# Homework 7

EECS/BioE C106A/206A
Introduction to Robotics

Due: October 27, 2020

**Problem 1. Jacobian for a 4DOF manipulator**

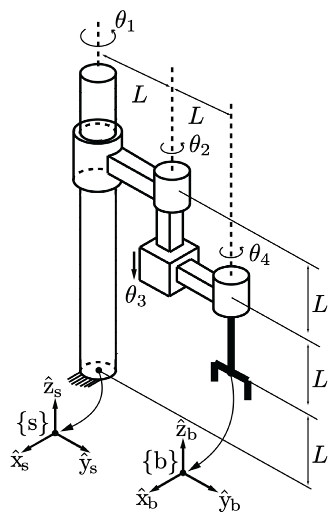


Figure 1: A four degree of freedom manipulator

Figure 1 shows a 4DOF manipulator with 3 revolute joints and 1 prismatic joint (joint 3) in its initial configuration $\theta = 0$.

(a) Compute the spatial Jacobian $J^s$ and the body Jacobian $J^b$ of the manipulator in the configuration shown.

(b) Now let the robot move so that $\theta_2 = \pi/2$, with all other joints remaining at zero. Compute the spatial Jacobian $J^s$ and body Jacobian $J^b$ in the new configuration.

(c) In which configurations, the one in part (a), or the one in part (b), is the robot in a singular configuration? Justify.

(d) During the execution of a smooth joint trajectory $\theta(t) \in \mathbb{R}^4$, the robot passes through the configuration from part(a) with joint velocities $\dot{\theta}(t) = (0, -1/L, 1, 1/L)$. Find the velocity of the origin of the end effector as seen from the spatial frame at that instant. Note that here we are asking for the velocity of the *point* at the origin of the tool frame, so your answer should just be a vector $\dot{p}_{sb} \in \mathbb{R}^3$.

**Problem 2. Singularities of Euler Angles**

(a) Write down the adjoint of a rotation about the origin by rotation matrix $R$.

(b) Using this, demonstrate that the singularity of ZYX (extrinsic) Euler angles occurs when $\theta_2 = \frac{\pi}{2}$. Intuitively, why does this occur?

*Hint: You should model these Euler angles as the product of three revolute joints along the $X, Y, Z$ axes and show that when the second angle is $\pi/2$ the Jacobian loses rank.*

(c) Prove that any rotation represented by three rotations about arbitrary axes ($R = e^{\hat{\omega}_1\theta_1} e^{\hat{\omega}_2\theta_2} e^{\hat{\omega}_3\theta_3}$) will have a singularity.
*Hint 1: First prove the case where $a\omega_1 + b\omega_2 = \omega_3$ (such as with ZYZ Euler angles) i.e. when the three axes are linearly dependent. Then prove the harder case where $\omega_1, \omega_2, \omega_3$ are linearly independent.*
*Hint 2: Doing the second part by brute force is very difficult*

**Problem 3. Kinematic Singularity: prismatic joint perpendicular to two parallel coplanar revolute joints**

A prismatic joint with twist $\xi_3 = (v_3, 0)$ is normal to a plane containing two parallel revolute axes $\xi_i = (q_i \times \omega_i, \omega_i),\ i = 1, 2$ if

- $v_3^T \omega_i = 0$
- $v_3^T (q_1 - q_2) = 0$
- $\omega_1 = \pm\omega_2$

Show that when this occurs, any six degree of freedom manipulator is at a singular configuration. Give an example of a manipulator exhibiting such a singularity.