Physics 5A, Fall 2017
Homework Set 9

KK Ch 8: 8.6

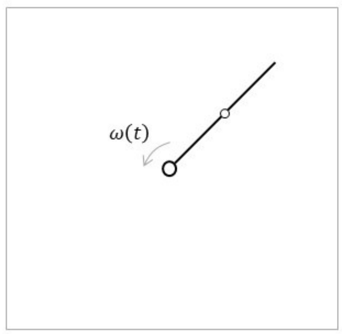
APF Ch 1: 1.9, 1.10, 1.11

APF Ch 2: 2.2, 2.4, 2.5

S 9.1 Let's go back to the bead on a rotating rod (see figure below). The rod is frictionless, and has negligible mass. The bead has mass, m, and at $t = 0$,

$$r(0) = r_0, \quad \dot{r}(0) = 0, \quad \omega(0) = \omega_0. \tag{1}$$

The rod is rotates freely without an applied torque.



(a) What is the energy E and angular momentum L (about the pivot point) of the bead? Express both in terms of m, r_0 and ω_0.

(b) Using energy and angular momentum conservation, show that

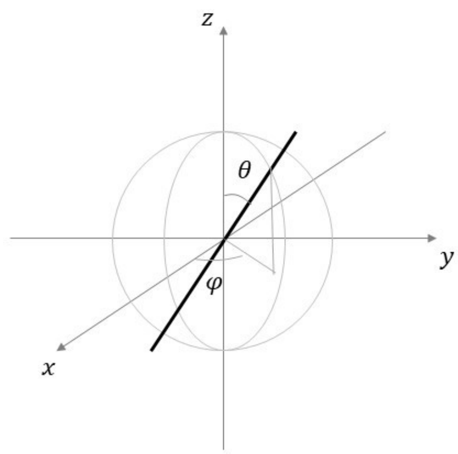
$$\frac{dr}{dt} = r_0\omega_0 \left(1 - \frac{r_0^2}{r^2}\right)^{1/2}. \tag{2}$$

(c) What is $r(t)$ and $\omega(t)$? Express both in terms of r_0 and ω_0.

S 9.2 In lecture, we looked at a rod with length l and mass M rotating about its center of mass (see figure below). The rod has an angular velocity $\vec{\omega} = \omega_x\hat{\imath} + \omega_y\hat{\jmath}$

(a) Find the components of the rotational inertia tensor $\overleftrightarrow{I}$.

(b) Find the angular momentum vector $\vec{L}$ of the rod. Are all three components of the vector linearly independent?



(c) Find kinetic energy of the rod in terms of the components of $\vec{\omega}$.

(d) Now find the kinetic energy of the rod in terms of L_x and L_y (you will have to solve a set of two simultaneous equations).

S 9.3 On pp 27-28 of AP French the effects of the superposition of many vibrations with the same frequency and amplitude is analyzed. In this problem we will do this analysis a different way.

Consider a series of N vibrations all with amplitude A_0 and frequency ω, but with differing phase shifts δ:

$$\begin{aligned} x(t) &= A_0 \cos(\omega t) + A_0 \cos(\omega t + \delta) + A_0 \cos(\omega t + 2\delta) + \ldots A_0 \cos(\omega t + (N-1)\delta), \\ &= \sum_{n=0}^{N-1} A_0 \cos(\omega t + n\delta). \end{aligned} \tag{3}$$

(a) Show that

$$x(t) = A_0 \mathrm{Re} \left\{ e^{j\omega t} \sum_{n=0}^{N-1} \left(e^{j\delta} \right)^n \right\}. \tag{4}$$

(b) Next, using the identity,

$$\sum_{n=0}^{N-1} x^n = \frac{1 - x^N}{1 - x}, \tag{5}$$

find $x(t)$ and show that it agrees with Eq. (2.9) of AP French.