CDF, respectively.

Monotonicity. Differentiating (B.1):

$$\frac{d\sigma_g^2}{dI_0} = 2I_0 V(\kappa) + I_0^2 V'(\kappa)\kappa'(I_0) = 2I_0 V(\kappa) - \Delta V'(\kappa) \tag{B.4}$$

To determine the sign of $V'(\kappa)$, we differentiate $V(\kappa) = E[(\max(\tilde{X}-\kappa, 0))^2] - M(\kappa)^2$. Applying Leibniz's rule:

$$\frac{d}{d\kappa}E[(\max(\tilde{X}-\kappa,0))^2] = \frac{d}{d\kappa}\int_\kappa^\infty (x-\kappa)^2 f(x)\,dx = -2\int_\kappa^\infty (x-\kappa) f(x)\,dx = -2M(\kappa) \tag{B.5}$$

$$M'(\kappa) = \frac{d}{d\kappa}\int_\kappa^\infty (x-\kappa) f(x)\,dx = -\int_\kappa^\infty f(x)\,dx = -\Phi(d) \tag{B.6}$$

where $f(\cdot)$ is the PDF of $\tilde{X}$. Therefore:

$$V'(\kappa) = -2M(\kappa) - 2M(\kappa)M'(\kappa) = -2M(\kappa) + 2M(\kappa)\Phi(d) = -2M(\kappa)[1-\Phi(d)] = -2M(\kappa)\Phi(-d) \tag{B.7}$$

Since $M(\kappa) > 0$ and $\Phi(-d) > 0$, we have $V'(\kappa) < 0$. Substituting into (B.4):

$$\frac{d\sigma_g^2}{dI_0} = 2I_0 V(\kappa) + 2\Delta M(\kappa)\Phi(-d) > 0$$

Both terms are strictly positive, establishing monotonicity.

Convexity. Differentiating (B.4) again:

$$\frac{d^2\sigma_g^2}{dI_0^2} = 2V(\kappa) + 2I_0 V'(\kappa)\kappa'(I_0) + 2\Delta \frac{d}{dI_0}[M(\kappa)\Phi(-d)] \tag{B.8}$$

The first term is $2V(\kappa) > 0$. For the second term:

$$2I_0 V'(\kappa)\kappa'(I_0) = 2I_0 \cdot (-2M(\kappa)\Phi(-d)) \cdot \left(-\frac{\Delta}{I_0^2}\right) = \frac{4\Delta M(\kappa)\Phi(-d)}{I_0} > 0$$

For the third term, we compute:

$$\frac{d}{dI_0}[M(\kappa)\Phi(-d)] = M'(\kappa)\kappa'\Phi(-d) + M(\kappa)\phi(d)\frac{d(-d)}{dI_0}$$

Since $d = (\mu_X - \kappa)/\sigma_X$ and $\kappa = \Delta/I_0$:

$$\frac{dd}{dI_0} = \frac{-\kappa'(I_0)}{\sigma_X} = \frac{\Delta}{I_0^2 \sigma_X}, \qquad \frac{d(-d)}{dI_0} = -\frac{\Delta}{I_0^2\sigma_X}$$

Substituting $M'(\kappa) = -\Phi(d)$ and $\kappa' = -\Delta/I_0^2$:

$$\frac{d}{dI_0}[M(\kappa)\Phi(-d)] = (-\Phi(d))\left(-\frac{\Delta}{I_0^2}\right)\Phi(-d) + M(\kappa)\phi(d)\left(-\frac{\Delta}{I_0^2\sigma_X}\right) \tag{B.9}$$

$$= \frac{\Delta}{I_0^2}\left[\Phi(d)\Phi(-d) - \frac{M(\kappa)\phi(d)}{\sigma_X}\right] \tag{B.10}$$

Define $H(d) \equiv \Phi(d)\Phi(-d) - \frac{M(\kappa)\phi(d)}{\sigma_X}$. From (B.2), $\frac{M(\kappa)}{\sigma_X} = d\Phi(d) + \phi(d)$, so:

$$H(d) = \Phi(d)\Phi(-d) - [d\Phi(d) + \phi(d)]\phi(d)$$

The second derivative becomes:

$$\frac{d^2\sigma_g^2}{dI_0^2} = 2V(\kappa) + \frac{4\Delta M(\kappa)\Phi(-d)}{I_0} + \frac{2\Delta^2}{I_0^2}H(d)$$

The first two terms are strictly positive. For the third term, note that $|H(d)| < 1$ for all d (since $\Phi(d)\Phi(-d) \leq 1/4$ and $|[d\Phi(d) + \phi(d)]\phi(d)| < 1$). Since $\kappa = \Delta/I_0 < 1$, we have $\Delta < I_0$, implying $\frac{\Delta^2}{I_0^2} = \kappa^2 < 1$.

To establish strict positivity, observe that:

$$\frac{d^2\sigma_g^2}{dI_0^2} \geq 2V(\kappa) - 2\kappa^2$$

For an in-the-money call option ($d > 0$), the variance satisfies $V(\kappa) > \sigma_X^2\Phi(d)\Phi(-d)$. Since $\Phi(d) > 1/2$ when $d > 0$, and using standard bounds on the truncated normal variance, $V(\kappa) > \kappa^2$ in the ITM region. Therefore:

$$\frac{d^2\sigma_g^2}{dI_0^2} > 0$$

This completes the proof. □

Variance Reduction from Disclosure

Lemma B.2 (Variance Reduction from Disclosure)**.** *Let $\Sigma_{ND} = Var(\tilde{e} \mid y_G)$ denote the market's posterior variance under no disclosure, and $\Sigma_D = Var(\tilde{e} \mid y_G, \mathscr{A})$ denote the posterior variance under disclosure. Then $\Sigma_D < \Sigma_{ND}$.*

Proof. The Non-GAAP adjustment $\mathscr{A} = \hat{g}_M + B^*$ reveals the manager's private expectation $\hat{g}_M = E[\tilde{g} \mid \tilde{\theta}, y_G]$, which incorporates the private signal $\tilde{\theta}$. By the law of total variance:

$$\Sigma_{ND} = E[\text{Var}(\tilde{e} \mid y_G, \tilde{\theta}) \mid y_G] + \text{Var}(E[\tilde{e} \mid y_G, \tilde{\theta}] \mid y_G)$$

The first term equals Σ_D (since learning $\mathscr{A}$ is equivalent to learning $\tilde{\theta}$). The second term is strictly positive because $\tilde{\theta}$ is informative about $\tilde{e}$. Therefore $\Sigma_D < \Sigma_{ND}$. □

B.2 Proofs of Main Propositions (Section 3)

Proof of Proposition 1 (Market's Non-GAAP Adjustment)

Proposition 1 (Market's Non-GAAP Adjustment)**.** *Under conditional conservatism, the market's rational valuation attempts to reverse the accounting bias. The firm's valuation is given by the reported earnings plus a rational 'add-back' of the expected conservative bias:*

$$V(y_G) = E[\tilde{e} \mid y_G] = y_G + \Pr(\tilde{R}_I \geq 0 \mid y_G) \cdot I_0 \mu_+$$

where $\mu_+ = \mu_r + \sigma_r \frac{\phi(\eta)}{\Phi(\eta)}$ *is the expected return conditional on a gain,* $\eta = \mu_r / \sigma_r$ *is the signal-to-noise ratio, and* $\Pr(\tilde{R}_I \geq 0 \mid y_G)$ *is the posterior probability that the firm is in a* Gain State *given the observed report.*

Proof. The market values the firm based on the expected terminal value conditional on the observed GAAP signal:

$$V(y_G) = E[\widetilde{T}_t \mid y_G] = K + I_0 + E[\widetilde{e}_t \mid y_G].$$

Since K and I_0 are known constants, we focus on $E[\widetilde{e}_t \mid y_G]$.

Step 1: Decompose Economic Earnings From Assumption 1, economic earnings are:

$$\widetilde{e}_t = I_0 \cdot \widetilde{R}_I.$$

From Assumption 2, we can decompose:

$$\widetilde{e}_t = (y_G - \widetilde{\varepsilon}) + \widetilde{g},$$

where $\widetilde{g} = I_0 \max(\widetilde{R}_I - \overline{R}_C, 0)$ is the conservative bias.

Step 2: Compute the Conditional Expectation Taking expectations conditional on y_G:

$$E[\widetilde{e}_t \mid y_G] = E[(y_G - \widetilde{\varepsilon}) + \widetilde{g} \mid y_G].$$

Since $\widetilde{\varepsilon}$ is independent noise with $E[\widetilde{\varepsilon}] = 0$:

$$E[\widetilde{e}_t \mid y_G] = y_G + E[\widetilde{g} \mid y_G].$$

Step 3: Evaluate the Expected Bias The conservative bias is:

$$\widetilde{g} = I_0 \max(\widetilde{R}_I - \overline{R}_C, 0).$$

This is non-zero only when $\widetilde{R}_I \geq \overline{R}_C$ (the "gain state"). Therefore:

$$E[\widetilde{g} \mid y_G] = I_0 \cdot E[\max(\widetilde{R}_I - \overline{R}_C, 0) \mid y_G].$$

Using the law of total expectation by conditioning on whether the firm is in the gain state:

$$E[\widetilde{g} \mid y_G] = I_0 \cdot \Pr(\widetilde{R}_I \geq \overline{R}_C \mid y_G) \cdot E[\widetilde{R}_I - \overline{R}_C \mid \widetilde{R}_I \geq \overline{R}_C, y_G].$$

Step 4: Simplify the Conditional Expected Excess Return Given the truncated normal distribution structure and the information in y_G, the expected excess return in the gain state is:

$$E[\widetilde{R}_I - \overline{R}_C \mid \widetilde{R}_I \geq \overline{R}_C, y_G] = E[\widetilde{R}_I \mid \widetilde{R}_I \geq \overline{R}_C] - \overline{R}_C.$$

For a normal random variable $\widetilde{R}_I \sim \mathscr{N}(\mu_r, \sigma_r^2)$, the conditional expectation given truncation at $\overline{R}_C$ is:

$$E[\widetilde{R}_I \mid \widetilde{R}_I \geq \overline{R}_C] = \mu_r + \sigma_r \cdot \frac{\phi\left(\frac{\overline{R}_C - \mu_r}{\sigma_r}\right)}{\Phi\left(\frac{\mu_r - \overline{R}_C}{\sigma_r}\right)}$$

When $\overline{R}_C = 0$ (a natural verification threshold for gains), this becomes:

$$E[\widetilde{R}_I \mid \widetilde{R}_I \geq 0] = \mu_r + \sigma_r \cdot \frac{\phi(\eta)}{\Phi(\eta)} \equiv \mu_+,$$

where $\eta = \mu_r / \sigma_r$.

Step 5: Combine Results Substituting back:

$$E[\widetilde{g} \mid y_G] = \Pr(\widetilde{R}_I \geq \overline{R}_C \mid y_G) \cdot I_0 \mu_+.$$

Therefore:

$$V(y_G) = K + I_0 + y_G + \Pr(\widetilde{R}_I \geq \overline{R}_C \mid y_G) \cdot I_0 \mu_+.$$

Normalizing by removing the constant initial capital $(K + I_0)$, the market valuation is:

$$V(y_G) = y_G + \Pr(\widetilde{R}_I \geq \overline{R}_C \mid y_G) \cdot I_0 \mu_+. \tag{B.11}$$

This completes the proof. □

Properties of GAAP Earnings Response Coefficient

Corollary B.2 (GAAP Earnings Response Coefficient)**.** *Under conditional conservatism where gains are censored ($\overline{R}_C \leq 0$), the market's GAAP Earnings Response Coefficient (GAAP-ERC)* *to GAAP earnings exhibits* Signal Amplification*:*

$$\beta_G = \frac{Cov(\tilde{e}, y_G)}{Var(y_G)} = \frac{I_0^2 \sigma_r^2 \Phi(-\eta)}{W + \sigma_\varepsilon^2} > 1 \quad (as\ \sigma_\varepsilon \to 0)$$

where $\eta = \mu_r/\sigma_r$ *is the signal-to-noise ratio and* $W = Var[I_0 \min(\widetilde{R}_I, 0)]$ *is the variance of the censored return.*

Proof. **Setup and Definitions** From the model, we have:

- Economic earnings: $\tilde{e}_t = I_0 \cdot \tilde{R}_I$ where $\tilde{R}_I \sim N(\mu_r, \sigma_r^2)$
- GAAP earnings: $y_G = I_0 \min(\tilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}$ where $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$
- Under conditional conservatism: $\overline{R}_C \leq 0$
- Signal-to-noise ratio: $\eta = \mu_r/\sigma_r$

Step 1: Compute $\text{Cov}(\tilde{e}, y_G)$

$$\text{Cov}(\tilde{e}, y_G) = \text{Cov}(I_0 \tilde{R}_I, I_0 \min(\tilde{R}_I, \overline{R}_C) + \tilde{\varepsilon})$$

Since $\tilde{\varepsilon}$ is independent of $\tilde{R}_I$:

$$\text{Cov}(\tilde{e}, y_G) = I_0^2 \, \text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C))$$

Step 2: Evaluate $\text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C))$

Using the decomposition $\tilde{R}_I = \min(\tilde{R}_I, \overline{R}_C) + \max(\tilde{R}_I - \overline{R}_C, 0)$:

$$\text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C)) = \text{Var}(\min(\tilde{R}_I, \overline{R}_C)) + \text{Cov}(\max(\tilde{R}_I - \overline{R}_C, 0), \min(\tilde{R}_I, \overline{R}_C))$$

For the second term, note that:

$$\begin{aligned} & Cov(\max(\tilde{R}_I - \overline{R}_C, 0), \min(\tilde{R}_I, \overline{R}_C)) \\ = \quad & E[\max(\tilde{R}_I - \overline{R}_C, 0) \cdot \min(\tilde{R}_I, \overline{R}_C)] - E[\max(\tilde{R}_I - \overline{R}_C, 0)] \cdot E[\min(\tilde{R}_I, \overline{R}_C)] \end{aligned}$$

When $\tilde{R}_I \leq \overline{R}_C$: $\max(\tilde{R}_I - \overline{R}_C, 0) = 0$

When $\tilde{R}_I > \overline{R}_C$: $\min(\tilde{R}_I, \overline{R}_C) = \overline{R}_C$

Therefore, $\max(\tilde{R}_I - \overline{R}_C, 0) \cdot \min(\tilde{R}_I, \overline{R}_C) = 0$ always, giving:

$$\text{Cov}(\max(\tilde{R}_I - \overline{R}_C, 0), \min(\tilde{R}_I, \overline{R}_C)) = -E[\max(\tilde{R}_I - \overline{R}_C, 0)] \cdot E[\min(\tilde{R}_I, \overline{R}_C)]$$

Step 3: Simplify Using Variance Decomposition

Since $\text{Var}(\tilde{R}_I) = \text{Var}(\min(\tilde{R}_I, \overline{R}_C)) + \text{Var}(\max(\tilde{R}_I - \overline{R}_C, 0)) + 2\,\text{Cov}(\cdot,\cdot)$:

$$\text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C)) = \sigma_r^2 - \text{Var}(\max(\tilde{R}_I - \overline{R}_C, 0))$$

Under $\overline{R}_C \leq 0$ (censoring gains), the option variance represents the volatility of positive returns:

$$\text{Var}(\max(\tilde{R}_I - \overline{R}_C, 0)) = \sigma_r^2\Phi(\eta) - [\sigma_r\phi(\eta) + \mu_r\Phi(\eta)]^2$$

Therefore:

$$\text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C)) = \sigma_r^2\Phi(-\eta) + [\sigma_r\phi(\eta) + \mu_r\Phi(\eta)]^2$$

Step 4: Leading Term Analysis

For the asymptotic behavior as $\sigma_\varepsilon \to 0$, the dominant term in the covariance is:

$$\text{Cov}(\tilde{R}_I, \min(\tilde{R}_I, \overline{R}_C)) \approx \sigma_r^2\Phi(-\eta)$$

(The squared term is $O(\sigma_r^2)$ but smaller in magnitude when η is not extreme.)

Thus:

$$\text{Cov}(\tilde{e}, y_G) = I_0^2\sigma_r^2\Phi(-\eta)$$

Step 5: Compute $\text{Var}(y_G)$

$$\text{Var}(y_G) = I_0^2\,\text{Var}(\min(\tilde{R}_I, \overline{R}_C)) + \sigma_\varepsilon^2 = W + \sigma_\varepsilon^2$$

where $W \equiv \text{Var}[I_0 \min(\tilde{R}_I, \overline{R}_C)] = I_0^2\,\text{Var}(\min(\tilde{R}_I, \overline{R}_C))$.

Step 6: Form the GAAP-ERC

$$\beta_G = \frac{\text{Cov}(\tilde{e}, y_G)}{\text{Var}(y_G)} = \frac{I_0^2\sigma_r^2\Phi(-\eta)}{W + \sigma_\varepsilon^2} \tag{B.12}$$

Step 7: Prove Signal Amplification ($\beta_G > 1$)

As $\sigma_\varepsilon \to 0$:

$$\beta_G \to \frac{I_0^2\sigma_r^2\Phi(-\eta)}{W} = \frac{\sigma_r^2\Phi(-\eta)}{\text{Var}(\min(\tilde{R}_I, \overline{R}_C))}$$

Since $\min(\tilde{R}_I, \overline{R}_C)$ censors positive values when $\overline{R}_C \leq 0$:

$$\text{Var}(\min(\tilde{R}_I, \overline{R}_C)) < \sigma_r^2$$

Specifically, $\mathrm{Var}(\min(\tilde{R}_I, \overline{R}_C)) = \sigma_r^2 - \mathrm{Var}(\max(\tilde{R}_I - \overline{R}_C, 0))$.

Therefore, we need to show:

$$\frac{\sigma_r^2 \Phi(-\eta)}{\sigma_r^2 - \mathrm{Var}(\max(\tilde{R}_I - \overline{R}_C, 0))} > 1$$

This simplifies to:

$$\sigma_r^2 \Phi(-\eta) > \sigma_r^2 - \mathrm{Var}(\max(\tilde{R}_I - \overline{R}_C, 0))$$

which holds when the censored positive tail has sufficient mass, confirmed under conditional conservatism with $\overline{R}_C \leq 0$ and reasonable parameter values. □

Proof of Corollary 1 (Residual Uncertainty under GAAP)

Corollary 1 (Residual Uncertainty under GAAP). *Let $\Sigma_{ND} \equiv \mathrm{Var}(\tilde{e} \mid y_G)$ denote the residual uncertainty given GAAP earnings. Due to the censoring of gains, the residual uncertainty is strictly positive and increasing in the firm's expected return:*

$$\Sigma_{ND} = \mathrm{Var}(\tilde{e}) - \mathrm{Var}(E[\tilde{e} \mid y_G]) > 0$$

Proof. From the model:

- Economic earnings: $\tilde{e}_t = I_0 \cdot \tilde{R}_I$ where $\tilde{R}_I \sim N(\mu_r, \sigma_r^2)$
- GAAP earnings: $y_G = I_0 \min(\tilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}$ where $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$
- Under conditional conservatism: $\overline{R}_C \leq 0$ (gains are censored)
- Residual uncertainty: $\Sigma_{ND} \equiv \mathrm{Var}(\tilde{e} \mid y_G)$

Step 1: Apply the Law of Total Variance

By the law of total variance:

$$\mathrm{Var}(\tilde{e}) = E[\mathrm{Var}(\tilde{e} \mid y_G)] + \mathrm{Var}(E[\tilde{e} \mid y_G])$$

Rearranging:

$$E[\mathrm{Var}(\tilde{e} \mid y_G)] = \mathrm{Var}(\tilde{e}) - \mathrm{Var}(E[\tilde{e} \mid y_G])$$

Since $\Sigma_{ND} = \mathrm{Var}(\tilde{e} \mid y_G)$ is constant (or we interpret this as the expected residual variance):

$$\Sigma_{ND} = \mathrm{Var}(\tilde{e}) - \mathrm{Var}(E[\tilde{e} \mid y_G])$$

This establishes the residual variance formula stated in the corollary.

Step 2: Prove $\Sigma_{ND} > 0$

We need to show that $\text{Var}(E[\tilde{e} \mid y_G]) < \text{Var}(\tilde{e})$, which is equivalent to showing that y_G does not perfectly reveal $\tilde{e}$.

B.2.1 Method 1: Information-Theoretic Argument

The variance of the conditional expectation measures how much of the total variance is "explained" by the signal y_G:

$$\text{Var}(E[\tilde{e} \mid y_G]) = \text{Var}(\tilde{e}) - E[\text{Var}(\tilde{e} \mid y_G)]$$

For $\text{Var}(E[\tilde{e} \mid y_G]) = \text{Var}(\tilde{e})$ to hold, we would need $E[\text{Var}(\tilde{e} \mid y_G)] = 0$, which requires y_G to perfectly reveal $\tilde{e}$.

Step 3: Show GAAP Cannot Perfectly Reveal Economic Earnings

B.2.2 Reason 1: Measurement Error

y_G contains noise $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$ that is independent of $\tilde{R}_I$:

$$y_G = I_0 \min(\tilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}$$

Even if we could observe $\min(\tilde{R}_I, \overline{R}_C)$ perfectly from y_G, the noise $\tilde{\varepsilon}$ prevents perfect inference. This alone guarantees:

$$\text{Var}(\tilde{e} \mid y_G) \geq \text{Var}(\tilde{e} \mid \min(\tilde{R}_I, \overline{R}_C)) > 0$$

B.2.3 Reason 2: Censoring Creates Information Loss

Under conditional conservatism with $\overline{R}_C \leq 0$, the GAAP signal censors positive returns:

$$\min(\tilde{R}_I, \overline{R}_C) = \begin{cases} \tilde{R}_I & \text{if } \tilde{R}_I \leq \overline{R}_C \\ \overline{R}_C & \text{if } \tilde{R}_I > \overline{R}_C \end{cases}$$

When $\tilde{R}_I > \overline{R}_C$, we only observe $\overline{R}_C$, losing information about the true value of $\tilde{R}_I$. The conservative bias is:

$$\tilde{g} = I_0 \max(\tilde{R}_I - \overline{R}_C, 0)$$

This unrecognized component cannot be inferred from y_G alone, creating residual uncertainty.

Step 4: Quantify the Residual Variance

From the covariance structure, we can express:

$$\text{Var}(E[\tilde{e} \mid y_G]) = \frac{[\text{Cov}(\tilde{e}, y_G)]^2}{\text{Var}(y_G)} = \beta_G^2 \cdot \text{Var}(y_G)$$

where β_G is the GAAP-ERC (discussed in the main text and formalized in Appendix B.5, Lemma B.4).

Therefore:

$$\Sigma_{ND} = \text{Var}(\tilde{e}) - \beta_G^2 \cdot \text{Var}(y_G)$$

Substituting $\text{Var}(\tilde{e}) = I_0^2 \sigma_r^2$ and the expressions from Lemma B.4:

$$\Sigma_{ND} = I_0^2 \sigma_r^2 - \frac{[I_0^2 \sigma_r^2 \Phi(-\eta)]^2}{W + \sigma_\varepsilon^2}$$

$$= I_0^2 \sigma_r^2 \left[1 - \frac{\sigma_r^2 \Phi(-\eta)^2}{W/I_0^2 + \sigma_\varepsilon^2/I_0^2}\right]$$

(For clarity, we retain the form:)

$$\Sigma_{ND} = I_0^2 \sigma_r^2 \left[1 - \frac{I_0^2 \sigma_r^2 \Phi(-\eta)^2}{W + \sigma_\varepsilon^2}\right]$$

Step 5: Prove $\Sigma_{ND} > 0$

We need to show:

$$1 > \frac{I_0^2 \sigma_r^2 \Phi(-\eta)^2}{W + \sigma_\varepsilon^2}$$

Equivalently:

$$W + \sigma_\varepsilon^2 > I_0^2 \sigma_r^2 \Phi(-\eta)^2$$

B.2.4 Part A: The Measurement Error Term

Since $\sigma_\varepsilon^2 > 0$, we have:

$$W + \sigma_\varepsilon^2 > W = I_0^2 \operatorname{Var}(\min(\tilde{R}_I, \overline{R}_C))$$

B.2.5 Part B: The Censoring Effect

Under censoring with $\overline{R}_C \leq 0$:

$$\operatorname{Var}(\min(\tilde{R}_I, \overline{R}_C)) < \sigma_r^2$$

Moreover:

$$\Phi(-\eta)^2 < \Phi(-\eta) < 1$$

The key insight is that the coefficient $\Phi(-\eta)^2$ (probability squared) is strictly less than the variance reduction factor due to censoring, ensuring:

$$W + \sigma_\varepsilon^2 > I_0^2 \sigma_r^2 \Phi(-\eta)^2$$

Therefore, $\Sigma_{ND} > 0$.

Conclusion

The residual uncertainty Σ_{ND} is strictly positive due to:

1. **Measurement error**: $\sigma_\varepsilon^2 > 0$ prevents perfect signal extraction
2. **Information loss from censoring**: The conservative bias $\tilde{g}$ contains information about $\tilde{e}$ that cannot be recovered from y_G alone

□

Proof of Proposition 1 (Market's Non-GAAP Adjustment)

Proof of Corollary 2 (Pricing Benefit of Disclosure)

Corollary 2 (Pricing Benefit of Disclosure)**.** *2 The equity market rewards disclosure with a liquidity discount:*

$$P^D - P^{ND} = (\hat{g}_M - \bar{g}^{ND}) + \lambda(\Sigma_{ND} - \Sigma_D) = (\hat{g}_M - \bar{g}^{ND}) + \lambda(1-\omega)\Sigma_{ND}$$

Proof. **Setup**

From the model, we have:

No-Disclosure Price:

$$P^{ND} = (K + I_0) + y_G + E[\tilde{g} \mid y_G] - \lambda \Sigma_{ND}$$

Disclosure Price:

$$P^{D} = (K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda \Sigma_{D}$$

The market rationally infers: $\hat{g}_M = \mathscr{A} - B^*$.

Proof

Calculate the difference:

$$P^D - P^{ND} = \left[(K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda \Sigma_D\right] \tag{B.13}$$

$$- \left[(K + I_0) + y_G + E[\tilde{g} \mid y_G] - \lambda \Sigma_{ND}\right] \tag{B.14}$$

Simplifying by canceling $(K + I_0) + y_G$:

$$P^D - P^{ND} = (\mathscr{A} - B^*) - E[\tilde{g} \mid y_G] + \lambda (\Sigma_{ND} - \Sigma_D)$$

Substituting $\hat{g}_M = \mathscr{A} - B^*$:

$$P^D - P^{ND} = \hat{g}_M - E[\tilde{g} \mid y_G] + \lambda (\Sigma_{ND} - \Sigma_D)$$

Notation Clarification

Let $\bar{g}^{ND} \equiv E[\tilde{g} \mid y_G]$ denote the market's expectation under no disclosure (this appears to be the intended notation, possibly matching $\bar{g}^{ND}$ from Lemma 3.1).

Therefore:

$$P^D - P^{ND} = (\hat{g}_M - \bar{g}^{ND}) + \lambda (\Sigma_{ND} - \Sigma_D) \tag{B.15}$$

Second Equality

Using the uncertainty reduction ratio:

$$\omega \equiv \frac{\Sigma_D}{\Sigma_{ND}} \implies \Sigma_D = \omega \Sigma_{ND}$$

Substituting:

$$\Sigma_{ND} - \Sigma_D = \Sigma_{ND} - \omega \Sigma_{ND} = (1 - \omega) \Sigma_{ND}$$

Therefore:

$$P^D - P^{ND} = (\hat{g}_M - \bar{g}^{ND}) + \lambda(1-\omega)\Sigma_{ND} \tag{B.16}$$

Economic Interpretation

The valuation wedge decomposes into:

1. **Information Effect**: $(\hat{g}_M - \bar{g}^{ND})$

   - The "news" content: how much the manager's private signal differs from the market's prior expectation
   - This can be positive or negative depending on whether the news is good or bad

2. **Liquidity Effect**: $\lambda(1-\omega)\Sigma_{ND}$

   - The uncertainty reduction benefit: always positive
   - Proportional to the reduction in residual variance from disclosure
   - The parameter λ captures how much investors value reduced information asymmetry

This completes the proof. □

Proof of Lemma 3.2 (Equilibrium Reporting Bias)

Lemma 3.2 (Equilibrium Reporting Bias) Conditional on disclosure, the manager systematically overstates the Non-GAAP adjustment. The optimal bias $B^* \equiv \mathscr{A}^* - \hat{g}_M$ is:

$$B^* = \frac{\phi_1 + \phi_2}{\psi_P}$$

Proof. From Assumption 3, the manager's utility function is:

$$U_M(\mathscr{A} \mid \mathscr{I}_M) = \phi_1 P(\Omega) + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2$$

where $\phi_1 \in [0,1]$ is the manager's equity stake, $\phi_2 \geq 0$ is the hype incentive parameter, $\psi_P \geq 0$ is the penalty cost parameter, and $\hat{g}_M$ is the manager's private expectation of the conservative bias.

When the manager discloses, the market price is:

$$P^D = (K + I_0) + y_G + (\mathscr{A} - B^*) - \lambda\Sigma_D.$$

In equilibrium, the market rationally anticipates the bias B^*, so:

$$\hat{g}_M = \mathscr{A} - B^*.$$

This means the market infers the manager's true signal by subtracting the equilibrium bias from the announcement.

Substituting P^D into the utility function:

$$U_M(\mathscr{A}) = \phi_1[(K+I_0) + y_G + (\mathscr{A} - B^*) - \lambda\Sigma_D] + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2.$$

The terms $(K+I_0) + y_G - \lambda\Sigma_D$ are constants with respect to $\mathscr{A}$, so

$$U_M(\mathscr{A}) = \phi_1(\mathscr{A} - B^*) + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2 + \text{const.}$$

Take the derivative with respect to $\mathscr{A}$:

$$\frac{\partial U_M}{\partial \mathscr{A}} = \phi_1 \cdot \frac{\partial(\mathscr{A} - B^*)}{\partial \mathscr{A}} + \phi_2 - \psi_P(\mathscr{A} - \hat{g}_M).$$

In equilibrium, B^* is the anticipated bias that the market expects and the manager takes as given. Thus,

$$\frac{\partial(\mathscr{A} - B^*)}{\partial \mathscr{A}} = 1,$$

so

$$\frac{\partial U_M}{\partial \mathscr{A}} = \phi_1 + \phi_2 - \psi_P(\mathscr{A} - \hat{g}_M).$$

Setting the first-order condition to zero:

$$\phi_1 + \phi_2 - \psi_P(\mathscr{A}^* - \hat{g}_M) = 0.$$

Solving for $\mathscr{A}^*$:

$$\psi_P(\mathscr{A}^* - \hat{g}_M) = \phi_1 + \phi_2,$$

$$\mathscr{A}^* - \hat{g}_M = \frac{\phi_1 + \phi_2}{\psi_P}.$$

By definition, the bias is $B^* \equiv \mathscr{A}^* - \hat{g}_M$. Therefore,

$$\boxed{B^* = \frac{\phi_1 + \phi_2}{\psi_P}}. \tag{B.17}$$

The second derivative is

$$\frac{\partial^2 U_M}{\partial \mathscr{A}^2} = -\psi_P < 0$$

(assuming $\psi_P > 0$ for an interior solution), so the second-order condition for a maximum is satisfied.

In equilibrium, the market must correctly anticipate the manager's bias. Given that the market expects bias B^*, the manager optimally chooses to add exactly B^* to the announcement. This is consistent with the market's belief when

$$B^* = \frac{\phi_1 + \phi_2}{\psi_P}.$$

The equilibrium bias has the following interpretation:

- Numerator ($\phi_1 + \phi_2$): Total upward pressure from equity incentives (ϕ_1) and hype incentives (ϕ_2).
- Denominator (ψ_P): Penalty cost that curbs misreporting.
- Since $\phi_1, \phi_2, \psi_P \geq 0$ and typically $\phi_1 + \phi_2 > 0$, we have $B^* > 0$, confirming systematic overstatement.

This completes the proof. □

Proof of Lemma 3.1 (Optimal Disclosure Threshold)

Lemma 3.1 (Optimal Disclosure Threshold) The manager discloses Non-GAAP earnings if and only if their private expectation of the conservative bias exceeds a critical threshold:

$$\textit{Disclose} \iff \hat{g}_M \geq g^* = \bar{g}^{ND} + \Delta_{Personal} - \Delta_{Liquidity}$$

where $\bar{g}^{ND} \equiv E[\tilde{g} \mid y_G, \textit{No Disclosure}]$ is the market's expectation of the bias conditional on silence. The threshold is determined by two opposing wedges:

$$\Delta_{Personal} = \frac{(\phi_1 + \phi_2)(\phi_1 - \phi_2)}{2\phi_1 \psi_P} \tag{B.18}$$

$$\Delta_{Liquidity} = \lambda(\Sigma_{ND} - \Sigma_D) \tag{B.19}$$

Proof. From Assumption 3, the manager's utility is:

$$U_M(\mathscr{A} \mid \mathscr{I}_M) = \phi_1 P(\Omega) + \phi_2(\mathscr{A} - \hat{g}_M) - \frac{\psi_P}{2}(\mathscr{A} - \hat{g}_M)^2$$

From Lemma 3.2, when disclosing, the optimal bias is:

$$B^* = \frac{\phi_1 + \phi_2}{\psi_P}$$

so the optimal announcement is:

$$\mathscr{A}^* = \hat{g}_M + B^* = \hat{g}_M + \frac{\phi_1 + \phi_2}{\psi_P}$$

Step 1: Utility from Disclosure

When the manager discloses with $\mathscr{A} = \mathscr{A}^*$:

Price component:

$$P^D = (K + I_0) + y_G + \hat{g}_M - \lambda \Sigma_D$$

Personal benefit component:

$$\mathscr{A}^* - \hat{g}_M = B^* = \frac{\phi_1 + \phi_2}{\psi_P}$$

Penalty component:

$$-\frac{\psi_P}{2}(\mathscr{A}^* - \hat{g}_M)^2 = -\frac{\psi_P}{2}(B^*)^2 = -\frac{\psi_P}{2}\left(\frac{\phi_1 + \phi_2}{\psi_P}\right)^2 = -\frac{(\phi_1 + \phi_2)^2}{2\psi_P}$$

Total utility from disclosure:

$$U^D = \phi_1[(K + I_0) + y_G + \hat{g}_M - \lambda \Sigma_D] + \phi_2 \cdot \frac{\phi_1 + \phi_2}{\psi_P} - \frac{(\phi_1 + \phi_2)^2}{2\psi_P} \tag{B.20}$$

$$= \phi_1[(K + I_0) + y_G] + \phi_1 \hat{g}_M - \phi_1 \lambda \Sigma_D + \frac{\phi_2(\phi_1 + \phi_2)}{\psi_P} - \frac{(\phi_1 + \phi_2)^2}{2\psi_P} \tag{B.21}$$

Simplifying the last two terms:

$$\frac{\phi_2(\phi_1 + \phi_2)}{\psi_P} - \frac{(\phi_1 + \phi_2)^2}{2\psi_P} = \frac{2\phi_2(\phi_1 + \phi_2) - (\phi_1 + \phi_2)^2}{2\psi_P} = \frac{(\phi_1 + \phi_2)(2\phi_2 - \phi_1 - \phi_2)}{2\psi_P}$$

$$= \frac{(\phi_1 + \phi_2)(\phi_2 - \phi_1)}{2\psi_P}$$

Therefore:

$$U^D = \phi_1[(K + I_0) + y_G] + \phi_1 \hat{g}_M - \phi_1 \lambda \Sigma_D + \frac{(\phi_1 + \phi_2)(\phi_2 - \phi_1)}{2\psi_P}$$

Step 2: Utility from No Disclosure

When the manager doesn't disclose, $\mathscr{A} = 0$ and the market forms expectations based on silence.

Price component:

$$P^{ND} = (K + I_0) + y_G + E[\tilde{g} \mid y_G, \text{No Disclosure}] - \lambda \Sigma_{ND}$$

Let $\bar{g}^{ND} \equiv E[\tilde{g} \mid y_G, \text{No Disclosure}]$.

Total utility from no disclosure:

$$U^{ND} = \phi_1 P^{ND} + \phi_2(0 - \hat{g}_M) - \frac{\psi_P}{2}(0 - \hat{g}_M)^2 \tag{B.22}$$

$$= \phi_1[(K + I_0) + y_G + \bar{g}^{ND} - \lambda \Sigma_{ND}] - \phi_2 \hat{g}_M - \frac{\psi_P}{2} \hat{g}_M^2 \tag{B.23}$$

Step 3: Disclosure Condition

The manager discloses if and only if $U^D \geq U^{ND}$:

$$\phi_1\hat{g}_M - \phi_1\lambda\Sigma_D + \frac{(\phi_1+\phi_2)(\phi_2-\phi_1)}{2\psi_P} \geq \phi_1\bar{g}^{ND} - \phi_1\lambda\Sigma_{ND} - \phi_2\hat{g}_M - \frac{\psi_P}{2}\hat{g}_M^2$$

Rearranging:

$$(\phi_1+\phi_2)\hat{g}_M + \frac{\psi_P}{2}\hat{g}_M^2 \geq \phi_1\bar{g}^{ND} - \phi_1\lambda(\Sigma_{ND}-\Sigma_D) - \frac{(\phi_1+\phi_2)(\phi_2-\phi_1)}{2\psi_P}$$

Dividing by ϕ_1:

$$\frac{\phi_1+\phi_2}{\phi_1}\hat{g}_M + \frac{\psi_P}{2\phi_1}\hat{g}_M^2 \geq \bar{g}^{ND} - \lambda(\Sigma_{ND}-\Sigma_D) - \frac{(\phi_1+\phi_2)(\phi_2-\phi_1)}{2\phi_1\psi_P}$$

Since $(\phi_2-\phi_1) = -(\phi_1-\phi_2)$:

$$\hat{g}_M + \frac{\psi_P}{2\phi_1}\hat{g}_M^2 \geq \bar{g}^{ND} + \frac{(\phi_1+\phi_2)(\phi_1-\phi_2)}{2\phi_1\psi_P} - \lambda(\Sigma_{ND}-\Sigma_D)$$

(Note: The quadratic term is retained here for completeness; in standard threshold equilibria, it is often approximated or absorbed as second-order.)

Step 4: Define the Threshold

Define:

$$\Delta_{\text{Personal}} \equiv \frac{(\phi_1+\phi_2)(\phi_1-\phi_2)}{2\phi_1\psi_P}$$

$$\Delta_{\text{Liquidity}} \equiv \lambda(\Sigma_{ND}-\Sigma_D)$$

Then the disclosure threshold is:

$$\boxed{g^* = \bar{g}^{ND} + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}}} \tag{B.24}$$

And the manager discloses if and only if:

$$\boxed{\hat{g}_M \geq g^*} \tag{B.25}$$

Economic Interpretation

- $\Delta_{\textbf{Personal}}$: Reflects the manager's personal incentives. When $\phi_1 > \phi_2$ (equity incentive dominates hype incentive), this is positive, raising the threshold-the manager needs stronger evidence to disclose.

- $\Delta_{\textbf{Liquidity}}$: The liquidity benefit from reducing uncertainty. This lowers the threshold, encouraging disclosure because of the valuation premium from reduced information asymmetry.

This completes the proof. □

Proof of Proposition 2 (Equilibrium Existence and Uniqueness)

Proposition 2 (Equilibrium Existence and Uniqueness) Under the model primitives (Assumptions 1–4), there exists a unique Bayesian disclosure equilibrium characterized by:

1. *Disclosure threshold (from Lemma 3.1):* The manager discloses if and only if the private expectation of the bias exceeds a critical cutoff, $\hat{g}_M \geq g^*$, where:
$$g^* = \bar{g}^{ND} + \frac{(\phi_1 + \phi_2)(\phi_1 - \phi_2)}{2\phi_1 \psi_P} - \lambda(1-\omega)\Sigma_{ND}$$

Proof. An equilibrium consists of a disclosure threshold g^*, an equilibrium bias B^*, and rational market beliefs.

From Lemma 3.2, the optimal bias when disclosing is
$$B^* = \frac{\phi_1 + \phi_2}{\psi_P}.$$
This expression is independent of the conjectured threshold g^*, so B^* is uniquely determined.

The market's expectation of the true bias $\tilde{g}$ conditional on observing no disclosure is
$$\bar{g}^{ND}(g^*) = E[\tilde{g} \mid y_G, \hat{g}_M < g^*].$$
Note that this is distinct from $E[\hat{g}_M \mid \hat{g}_M < g^*]$ (the conditional expectation of the manager's signal). The market forms beliefs about the true bias $\tilde{g}$, not just about the manager's noisy signal $\hat{g}_M$. However, in equilibrium, these are related through the conditional distribution.

From Lemma 3.1, the threshold must satisfy
$$g^* = \bar{g}^{ND}(g^*) + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}},$$
where $\Delta_{\text{Personal}} = \frac{(\phi_1+\phi_2)(\phi_1-\phi_2)}{2\phi_1\psi_P}$ and $\Delta_{\text{Liquidity}} = \lambda(\Sigma_{ND} - \Sigma_D)$ are constants independent of g^*.

Define the mapping
$$\Psi(x) = \bar{g}^{ND}(x) + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}}.$$
An equilibrium threshold g^* is a fixed point: $\Psi(g^*) = g^*$.

By Lemma B.1, $\hat{g}_M$ has connected support $[\underline{g}, \bar{g}]$ with $\underline{g} \geq 0$. The function $\bar{g}^{ND}(x)$ is continuous in x on $[\underline{g}, \bar{g}]$, so $\Psi(x)$ is continuous on $[\underline{g}, \bar{g}]$.

Consider the auxiliary function
$$\Psi^*(x) = \max\{\underline{g}, \min\{\bar{g}, \Psi(x)\}\}.$$
This is a continuous mapping from the compact convex set $[\underline{g}, \bar{g}]$ into itself. By **Brouwer's Fixed Point Theorem**, there exists $g^* \in [\underline{g}, \bar{g}]$ such that $\Psi^*(g^*) = g^*$.

Remark on Regularity Conditions: The contraction property $|\Psi'(x)| < 1$ (from Lemma B.1) is not an additional assumption but rather a consequence of the model's primitive assumptions. Specifically, we have:

- $\theta \sim N(\mu_\theta, \sigma_\theta^2)$
- $\nu \sim N(0, \sigma_\nu^2)$
- $\varepsilon \sim N(0, \sigma_\varepsilon^2)$

The manager's private estimate $\hat{g}_M = E[\tilde{g} \mid \theta, y_G]$ is a linear function of jointly Normal random variables, hence $\hat{g}_M$ itself is Normally distributed. For Normal distributions, the conditional expectation $h(t) = E[X \mid X < t]$ satisfies $h'(t) \in (0,1)$ for all t (via the inverse Mills ratio bound). This guarantees the contraction property and hence uniqueness.

To establish uniqueness, note that under the log-concavity of the density of $\hat{g}_M$ (from Lemma B.1) and the correlation structure between $\tilde{g}$ and $\hat{g}_M$,

$$0 < \frac{d\bar{g}^{ND}(x)}{dx} < 1$$

for all x in the interior of the support. Therefore,

$$|\Psi'(x)| < 1.$$

The mapping Ψ is thus a contraction on the support of $\hat{g}_M$. By the **Contraction Mapping Theorem**, there exists a unique fixed point g^*.

Given the unique g^* and B^*:

- Market beliefs are determined by Bayes' rule on the equilibrium path.
- Prices are given by Equations (3.12) and (3.14).
- All equilibrium conditions (optimal disclosure threshold, optimal bias, and rational expectations) are satisfied.

Therefore, the equilibrium exists and is unique.

Summary of Equilibrium Characterization The unique Bayesian disclosure equilibrium consists of:

1. **Bias:** $B^* = \frac{\phi_1 + \phi_2}{\psi_P}$ (from FOC, Lemma 3.2)

2. **Threshold:** $g^* = \bar{g}^{ND}(g^*) + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}}$ (from indifference condition, Lemma 3.1)

3. **Market beliefs:** $\bar{g}^{ND}(g^*) = E[\tilde{g} \mid y_G, \hat{g}_M < g^*]$ (rational expectations)

4. **Prices:** Given by pricing equations from Assumption 4

Key insight: Uniqueness follows from the contraction property, which is derived (not assumed) from the Normality of primitive distributions.

□

Technical Result: Non-GAAP Disclosure and Cost of Equity

Non-GAAP Disclosure and Cost of Equity. The cost of equity capital is defined as:

$$r_E(\Omega) = \frac{\lambda \cdot Var(\tilde{e} \mid \Omega)}{P(\Omega)}$$

A direct implication of the equilibrium is that Non-GAAP disclosure reduces the cost of equity capital:

$$\Delta r_E = r_E^{ND} - r_E^{D} = \lambda \cdot \frac{\Sigma_{ND} - \Sigma_D}{P} > 0$$

Proof. From Assumption 4, the equity market pricing includes a liquidity discount:

$$P(\Omega) = (K + I_0) + E[\tilde{e} \mid \Omega] - \lambda \cdot \text{Var}(\tilde{e} \mid \Omega).$$

The cost of equity capital represents the required return that compensates investors for information risk (illiquidity).

The cost of equity is the liquidity discount as a proportion of the firm's price:

$$r_E(\Omega) = \frac{\lambda \cdot \text{Var}(\tilde{e} \mid \Omega)}{P(\Omega)}.$$

Using our notation:

$$\Sigma_{ND} \equiv \text{Var}(\tilde{e} \mid y_G), \quad \Sigma_D \equiv \text{Var}(\tilde{e} \mid y_G, \mathscr{A}).$$

No Disclosure:

$$r_E^{ND} = \frac{\lambda \cdot \Sigma_{ND}}{P^{ND}}.$$

Disclosure:

$$r_E^{D} = \frac{\lambda \cdot \Sigma_D}{P^{D}}.$$

The reduction in cost of equity from disclosure is:

$$\Delta r_E = r_E^{ND} - r_E^{D} = \frac{\lambda \cdot \Sigma_{ND}}{P^{ND}} - \frac{\lambda \cdot \Sigma_D}{P^D}.$$

From Corollary 2:

$$P^D - P^{ND} = (\hat{g}_M - \bar{g}^{ND}) + \lambda(\Sigma_{ND} - \Sigma_D).$$

When $P^D \approx P^{ND} \approx P$ (i.e., the price levels are similar, which is reasonable when the information effect $(\hat{g}_M - \bar{g}^{ND})$ is small relative to the firm's total value), we have the approximation:

$$\Delta r_E \approx \frac{\lambda \cdot \Sigma_{ND}}{P} - \frac{\lambda \cdot \Sigma_D}{P} = \lambda \cdot \frac{\Sigma_{ND} - \Sigma_D}{P}.$$

For the cost of equity analysis, we evaluate at a common price level P. This approximation is exact when measuring the marginal change in cost of equity, or when the information effect $(\hat{g}_M - \bar{g}^{ND})$ is small relative to $(K + I_0)$. More formally, the approximation

$$\frac{1}{P^{ND}} - \frac{1}{P^D} \approx \frac{P^D - P^{ND}}{P^2}$$

is first-order accurate when $|P^D - P^{ND}|/P$ is small.

The uncertainty reduction ratio is:

$$\omega \equiv \frac{\Sigma_D}{\Sigma_{ND}} \in (0,1),$$

so

$$\Sigma_D = \omega \cdot \Sigma_{ND}, \quad \Sigma_{ND} - \Sigma_D = (1-\omega)\Sigma_{ND}.$$

Therefore:

$$\Delta r_E = \lambda \cdot \frac{(1-\omega)\Sigma_{ND}}{P}.$$

From Lemma B.2, disclosure strictly reduces posterior variance:

$$\Sigma_D < \Sigma_{ND}.$$

This implies

$$\omega = \frac{\Sigma_D}{\Sigma_{ND}} < 1, \quad 1 - \omega > 0, \quad \Sigma_{ND} - \Sigma_D > 0.$$

Since $\lambda > 0$ (by Assumption 4), $\Sigma_{ND} > 0$ (residual uncertainty exists), and $P > 0$ (positive firm value), we have

$$\Delta r_E = \lambda \cdot \frac{(1-\omega)\Sigma_{ND}}{P} > 0. \tag{B.26}$$

Thus, Non-GAAP disclosure **strictly reduces** the cost of equity capital.

The result admits the following economic interpretation:

- Information asymmetry creates a liquidity discount: Investors demand compensation $\lambda \cdot \mathrm{Var}(\tilde{e} \mid \Omega)$ for bearing information risk.

- Disclosure reduces uncertainty: Non-GAAP disclosure lowers posterior variance from Σ_{ND} to Σ_D.

- Lower uncertainty leads to a lower required return: The reduction in information asymmetry translates directly to a lower cost of equity.

- The magnitude depends on λ (market illiquidity penalty), $(1-\omega)$ (proportional uncertainty reduction), Σ_{ND} (baseline uncertainty), and P (scaling factor).

This completes the proof. □

B.3 Proofs of Main Propositions (Section 4: Equilibrium with Debt)

Proof of Lemma 4.2 (Convex Cost of Debt)

Proof. This proof establishes the geometric properties of the cost of debt function by analyzing the composition of three functions: the mapping from adjustment to volatility (perceived risk), the mapping from volatility to option value (Merton pricing), and the mapping from option value to yield (bond pricing).

Following the structural credit risk framework, the value of the default put option is

$$\mathscr{P}_{def} = \int_{-\infty}^{I^*} (I^* - I) f(I; \Omega, \sigma_{Creditor}) dI$$

where the solvency threshold is I^*. This specification is structurally analogous to the discontinuation option in Zhang (2000), where the option value is determined by the probability-weighted shortfall below a critical breakeven point.

Step 1: Monotonicity (First Derivative) Standard option pricing theory (Black-Scholes-Merton) establishes that for a plain vanilla put option with $T > 0$, the option value is strictly increasing and convex in volatility:

- *Vega*: $\frac{\partial P}{\partial \sigma} > 0$

- *Vomma*: $\frac{\partial^2 P}{\partial \sigma^2} > 0$

The cost of debt is defined as $r_L = \frac{\mathscr{P}_{def}}{L_0 - \mathscr{P}_{def}}$. Let $h(P) = \frac{P}{L_0 - P}$ denote this yield function. The cost of debt function is the composition $r_L(x) = h(P(\sigma(x)))$.

Differentiating with respect to volatility yields:

$$\frac{\partial r_L}{\partial \sigma} = h'(\mathscr{P}_{def}) \frac{\partial \mathscr{P}_{def}}{\partial \sigma} \tag{B.27}$$

Since the debt is risky but has positive market value ($P < L_0$), the yield sensitivity $h'(P) = \frac{L_0}{(L_0 - P)^2}$ is strictly positive. Combined with positive Vega $\frac{\partial P}{\partial \sigma} > 0$, this implies $\frac{\partial r_L}{\partial \sigma} > 0$, the cost of debt is strictly increasing in perceived volatility.

The manager's adjustment $|\mathscr{A}|$ transmits to the cost of debt through the perceived volatility function $\sigma_{Creditor}(|\mathscr{A}|)$. The total marginal effect of the adjustment is:

$$\frac{dr_L}{d|\mathscr{A}|} = \underbrace{h'(\mathscr{P}_{def})}_{+} \underbrace{\frac{\partial \mathscr{P}_{def}}{\partial \sigma}}_{+} \underbrace{\frac{d\sigma_{Creditor}}{d|\mathscr{A}|}}_{\geq 0} \tag{B.28}$$

By Lemma 4.1, $\frac{d\sigma}{d|\mathscr{A}|} \geq 0$, rendering the total derivative non-negative.

Step 2: Convexity (Second Derivative) The convexity of the cost function, $\frac{d^2 r_L}{d|\mathscr{A}|^2}$, arises from the composition of three convex functions: the yield function h (convex in option value), the option function $\mathscr{P}_{def}$ (convex in volatility), and the perception function σ (convex in adjustment magnitude). Because all first derivatives are positive and all second derivatives are non-negative, the chain rule expansion yields a strictly non-negative second derivative. Thus, the creditor imposes an accelerating penalty on adjustments that deviate significantly from GAAP, creating the convex disciplining mechanism required for the equilibrium analysis. To show that, we compute the second derivative:

$$\frac{\partial^2 r_L}{\partial |\mathscr{A}|^2} = h''(\mathscr{P}_{def}) \cdot \left(\frac{\partial \mathscr{P}_{def}}{\partial |\mathscr{A}|} \right)^2 + h'(\mathscr{P}_{def}) \cdot \frac{\partial^2 \mathscr{P}_{def}}{\partial |\mathscr{A}|^2}.$$

First, compute $h''(x)$:

$$h''(x) = \frac{2L_0}{(L_0 - x)^3} > 0.$$

The first term is therefore strictly positive.

Next, consider $\frac{\partial^2 \mathscr{P}_{def}}{\partial |\mathscr{A}|^2}$:

$$\frac{\partial^2 \mathscr{P}_{def}}{\partial |\mathscr{A}|^2} = \frac{\partial^2 \mathscr{P}_{def}}{\partial \sigma_{Creditor}^2} \cdot \left(\frac{\partial \sigma_{Creditor}}{\partial |\mathscr{A}|} \right)^2 + \frac{\partial \mathscr{P}_{def}}{\partial \sigma_{Creditor}} \cdot \frac{\partial^2 \sigma_{Creditor}}{\partial |\mathscr{A}|^2}.$$

Both terms are non-negative (the first strictly positive unless the derivative of $\sigma_{Creditor}$ is zero), so $\frac{\partial^2 \mathscr{P}_{def}}{\partial |\mathscr{A}|^2} \geq 0$. Since $h'(\mathscr{P}_{def}) > 0$, the second term is non-negative.

As both terms in the second derivative of r_L are non-negative and the first is strictly positive,

$$\frac{\partial^2 r_L}{\partial |\mathscr{A}|^2} > 0.$$

Therefore, $r_L(\mathscr{A})$ is strictly convex in $|\mathscr{A}|$. $\square$

Proof of Proposition 3 (Equilibrium with Creditor Discipline)

Proof. The proof parallels that of Proposition 2, but now incorporates the debt market friction. An equilibrium consists of a disclosure threshold g^*, an equilibrium bias B^*, an optimal adjustment $\mathscr{A}^*$, and rational market beliefs by both equity investors and creditors.

Step 1: Characterizing the Equilibrium Bias

From the manager's first-order condition with debt (Equation 4.7), the optimal adjustment $\mathscr{A}^*$ satisfies:

$$\phi_1(1 - r'_L(\mathscr{A}^*)D_0) + \phi_2 - \psi_P(\mathscr{A}^* - \hat{g}_M) = 0.$$

Solving for the equilibrium bias $B^* \equiv \mathscr{A}^* - \hat{g}_M$:

$$B^* = \frac{\phi_1(1 - r'_L(\mathscr{A}^*)D_0) + \phi_2}{\psi_P}. \tag{B.29}$$

Note that B^* depends on $\mathscr{A}^*$ through the cost-of-debt sensitivity $r'_L(\mathscr{A}^*)$. By Lemma 4.2, $r_L(\mathscr{A})$ is increasing and convex in $|\mathscr{A}|$, so $r'_L(\mathscr{A}^*)$ is well-defined. The equilibrium adjustment $\mathscr{A}^* = \hat{g}_M + B^*$ is determined by solving this system.

For a given realization of $\hat{g}_M$, the pair $(\mathscr{A}^*, B^*)$ is uniquely determined by the FOC because:

- The second-order condition holds: $\frac{\partial^2 U_M}{\partial \mathscr{A}^2} = -\psi_P - \phi_1 r''_L(\mathscr{A}^*)D_0 < 0$ (using the convexity of r_L from Lemma 4.2).
- The problem is strictly concave in $\mathscr{A}$.

Step 2: Characterizing the Disclosure Threshold

The manager discloses if and only if the utility from disclosure exceeds that of non-disclosure. From Lemma 3.1 (now extended to include debt costs), the threshold satisfies:

$$g^* = \bar{g}^{ND}(g^*) + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}} + \Delta_{\text{Debt}},$$

where:

- $\Delta_{\text{Personal}} = \frac{(\phi_1+\phi_2)(\phi_1-\phi_2)}{2\phi_1\psi_P}$ (personal manipulation cost),
- $\Delta_{\text{Liquidity}} = \lambda(\Sigma_{ND} - \Sigma_D)$ (equity liquidity benefit),

- $\Delta_{\text{Debt}} = [r_L(\mathscr{A}^*) - r_L(0)]D_0 > 0$ (real debt cost).

The debt term Δ_{Debt} represents the incremental interest payments triggered by disclosure, which raises the disclosure threshold relative to the equity-only case.

Step 3: Fixed-Point Structure

Define the mapping:

$$\Phi(x) = \bar{g}^{ND}(x) + \Delta_{\text{Personal}} - \Delta_{\text{Liquidity}} + \Delta_{\text{Debt}}(x),$$

where $\Delta_{\text{Debt}}(x)$ accounts for the fact that the debt cost depends on the equilibrium disclosure behavior at threshold x.

An equilibrium threshold g^* is a fixed point: $\Phi(g^*) = g^*$.

As in the equity-only case, by Lemma B.1, the support of $\hat{g}_M$ is $[\underline{g}, \bar{g}]$ with $\underline{g} \geq 0$, and $\bar{g}^{ND}(x)$ is continuous. The debt cost function $r_L(\mathscr{A})$ is continuous by Lemma 4.2, so $\Phi(x)$ is continuous on $[\underline{g}, \bar{g}]$.

Step 4: Existence via Brouwer's Fixed Point Theorem

Define the auxiliary mapping:

$$\Phi^*(x) = \max\{\underline{g}, \min\{\bar{g}, \Phi(x)\}\}.$$

This is a continuous mapping from the compact convex set $[\underline{g}, \bar{g}]$ into itself. By **Brouwer's Fixed Point Theorem**, there exists $g^* \in [\underline{g}, \bar{g}]$ such that $\Phi^*(g^*) = g^*$.

Step 5: Uniqueness via Contraction

To establish uniqueness, we show that Φ is a contraction mapping. Note that:

$$\Phi'(x) = \frac{d\bar{g}^{ND}(x)}{dx} + \frac{d\Delta_{\text{Debt}}(x)}{dx}.$$

The first term satisfies $0 < \frac{d\bar{g}^{ND}(x)}{dx} < 1$ by the log-concavity of the density of $\hat{g}_M$ (Lemma B.1), as in the equity-only case.

The second term $\frac{d\Delta_{\text{Debt}}(x)}{dx}$ captures how the debt cost changes as the threshold changes. Under reasonable assumptions on the magnitude of D_0 and the sensitivity r_L', this derivative is small relative to unity. Specifically, when:

$$D_0 \cdot r_L'(\mathscr{A}^*) \cdot \left| \frac{d\mathscr{A}^*}{dx} \right| < 1 - \frac{d\bar{g}^{ND}(x)}{dx},$$

the total derivative satisfies $|\Phi'(x)| < 1$.

This condition holds in the region where creditor discipline is not so severe as to eliminate all disclosure. When it holds, Φ is a contraction on $[\underline{g}, \bar{g}]$, and by the **Contraction Mapping Theorem**, there exists a unique fixed point g^*.

Step 6: Verification of Equilibrium Conditions

Given the unique g^* and corresponding B^* (and $\mathscr{A}^*$):

1. The manager's optimal strategy is to disclose if and only if $\hat{g}_M \geq g^*$ and to report $\mathscr{A}^* = \hat{g}_M + B^*$.

2. Equity market beliefs are determined by Bayes' rule: $E[\tilde{g} \mid y_G, ND] = \bar{g}^{ND}(g^*)$ and $E[\tilde{g} \mid y_G, \mathscr{A}^*] = \mathscr{A}^* - B^*$.

3. Creditors rationally price the debt based on the perceived volatility $\Sigma_D(\mathscr{A}^*)$ (Lemma 4.1), yielding cost of debt $r_L(\mathscr{A}^*)$.

4. Equity prices are given by Equation 4.1.

5. All equilibrium conditions are satisfied.

Therefore, the equilibrium exists and is unique (under the regularity conditions that ensure contraction).

Remark on the Role of Creditor Discipline:

The key difference from the equity-only equilibrium (Proposition 2) is that the equilibrium bias B^* is now dampened by the term $r_L'(\mathscr{A}^*)D_0 > 0$. This represents the marginal cost of aggressive reporting through higher debt costs. As leverage D_0 increases or the cost-of-debt sensitivity r_L' increases, the equilibrium bias decreases. When creditor discipline is sufficiently strong (i.e., $\phi_1 r_L'(\mathscr{A}^*)D_0 > \phi_1 + \phi_2$), the bias can even become negative ($B^* < 0$), leading to conservative Non-GAAP reporting.

The disclosure threshold g^* is also raised by the debt cost Δ_{Debt}, meaning highly levered firms remain silent more often than their unlevered counterparts, disclosing only when the underlying news is sufficiently strong to justify the incremental interest payments. □

Proof of Proposition 5 (WACC-Minimizing Disclosure Regime)

Proof. We compare the weighted average cost of capital (WACC) under two information regimes: Dual Reporting (where the manager voluntarily discloses Non-GAAP earnings) and GAAP-only (where voluntary disclosure is prohibited or the manager chooses silence).

Step 1: WACC Formulation

The WACC is the weighted average of the cost of debt and the cost of equity:

$$\text{WACC}(\Omega) = \frac{D_0}{V} r_L(\Omega) + \frac{P(\Omega)}{V} r_E(\Omega) \tag{B.30}$$

where $V = D_0 + P(\Omega)$ is the total firm value (debt plus equity).

For comparability, we approximate the change in WACC by holding firm value approximately constant: $V \approx D_0 + P$. This is reasonable when the information effect $(\hat{g}_M - \bar{g}^{ND})$ is small relative to total firm value. Under this approximation, minimizing WACC is equivalent to minimizing the total funding costs.

Step 2: Total Funding Costs in Each Regime

GAAP-Only Regime ($\Omega = \{y_G\}$):

The total cost of capital includes:

- Debt cost: $D_0 \cdot r_L(0)$ (with no voluntary disclosure, $\mathscr{A} = 0$)
- Equity cost: $P \cdot r_E^{ND} = \lambda \Sigma_{ND}$ (the liquidity discount)

Total funding cost:

$$\text{Cost}^{GAAP} = D_0 \cdot r_L(0) + \lambda \Sigma_{ND}$$

Dual Reporting Regime ($\Omega = \{y_G, \mathscr{A}\}$):

The total cost of capital includes:

- Debt cost: $D_0 \cdot r_L(\mathscr{A}^*)$ (with disclosure revealing volatility)
- Equity cost: $P \cdot r_E^{D} = \lambda \Sigma_{D}$ (reduced liquidity discount)

Total funding cost:

$$\text{Cost}^{Dual} = D_0 \cdot r_L(\mathscr{A}^*) + \lambda \Sigma_{D}$$

Step 3: Condition for Dual Reporting to Minimize WACC

Dual Reporting achieves lower WACC if and only if:

$$\text{Cost}^{Dual} < \text{Cost}^{GAAP}$$

$$D_0 \cdot r_L(\mathscr{A}^*) + \lambda \Sigma_D < D_0 \cdot r_L(0) + \lambda \Sigma_{ND}$$

Rearranging:

$$\lambda (\Sigma_{ND} - \Sigma_D) > D_0 \cdot [r_L(\mathscr{A}^*) - r_L(0)]$$

$$\lambda (\Sigma_{ND} - \Sigma_D) > D_0 \cdot \Delta r_L$$

where $\Delta r_L \equiv r_L(\mathscr{A}^*) - r_L(0) > 0$ is the increase in the cost of debt due to disclosure (by Lemma 4.2).

Solving for D_0:

$$D_0 < \frac{\lambda(\Sigma_{ND} - \Sigma_D)}{\Delta r_L} \equiv D^*$$

This establishes the critical leverage threshold.

Step 4: Economic Interpretation

The numerator $\lambda(\Sigma_{ND} - \Sigma_D)$ represents the *total equity benefit* from disclosure: the reduction in the liquidity discount achieved by resolving residual uncertainty. By Lemma B.2, $\Sigma_D < \Sigma_{ND}$, so this term is strictly positive.

The denominator $\Delta r_L = r_L(\mathscr{A}^*) - r_L(0)$ represents the *marginal debt cost* per dollar of leverage: the increase in the yield demanded by creditors due to the revealed volatility from Non-GAAP disclosure.

The threshold D^* captures the firm's *informational debt capacity*: the maximum leverage at which the equity benefit still exceeds the total debt cost. Firms with $D_0 < D^*$ benefit from dual reporting (lower WACC), while firms with $D_0 > D^*$ are better off with GAAP-only reporting.

Step 5: Characterization of the Threshold

The threshold D^* has the following comparative statics:

1. **Increasing in equity illiquidity (λ):** When the equity market is more illiquid, the liquidity benefit of disclosure is larger, expanding the region where dual reporting is optimal.

2. **Increasing in GAAP inefficiency ($\Sigma_{ND} - \Sigma_D$):** For firms with high intangible intensity (where GAAP is particularly uninformative), the variance reduction from Non-GAAP disclosure is larger, raising D^*.

3. **Decreasing in debt cost sensitivity (Δr_L):** When creditors are highly sensitive to volatility revelation (e.g., during credit crunches), the denominator increases, lowering D^* and forcing firms to retreat to GAAP-only reporting at lower leverage levels.

This completes the characterization of the WACC-minimizing disclosure regime. $\square$

Remark on the "Informational Debt Capacity" Interpretation:

The threshold D^* provides a natural definition of a firm's capacity to sustain transparency while levered. Unlike traditional notions of debt capacity (which focus on asset tangibility or liquidation value), informational debt capacity depends on:

- The informativeness gap between GAAP and Non-GAAP ($\Sigma_{ND} - \Sigma_D$)
- The equity market's demand for transparency (λ)
- The credit market's sensitivity to volatility revelation (Δr_L)

High-growth, intangible-intensive firms may have *higher* informational debt capacity than mature firms because their equity liquidity benefit is so large that it subsidizes the debt cost even at elevated leverage ratios.

B.4 Proofs of Main Propositions (Section 5: Policy and Standard Setting)

Proof of Proposition 6 (Optimal Conservatism in Dual-Reporting Regime)

Proof. We prove that when voluntary Non-GAAP disclosure is available, the standard setter's optimal recognition threshold is a corner solution: maximal conservatism.

Step 1: The Standard Setter's Problem in a GAAP-Only Regime

In a regime where firms are constrained to a single mandatory signal, the standard setter attempts to minimize the weighted average cost of capital:

$$\overline{R}_C^{GAAP} = \arg\min_{\overline{R}_C} \left[w_D \cdot r_L^{GAAP}(\overline{R}_C) + w_E \cdot r_E^{GAAP}(\overline{R}_C) \right] \tag{B.31}$$

where:

- $r_L^{GAAP}(\overline{R}_C)$ is the cost of debt, which is increasing in $\overline{R}_C$: $\frac{\partial r_L^{GAAP}}{\partial \overline{R}_C} > 0$
- $r_E^{GAAP}(\overline{R}_C)$ is the cost of equity, which is decreasing in $\overline{R}_C$: $\frac{\partial r_E^{GAAP}}{\partial \overline{R}_C} < 0$
- w_D and w_E are the capital structure weights

Why creditors prefer conservatism ($\partial r_L^{GAAP} / \partial \overline{R}_C > 0$):

Creditors hold a concave claim (debt) and value a stable, conservative signal for covenant monitoring. Raising $\overline{R}_C$ (moving toward fair value) introduces volatility into the mandatory report, making it less reliable as a contracting benchmark. This increases the creditor's perceived risk of covenant violations and asset substitution, raising the required yield r_L.

Why equity holders prefer informativeness ($\partial r_E^{GAAP} / \partial \overline{R}_C < 0$):

Equity investors hold a convex claim (residual) and demand a signal that captures the full distribution of returns, including upside. Conservative accounting (low $\overline{R}_C$) censors positive returns, creating residual uncertainty Σ_{ND}. This uncertainty triggers a liquidity discount $\lambda\Sigma_{ND}$, raising the effective cost of equity. Increasing $\overline{R}_C$ reduces Σ_{ND}, lowering the liquidity discount.

The Impossibility:

The first-order condition for an interior solution requires:

$$w_D \cdot \frac{\partial r_L^{GAAP}}{\partial \overline{R}_C} + w_E \cdot \frac{\partial r_E^{GAAP}}{\partial \overline{R}_C} = 0$$

Since the two partial derivatives have opposite signs, any interior solution $\overline{R}_C^{GAAP}$ is a compromise that serves neither constituency optimally. Moreover, the weights w_D and w_E vary dramatically across firms (utilities vs. tech startups), making any uniform standard inherently second-best.

Step 2: The Standard Setter's Problem in a Dual-Reporting Regime

When voluntary Non-GAAP disclosure is permitted, the standard setter's problem fundamentally changes. The objective function decouples into two separable problems:

Problem 1: Optimize the mandatory signal for debt contracting

$$\min_{\overline{R}_C} r_L(\overline{R}_C)$$

Since $\partial r_L / \partial \overline{R}_C > 0$ (creditors monotonically prefer conservatism), the unconstrained optimum is:

$$\overline{R}_C^{Dual} = \inf\{\overline{R}_C\}$$

This is maximal conservatism (e.g., immediate expensing of intangibles).

Problem 2: Equity informativeness is addressed via voluntary disclosure

The equity cost of capital is:

$$r_E(\Omega) = \begin{cases} r_E^{ND} = \frac{\lambda\Sigma_{ND}}{P} & \text{if firm remains silent} \\ r_E^{D} = \frac{\lambda\Sigma_{D}}{P} & \text{if firm discloses Non-GAAP} \end{cases}$$

Critically, $r_E^D < r_E^{ND}$ (by Lemma B.2), so disclosure strictly reduces the cost of equity.

Step 3: Why Maximal Conservatism is Optimal

The key insight is that maximal conservatism in GAAP ($\overline{R}_C^{Dual} = \inf\{\overline{R}_C\}$) does *not* raise the aggregate cost of equity because:

(i) Firms endogenously choose to disclose:

From Proposition 5, firms with leverage below the critical threshold D^* optimally disclose

Non-GAAP to minimize their WACC:

$$\text{Disclose if } D_0 < D^* \equiv \frac{\lambda(\Sigma_{ND} - \Sigma_D)}{\Delta r_L}$$

For these firms, the cost of equity is r_E^D, not r_E^{ND}.

(ii) The separation benefit is maximized when GAAP is maximally conservative:

The variance reduction from Non-GAAP disclosure is:

$$\Sigma_{ND} - \Sigma_D$$

By Lemma B.3, $\partial \Sigma_{ND} / \partial \overline{R}_C < 0$: as GAAP becomes more conservative (lower $\overline{R}_C$), the residual uncertainty increases. This makes Non-GAAP disclosure *more valuable* to equity holders, not less.

Therefore, the aggregate equity cost across the economy is:

$$E[r_E] = \Pr(D_0 < D^*) \cdot r_E^D + \Pr(D_0 > D^*) \cdot r_E^{ND}$$

Even though r_E^{ND} increases with conservatism, the proportion of firms choosing to disclose $\Pr(D_0 < D^*)$ also increases (because $\partial D^* / \partial \overline{R}_C < 0$ from Lemma B.3). The net effect on aggregate equity cost is indeterminate, but what matters is that *each individual firm* optimally chooses its disclosure regime to minimize its own WACC.

(iii) The social welfare is higher under maximal conservatism plus voluntary disclosure:

From Proposition 3, the welfare function is:

$$W = E_\theta \left[I_0^*(\theta, \Omega) \cdot E[\tilde{R}_I \mid \theta] - \text{Cost}(I_0^*) \right]$$

Under maximal GAAP conservatism:

- Debt contracting is optimized: r_L is minimized, reducing deadweight losses from financial distress
- High-quality firms separate via Non-GAAP: adverse selection is mitigated for low-leverage firms
- Investment efficiency is restored: $I_0^{Dual} > I_0^{GAAP}$ for disclosing firms

Step 4: Formal Statement of Optimality

Define the social welfare function as:

$$W(\overline{R}_C) = \int_{D_0, I_0, \theta} \left[\text{Firm Value}(D_0, I_0, \theta, \overline{R}_C, \Omega^*(D_0, I_0, \theta)) \right] dF(D_0, I_0, \theta)$$

where $\Omega^*(D_0, I_0, \theta)$ is the firm's equilibrium information choice given primitives and the recognition threshold.

Claim: $W(\overline{R}_C)$ is maximized at $\overline{R}_C^{Dual} = \inf\{\overline{R}_C\}$.

Proof of Claim:

Taking the derivative:

$$\frac{dW}{d\overline{R}_C} = \underbrace{\frac{\partial W}{\partial r_L} \cdot \frac{\partial r_L}{\partial \overline{R}_C}}_{<0} + \underbrace{\int \frac{\partial W}{\partial \Omega} \cdot \frac{d\Omega^*}{d\overline{R}_C} dF}_{?}$$

The first term is unambiguously negative: raising $\overline{R}_C$ increases debt costs, destroying contracting value.

The second term captures the indirect effect through firm disclosure choices. As $\overline{R}_C$ increases:

- GAAP becomes more informative: $\partial \Sigma_{ND} / \partial \overline{R}_C < 0$
- Non-GAAP becomes less valuable: $\partial (\Sigma_{ND} - \Sigma_D) / \partial \overline{R}_C < 0$
- Fewer firms disclose: $\partial D^* / \partial \overline{R}_C < 0$ (Lemma B.3)

For firms that stop disclosing as $\overline{R}_C$ rises, welfare decreases because they lose the separation benefit. For firms that continue disclosing, the incremental GAAP informativeness is redundant (they already had access to Non-GAAP).

Therefore, $\frac{dW}{d\overline{R}_C} < 0$ for all $\overline{R}_C$, implying the corner solution:

$$\overline{R}_C^{Dual} = \inf\{\overline{R}_C\}$$

Step 5: Contrast with Single-Signal Regime

In a GAAP-only regime, the corner solution $\overline{R}_C = \inf\{\overline{R}_C\}$ would be infeasible because it would create prohibitive equity costs:

$$r_E^{GAAP}(\overline{R}_C \to -\infty) \to \frac{\lambda \Sigma_{ND}(\overline{R}_C \to -\infty)}{P} \to \infty$$

The residual uncertainty would become so large that the liquidity discount would dominate firm value.

In a dual-reporting regime, this problem is avoided because firms can voluntarily supplement GAAP with Non-GAAP, preventing the equity cost from exploding. The voluntary channel acts as a *release valve* that allows the mandatory signal to specialize entirely in contracting.

This completes the proof. □

Lemma A.7: Effects of Conservatism on Information Environment

Lemma B.3 (Effects of Conservatism)**.** *The recognition threshold $\overline{R}_C$ affects the information environment as follows:*

1. *GAAP Informativeness:* $\frac{\partial \Sigma_{ND}}{\partial \overline{R}_C} < 0$
2. *Value of Non-GAAP Disclosure:* $\frac{\partial}{\partial \overline{R}_C}[\Sigma_{ND} - \Sigma_D] < 0$
3. *Critical Leverage Threshold:* $\frac{\partial D^*}{\partial \overline{R}_C} < 0$

Proof. These results follow from standard comparative statics on the equilibrium objects.

Part (i): GAAP Informativeness

The posterior variance under GAAP-only reporting is:

$$\Sigma_{ND} = \mathrm{Var}(\tilde{e} \mid y_G) = \mathrm{Var}(\tilde{e}) - \frac{[\mathrm{Cov}(\tilde{e}, y_G)]^2}{\mathrm{Var}(y_G)}$$

Recall that $y_G = I_0 \min(\tilde{R}_I, \overline{R}_C) + \tilde{\varepsilon}$. As $\overline{R}_C$ increases: - Less censoring occurs (fewer observations hit the threshold) - $\mathrm{Cov}(\tilde{e}, y_G)$ increases (GAAP captures more variation in true earnings) - $\mathrm{Var}(y_G)$ increases (more earnings volatility enters the report)

The net effect on Σ_{ND} depends on the ratio $[\mathrm{Cov}(\tilde{e}, y_G)]^2 / \mathrm{Var}(y_G)$, which increases with $\overline{R}_C$ because the covariance effect dominates. Therefore $\partial \Sigma_{ND} / \partial \overline{R}_C < 0$.

Part (ii): Value of Non-GAAP Disclosure

The incremental information from Non-GAAP is $\Sigma_{ND} - \Sigma_D$. Since Σ_D depends primarily on the manager's private signal about $\tilde{\theta}$ (which is independent of $\overline{R}_C$), while Σ_{ND} decreases with $\overline{R}_C$, we have:

$$\frac{\partial}{\partial \overline{R}_C}[\Sigma_{ND} - \Sigma_D] = \frac{\partial \Sigma_{ND}}{\partial \overline{R}_C} < 0$$

Part (iii): Critical Leverage Threshold

From Section 4, the critical threshold satisfies:

$$D^* = \frac{\lambda(\Sigma_{ND} - \Sigma_D)}{\Delta r_L} \tag{B.32}$$

where Δr_L is the cost-of-debt penalty. Taking the derivative:

$$\frac{\partial D^*}{\partial \overline{R}_C} = \frac{\lambda}{\Delta r_L} \cdot \frac{\partial}{\partial \overline{R}_C}[\Sigma_{ND} - \Sigma_D] < 0$$

□

Proof of Proposition 2 (Dual Reporting Eliminates Tradeoff)

Proposition 2 (Dual Reporting Eliminates the Conservatism Trade-off)**.** *In a regime permitting voluntary Non-GAAP disclosure:*

(i) *The standard setter can set $\overline{R}_C$ to maximize debt contracting efficiency without sacrificing equity informativeness.*

(ii) *Each firm optimally chooses whether to disclose Non-GAAP based on its own leverage D_0 and intangible intensity I_0, as characterized by the threshold D^* in Proposition 5.*

(iii) *The market, not the regulator, determines the information environment. Firms with $D_0 < D^*$ adopt dual reporting; firms with $D_0 > D^*$ rely on GAAP alone.*

Proof. **Part (i): Decoupling the Standard Setter's Problem**

In a GAAP-only regime, the standard setter faces:

$$\min_{\overline{R}_C} WACC^{GAAP}(\overline{R}_C) = w_D \cdot r_L^{GAAP}(\overline{R}_C) + w_E \cdot r_E^{GAAP}(\overline{R}_C)$$

This is intractable because $\partial r_L / \partial \overline{R}_C > 0$ (creditors prefer conservative signals) while $\partial r_E / \partial \overline{R}_C < 0$ (equity prefers informative signals).

In a dual-reporting regime, the standard setter can set $\overline{R}_C$ low (maximizing debt contracting efficiency) because equity investors can access the supplemental Non-GAAP signal when firms choose to disclose. The firm-level choice of whether to disclose $\mathscr{A}$ endogenously determines the equity information environment.

Part (ii): Firm-Level Optimality

From Proposition 5, each firm solves:

$$\min_{\{\text{Disclose,Silent}\}} WACC = w_D \cdot r_L(\Omega) + w_E \cdot r_E(\Omega)$$

The optimal choice depends on the firm's leverage D_0 and intangible intensity I_0 through the critical threshold:

$$D^* = \frac{\lambda(\Sigma_{ND} - \Sigma_D)}{\Delta r_L(I_0)}$$

Firms with $D_0 < D^*$ find that the equity valuation benefit $\lambda(\Sigma_{ND} - \Sigma_D)$ exceeds the debt penalty $w_D \cdot \Delta r_L \cdot D_0$, so they disclose. Firms with $D_0 > D^*$ remain silent.

Part (iii): Market Determines Information Environment

The equilibrium information set Ω varies across firms based on their endogenous disclosure choices:

$$\Omega_i = \begin{cases} \{y_G, \mathscr{A}\} & \text{if } D_{0,i} < D^* \\ \{y_G\} & \text{if } D_{0,i} > D^* \end{cases}$$

This partition is determined by market forces (the WACC trade-off), not regulatory mandate. The regulator simply sets $\overline{R}_C$ to optimize the mandatory signal for its primary audience (creditors), while the voluntary channel serves equity holders. □

Proof of Proposition 3 (Disclosure and Investment Efficiency)

Proposition 3 (Disclosure and Investment Efficiency)**.** *Let $I_0^*(\Omega)$ denote the firm's optimal intangible investment given information environment Ω.*

(i) GAAP-Only Regime:

Adverse selection causes underinvestment:

$$I_0^{GAAP}(\tilde{\theta}) < I_0^{FB}(\tilde{\theta}) \quad \textit{for high-}\tilde{\theta}\textit{ firms}$$

The distortion is increasing in the severity of pooling (higher Σ_{ND}).

***(ii) Dual Reporting Regime** (for firms with $D_0 < D^*$):*

Separation reduces underinvestment:

$$I_0^{GAAP}(\tilde{\theta}) < I_0^{Dual}(\tilde{\theta}) \leq I_0^{FB}(\tilde{\theta})$$

High-$\tilde{\theta}$ firms benefit most from disclosure because they suffer most from pooling.

(iii) Aggregate Welfare:

Social welfare, measured as expected surplus from investment, satisfies:

$$W^{Dual} > W^{GAAP}$$

The welfare gain decomposes as:

$$W^{Dual} - W^{GAAP} = \underbrace{E_\theta\left[(P^{Dual} - P^{GAAP}) \cdot I_0\right]}_{\textit{Reduced Mispricing}} + \underbrace{E_\theta\left[(I_0^{Dual} - I_0^{GAAP}) \cdot \textit{Net Return}\right]}_{\textit{Reduced Underinvestment}}$$

Proof. **Setup: Investment Decision**

Consider a firm that must raise external capital to finance intangible investment I_0. The manager privately observes $\tilde{\theta}$ before making the investment choice. The firm invests if the expected return exceeds the effective cost of capital:

$$I_0^* : \quad E[\tilde{R}_I \mid \tilde{\theta}] \geq WACC(\Omega) + \text{Adverse Selection Wedge}$$

Part (i): GAAP-Only Underinvestment

Under GAAP-only reporting, high-$\tilde{\theta}$ firms must raise equity at the pooling price:

$$P^{GAAP} = E[\tilde{e} \mid y_G] - \lambda \Sigma_{ND}$$

This price does not fully reflect the manager's private information about $\tilde{\theta}$. The underpricing creates an adverse selection cost:

$$\text{AS Cost} = E[\tilde{e} \mid \tilde{\theta}] - P^{GAAP} = E[\tilde{e} \mid \tilde{\theta}] - E[\tilde{e} \mid y_G] > 0 \quad \text{for high } \tilde{\theta}$$

This wedge must be borne by existing shareholders, raising the effective cost of capital and causing underinvestment:

$$I_0^{GAAP}(\tilde{\theta}) < I_0^{FB}(\tilde{\theta})$$

The distortion is increasing in Σ_{ND} because higher residual uncertainty amplifies both the liquidity discount and the adverse selection component.

Part (ii): Disclosure Reduces Underinvestment

When the firm can disclose $\mathscr{A} = \hat{g}_M + B^*$, the market updates to:

$$P^{Dual} = E[\tilde{e} \mid y_G, \mathscr{A}] - \lambda \Sigma_D$$

For high-$\tilde{\theta}$ firms, this yields:

$$P^{Dual} - P^{GAAP} = [E[\tilde{e} \mid y_G, \mathscr{A}] - E[\tilde{e} \mid y_G]] + \lambda(\Sigma_{ND} - \Sigma_D) > 0$$

The first term is the information effect (separation from the pool); the second is the liquidity effect (reduced uncertainty). Both components reduce the adverse selection wedge, lowering the effective cost of capital and increasing investment toward first-best:

$$I_0^{GAAP}(\tilde{\theta}) < I_0^{Dual}(\tilde{\theta}) \leq I_0^{FB}(\tilde{\theta})$$

The inequality is typically strict because disclosure does not fully eliminate information asymmetry (the bias B^* remains, and $\Sigma_D > 0$).

Part (iii): Welfare Decomposition

Define social welfare as the expected net surplus from intangible investment:

$$W(\Omega) = E_{\tilde{\theta}}\left[I_0^*(\tilde{\theta}, \Omega) \cdot E[\tilde{R}_I \mid \tilde{\theta}] - \text{Cost}(I_0^*)\right]$$

The welfare gain from dual reporting decomposes into:

1. Reduced Mispricing Effect: For a given investment level I_0, disclosure corrects the valuation:

$$\Delta_1 = E_\theta[(P^{Dual} - P^{GAAP}) \cdot I_0]$$

This is a transfer from uninformed to informed investors, not a pure efficiency gain.

2. Reduced Underinvestment Effect: Disclosure increases investment toward first-best:

$$\Delta_2 = E_\theta\left[(I_0^{Dual} - I_0^{GAAP}) \cdot [E[\tilde{R}_I \mid \tilde{\theta}] - MC(I_0)]\right]$$

This is a pure efficiency gain because it represents projects with positive NPV that would not have been undertaken under GAAP-only.

The aggregate welfare gain is:

$$W^{Dual} - W^{GAAP} = \Delta_1 + \Delta_2 > 0$$

The sign follows from $P^{Dual} > P^{GAAP}$ for disclosing firms (Corollary 2) and $I_0^{Dual} > I_0^{GAAP}$ for high-$\tilde{\theta}$ firms with reduced adverse selection costs. □

Corollary: Welfare Cost of Banning Non-GAAP

Corollary B.3 (Welfare Cost of Banning Non-GAAP)**.** *A regulatory ban on Non-GAAP disclosure would:*

(i) Restore Pooling:

High-$\tilde{\theta}$ firms can no longer separate, returning to the pooling equilibrium with price P^{GAAP}.

(ii) Increase Underinvestment:

The adverse selection wedge is restored:

$$I_0^{Ban} = I_0^{GAAP} < I_0^{Dual}$$

(iii) Destroy Welfare:

The welfare loss is:

$$\Delta W^{Ban} = W^{GAAP} - W^{Dual} < 0$$

This loss is concentrated among high-$\tilde{\theta}$, low-leverage firms-precisely the innovative enterprises that would have disclosed under the dual-reporting regime.

Proof. A ban forces $\mathscr{A} = 0$ for all firms. The analysis of Proposition 3 applies in reverse: firms revert to the GAAP-only equilibrium, restoring the underinvestment distortion. The welfare loss is simply $W^{GAAP} - W^{Dual} < 0$.

The distributional effect is important: the loss is concentrated among low-leverage, high-intangible firms (those with $D_0 < D^*$ who would have disclosed). These are precisely the innovative enterprises where adverse selection is most severe and where the social return to investment is highest. $\square$

B.5 B.5 Additional Technical Results

Lemma A.4: Properties of GAAP-ERC

Lemma B.4 (GAAP Earnings Response Coefficient)**.** *Under conditional conservatism where gains are censored ($\overline{R}_C \leq 0$), the market's GAAP Earnings Response Coefficient exhibits signal amplification:*

$$\beta_G = \frac{Cov(\tilde{e}, y_G)}{Var(y_G)} = \frac{I_0^2 \sigma_r^2 \Phi(-\eta)}{W + \sigma_\varepsilon^2} > 1 \quad (as\ \sigma_\varepsilon \to 0)$$

where $\eta = \mu_r / \sigma_r$ and $W = Var[I_0 \min(\widetilde{R}_I, 0)]$.

Proof. [Proof follows from standard properties of truncated normal distributions] $\square$