Statistical Inference Project Part 1 (Simulation)

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We will be simulating the exponential distribution in R with `rexp(n, rate)` (from the documentation rexp(n, rate = 1)). The rate will be signified by $\lambda.

- \(1/\lambda \) is the mean of the expoential distribution.
- The standard deviation is \(1/\lambda \).

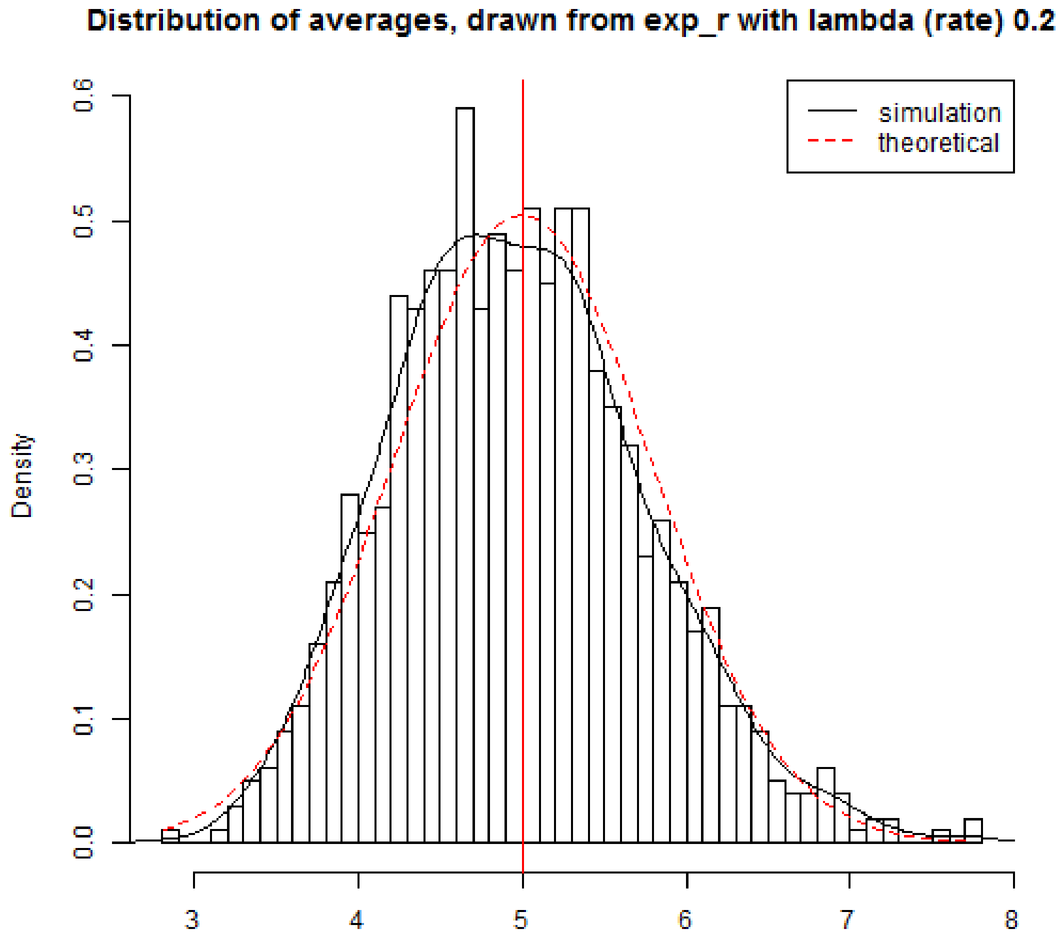
Simulation:

we set \(\lambda \) (the rate) to 0.2. In this simulation, will take the averages of 40 numbuers sampled from the distribution. We will do 1000 simulated averages of the 40 exponentials.

```
set.seed(1234)
lambda <- 0.2
number_of_simulation <- 1000
size_of_sample <- 40

# Create the simluation
sim <- matrix(rexp(number_of_simulation*size_of_sample,
rate=lambda), number_of_simulation, size_of_sample)

#now take an average of the simluations
row_avgs <- rowMeans(sim)
```

Now we will plot the sample means

```
# plot the histogram of averages
hist(row_avgs, breaks=50, prob=TRUE,
     main="Distribution of averages, drawn from exp_r with
lambda (rate) 0.2", xlab="")

# Now we will add the density
lines(density(row_avgs))


# plot the theoretical center of distribution
abline(v=1/lambda, col="red")

# plot theoretical density of the averages of samples
xfit <- seq(min(row_avgs), max(row_avgs), length=100)

yfit <- dnorm(xfit, mean=1/lambda, sd=
(1/lambda/sqrt(size_of_sample)))

lines(xfit, yfit, pch=22, col="red", lty=2)
# Place legend on the screen
legend('topright', c("simulation", "theoretical"),
lty=c(1,2), col=c("black", "red"))
```

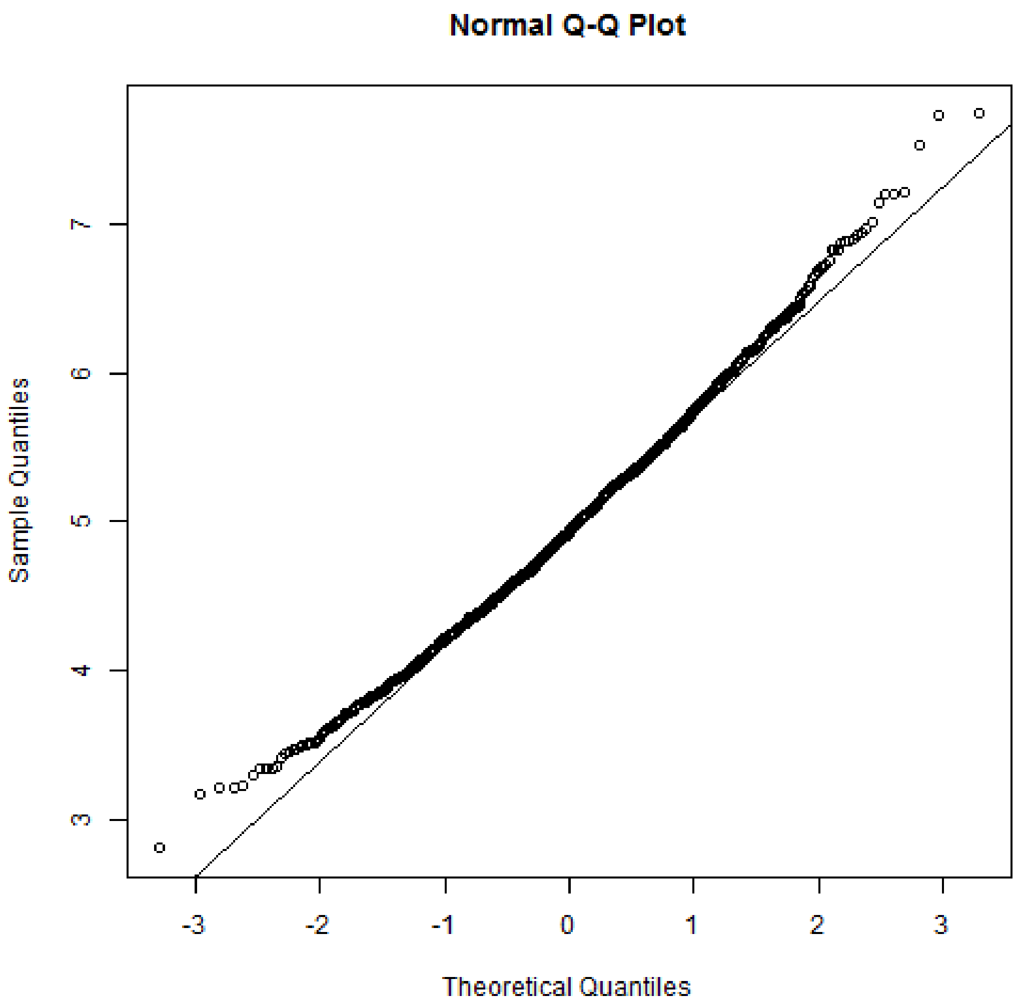


- The sample means is centered at 4.9742388
- The theoretical center of the distribution is \(\lambda^{-1} \) = 5.
- The variance of sample means is 0.5949702
- The theoretical variance is \(\sigma^2 / n = 1/(\lambda^2 n) = 1/(0.04 \times 40) \) = 0.625.

** We can use the central limit therom to find the averages.

We will not look at normality.

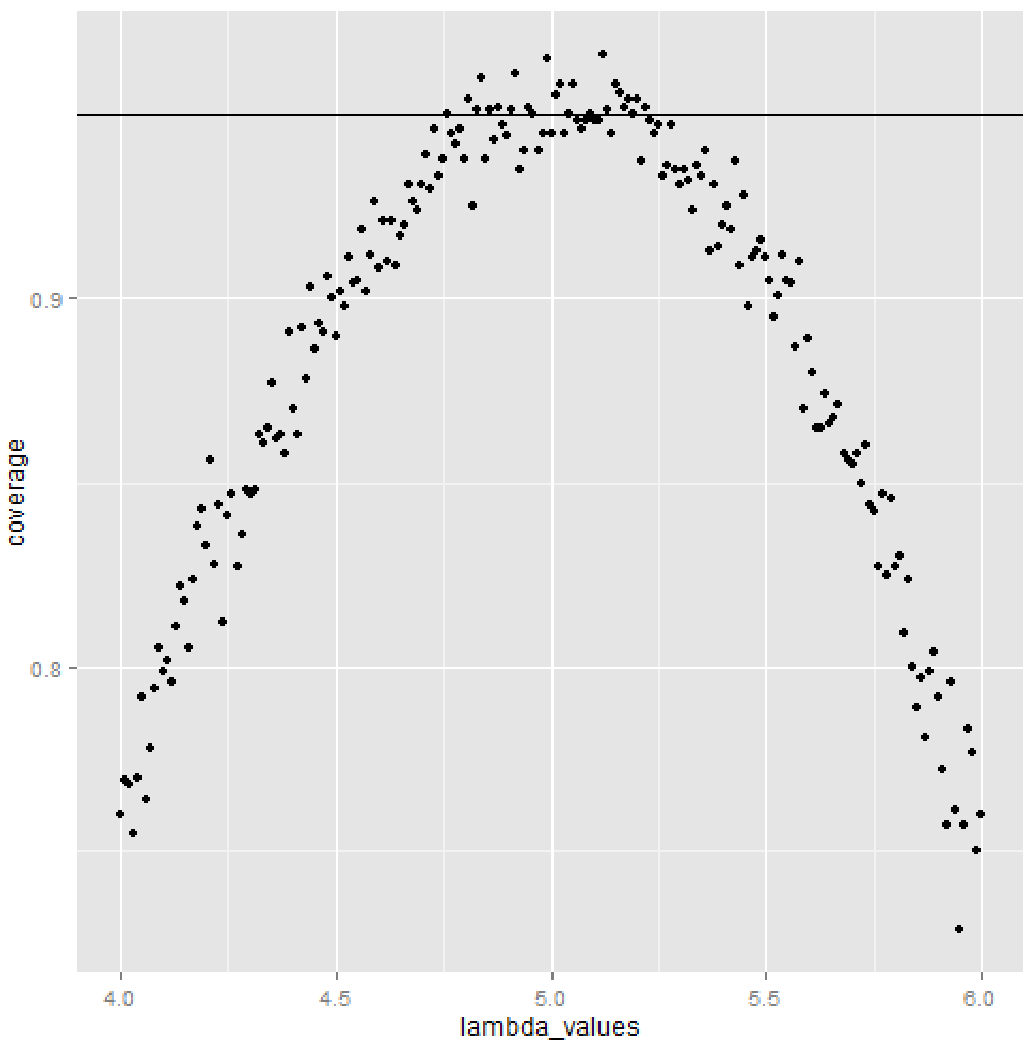
```
qqnorm(row_avgs)
qqline(row_avgs)
```



We will then look at the coverage of the confidence interval for

\(1/\lambda = \bar{X} \pm 1.96 \frac{S}{\sqrt{n}} \)

```
lambda_values <- seq(4, 6, by=0.01)

## Find coverage using sapply from the lamda_values
coverage <- sapply(lambda_values, function(lamb) {
    mu_h <-
rowMeans(matrix(rexp(size_of_sample*number_of_simulation,
rate=0.2),
                            number_of_simulation,
size_of_sample))

    ul <- mu_h + qnorm(0.975) *
sqrt(1/lambda**2/size_of_sample)

    ll <- mu_h - qnorm(0.975) *
sqrt(1/lambda**2/size_of_sample)

    mean(ll < lamb & ul > lamb)
})

library(ggplot2)
qplot(lambda_values, coverage) + geom_hline(yintercept=0.95)
```



Summary

- The 95% confidence intervals for the rate parameter (λ) to be estimated ($\hat{\lambda}$) are $\hat{\lambda}_{low} = \hat{\lambda}(1 - \frac{1.96}{\sqrt{n}})$ agnd $\hat{\lambda}_{upp} = \hat{\lambda}(1 + \frac{1.96}{\sqrt{n}})$.
- For selection of $\hat{\lambda}$ around 5, the average of the sample mean falls within the confidence interval at least 95% of the time.
- The true rate, λ is 5.