MATH 3012-QHS, Homework Assignment 2, 2016, WTT

Due Date: Friday, September 30, 2016 by 4:35 pm, to be uploaded to T-Square. Note that Test 1 has been rescheduled to Wednesday, October 5, 2016.

Note: To receive credit, you must write your homework legibly (or type it) and have your name written on the front page at the top of the assignment. You must show your work. You may work together, but your write-up must be your own. If you use an outside source, cite it.

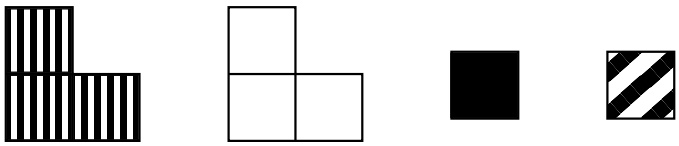
1. Consider the recurrence, $r(n) = 2r(n-1)+1$, with $r(0) = 0$. Determine a closed-form solution for $r(n)$, and use math induction to prove your answer is correct.

2. Prove that $11^n - 7^n$ is divisible by 4 for all $n \geq 1$.

3. Find the integer $d = \gcd(5390, 97461)$, and find integers a and b such that

$$d = 5390a + 97461b.$$

4. Consider filling a $2 \times n$ checkerboard with the following non-rotatable pieces:



Let $g(n)$ be the number of ways to fill the $2 \times n$ board by using any number of each piece above so that no two black square pieces are stacked on top of each other. Find a recursive formula and base cases for $g(n)$.

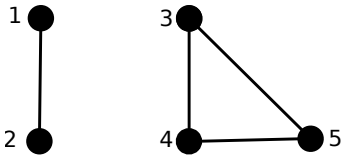
5. What is the coefficient of $x^{15}y^{120}z^{25}$ in $(2x + 3y^2 + z)^{100}$?

6. Consider putting 301 points down anywhere on a 10×10 square (where "anywhere" includes both the boundary and interior of the square). Prove that no matter how the points are placed, there is always a collection of 4 points, all of which are within at most $\sqrt{2}$ of each other.

7. How many graphs have vertex set $\{1, 2, \ldots, 100\}$?

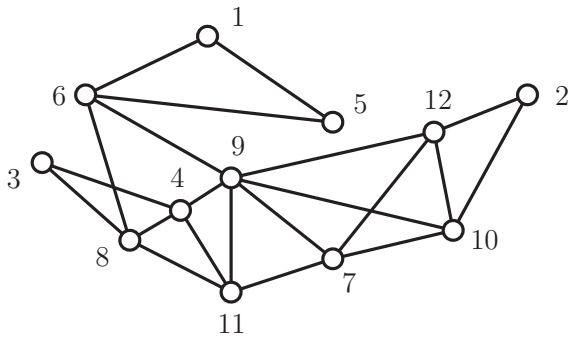
8. Count the number of unlabelled graphs on 5 vertices.

9. How many subgraphs does the following graph have?

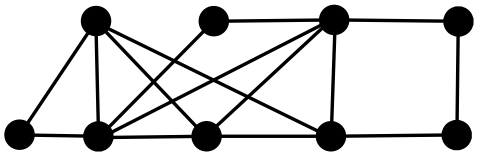


10. Find two non-isomorphic trees having the same number of vertices and the same multiset of degrees. Note: When G is a graph, the multiset of degrees is the multiset which contains the degree of x for every vertex x in G. For example, the multiset of degrees for a complete graph on 4 vertices is $\{3, 3, 3, 3\}$.

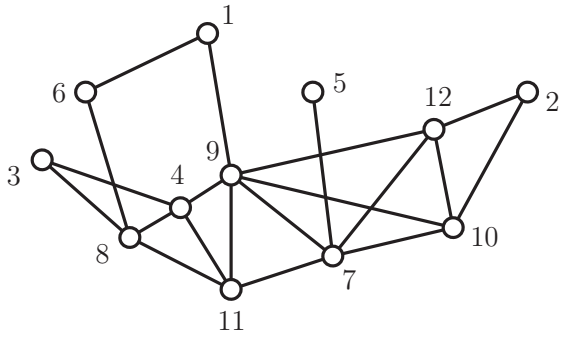
11. Use the lexicographic algorithm developed in the text (always proceed to the lowest legal vertex) to find an Euler circuit in the graph G shown below (use node 1 as root):



12. Is the following graph Hamiltonian? Why or why not?



13. For the graph below,



(a) Find a maximal clique of size 2.

(b) Find a maximal clique of size 3.

(c) Find a maximal clique of size 4.

(d) Find an induced cycle of size 5.

(e) Show that $\chi(G) \leq 4$ by producing a proper coloring using the elements of $\{1, 2, 3, 4\}$ as colors.

14. Verify Euler's formula for the following graph drawn in the plane with no crossing edges: