Modeling variably saturated flow using kinematic waves in MODFLOW

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Modeling Variably Saturated Flow Using Kinematic Waves in MODFLOW

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As human populations grow and demand for groundwater increases, so does the need for improving estimates of storage in and flow through aquifers. Numerical models are useful tools for assessing potential effects caused by groundwater withdrawals and for evaluating the sensitivity of groundwater flow and storage to climate change. Because demand for water is increasing, the relation of surface water to groundwater is becoming more important. Most commonly used groundwater models assume instantaneous recharge between a stream and groundwater even in areas where the stream is separated from groundwater by a vadose zone. Accounting for storage and the time for water to move through these vadose zones is important to effectively estimate long-term impacts of groundwater withdrawals and climate change. Because traditional modeling of variably saturated flow at basin-size scales is computationally demanding, a simplified approach was developed for the U.S. Geological Survey's modular three-dimensional groundwater flow model (MODFLOW) to account for flow and storage through a vadose zone beneath a stream. This new module to MODFLOW simulates one-dimensional vadose zone flow between streams and groundwater using a kinematic wave solution to Richards' equation. Comparisons between the kinematic wave and numerical solutions to Richards' equation show good agreement for both time-variable flux and head-boundary conditions. The method allows for simulation of flow between streams and groundwater over large regions where the stream and groundwater are or may become disconnected either from groundwater withdrawals or from changing climate patterns with little increase in computational time relative to the instantaneous recharge approach.

INTRODUCTION

For several decades researchers have been modeling surface and subsurface flow as a continuum, trying to consider all exchanges of water between the land surface and the aquifers below. To this end, modelers have been able to estimate the various components of the water budget within a watershed. This work has been inspired, in part, from seminal

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papers published by *Freeze* [1969], *Freeze and Harlan* [1969], and *Freeze and Banner* [1970] who present the framework for building models to simulate the coupled effects of watershed runoff and groundwater flow. Due to recent advancements in computer technology and numerical methods, researchers have been able to develop physically based models to simulate runoff and groundwater flow on a catchment scale. This modeling has been a challenge in the past partly due to the computational difficulty in solving Richards' equation for vadose zone flow (VZF). In the last decade, new models have been developed and larger scale problems have been analyzed using physically based governing equations. In a modeling study applied to a 1-km^2 catchment, *VanderKwaak and Loague* [2001] were able to simulate aspects of the hydrologic cycle and validate theoretical models that had not yet been tested on systems larger than an experimental plot. They used three-dimensional (3d) numerical solutions to Richards' equation coupled with two-dimensional (2d) surface-water routing equations. Similar to the approach taken by *VanderKwaak and Loague* [2001], *Bixio et al.* [2000] introduced a model that solves Richards' equation in 3d to simulate catchment scale, hydrologic flow processes.

Modeling studies performed on a basin scale (>1000 km^2) do not routinely simulate coupled interaction between regional aquifers and the vadose zone. Studies that incorporate the interaction often assume that infiltration at land surface reaches the underlying aquifer instantaneously [*Sophocleous and Perkins*, 2000] or else account for changes in vadose-zone storage and aquifer recharge by representing the vadose zone as a linear reservoir [*Montgomery and Watson*, 1993]. Another approach, such as used by MIKE SHE [*Refsgaard and Storm*, 1995], is to estimate recharge using a one-dimensional (1d) VZF model independent of the regional groundwater flow model. Recharge estimates from the VZF model are then incorporated as surface boundary conditions in a strictly saturated groundwater flow model.

The overall goal is to develop a module for MODFLOW [*McDonald and Harbaugh*, 1988] that is capable of simulating VZF beneath land surface wherever infiltration occurs. MODFLOW and this VZF module will be coupled to watershed runoff models such as MMS (*Leavesely et al.*, 1983; *Leavesely et al.*, 1996) and HEC-HMS (Army Corps of Engineers, 2000) so that surface and subsurface flow can be modeled together within large watershed areas. These models will be coupled such that water storage in the subsurface will constrain flow to and from the surface.

The purpose of this paper is to describe a method for simulating VZF as a result of seepage loss beneath ephemeral streams [*Prudic*, 1989]. In semiarid and arid regions streams are often hydraulically disconnected from the groundwater by a vadose zone and modeling recharge from these streams requires simulation of VZF. This work is presented as an example to demonstrate the utility of this VZF module with MODFLOW. The solution to Richards' equation for simulating VZF is based on the method of characteristics described by *Smith* [1983]. Richards' equation is solved assuming 1d gravity-driven flow, ignoring matric potential gradients, which may be an over-simplification for some natural systems. However, this approach for simulating VZF allows MODFLOW to simulate recharge accounting for the time delay that occurs for water to travel through the vadose zone while still maintaining the applicability of MODFLOW to large-scale problems.

Column experiments presented in the literature [*Mein and Larson*, 1973; *Childs and Bybordi*, 1969] document infiltration into homogeneous soils that correspond with theory where infiltration into an initially dry soil begins rapidly due to matric potential gradients but this force dissipates quickly. *Vauclin et al.* [1979] provide experimental evidence of gravity-dominated infiltration using a 3d laboratory model. Another example of gravity-dominated infiltration (Fig. 1) shows changes in infiltration across a streambed from a stream in western Nevada [*Ronan et al.*, 1998]. In Figure 1, the infiltration rate is shown as a function of time where infiltration into an initially dry streambed begins at a rate of 15 cm/hr and after 4 hrs drops to the average streambed hydraulic conductivity equal to 6 cm/hr. Enhancement of the infiltration rate by matric potential gradients at the onset of infiltration only resulted in an additional 27 cm of water as compared to a total of 600 cm during the first 4 days of the experiment.

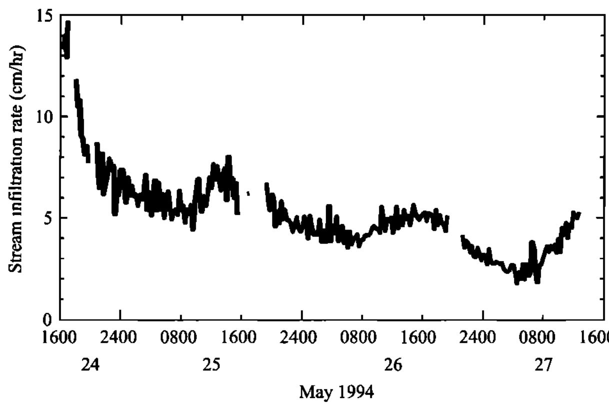

Figure 1. Infiltration across a streambed along an ephemeral stream in western Nevada with enhanced seepage due to matric potential gradients for 4 hours after onset of flow [from *Ronan et al.*, 1998].

METHODS

Method of Characteristics

Theory associated with the method of characteristics states that if there is an approximate functional relation between water flux and sediment-water content, then the movement of water through the vadose zone can be represented by kinematic waves. The method can be used to approximate Richards' equation, which can be written as:

$$\frac{\partial \theta}{\partial t} = \frac{\partial q}{\partial z} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z} - K(\theta)\right] \tag{1}$$

where θ is volumetric water content, q is water flux (positive upward), z is elevation in vertical direction, $D(\theta)$ is hydraulic diffusivity as a function of water content, $K(\theta)$ is hydraulic conductivity as a function of water content, and t is time.

When applying the method of characteristics to equation (1), the equation is simplified by assuming water flux is only dependent on water content such that

$$q = -K(\theta)\,. \tag{2}$$

Substituting equation (2) into equation (1) yields:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial K(\theta)}{\partial z}\,. \tag{3}$$

Applicability of Assumptions

If one assumes that an individual wetting front does not change as it travels down through a vadose zone, then by applying the chain rule to equation (3) and dividing by $\partial\theta/\partial z$ one arrives at equation (4), which is the gravitational characteristic velocity for a point of given θ on a wetting front:

$$\frac{\partial q}{\partial \theta} = \frac{\partial z}{\partial t} = v(\theta) \tag{4}$$

where $v(\theta)$ is the gravitational characteristic velocity.

An increase in the surface infiltration rate (i) from i_1 to i_2 at t_1 will increase the water content at the surface from θ_1 *to* θ_2 creating a wetting front that will infiltrate down through the vadose zone (Fig. 2). The wetting front has a tendency to stay sharp due to the force of gravity. This sharp wetting front will travel down through the vadose zone at a constant speed assuming matric potential gradients have dissipated and the water content ahead of the wetting front is constant at θ_1. By integrating the characteristic velocity over the sharp wetting front from θ_1 *to* θ_2 one arrives at the average velocity of a wetting front through a homogeneous vadose zone:

$$u_s(\theta_2,\theta_1) = \frac{K(\theta_2) - K(\theta_1)}{\theta_2 - \theta_1} \tag{5}$$

where u_s is average shock wave velocity, θ_2 is volumetric water content behind a shock wave, and θ_1 is volumetric water content below a shock wave.

When the surface infiltration decreases, from i_2 to i_3 at t_2, the water content at the surface decreases from θ_2 *to* θ_3 creating a trailing front (Fig. 2). Unlike wetting fronts, trailing fronts elongate steadily with time and must be

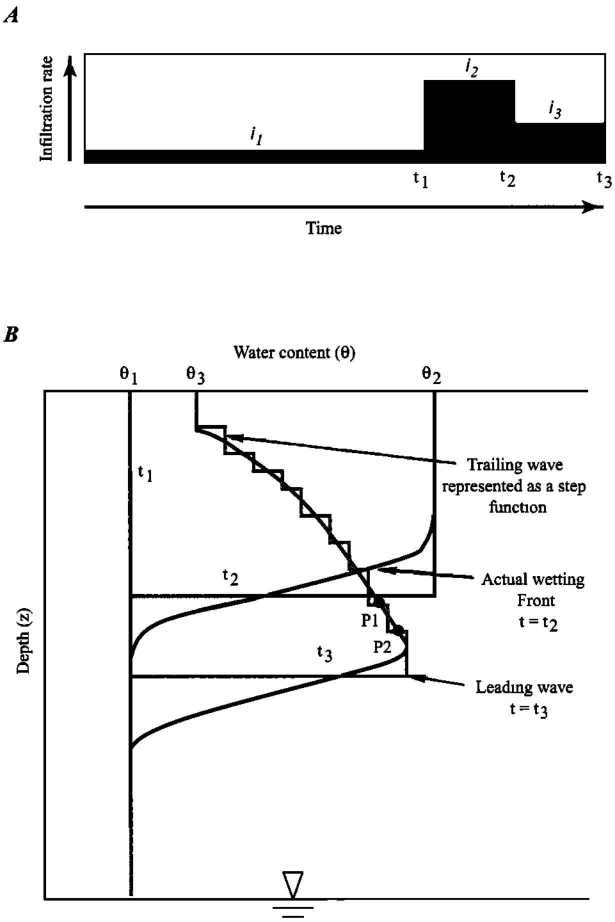

Figure 2. Depiction of a wetting front (black lines) through a uniform column of soil affected first by an increase in infiltration followed by a decrease and results from a kinematic wave model (gray lines) that approximate the wetting front using leading and trailing step waves [modified from *Smith and Hebbert, 1983*].

approximated by a series of incremental step waves (trailing step waves) each having their own water content and velocities [*Smith and Hebbert*, 1983]. The velocities of the individual trailing step waves that are used to represent a trailing front are determined based on the characteristic velocities at points along the trailing front. For example, P1 and P2 in Figure 2 are used to represent two points along the trailing front where the velocities may be determined. Velocities of each trailing wave used to represent a trailing front at P1 and P2 are equal to the slope of the trailing front at these points. The depths of these points below land surface at any point in time can be related by the ratio of their velocities, assuming the characteristic velocity is a constant function of water content at any point along the trailing front:

$$\frac{z_2}{z_1} = \frac{v(\theta_2)}{v(\theta_1)} = \frac{\partial K(\theta)/\partial\theta\big|_{\theta=\theta_2}}{\partial K(\theta)/\partial\theta\big|_{\theta=\theta_1}} \tag{6}$$

where z_1 is depth of a trailing step wave below land surface represented by P1 in Figure 2, and z_2 is depth of a trailing step wave below land surface represented by P2 in Figure 2.

The Brooks and Corey [1966] function relating hydraulic conductivity to water content was used to evaluate $\partial K(\theta)/\partial\theta$, which can be written as:

$$K(\theta) = K_{sv}\left[\frac{\theta-\theta_r}{\theta_s-\theta_r}\right]^{\varepsilon} \tag{7}$$

where K_{sv} is saturated vertical hydraulic conductivity, θ_r is residual water content, θ_s is saturated water content, and ε is parameter, usually between 3 and 4.

After taking the derivative of the Brooks and Corey function, a relation between the depth of the deepest trailing step wave and all other trailing step waves is written as:

$$z(\theta) = z_o\left[\frac{\theta-\theta_r}{\theta_o-\theta_r}\right]^{\varepsilon-1} \tag{8}$$

where $z(\theta)$ and θ are depth and water content of a point on the trailing front, respectively, and z_0 and θ_0 are depth and water content of trailing front's leading edge, respectively. And the velocity of the lead trailing step wave (v) is written as:

$$v = \frac{\varepsilon K_s}{\theta_s-\theta_r}\left[\frac{\theta-\theta_r}{\theta_s-\theta_r}\right]^{\varepsilon-1}. \tag{9}$$

An algorithm was developed in FORTRAN relying on equations (5), (8), and (9) in order to simulate vertical VZF within MODFLOW. This algorithm keeps track of the location, water content, and flux of all waves moving through the vadose zone through time and the interaction of waves overtaking one another. Additionally, increases and decreases in the thickness of the vadose zone due to changes in the groundwater level are considered with respect to waves reaching the water table and contributing to groundwater storage. The result is an explicit solution for VZF coupled to an implicit groundwater flow model, allowing the two models to run at independent temporal and spatial scales.

Boundary conditions used to represent infiltration from the land surface usually are formulated as a water flux or hydrostatic pressure head acting on a streambed. Because the velocity of a wave moving through the vadose zone is related to the water content in the approach used herein, a relation between the infiltration rate or hydrostatic pressure head and water content must be included within the model. A relation between water content and infiltration rate based on equations (7) and (2) is written as:

$$\theta(q) = \left(\frac{q}{K_{sv}}\right)^{1/\varepsilon}(\theta_s-\theta_r)+\theta_r \tag{10}$$

where $\theta(q)$ is water content of a wave generated from an infiltration event, and q is infiltration rate.

Flow through the vadose zone is assumed to be driven strictly by gravitational gradients; however, the infiltration rate through the land surface or streambed can be calculated based on the hydraulic gradient, which may include matric potential gradients. Long after the onset of infiltration, matric potential gradients can still be a significant component of the hydraulic gradient driving infiltration. This situation occurs when a low permeability layer of sediment exists on the land or streambed surface due to the compaction of soil, the deposition of fine sediments and biomass, the formation of a hardpan, and the natural soil heterogeneity [*Moore and Jenkins*, 1966; *Vauclin et al.*, 1979; *Peterson,* 1989, and *Peterson and Zhang*, 2000]. The matric potential in the underlying higher permeability sediment will be less than zero due to the limited water flux from the overlying lower permeable sediment [*Childs and Bybordi*, 1969]. Matric potential gradients also may enhance flow deeper in the vadose zone for highly heterogeneous soils but these conditions are not represented in the model presented herein.

The matric potential directly below the land surface or streambed is calculated based on the Brooks and Corey [1966] retention function and the water content just below the streambed at the previous time step as:

$$h_{mp} = h_a \left[\frac{(\theta_l - \theta_r)}{\theta_s - \theta_r} \right]^{-1/\varepsilon} \tag{11}$$

where h_{mp} is matric potential head, h_a is air entry pressure, and θ_l is water content below surface crust or semipervious streambed.

The seepage flux is limited by the vertical hydraulic conductivity of the vadose zone if the stream stage causes the seepage through a semipervious streambed to exceed the saturated hydraulic conductivity of the vadose zone.

Routing Surface-Water Flow

Surface water is routed through a series of reaches using the streamflow routing package (STR1; *Prudic*, 1989). Routing of streamflow is based on the continuity equation and assuming steady (non-changing in time), uniform (non-changing in location), constant density channel flow such that during all times, volumetric inflow and outflow rates are equal and no water is added to or removed from storage in the surface channels.

STR1 is designed to route flows through a network of channels (which include rivers, streams, canals, and ditches) where flows are always in the same direction along the channels, and where flows are constant for each time period used in the groundwater flow model. However, with the addition of an unsaturated zone, flow and seepage in the stream may change within a MODFLOW time step and all water reaching the water table is totaled during this time step. If changes in channel storage during each time period are important, then VZF could be added to more complicated, 1d unsteady or diffusion based streamflow models used with MODFLOW [*Swain and Wexler*, 1996 and *Jobson and Harbaugh*, 1999).

STR1 has been modified to allow several methods for estimating stream stage and stream width. The representative width of the stream may be held as a constant or determined based on the stream cross-section dimensions, discharge, and stage. The relation between stage and discharge may be determined based on a normal flow condition calculated using Manning's equation or based on a user specified stage-discharge relation. The option to simulate unsaturated flow between streams and aquifers may be used for a stream of constant width or based on an eight-point cross-section (cross-sectional dimensions specified by eight coordinate pairs). If the stream width is variable then it is segmented into seven separate widths corresponding to the horizontal discretization of the vadose zone. Seven widths were used here because an eight-point cross-section delineates seven incremental distances.

When the aquifer head directly below the streambed is less than the streambed elevation, and the streambed hydraulic conductivity is greater or equal to that of the aquifer, the seepage is not head dependent and is set equal to the hydraulic conductivity of the aquifer. The stream and vadose zone are coupled to the MODFLOW solver via the right-hand side vector in the MODFLOW conductance equations, similar to how sources and sinks are included, and can be expressed as:

$$AX=B-Q \tag{12}$$

where A is a matrix containing coefficients of the conductance equations that are solved by MODFLOW, X is a 1d vector containing groundwater heads that are solved by MODFLOW, B is a 1d vector containing all known terms in conductance equations that are not multiplied by unknown head values, and Q is volume of water reaching the water table in a given MODFLOW cell from the vadose zone.

The quantity of water the aquifer receives from the vadose zone also is dependent on fluctuations of the water table. If the water table is rising into a vadose zone near saturation, then the water table rise will be exaggerated due to the increased yield from the vadose zone. Alternatively, if the water table is falling due to groundwater discharge or pumping at some other location, then it will take longer for a wetting front from the vadose zone to reach the aquifer. This describes a coupled interaction between aquifer storage and recharge; a phenomenon that has been recognized in natural systems as being important in predicting fluctuations in the water table due to recharge [*Freeze*, 1969].

Because the VZF model presented here acts as a storage reservoir, an additional mass balance must be calculated beyond the mass balance equations within MODFLOW and STR1. A separate mass-balance equation is used to account for all water that is stored within, has entered or left the vadose zone. Once water has reached the water table it is then accounted for within the MODLFLOW mass-balance equations. The mass-balance error for the vadose zone is calculated individually for each MODFLOW cell containing a vadose zone and is printed within the standard MODFLOW output file.

RESULTS AND DISCUSSION

Model Verification

Model verification consisted of comparing results from the kinematic wave model to those from the USGS variably saturated flow model VS2DT [Healy, 1990] that solves

Table 1. Input parameters used to compare results between kinematic wave and VS2DT models of vertical flow through a column.

Model input parameter	Value
Saturated vertical hydraulic conductivity, K_{sv} (m/s)	$3.4e^{-6}$
Saturated water content, θ_s (m^3/m^3)	0.35
Residual water content, θ_r (m^3/m^3)	0.079
Brooks-Corey exponent, ε	3.5
Air entry pressure, h_a (m)	–0.1

Richards' equation numerically in terms of matric and gravitational potentials. Two different types of 1d simulations were used for the model verification. The first model verification consisted of simulating a 100-m tall column of sediments having hydraulic properties typical of a sandy clay loam. Both the kinematic wave and VS2DT models were run using the Brooks and Corey functions to relate hydraulic conductivity to water content (K vs. θ) and the matric potential to water content (Ψ vs. θ). Table 1 lists the input parameters used for this model comparison. The model consisted of a variable flux boundary at the top of the column and a zero pressure head boundary at the bottom to approximate a water table. The initial water content was set equal to the residual water content throughout the profile. Figure 3 shows a comparison of the water content profile after 50, 75 and 100 days of simulation. Both water content and advancement of the wetting front for the kinematic wave model agree with VS2DT.

The second model verification addresses a specific condition referred to as a 'semipervious' streambed where the streambed hydraulic conductivity is less than that of the underlying sediments. The model presented herein accounts for the increased hydraulic gradient due to matric potential in order to calculate the flux over the interface between the semipervious streambed and the underlying sediments. In order to test the effectiveness of the kinematic wave model to properly simulate this situation, a hypothetical model was constructed. Both the kinematic wave and VS2DT models were used to simulate seepage from this hypothetical stream for a time varying stream stage (variable pressure head boundary condition) and the

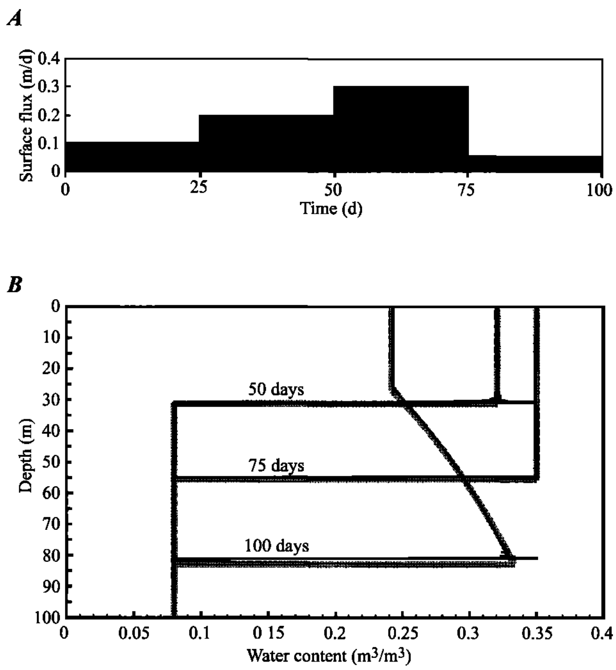

Figure 3A. Time varying surface flux applied to both kinematic wave and VS2DT models assuming vertical flow through a column of a sand-silt-clay loam. **B.** Comparison of results between kinematic wave (gray lines) and VS2DT (black lines) simulations after 50, 75, and 100 days. VS2DT oscillations at wetting front are from numerical instability.

results were compared. Table 2 lists the input parameters used for this modeling. A comparison of the flux through the streambed for four different stream stages (Table 3) and the corresponding advancement of the wetting front after 12, 23, and 35 days of infiltration (Fig. 4) show good agreement between the two models. Results indicate that seepage loss from the stream is dependent on stream depth and matric potential beneath the streambed when the streambed has a lower permeability than the vadose zone and the stream remains disconnected from the underlying aquifer. Correspondence in the depth of the wetting fronts in time verifies that infiltration through the vadose zone

Table 2. Input parameters used to compare results between kinematic wave and VS2DT models of vertical flow beneath a stream with a semipervious streambed.

Model input parameter	Semipervious streambed	Vadose zone
Saturated vertical hydraulic conductivity, K_{sv} (m/s)	$1.0e^{-7}$	$1.0e^{-6}$
Saturated water content, θ_s (m^3/m^3)	0.30	0.30
Residual water content, θ_r (m^3/m^3)	0.10	0.10
Brooks-Corey exponent, ε	3.5	3.5
Air entry pressure, h_a (m)	-0.2	–0.2

Table 3. Comparison of varying stream stage on seepage rates across a semipervious streambed between kinematic wave and VS2DT models.

Stream stage (m)	Seepage rate in kinematic wave model (m/s)	Seepage rate in VS2DT (m/s)
0.70	$9.54e^{-7}$	$9.44e^{-7}$
2.46	$1.83e^{-6}$	$1.78e^{-6}$
1.59	$1.40e^{-6}$	$1.37e^{-6}$
1.04	$1.12e^{-6}$	$1.11e^{-6}$

is controlled by gravity for the conditions used in this simulation.

Infiltration from wide streams is predominately vertical and so the kinematic wave model can provide a good estimate of recharge; however, narrow deep channels may exhibit a substantial amount of horizontal seepage thus changing the characteristics of the wetting fronts. Future work will involve verification of the kinematic wave model to 2d simulations or a stream cross-section to assess the magnitude of error associated with assuming strictly vertical flow.

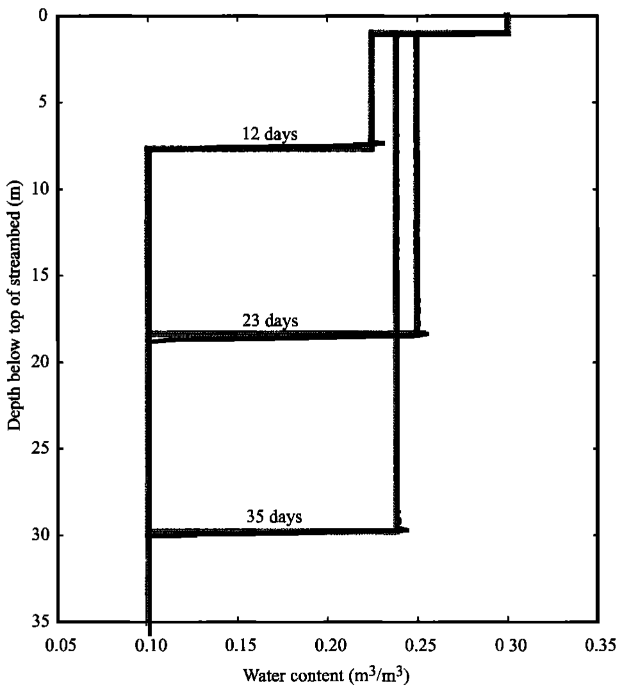

Figure 4. Comparison of results between kinematic wave (gray lines) and VS2DT (black lines) simulations after 50 days assuming a time varying pressure head at surface and a semipervious streambed. VS2DT oscillations at wetting front are from numerical instability.

Model Demonstration

MODFLOW Simulation 1

Two simulations are presented in which the kinematic wave model has been implemented for seepage beneath streams in MODFLOW. This is different than in the previous section where results of flow in the vadose zone from the kinematic wave model were verified against those from the VS2DT model.

For the first simulation, a hypothetical aquifer was modeled assuming groundwater separated from a stream by a vadose zone. The elevation of the stream ranged from 140 m in the upstream end to 110 m in the downstream end (Fig. 5). The hypothetical aquifer was divided into seven cells in the Y-direction (along columns) each 90 m long, 100 cells each 200 m long in X-direction (along rows) and one layer in the Z-direction (vertical; Fig. 5*A*). A stream was placed along the middle row in the x-direction and had a maximum width at peak flow equal to 14 m and a depth of 6 m and was represented by an eight-point cross-section. A total of 200 stream reaches (corresponding to the 200 aquifer cells in the X-direction) connected the individual cells. The initial water table was at 104 m resulting in an initial vadose zone that ranged from 36 m at the upstream end to 6 m at the downstream end of the model. No-flow boundaries were assigned along all sides of the model, except the down-stream side where a general head boundary was used to mimic a natural regional gradient. The vadose zone for each cell with a stream was divided into seven vertical cells beneath the stream (Fig. 5*B*). Unsaturated flow was not considered in any other cells.

Aquifer flow properties were considered homogeneous throughout the model with a horizontal hydraulic conductivity of $8.0e^{-4}$ m/s, a vertical hydraulic conductivity of $6.0e^{-6}$ m/s, and a specific yield of 0.20. The specific yield of the aquifer was set equal to the difference between residual and saturated water contents in the vadose zone in order to maintain a continuum between the vadose zone and the

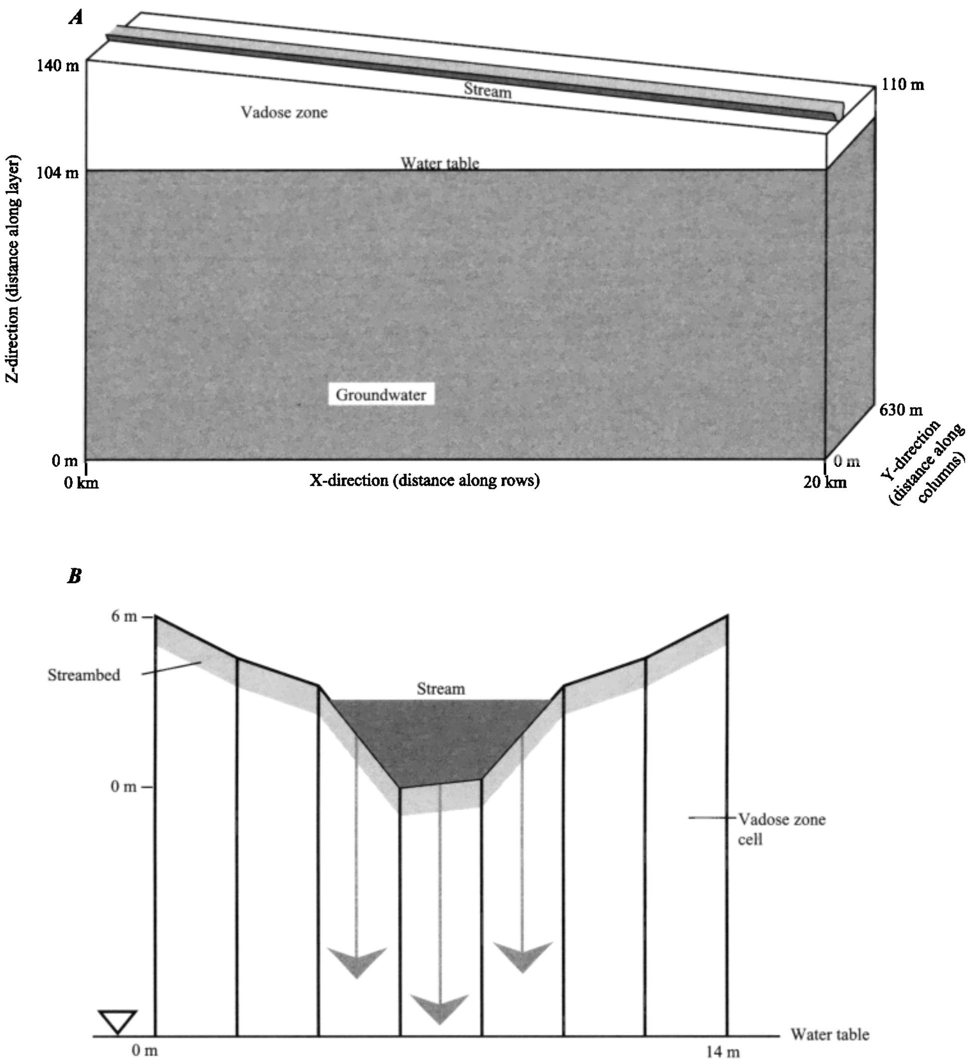

Figure 5A. Hypothetical stream and aquifer used in first MODFLOW simulation. **B.** Modeled vadose zone beneath stream channel.

aquifer. The streambed and vadose zone hydraulic properties are listed in Table 4. Flow into the uppermost stream cell was specified and resembles a natural mountain stream that does not flow for short periods in the late summer.

The semipervious streambed thickness was varied between 0.5 and 1.5 m and the resulting effects on seepage, recharge, stream stage, and water table beneath an individual MODFLOW cell were analyzed. As expected seepage, recharge, and water table decreased with increased streambed thickness (Fig. 6). Seepage increases with stream depth when the streambed is thin and seepage is less affected by changes in stream depth when the streambed is thick and has a low permeability.

Model sensitivity to unsaturated flow parameters was analyzed by comparing recharge rates within the MODFLOW cell in the center of the model domain (row 4 and column 100). A Brooks and Corey exponent equal to 4.0 held water in the vadose zone longer, resulting in 2% decrease in recharge over the simulation period as compared to an exponent value equal to 3.0 (Fig. 7*A*). A lower exponent also resulted in a slightly greater increase in the water table elevation. When the air entry pressure (h_a) was decreased from –0.05 to –0.25 m, the recharge rate increased by 6% resulting in a greater increase in the water table elevation as compared to higher air entry values (Fig. 7*B*).

MODFLOW Simulation 2

The second simulation was designed to test model performance when applied to a stream that changes from losing to gaining conditions. In this simulation, the number of cells and dimensions along rows and the number of layers in the vertical direction were the same as those used in the first simulation (Fig. 8). However, the number of cells and dimensions were reduced along columns where the cell corresponding to the stream was 10 m long and the two adjacent cells were only 5 m long. Also, the elevation of the streambed ranged from 125 m on the upstream end to 50 m on the downstream end, and the initial water table was set at an elevation of 51 m. Thus, the vadose zone ranged from 74 m at the upstream end to zero at the downstream end. No-flow boundaries were assigned along all external sides of the model domain. Recharge was only from stream seepage loss and groundwater discharge was only to the stream.

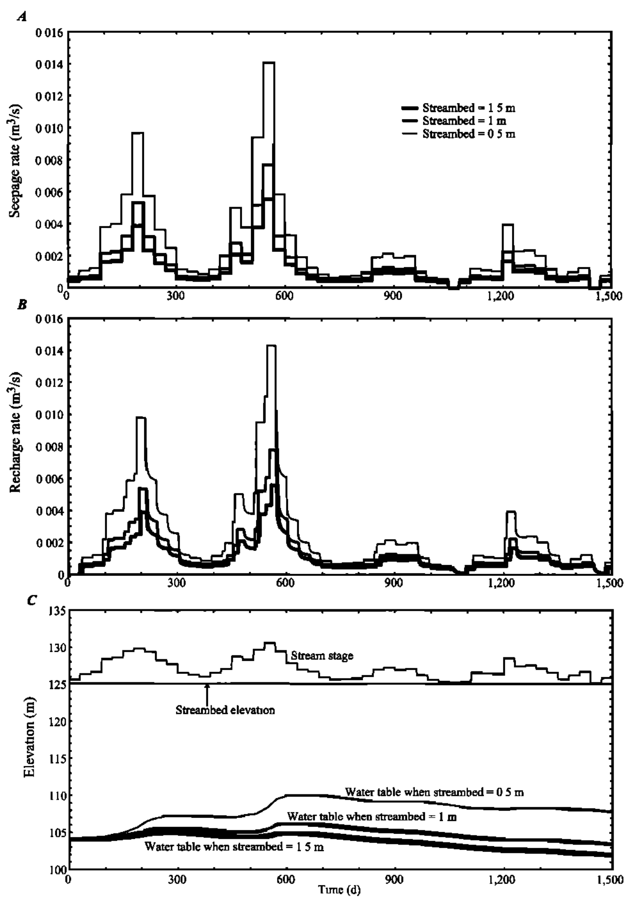

Figure 6. Effects of variations in thickness of a semipervious streambed on: **A.** infiltration rate; **B.** recharge rate; and **C.** water table for first MODFLOW simulation.

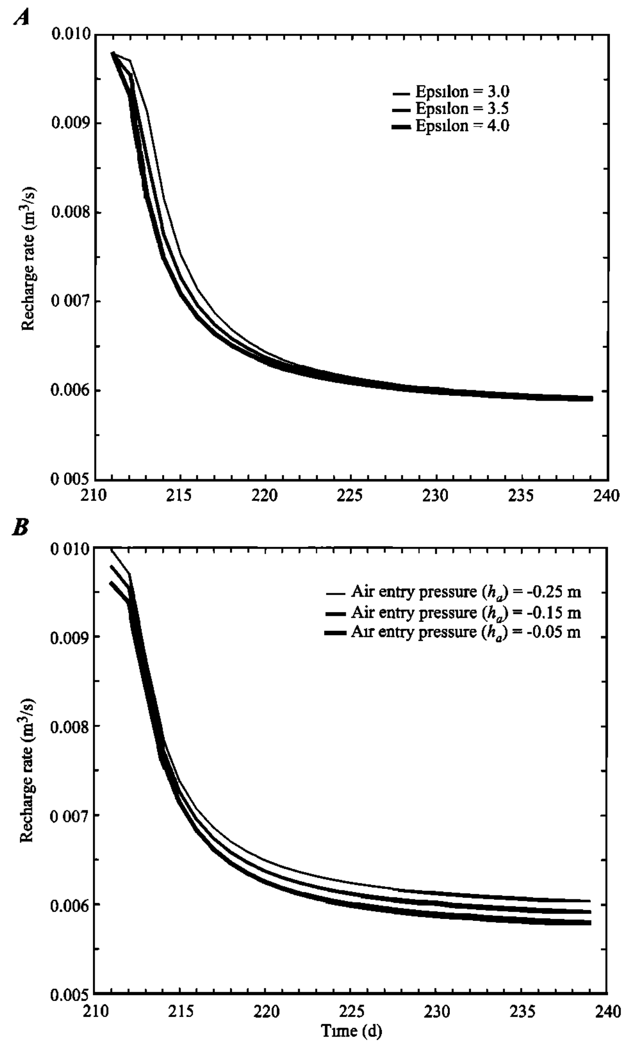

Figure 7. Effects of unsaturated flow parameters on recharge from a stream with a semipervious streambed: **A.** Brooks and Corey exponent and **B.** air entry pressure for first MODFLOW simulation.

In order to test model performance under a rapidly rising water table, high values of vertical and horizontal hydraulic conductivity of the vadose zone and aquifer were used, equal to $4.0e^{-5}$ and $2.0e^{-3}$ m/s, respectively. The streambed was 1 m thick and had a vertical hydraulic conductivity of $8.0e^{-7}$ m/s. The other streambed and vadose zone hydraulic properties are the same as those listed in Table 4.

Simulation 2 is designed to illustrate reconnection of groundwater to a stream after a prolonged period of no flow in the stream. The simulation is similar to an ephemeral stream that produces runoff from snowmelt in the adjacent mountains and flow only reaches the lower basin during the melting of an unusually large snow pack. During the simula-

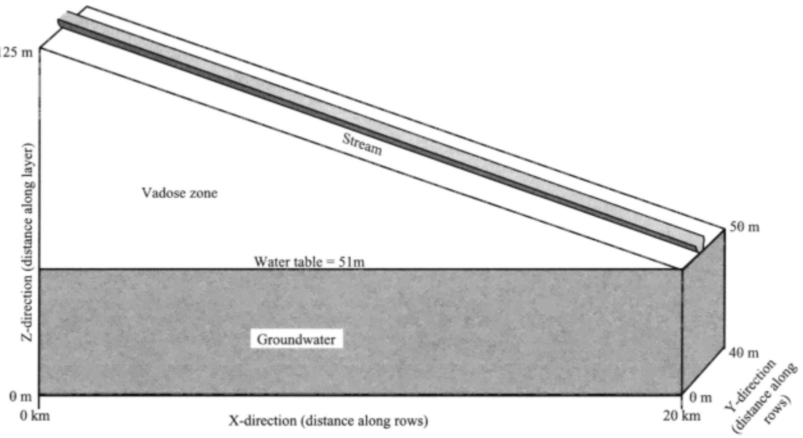

Figure 8. Hypothetical stream and aquifer used in second MODFLOW simulation.

tion, seepage through the streambed and flow through the vadose zone resulted in a rapid water-table rise (Fig. 9). After only 5 days, the water table was higher beneath the lower end of the stream because flow through the thin or nonexistent vadose zone was recharging groundwater. After 15 days, the water table was above the stream stage at the lower end and groundwater began to discharge to the stream. The water table continued to rise until after 180 days, the aquifer was saturated and the simulation had reached steady state. Stream loss was highest and constant for the first 4 days and thereafter decreased nearly linearly until it equaled groundwater discharge at 160 days (Fig. 10). The peak in vadose-zone storage was at 8 days, whereas recharge through the vadose zone peaked at 10 days. Following the peaks, both decreased to zero after 155 days indicating that the aquifer had become fully saturated (Fig. 10).

These modeling results are based on hypothetical streams that share many of the common hydraulic characteristics of streams in semi-arid to arid regions. These characteristics include high transmission rates to and from the stream, ephemeral flow behavior, and periodic disappearance and reappearance of a vadose zone between the stream and underlying groundwater. Simulating these conditions usually poses a highly non-linear mathematical problem that can be difficult to solve for large-scale applications. The modeling approach presented herein was able to simulate these aquifer-stream relations with little difficulty and at a very low cost in computation time. Simulations using the kinematic wave model combined with MODFLOW required little more computation time than applying similar recharge rates to MODFLOW alone, as internal boundary conditions. Future work with this model will involve basin scale applications where this model will be linked to watershed runoff models. A VZF package is being developed to simulate water infiltrating into all partially or completely unsaturated MODFLOW cells. This model will be used to simulate the feedback among groundwater storage, recharge, and runoff for basin-scale applications.

SUMMARY AND CONCLUSIONS

A new model is presented that was combined with MODFLOW and the MODFLOW stream package (STR1) to simulate vadose zone flow (VZF) beneath ephemeral streams hydraulically disconnected from groundwater by a vadose zone. The VZF model was formulated based on a kinematic wave solution to Richards' equation that considers vertical, gravity driven infiltration. This model was tested

Table 4. Vadose-zone input parameters used in first MODFLOW simulation.

Model input parameter	Semipervious streambed	Vadose zone
Saturated vertical hydraulic conductivity, K_{sv} (m/s)	5.0e^{-7}	6.0e^{-6}
Saturated water content, θ_s (m^3/m^3)	0.30	0.30
Residual water content, θ_r (m^3/m^3)	0.10	0.10
Brooks-Corey exponent, ε	3.5	3.5
Air entry pressure, h_a (m)	–0.15	–0.15

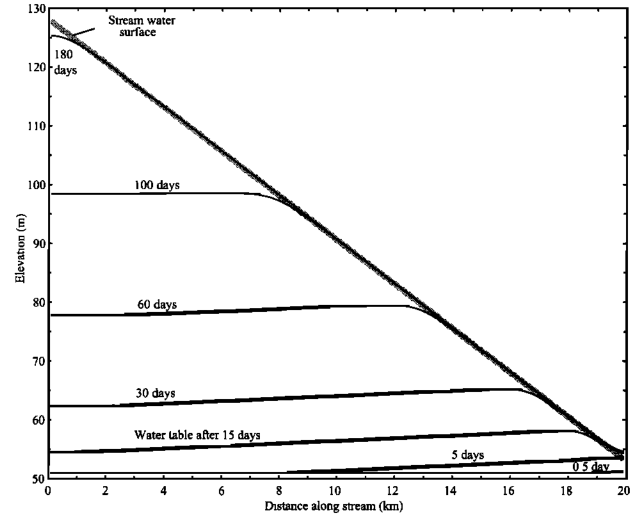

Figure 9. Increase in water table beneath stream due to seepage for second MODFLOW simulation.

against a numerical solution to Richards' equation (VS2DT) and was able to match both water content and advancement of the infiltrating front. The model also was verified against VS2DT in a simulation involving a semipervious streambed. The kinematic wave model was developed so that matric potential gradients just below the streambed can be included in the calculation of the stream seepage loss rate. With this formulation, the kinematic wave model agreed well with the VS2DT model for a simulation involving a variable stream stage (pressure head) boundary and a semipervious streambed.

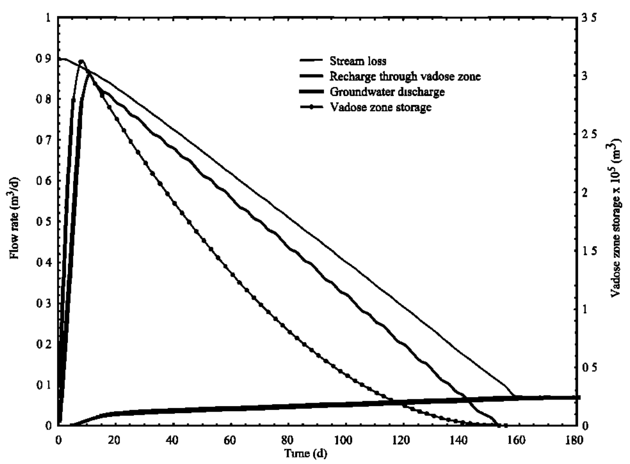

Figure 10. Stream loss, recharge, groundwater discharge, and vadose-zone storage changes resulting from stream seepage and rising water table during second MODFLOW simulation.

Simulations involving hypothetical streams were carried out to evaluate model performance and model sensitivity to VZF parameters. The kinematic wave model combined with the MODFLOW stream package was able to easily simulate complex stream-aquifer relations including the appearance and disappearance of a vadose zone between the stream and aquifer and the conversion from a losing to gaining stream and vice versa. The model was able to simulate these processes without greatly increasing the computation time beyond what would be required by MODFLOW alone.

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