Midterm 2

EECS/BioE C106A/206A
Introduction to Robotics
Fall 2020

Issued: November 18, 2020, 8 PM
Due: November 19, 2020, 8 PM

Problem	**Max. Score**
Short Answer	10
Essential Matrices for Planar Motion	10
Da Vinci Robot	10
Pick and Place	12
Singularities	6
Ball and Beam	7
Properties of Adjoints	13
Gradient of Jacobian	7
Total	75
Max Score	70

Instructions

1. This exam is open-book, open-notes and open-internet.
2. The exam is, however, NOT open-peer, and any collaboration between students on the exam will be considered academic misconduct.
3. There are a total of 75 points worth of problems in the exam, but 70 will be considered a perfect score.
4. Please include all your work so that you may be eligible for partial credit.
5. You do not need to print out the exam or annotate on it. Feel free to write out your solutions the way you would for a homework assignment.
6. You have 24 hours to take this exam. You should submit your solutions to the Gradescope assignment by ***Thursday, November 19th, 8:00:00 PM PST.***

Problem 0. Honour Code

Copy down the following honour code in your submission and sign and date it. *Failure to do so will result in an automatic 0 on the exam.*

I swear on my honour that:

1. *I alone am taking this exam.*
2. *I will not have assistance from anyone while taking this exam.*
3. *I will not discuss this exam with anyone else until exam solutions have been released by course staff.*

Problem 1. Short Answers (10 points)

(a) (4 points) Consider two cameras $\{1\}$ and $\{2\}$ arranged so that the transform g_{21} is a pure translation. Let x_1 be the normalized image coordinates of some point in image 1. Likewise, let x_2 be the normalized image coordinates of some (possibly different) point in image 2. If $x_1 = x_2$, show that (x_1, x_2) satisfy the epipolar constraint.

(b) (6 points) You know that the generalized velocity of the tool frame of a manipulator $V_{st} \in \mathbb{R}^6$ is related to the joint velocities of a manipulator $\dot{\theta} \in \mathbb{R}^n$ by the Jacobian formula

$$V_{st} = J(\theta)\dot{\theta}$$

You are told that the robot is redundant, that is $n > 6$, You are given V_{st} and asked to solve for $\dot{\theta}$. Your MIT friend tells you that if $J(\theta)$ has rank 6, then $J(\theta)J^T(\theta)$ is invertible.

(i) (4 points) Is that true? Justify your answer.

(ii) (2 points) If there is a solution to the equation for $\dot{\theta}$ is it unique? Justify.

Problem 2. Essential Matrix for Planar Motion (10 points)

Consider a camera whose motion is confined to its XY plane. Note that this also means that it can only rotate about its Z axis.

(a) (4 points) Show that the essential matrix $E = \hat{T}R$ between two distinct frames taken by such a camera has the special form

$$E = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{bmatrix}, \quad a, b, c, d \in \mathbb{R}$$

(b) (6 points) Given that the camera is restricted to move in this way, develop a way to solve for (R, T) given an essential matrix of the above form in terms of a, b, c, d, without using the SVD decomposition introduced in lecture. Make sure to highlight any conditions under which a unique solution does not exist.

Problem 3. Jacobian of an RRPRRP Manipulator (10 points)

Figure 1 shows a kinematic model for the Da Vinci surgical robot arm, with four revolute joints and two prismatic joints (joints 3 and 6).

(a) da Vinci S Surgical System instrument arm, © 2016 Intuitive Surgical, Inc.

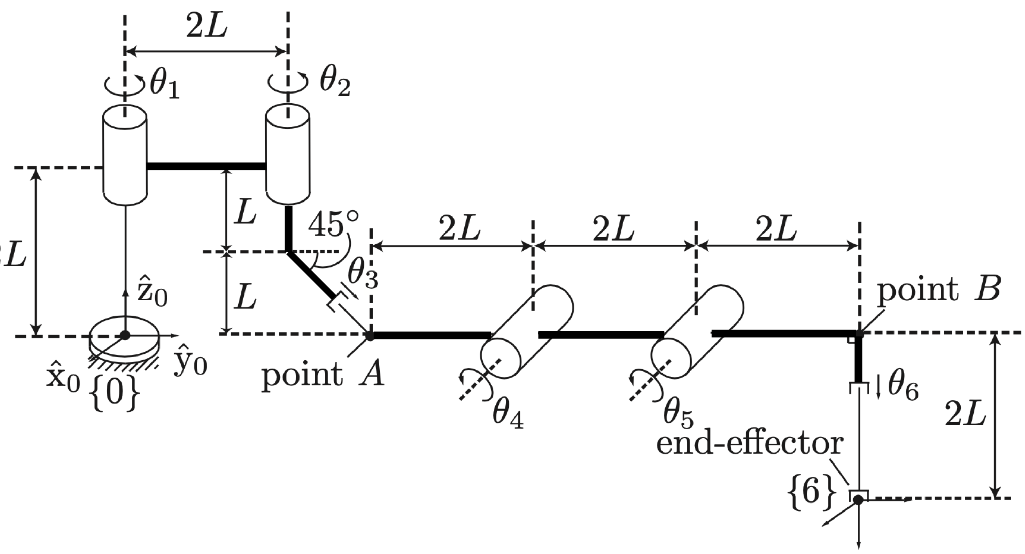


Figure 1: RRPRRP DaVinci Robot in its initial configuration. All lines connecting joints lie in the YZ plane.

(a) (6 points) Compute the spatial Jacobian J^s in the initial configuration. Argue that in this configuration, the Jacobian is not full rank.

(b) (4 points) Are there any configurations where the Jacobian of this robot is full rank? If yes, explicitly specify a set of joint angles at which the Jacobian is full-rank. If no, justify your answer.

Problem 4. Pick and Place (12 points)

You are designing a system that will let Baxter locate an object on a table in front of it and will then command the arm to move to the required location and pick it up. You have a computer vision system that is capable of detecting where the object is in the robot's base frame.

(a) (4 points) All the objects in the scene are cubes that are lying flat on one of their sides on the table. Your computer vision node is capable of detecting the side-length and the 3D location and orientation of each cube (where the location is the location of the center of mass). Design a ROS Message type that will allow the vision node to communicate information about the various objects in the scene to the planning and control nodes. At each timestep, the vision node will publish a message of this type to summarize the set of all such objects currently in the scene.

You may assume access to any standard `geometry_msgs` or `std_msgs` message types. **Briefly explain the functionality of your new message (what fields you have included, and what the purpose of each field is), and specify the exact contents of the `.msg` file you would create.**

Make sure your message type is capable of conveying all of the following information:

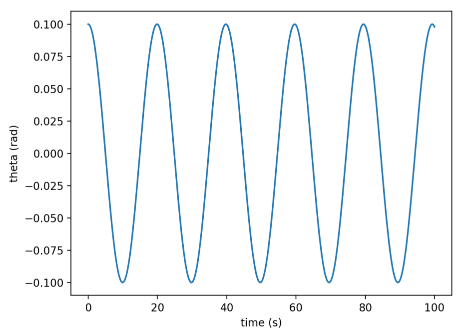
- The name of the TF frame with respect to which the objects were measured.
- The number of objects in the scene.
- The sidelengths and poses (position and orientation) of each object.

Note: You may also choose to create other custom helper message types that you then use in this message type. If so, make sure to describe each `.msg` file you would create, and specify which message type is the main one that your node will publish at each timestep.

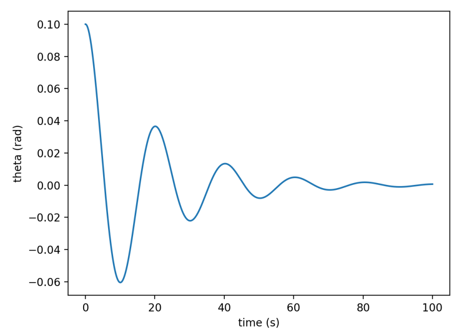
(b) (4 points) You are using a computed-torque control law to drive the 7 joint angles of the robot to their desired values. Your planner tells you that the desired value for joint 1 is $\theta_{1,d}(t) = 0$. The joint starts off at $\theta_1(0) = 0.1$. You use the gain matrices $K_p = \mathrm{diag}(K_{p,1}, \cdots, K_{p,7})$ and $K_v = \mathrm{diag}(K_{v,1}, \cdots, K_{v,7})$. You are trying to tune the gains $K_{p,1}$ and $K_{v,1}$ so that first joint angle is driven to its target value of 0 with minimal oscillations or overshoot.

For each set of gains (i) to (iv) below, state which plot from (a) to (d) corresponds to how the first joint angle would respond under a computed torque controller with those values of $K_{p,1}$ and $K_{v,1}$. Also state which set of gains you would choose of the provided options.

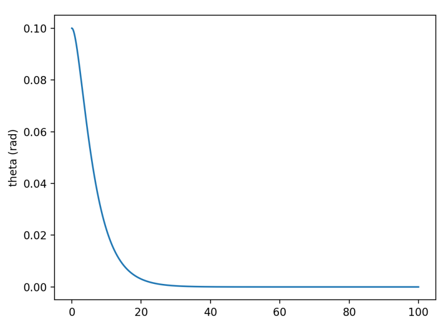
index	$K_{p,1}$	$K_{v,1}$
(i)	0.1	0.0
(ii)	0.5	0.0
(iii)	0.1	0.1
(iv)	0.1	0.7



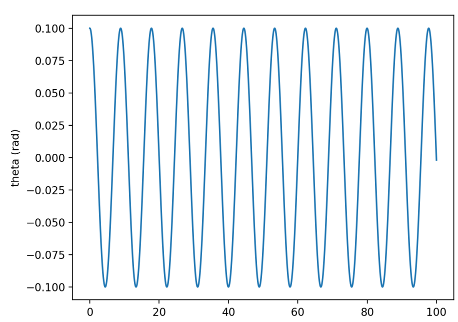
(a)



(b)



(c)



(d)

(c) (4 points) You write the following program to control the right arm of the robot. The script is given a list of desired joint angles and commands Baxter to move to them.

```
done = False
while not done:
    # Get the current joint angles from the right arm
    measured_angles = get_joint_angles(right)

    # Compute the required torques to be sent as input to the robot.
    torques = control_law(right, measured_angles, desired_angles)

    # Send the computed torques to the right arm.
    set_joint_torques(right, torques)

    done = True
    for i in range(0,len(desired_angles)):
        if desired_angles[i] != measured_angles[i]:
            done = False
            break
```

Assume that the code has no syntactical errors and runs without throwing any exceptions. All called functions operate correctly. Do you expect the code to correctly bring the arm to its desired position in normal operation, on a real robot? Explain.

Problem 5. Kinematic Singularity (6 points)

Four revolute joint axes with twists $\xi_i = [q_i \times \omega_i, \omega_i]^T$, $i = 1, \ldots, 4$ are said to be parallel if

$$\omega_i = \pm\omega_j,\ \ i, j = 1, \ldots, 4$$

Show that a six degree of freedom manipulator with four parallel axes is at a singular configuration, that is that $J(\theta)$ at that configuration is singular.

Problem 6. Dynamics of the Ball and Beam System (7 points)

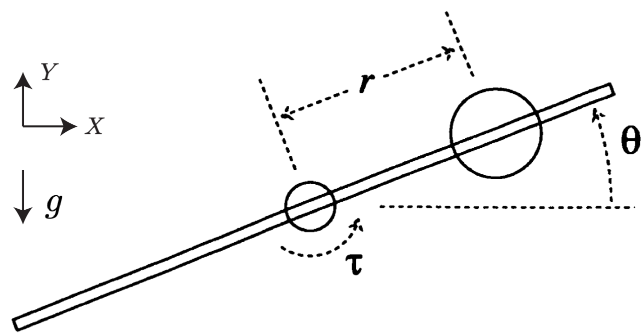


Figure 2: The ball and beam system.

Consider the ball and beam system depicted in figure 2. A ball is free to slide without resistance along the beam (like a bead threaded on a wire). A torque τ is applied by a motor at the center of the beam which allows the beam to rotate in a vertical plane.

Let the moment of inertia of the beam (relative to the center of the beam, about an axis pointing out of the page) be J and the mass of the ball be m. You may treat the ball as a point mass so that it has no moment of inertia about its center of mass. Let g be the acceleration due to gravity. Choose as generalized coordinates for this system $r \in \mathbb{R}$ the position of the ball on the beam and $\theta \in \mathbb{R}$ the angle of the beam with the horizontal. That is $q = (r, \theta)^\top \in \mathbb{R}^2$. Write down a formula for the total Kinetic Energy of the system (this includes the kinetic energy of the beam and the ball). Write the potential energy of the system and the Lagrangian of the system. Use the method of Lagrange to derive the equations of motion of this system. The only external force is the torque τ about the beam center. Identify the inertia matrix, the gravity vector, the Coriolis terms, and a suitable Coriolis matrix.

Problem 7. Properties of Adjoints and Commutators (13 points)

Consider twists as 6 dimensional vectors, which are identified with elements of $\mathfrak{se}(3)$ by the *hat* and *vee* operators. In this problem we will consider the *Lie bracket* operation on $\mathfrak{se}(3)$. For $X, Y \in \mathbb{R}^6$, the bracket $[X, Y]$ is an operation that outputs a new twist as a vector $\in \mathbb{R}^6$, and should be thought of as a higher dimensional analog of the cross product. It is defined as follows

$$[X, Y] = \left(\hat{X}\hat{Y} - \hat{Y}\hat{X}\right)^\vee$$

Further recall the defining property of the Adjoint. For $g \in SE(3)$, Ad_g is a 6×6 matrix with the property that for any $\xi \in \mathbb{R}^6$ we have

$$\mathrm{Ad}_g \, \xi = \left(g\hat{\xi}g^{-1}\right)^\vee$$

(a) (2 points) For $\xi_1, \xi_2 \in \mathbb{R}^6$ show that $[\xi_1, \xi_2] = -[\xi_2, \xi_1]$.

(b) (4 points) For a rigid body transform $g \in SE(3)$ and $\xi_1, \xi_2 \in \mathbb{R}^6$, show that

$$\mathrm{Ad}_g[\xi_1, \xi_2] = [\mathrm{Ad}_g \, \xi_1, \mathrm{Ad}_g \, \xi_2]$$

(c) (2 points) For $\xi \in \mathbb{R}^6$ and any scalar θ, show that

$$\mathrm{Ad}_{e^{\hat{\xi}\theta}} \, \xi = \xi$$

(d) (5 points) Let $X, \xi \in \mathbb{R}^6$ be constant twists and let $\theta \in \mathbb{R}$ be a scalar variable. Let $g = e^{\hat{\xi}\theta}$. Define

$$Y(\theta) = \mathrm{Ad}_g \, X$$

Show that Y satisfies

$$\frac{\partial Y}{\partial \theta} = [\xi, Y]$$

Problem 8. Gradient of the Manipulator Jacobian (7 points)

Let $J(\theta)$ be the spatial manipulator jacobian of an open chain manipulator with n degrees of freedom. $\theta = (\theta_1, ..., \theta_n)^T \in \mathbb{R}^n$ is the vector of joint angles of the manipulator. The goal of this problem is to compute the gradient of J with respect to the joint angles. Let $J^{(i)}$ be the i-th column of J.

Recall that the columns of J are vectors of dimension 6 that represent twists, or elements of $\mathfrak{se}(3)$. Further recall the Lie bracket on $\mathfrak{se}(3)$, defined as follows. Let $X, Y \in \mathbb{R}^6$. Then

$$[X, Y] = \left(\hat{X}\hat{Y} - \hat{Y}\hat{X}\right)^\vee$$

Show that for $j < i$

$$\frac{\partial J^{(i)}}{\partial \theta_j} = [J^{(j)}, J^{(i)}]$$

and for $j \geq i$, $\frac{\partial J^{(i)}}{\partial \theta_j} = 0$.

Note: You may use any of the properties stated in the previous problem without proof.