Name_____________________________________

Quantum Mechanics 137B Fall 2020 Final Exam

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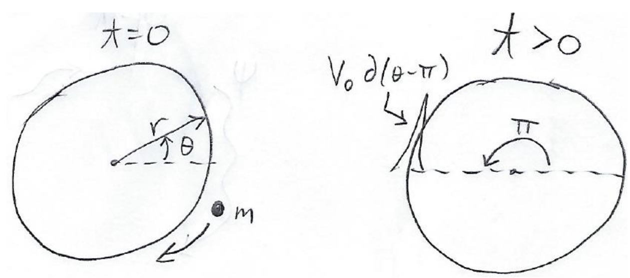
The testing period for this exam is 24 hours: 12/17, 7:00 pm PDT to 12/18, 7:00 pm PDT. During those 24 hours choose a 7 hour period in which to take the exam in one sitting and turn it in. Exams must be submitted via Gradescope (Gradescope will keep track of your time, so be mindful of this or else you will be penalized for being late). Please make sure all of your work is legible after scanning and uploading.

This is an open-book (Griffiths *Quantum Mechanics*), open-note exam. You may *not* use the internet or any other books or calculators or mathematical software/symbolic evaluators (WolframAlpha, Mathematica etc.). You can write your answer as a definite integral or a sum. All quantities, including limits of integration or summation, must be well specified in terms of the given parameters in each problem or fundamental constants. Please justify your work. There are 8 problems, check that you have them all. If you have a question, please send an email to the Professor *and* the GSI.

#1) (30 pts) A particle having mass = m can move freely on a ring of radius = r. Suppose the particle is initially in the lowest energy state that moves clockwise on the ring. At $t = 0$ a delta-function perturbation is turned on at the location $\theta = \pi$ that has the form $V(\theta) = V_0 \partial(\theta - \pi)$, where V_0 is a constant.



(a) (20 pts) What is the probability that the particle will continue to have the same energy but reverse its direction at time $t > 0$? (Use time dependent perturbation theory).

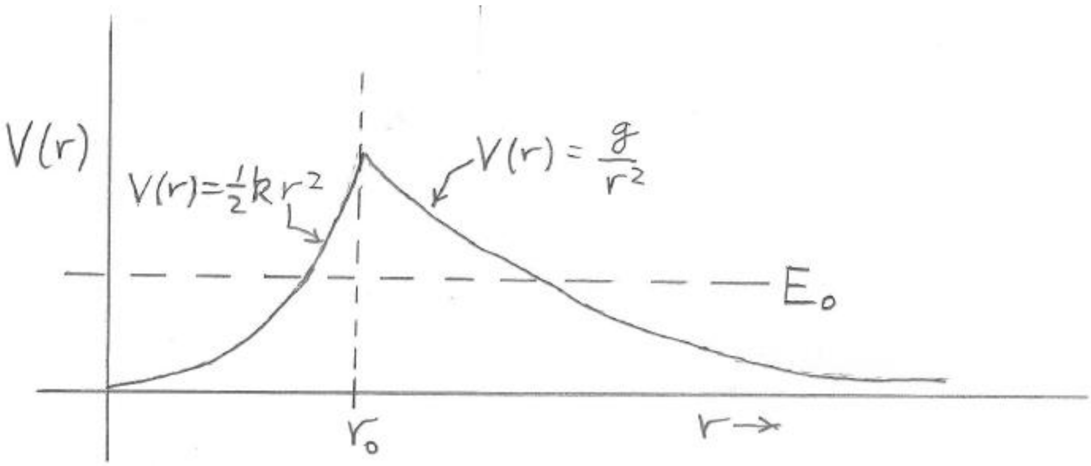
(b) (10 pts) At what times ($t > 0$) will the particle be LEAST LIKELY to be found in the next excited state of the ring? (Use time dependent perturbation theory again).

#2) (25 pts) Consider a particle having mass = m. Suppose the particle is dropped onto a hard surface where the force of gravity points down. If we assume that the particle falls straight down and elastically bounces from the surface, what are the allowed energy levels of the particle? (It's ok to leave your answer in terms of an unsolved integral).

[Hint: Use the WKB approximation.]

#3) (25 pts) Consider a fictitious particle of mass = m called a "quack" that is bound to atomic nuclei by a spring potential $V(r) = 1/2\, k\, r^2$ (where $k > 0$ is known). At a distance of $r = r_0$ (where r_0 is the size of a nucleus and is known) the potential felt by a quack becomes repulsive and follows a different functional dependence: $V(r) = g/r^2$ (where g > 0 is known). Suppose the ground-state energy of a quack inside the nucleus is E_0 and the average velocity of the quack as it rattles around in the nucleus is v_0. **What is the rate (probability/time) that a quack is emitted by a nucleus?**

[*Hint:* Use WKB]



4) (30 pts) Suppose that the ground-state, $\psi_0(x)$, of a 1D Hamiltonian, $\hat{H}$, is known. Show that the expectation value of $\hat{H}$ for any trial wave-function orthogonal to $\psi_0(x)$ is an upper bound on the energy of the first-excited-state of this system.

#5) (30 pts) What is the total cross-section for a particle of mass = m and energy = E to scatter off of the potential $V(r) = V_0 e^{-\alpha r^2}$ where V_0 and α are known constants?

[*Hint*: use the Born approximation]

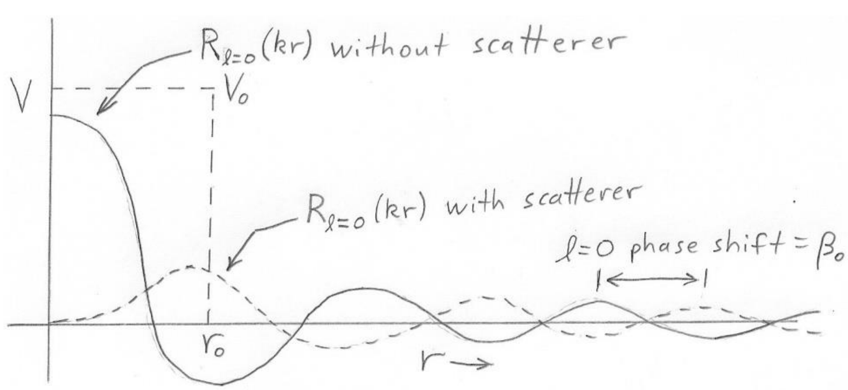
#6) (30 pts total) Consider a very low energy particle having energy = E that scatters off of a "soft sphere" potential: $V(r) = \begin{cases} V_0 & r \le r_0 \\ 0 & r > r_0 \end{cases}$. The $l = 0$ spherically symmetric radial wave function of the particle in the presence of the scattering potential is phase shifted by an amount = β_0 when compared to the free particle radial wave function for $r \to \infty$.

(a) (5 pts) What is the differential cross section, $\frac{d\sigma}{d\Omega}(\theta)$, for this low energy scattering process? (Assume that "low energy" means $kr_0 \ll 1$).

(b) (5 pts) What is the total cross section = σ_T for this process?

(c) (20 pts) Suppose the "soft sphere" is turned into a "hard ball" scatterer by making $V_0 \to \infty$, causing the $l = 0$ phase shift to become δ_0. Find the range = r_0 (i.e. the radius) of the new hard potential in terms of δ_0 and E (assume the range is very small).

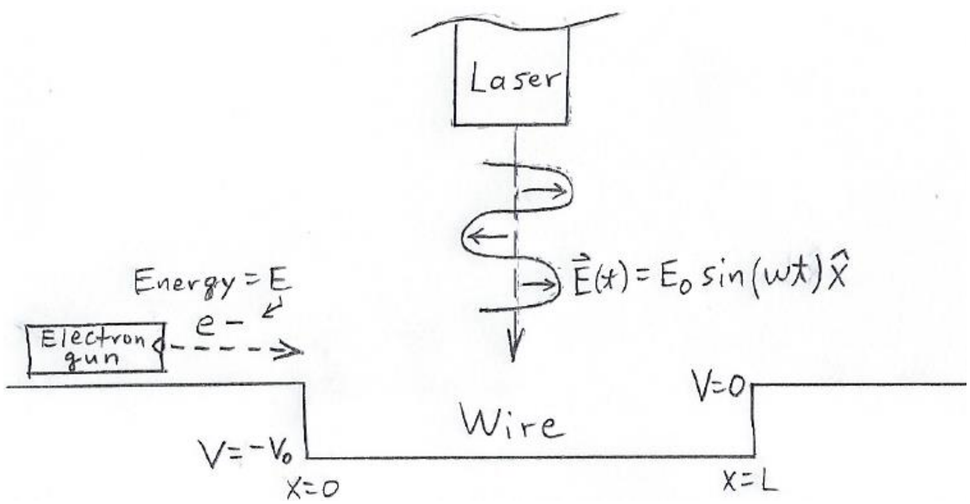
[**<u>Some useful identities are as follows</u>:**
If $f(x) = a\sin x + b\cos x$, then $f(x) = g\sin(x + \partial)$ where $\partial = \tan^{-1}\left(\frac{b}{a}\right)$ and $g = \sqrt{a^2 + b^2}$
For large r: $j_0(kr) = \sin(kr)/kr$ and $\eta_0(kr) = -\cos(kr)/kr$
For small r: $j_0(kr) = 1$ and $\eta_0(kr) = -1/kr$]



#7) (30 pts) **An "inverse photoemission" problem**: Consider a wire that can be approximated by a 1D square potential of length = L and depth = $-V_0$ (L is very large so think of this as an infinitely long 1D system with periodic boundary conditions. L is the only important length in this problem). Suppose that an electron is shot along the wire (parallel to it) from an electron gun with energy = E (see sketch). Light is shined on the wire from above so that it is bathed in an electric field $\vec{E} = E_0 \sin(\omega t)\hat{x}$.

(a) (25 pts) What is the probability/time that the electron falls into the wire via stimulated absorption? (Ignore issues such as work function and the Pauli exclusion principle. Use Fermi's golden rule and assume $E - \hbar\omega < V_0$).

(b) (5 pts) If 10^6 electrons are shot at the wire at the same time then what electrical current would they produce in the wire?



Hint: 1D density of states should be useful:

$$D(E)\,dE = D(k)\,dk$$

$$D(E) = D(k)\frac{dk}{dE}$$

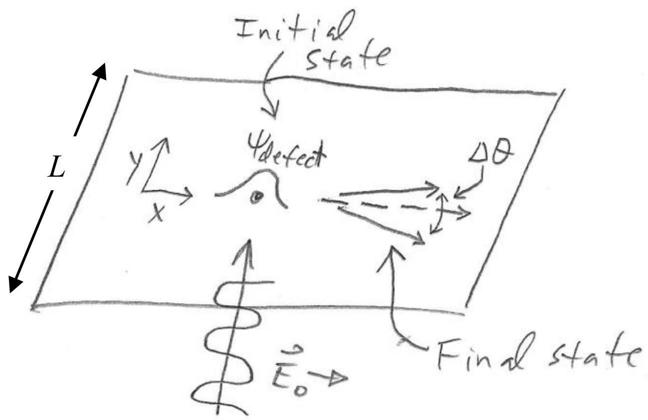
Forward + backward states

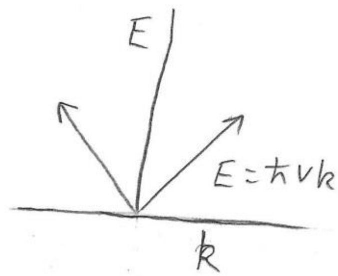
$$D(E) = \left(L/2\pi\right)\left(\frac{dE}{dk}\right)^{-1} \cdot 2$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m} \rightarrow \frac{dE}{dk} = \frac{\hbar^2 k}{m} \quad \left(k = \frac{\sqrt{2mE}}{\hbar}\right)$$

$$\Rightarrow D(E) = \frac{L}{2\pi} \cdot \frac{m}{\hbar^2 k} \cdot 2 = \frac{mL}{\pi \hbar^2 k} = \frac{mL}{\pi\hbar} \cdot \frac{1}{\sqrt{2mE}}$$

#8) (30 pts) **A photoemission problem:** Graphene is a 2D material made of carbon. Free electrons in graphene act like 2D plane waves that have a linear dispersion relation: $E(\vec{k}) = \hbar \mathrm{v} |\vec{k}|$ (where v = velocity is a constant). Consider an electron in graphene that is trapped in a localized defect state having the form $\psi(x, y) = \beta e^{-\alpha(x^2+y^2)}$ (β and α are positive constants). Suppose that light (polarized in the $\vec{x}$ direction) with an amplitude = E_0 and frequency ω is shined on the defect (assume $\hbar\omega$ is much bigger than the binding energy of the electron in the defect state and so the defect state energy ≈ 0).





a) (15 pts) What is the overall 2D density of states of graphene for a square sheet of length = L to a side?

b) (15 pts) Use the 2D density of states of graphene to find the rate at which light causes an electron trapped by a graphene defect to escape (i.e., to "photo-emit") into the graphene within a small angle Δθ along the $+\vec{x}$ direction. (Use Fermi's golden rule.)