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Probability Assignment 4

1)

ⓐ It is a repeated Bernoulli experiment where tests can be either positive (true) or negative (false).

Parameters: $X \sim \text{Binomial}(n=12, p)$

Where n is number of trials and p is probability of success

ⓑ $$L(p) = P(X=x_1)\dots P(X=x_k) = \prod_{i=1}^{k} P(X=x_i)$$

$$P(X=x) = \binom{k}{x} p^x (1-p)^{k-x}$$

$$L(p) = \prod_{i=1}^{k} \binom{n}{x} p^{x_i} (1-p)^{n-x_i}$$

$$= \left[p^{\sum_{i=1}^{k} x_i}\right]\left[(1-p)^{nk-\sum_{i=1}^{k} x_i}\right]\left[\prod_{i=1}^{k} \binom{n}{x_i}\right]$$

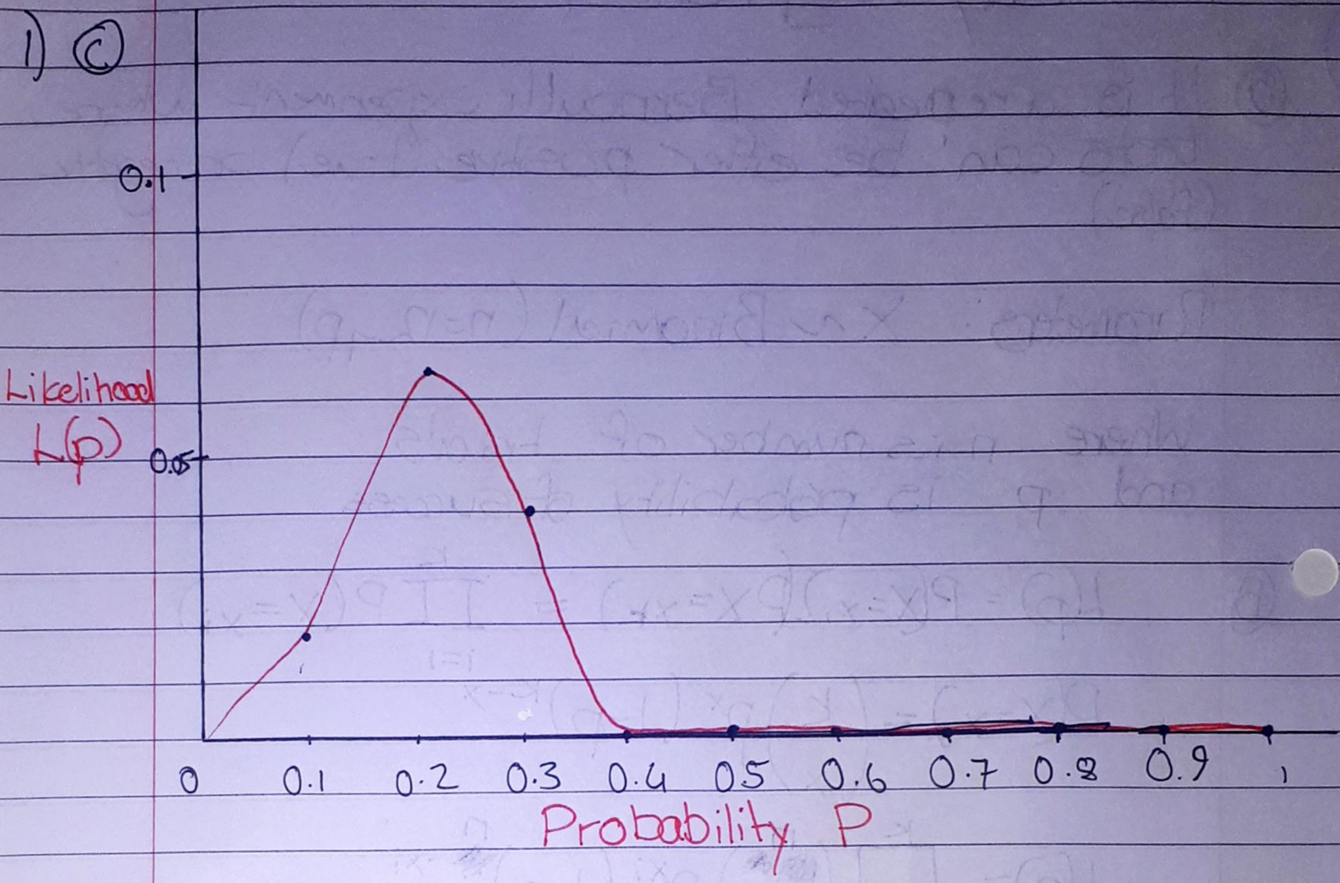
$$= \left[p^{3+2}\right]\left[(1-p)^{24-5}\right]\left[\binom{12}{3}\binom{12}{2}\right]$$

$$= \left[p^5\right]\left[(1-p)^{19}\right]\left[(220)(66)\right]$$

$$= \left[p^5\right]\left[(1-p)^{19}\right]\left[14520\right]$$

$$= 14520\, p^5 (1-p)^{19}$$

1) (c)



0 = 0

0.1 = 0.01961

0.2 = 0.06696

0.3 = 0.04022

0.4 = 0.000906

0.5 = 0.00008655

0.6 = 0.000003104

0.7 = 0.0000000284

0.8 = 0.000000000002495

0.9 = 0.0000000000000000008574

1 = 0

1) ⓐ MLE

$$L(p) = 14520\, p^5 (1-p)^{19}$$

$$l(p) = \log\left[14520\, p^5 (1-p)^{19}\right]$$

$$= \log(14520 p^5) + \log((1-p)^{19})$$

Derivative

$$\frac{d}{dp}\left[\log(14520 p^5) + \log((1-p)^{19})\right]$$

$$= \frac{\cancel{14520}(5)p^4}{\cancel{14520}\,p^5} + \frac{-19\,\cancel{(1-p)^{18}}}{(1-p)^{\cancel{19}}}$$

$$= \frac{5\cancel{p^4}}{p^5} - \frac{19}{(1-p)}$$

$$= \frac{5}{p} - \frac{19}{1-p}$$

$$= 5 - 5p - 19p$$

$$= 5 - 24p$$

$$0 = 5 - 24p$$

$$5 = 24p$$

$$p = \frac{5}{24} = \underline{0.20833} \quad \text{MLE}$$