Homework 8

EECS/BioE C106A/206A
Introduction to Robotics

Due: October 27, 2020

Problem 1. Feedforward Trajectory Tracking with Jacobians

Say we are working with a Baxter robot and we would like the end effector of our robot arm to perform some trajectory in its workspace. This trajectory is given to us as a smooth trajectory of rigid transforms, $g(t) \in SE(3)$. Assume we have access to both $g(t)$ and $\dot{g}(t)$ for $t \in [0, T]$, and that the trajectory starts from the current position of the robot, $g(0)$, and ends up at some desired position $g(T)$ in T seconds. Recall that we can only give the robot jointspace commands, so we want to convert this workspace trajectory into a jointspace trajectory $\theta(t)$ such that $g_{ST}(\theta(t)) = g(t)$ for $t \in [0, T]$ where $g_{ST}(\theta)$ is the forward kinematics map.

(a) How would you solve this problem using an inverse kinematics solver? Why might this be undesirable?

(b) Write down an expression for the desired spatial workspace velocity $\hat{V}$ that we want the end effector to perform at time t, in order to perform the trajectory g.

(c) Assume we also have efficient access to the spatial jacobian $J(\theta)$. Let $\theta(t)$ be some jointspace trajectory that satisfies the required constraints. Write down an expression for $\dot{\theta}(t)$ in terms of the velocity you computed in the previous part (recall the Moore-Penrose pseudoinverse that we spoke about in lecture).

(d) Explain how we may compute $\theta(t)$ from your answers to the previous two parts.

(e) Consider the following strategy to make the robot arm track the desired trajectory. At each timestep, we use (b) to compute the desired spatial velocity, and then we use (c) to compute the desired joint velocity. Then, we send this joint velocity as a command to the robot. Do you think this strategy will allow us to track the desired trajectory well?

Hint: Remember to consider when the solution to (c) *becomes ill-conditioned, in addition to other factors that may lead to poor tracking.*

Problem 2. Dynamics of a Mass-Spring System

Figure 1 shows a system involving a mass m and a spring with spring constant k on an incline. Pick a suitable set of generalized coordinates (you should only need one), and use Lagrangian dynamics to find the equations of motion of the mass-spring system. State the Inertia matrix, Coriolis matrix, and Gravity vector for this system (since this is a one dimensional problem, these will all just be scalars). What is the physical meaning of the generalized forces Υ in this case?

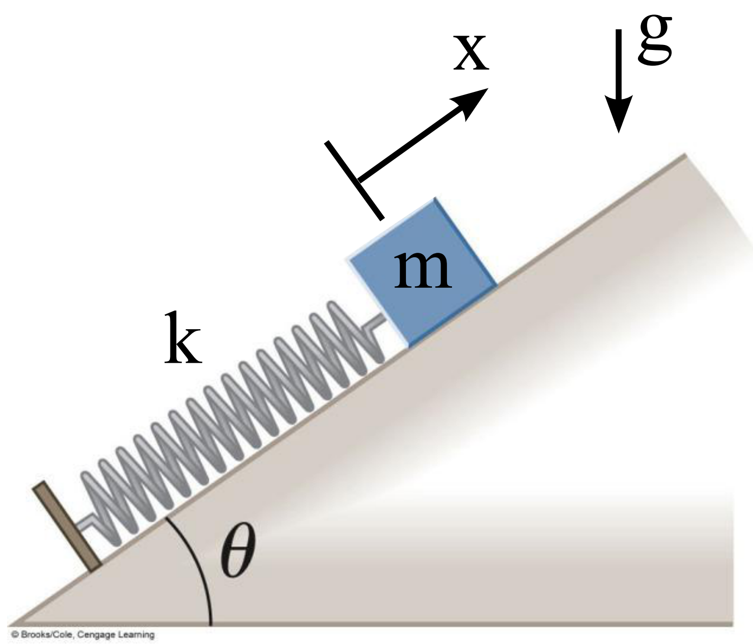

Figure 1: Mass-spring system on a slope.

Problem 3. Dynamics of a Double Pendulum

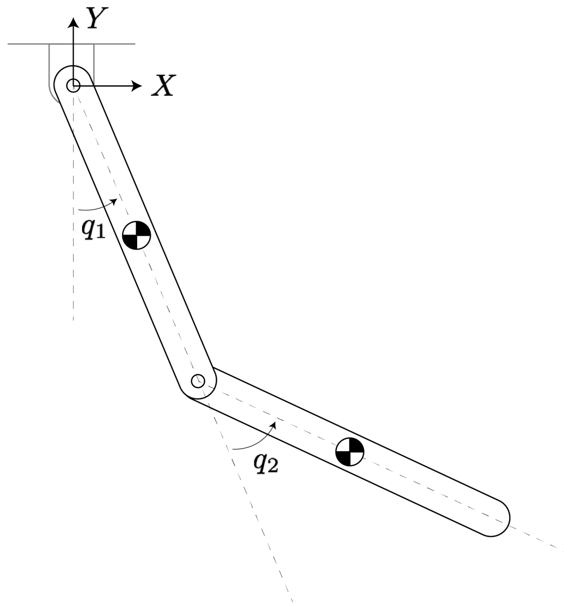

Figure 2: Double pendulum. $z-$axis points directly out of the plane of the paper.

Figure 2 shows a double pendulum hanging from a rigidly fixed pivot. Both arms have length L, mass m, and moments of inertia I about the $z-$axis (pointing out of the plane of the paper) at the center of mass. Both arms have a uniform mass density so that their center of masses are at their centers. Gravity points in the negative y direction.

Pick a suitable set of generalized coordinates (you should need two), and use Lagrangian dynamics to find the equations of motion of the double pendulum. State the Inertia matrix, Gravity vector, and a possible Coriolis matrix for this system. What is the physical meaning of the generalized forces Υ in this case?

If you so choose, you may use software like the the MATLAB symbolic toolbox or SymPy to help you simplify your expressions or compute derivatives. Please remember to include your final expressions for the following quantities in your solutions:

1. The kinetic energy T of the system.
2. The potential energy V of the system.
3. The necessary derivatives of the Lagrangian: $\frac{\partial L}{\partial q}$ and $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$.
4. The inertia matrix $M(q)$ of the system.
5. The Coriolis term $C(q, \dot{q})\dot{q}$ and your choice of Coriolis matrix $C(q, \dot{q})$.
6. The gravity vector $N(q, \dot{q})$.