Practice Midterm 2

EECS/BioE C106A/206A
Introduction to Robotics

Due: November 18, 2020

Problem	**Max. Score**
Lin Alg Review	20
Tidying up your workspace	15
Interpreting Jacobians	15
Essential Matrices	5
Velocities and Reference Frames	15
Dynamics of Pendulum with Spring	10
Jacobian of a Manipulator	10
Kinematic Singularity	10
Total	100

Problem 1. Linear Algebra Review (20 points)

(a) (4 points) If B is invertible, show that $\text{rank}(AB) = \text{rank}(A)$.

(b) (4 points) If A is invertible, show that $\text{rank}(AB) = \text{rank}(B)$.

(c) (4 points) For fixed A, B define $Y(t) = e^{At}e^{-Bt}$.

   (i) Show that $\dot{Y}(t) = e^{At}(A - B)e^{-Bt}$.

   (ii) Conclude that when $A = B$, $Y(t)$ is a constant matrix for all t. Hence, show that the inverse of e^{At} is e^{-At} for any matrix A.

(d) (4 points) Let B be arbitrary and let $C = B^\top B$. Show that for any vector v we have $Cv = 0$ if and only if $v^\top Cv = 0$.

(e) (4 points) Given a differential equation of the form $\dot{x} = Ax + f(t)$ where $x \in \mathbb{R}^n$ and $f : \mathbb{R} \to \mathbb{R}^n$ is a differentiable function of time, show that the general solution can be written as

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)} f(s) ds$$

Hint: Define $z(t) = e^{-At}x(t)$ *and evaluate* $\dot{z}(t)$.

Problem 2. Tidying up your Workspace (15 points)

You're in lab section preparing to get checked off by a wise member of course staff. The lab you are working on requires the use of many nodes; you must start up each one in a specific order, but do this across several terminal windows. You get to terminal window 2, then 3, then 4, and then 5 when the 5th node crashes because of a bug in your code. Yikes. You fix the bug in your code, and then realize you actually didn't and need to restart everything again. There must be a better way ...

(a) (5 points) The nodes of interest are `camera_node`, `planning_node`, `vision_node`, `teleop_node`, `logging_node`, part of the `messy_pkg` package. What ROS file can you use to make the process of bringing up all these nodes at once a lot less cumbersome? Construct the file outright or at least "psuedocode" for the information you would need to specify.

(b) (5 points) Your ROS workspace directory currently looks like

```
midterm2_practice_ws
├── build
├── devel
└── src
    └── messy_pkg
        └── src
            ├── camera_node.py
            ├── planning_node.py
            ├── vision_node.py
            ├── teleop_node.py
            └── logging_node.py
```

Where do you need to place the file, and then what would be the sequence of commands to run in order to execute your new file?

(c) (5 points) Nice! It's now a lot easier to start up everything after making small changes. You realize that there are a set of parameters that define the initial configuration of your whole system, which you would like to easily customize. In particular, all nodes need to know that we have an $n \times m$ image, and that the system should shutoff after `timeout` seconds. These values are shared across many nodes, so it's in your best interest to have one central place to customize them from. What feature in ROS allows you to utilize this functionality? What files do you need to modify to enable this functionality? Explain the modifications you must make in code or psuedocode.

Problem 3. Interpreting Jacobians (15 points)

Let $J^s(\theta_1, \theta_2, \theta_3)$ be the manipulator Jacobian of a robot parameterized by the joint position vector θ. In the current configuration, all joint positions are 0. All of the robot's joints are either revolute or prismatic. The Jacobian is

$$J^s(0,0,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) (1 point) How many joints does this robot have?

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8

(b) (2 point) How many revolute joints?

○ 0 ○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8

(c) (2 point) This robot is in a singular configuration.

○ True ○ False ○ Not enough info

(d) (2 points) In this configuration, in which directions is it possible to induce a nonzero velocity?

○ v_x ○ v_y ○ v_z ○ ω_x ○ ω_y ○ ω_z

(e) (2 points) Which column(s) of the spatial Jacobian are constant for all values of θ_1?

○ First column ○ Second column ○ Third column

(f) (2 points) Which column(s) of the spatial Jacobian are constant for all values of θ_2?

○ First column ○ Second column ○ Third column

(g) (2 points) Which column(s) of the spatial Jacobian are constant for all values of θ_3?

○ First column ○ Second column ○ Third column

(h) (2 points) How many singular configurations does this robot have?

○ 0 ○ 3 ○ 18 ○ ∞

○ 1 ○ 6 ○ 36 ○ Not enough info

Problem 4. Essential Matrices (5 points)

Recall that the essential matrix between two camera frames $\{1\}$ and $\{2\}$ of frame $\{1\}$ with respect to $\{2\}$ is given by $E = \hat{T}R$ where $(R, T) = g_{21}$. Show that the essential matrix of frame $\{1\}$ with respect to frame $\{2\}$ is $E^\top$.

Problem 5. Velocities and Reference Frames (15 points)

Consider the motion $g_{AB}(t) \in SE(3)$ of a rigid body with a body reference frame B attached rigidly to it and a fixed spatial frame A.

(a) Say we switch to a new fixed spatial frame A':

  (i) (3 points) Show that the body velocity remains unchanged.

  (ii) (4 points) Show that the new spatial velocity is $\mathrm{Ad}_{g_{AA'}}^{-1} V_{AB}^{s}$.

(b) Now say we keep the spatial frame fixed as A, but we switch to a new body frame B' which is also fixed rigidly onto the body.

  (i) (4 points) Show that the new body velocity is $\mathrm{Ad}_{g_{BB'}}^{-1} V_{AB}^{b}$.

  (ii) (4 points) Show that the spatial velocity remains unchanged.

Problem 6. Dynamics of Pendulum with Spring (10 points)

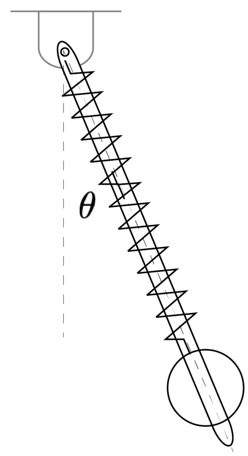

Figure 1: Pendulum with spring.

Figure 1 shows a massless rod with a bob of mass m threaded on it so that it is free to slide without friction along the rod. There is also a spring with spring constant k connecting the pivot of the pendulum to the bob. In it's neutral position, the spring has length l. By picking a suitable set of generalized coordinates, use the method of Lagrange to find the equations of motion for this system. Identify the inertia matrix, coriolis terms, and gravity vector.

Problem 7. Jacobian of a manipulator (10 points)

Please write down the spatial Jacobian of the SCARA manipulator in Figure 2 in the configuration shown. Please make sure to show your work

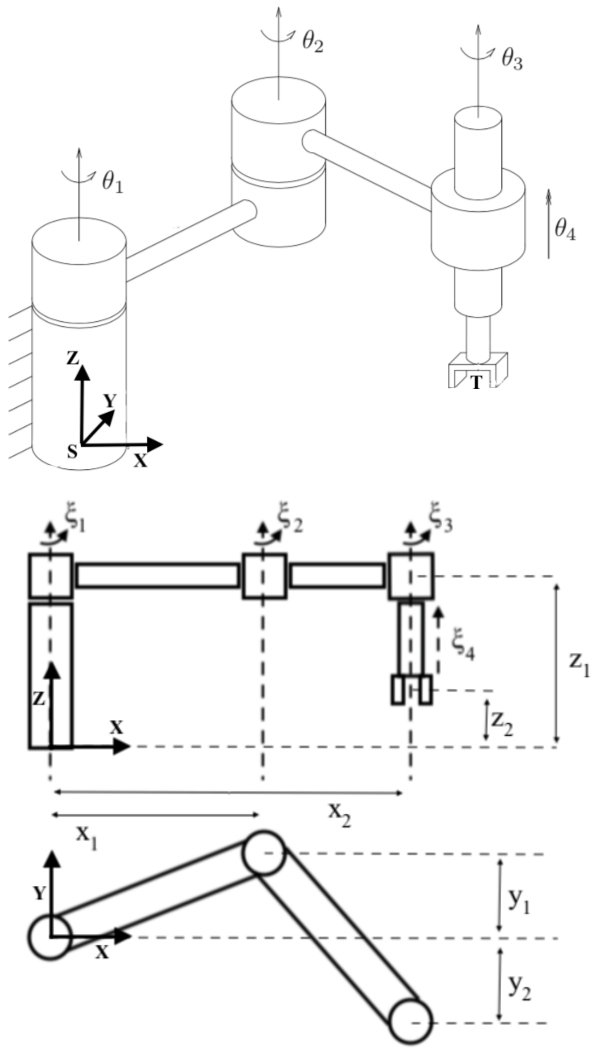

Figure 2: Isometric, side, and top views of a SCARA manipulator

Problem 8. Singular Configuration (10 points)

Three revolute joint axes with twists $\xi_i = [q_i \times \omega_i, \omega_i]^T, i = 1, 2, 3$ are said to be parallel if

$$\omega_i = \pm\omega_j,\ \ i, j = 1, 2, 3$$

A prismatic joint with twist $\xi_4 = [v, 0]^T$ is said to be perpendiuclar to these revolute joints if

$$v^T \omega_i = 0,\ \ i = 1, 2, 3$$

Show that a six degree of freedom manipulator with three parallel revolute axes and a prismatic axis perpendicular to all three is at a singular configuration, that is, that $J(\theta)$ is singular.