Basic Colorimetry & Tristimulus Theory

Computational Color
UC Berkeley CS294-164

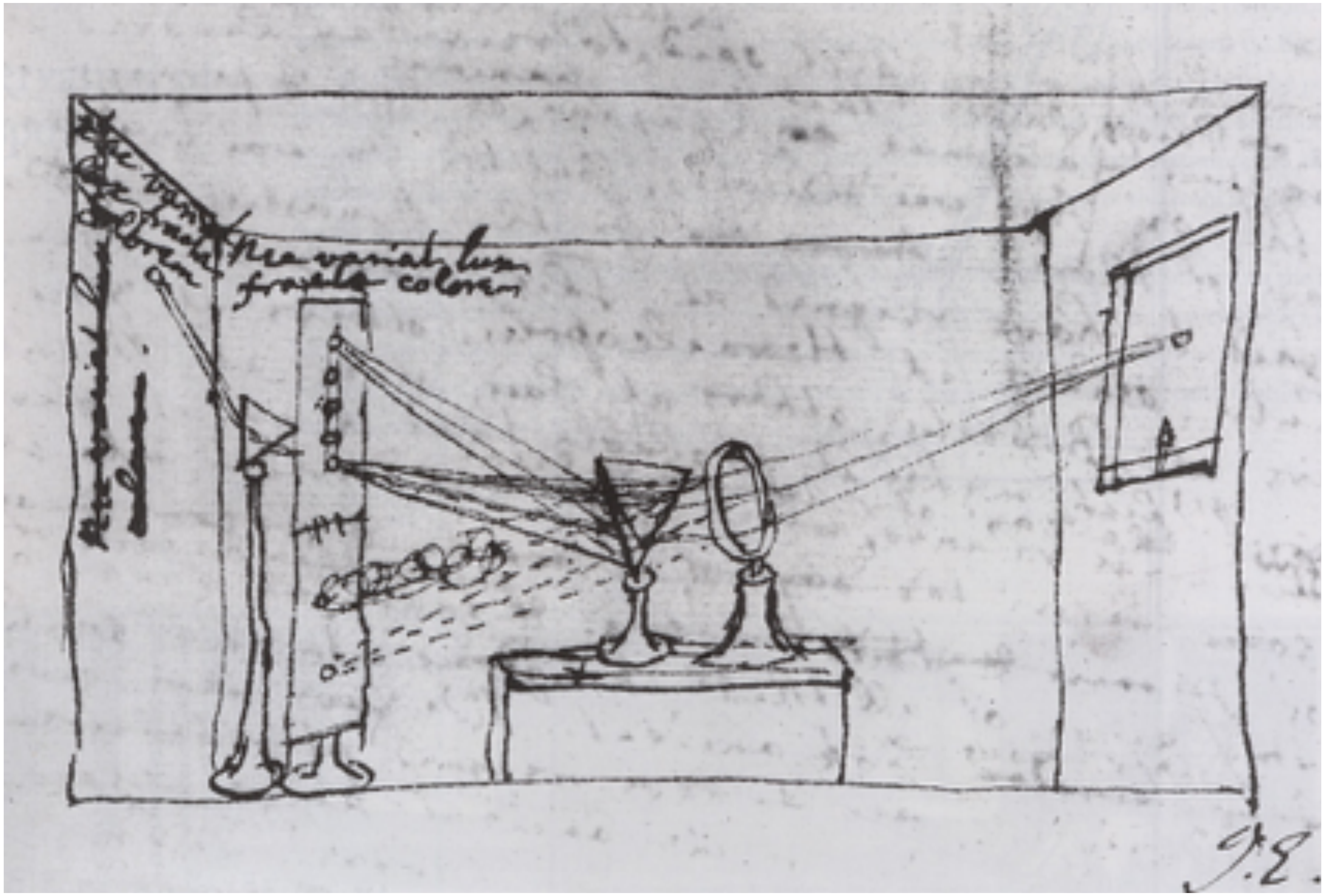
credit: Science Media Group.

Physical Basis of Color

Isaac Newton's Experimentum Crucis

Isaac Newton performing his crucial prism experiment – the 'experimentum crucis' – in his Woolsthorpe Manor bedroom.
Acrylic painting by Sascha Grusche (17 Dec 2015)

The Fundamental Components of Light



- Newton showed sunlight can be subdivided into a rainbow with a prism
- Resulting light cannot be further subdivided with a second prism

The Visible Spectrum of Light

Electromagnetic radiation

- Oscillations of different frequencies (wavelengths)

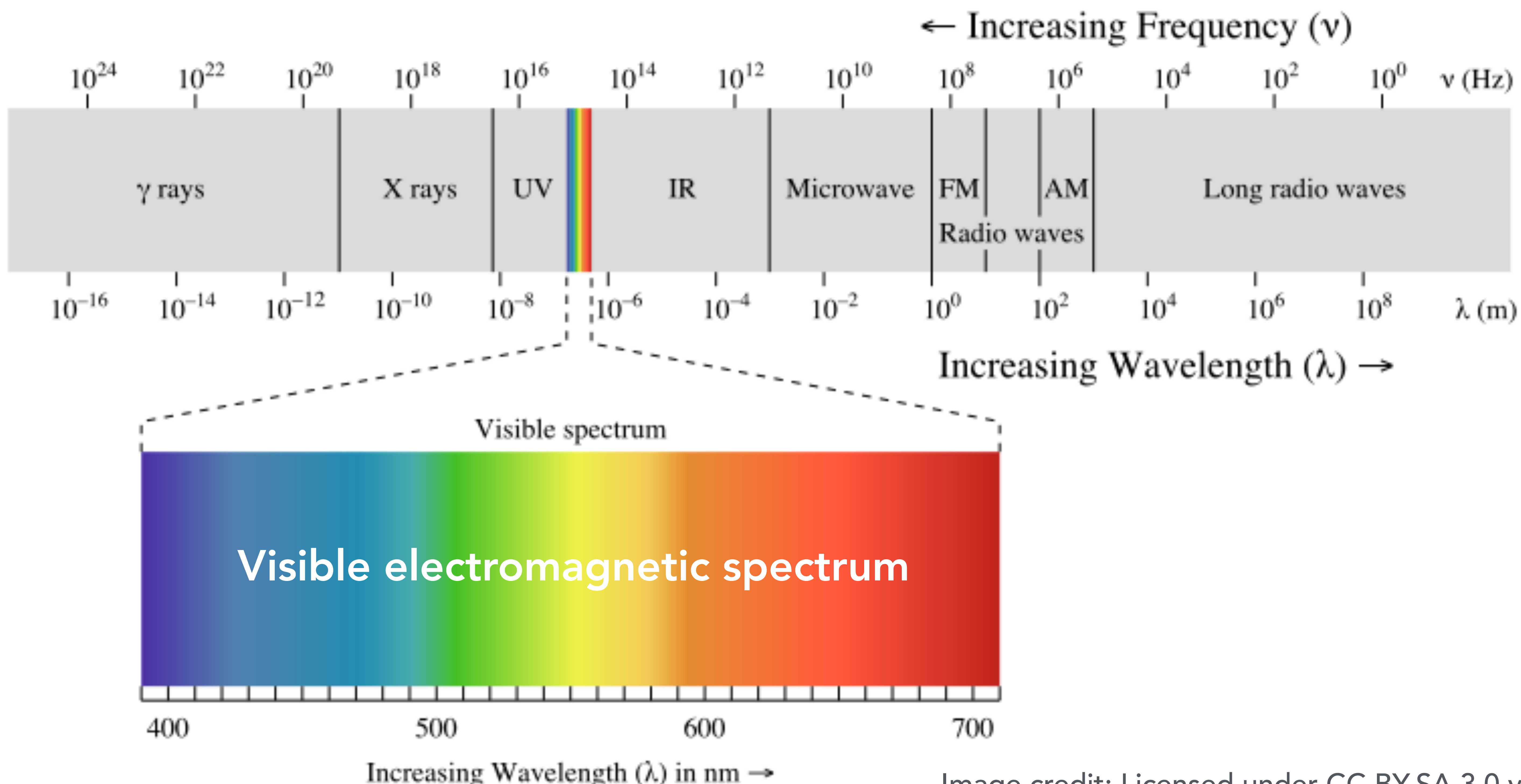
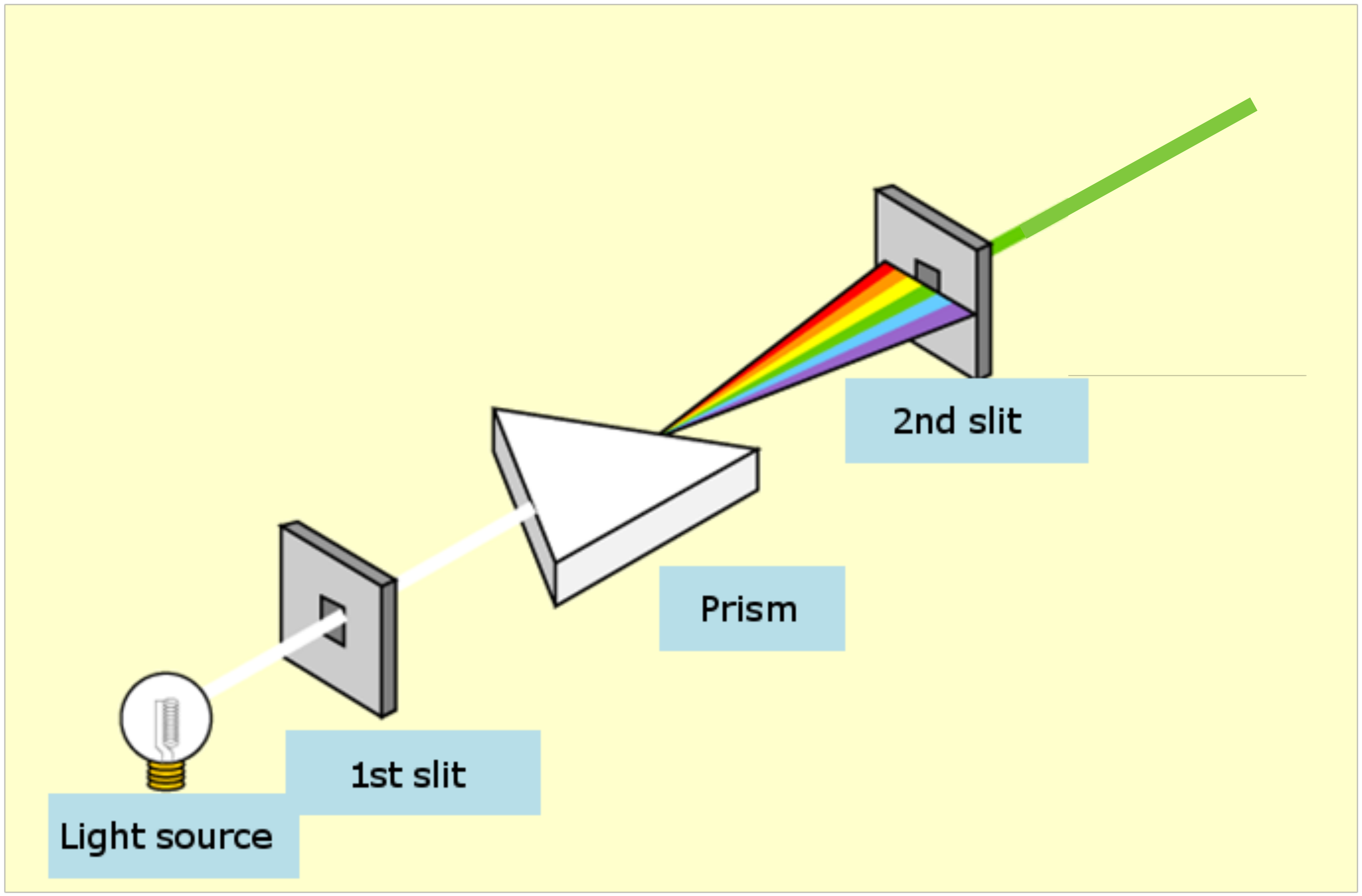


Image credit: Licensed under CC BY-SA 3.0 via Commons
https://commons.wikimedia.org/wiki/File:EM_spectrum.svg#/media/File:EM_spectrum.svg

Monochromator



A monochromator delivers light of a single wavelength from a light source with broad spectrum.
Control which wavelength by angle of prism.

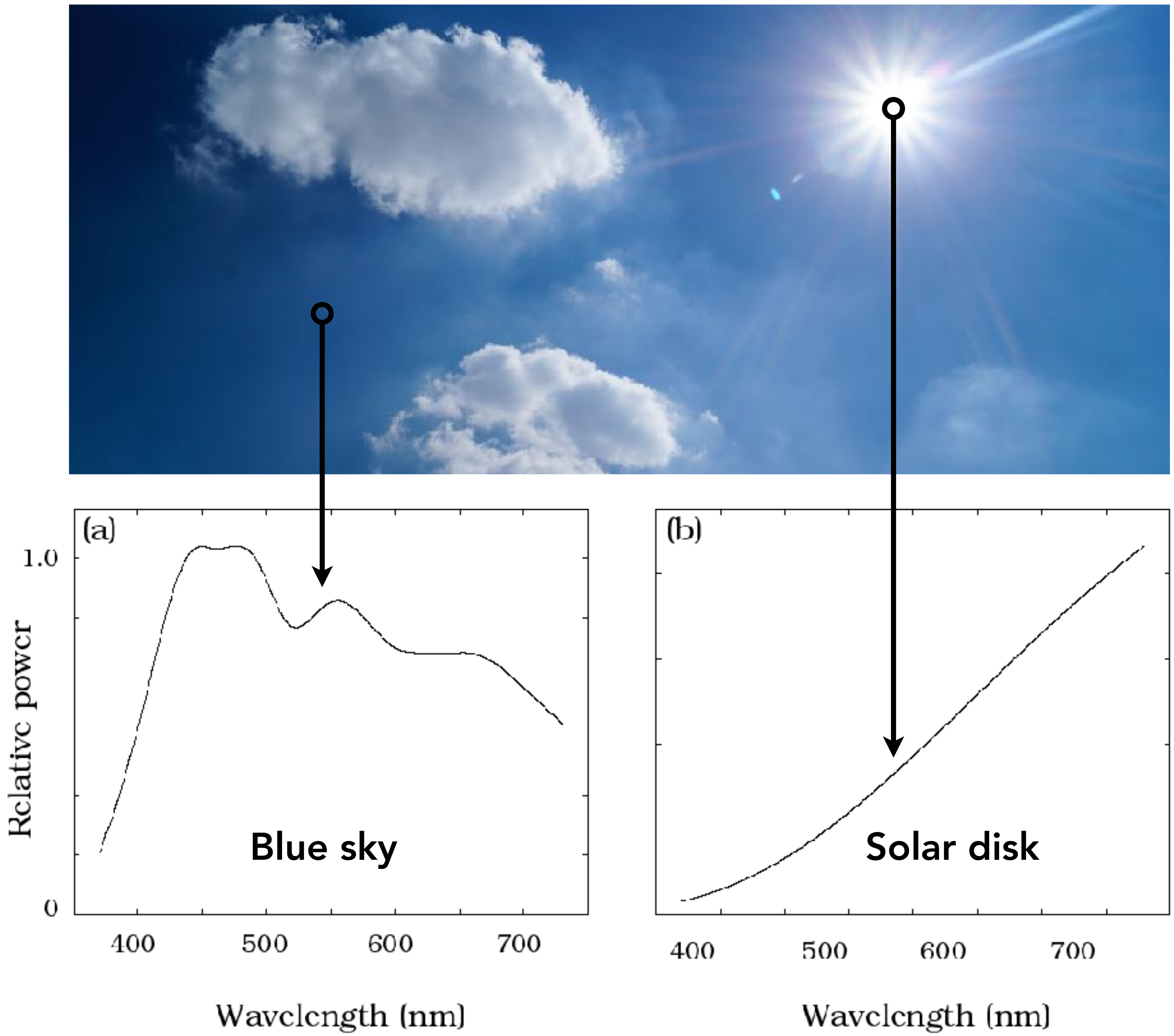
Hegyi et al. "Introduction to Practical Biochemistry"
http://elte.prompt.hu/sites/default/files/tananyagok/IntroductionToPracticalBiochemistry/ch04s07.html

Spectral Power Distribution (SPD)

Salient property in measuring light

- The amount of light present at each wavelength
- Units:
  - radiometric units / nanometer (e.g. watts / nm)
  - Can also be unit-less
- Often use "relative units" scaled to maximum wavelength for comparison across wavelengths when absolute units are not important

Daylight Spectral Power Distributions Vary



[Brian Wandell]

Spectral Power Distribution of Light Sources

Describes distribution of energy by wavelength

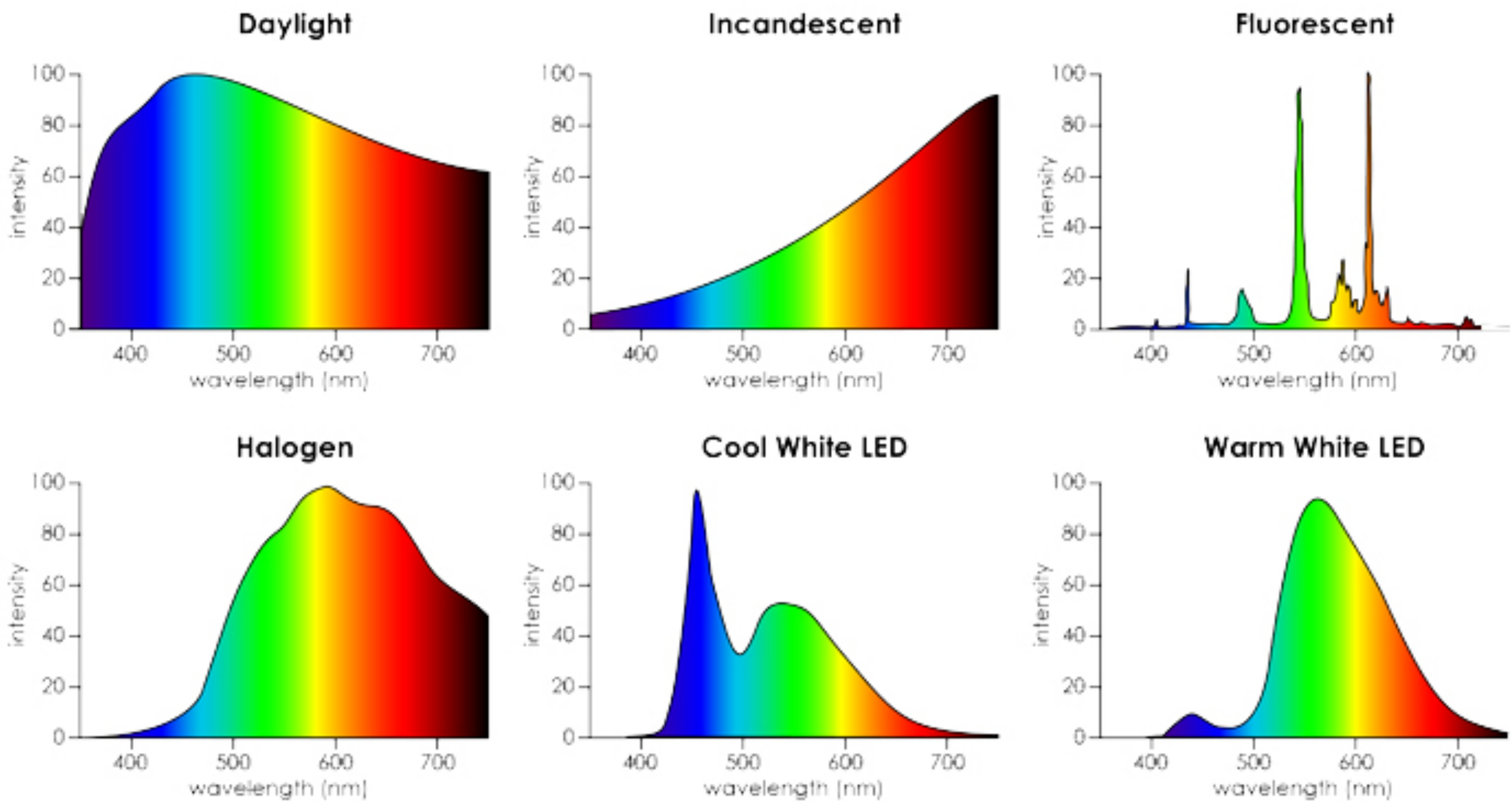
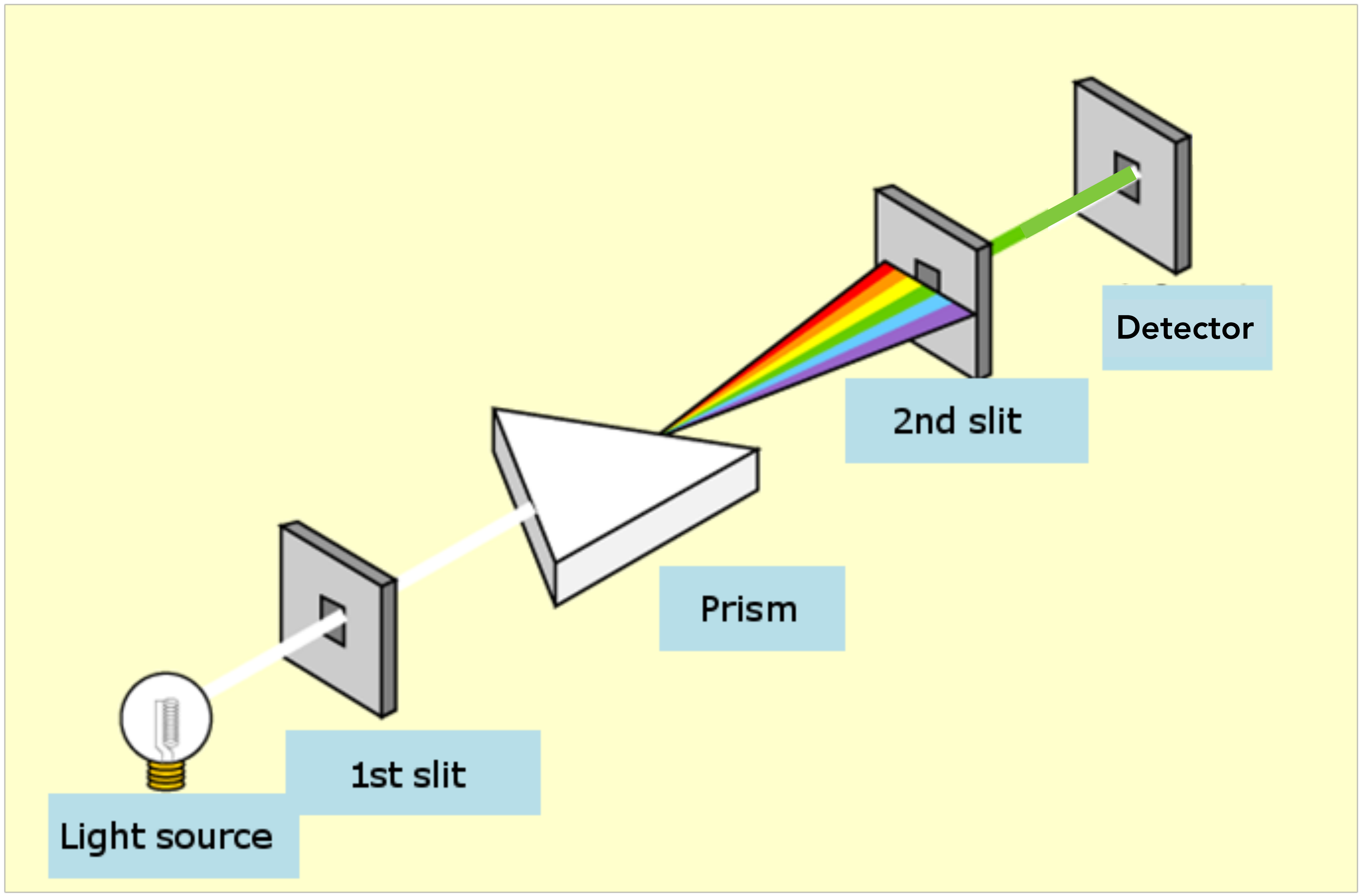


Figure credit: admesy ADVANCED MEASUREMENT SYSTEMS

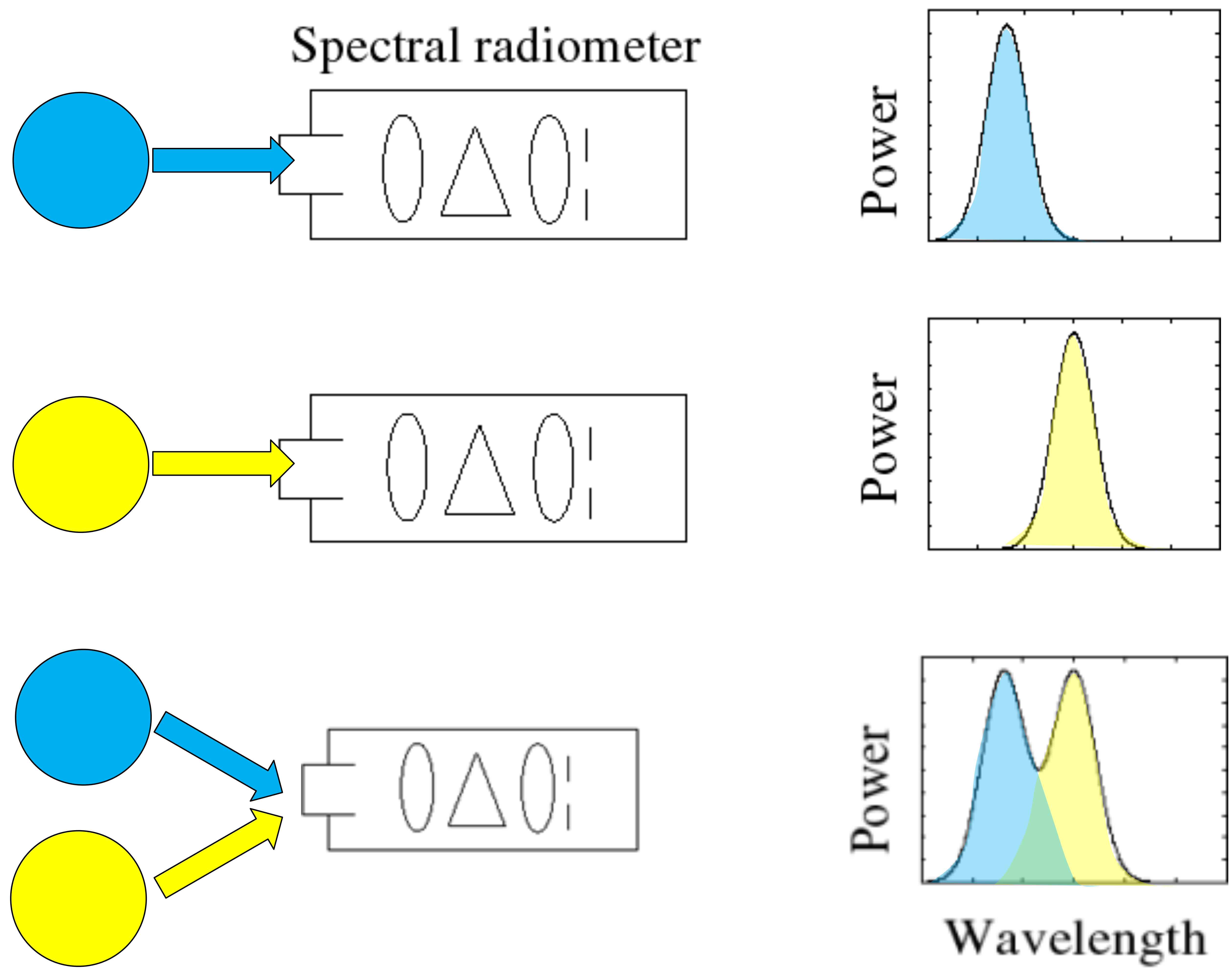
Spectrometer



http://elte.prompt.hu/sites/default/files/tananyagok/IntroductionToPracticalBiochemistry/ch04s07.html
Hegyi et al. "Introduction to Practical Biochemistry"

For unknown light source, use a monochromator to isolate each wavelength of light for measurement

Superposition (Linearity) of Spectral Power Distributions



[Brian Wandell]

Measuring Light

A Simple Model of a Light Detector

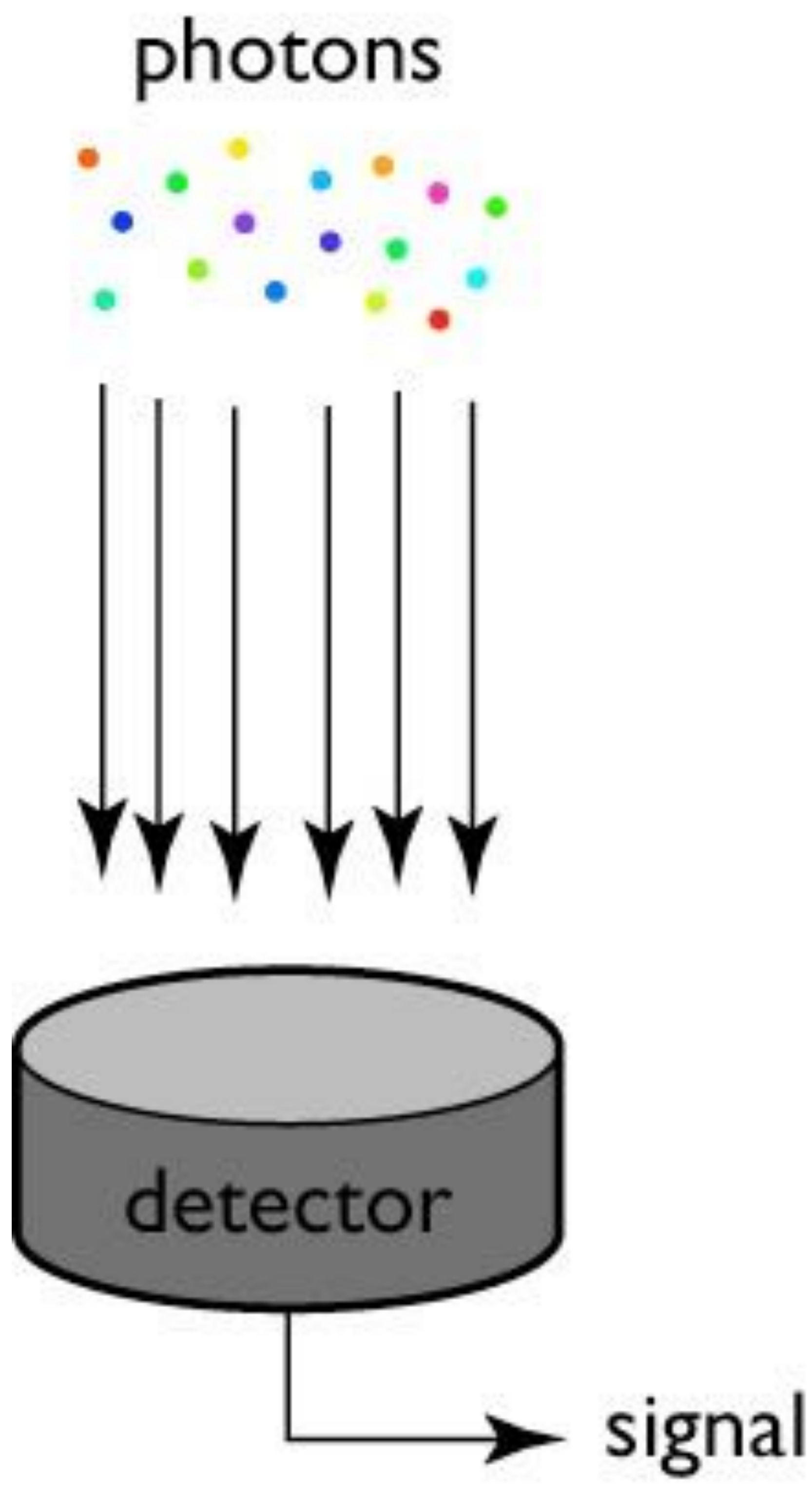
Produces a scalar value (a number) when photons land on it

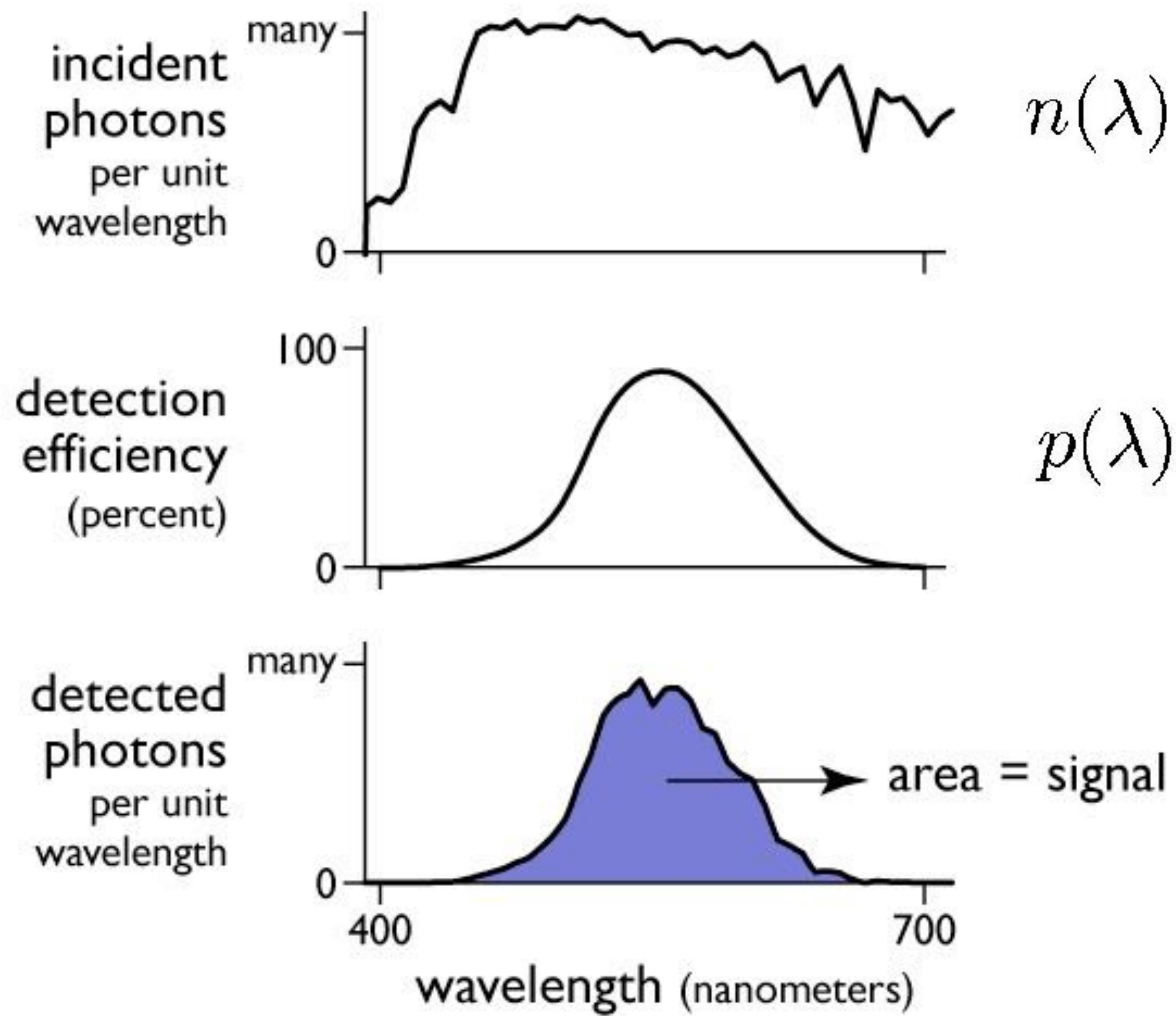
- Value depends only on the number of photons detected
- Each photon has a probability of being detected that depends on the wavelength
- No way to distinguish between signals caused by light of different wavelengths: there is just a number

This model works for many detectors:

- based on semiconductors (such as in a digital camera)
- based on visual photopigments (such as in human eyes)

A Simple Model of a Light Detector





Credit: Marschner

$$X = \int n(\lambda) p(\lambda) \, d\lambda$$

Mathematics of Light Detection

Same math carries over to spectral power distributions

- Light entering the detector has its spectral power distribution, s(λ)
- Detector has its spectral sensitivity or spectral response, r(λ)

$$\underbrace{X}_{\text{measured signal}} = \int \underbrace{s(\lambda)}_{\text{input spectrum}}\underbrace{r(\lambda)}_{\text{detector's sensitivity}}\, d\lambda$$

Mathematics of Light Detection

If we think of s and r as discrete, sampled representations (vectors) rather than continuous functions, this integral operation is a dot product:

$$X = s \cdot r$$

We can also write this in matrix form:

$$X = \begin{bmatrix} \text{—} & s & \text{—} \end{bmatrix} \begin{bmatrix} | \\ r \\ | \end{bmatrix}$$

Dimensionality Reduction From ∞ to 1

At the detector:

- SPD is a function of wavelength (∞ - dimensional signal)
- Detector result is a scalar value (1 - dimensional signal)

Discussion: What is The Dimensionality of Color?

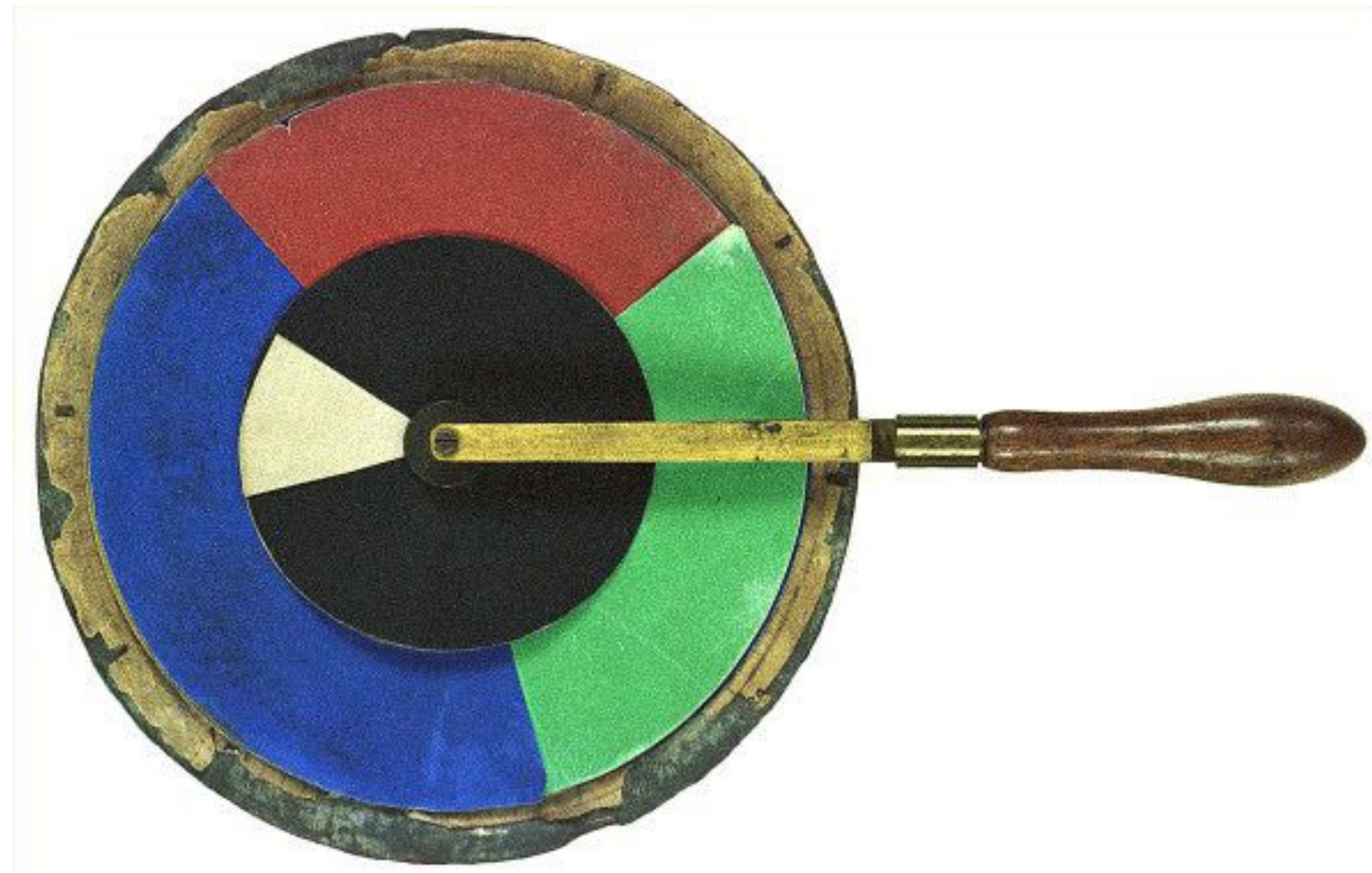
How do we know?

- Depends on representations: RGB (3 dim), visible spectrum (infinite dimensional) — discretized; but intrinsically a infinite dimensional object
- Human visual system has three color receptor cones — can consider for humans, color is three-dimensional. Many of our representations are 3D, and reflects our perception system.
- HDR imaging: merging multiple photos, we can get higher dynamic range..
- Some computer systems also use opacity as a fourth dimension in color representation, e.g. RGBA.
- Clue? In photo editing app, we can change saturation, brightness
- HSV: hue, saturation, value: 3D color model — continuous. RGB discretized.
- CIELAB color space, and CIEXYZ — different than RGB model. CMYK??
- Or if we think of color as a mixture of other colors, you could have 50 paint tubes. Or if you have red, green, blue primary colors, there is a certain range of colors you can get. Versus other primary sets…

Tristimulus Theory of Color

Searching for a Basis for Colors: Color Matching Experiment

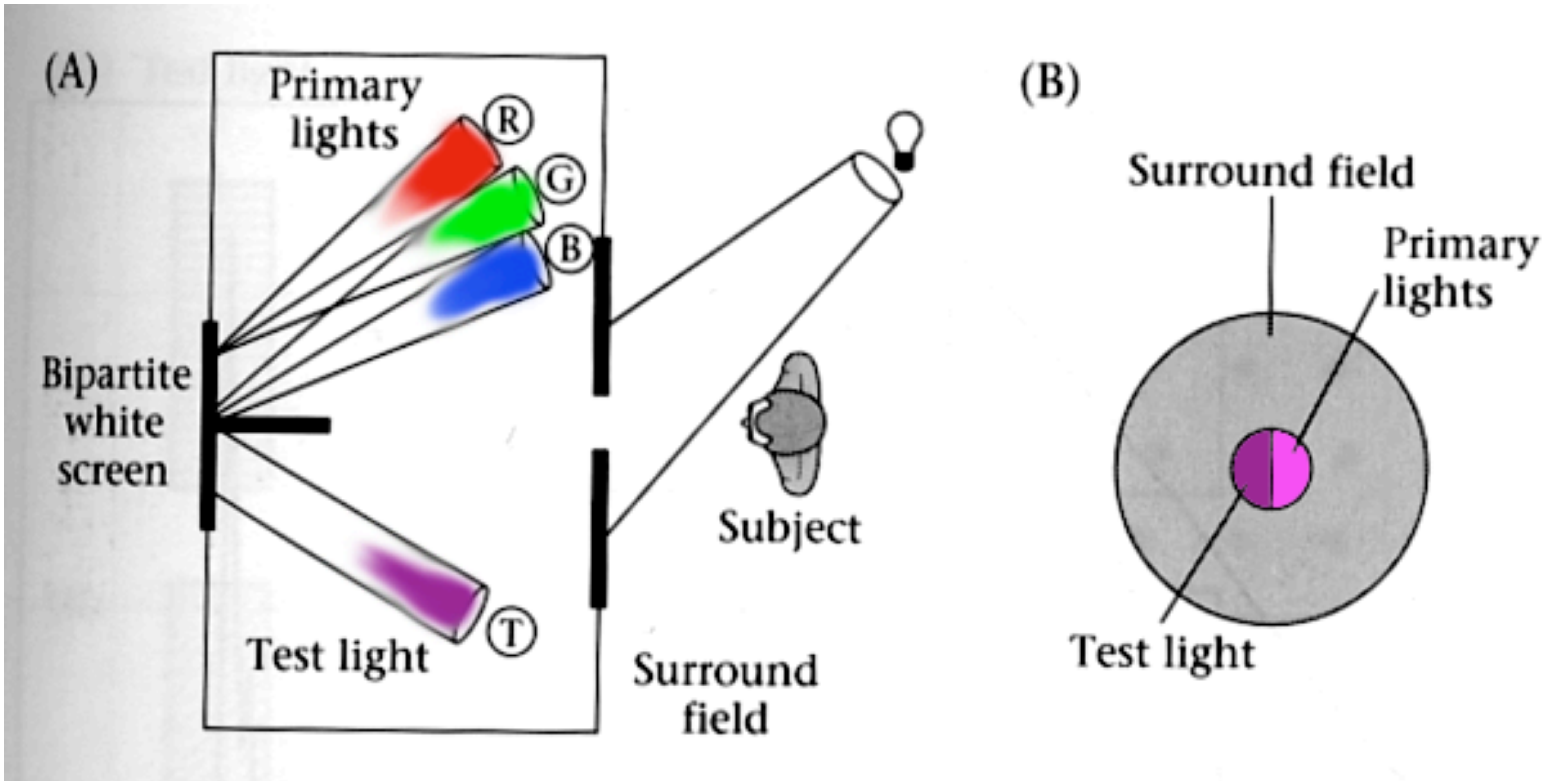
Maxwell's Crucial Color Matching Experiment



http://designblog.rietveldacademie.nl/?p=68422

Portrait: http://rsta.royalsocietypublishing.org/content/366/1871/1685

Color Matching Experiment



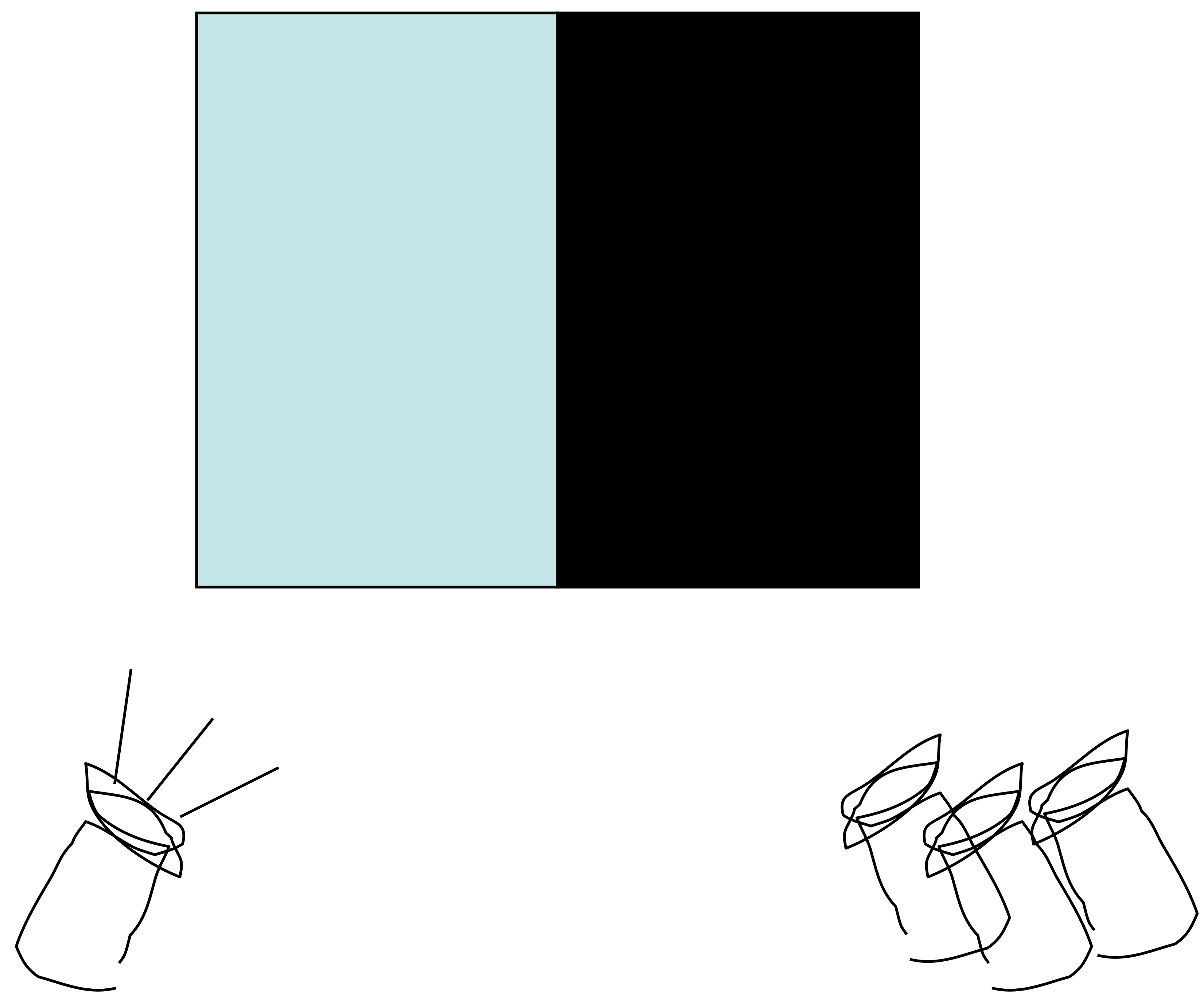
Same idea as spinning top, fancier implementation (Maxwell did this too)

Show test light spectrum on left

Mix "primaries" on right until they match

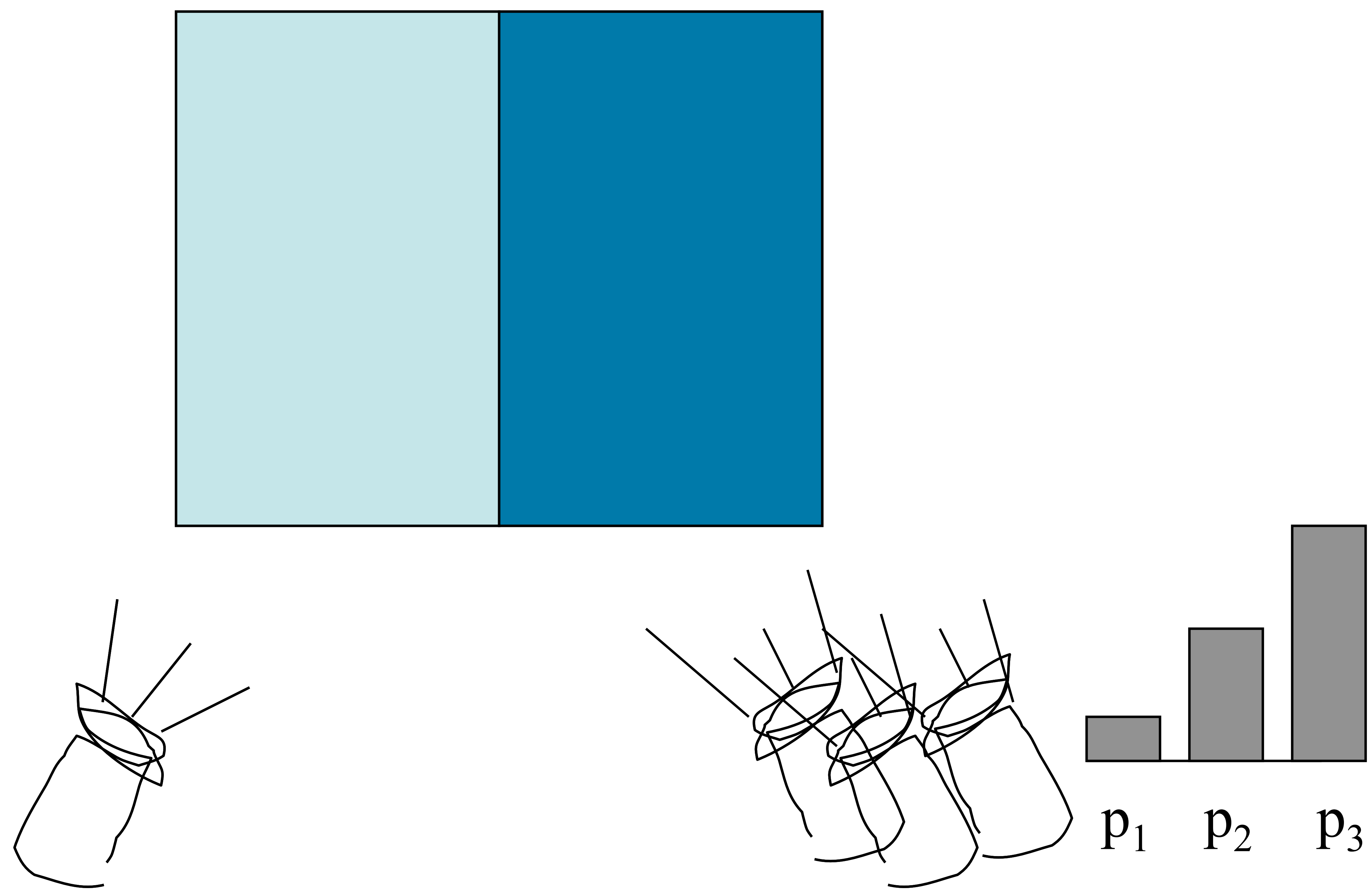
The primaries need not be RGB

Example Experiment



Slide from Durand and Freeman 06

Example Experiment



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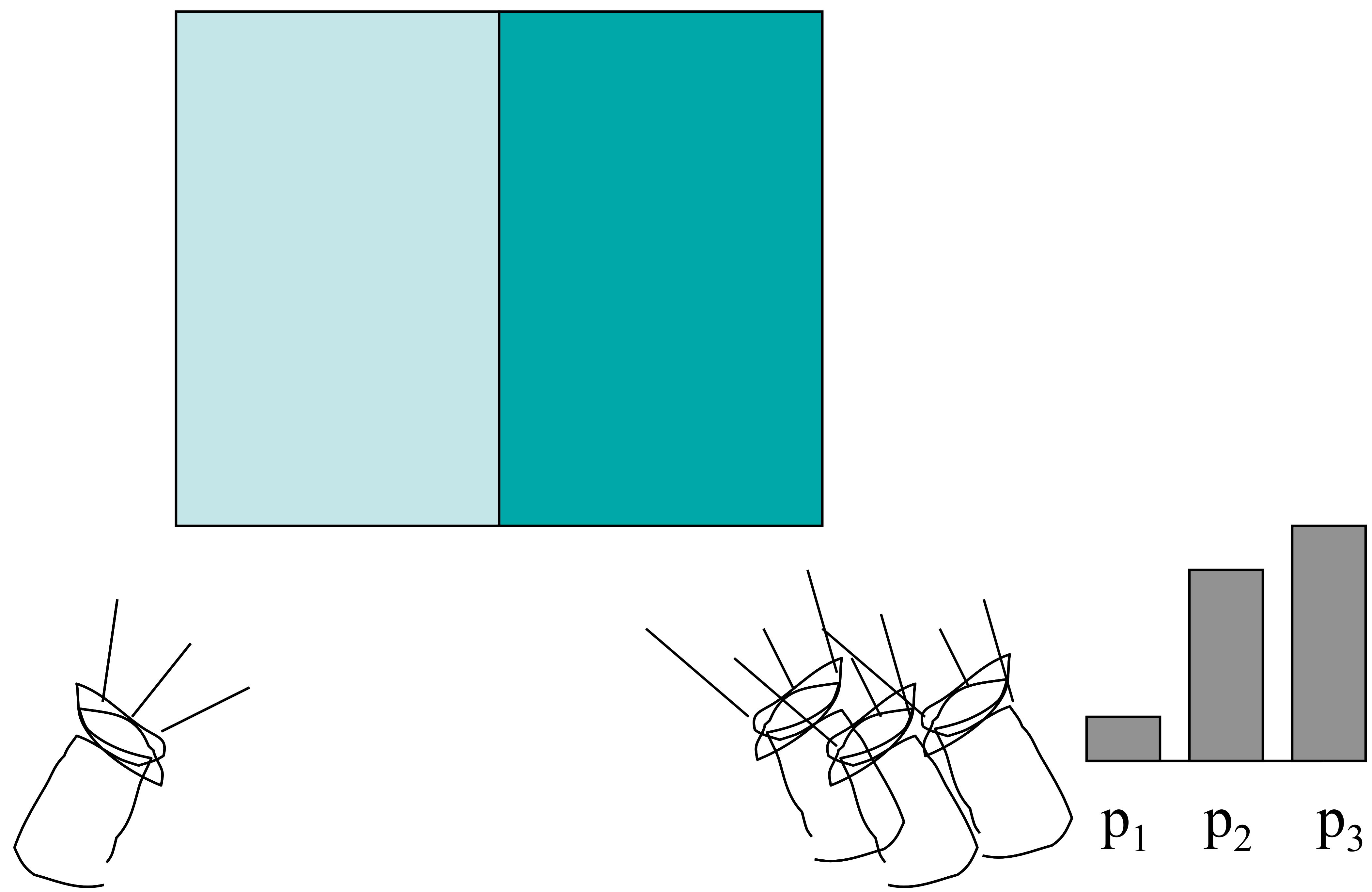
Example Experiment



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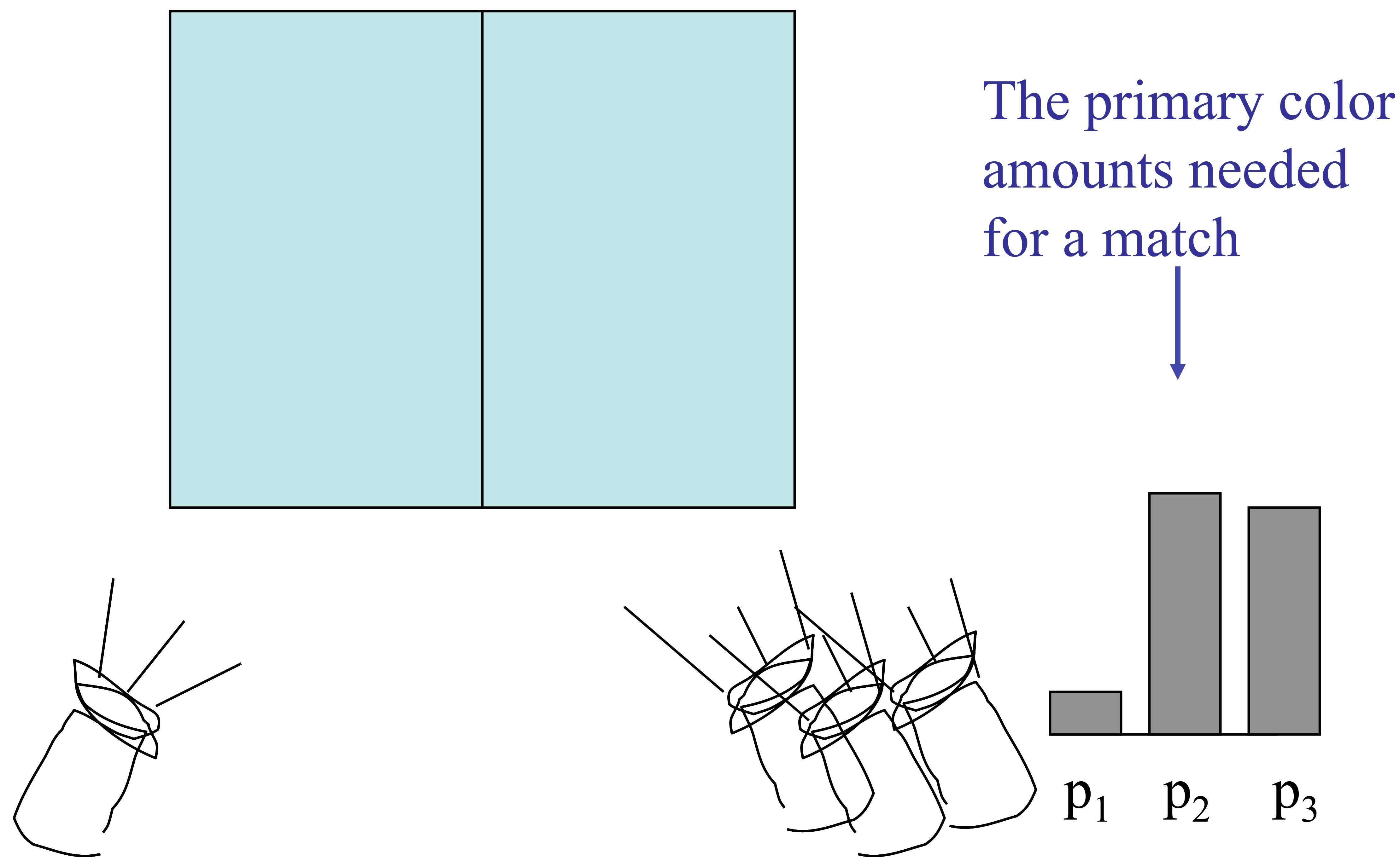
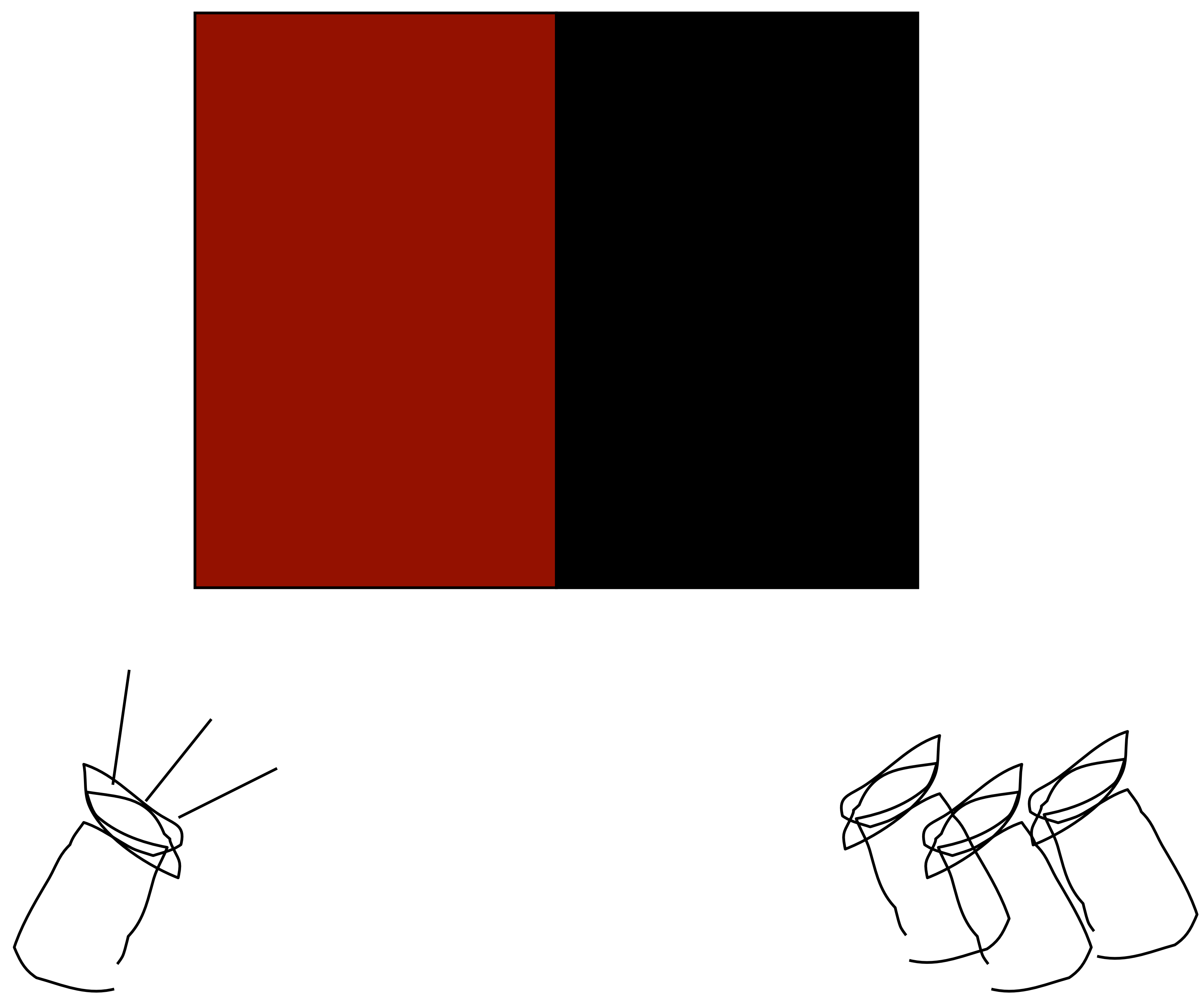
Example Experiment



Slide from Durand
and Freeman 06

Experiment 2: Out of Gamut



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and Freeman 06

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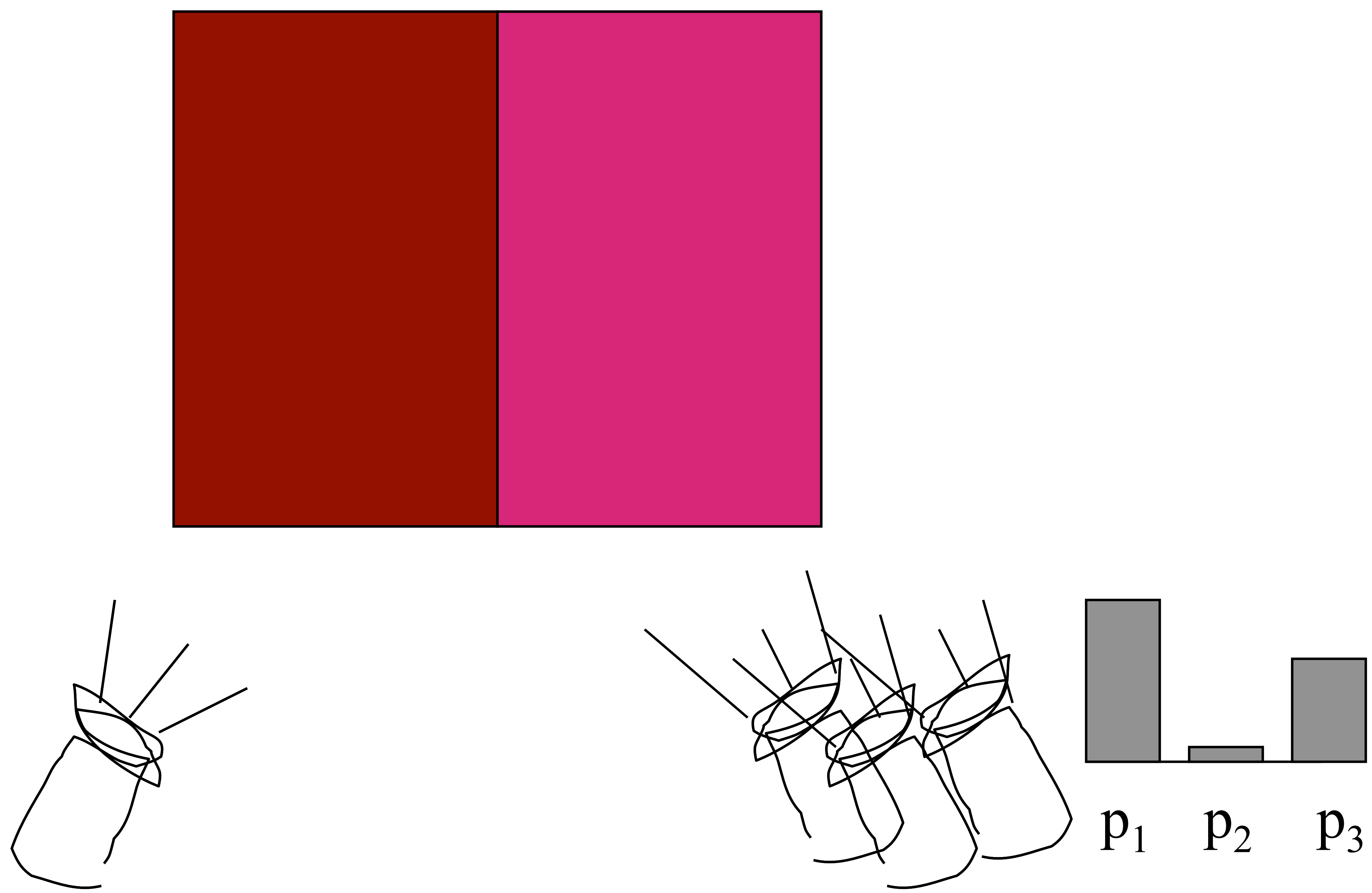

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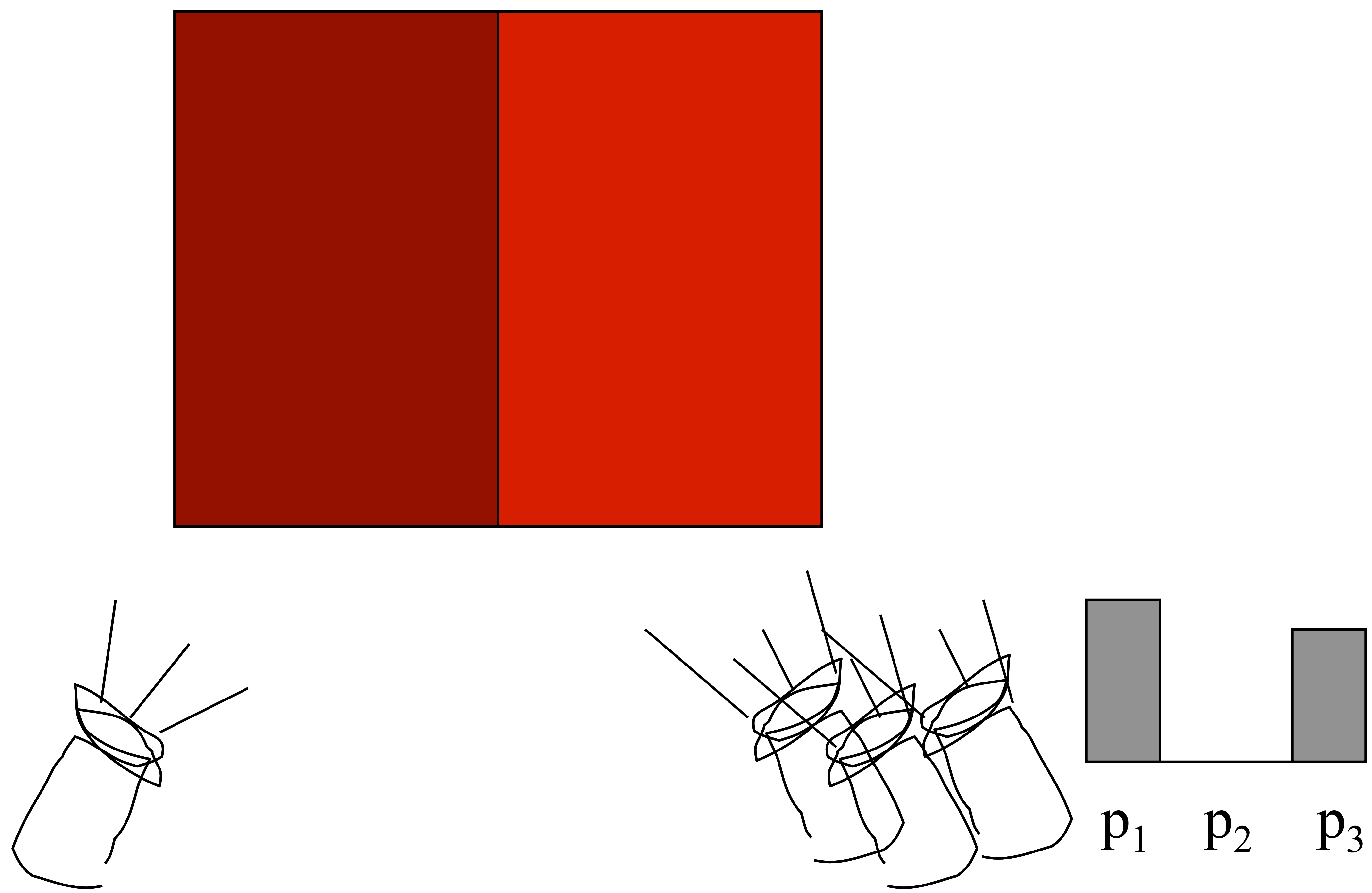
Experiment 2: Out of Gamut



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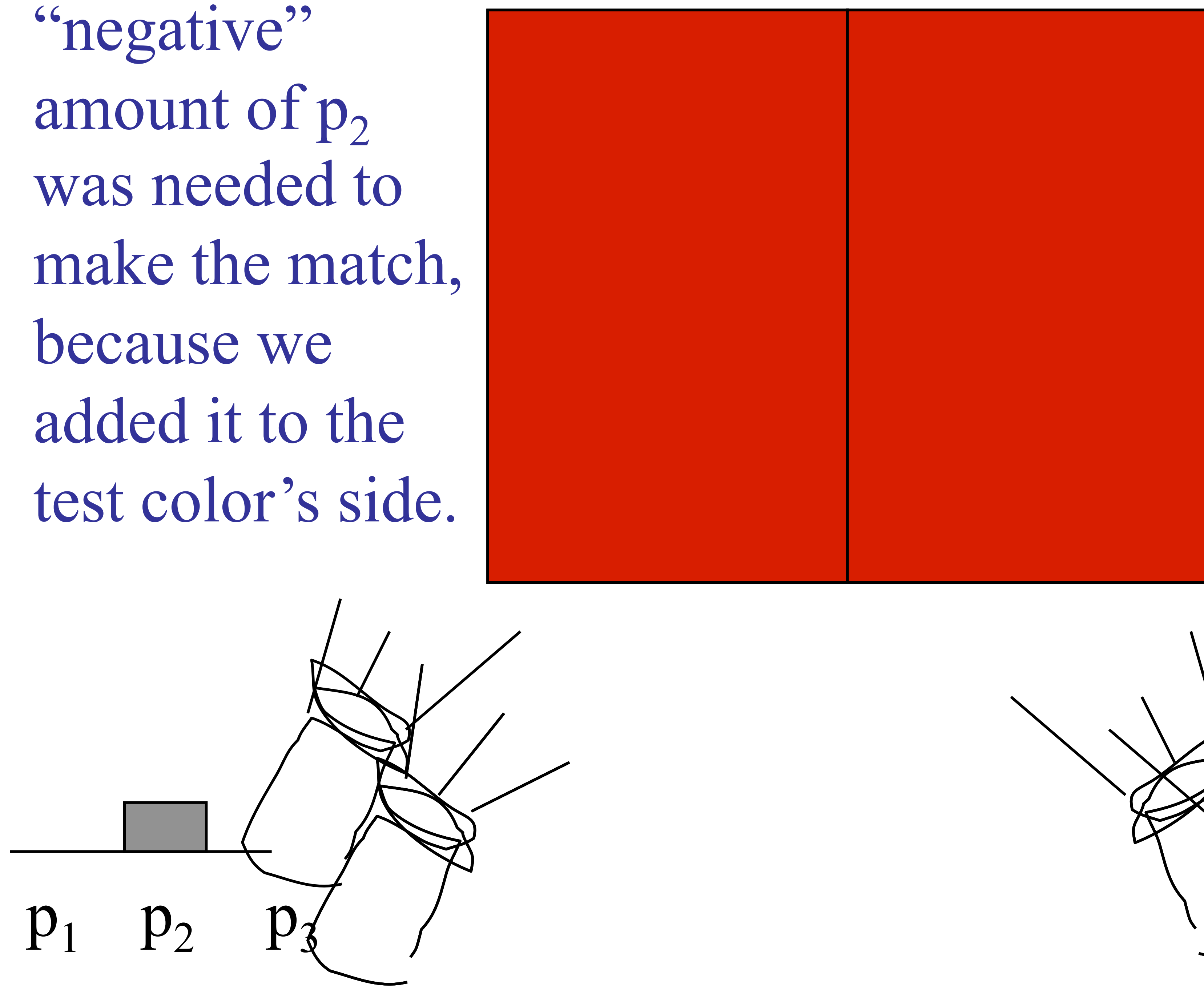
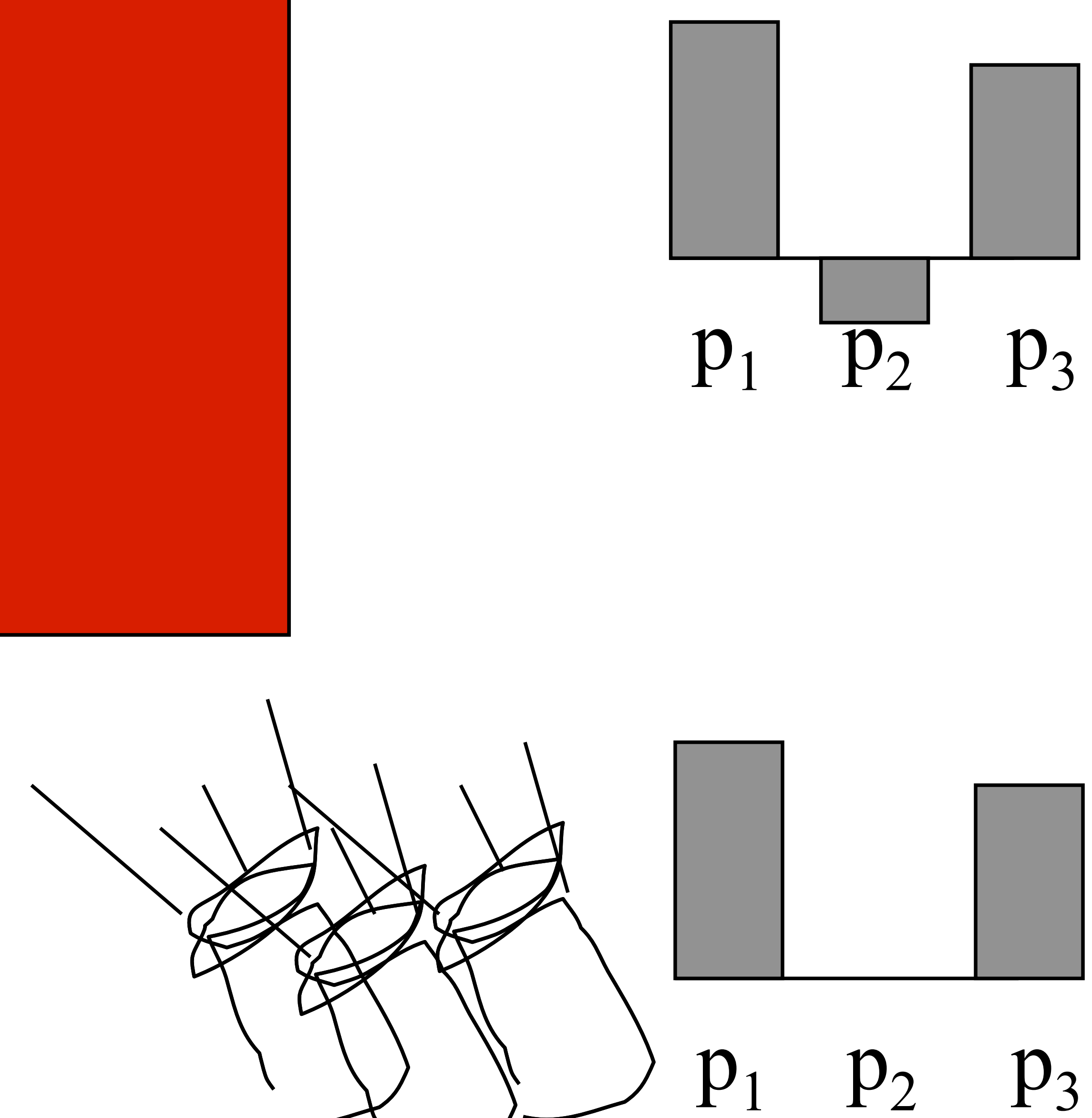
Experiment 2: Out of Gamut

We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.

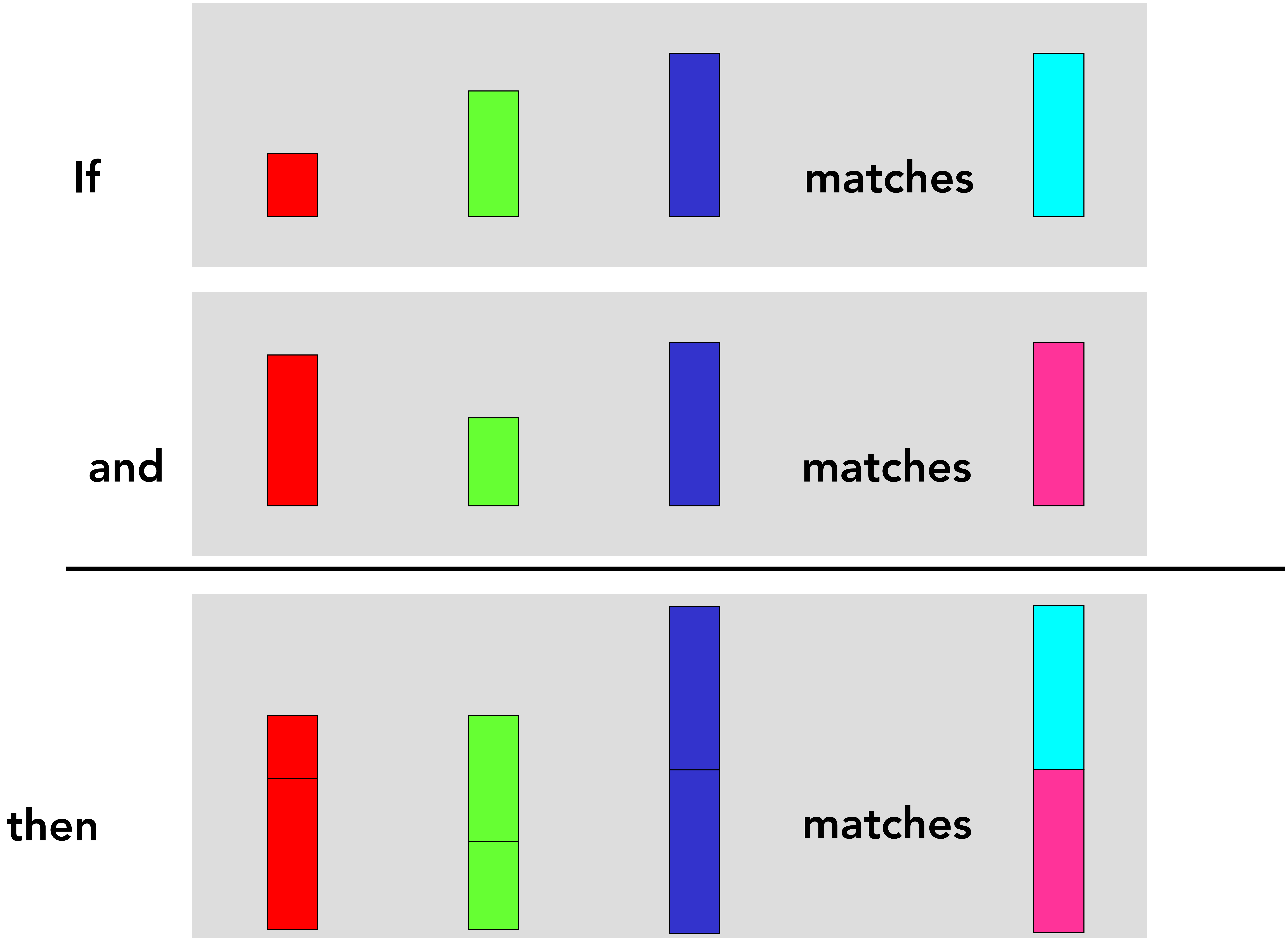


The primary color amounts needed for a match:



Slide from Durand and Freeman 06

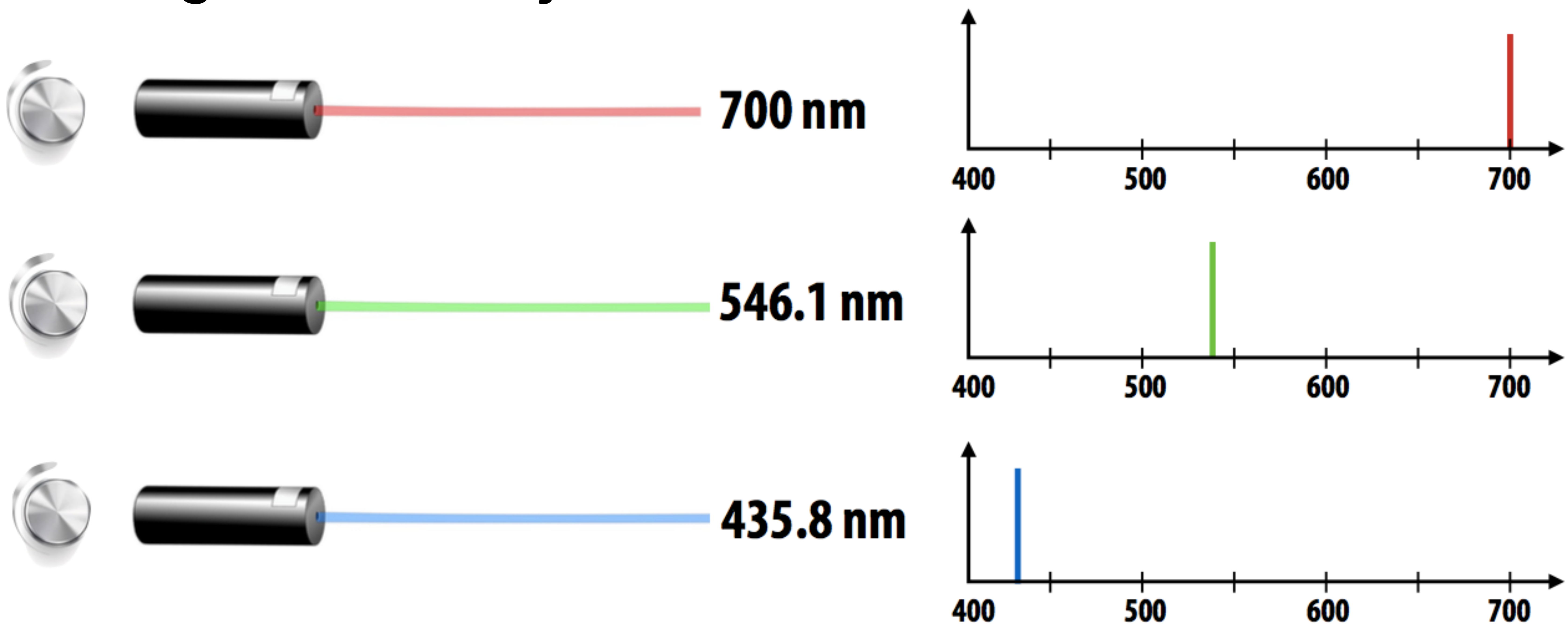
The Color Matching Experiment is Linear



Brian Wandell

CIE RGB Color Matching Experiment

Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE RGB standard

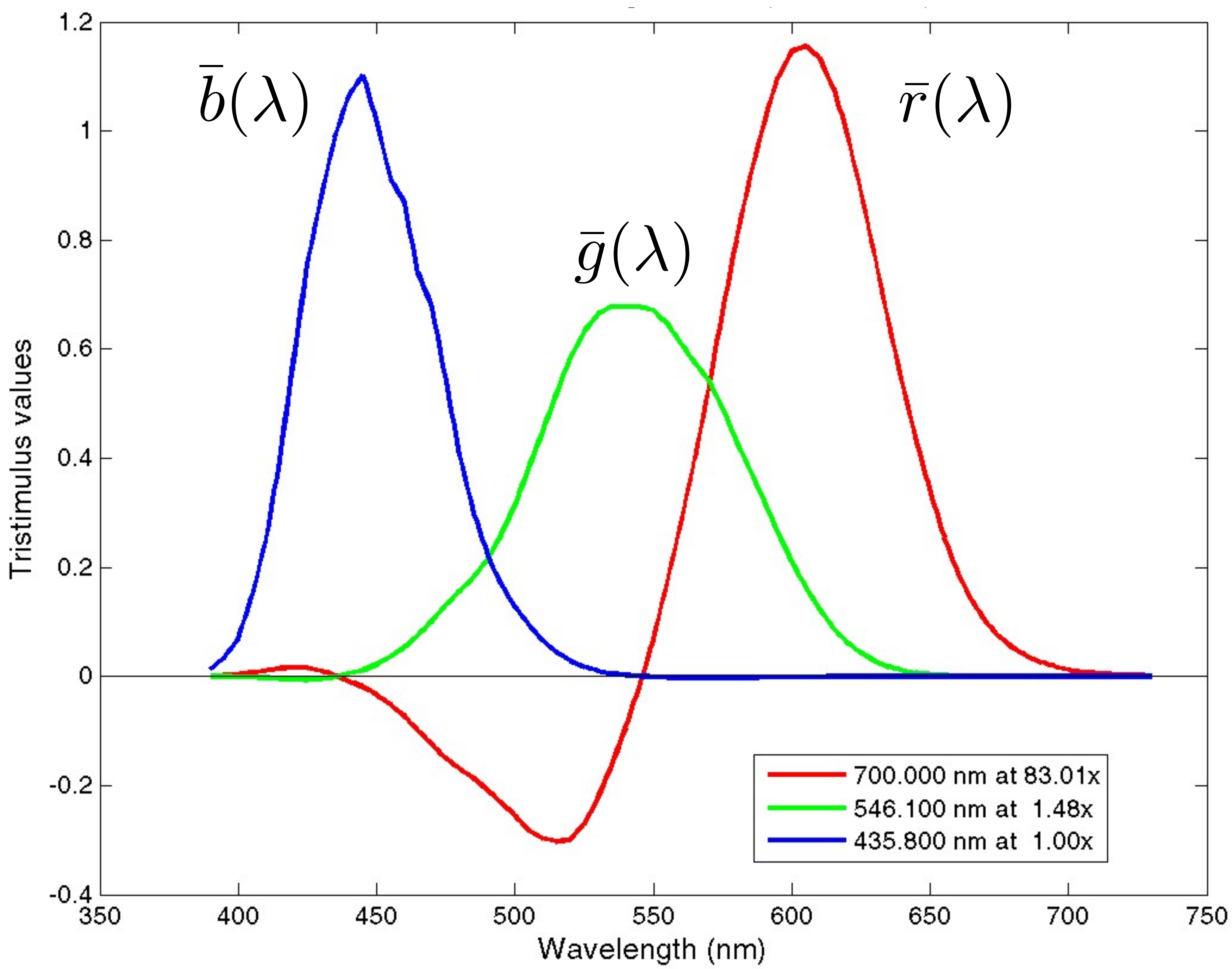


The test light is also a monochromatic light

Kayvon Fatahalian

CIE RGB Color Matching Functions

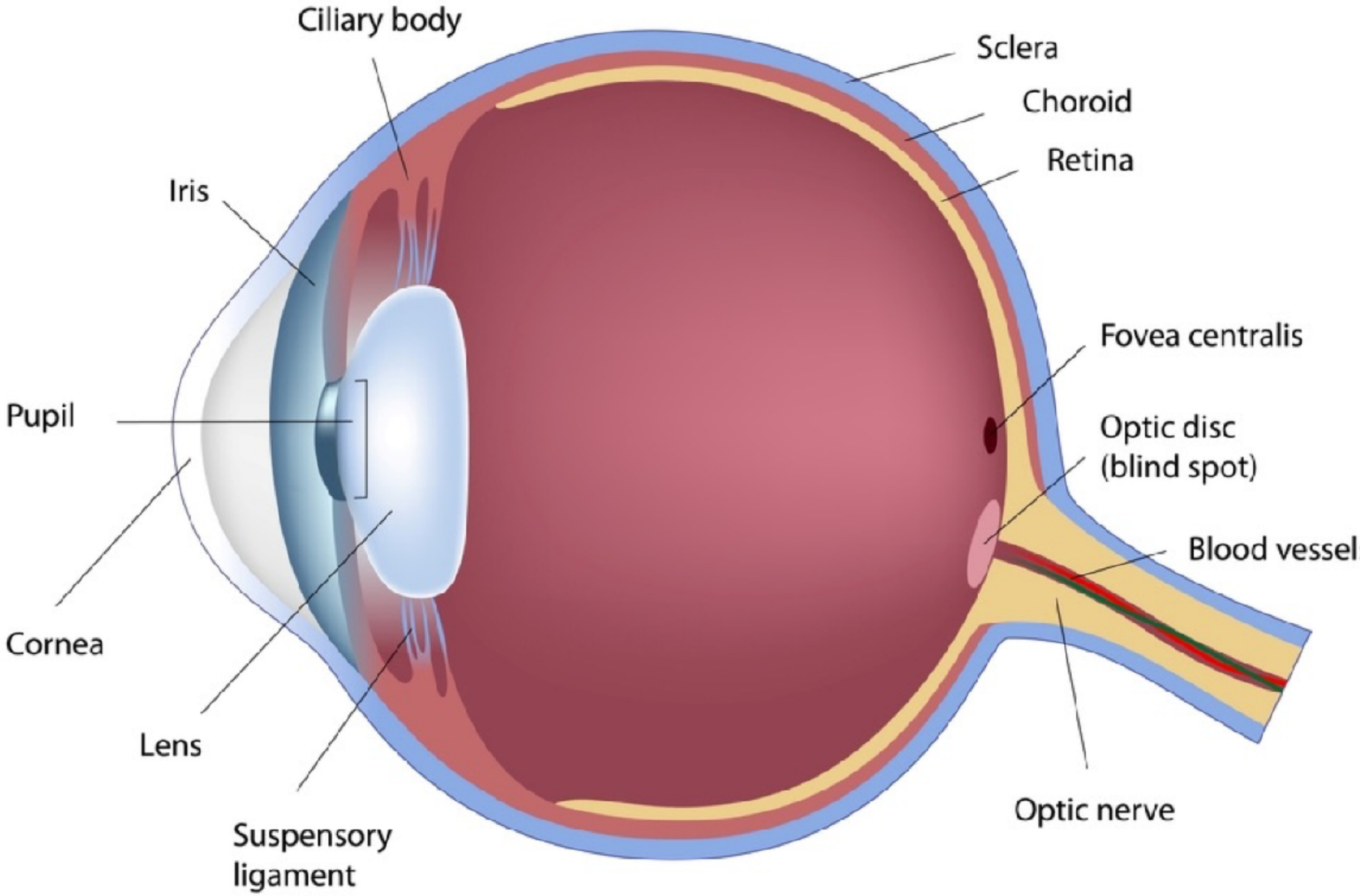
Graph plots how much of each CIE RGB primary light must be combined to match a monochromatic light of wavelength given on x-axis



 Careful: these are not response curves or primary spectra!

Biological Basis of Color

Anatomy of The Human Eye



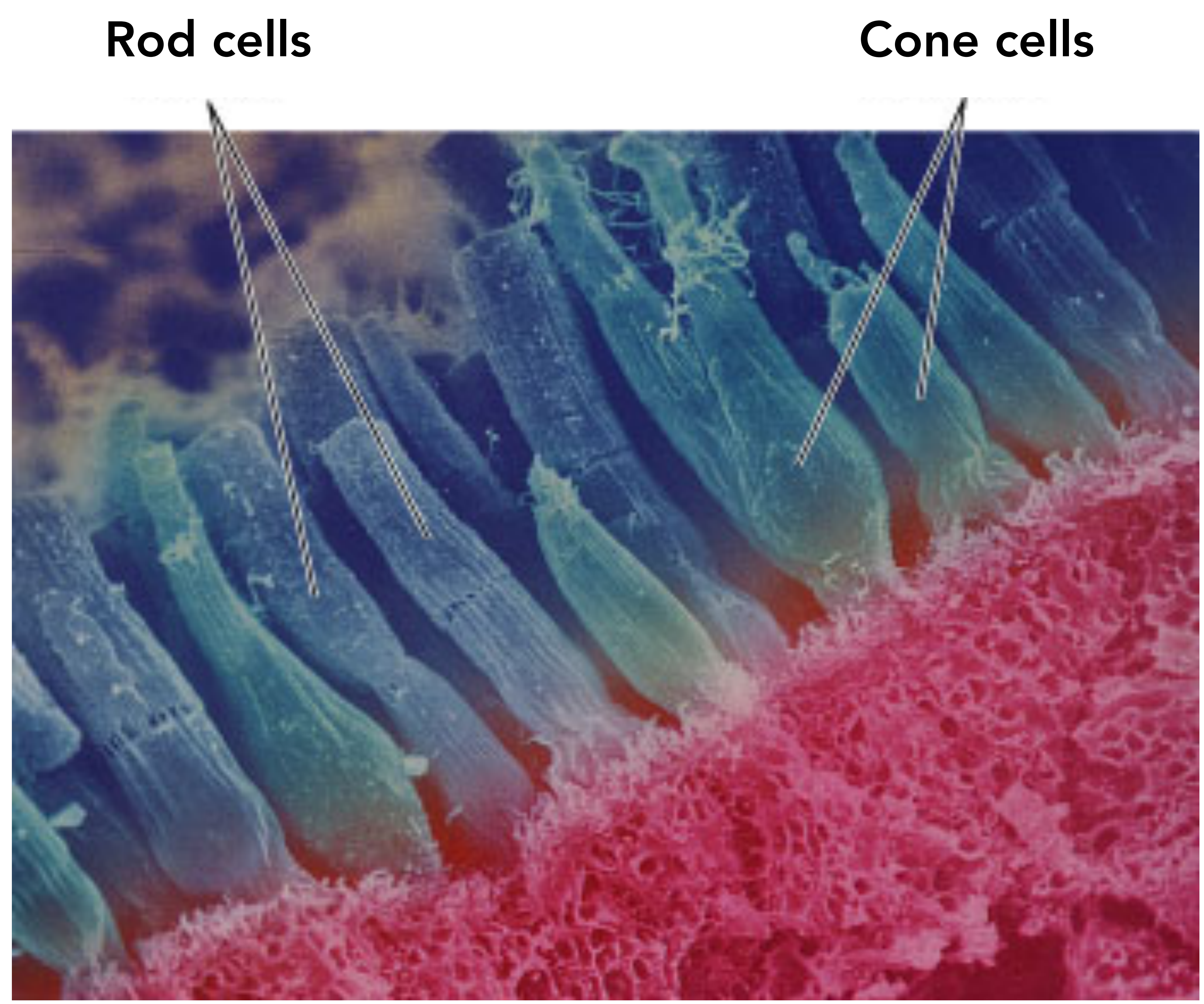
Retinal Photoreceptor Cells: Rods and Cones

Rods are primary receptors in very low light (“scotopic” conditions), e.g. dim moonlight

- ~120 million rods in eye
- Perceive only shades of gray, no color

Cones are primary receptors in typical light levels (“photopic”)

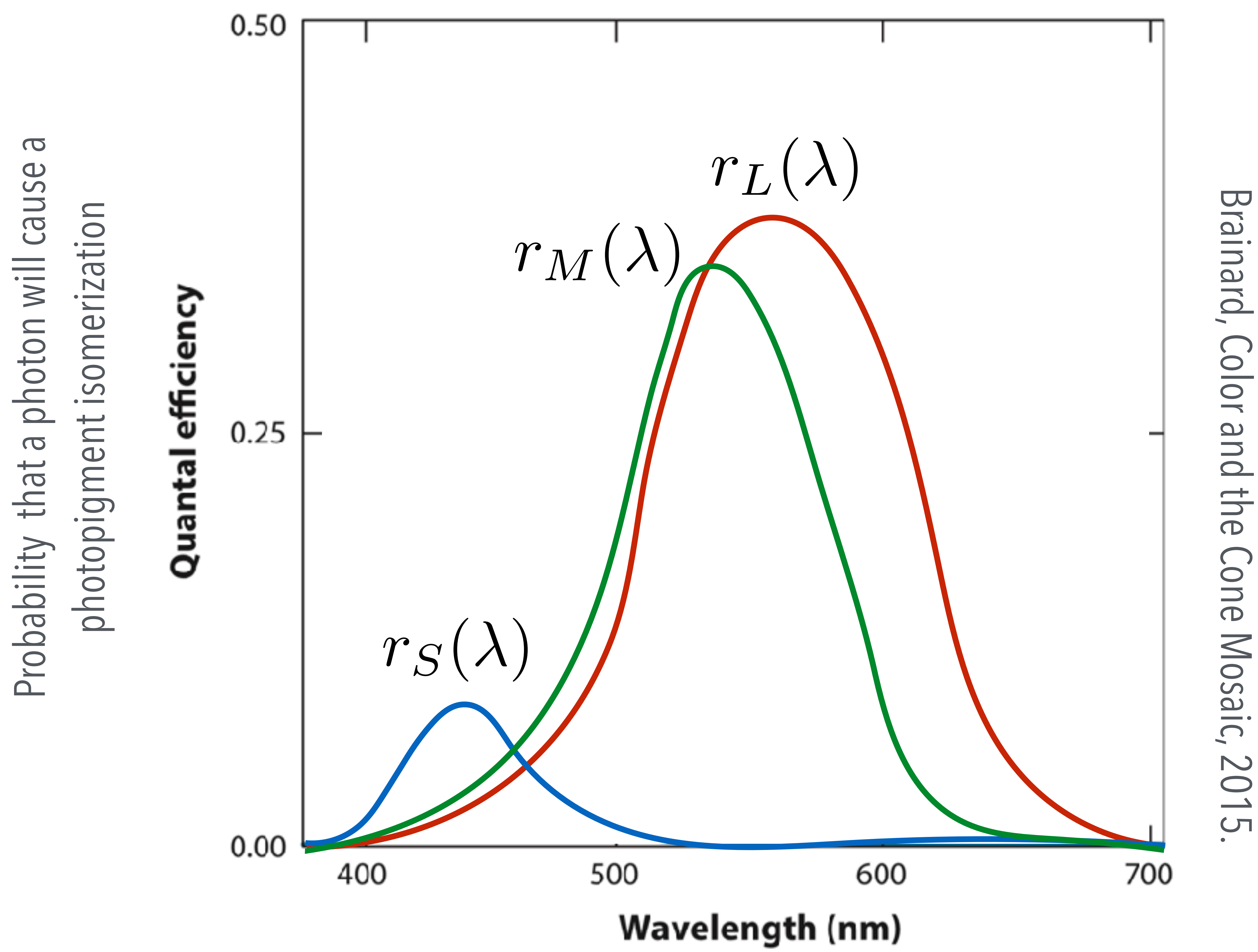
- ~6-7 million cones in eye
- Three types of cones, each with different spectral sensitivity
- Provide sensation of color



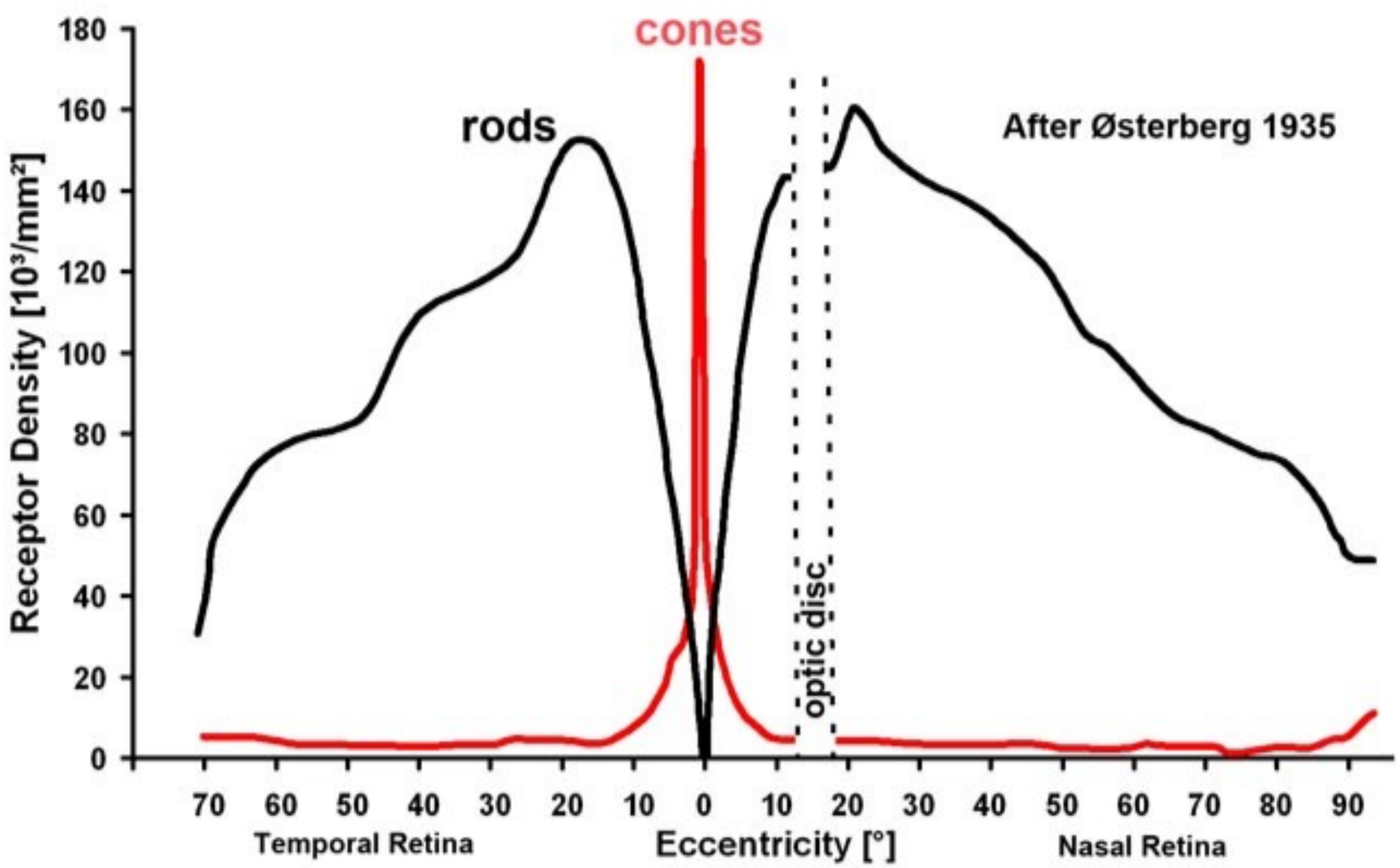
http://ebooks.bfwpub.com/life.php Figure 45.18

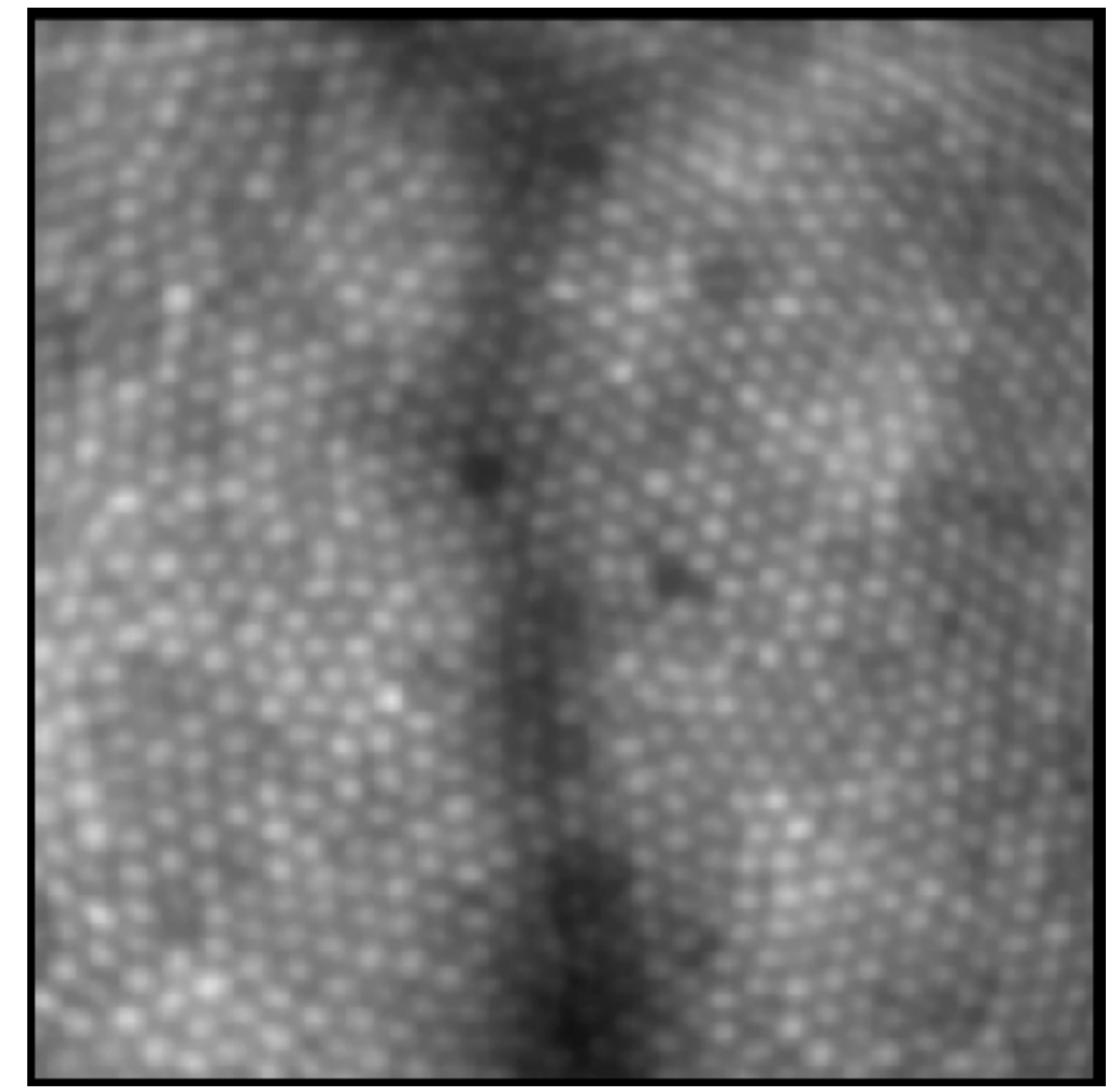
Human Retinal Cone Cell Response Functions (L, M, S Types)

Three types of cone cells: S, M, and L (corresponding to peak response at short, medium, and long wavelengths)



An Aside: Spatial Resolution of Rods and Cones in the Retina



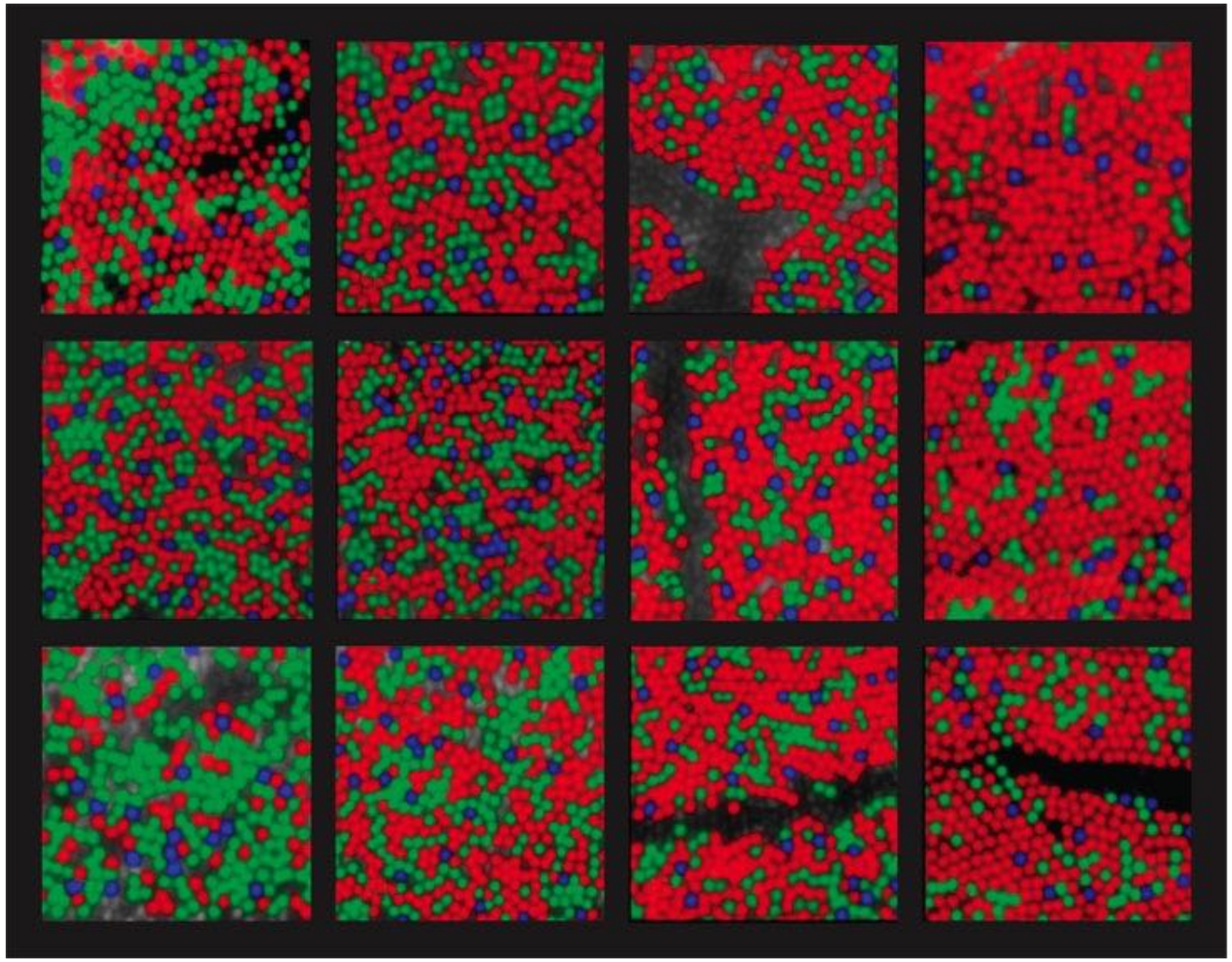


[Roorda 1999]

- Highest density of cones is in fovea (and no rods there)
- "Blind spot" at the optic disc, where optic nerve exits eye

Fraction of Three Cone Cell Types Varies Widely

Hofer, H. et al. J. Neurosci. 2005;25:9669-9679



© 2007 Thomson Higher Education

Distribution of cone cells at edge of fovea in 12 different humans with normal color vision. Note high variability of percentage of different cone cell types. (false color image)

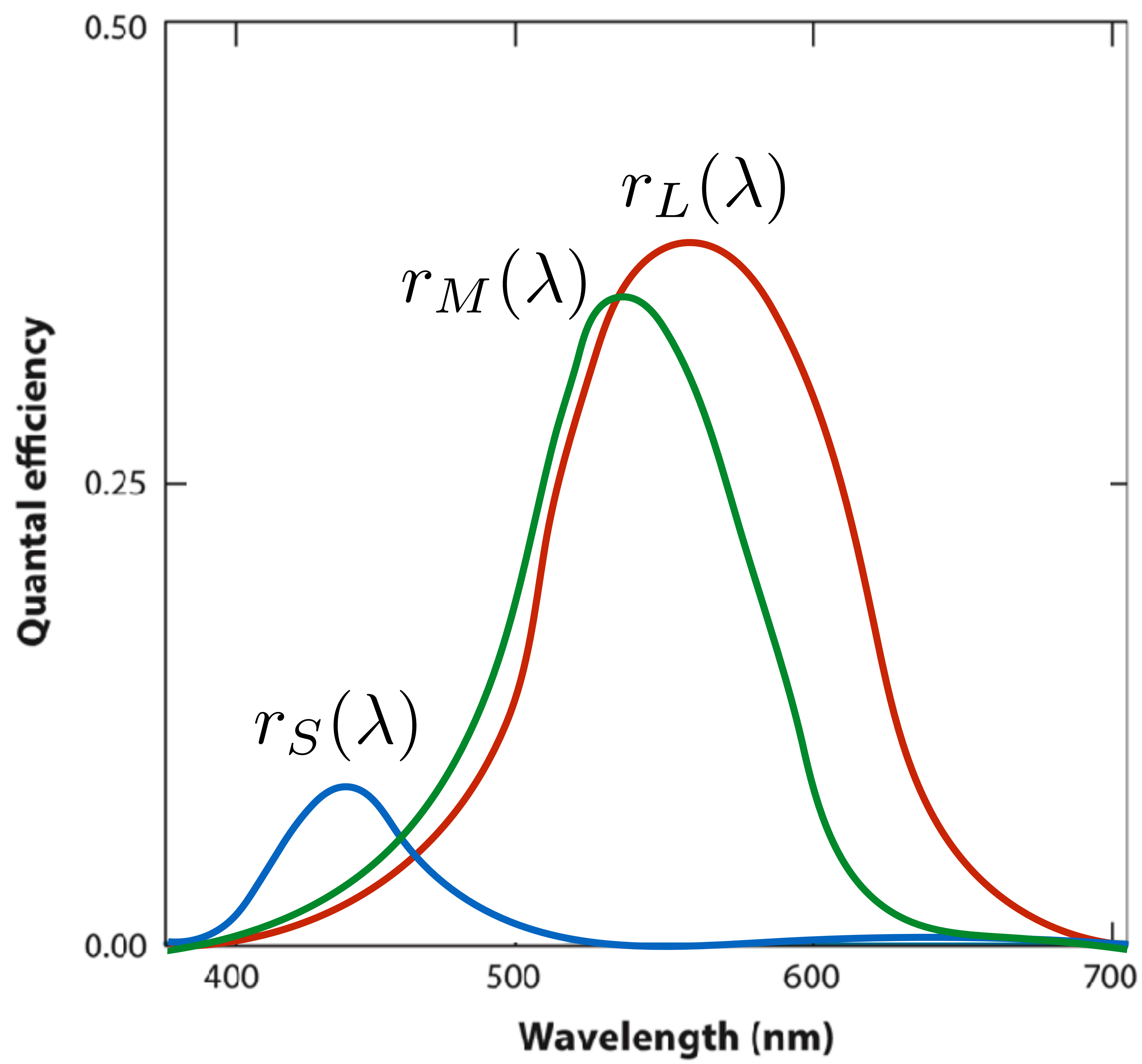
Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

$$S = \int r_S(\lambda)s(\lambda)\,d\lambda$$

$$M = \int r_M(\lambda)s(\lambda)\,d\lambda$$

$$L = \int r_L(\lambda)s(\lambda)\,d\lambda$$



Brainard, Color and the Cone Mosaic, 2015.

Example: Spectral Response of Human Cone Cells

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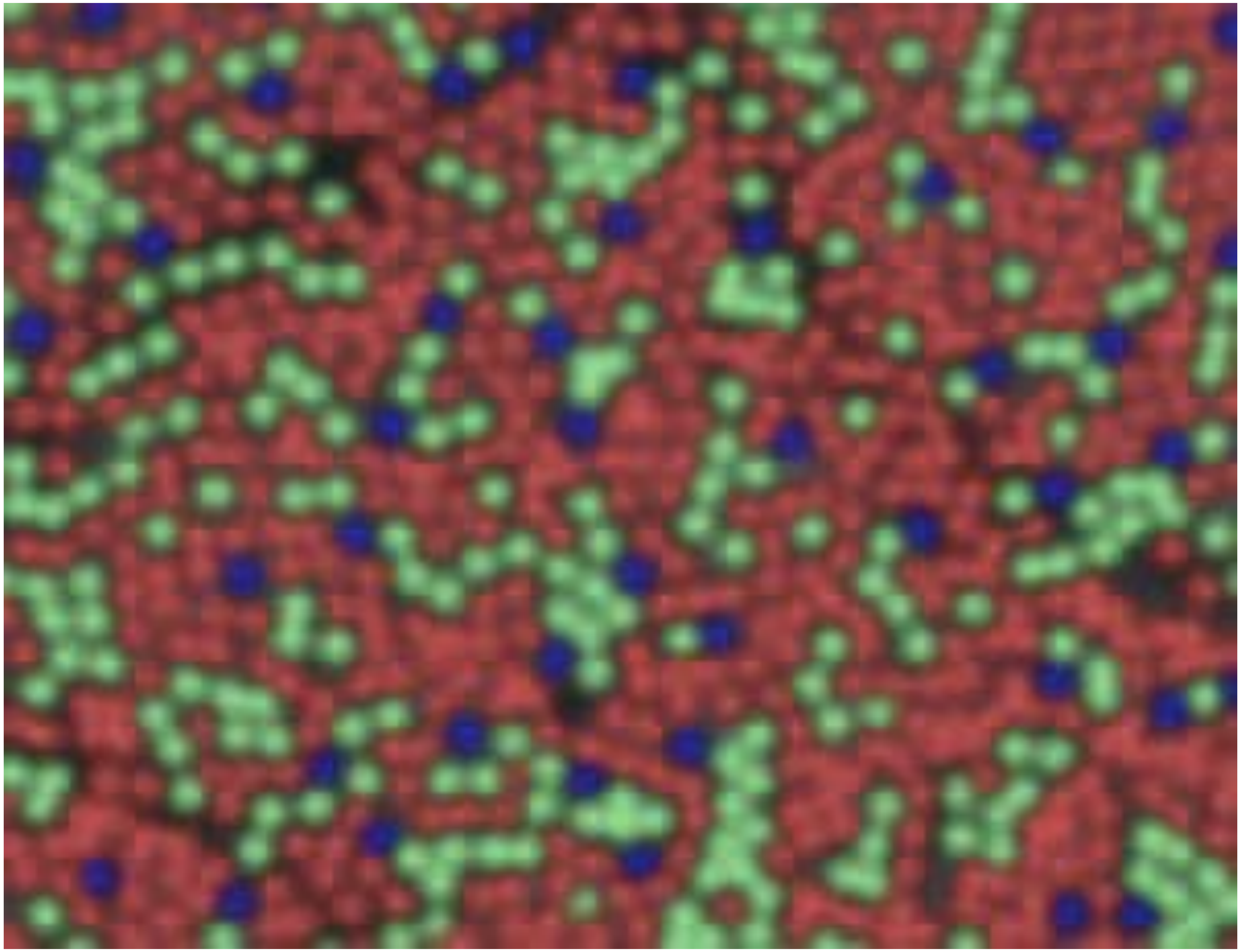
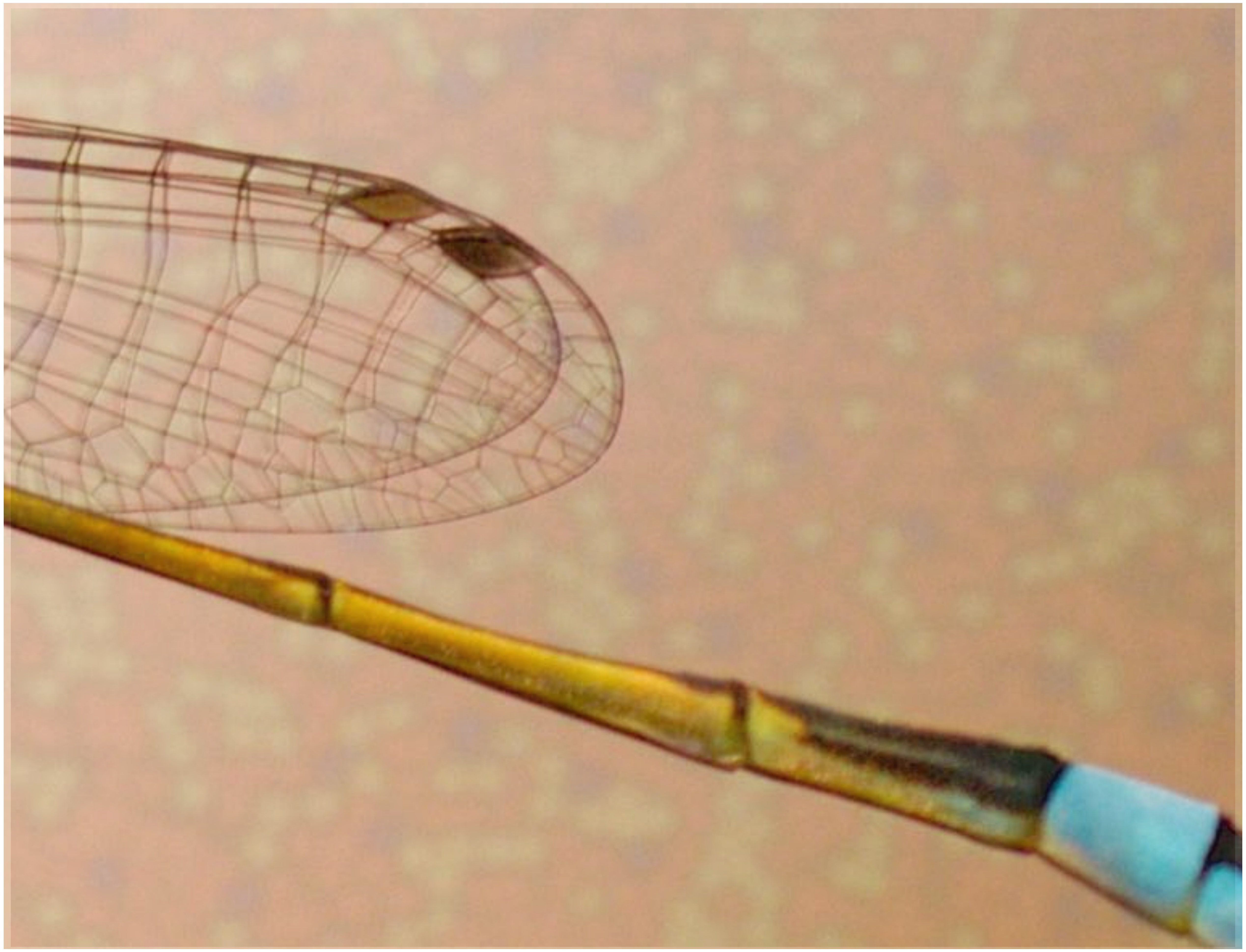


Scene projected onto retina

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: Spectral Response of Human Cone Cells



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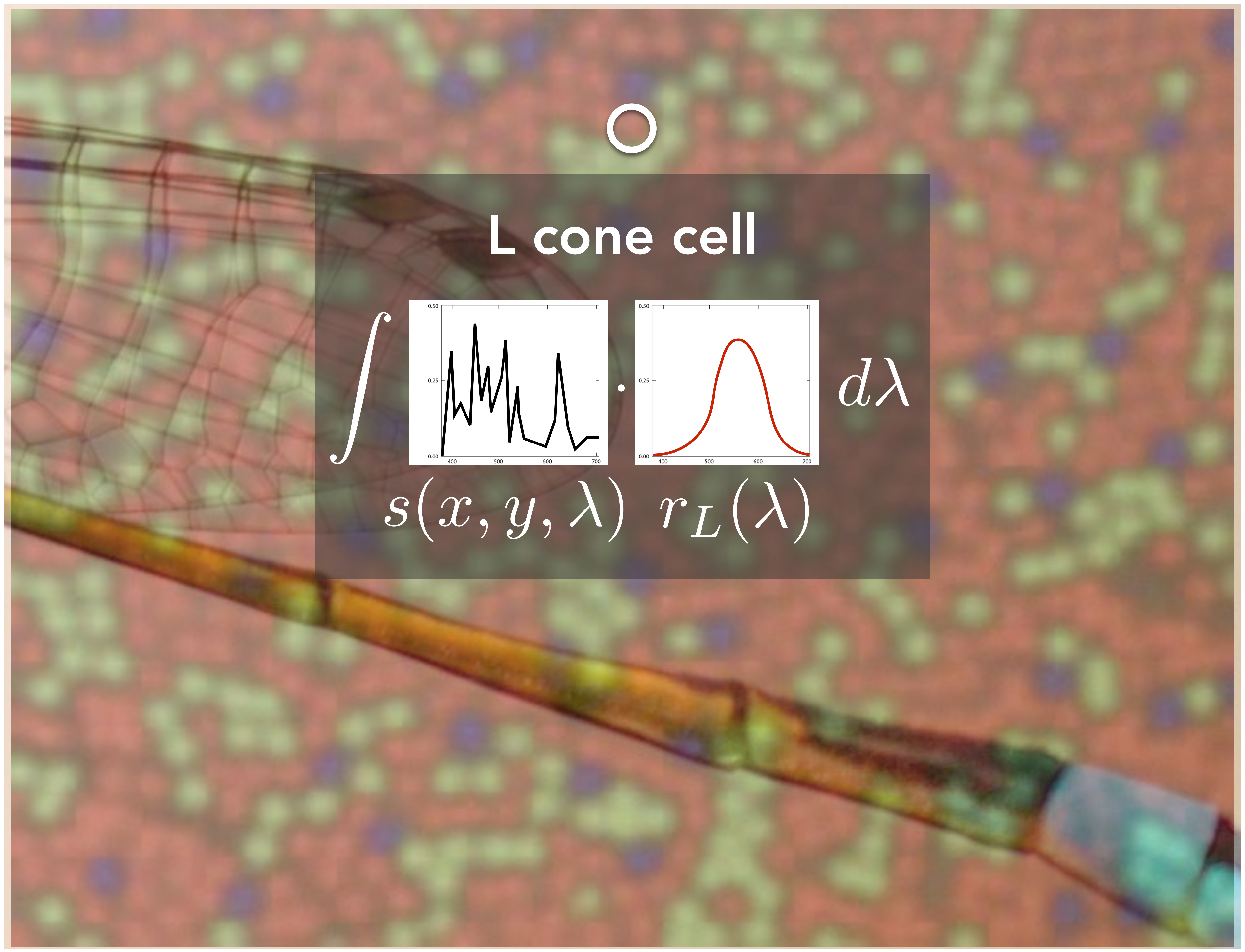

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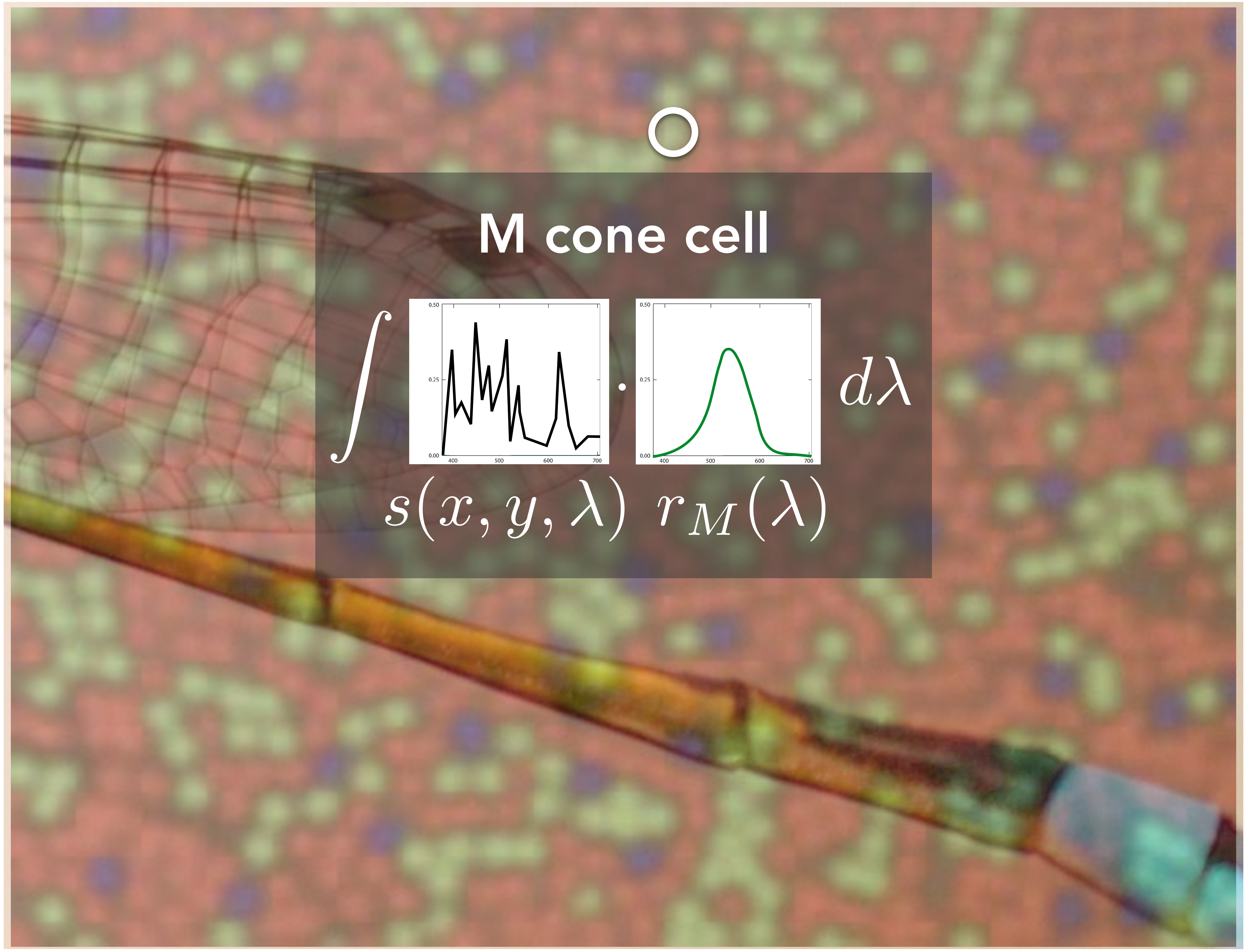
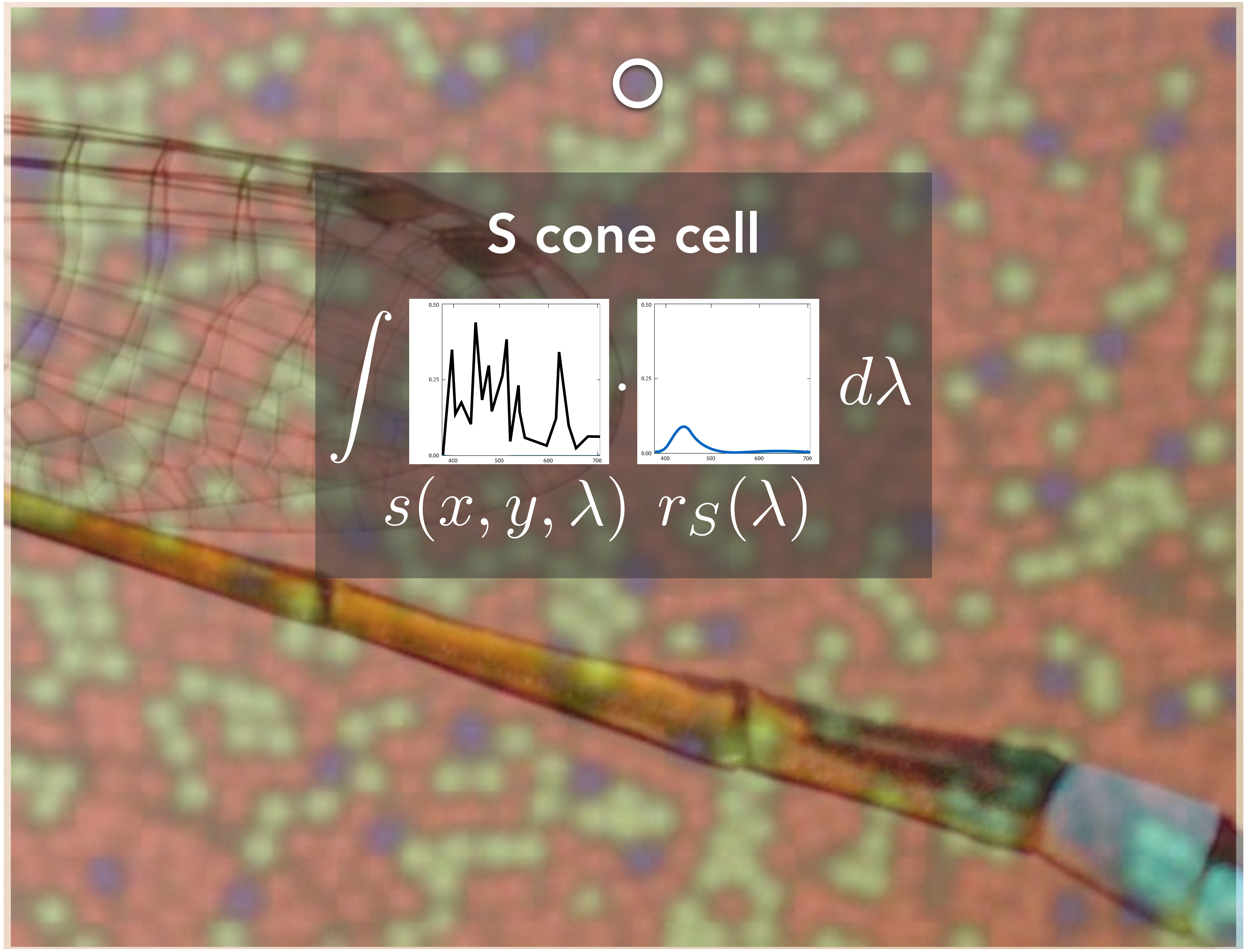


$$\int s(x,y,\lambda) \cdot r_M(\lambda)\, d\lambda$$

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Example: Spectral Response of Human Cone Cells



$$\int s(x,y,\lambda) \cdot r_S(\lambda)\, d\lambda$$

Credit: Sabesan, http://depts.washington.edu/sabaolab/

Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

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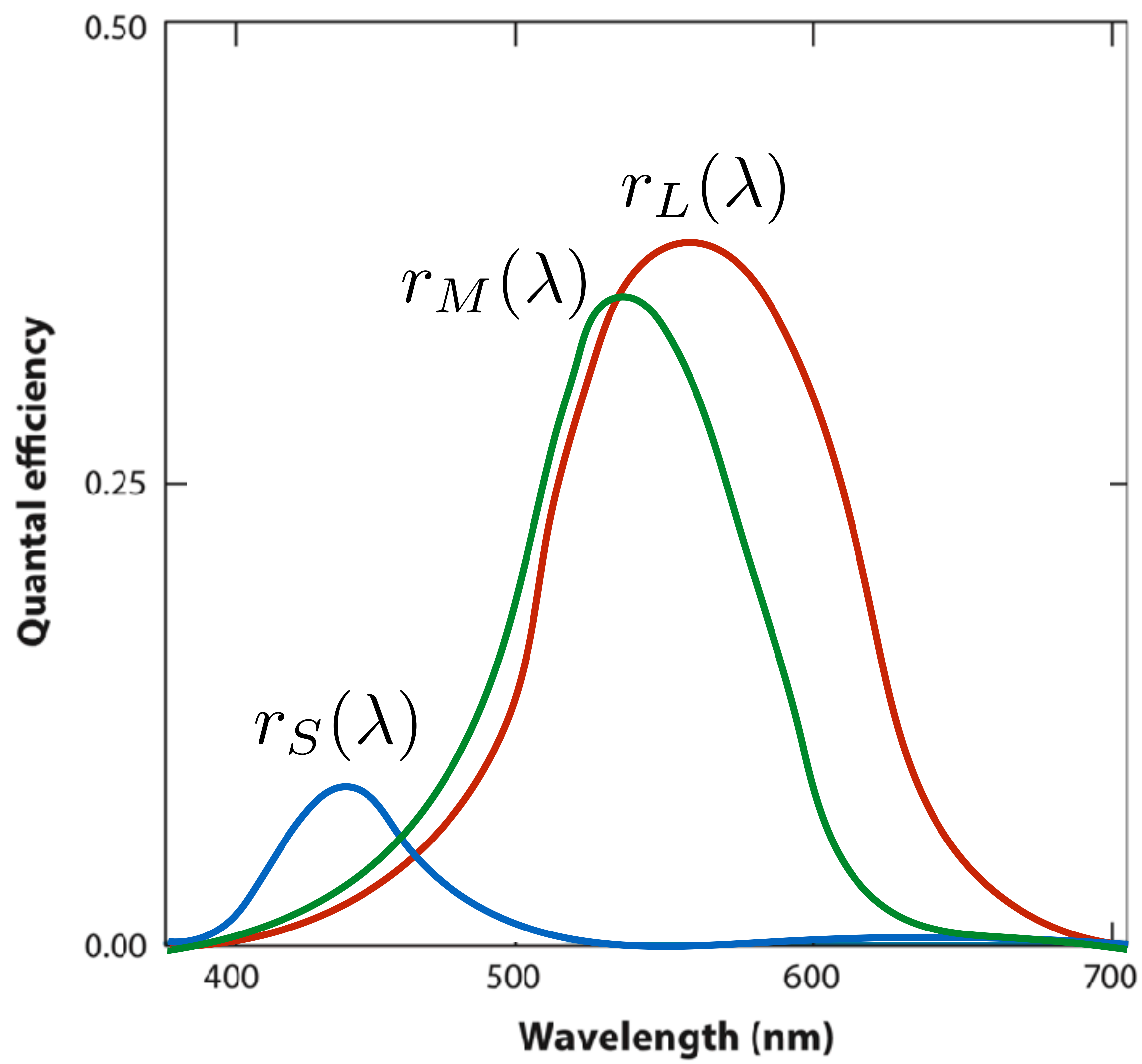


Brainard, Color and the Cone Mosaic, 2015.

Spectral Response of Human Cone Cells

Instead of one detector as before, now we have three detectors (S, M, L cone cells), each with a different spectral response curve

Written as vector dot products:

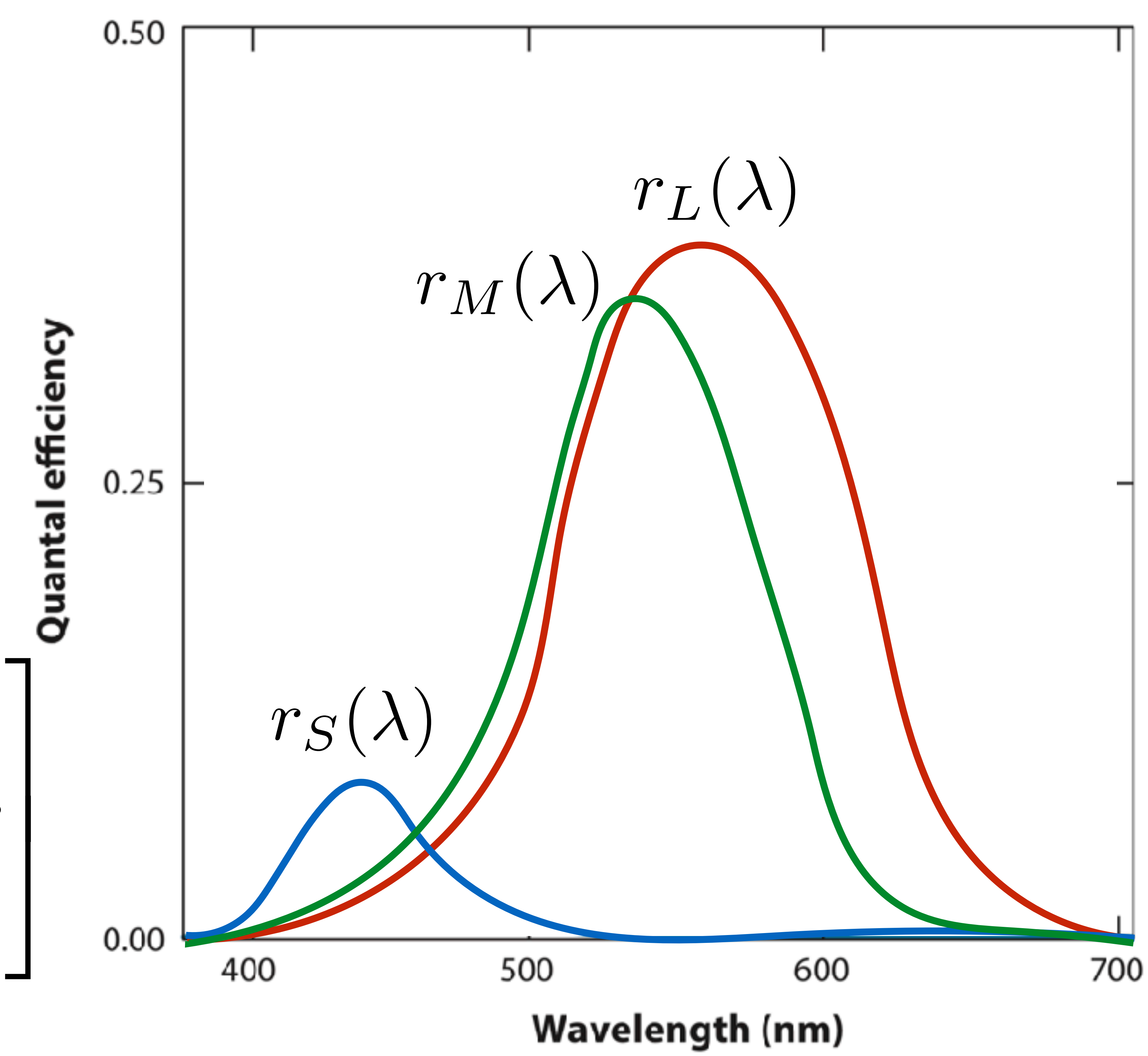
$$S = r_S \cdot s$$

$$M = r_M \cdot s$$

$$L = r_L \cdot s$$

Matrix formulation:

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$



Brainard, Color and the Cone Mosaic, 2015.

Dimensionality Reduction From ∞ to 3

At each position on the human retina:

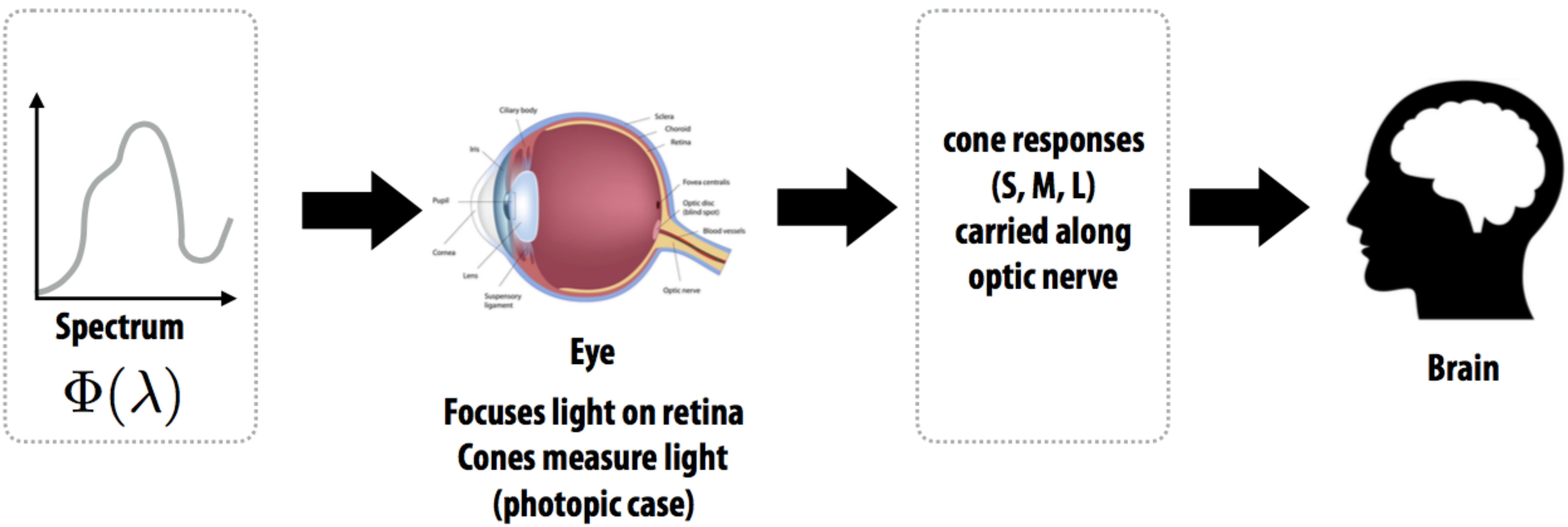
- SPD is a function of wavelength (∞ - dimensional signal)
- 3 types of cones near that position produce three scalar values (3 - dimensional signal)

What about 2D images?

- The dimensionality reduction described above is happening at every 2D position in our visual field

The Human Visual System

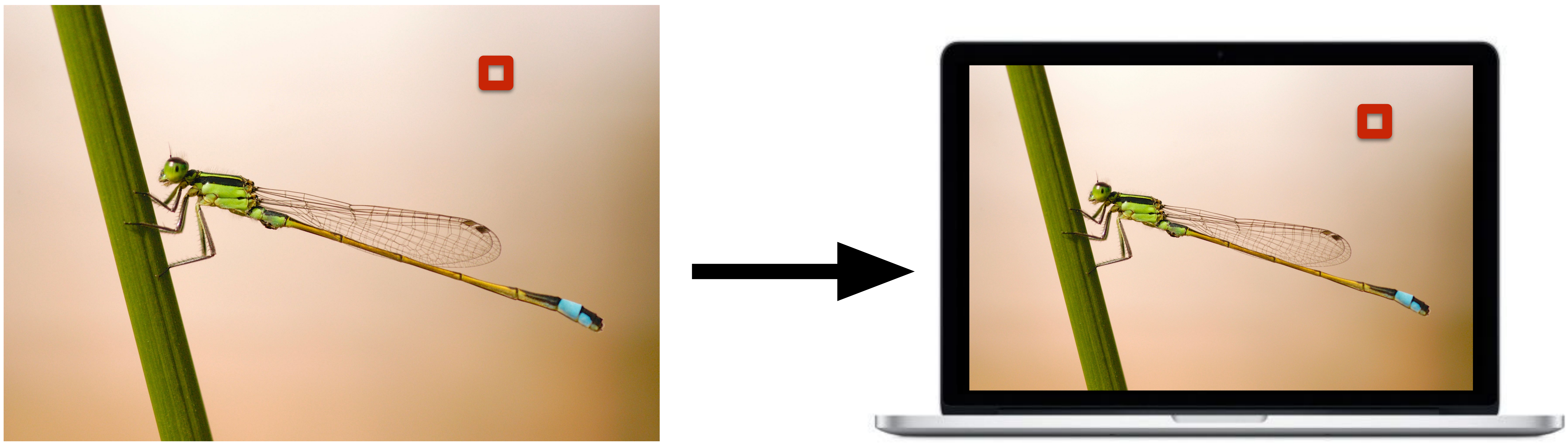
- Human eye does not measure and brain does not receive information about each wavelength of light
- Rather, the eye measures three response values only (S, M, L) at each position in visual field, and this is only spectral info available to brain
  - This is the result of integrating the incoming spectrum against response functions of S, M, L cones



Kayvon Fatahalian

Color Reproduction Problem (Simplified)

Color Reproduction Problem



Target real spectrum $s(\lambda)$

Display outputs spectrum
$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

Metamerism

Metamers

Metameters are two different spectra (∞-dim) that project to the same (S,M,L) (3-dim) response.

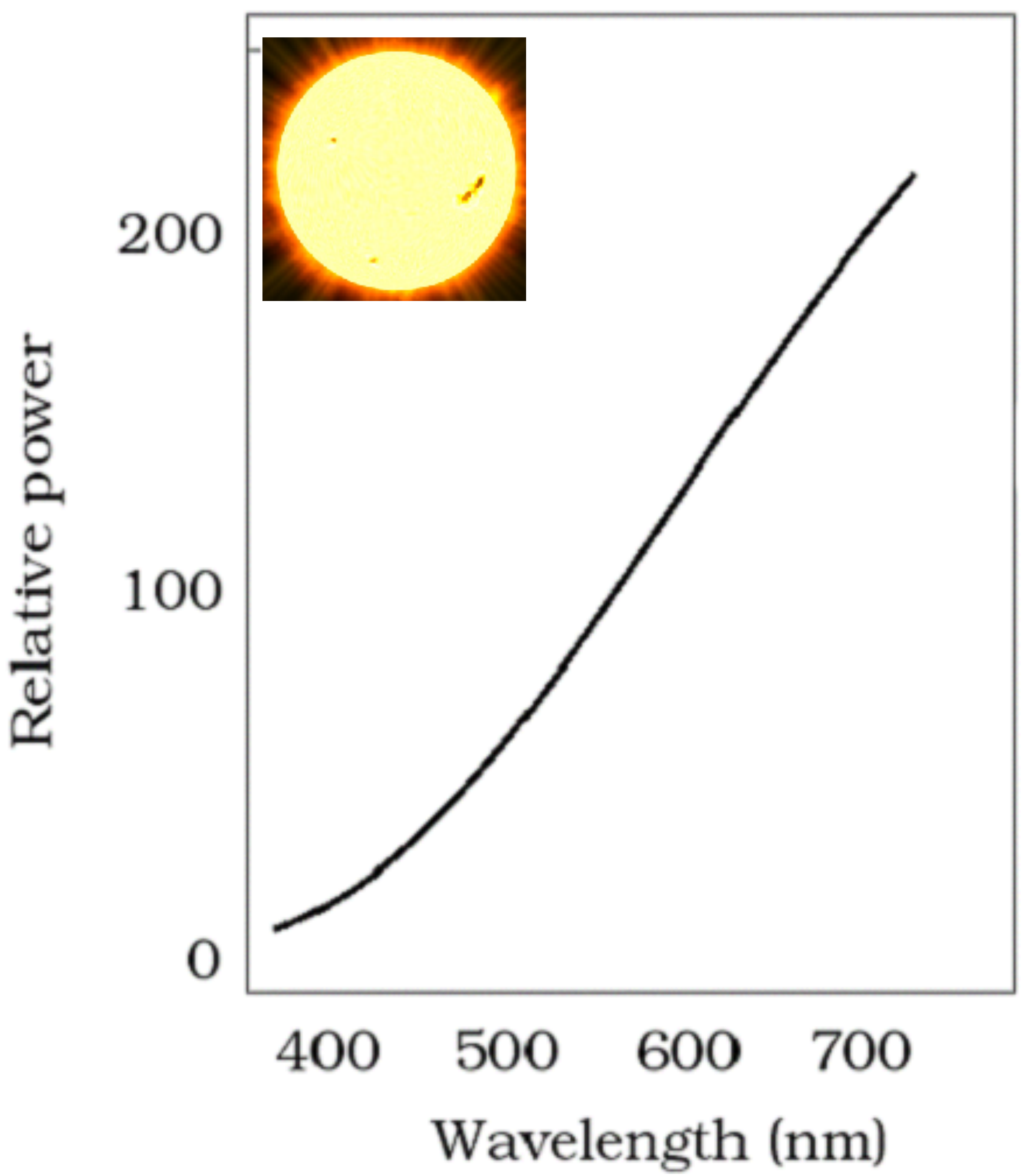
- These will appear to have the same color to a human

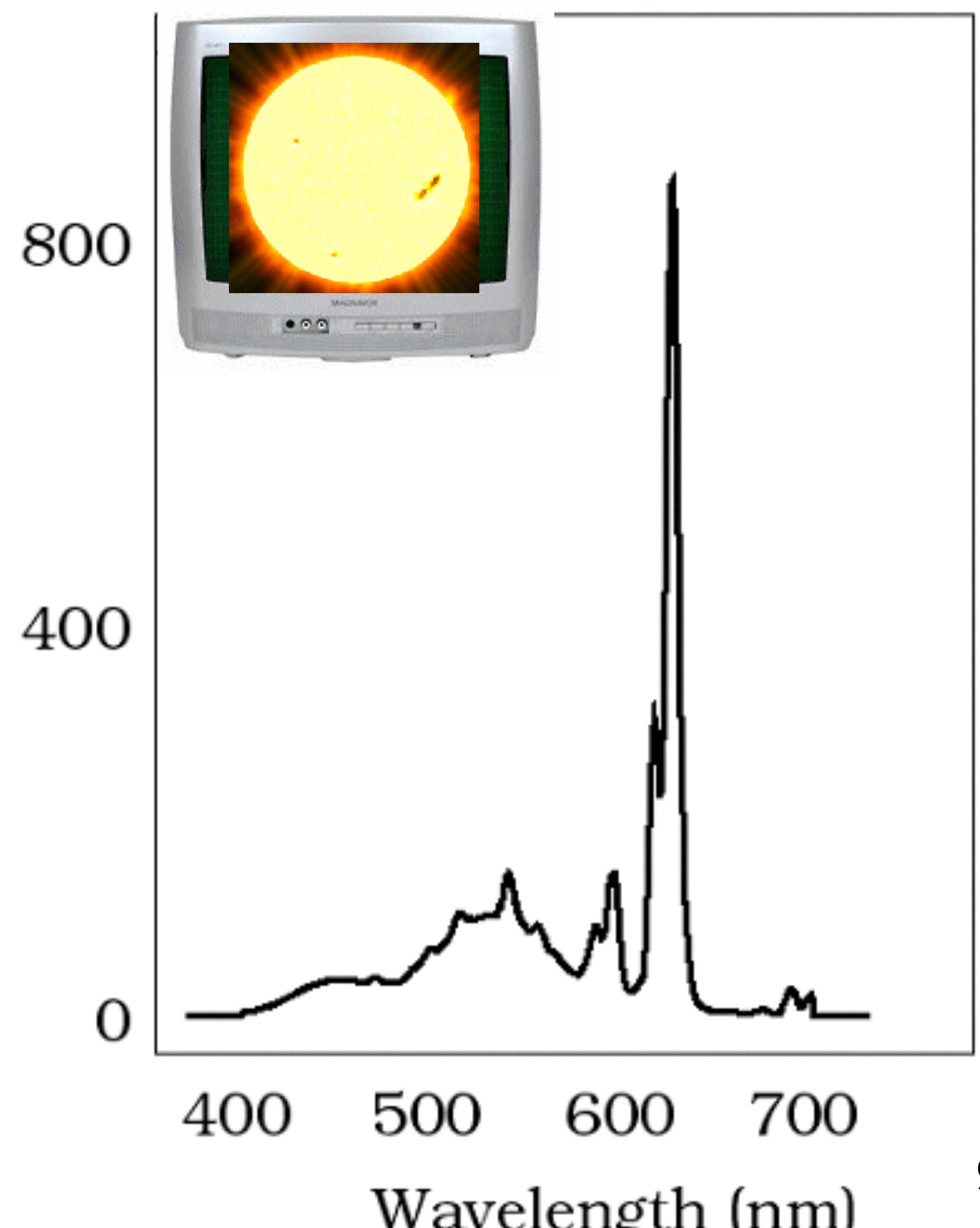
The existence of metamers is critical to color reproduction

- Don’t have to reproduce the full spectrum of a real world scene
- Example: A metamer can reproduce the perceived color of a real-world scene on a display with pixels of only three colors

Metamerism

Color matching is an important illusion that is understood quantitatively





Brian Wandell

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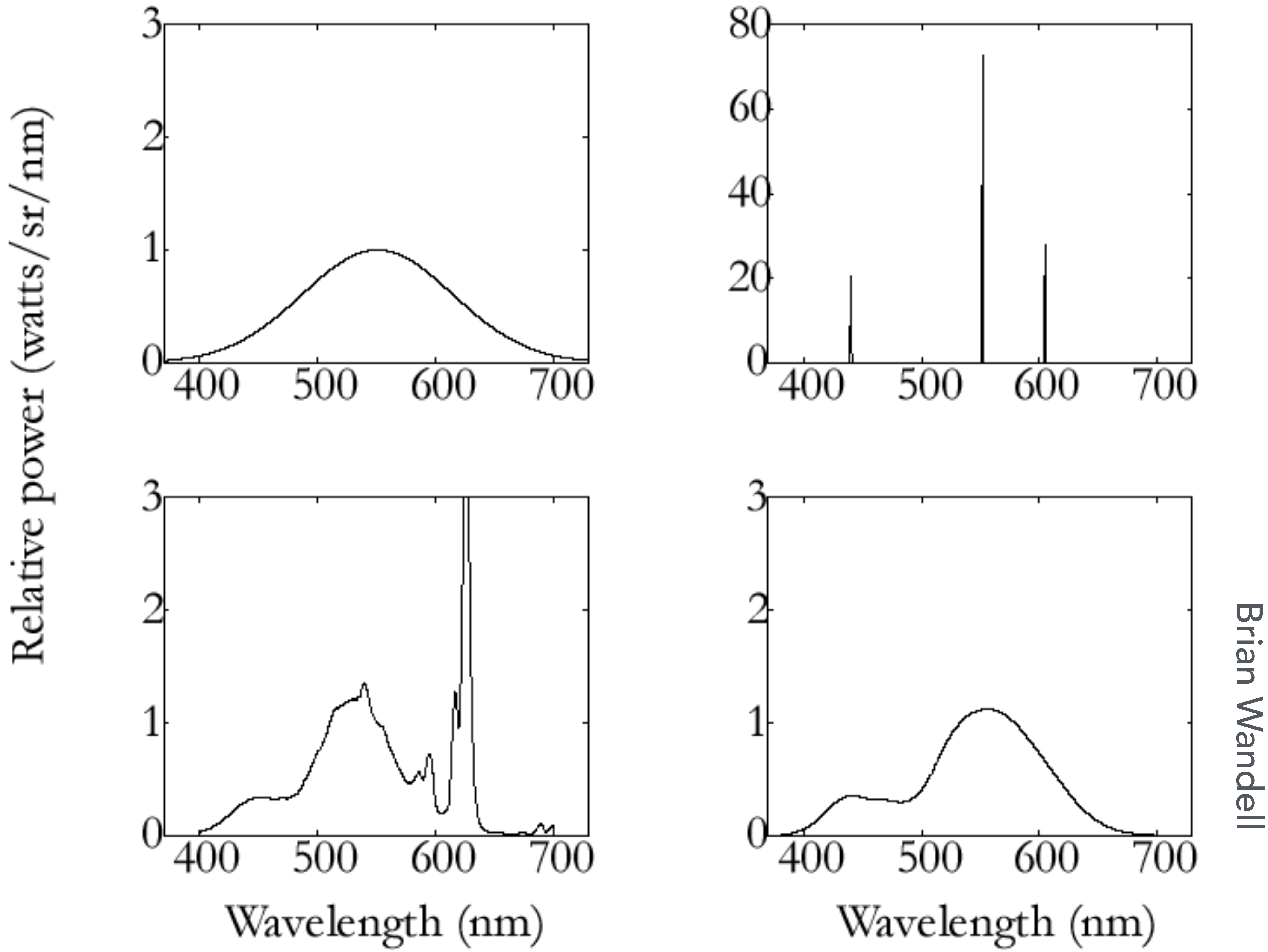
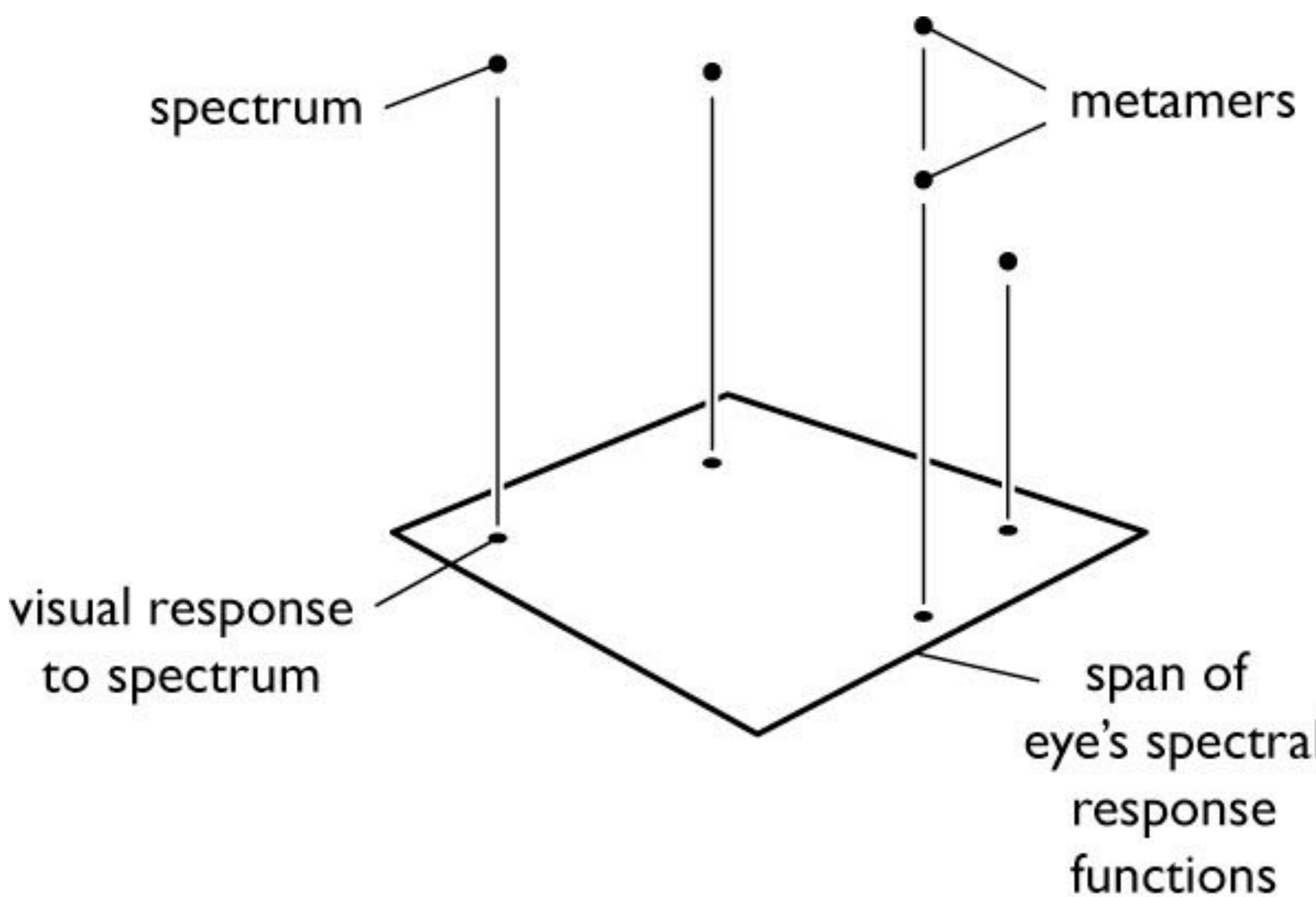
Metamerism is a Big Effect



Pseudo-Geometric Interpretation

We are projecting a high dimensional vector (wavelength spectrum function) onto a low-dimensional subspace (SML visual response)

- Differences that are perpendicular to the basis vectors of the low-dimensional space are not detectable



Slide credit: Steve Marschner

Color Reproduction

Additive Color

- Given a set of primary lights, each with its own spectral distribution (e.g. R,G,B display pixels):

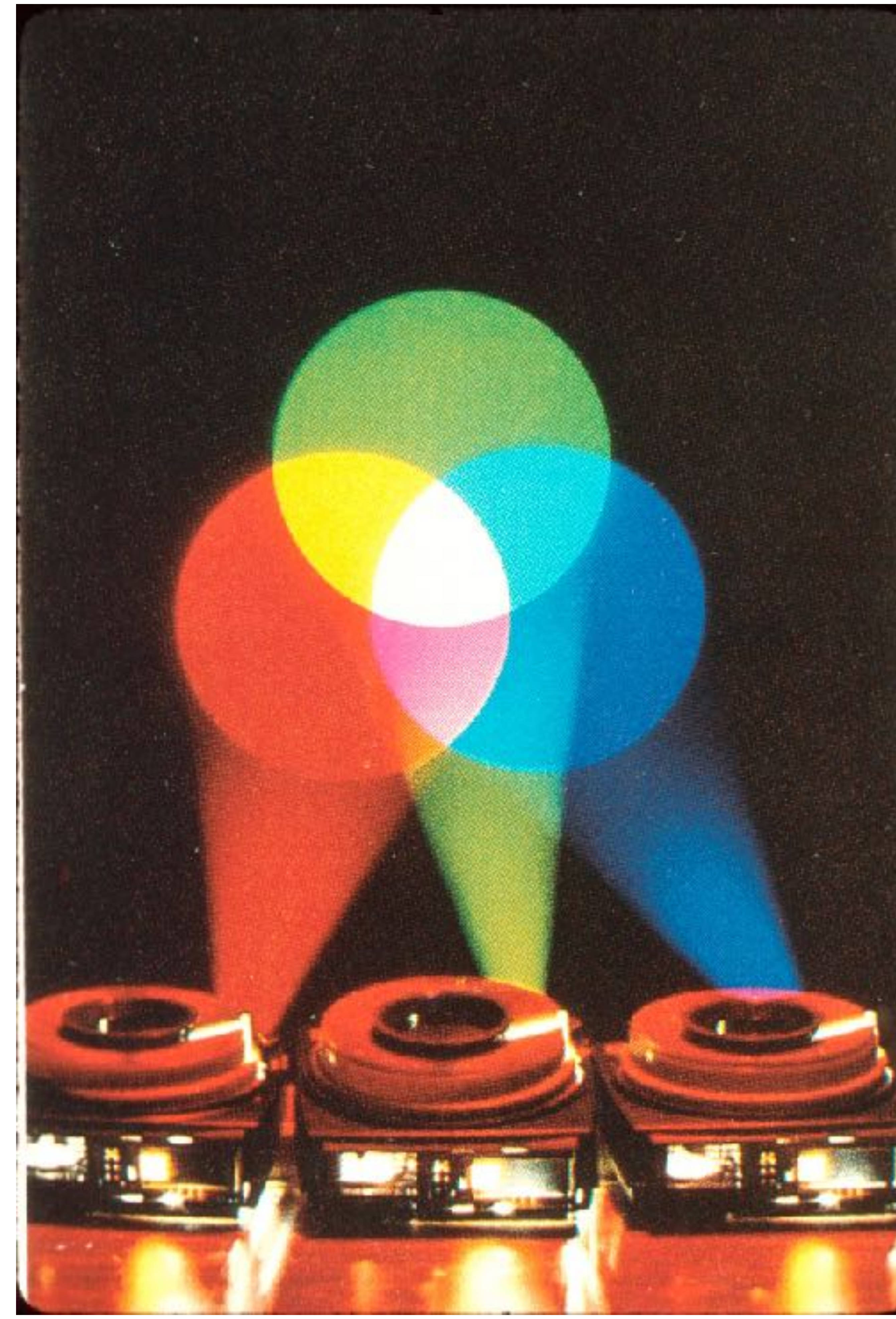
$$s_R(\lambda),\ s_G(\lambda),\ s_B(\lambda)$$

- We can adjust the brightness of these lights and add them together to produce a linear subspace of spectral distribution:

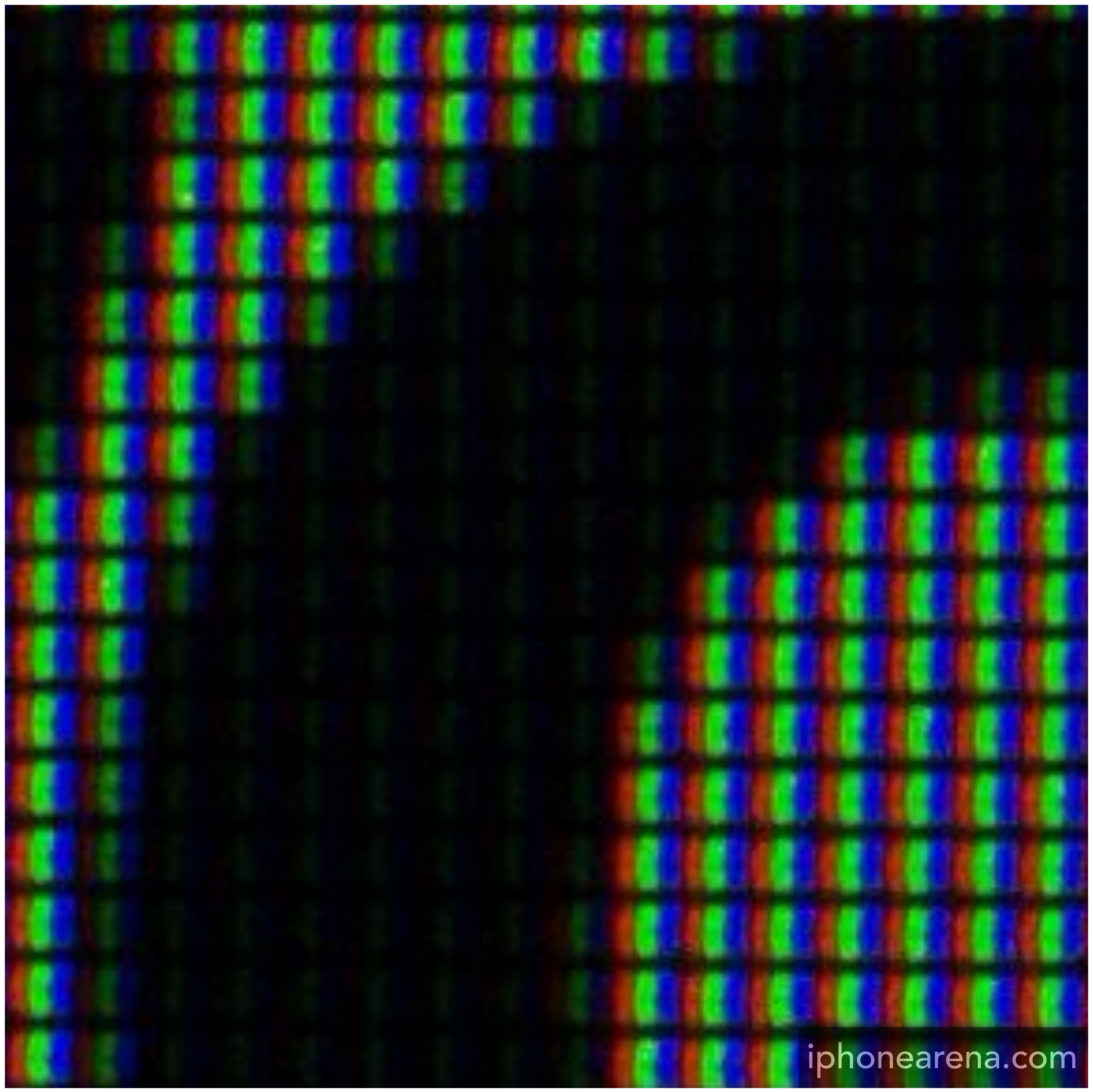
$$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

- The color is now described by the scalar values:

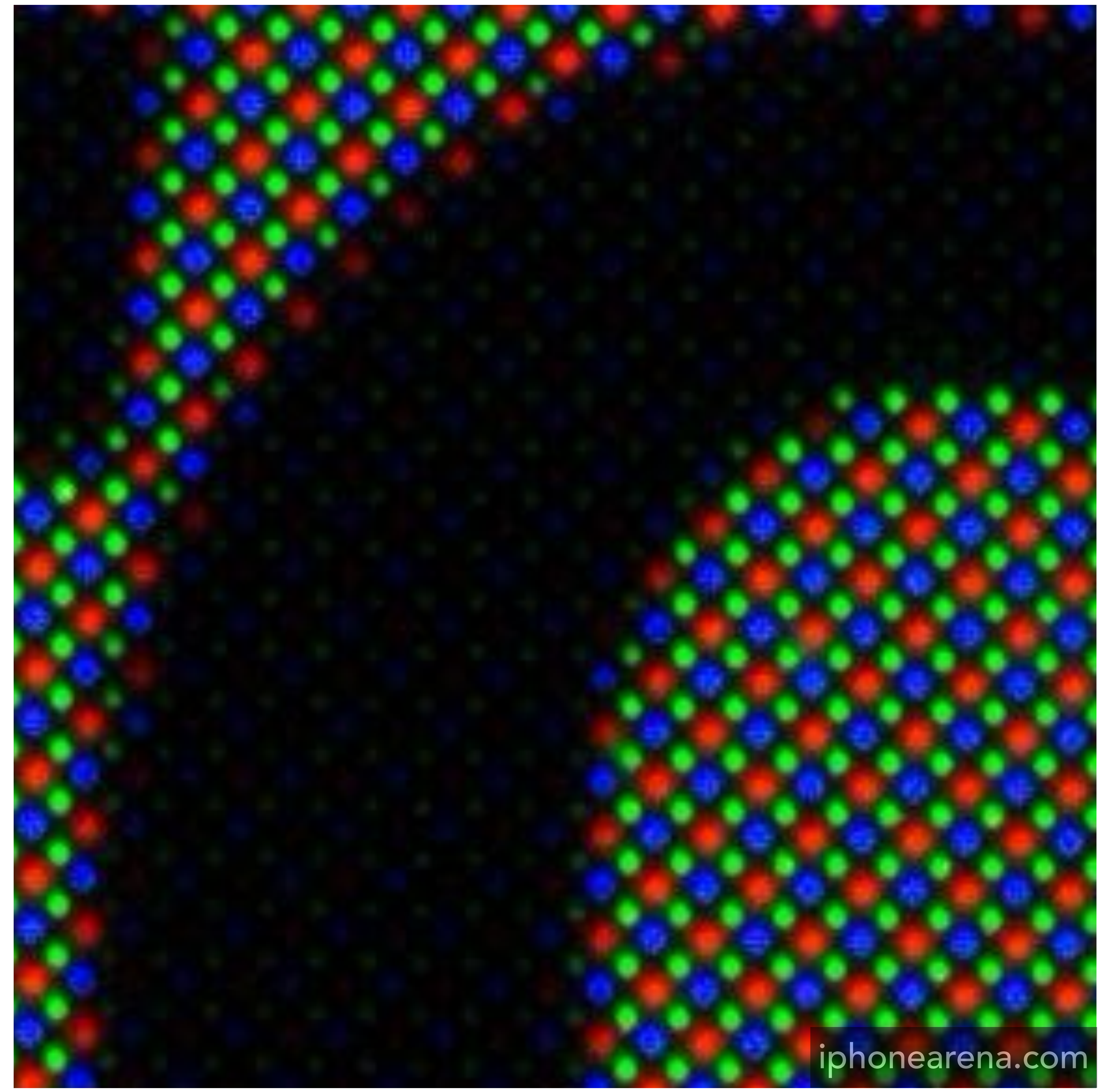
$$R,\ G,\ B$$



Real LCD Screen Pixels (Closeup)



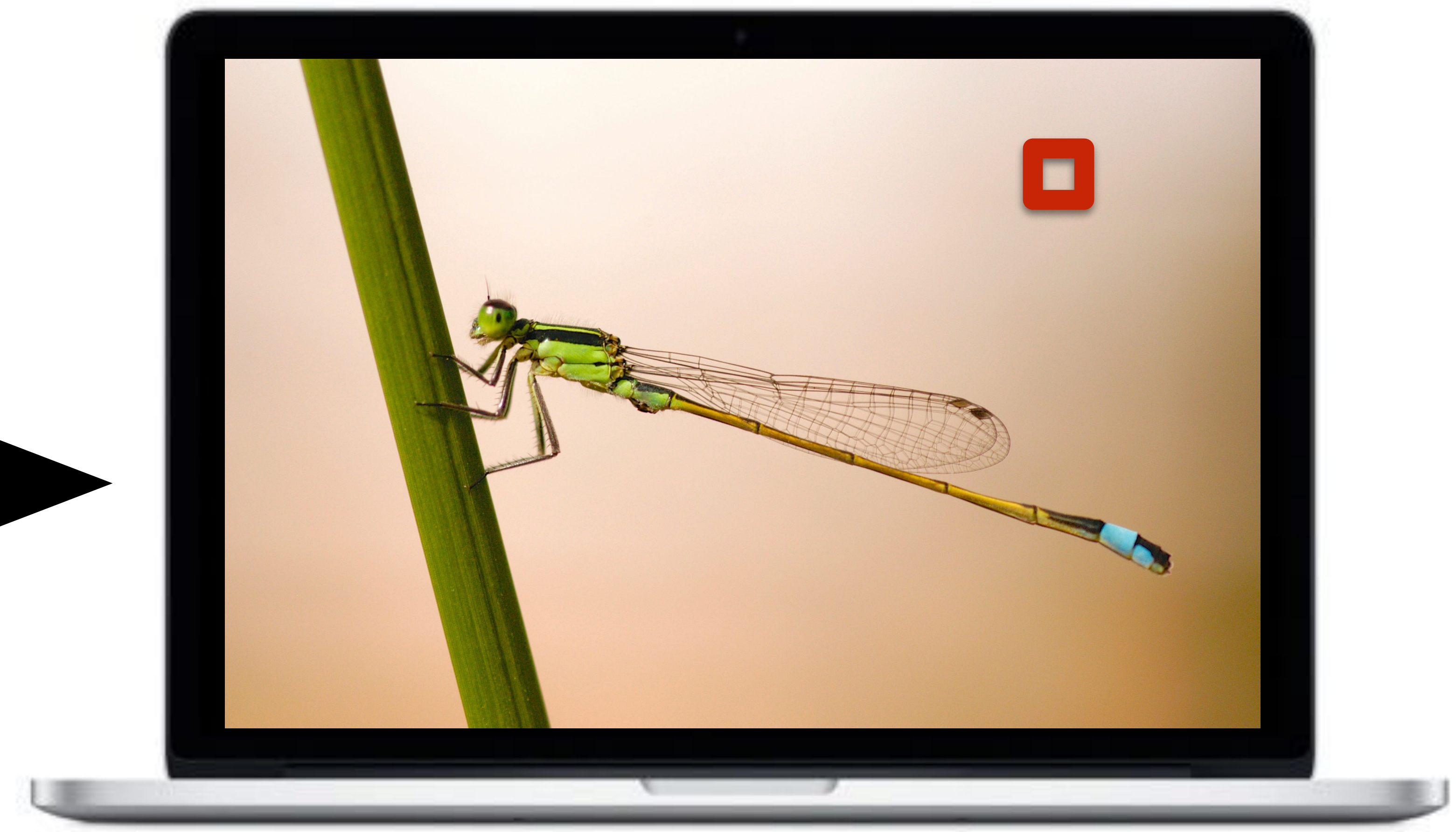
iPhone 6S



Galaxy S5

Notice R, G, B sub-pixel geometry.
Effectively three lights at each (x,y) location.

Color Reproduction Problem



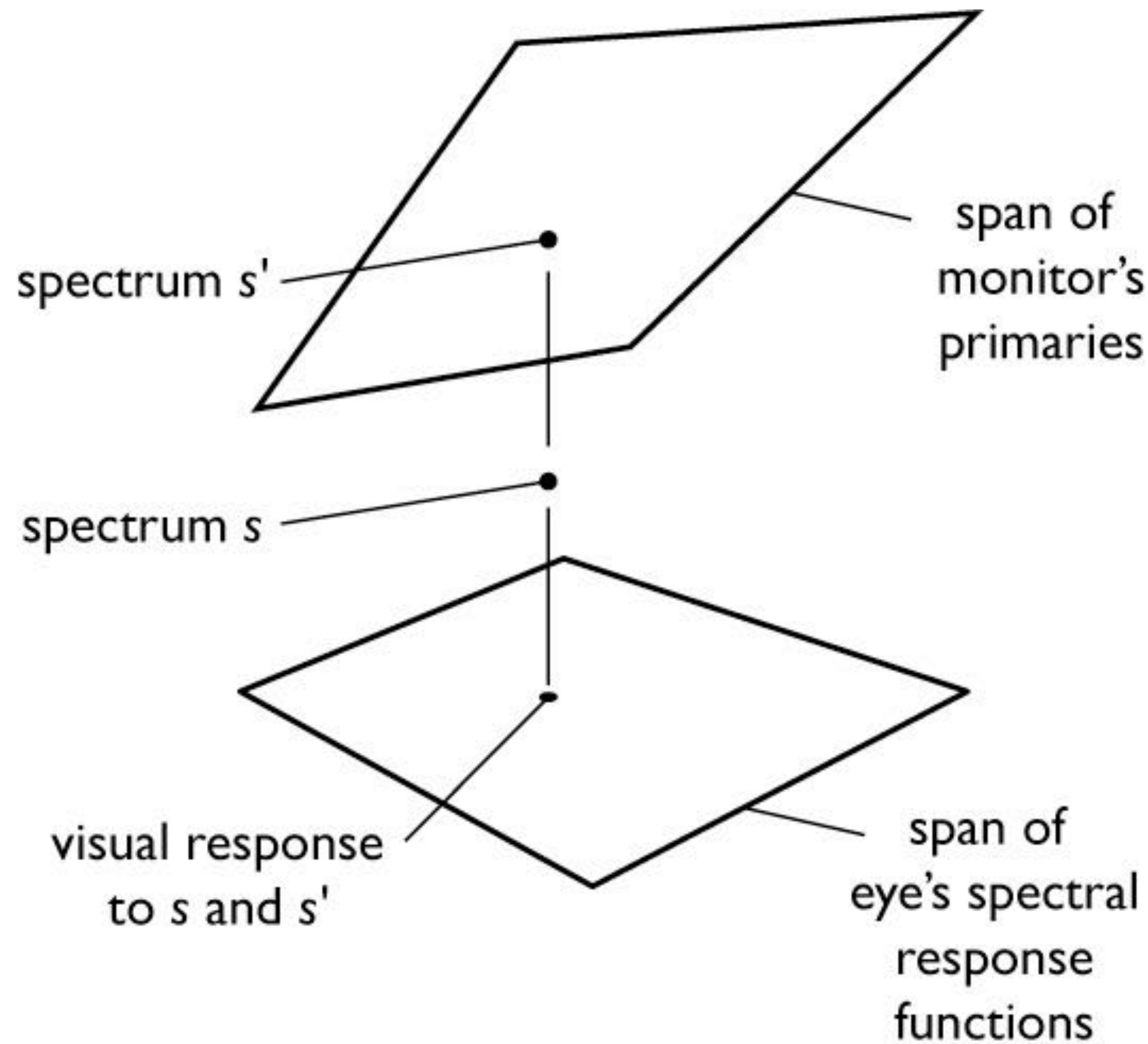
Target real spectrum $s(\lambda)$

Display outputs spectrum
$R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$

Goal: at each pixel, choose R, G, B values for display so that the output color matches the appearance of the target color in the real world.

Pseudo-Geometric Interpretation of Color Reproduction

- The display can only produce a low-dimensional subspace of all possible (linear combinations of display primaries)
- In color reproduction, for a given spectrum s (high dimensional), we want to choose a spectrum s' in the display's low-dimensional subspace, such that s' and s project to the same response in the low-dimensional subspace of the eye's SML response



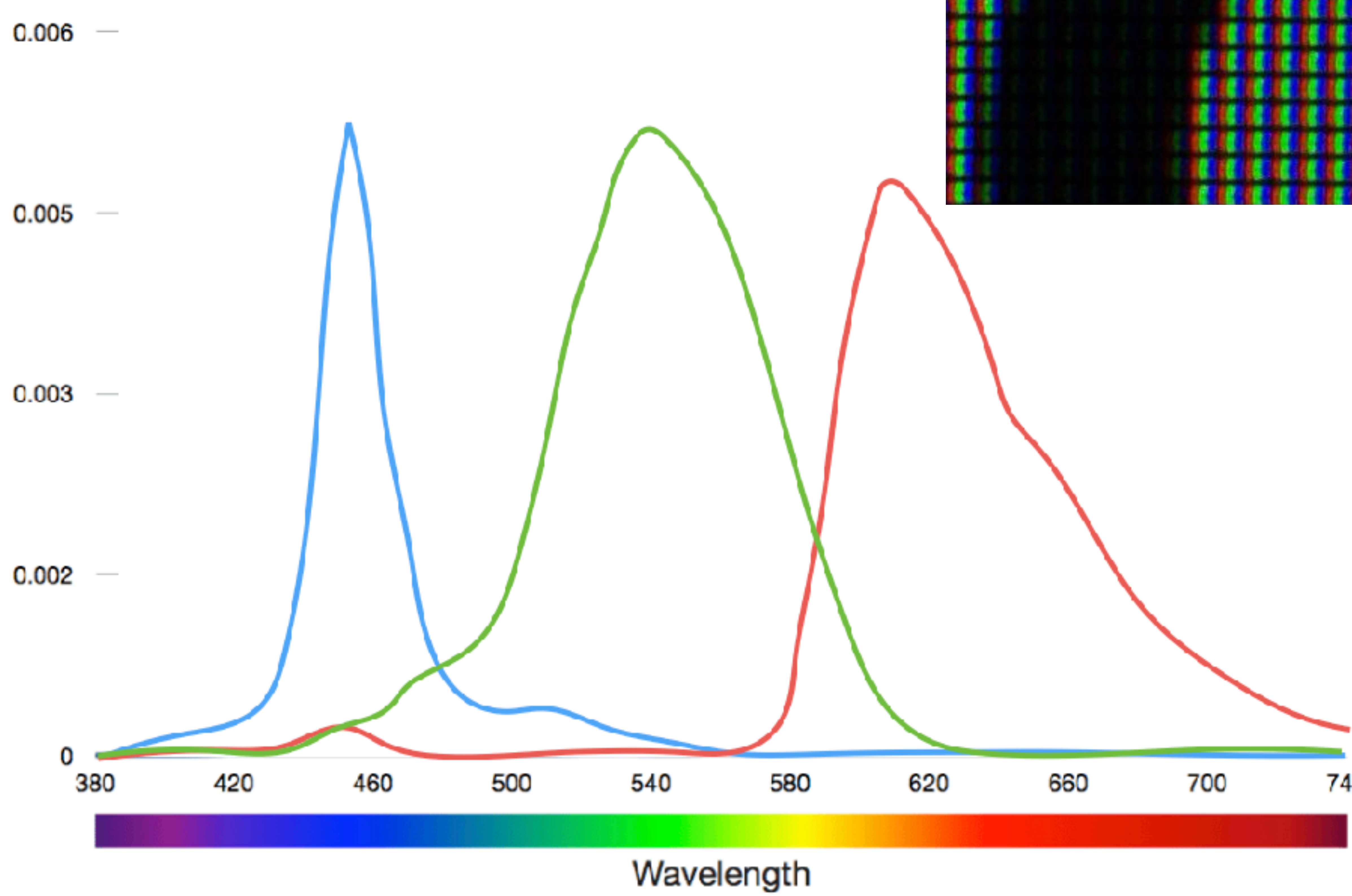
Slide credit: Steve Marschner

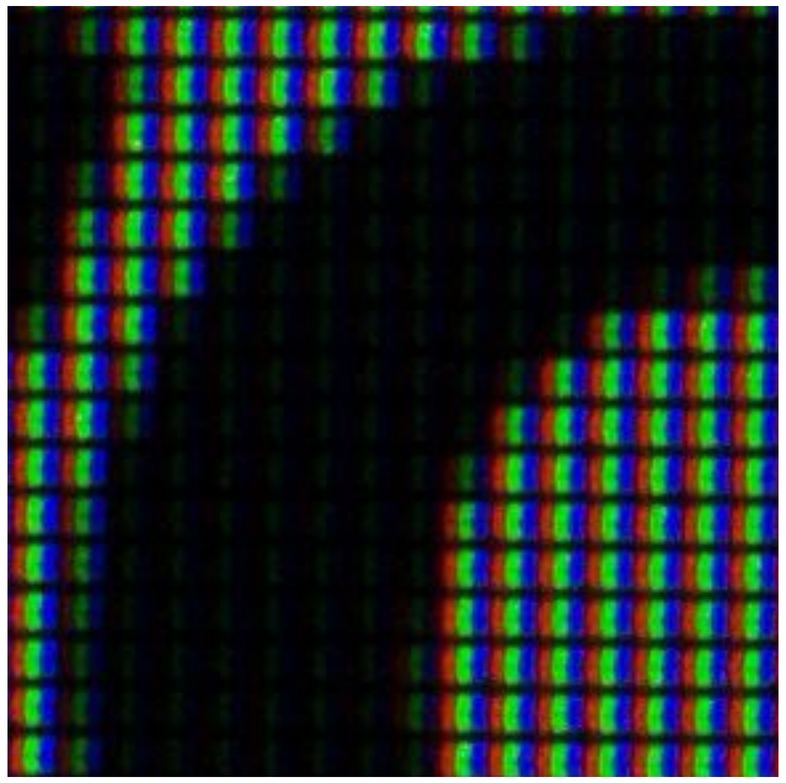
Example RGB Emission Spectra ("Color Primaries") for Phone Display

https://www.macrumors.com/roundup/iphone-5s/

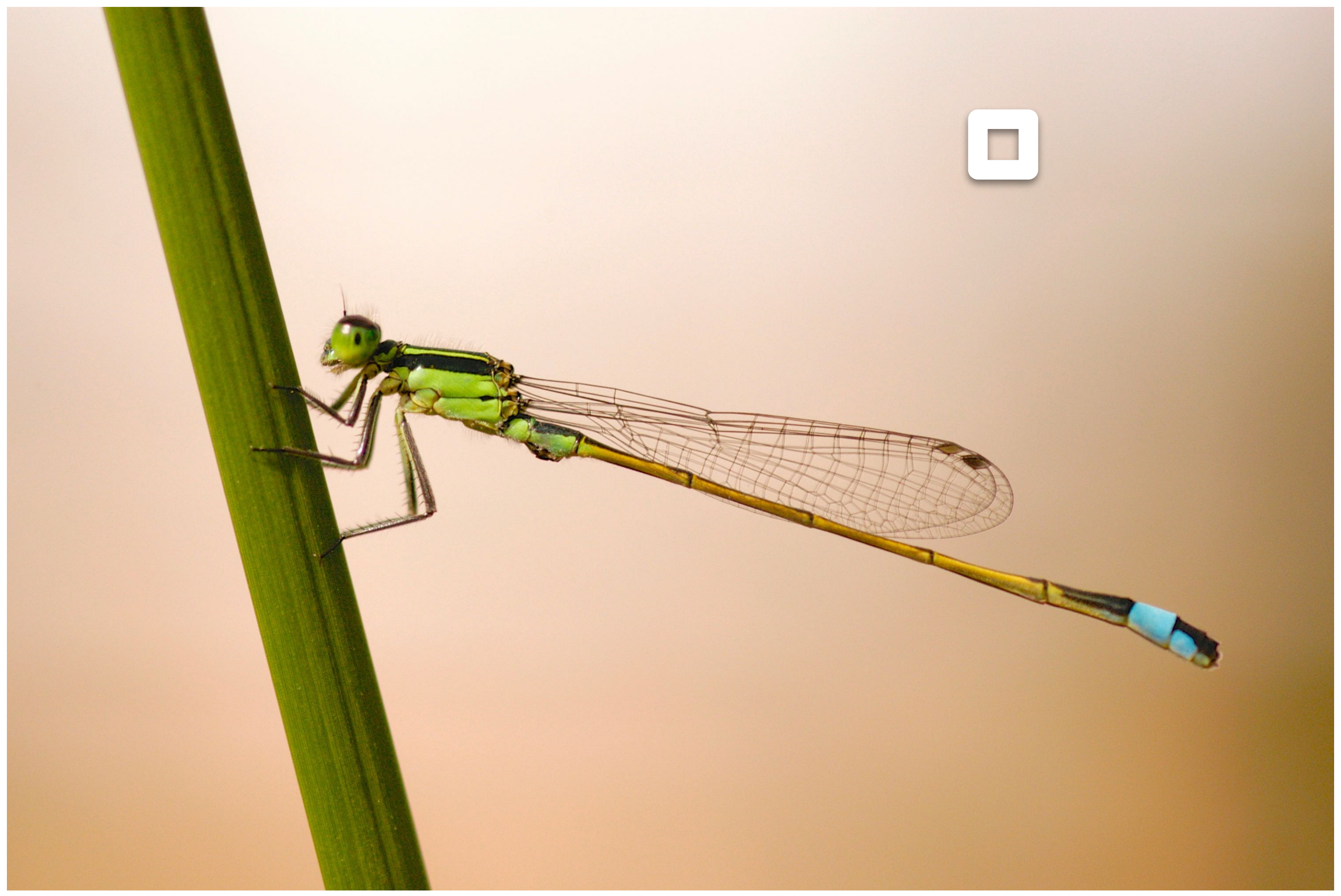
RGB pixel spectra (iPhone 5)

Credit: Yurek, https://dot-color.com/tag/color-2/page/2/

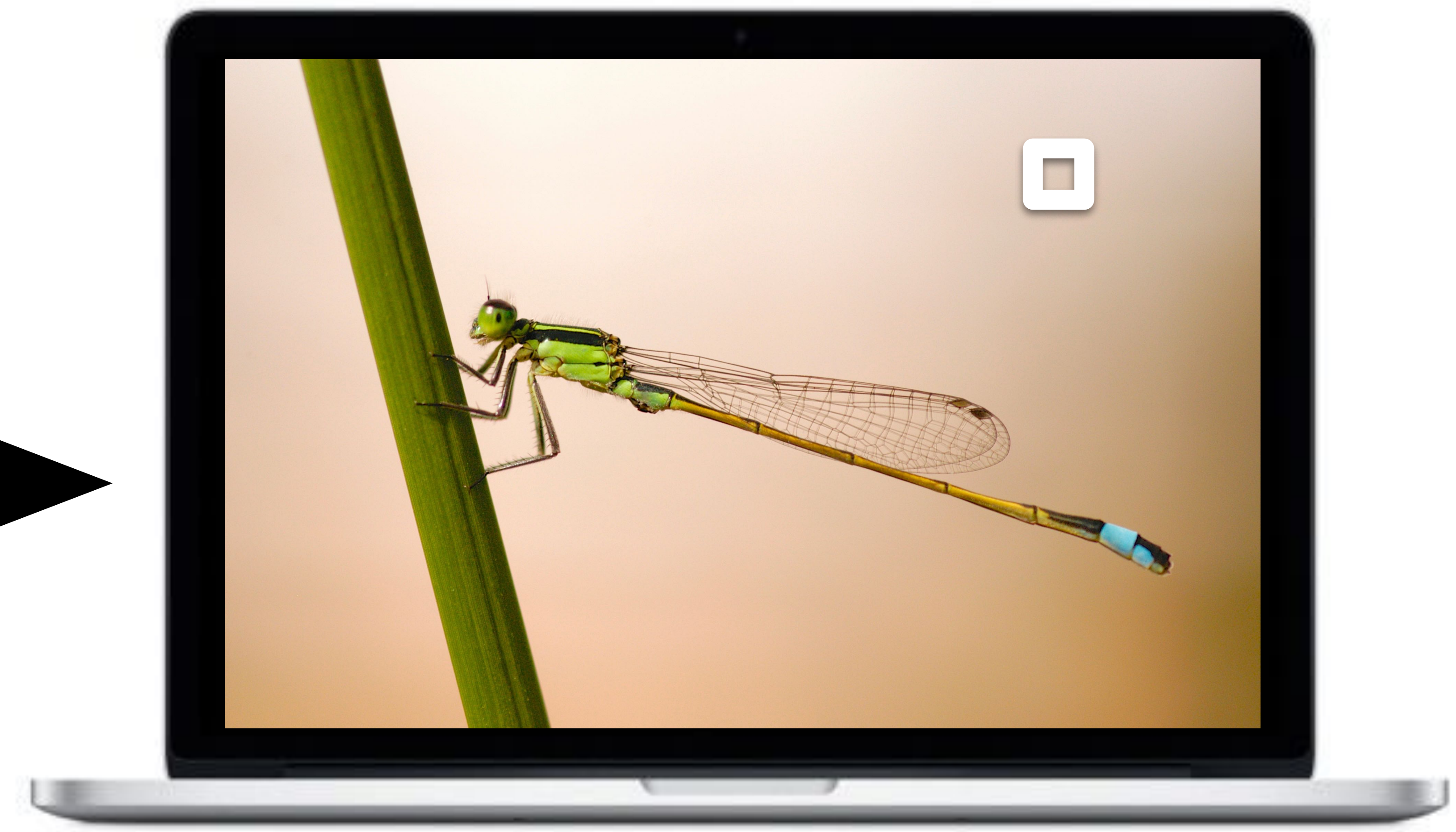




Color Reproduction as Linear Algebra



Input spectrum *s*



What R, G, B values?

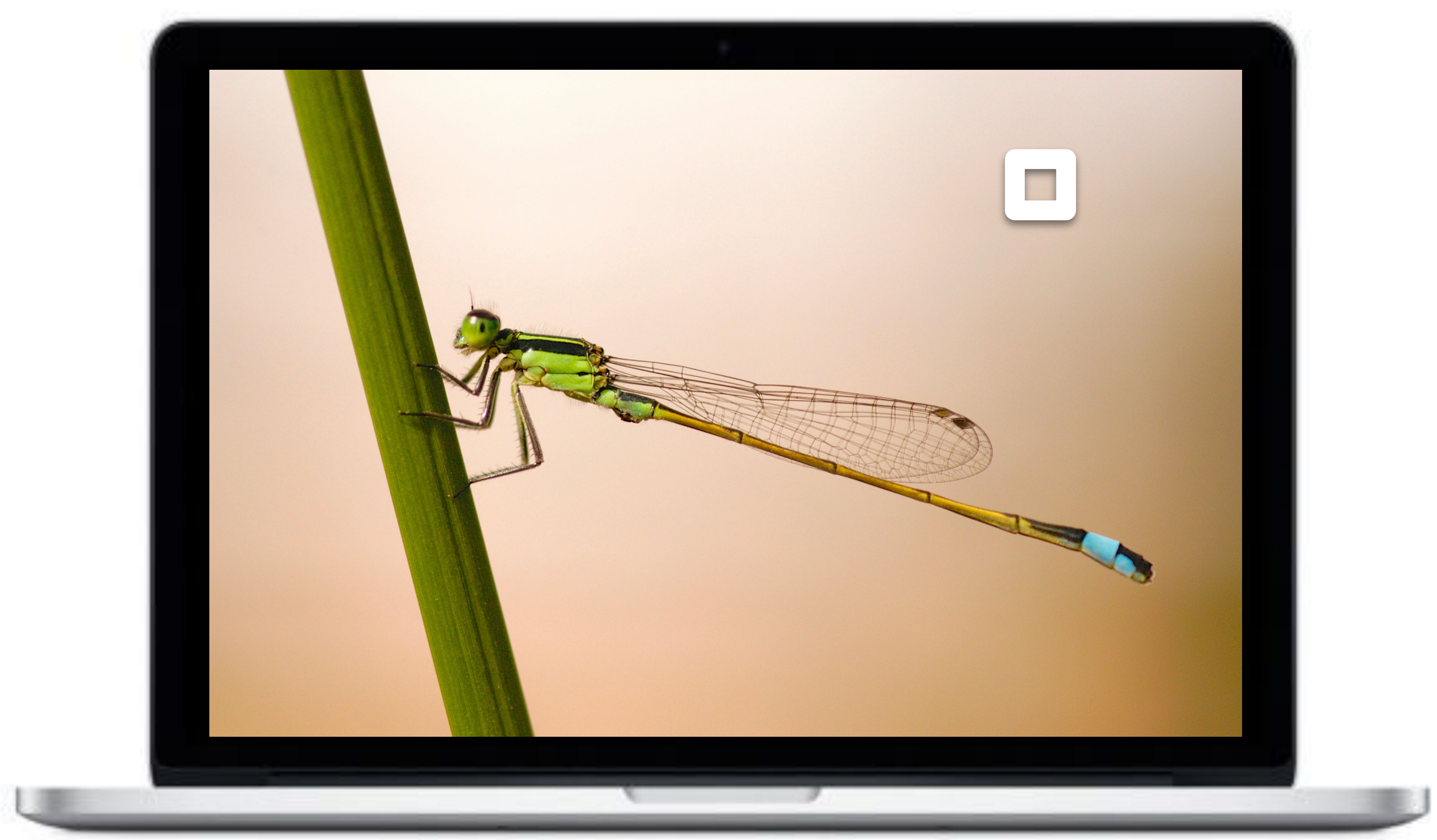
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} \text{---} & ? & \text{---} \\ \text{---} & ? & \text{---} \\ \text{---} & ? & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Color Reproduction as Linear Algebra

Spectrum produced by display given values R,G,B:

$$s_{\text{disp}}(\lambda) = R\, s_R(\lambda) + G\, s_G(\lambda) + B\, s_B(\lambda)$$

$$\Longrightarrow \begin{bmatrix} | \\ s_{\text{disp}} \\ | \end{bmatrix} = \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



Color Reproduction as Linear Algebra

What color do we perceive when we look at the display?

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\mathrm{disp}} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s_{\mathrm{disp}} \\ | \end{bmatrix}$$

$$= \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

We want this displayed spectrum to be a metamer for the real-world target spectrum.

Color Reproduction as Linear Algebra

Color perceived for display spectra with values R,G,B

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{disp}} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Color perceived for real scene spectra, s

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix}_{\text{real}} = \begin{bmatrix} \text{---} & r_S & \text{---} \\ \text{---} & r_M & \text{---} \\ \text{---} & r_L & \text{---} \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

How do we reproduce the color of s? Set these lines equal and solve for R,G,B as a function of s!

Color Reproduction as Linear Algebra

Solving:

$$\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix} \begin{bmatrix} | \\ s \\ | \end{bmatrix}$$

Solution:

$$\underbrace{\begin{bmatrix} R \\ G \\ B \end{bmatrix}}_{\text{1x3}} = \left(\underbrace{\underbrace{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}}_{\text{Nx3}} \underbrace{\begin{bmatrix} | & | & | \\ s_R & s_G & s_B \\ | & | & | \end{bmatrix}}_{\text{3xN}}}_{\text{3x3}} \right)^{-1} \underbrace{\underbrace{\begin{bmatrix} — & r_S & — \\ — & r_M & — \\ — & r_L & — \end{bmatrix}}_{\text{Nx3}} \underbrace{\begin{bmatrix} | \\ s \\ | \end{bmatrix}}_{\text{1xN}}}_{\text{1x3}}$$

Color Reproduction Issue: No Negative Light

R,G,B values must be positive

- Display primaries can't emit negative light
- But solution formulas can certainly produce negative R,G,B values

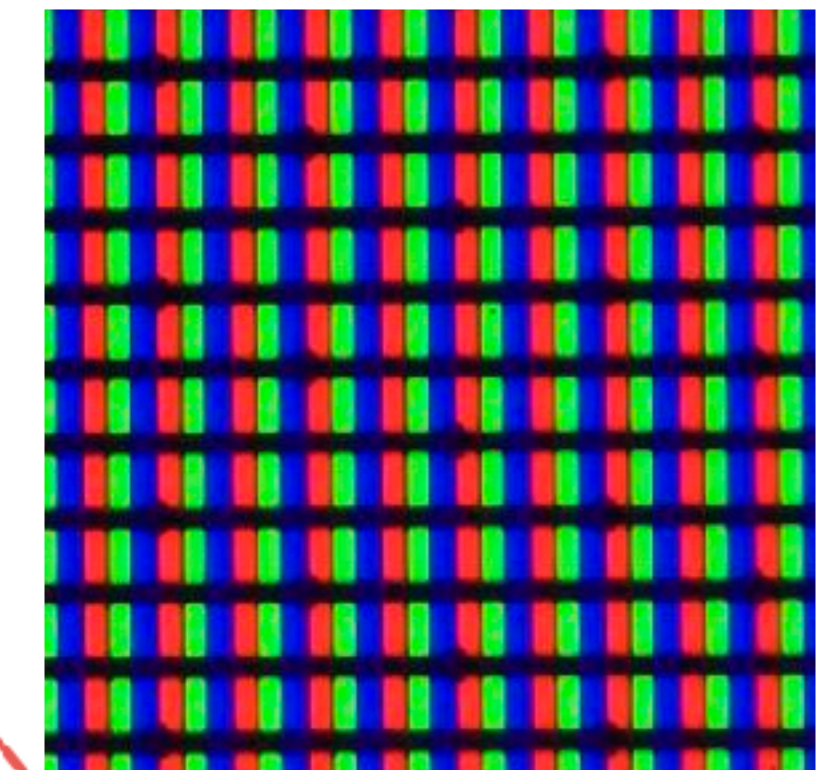
What do negative R,G,B values mean?

- Display can't physically reproduce the desired color
- Desired color is outside the display's color gamut

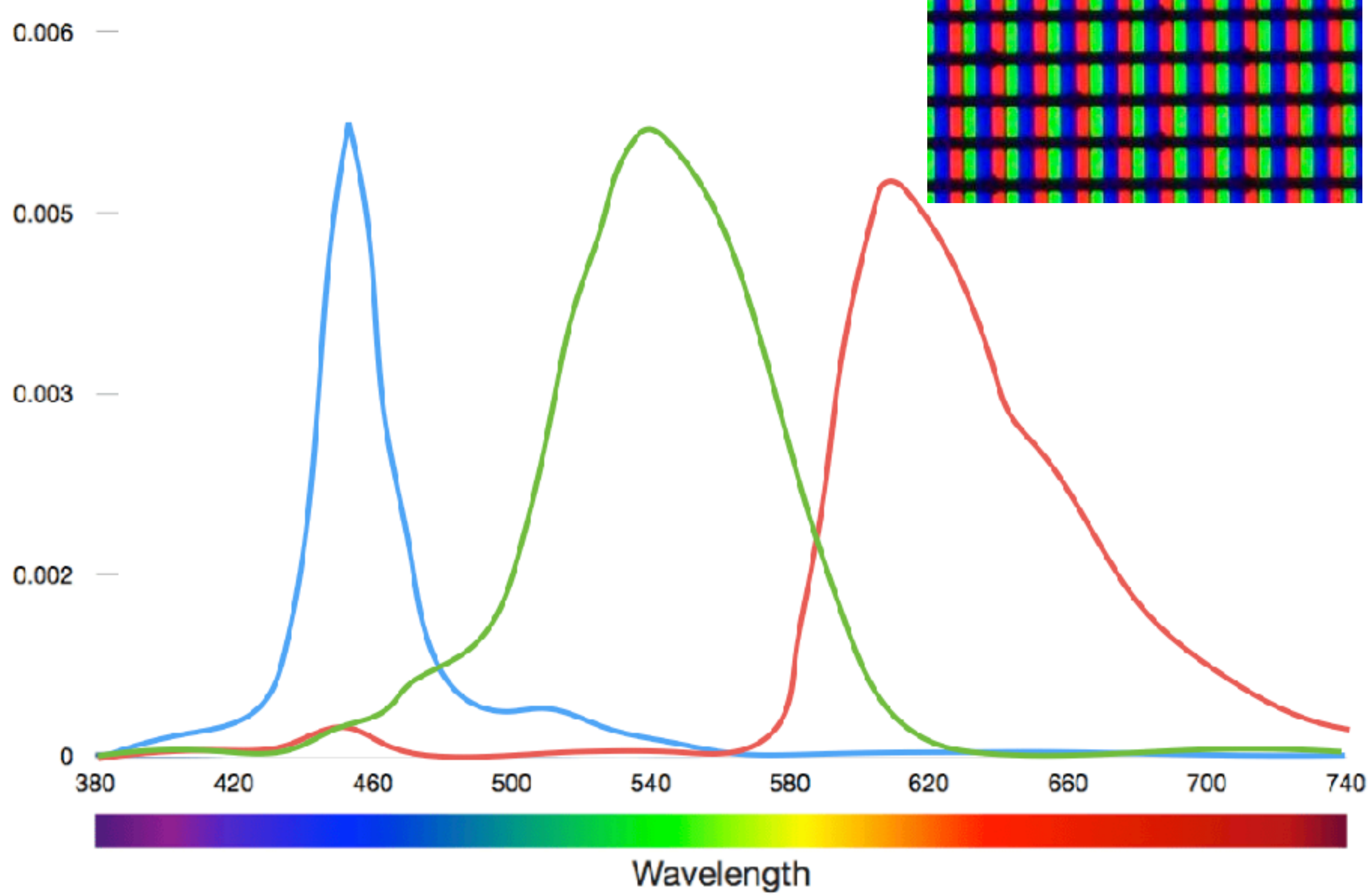
Phone Display Pixel Emission Spectra (Device RGB)

https://www.macrumors.com/roundup/iphone-5s/

RGB pixel spectra (iPhone 5)



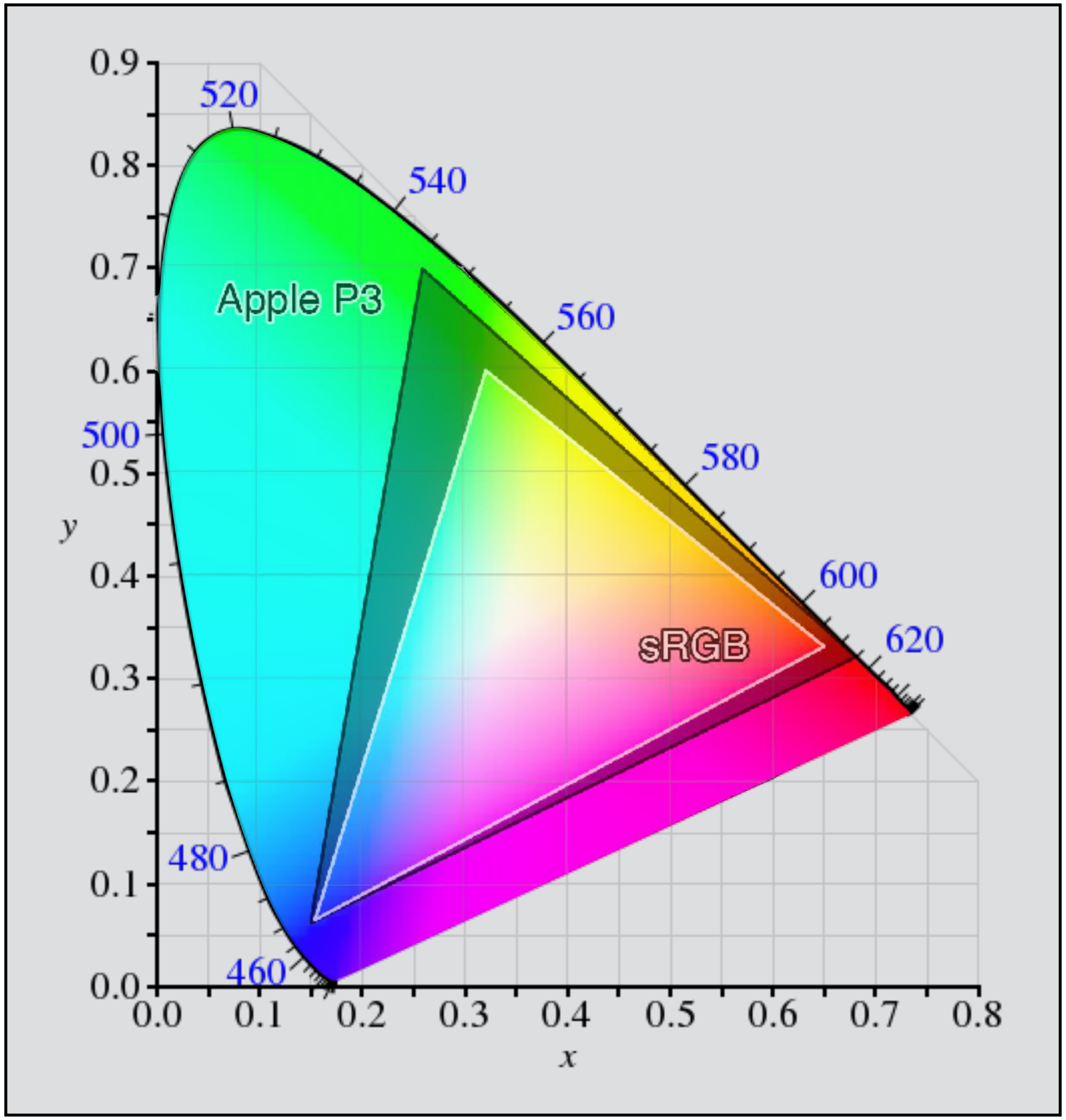
Credit: Jones, https://prometheus.med.utah.edu/~bwjones/2012/09/iphone-5-display-vs-iphone-4-display/



Credit: Yurek, https://dot-color.com/tag/color-2/page/2/

Gamut

Example: Color Gamut for sRGB and Apple P3



Wikipedia

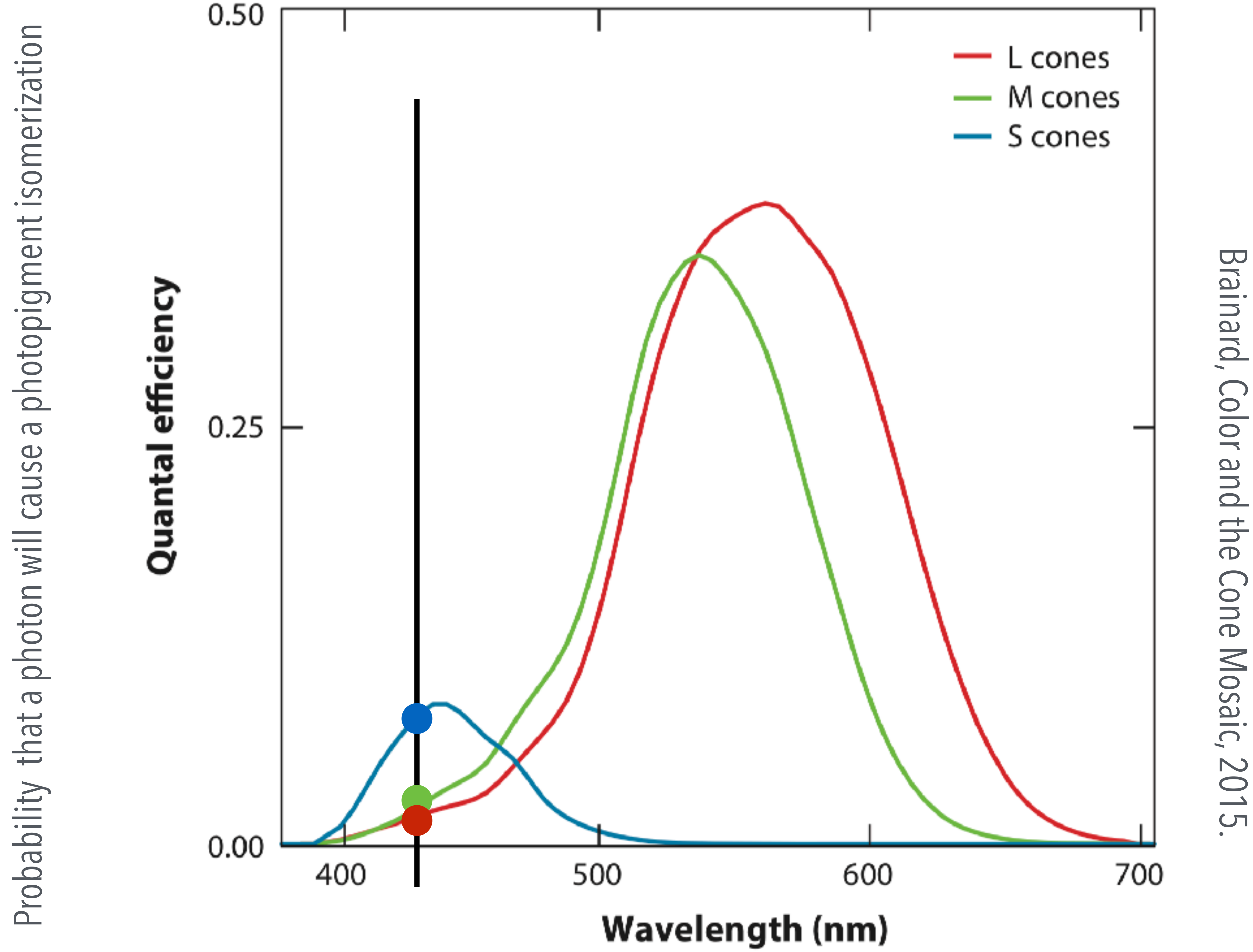
Comparing sRGB and Wide Gamut P3 Color Spaces

Interactive Color Space Comparison:

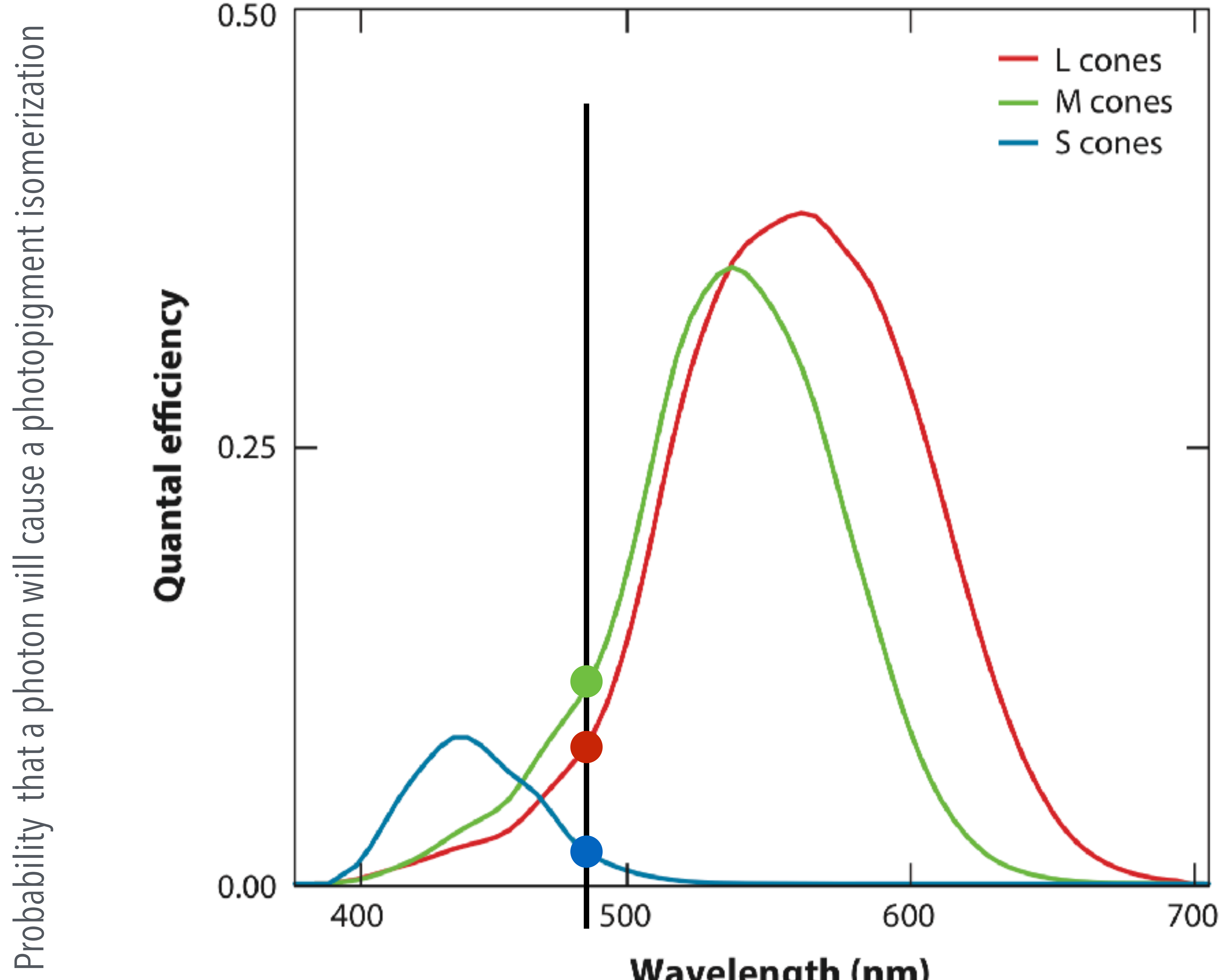
https://webkit.org/blog-files/color-gamut/comparison.html

- Needs a wide-gamut physical display
- I can see differences clearly on my MacBook Pro 2017

LMS Response Values for Each Wavelength



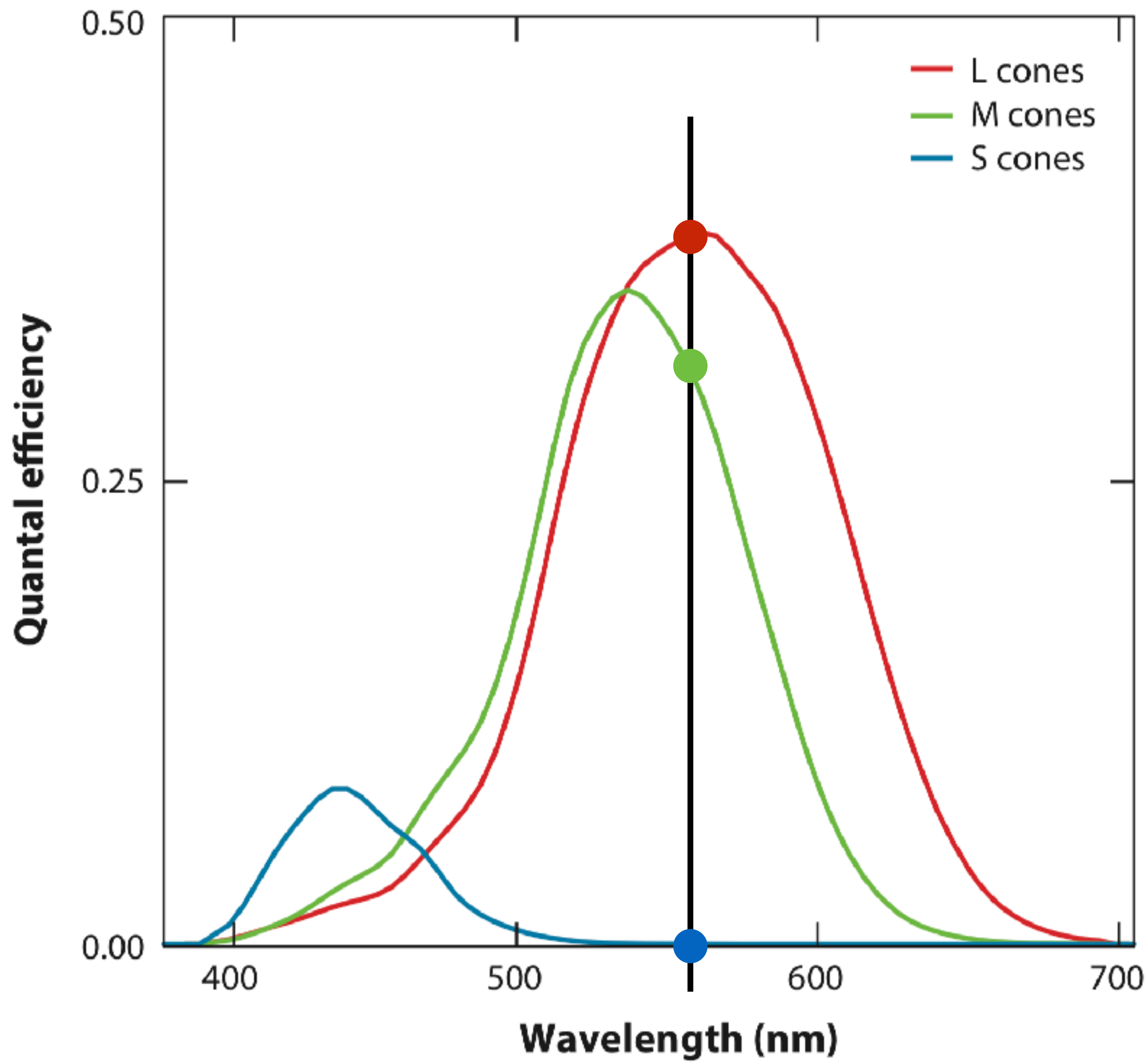
LMS Response Values for Each Wavelength



Brainard, Color and the Cone Mosaic, 2015.

LMS Response Values for Each Wavelength

Probability that a photon will cause a photopigment isomerization



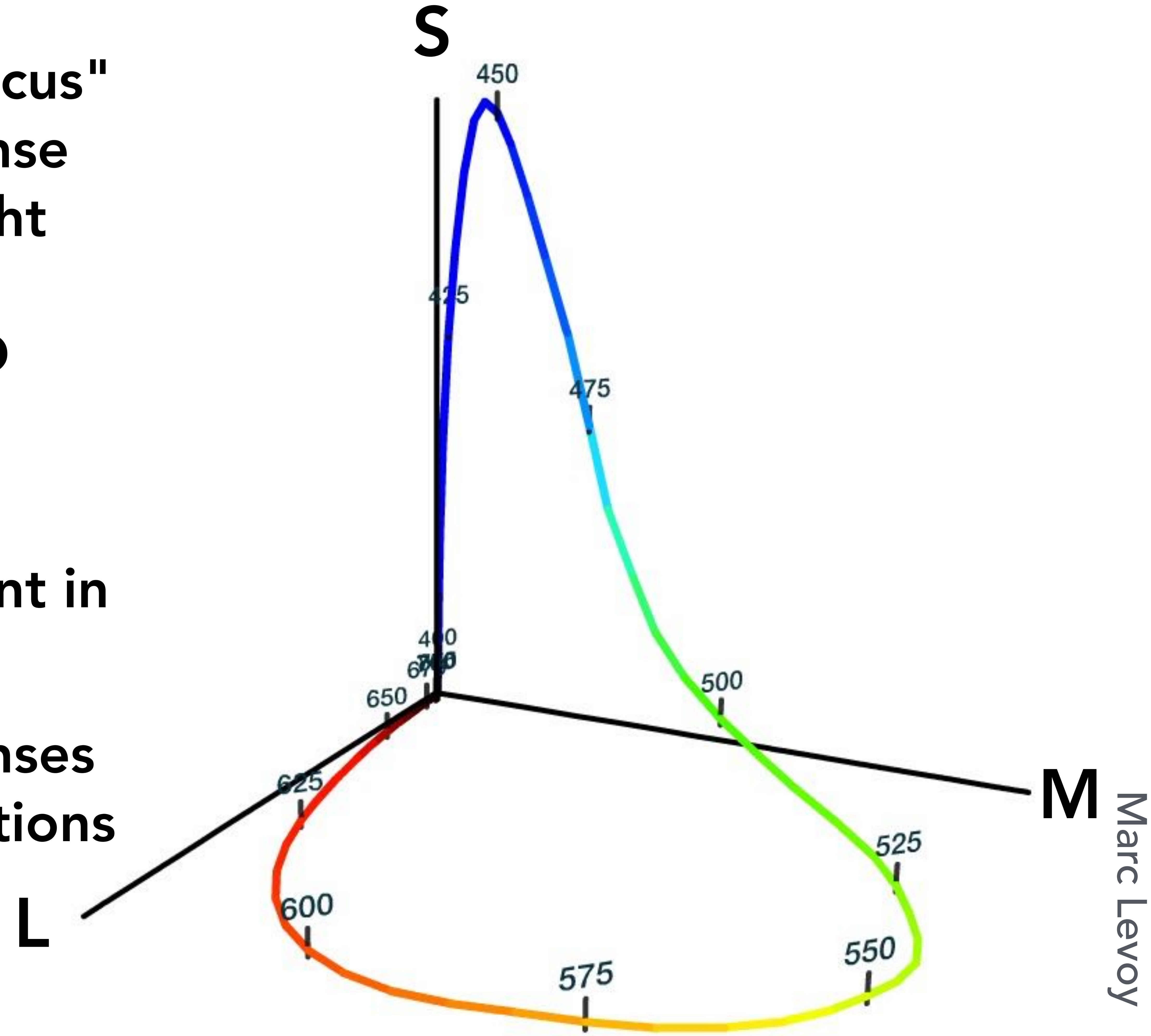
Brainard, Color and the Cone Mosaic, 2015.

LMS Responses Plotted as 3D Color Space

Visualization of "spectral locus" of human cone cells' response to monochromatic light (light with energy in a single wavelength) as points in 3D space.

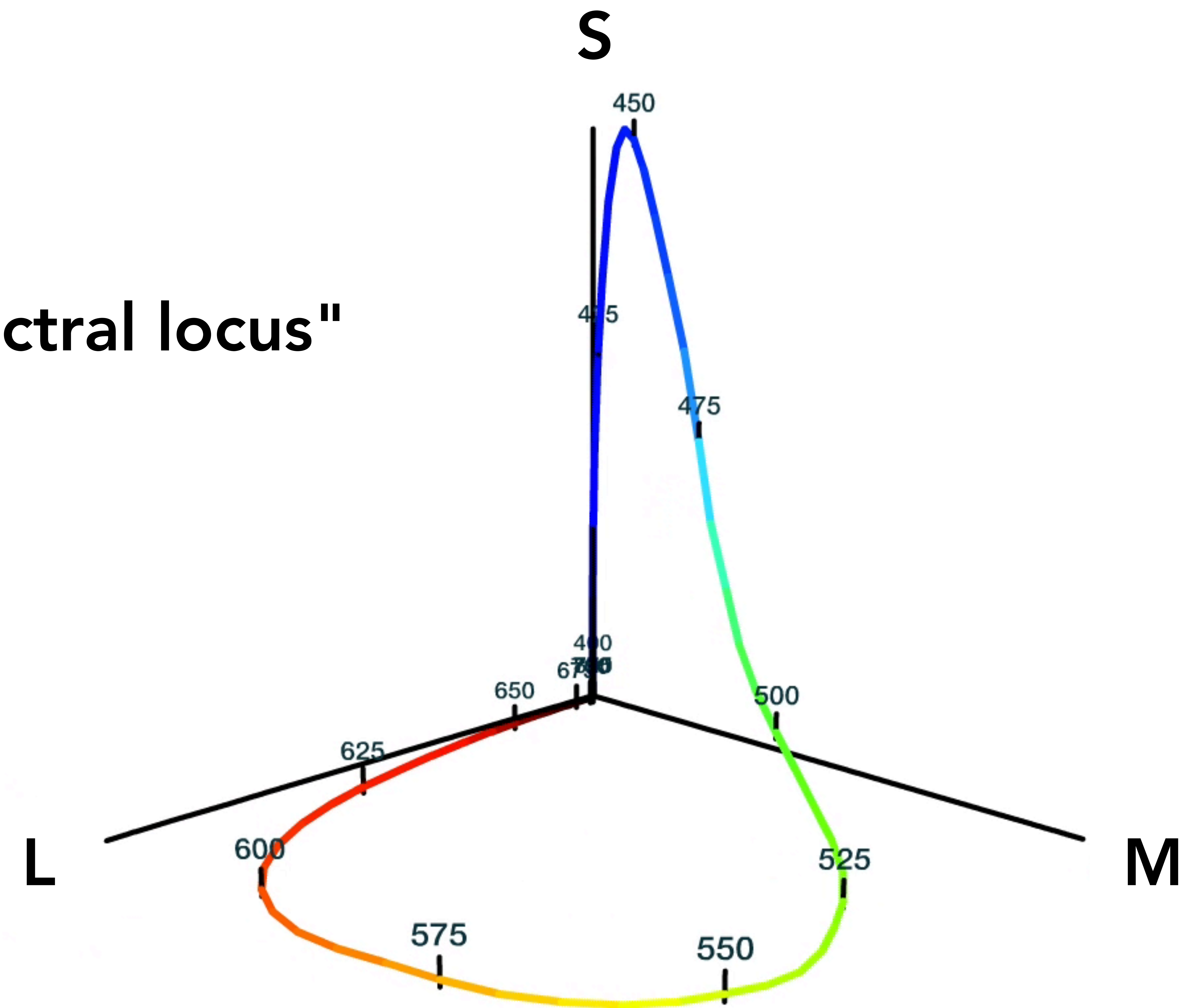
This is a plot of the S, M, L response functions as a point in 3D space.

Space of all possible responses are positive linear combinations of points on this curve.



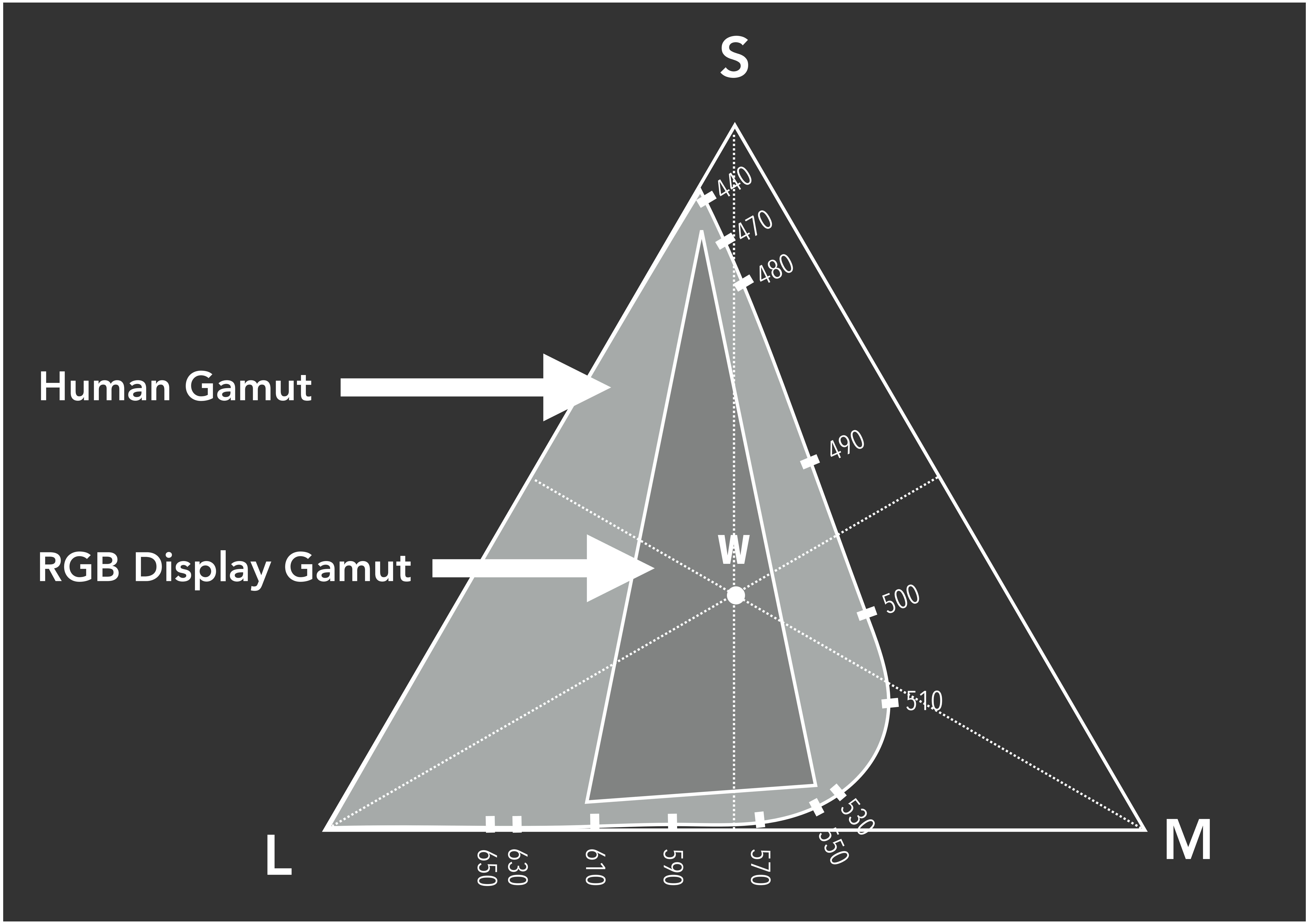
LMS Responses Plotted as 3D Color Space

"Spectral locus"



Dektar, Adams, Levoy
https://graphics.stanford.edu/courses/cs178-10/applets/locus.html

Chromaticity Diagram (Maxwellian)



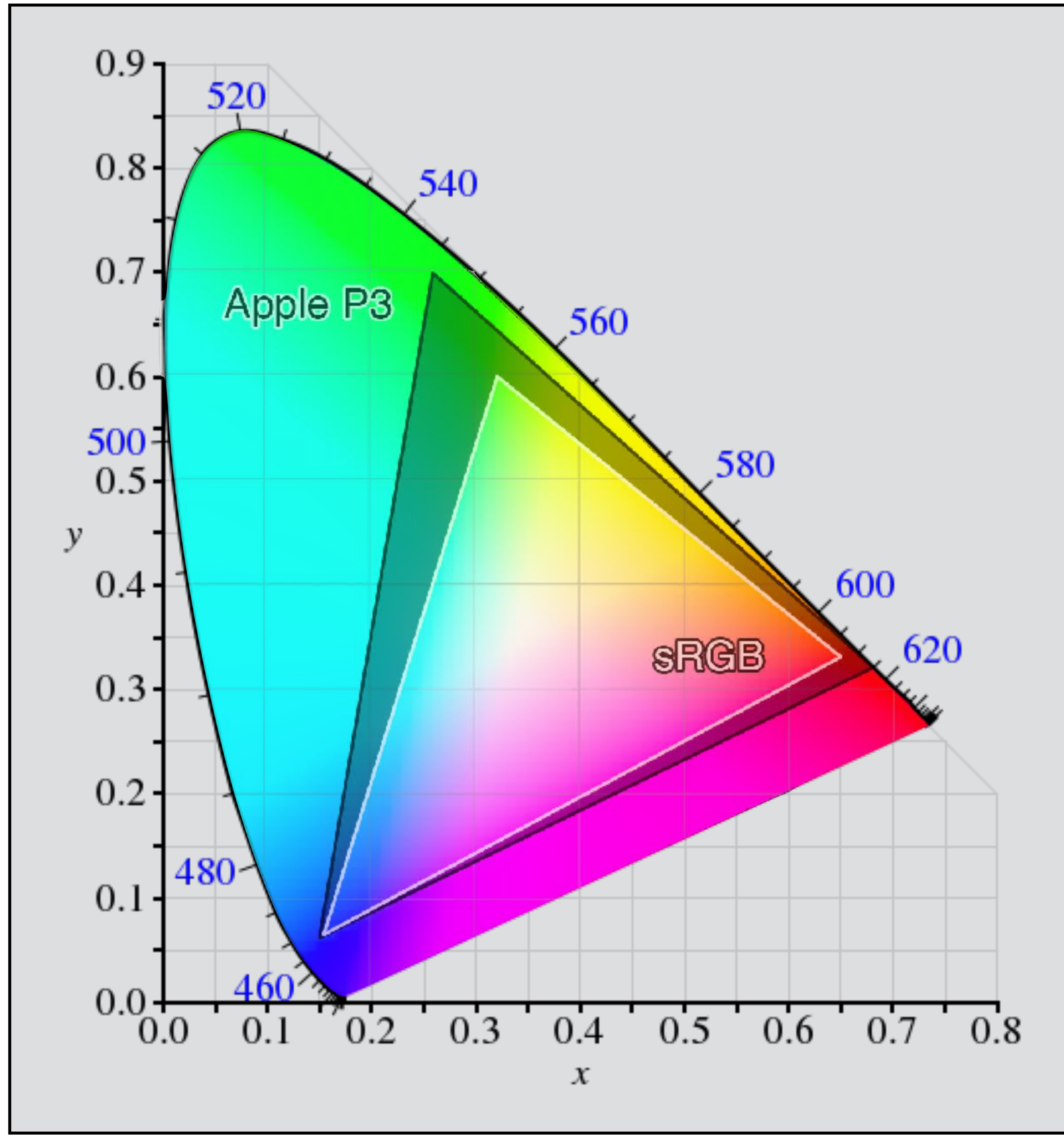
Perspective projection of spectral locus
looking diagonally down at origin from (1,1,1)

Chromaticity Diagram (CIE 1931 xy)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9121 & -1.1121 & 0.2019 \\ 0.3709 & 0.6291 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

$$x = \frac{X}{|X| + |Y| + |Z|}$$

$$y = \frac{Y}{|X| + |Y| + |Z|}$$

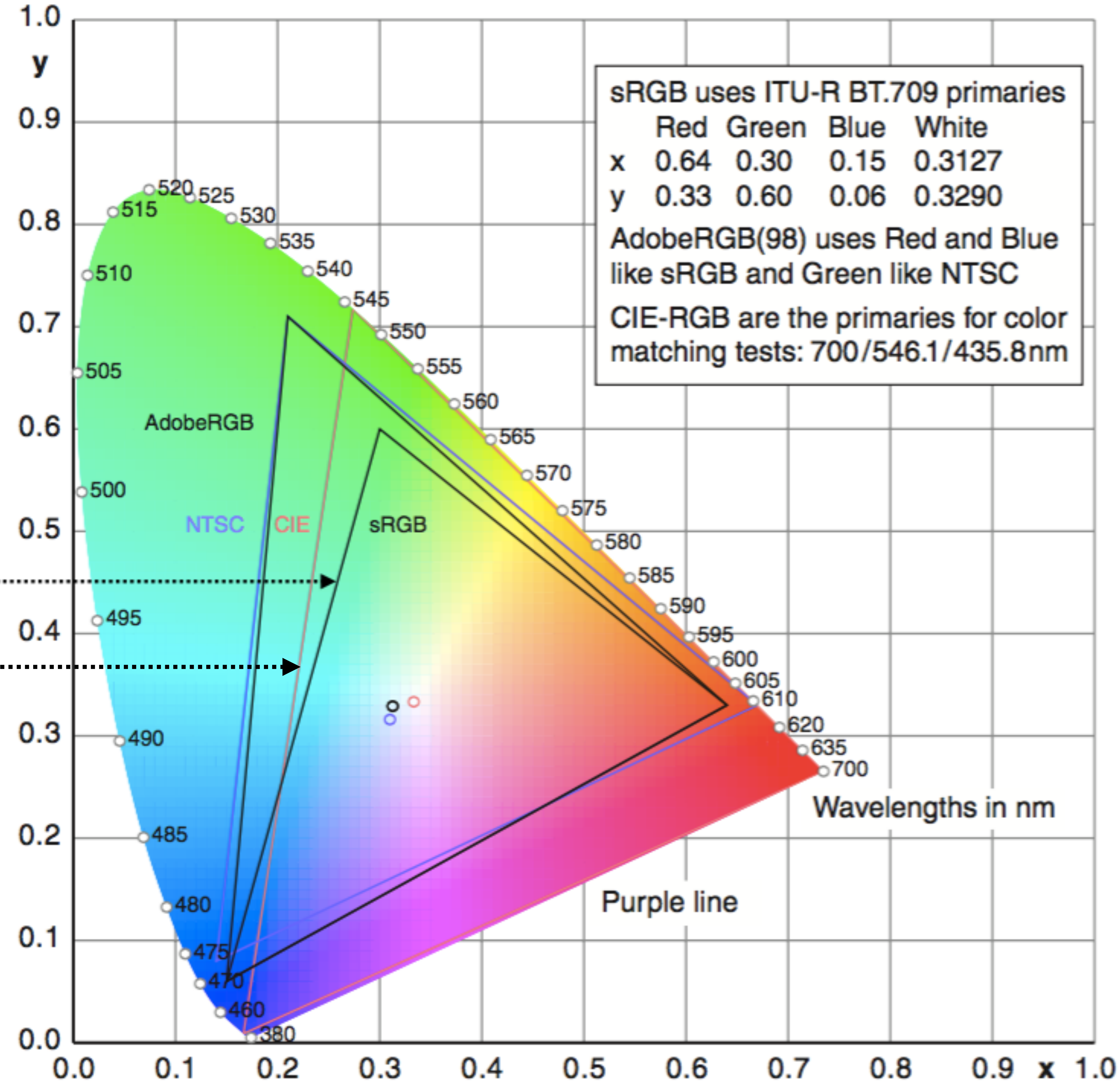


Wikipedia

Color Gamut

sRGB is a common color space used throughout the internet

CIE RGB are the monochromatic primaries used for color matching tests described earlier



Things to Remember

Physics of Light

- Spectral power distribution (SPD)
- Superposition (linearity)

Tristimulus theory of color

- Spectral response of human cone cells (S, M, L)
- Metamers - different SPDs with the same perceived color
- Color reproduction mathematics
- Color matching experiment, per-wavelength matching functions

Color spaces

- CIE RGB, XYZ, xy chromaticity
- Gamut

Acknowledgments

Many thanks and credit for slides to Steve Marschner, Kayvon Fatahalian, Brian Wandell, Marc Levoy, Katherine Breeden, Austin Roorda and James O'Brien.


calvin and HobbEs
by WATTERSON
WOW, HONEY, YOU'RE MISSING A BEAUTIFUL SUNSET OUT HERE!
I'LL COUNT TO 10, AND THEN... POW!
DAD, HOW COME OLD PHOTOGRAPHS ARE ALWAYS BLACK AND WHITE? DIDN'T THEY HAVE COLOR FILM BACK THEN?
SURE THEY DID. IN FACT, THOSE OLD PHOTOGRAPHS ARE IN COLOR. IT'S JUST THE WORLD WAS BLACK AND WHITE THEN.
REALLY?
YEP. THE WORLD DIDN'T TURN COLOR UNTIL SOMETIME IN THE 1930s, AND IT WAS PRETTY GRAINY COLOR FOR A WHILE, TOO.
THAT'S REALLY WEIRD.
WELL, TRUTH IS STRANGER THAN FICTION.
BUT THEN WHY ARE OLD PAINTINGS IN COLOR?! IF THE WORLD WAS BLACK AND WHITE, WOULDN'T ARTISTS HAVE PAINTED IT THAT WAY?
NOT NECESSARILY. A LOT OF GREAT ARTISTS WERE INSANE.
BUT... BUT HOW COULD THEY HAVE PAINTED IN COLOR ANYWAY? WOULDN'T THEIR PAINTS HAVE BEEN SHADES OF GRAY BACK THEN?
OF COURSE, BUT THEY TURNED COLORS LIKE EVERYTHING ELSE DID IN THE '30s.
SO WHY DIDN'T OLD BLACK AND WHITE PHOTOS TURN COLOR TOO?
BECAUSE THEY WERE COLOR PICTURES OF BLACK AND WHITE, REMEMBER?
THE WORLD IS A COMPLICATED PLACE, HOBBES.
WHENEVER IT SEEMS THAT WAY, I TAKE A NAP IN A TREE AND WAIT FOR DINNER.