F20 PHYSICS 137B: HW 2

Due September 11 at 5 pm

September 1, 2020

1 Griffiths problems

Do the following problems from Griffiths: 4.37, 4.38, 5.14.

2 Other problems

2.1

For two angular momenta of quantum numbers j_1 and j_2, there are $(2j_1+1)\times(2j_2+1)$ possible products $|j_1, m_{j_1}\rangle\,|j_2, m_{j_2}\rangle$ of eigenstates of the individual angular momenta. Count all the possible eigenstates $|j, m_j\rangle$ of the total angular momentum, and show that there are exactly $(2j_1+1)\times(2j_2+1)$ such eigenstates.

2.2

Consider two free electrons, with single-particle wavefunctions $e^{i\mathbf{p}_1\cdot\mathbf{r}_1/\hbar}\,|\pm\rangle$ and $e^{i\mathbf{p}_2\cdot\mathbf{r}_2/\hbar}\,|\pm\rangle$.

(a) Construct the antisymmetric two-electron wavefunction of net spin zero.

(b) Construct the antisymmetric two-electron wavefunction of net spin one. Assume that both spins are up.

2.3

Consider the following state constructed out of products of eigenstates of two individual angular momenta with $j_1=\frac{3}{2}$ and $j_2=1$:

$$\sqrt{\frac{3}{5}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle\,|1,-1\rangle+\sqrt{\frac{2}{5}}\,|\tfrac{3}{2},-\tfrac{3}{2}\rangle\,|1,0\rangle\,.$$

(a) Show that this isan eigenstate of the total angular momentum. What are the values of j and m_j for this state?

(b) Construct a (normalized) state of the same j, but a value of m_j larger by 1.

2.4

Suppose that five electrons are placed in a one-dimensional infinite potential well of length L. What is the energy of the ground state of this system of five electrons? What is $\langle S_z\rangle$ of the ground state? Take the exclusion principle into account, and ignore the Coulomb interaction of the electrons with each other.