Name ___________________________

Quantum Mechanics 137B Fall 2020 Midterm

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Total	

The testing period for this exam is 24 hours: 10/8, 10:30 am PDT to 10/9, 10:30 am PDT. During those 24 hours, choose a 2.5 hour period in which to take this exam in one sitting. Please observe the Honor Code in doing so. Exams should be submitted via Gradescope. Please make sure all you work is legible after scanning and uploading.

This is an open-book (Griffiths Quantum Mechanics), open-note exam. You may *not* use the internet, any other books, calculators or mathematical software/symbolic evaluators (WolframAlpha, Mathematica etc.). You can write your answer as a definite integral or a sum. All quantities, including limits of integration or summation, must be well specified in terms of the given parameters in each problem. Please justify your work. There are 5 problems, check that you have them all. If you have a question, please send an email to the Professor *and* the GSI.

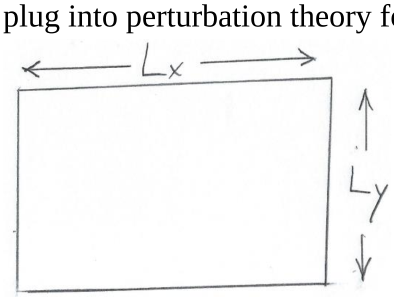
#1) (25 pts) 5 identical particles having mass = m are put in a 2D rectangular box where $L_x = 1.2\, L_y$. The energy eigenstates of the box look like this: $\psi_{n_x n_y}(x, y) = A\, sin\left(\frac{n_x \pi}{L_x} x\right) sin\left(\frac{n_y \pi}{L_y} y\right)$.

(a) (6 pts) What is the ground state energy of this system if the particles each have spin = 1/2 ?

(b) (6 pts) What is the ground state energy of this system if the particles each have spin = 1 ?

(c) (6 pts) What is the ground state energy of this system if the particles each have spin = 3/2 ?

(d) (7 pts) Suppose only 2 electrons are placed in the box. Write down one allowed wavefunction of the first excited state (i.e., the lowest-energy excited state). Take into account the fact that electrons repel each other (this leads to Hund's rule) and explain how it affects the energy of the state. (Do not plug into perturbation theory formulas).



#2) (25 pts) Consider a system comprised of three electrons.

(a) (7 pts) Suppose that the three electrons are each held fixed at different nearby locations. What are all the different possible total spin states for this system? How many *different* states are there for each total spin value?

(b) (9 pts) Suppose that the three electrons are now sitting in the lowest possible radial states of a lithium atom (i.e., the lowest possible "n" states) (lithium has Z=3). What is the *maximum possible total angular momentum* for all three electrons within these energy levels (include both spin and orbital angular momentum and ignore all perturbations)? Clarify how many different states there are that have this value of total angular momentum.

(c) (9 pts) What is the expectation value of the spin-orbit coupling, $\hat{A} = \gamma\, \hat{\vec{L}}_T \cdot \hat{\vec{S}}_T$ (where γ is a constant) , for the system described in (b)?

#3) (25 pts) An electron is placed in a 1D box of length = L. The box is then placed into a region where an electric field exists that has the form $\vec{E} = E_0\, x^3\, \hat{x}$ (here $\hat{x}$ is a unit vector, x is position, and the left edge of the box is at $x = 0$).

(a) (10 pts) How does the electric field change the energy of the nth level of the box? (To first order.)

(b) (15 pts) By what amount does the E-field change the probability of finding the particle on the left side of the box for the n^{th} state (to first order)? (**Remember**: it is ok to write all answers in this test as definite integrals where every term, including limits, are well-defined in terms of given parameters. You do not need to work out the integrals.)

#4) (30 pts) Consider an electron that is free to move in a 1-d wire of length = L $\left(-\frac{L}{2} \geq x \geq +\frac{L}{2}\right)$. The electron has energy $E_0 = {\hbar^2 k_0}^2 / 2m$. The two ends of the wire are connected so that the electron can either be moving forward in the eigenstate $\psi_1(\vec{r}) = \frac{1}{\sqrt{L}} e^{ik_0 x}$ or backward in the eigenstate $\psi_2(\vec{r}) = \frac{1}{\sqrt{L}} e^{-ik_0 x}$. Suppose a weak periodic potential $V(x) = V_0 \sin(2k_0 x)$ is turned on in the wire (where $V_0 > 0$).

(a) (20 pts) Calculate how the states mentioned above split in energy in response to the perturbation. (This is the mechanism that causes bandgaps to form in semiconductors. The perturbation comes from the corrugation of the atoms and the energy splitting is the bandgap).

(b) (10 pts) What are the new eigenstates of the perturbed system? (In a semiconductor these are called the "band-edge" states).

#5) (30 pts) Suppose that the electron in a hydrogen atom is subjected to a perturbation of the form $H' = b\left(\hat{S}_z + \hat{L}_z\right) + c\hat{L}^2 + d\hat{\vec{L}} \cdot \hat{\vec{S}}$ (where b, c, and d are constants).

(a) (20 pts) This causes the degeneracy of the unperturbed hydrogen groundstate to split. Calculate the amount of that splitting.

(b) (10 pts) How many levels will the degenerate n = 2 level split into?