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Discussion #2

Exercise 1 (Eigenvalues) Let $A \in \mathbb{R}^{n,n}$ and $B^2 = A^2 + I$.

1. Prove that if λ is an eigenvalue of A then $\lambda^2 + 1$ is an eigenvalue of B.
2. Prove that if A has an eigenvalue decomposition, then B has one as well.

Exercise 2 (Eigenvectors of a symmetric matrix) Let $p, q \in \mathbb{R}^n$ be two linearly independent vectors, with unit norm ($\|p\|_2 = \|q\|_2 = 1$). Define the symmetric matrix $A \doteq pq^\top + qp^\top$. In your derivations, it may be useful to use the notation $c \doteq p^\top q$.

1. Show that $p + q$ and $p - q$ are eigenvectors of A, and determine the corresponding eigenvalues.
2. Determine the nullspace and rank of A.
3. Find an eigenvalue decomposition of A, in terms of p, q. *Hint:* use the previous two parts.
4. What is the answer to the previous part if p, q are not normalized?

Exercise 3 (Maximum singular value) Prove $\max_{\|u\|_2=1} \|Au\|_2 = \sigma_1(A)$, where $\sigma_1(A)$ is the maximum singular value of A.