EECS C106A: Remote Lab 3 Prelab: Kinematic Functions

Fall 2020

Goals

By the end of this lab you should be able to:

- Write kinematic functions in Python using Numpy and use them for Lab 3 and Homework 3.

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1 Implementing Kinematic Functions

In this prelab, you will be writing a few function in Python using Numpy which you will use to implement Forward Kinematics in Lab 3 and Homework 3. Fill in the provided `kin_func_skeleton.py` file to implement the following formulas using numpy. Test your implementation with the provided test cases by simply running `python kin_func_skeleton.py` in the command line.

(a) The "hat" $(\cdot)^\wedge$ operator for rotation axes in 3D.

- Input: 3×1 vector, $\omega = [\omega_x, \omega_y, \omega_z]^T$
- Output: 3×3 matrix,

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{1}$$

(b) Rotation matrix in 3D as a function of ω and θ

- Input: 3×1 vector, $\omega = [\omega_x, \omega_y, \omega_z]^T$ and scalar, θ
- Output: 3×3 matrix,

$$R(\omega, \theta) = e^{\hat{\omega}\theta} = I + \frac{\hat{\omega}}{\|\omega\|}\sin(\|\omega\|\theta) + \frac{\hat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|\theta)) \tag{2}$$

(c) The "hat" $(\cdot)^\wedge$ operator for Twists in 3D.

- Input: 6×1 vector, $\xi = \begin{bmatrix} v^T, w^T \end{bmatrix}^T = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^T$
- Output: 4×4 matrix,

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_z & \omega_y & v_x \\ \omega_z & 0 & -\omega_x & v_y \\ -\omega_y & \omega_x & 0 & v_z \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

(d) Homogeneous transformation in 3D as a function of twist ξ and joint angle θ.

- Input: 6×1 vector, $\xi = \left[v^T, w^T\right]^T = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^T$ and scalar θ
- Output: 4×4 matrix,

$$g(\xi, \theta) = e^{\hat{\xi}\theta} = \begin{cases} \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix} & w = 0 \\ \\ \begin{bmatrix} e^{\hat{\omega}\theta} & \frac{1}{\|w\|^2}\left(\left(I - e^{\hat{\omega}\theta}\right)(\hat{\omega}v) + \omega\omega^T v\theta\right) \\ 0 & 1 \end{bmatrix} & \omega \neq 0 \end{cases} \tag{4}$$

(e) Product of exponentials in 3D.

- Input: n 6D vectors, $\xi_1, \xi_2, \ldots, \xi_n$ and scalars, $\theta_1, \theta_2, \ldots, \theta_n$
- Output:

$$g\left(\xi_1, \theta_1, \xi_2, \theta_2, \ldots, \xi_n, \theta_n\right) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \ldots e^{\hat{\xi}_n\theta_n} \tag{5}$$