MATH 3012-QHS, Homework Assignment 1, 2016, WTT

Due Date: Tuesday, September 6, 2016 by 4:35 pm, to be uploaded to T-Square.

Note: To receive credit, you must write your homework legibly (or type it) and have your name written on the front page at the top of the assignment. You must show your work. You may work together, but your write-up must be your own. If you use an outside source, cite it.

1. A multiset is like a regular set, except that elements can be repeated. For example, $\{1,1,2,3,4,4\}$ is a multiset. How many possible multisets are there formed from some subset of the digits $\{1,2,3,\ldots,n\}$, if each digit can appear at most twice?

2. Let a_n be the number of ternary strings of length n which avoid the sequence 00. Find a recurrence relation and initial conditions for a_n.

3. Give a combinatorial proof of the following identity:

$$k\binom{n}{k} = n\binom{n-1}{k-1}$$

Then, give an algebraic proof of the same identity.

4. Give a combinatorial proof of the following identity:

$$\sum_{k=0}^{n}\binom{n}{k}2^{n-k} = 3^n$$

To do so, consider where the zeroes can be placed in a ternary string of length n.

5. How many lattice paths from $(3,2)$ to $(12,14)$ avoid the points $(4,5)$ and $(6,9)$?

6. How many integer solutions are there to the equation,

$$x_1 + x_2 + x_3 + x_4 = 493$$

where $x_1 \geq 2, x_2 \geq 15, x_3 > 0$, and $24 < x_4 \leq 39$? Explain your answer.

7. Define the following sequences recursively, specifying appropriate initial conditions:

1. $2, 8, 14, 20, 26, 32, 38, \ldots$
2. $8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$
3. $1, 3, 7, 15, 31, 63, 127, \ldots$