COLOUR IN THE WILD

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De Clootcrans Press

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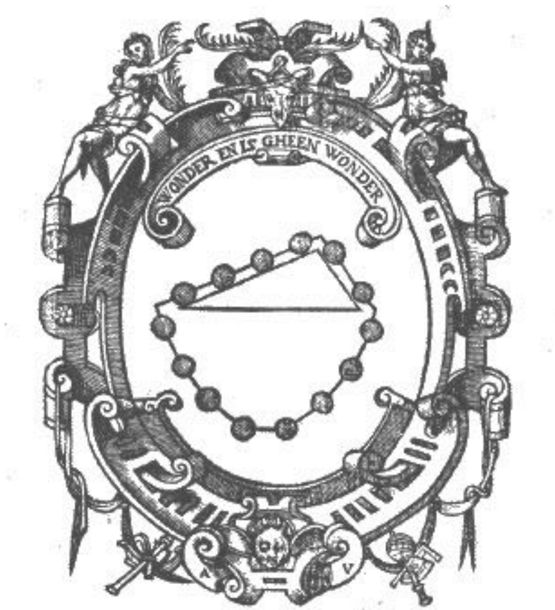

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Front cover: Mikhail Vrubel (1856–1910) was a Russian Symbolist painter. This is his painting *Demon seated* (1890). In his own description: "*A half-naked, winged, young, moody and thoughtful figure sits, hugging his knees against the sunset and looks at a flowering field, where branches rotting under flowers stretch.*" The image symbolises the interactions between mind, body and environment, a topic "Colour in the Wild" is about.

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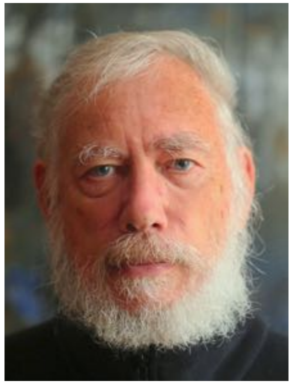
Colour in the Wild

The proper perspective on colour vision is biology. After all, humans are mammals, our visual system is not really different from that of various monkeys. It evolved in the wild and is likely to show all the signs of that. Perhaps strange to say, but much of colour science tends to ignore that fact. One thinks of Goethe as a misguided romantic, because he stressed the importance of daylight and the sun, instead of physics and physiology. But, of course, daylight is exactly what our vision has evolved to work with! It is *part of our vision*, not less so than the eye. Here I try to draw the consequences of such an offbeat view. It is not only fundamentally interesting, it has applications in areas of image technology, that are not offered by standard colorimetry.

Utrecht, june 2018 — Jan Koenderink



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ODILON REDON.

The various “in between” images are by the French painter Odilon Redon (1840–1916), an artist who understood (at the gut level, I mean) how to use colour to his best advantage. He is usually categorised as a “symbolist painter”.
Redon was hardly known till the publication of *À rebours* by Joris–Karl Huysmans, the protagonist Jean Des Esseintes being described as an avid collector of his work. This cult novel was an éclantant success and Redon enjoyed an unexpected free ride.
Redon’s work in colour (mainly using pastels) mostly dates from after 1890. In 1899 he exhibited with the *Nabis*, which—I guess—stamps him a symbolist.
According to Redon himself:

> ***My drawings inspire, and are not to be defined. They place us, as does music, in the ambiguous realm of the undetermined.***

Redon had an unusual way with colour. That there is no way to catch his images in terms of language makes him an apt illustrator for this book. So don’t try to understand his images, although you will no doubt identify some classical topics—“Birth of Venus”, “Ophelia” and the like—and even a portrait of the artist’s cat. Mentioning a topic hardly exhausts imaginative meaning. Just look at the colour!

Why "Colour in the Wild"? — The Argument

WHAT I argue is that many common questions concerning colour are most naturally answered from a biological perspective. For instance, the simple question HOW MANY COLOURS ARE THERE?, so often asked by the general public, fails to find an answer in *colorimetry*, which purports to be the "science of colour measurement".

Naturally, a "biological perspective" implies an evolutionary perspective. And, of course, that again implies both ecological (▷ p.83)[*] and ethological (▷ p.84) perspectives.

The ethological perspective has to do with what we *are*, I mean, what makes us tick. Your cat or dog lives in the same environment as you, yet it is obvious that they see very differently from you. This is partly because their physiology (▷ p.85) is different, but mostly because what makes them tick is different from what makes us tick. And with respect to the physiology, they had their chance to develop their systems just as we had ours. Or *vice versa*. For *what we are* physiologically also derives from *what makes us tick*. Everything is tightly interrelated, there are no simple, linear causal chains.

Different as our vision might be from that of the cat or dog, it is probably very similar to the Neanderthal[†] or the early AMH (Anatomically Modern Humans; see page 81) of at least three-hundred-thousand years ago.[‡] That is, with respect to colour vision,[§] for, of course, the Neanderthal doubtless had little eye for what might interest a modern Western intellectual in a landscape. They had to make a living, to stay alive and to procreate. In order to be able to do that their vision was of great importance. Human vision developed (▷ p.84) as a means of survival in a hunter–gatherer group in the African savannah or the tundras of the bleak North (see page 76). The ecological optics (▷ p.83) was much as it is now, at least "in the wild", were there are no traffic lights, neon signs or discos.

What makes us tick? There is a prime mover that generates urges in us, I'll call it the Will to Power, borrowing the term from Friedrich Nietzsche.[¶] All life strives to increase its dominion. This is so basic we never notice, we're just here to do and die. But the tiger has urges that differ from those of the lamb. We (sentient beings I mean) are all different, even if we live in the same environment and even if our basic physiologies are similar. The main differences are the primordial urges, Jakob von Uexküll's *Bauplan*.

The ecological perspective is important, but you should remember that each sentient being lives in its "umwelt", rather than the "physical world". The vital concept of *Umwelt* is also due to Jakob von Uexküll and is really the corner stone of ethology.[‖] The umwelt may be factored in a *Merkwelt* (World of Signs) and a *Wirkwelt* (World of Actions). Both merkwelt and wirkwelt differ greatly among sentient beings.[**]

When considering colour vision the merkwelt appears to be most important, but notice that what is worth to be noticed depends crucially on how you are prepared to deal with the world. The merkwelt and wirkwelt are so intimately coupled, that the notion of umwelt is perhaps preferable. Nevertheless, the perspectives of noticing and acting are often individually useful.

My arguments focus on two generic tasks that apply to a variety of biologically important interactions with the physical environment:

SEGMENTATION of the the visual field in optically distinct segments;

RECOGNITION of segments in the visual field on the basis of optical cues.

[*] Such references as these (▷ p.87) are to the technical annex.

[†] In this book I frequently refer to the Neanderthals, because they make a convenient and easy to illustrate example. I do not suggest that they are our direct precursors in our evolutional path. Most likely they are not, although you certainly inherit some of their genes. We may have split ways between a million and have a million years ago. Anyway there is likely to have been interbreeding. The precise pattern is irrelevant to my argument.

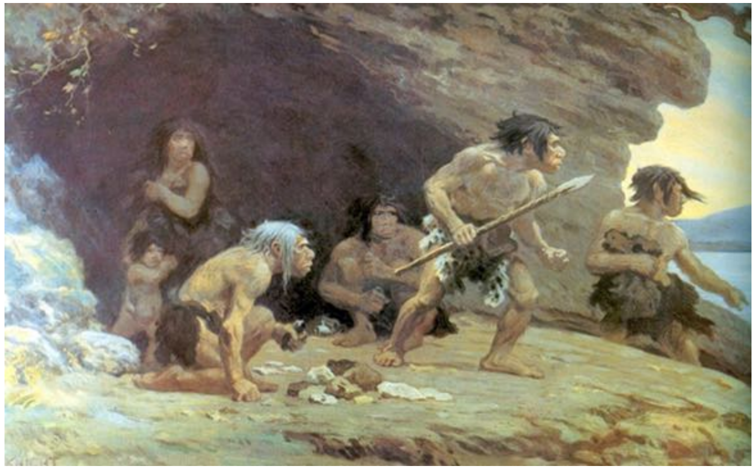
[‡] *Home erectus* had walked the earth for over a million years then. It seems unlikely that its vision was much different from ours.

[§] We have preciously little factual knowledge about the colour vision of early humans. This is not of much relevance to the main thrust of this book though.

[¶] Nietzsche's Will to Power (*Wille zur Macht* in *Also sprach Zarathustra: Ein Buch für Alle und Keinen* (1883–91)) is no doubt an echo of Arthur Schopenhauer's Will, as in his *Die Welt als Wille und Vorstellung* (1818).

[‖] Jakob von Uexküll *Umwelt und Innenwelt der Tiere* (1921).

[**] From here on I'll treat the German *Merkwelt*, *Wirkwelt* and *Umwelt* as technical terms in English, thus merkwelt, wirkwelt and umwelt.



Charles Robert Knight (1874–1953): Neanderthals (1920). Here the illumininant is "average daylight".

Charles Robert Knight: Cro–Magnon artists painting in Font-de-Gaume (1920). Here the illuminants are flames, the only "artificial illuminant" known to early man.

Segmentation allows you to distinguish optical patches from each other, recognition allows you to label visual objects and see them as the same at another time. Notice that both abilities are extremely useful. Also notice that they are categorically different. (See page 76.)

Segmentation allows you to notice yellow, orange or red berries among mostly green leaves.

Recognition allows you to spot *reliably* (meaning any time of the day, whether clear skies, overcast, direct sunlight, sunset, ...) berries that are somewhat yellowish as compared to other berries that are slightly bluish.

Finding berries is useful. Distinguishing reliably between types of berries might be a life saver, some make good nutrition others are poisonous.

Segmentation is much easier than recognition. Two types of berries might be easy to segment from the foliage background. But type $\mathcal{A}$ may appear purplish as compared to type $\mathcal{B}$ at noon, but the same type $\mathcal{A}$ may appear reddish as compared to type $\mathcal{B}$ at the end of the day.

Berries are hard to find if you are a monochromat. Red–Green segregation really helps a lot.

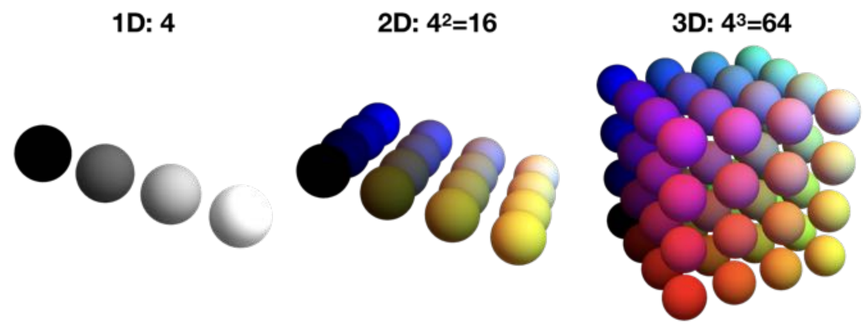
Segmenting is not recognition. A single set of objects may change its internal colour relations according the illumination, say time of day. This may appear magical, since one silently assumes that objects will retain their physical properties (or rather *Merkmale*, "signs") on the short term. But *magick and is no magick*.*

*This echoes Simon Stevin's *Wonder en is gheen wonder*. See the back cover.

The point is that vision does not take the full physical properties into account. That is why objects $\mathcal{A}$, $\mathcal{B}$ under illumination $\mathcal{P}$ may look the same, whereas the same objects $\mathcal{A}$, $\mathcal{B}$ under illumination $\mathcal{Q}$ may look different from each other. The phenomenon is known as metamerism. It is nothing special,* but will need a little coaching if unfamiliar. What is important is how it pops up in the biological account of vision. It will yield one important answer to the question of "how many colours are there?".

Berries of different kinds, or in different stages of ripening are hard to distinguish without trichromacy.

A trivial answer to that question would be to consider mere dimensionality. Suppose a species can distinguish N shades of grey. Now suppose it evolves M (instead of just one, the grey dimension) colour channels, each equally good as the single grey channel. Then it would be able to distinguish N^M colours, instead of just N greys, a boost by a factor of N^{M-1}. That is highly significant! Suppose $N \approx 10$ (the human condition) and $M = 3$ (the present human condition, so called "trichromacy"), then we have a boost of $10^{3-1} = 100$, so as much as a factor of a hundred. That would indeed be a huge evolutionary advantage (small wonder we evolved like that!).



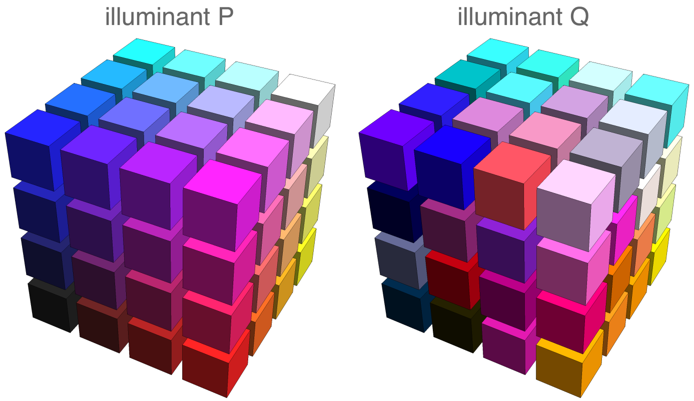
The effect of dimensionality. Here three-dimensionality yields a boost factor of $4^{3-1} = 16$. At least when all samples can be differentiated from each other.

This argument was the driving force in the theoretical work of the mathematician Christine Ladd–Franklin.† She somehow vanished from the colour science literature, but I see no valid reasons for that. Most of her early ideas on the evolution of colour vision are actually quite in line with modern ideas—of course, give or take some slop taking account of the periods. One hopes that scientists would have at least *some* historical sense, but apparently only a few do. Yet such a sense is of importance to their own field too!‡

*For instance, nobody is surprised that two identically looking pieces of metal may differ in that one is hot, the other not. If you do soldering work you need to remember whether the soldering iron is hot or cold, you can't see it. I often use my hand or my cheek to feel whether it emits thermal radiation. Likewise, you might test a colour by viewing it through a set of filters. I'll discuss that later.

†Christine Ladd-Franklin *A New Theory of Light- Sensation*, originally published in Proceedings of the International Congress of Experimental Psychology, London, 1892.

‡Why not be interested? After all, each of us is only a link in a long chain. You can't know yourself if you don't consider the chain.



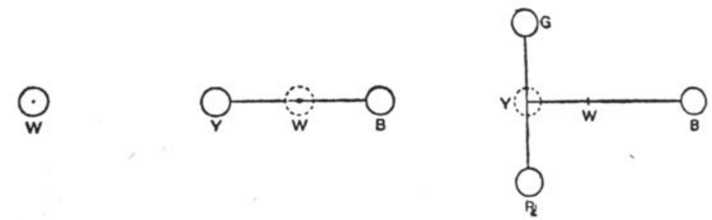
As the illumination changes, this might happen to the colours. Segregation is not affected, but individual objects can no longer be identified since they may "switch identity". The boost factor falls, in this case dramatically so. This effect depends on ecological, ethological and physiological factors. It makes that for an answer to "How many colours are there?" you must turn to biology.

The boost in segregation ability is a strong idea. However, consider the silent assumptions:

- are there enough dimensions available? (If so, then why not $M = 10$ instead of $M = 3$?);
- are the new states mutually identifiable? If they're all the same, they obviously don't count! This needs checking.

These are important and nontrivial problems. Their resolutions are to be traced in physics (ecology) and ethology. Conventional colorimetry is a tool, but no more than that. I'll explain it on the way.

These problems are in no way simple to resolve. In fact, I do not think that there is such a thing as a final resolution. The best one can do is something "reasonable". I'll sketch such an attempt in this book. I think it makes good sense, it is rather different from attempts I am aware of, and it appears to fit the facts. The arguments and methods are thoroughly based on ecological/ethological considerations.



Christine Ladd-Franklin (1892) "the colour molecule (hypothetical) in three successive stages of development". At left just WHITE, *at centre the* WHITE *has split in* BLUE *and* YELLOW, *at right the* YELLOW *has split in* RED *and* GREEN.

Finally, although this may perhaps seem trivial to you, the question of "how many colours are there" also allows of an *empirical* answer. Such an answer is highly informative, although it yields no understanding. No doubt Johann Wolfgang von Goethe would have complained, but I am using "understanding" in the sense common to the sciences here.*

But suppose you did not have the empirical facts, but conceptually understood the problem, then you would long for the facts. I am in no way downplaying the importance of empirical fact. It is vital. But knowing empirical facts as such does not implicate conceptual understanding.

I found the search for an empirical answer enlightening because it turns out to be so different from the mainstream (largely theoretical) notions. The pleasure in empirical research is in stumbling into surprises. These are where the process of induction begins.

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SO HERE IS THE ARGUMENT IN A NUTSHELL: In concrete actuality, that is awareness, one cannot differentiate mind, body and environment. These are

*I am in full sympathy with Goethe's notions concerning intuitive understanding. "*Das Höchste wäre, zu begreifen, da alles Faktische schon Theorie ist. Die Bläue des Himmels offenbart uns das Grundgesetz der Chromatik. Man suche nur nichts hinter den Phänomenen; sie selbst sind die Lehre.*"
I feel that a scientist not striving for intuitive understanding of the phenomena of his science is a poor creature. Most scientists I value highly are of the same opinion.

intimately interrelated through biological evolution and personal sedimented experience. Do animals see because they have eyes, or do they have eyes because they want to see, do they want to see because there are optical possibilities, or are there optical possibilities because there are seeing beings? One may find examples of all these.* There are no simple causal connections, the questions are like the chicken–and–egg dilemma.

In the chicken–and–egg dilemma we have a fast, circular process, superimposed on the slow drift of evolution, like a vortex on a stream. We only experience the fast, circular process.

That renders us blind to causation. In circular processes there is no before or after, no causation. This resolves the "vitalist" problems that troubled biology in the early twentieth century. For instance, von Uexküll was forced out of biology for this reason (it is quite painful to read the accusations by Lorenz, who acted as a kind of inquisitor). Some relations appear spooky, but only because one fails to take *all* relations into account. At the time (mid 20th century!) people were unable to see that.

Why is water tasteless? Obviously because we were born with water in our mouths. Water is a special fluid, there would be no biology as we know it without water. Yet water is not special to the physicist. Helium II or mercury are more interesting. Nor is it all that special to the chemist, nitroglycerin or sulfuric acid offering more adventure. As a liquid, water is just one incident among many for physics and chemistry, whereas it is THE fluid to the biologist. Our human system is totally tuned to water.

Why is daylight colourless? Same story! Yet daylight is only an incident to physics and colorimetry, of no special interest. But our human system is totally tuned to daylight. Our body is built to deal with daylight, our mind is organised in terms of interactions between our body and varieties of daylight.

One should consider mind, body and environment as a single system and daylight as a major strand in the texture (like water and many other things). This means that optics (physics) and colorimetry, but also the physiology of the eye, neuroanatomy, or neurophysiology are as such unfit to "explain" what we mean by "seeing colours". Unfit, because they abstract from mind, that is phenomenology (including ethology) and the environment (or ecology). Any somewhat coherent account must involve all of these. Simultaneously and coherently.

This involves many changes of emphasis, for instance, the recognition of daylight as a decisive factor instead of a mere incident. That is why "colour in the wild" is a more promising entry–point than "colour vision" in the usual abstract, formal sense.

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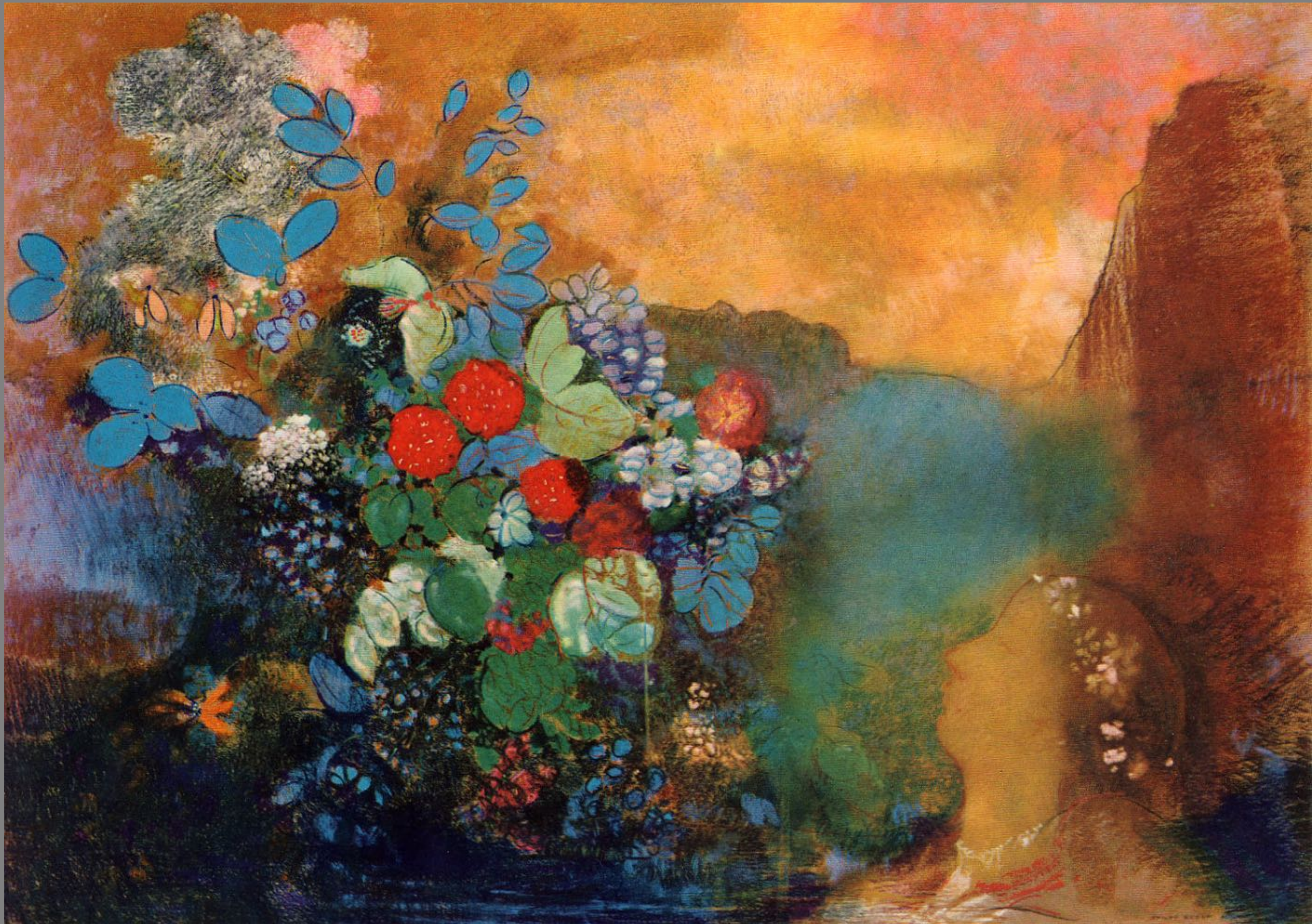
*You'll find it easy enough to come up with answers yourself. The last case is perhaps somewhat difficult ("are there optical possibilities because there are seeing beings?"). But think of flowers that lure insects to pollinate them. The flower patterns are very specifically aimed at the visual systems of particular insects. They evidently evolved because there are seeing beings, but plants themselves have no optical apparatus to see themselves.



A sprinkle of background facts and methods

IN THIS chapter I discuss various facts and concepts that I will freely use throughout the book. I mention historical facts, empirical facts, concepts, theories and common misunderstandings. I also touch on the formal, technical matters needed in order to do real life analyses and computations. The topics spread over a wide range of sciences. I have tried to put a modicum of order on them.

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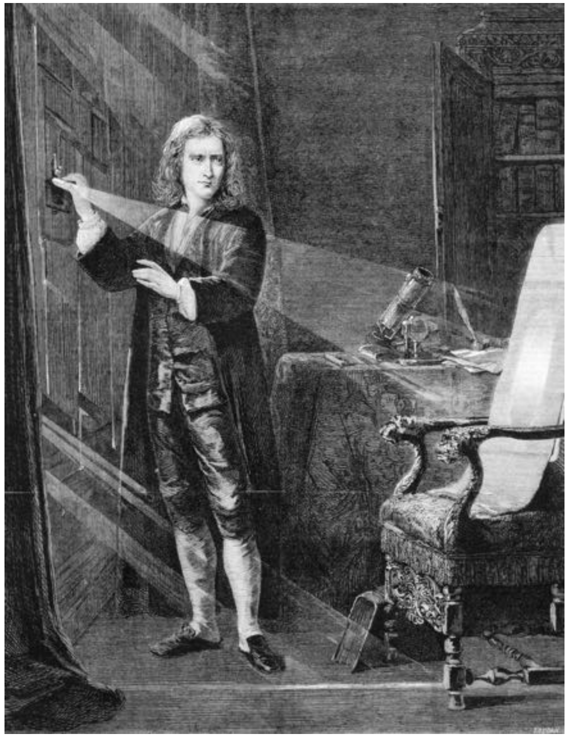


Newton and the spectrum. Mainstream science holds that the Neanderthals had eyes in order to distinguish wavelengths best as they could. Echo from Newton!

Physics

OPTICS starts with the Greeks, but for our purposes it starts with Newton. There were major developments during the nineteenth ("classical physics") and twentieth ("modern physics") centuries and optics is still in full development. For this book some "technical optics" suffices.

As everyone knows, Newton was the first to investigate the *spectrum* of sunlight. What most people accept for facts are (here "colours" are *qualia* and "monochromatic parts" are spectral selections termed "homogeneous lights" by Newton)

- sunlight is composed of monochromatic parts
- monochromatic parts and colours stand in a one–to–one relation
- there are no colours that are not in the spectrum

although, perhaps unfortunately, all these are misconceptions. These, almost universal, misconceptions unfortunately still determine philosopher's talk (I'm writing 2018!). Most modern intellectuals obtain their understanding of science (the fields they're not familiar with) from pop-philosophy and pop-science, which makes some sense. But not all pop-writers are equally good. Some are, most are not.

Newton indeed decomposed sunlight and put it together again. But the fact that you can decompose an entity does not imply that that entity would in any sense be "composed" of the resulting parts. Indeed, you can slice a sausage, but that by no means proves that a sausage is composed of slices. You have that

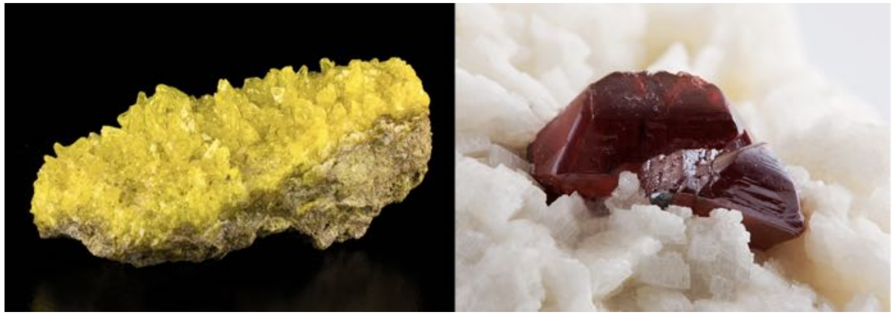
- $16 = 1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1$,
- $16 = 2+2+2+2+2+2+2+2$,
- $16 = 4+4+4+4$ and
- $16 = 8+8$.

So is 16 composed of 1's, 2's, 4's, or 8's? There is no answer to that. The number 16 is not composed of natural parts.*

*In case you are familiar with linear algebra: A linear space has no "natural parts". Any complete basis is as good as any other. There are infinitely may perspectives on "parts", none is uniquely important.

A monochromatic yellow can be matched by a mixture of a monochromatic red and a monochromatic green, the so-called Rayleigh match,* thus showing that colours do not stand in a one-to-one relation with monochromatic parts.

There is no such a thing as a monochromatic purple. Mixtures of red and blue monochromatic parts look purple, but they do not match any single monochromatic part. Thus there are colours that are not in the spectrum, thus "extra–spectral".



Sulphur crystals (the element, bright yellow) and cinnabar (a deep red mercury(II) sulphide) on Dolomite.

The belief in these misconceptions is still widespread. A philosopher can say "colour is either seeing by wavelength or it is mental paint" (▷ p.87) and will not be laughed out of the room. Perhaps hard to believe with Newton having been kicking up daisies for centuries, but (perhaps sadly) true.

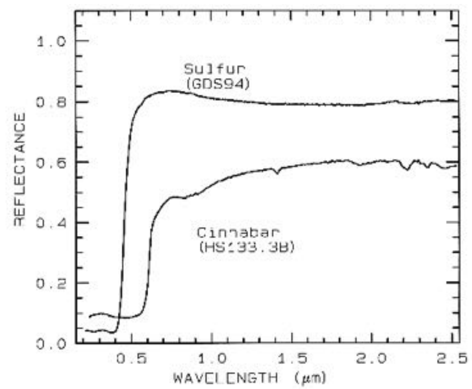
It is important to remember once and for all that

COLOUR HAS NOTHING TO DO WITH WAVELENGTH

Given the impact of the pop–science literature you may have to read this twice over. Anyhow, remember it!

Spectral representations of radiant power are often quite useful in computations, but that is an entirely different matter. The spectrum of a good yellow like sulphur is in no way like "100% at the wavelength of yellow and 0% anywhere else". Rather, sulphur reflects about half of the physical spectrum, it merely removes the short-wavelength range.

Indeed, if sulphur would only reflect "the yellow wavelength" (there is really no such a thing!) it would look black, because it would effectively reflect *nothing*. You will be rightly surprised when I tell you that this misunderstanding is not at all rare among academics.



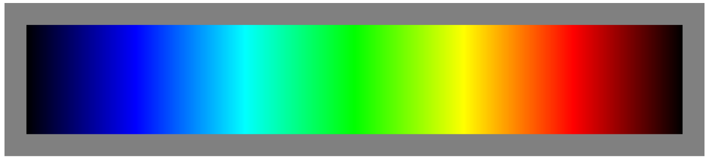
Reflection (range 0–100%) spectra of sulphur and cinnabar. The wavelength range involves the infrared, the visual range is about 0.4–0.75μm. Notice that these spectra are roughly of an all-or-none type. There are no signs of anything special at some "yellow or red wavelength" as many naive persons are wont to think.

Although the spectrum is of no particular interest to vision, it is good to be familiar with it. The first colours that strike the eye are (obvious, broad regions) red, green and (weaker) blue, separated by a marked narrow yellow band and (for people who look closely) a thin turquoise band that separates blue from green. (▷ p.82)

With some scrutiny you will be able to spot a few more colours when you look for them. For instance, it is not hard to locate orange.

However, there are many colours that you will not be able to find, for instance earth colours such as brown or olive, metallic colours such as gold or copper, or strange colours such as "anthracite". You can basically only find colours "from the air", not "from the earth" (see page 20), to use the ancient notion (usually connected with Aristotle) of the four elements.

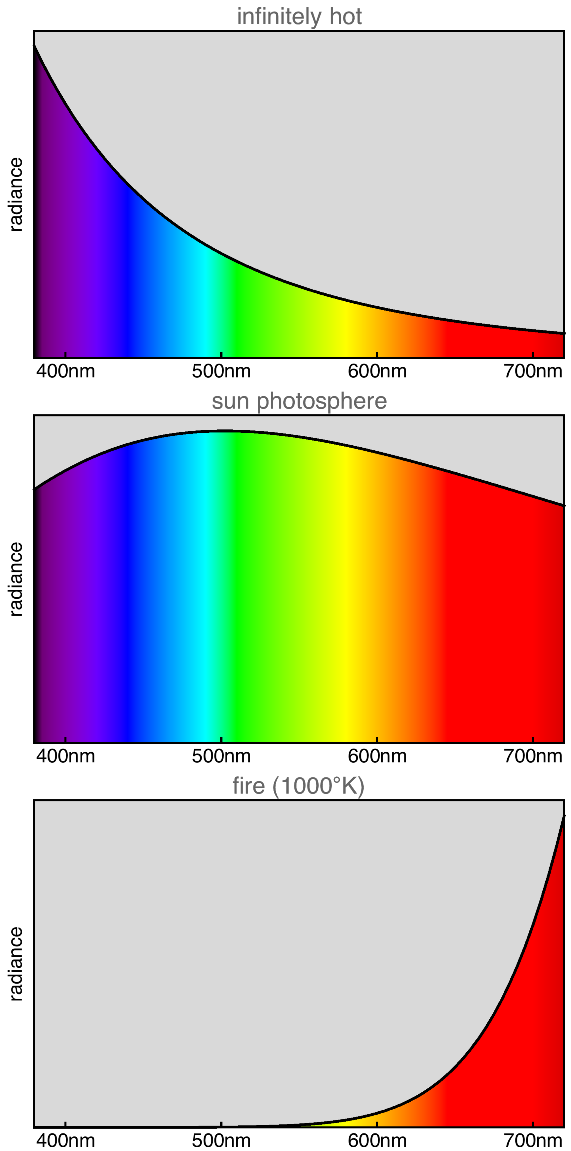
*It implies the Nagel–anomaloscope, you may want to Google it.



This is the spectrum as you might see it in a spectroscope. You need a very bright source for that. The resolution of hand-spectroscopes can be adjusted by setting the slit-width. Notice that, in order to see the best colours, you need a very wide slit, about a third of the spectrum length. Think about the implications of this observation!

How many colours has the rainbow? Most people first see red and green bands, mutually separated by a thin yellow band and then a blue band, sometimes seen separated from the green by a narrow turquoise (technically "cyan") band.

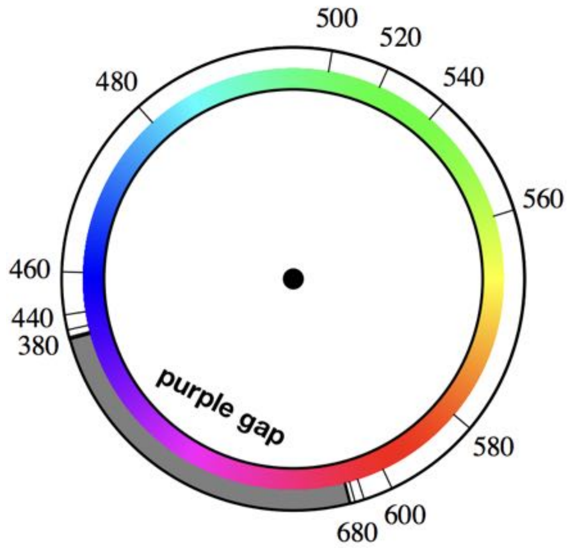
But a bright purple is also "a colour of the air", although you will not be able to locate it in the spectrum. This was reported by Goethe, who really *looked*, but only acknowledged in science through the work of James Clerk Maxwell and Hermann von Helmholtz. In the mid–nineteenth century the mathematician Hermann Graßmann, who also occasionally speculated on colour, could accuse Helmholtz of erroneous observations because he felt purple *had to be* a spectral colour because Newton had said so. But Helmholtz was never wrong in observations. purple is "an extra–spectral hue".



Varieties of thermal radiation. The infinitely hot source is very blue, with only some green and hardly any red. The sun's photosphere, at $5777°\mathrm{K}$ is similar to natural daylight. Firelight is almost "monochromatic", only red.

This implies that the spectrum is in no way to be confused by the "colour wheel" as often used by artists. It is not clear why Newton introduced the colour circle at all. There was clearly neither a phenomenological, nor a physical reason for it. He possibly tried to emulate the structure of the musical octave. Anyway, he created a confusion that lasts till the present day.

Phenomenologically, the colour wheel is an excellent way to represent the totality of hues. It evidently includes purple, which lives between blue and red. The spectrum may be mapped on the colour wheel, but fails to cover a "purple gap", thus is is a horseshoe–shape as discovered by Maxwell and Helmholtz. How the colour wheel derives from the spectrum became only slowly clear through the work of Johann Wolfgang von Goethe, Arthur Schopenhauer, Wilhelm von Ostwald and Erwin Schrödinger. By then we're in the 1920's, thus the story covers a full century.

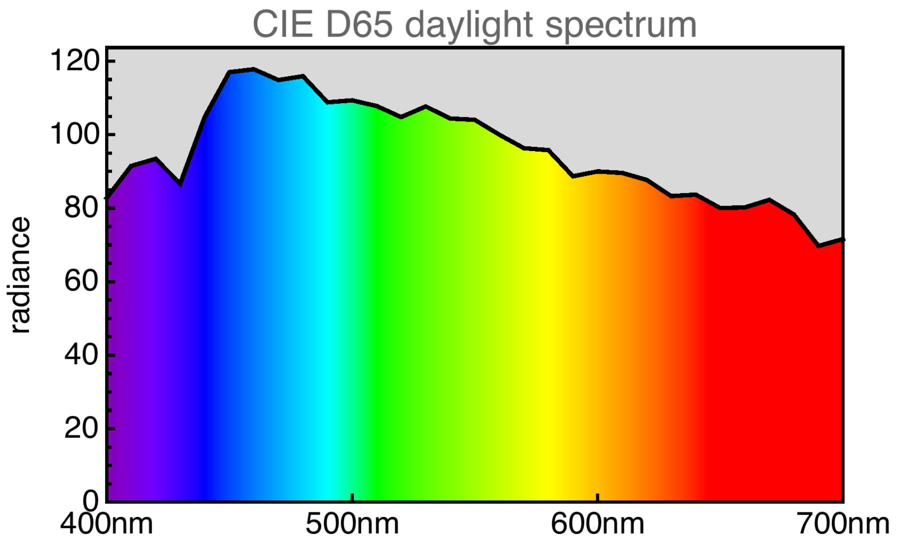


The spectrum mapped upon the colour wheel. Opposites can be mixed to white (the centre dot), thus are called "complementary". Purple is the complementary of green, but is not "spectral". The wavelengths here are merely used as labels, in a technical sense no colour on the colour wheel is "spectral". (Remember the sulphur and cinnabar).

This is an important story. I'll relate it piecewise, because I first need to introduce additional concepts and facts.

I'm not done with physics yet, although I will discuss only the absolute necessities. You will find it easy to mine the internet and pop-science books for more information. I will mostly touch on topics that tend to be universally misunderstood.

Most of the electromagnetic radiant power that enables our vision derives from the sun. Because I focus on *Homo Erectus* biology, I will mention only (varieties of) daylight and fire light (for which I will substitute candle light for concreteness). Both are thermal sources. Hot objects emit radiant power, the spectrum depends on their temperature. The sun has an effective temperature of about $5777°K$,* a fire flame about $1000°K$. Physics predicts the spectral radiant power from first principles (▷ p.82).†



This is CIE *illuminant* D65, *which is an "average daylight" spectrum. Notice that the visual range is approximately 400–700nm, so only a very minor piece of the radiant power spectrum is depicted here. At least from the physicist's perspective: for us humans it is all there is, at least optically. We feel the infrared radiation on our skins, whereas the ultraviolet* burns *it. Notice that the radiance is roughly constant over the full visual range.*

*This is an "effective" temperature. The temperature various widely over the various outer layers of the sun.

†This is the famous equation discovered by Max Planck (1900/01).

Thermal radiation is thoroughly chaotic noise, deriving from numerous short events, each of short duration and occurring at some arbitrary time.[*] The basic events are very localised both in time and in space.[†] So short that most "monochromatic parts" of sunlight in the visual range are roughly one wavelength or shorter. Thus the "physical parts" of thermal radiation are very far from being monochromatic. The spectrum is mere a convenience.[‡]

The sunlight spectrum is so broad that it is almost of constant radiant power density over the visual range. In practice, its precise spectrum is somewhat rough due to processes in the sun and in the atmosphere of the earth. For most purposes the notion of a thermal radiator amply suffices though.

A candle flame is a remarkable object, chemically, hydrodynamically and thermodynamically. Here I am only interested in the use as a source of radiation. That is taking care of by burning carbon particles ("soot") in the brightest part of the flame. It is the "business part" of the flame. Its temperature is about a thousand degrees Kelvin. In modern candles up to 1850°K.

In contradistinction, the candle light spectrum covers mainly the longest wavelength range of the visual band. This will prove to have important consequences for vision under candle light. Recruiting more candles hardly helps, it is the shape of the spectrum that counts, and does not change by recruiting more candles.

[*] A bit like the audio-spectrum of the applause from a huge audience after a magnificent aria presentation by some opera diva.

[†] Temporal and spatial size are immediately related by way of the speed of light.

[‡] From a formal perspective, the variance of the power spectral density diverges to infinity if the spectral resolution is arbitrarily increased.

There are many paintings showing the effect of candle light. The famous case in cinema is Stanley Kubrick's *Barry Lyndon*, largely done in natural illumination (a rare case!).

Of course, sunlight (or daylight) and candlelight are *sources*, that is, emitters of radiant power. Most things in your ken are not. You would not be able to see them in a closed room without windows, unless the lights were on. They become visible because they scatter radiation to your eye, radiation that derives from sources.

We tend to assume that, if there is sufficient daylight around, we can see an object from wherever we are. This is indeed usually true. But consider how special it is. It means that the object should scatter the radiation towards *all directions*! This is special because various surfaces don't achieve that. A key example is a mirror. A mirror *has no colour* because it shows whatever it reflects. A mirror may change from bright to black at the drop of a hat!

Still from Stanley Kubrick's Barry Lyndon. *It took a* BNC*–camera and Zeiss f/0.7 lenses to shoot it in natural light. It is hard to imagine that this was the best illumination we had for many centuries. Hardly better that what the Neanderthal could muster up. Notice you don't see many colours. Candle light is close to being effectively "monochromatic".*

Ideal surfaces that achieve to scatter all types of radiation over all directions, absorbing none, are named white "Lambertian" surfaces after Johann Heinrich Lambert (1728–1777) a Swiss mathematician—or, really, universal scientist.

Indeed, the definition of a "white" object is that it scatters *all* radiation (that, a perfect mirror does too) over *all directions* (that, a perfect mirror does not). Thus a white surface looks the same from wherever you are!

If it looks different it might be because its relation to the sources has changed. For instance, it looks different in the sun from what it looks like in the shadows.

Lambertian white surfaces are such materials as paper, foam on beer, milk (oil droplets in water), snow, chalk, and so forth. That is to say, the definition is not of a chemical nature.

The definition states that all incident radiation is scattered over all directions, independent of the nature of the incident beam. For the moment, I'll take a piece of chalk as the primordial instance. It is the most "object–like" thing imaginable, in the sense that mirrors or glass are far less object–like. They change appearance as you change your view point. Lambertian objects never behave capriciously, they behave as generic materials "should". No doubt a Neanderthal would bet on that, I mean *objects behaving as objects should*.

The "Seven Sisters", Sussex. Such white chalk cliffs are the primordial objects.

The white objects simply scatter the incident radiation back at you. All of it, I mean, the full spectrum. There are many objects that are Lambertian, but do not scatter all wavelengths equally well. They absorb some. Such objects appear "coloured".

We describe this through comparison with a truly white object. For some wavelength regions the coloured object will scatter less radiation than a white object does. The ratio is called the "spectral reflectance factor",* a number between zero and one. I already gave an example for the sulphur and cinnabar crystals.

The spectral reflectance factor is the physical causal factor in determining "colour" as a visual quality (quale). It is only one factor, but it is the physical factor. The spectral reflectance function of various materials will be an important item in this book.

Of course, there are numerous physical factors that need to be minded at various times. However, this is pretty much a solid rock bottom start.

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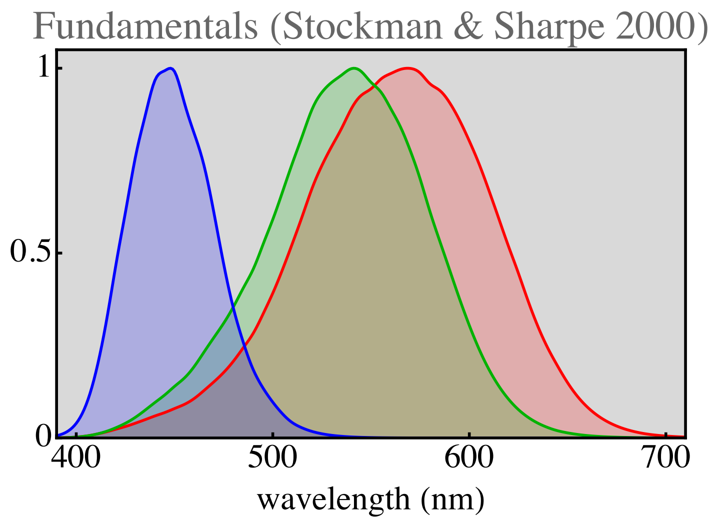
*An unfortunate term, for scattering is not (mirror–)reflection. This is often forgotten by the pop–science authors. Look at their illustrations and you will often see the mirror reflection law in action.

Physiology

THERE IS A LONG STORY to relate about eyes and the retina. I'll leave that to the pop-science books. If you don't know about these things, by all means read up on it (▷ p.85). It is interesting.



These are the cone fundamentals, best bets as to the cone receptor action spectra. Notice that the short wavelength sensitive curve stands far apart from the other two. The middle and long wavelength pigments overlap heavily and are hardly distinct. (Remember Christine Ladd–Franklin, page 8!)
Of course, any linear combination will be equally effective. There certainly are combinations that appear to "make more sense", or, more importantly, fit the phenomenology way better.

For this book all that is needed are the basic spectral sensitivities of the photopigments. In fact, even that is overkill. All that is really important are *arbitrary linear combinations* of these. I'll explain that later.

Then there are are brain centres that "process" what the retina sends into the skull by way of the optic nerve. What is important here is that the optic nerve carries only one–way traffic, eye to brain, not brain to eye.

However, we should not forget that the brain controls what is looked at, it controls the eye–movements, the fixations.

In essence, the eye fills the blackboard. This goes outside awareness. You can be blind even when your eyes and your "visual brain" are doing fine. Awareness* is due to the "questioning" of the blackboard by psychogenesis.†

Here almost anything might happen! A colour is not a simple transformation of the retinal excitation. No way!

It is a "hallucination" that fits the blackboard structure, or, at least, does not greatly contradict it. The "resistance" offered by the environment to the free generation of dreamlike imagery is what renders visual presentations "controlled hallucinations". Notice how this does right to the "objective structure of the world" as well as to the fact that this world (Kant's *Ding an sich*) is different in different observers sharing the same environment, say your dog and you.

Visual awareness is a user interface that screens the user from irrelevant complexity and cues the user (the interface elements are von Uexküll's *Merkmale*) to what is important for immediate, or future action.

Our immediate visual awareness today is essentially the user interface of a hunter–gatherer hominid. It is good to understand that, even better to enjoy it. It is concrete actuality as distinct from our "consciousness", which implies a self and reflective (involving concepts and language) thought. The latter is an abstraction that uses only remote references to visual awareness, it is essentially a social construct.

That is why artists spend years to "learn to see". Concrete actuality is so close to the chest that many (I guess most) modern people are blind to it. They simply live it, which is not necessarily a bad thing.

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*I use "awareness" in preference to "consciousness" because it is the simpler concept. Consciousness implies the notion of a "self", awareness proper is prior to that distinction.

†An account on the psychogenesis of awareness, the blackboard and so forth can be found on de Clootcrans Press and can be downloaded for free. See the inside of the back-cover.

Psychophysics

PSYCHOPHYSICS, is a term due to Gustav Theodor Fechner (1801–1887), a German physicist and experimental psychologist. It is essentially dry physiology. Psychophysics is important because it yields fully objective data, much like anatomy. Although indeed objective, the data may well prove irrelevant. That is because objective data is per definition entirely meaningless. The least trace of "meaning" would render it *subjective*.

Remember that meaning needs to be imposed. You cannot detect meaning in, or compute meaning from physical structure. Objectivity implies *observations without an observer*, like Alexius Meinong's "objects for which it is true that there are no such objects". This is fully at odds with phenomenology.

There are (at least) four kinds of psychophysical data that might interest us.

The first are absolute thresholds. It is perhaps interesting that the visual system is at the edge of physical possibilities. The minimum of radiant power is the absorption of a photon, and, indeed, the visual system registers such an event.

The second type are increment thresholds. The Weber-Fechner Law has that increments are detected when they exceed a certain "Weber-fraction". For instance, an increment ΔR in the radiant power density R is detected when $\Delta R/R > W_R$, where W_R is called the "Weber fraction". For radiant power the Weber fraction is about 0.2–1%.* Thus one–bit increments (1/255 of the total radiant power on the generic display) are often visible.

Discrimination thresholds are very similar. One compares radiant power R_1 with radiant power R_2. The difference with increment thresholds is mainly the type of presentation.

The most relevant psychophysical data for this book have to do with the discrimination of spectral compositions. This largely suffices for this book (▷ p.85). I'll discuss it further in the section on colorimetry.

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*That is to say, at daylight levels of radiance.

Phenomenology

WHEN WE SAY "COLOUR", what do we mean? The only sensible definition is what we see. However, we rarely see colour as such. We usually see objects of such and so a colour. Then the colour we see is the colour *of the object*. Sulphur powder and a banana peel may conceivably have the same spectral reflectance functions. Will they look the same then? Of course not! It will be this-or-that (same!) yellow. There are "colours" that really depend on the nature of the "this or that", in the case of the yellow, just think of gold. Is gold yellow? Probably yes. It that all it is? No, of course, not.

Monet's studio at the Hotel Baudy. Even diffuse light. Room so large scattering from the walls is not likely to be important.

One way to get around is to standardise the situation. This is somewhat of a cop out of course, but it is very useful from a formal, scientific perspective.

Think of it as a point of departure. Starting from the canonical situation you can then change it any way you want.

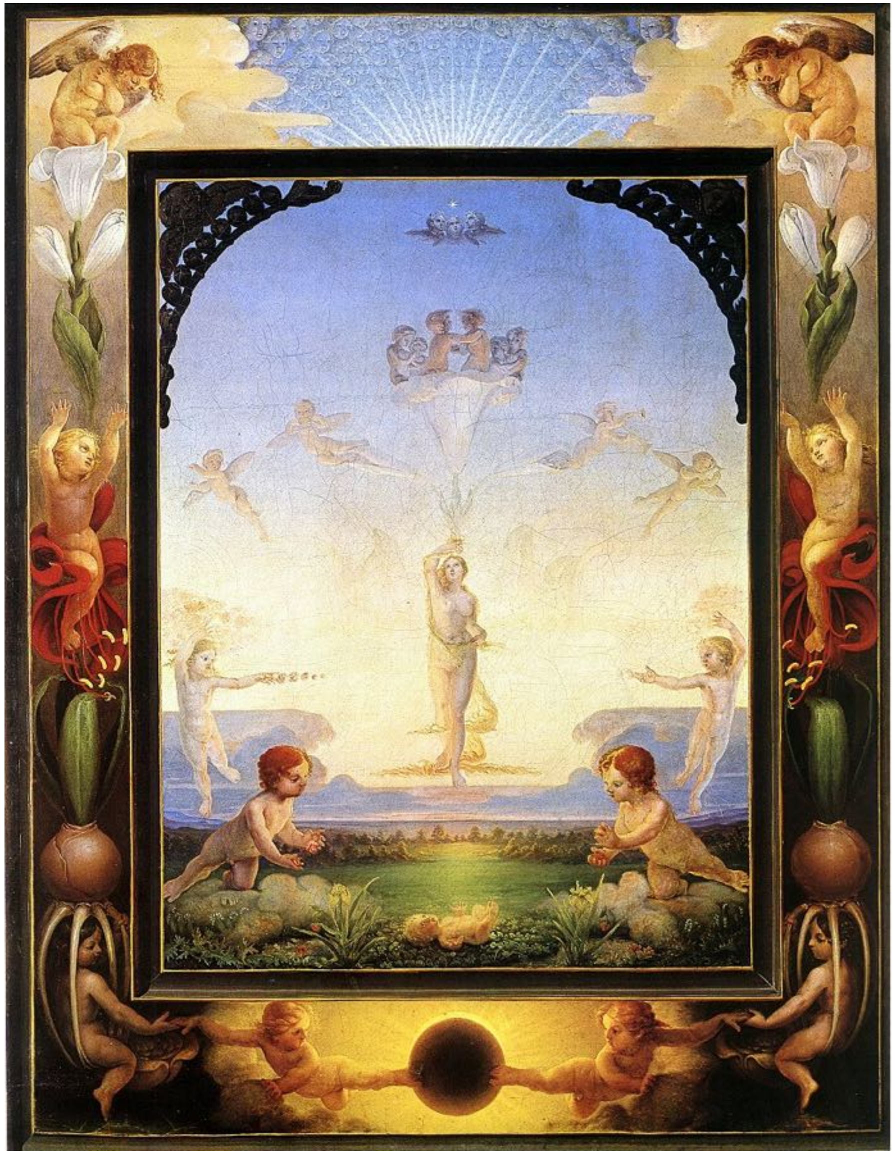
What is a good standard?

- the observer should be generic. This will usually be okay, but remember that there are various kinds of anomalies in vision. The worst case would be a blind person;
- the illumination should be standard. Something like the old-fashioned painter's studio with a large window on the north would be perfect;
- the objects to view should be Lambertian. This implies that the viewing geometry is irrelevant. Matte coloured papers or chalks (pastels say) are perfect.

As you see, the standard is not so bad that it might be considered irrelevant.

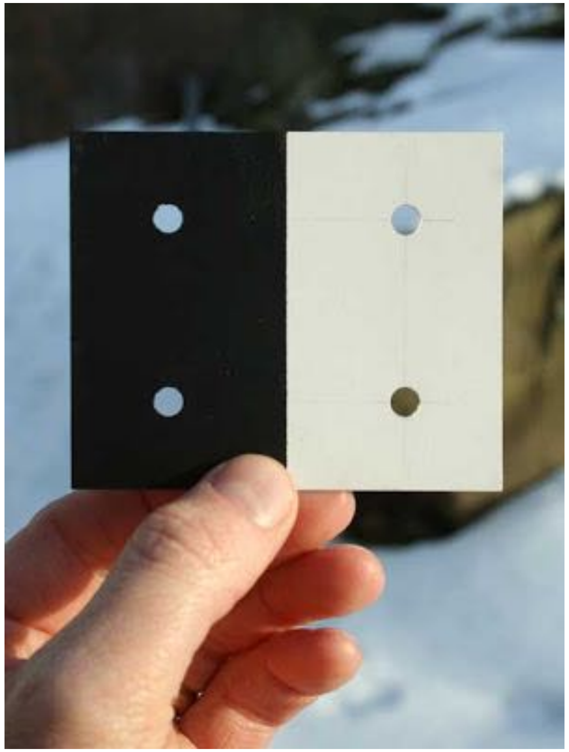
Pastel sticks offer something close to the Lambertian surfaces of the standard set up. They look "coloured" and they look "material", that is to say primordial objects.

Notice that all this applies to the colours of objects, which is largely what this book is about. It doesn't apply to the blue sky and so forth. The blue sky is not surface-like, you can't focus on it, it does not appear "material". Runge's painting "the small morning" (the large one remained unfinished) illustrates this very well.



Philip Otto Runge, the small Morning *(1808). Notice that Runge shows (among more) the difference between the colours of the earth and the colours of the air, that is to say, object colours and aperture colours.*

In the technical jargon such colours are know as "aperture colours". You can make any colour appear as an aperture colour by dematerialising it. Squinting often does the trick.



This is the way to see aperture colours. This is an easy DIY *project. Notice that this particular machine is a luxury item in that it allows you to compare the aperture colour to either the white or the black card. Holding the card at oblique angles to the light yields even more possibilities.*

A good technique is to carry a grey card with a smallish hole punched out in the middle. Hold the card at reading distance and look *at* (as opposed to *through*!) the hole. The hole will appear to be filled with an "aperture colour"—a very apt term.

You will find it enlightening to try this. For instance, you will notice that a brown earth colour (how material can you get?) will appear as a yellow or orange aperture colour.

The difference between the colours of surfaces and the colours of "lights" is categorical. The cause of the difference is that object colours are "related", whereas aperture colours are "unrelated". The hole in the card method serves to remove relations in the visual field, which it why it helps transform object colours into aperture colours.

What is "related"? Essentially the comparison with WHITE. All object colours are caused by the fact that the spectral reflectance factor is less than that of white in some spectral ranges. For instance the yellow of sulphur is caused by the fact that a short wavelength range is not scattered, but absorbed. All object colours are in that sense strictly less than white. Goethe called colours "shadow–like" and he was quite right in noticing their relation to white. All colours only exist in their relation to white.*

For the relation to be manifest it has to be visually present. If it is not, the situation is ambiguous and psychogenesis has to make a best guess as to what the current white might be. Painters and photographers intentionally introduce black and white anchors in their images. This establishes the luminous atmosphere by visually setting the scale. Intentional deviations from this practice yield various obvious "effects" that may be important to suggest certain moods, but make it hard to assess "object colours".

Without anchoring, there is neither "white", nor "black". A so called "black object" can be made to look "white" through some appropriate anchoring and vice versa.† A less extreme case is presented in the image presented on page 35.

Aperture colours cannot be related, there is nothing like an aperture colour "white", nor such a colour "black". Close your eyes, what do you see? Be honest: not *black* (the usual, easy answer). Black is not the absence of radiation, it is a relation. Aperture colours may appear un-coloured ("achromatic") alright, but to speak of "white light" is strictly wrong. I'll call such aperture colours "achromatic".

More on such important issues later in the book.

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*This is a technical remark, but the white can be implicitly present. For instance, in an RGB–image the maximum red, green and blue over all pixels yields a good estimate of white.

†See: Gilchrist, A., Kossifydis, C., Bonato, F., Agostini, T., Cataliotti, J., Li, X., Spehar, B., Annan, V., and Economou, E. (1999). *An anchoring theory of lightness perception*. Psychological Review 106(4): 795–834.

Colorimetry

COLORIMETRY is the science of measuring colour. It was pioneered by James Clerk Maxwell, developed by Hermann von Helmholtz, formalised by the mathematician Hermann Graßmann.

Colorimetry (▷ p.85) is based on just two important principles and techniques:

- radiant power spectra can be added to each other by superimposing them physically. The power spectra add on a per wavelength basis. This works because we deal with chaotic radiation fields, such as thermal radiation. It prevents tricky problems with "coherent" beams of radiation, due to interference and polarisation. The upshot is that the totality of radiant power spectra can be formalised as a linear space. The only proviso here is that actual spectral power density is necessarily non-negative, or, perhaps better, strictly positive.[*] Another thing to notice is that the space of radiant power spectra has no natural metric.[†]
- human observers can detect whether two beams of radiant power are *distinguishable*. This is the task of psychophysics. You simply set up a situation in which you switch beams on an observer and test whether the switch can be noticed. One finds that beams that cannot be distinguished are most likely to be spectrally different from each other. One calls this *metamerism*. Given any beam, there exist infinitely many metameric beams. Notice that the human observer only judges indiscriminability, but is never asked for an opinion on the quality of the radiation. Thus the judgment is fully *objective*.

The formal trick to construct colorimetry upon this, is to define the (colorimetric) colours as the equivalence classes of beams of radiant power under the discriminability criterion. Then a "colour" stands for an infinite set of mutually metameric beams, or spectral reflectance factors under standard illumination.

Notice that colorimetric colours and what you would normally be prepared to call "colours" (that is certain *qualia*) are ontologically distinct. One should *never* confuse them!

The science of colorimetry starts from here. All the rest is mere technique, for the larger part simply linear algebra.

Does this suggest an answer to the question "How many colours are there?". Well, only partially.

The partial answer is that the space of colorimetric colours turns out to be three-dimensional. This is the "trichromacy"' of human vision. Its physiological "explanation" are the three types of cone action spectra I mentioned before.

Historically trichromacy was clearly stated as a hypothesis by Thomas Young in his Bakerian lecture to the Royal Society of 1801:

> Now it is almost impossible to conceive each sensitive point of the retina to contain an infinite number of particles, each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited, for instance, to the three principal colours, red, yellow, and blue, of which the undulations are related in magnitude nearly as the numbers 8,7 and 6...

Maxwell and Helmholtz established trichromacy empirically. It became accepted by the mid nineteenth century.

So the answer from colorimetry is:

SOLUTION#1 [Colorimetry]

THERE ARE INFINITELY MANY COLOURS, THEY CAN BE MAPPED ONE-TO-ONE ON A THREE-DIMENSIONAL CONTINUUM

That is one way to measure a set, although it does not yield a number. The elements in a linear (sub–)space cannot be counted.

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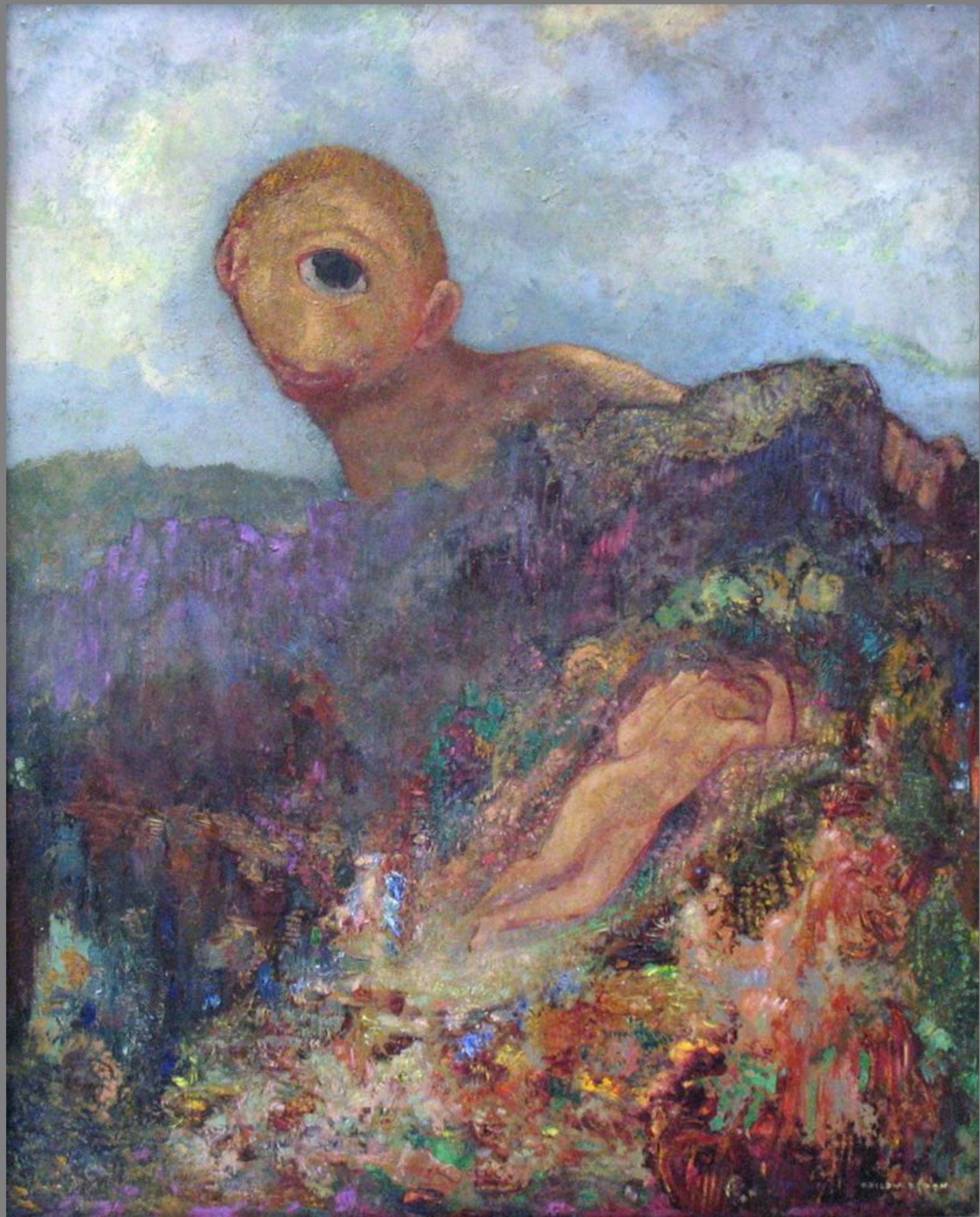
[*] For the physicist "non–negative" and "positive" are not essentially different, as they are for a mathematician.

[†] Technically speaking, it is a Hausdorff space, but not a Hilbert space. This has important consequences, leading to frequent errors in the literature.



Colorimetric Structures

In order to make progress on the question "How many colours are there?" I need to introduce a basic framework. It is different from what you will find in standard textbooks. The reason is the biological perspective, which is absent from colorimetry proper.

At the very start one defines the CIE XYZ–colorimetric coordinates of a radiant power spectrum. CIE* stands for Commission International d'Éclairage, it is the *keeper* of colorimetric conventions. In order to get into business, download their "Selected Colorimetric Tables", a free Excel file. It contains more than you need. Select the "CIE 1964 supplementary standard colorimetric observer". It is a table of three functions on wavelength basis, the "colour matching functions". The wavelengths run from 380nm to 780nm in steps of 5nm. Thus there are three 80–dimensional vectors.

You also want the "Relative spectral power distribution of CIE Standard Illuminant D65", which is an average daylight spectrum. It is listed from 300nm to 830nm in 5nm steps.

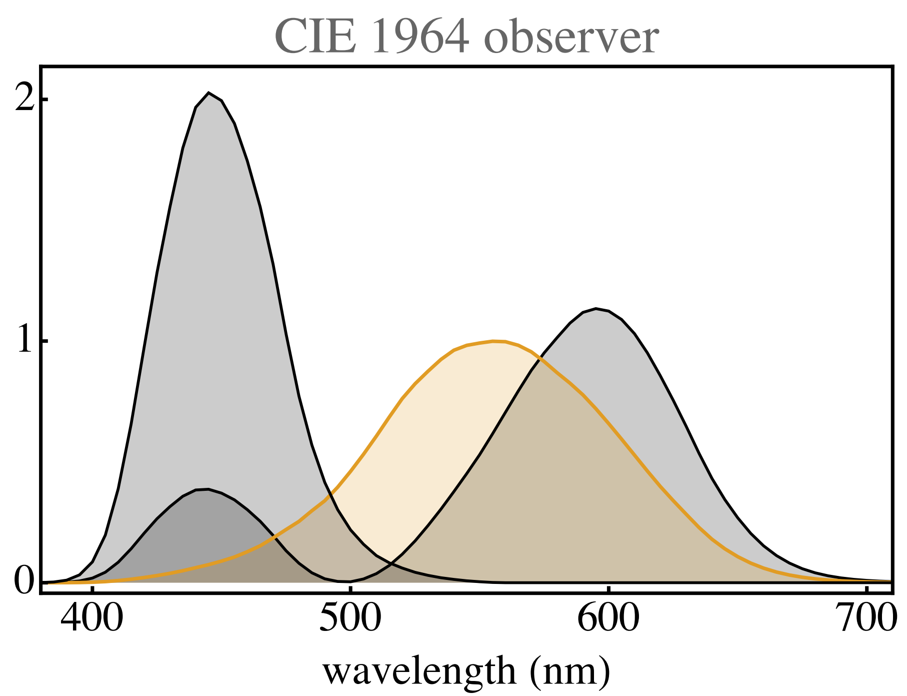
It is convenient to cut these tables down to (the same!) size. I suggest keeping 380–720nm in 5nm steps. Then you obtain 68–dimensional vectors.

Thus the space of radiant power spectra, which is really infinitely dimensional, is downsampled to a 68–dimensional linear space. This accuracy is really overkill, but even small computers handle such data with ease nowadays.

The colour matching functions define a linear functional from the space of radiant power spectra to a space of colorimetric colours. You simply put the vectors in a list and call it the "colour matching matrix $\mathbf{C}$" (say). The radiant power spectra are vectors, let's call the standard daylight $\mathbf{d65}$. Then $\mathbf{C}^{\mathsf{T}} \cdot \mathbf{d65}$ is a three dimensional vector, in this basis its components are conventionally called $\{X, Y, Z\}_{\mathrm{d65}}$. The "colorimetric coordinates of average daylight".

You get three numbers. So what? At this stage these are entirely meaningless. Colorimetric colours are equivalence classes of radiant power spectra. In this case the only spectrum is average daylight. So what to conclude? At best "average daylight looks like average daylight". Are you enlightened in any way? I guess not. The three numbers are meaningless as such. They only become meaningful when different from or equal to another set of three numbers.

*http://www.cie.co.at/publications/colorimetry



Graphs of the CIE colour matching functions. These are essentially arbitrary linear combinations of the cone action spectra from physiology. Colour matching functions (as different from the cone action spectra!) can be measured by psychophysical means, no physiology needed.

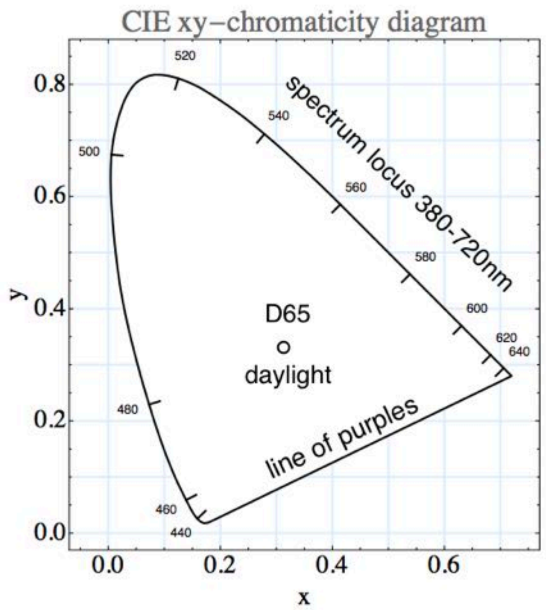
Here is a way to see something more interesting. Instead of computing $\mathbf{C}^{\mathsf{T}} \cdot \mathbf{d65}$, you compute a list $\mathbf{C}^{\mathsf{T}} \cdot \mathbf{d65}[\lambda]$, where $\mathbf{d65}[\lambda]$ is a "monochromatic" (Newtonian!) component, all of the 68 coefficients zero except one. This yields a curve in colour space.

Since most people intuit 2D but not 3D, the convention is to convert the $\{X, Y, Z\}$ coordinates to $\{x, y\}$, the coordinates of the "chromaticity", where

$\{x, y\} = \{X, Y, Z\}/(X + Y + Z)$. The chromaticity plane is a projective transformation of colour space.

Projective spaces are not linear spaces. This causes numerous erroneous conclusions in the literature, yet the "chromaticity diagram" has acquired the status of THE representation of colour space. One *has* to get used to it. It works, as long as you do not assign any metrical (comparing distances or angles) or affine (judging parallelity or bisection of linear stretches) properties.

All that makes sense are collinearities, which are relations between at least three points.



The CIE *xy-chromaticity diagram with the spectrum locus, the line of purples (**▷ p.88**) and the average daylight location.*

In the chromaticity diagram the daylight power spectrum appears like a curve, the spectrum locus, which is the same whatever the illuminant is. It depends only upon the colour matching functions. Average daylight appears as a point within the convex hull of the spectrum locus. The spectrum locus forms part of the boundary of its convex hull, the missing part is the "line of purples". It is good to get used to this figure, you will often meet it in the literature and—if you know how to interpret it—it is very useful and informative.

Here are a few things that the chromaticity diagram is often used for:

- if two beams have the same chromaticity, they differ only by the overall radiant power, their spectral distributions being equal. One often says that the beams have the same colours, although different intensity, whatever that may be construed to mean;
- if a chromaticity lies on the linear segment between the daylight chromaticity and a spectral chromaticity, the beam is said to have a "dominant wavelength" given by the wavelength of the spectral chromaticity;
- if the two beams have chromaticities that are collinear with the chromaticity of average daylight, they either have the same dominant wavelengths, or they are mutually "complementary".

This is useful in our setting, where average daylight indeed plays a key role. In colorimetry proper, the daylight spectrum is nothing special, thus notions of dominant wavelength or complementarity have only incidental meanings. One often uses a flat, or "equi-energy" spectrum, that is just as arbitrary, but somehow seems "less committed" to many—at least one gets rid of the merely incidental choice of daylight. It doesn't make much difference, which is why I cheerfully used average daylight here.

What you can see immediately is that not all wavelengths possess a complementary wavelength. So much is obvious because the spectrum locus is not a closed curve, as suggested by Newton. Goethe saw that there are no purples in the spectrum. Maxwell found empirically that the spectrum locus in the chromaticity diagram is not closed. Helmholtz settled this understanding once and for all, then we are already early in the second half of the nineteenth century. The simple graph of the chromaticity diagram represents at least a century of scientific effort by various persons. We should be thankful for that.

That such a graph is possible at all, is due to the fact that all humans are physiologically similar. An "average observer" does fine for the bulk of us. That should not blind us to variations and exceptions of various kinds. These exist! However, I will not dwell on that in this book.

The upshot is that you do not need to do intricate psychophysical experiments in order yourself to calibrate the colour vision of specific people. For almost any engineering purpose the CIE tables are really all that is needed.

This is nice, because it means that you can experiment with colour vision using just your computer after you downloaded the CIE–tables I mentioned. Don't download anything more, you won't need it and it might confuse you.*

I cannot urge you strong enough to indeed do this. Concepts that appear unclear to you will gain a meaning when you experiment with examples. Since all this takes is the simplest linear algebra, there is no reason to shrink away from it.

Anything I show in this text was computed using only these CIE–tables and a few spectral databases I scared up on the Internet (▷ p.87). So do yourself a favour and have fun! It will teach you more than a course or a book.

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*Judged by what one encounters in the literature, it confuses many people.

A biological focus

MAMMALS were originally smallish nocturnal animals. They came only out into the daylight after the demise of the dinos. This is still visible in the anatomy of the human retina, which is rod–dominated. The rod system is the original apparatus for night vision.

The cone system evolved as a system for use during daylight. No doubt, *homo erectus* developed as hunter–gatherer under average daylight irradiation, looking for animals, fruits, vegetable and roots, dealing with meat, fat and blood, occasionally with mud, earth and stones. It is only natural to model this as "object colours under standard daylight". Of course, both daylight and objects vary a lot, I'll consider that in due time. But the mode is definitely average daylight. The primordial OBJECT has a visible SURFACE, which is roughly LAMBERTIAN. One quality that helps to distinguish objects is their colour, due to their spectral reflection factors.

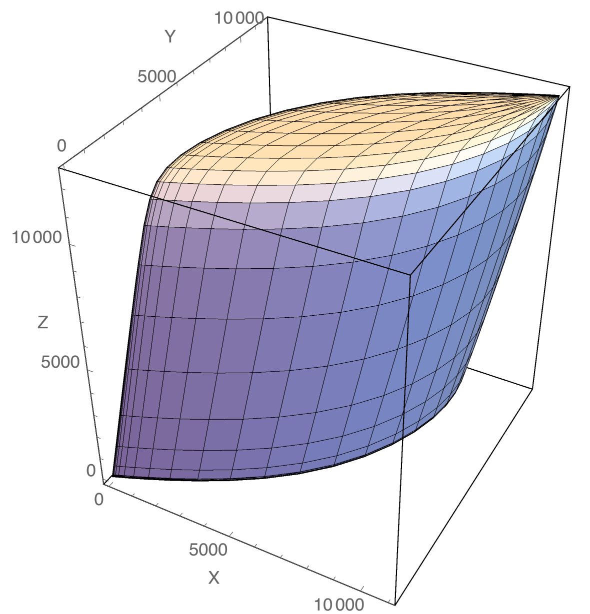
The reference is the white object. As the day progresses both the radiant power and the spectral composition deviate from the average daylight. The white object is perceived as unchanging, it calibrates vision over change. How that works is complicated, I'll come to that. For the moment, stick to the average daylight case and consider how to best arrange a colour estimator, "given" the cone action spectra.

The vertebrate brain is a great machine, it adapts to circumstances much faster than evolution can. That is why I consider the cone action spectra "given", they evolve much slower than the brain structures that make use of them.

The search for a "best" representation is the search for a "best" basis of colour space. What do we mean by "best"? Here is my best bet.

A basis $\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$ is "good" if it captures most of the action in its "crate", that is the linear hull $\mu_1\mathbf{f}_1 + \mu_2\mathbf{f}_2 + \mu_3\mathbf{f}_3$ with $\mu_{1,2,3} \in \mathbb{I}$ ($\mathbb{I}$ the unit interval $[0, 1]$). The "actions" here are the object colours. The only invariant features in linear spaces are ratios of volumes, so these I use.

The reflection factors are all in the range $[0, 1]$. This implies that the object colours are contained in an infinitely dimensional hypercube in the space of radiant spectra. In the space of colours they thus fill a convex, centrally symmetric volume (▷ p.85). Since $\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$–crate should catch as many object colours as possible, it should be an inscribed paralellopiped in this volume. Evidently, the white standard $\mathbf{w}$ should be $\mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3$ and since black is simply $0\mathbf{f}_1 + 0\mathbf{f}_2 + 0\mathbf{f}_3 = \mathbf{0}$, all object colours ideally should be in the crate.

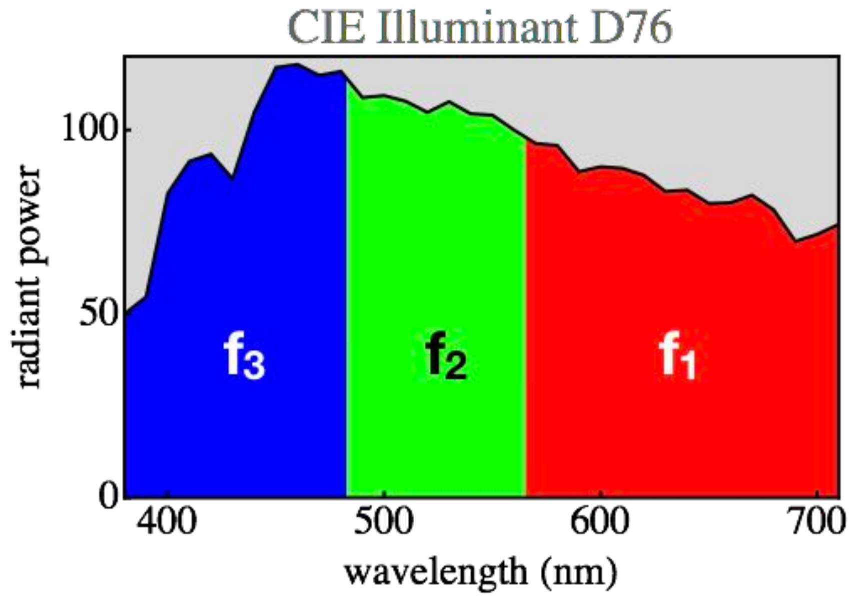


The colour solid in RGB–*space. This is the volume of all colours whose spectral power is limited by the spectral power of average daylight. As intuitively expected, it is a convex body of central symmetry. Notice that it is smooth, except for a crease and two conical singularities (at the black and white points). (See page 85 on the theory of the "Schrödinger color solid" and page 93 on how to compute it cheaply.*

So we look for a basis that maximises the crate volume under the constraint $\mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 = \mathbf{w}$ ($\mathbf{w}$ the colour of the white object). The solution is a simple one (method to find it suggested on page 93). The vectors $\mathbf{f}_i$ are "parts of white":

- the colour $\mathbf{f}_1$ reflects only the wavelength range of 565.43nm to the infrared spectrum limit, it is the "red part";
- the colour $\mathbf{f}_2$ reflects only the wavelength range from 482.65nm to 565.43nm, it is the "green part";
- the colour $\mathbf{f}_3$ reflects only the wavelength range from the ultraviolet spectrum limit to 482.65nm, it is the "blue part".

The names "red, green or blue part" make good phenomenological sense, because that is indeed how these colours *look*.



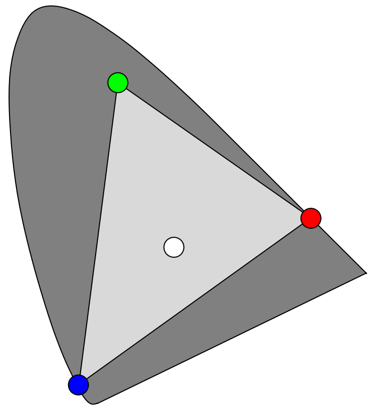
The Parts of White. In the remainder of the book I will use $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$ *for the parts* $\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$*, because that is what they look like to me. (A more intuitive wavelength scale is suggested on pade 93.)*

Notice that we have established a relation between two ontologically distinct realms! The colorimetric colours have nothing to do with qualia. The phenomenological colours (the qualia) have no relation to colorimetry.

But here we have a connection, call it a heuristic if you want, for there is no science of the matter. How can that be?

The reason is we are not doing colorimetry as such, we have introduced the average daylight spectrum as special. The "parts" depend upon this choice, of course. This is where biology enters the equation.

In terms of the parts of white object colours have coordinates $\{\mu_1, \mu_2, \mu_3\}$ in the unit cube. It is convenient to change the terminology and use $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$ for $\{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$, and $\{r, g, b\}$ for $\{\mu_1, \mu_2, \mu_3\}$. The unit cube will then be called the "RGB–cube". In the CIE xy-chromaticity diagram the RGB–cube appears as an RGB–triangle (the cube projected from the origin).

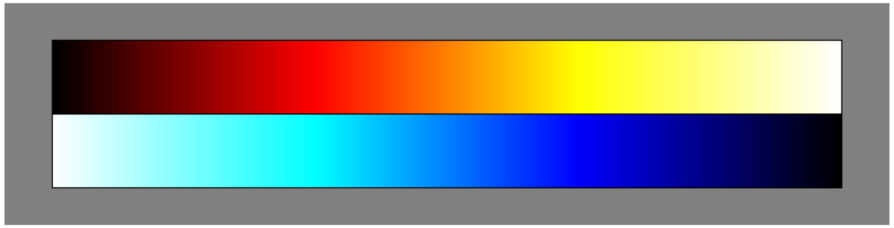


In the CIE *xy-chromaticity diagram the* RGB*–cube appears as an* RGB*–triangle. (It is a recommended exercise to try to imagine that geometrically!)*

Historically, Arthur Schopenhauer came very close to this notion of "parts of white" (▷ p.87). This was very early, *Über das Sehn und die Farben* was published 1815, a few years after Goethe's *Zur Farbenlehre* of 1810. Schopenhauer considered himself a student of Goethe, but attempted to "out–science" him. Both Goethe and Schopenhauer considered daylight (or sunlight) as the most basic entity in colour science. Object colours were seen as *Schattenhaft*, shadows of white. They considered Newton's spectral approach as seriously misguided.

Goethe once looked through a prism at a white wall and noticed that he did not see the Newtonian spectrum, at which moment he *knew* (on the gut level!) that Newton's account of colours was *seriously wrong*.

Looking more closely, Goethe noticed coloured bands at the edges of light or dark objects. He called them *Kantenfarben*, I'll translate that as "edge colours". Nowadays we have no scruples to describe the edge colours in spectral terms (Goethe would have shuddered at the thought). They are the cumulated spectral power as you start from the infrared spectrum limit (the "warm edge colour series"), or from the ultraviolet spectrum limit (the "cool edge colour series"). Both series start at black and end in white (▷ p.88). The warm edge colour series goes as black–red–yellow–white, where the cool edge colour series goes as black–blue–turquoise–white (turquoise is called "cyan" in the literature).



The Goethe Kantenfarben. *At top the warm edge colours, at bottom the cool edge colours. Edge colours that lie on a common vertical add to white. This implies that they are colorimetric complementaries with respect to average daylight. The edge colours are among the brightest colours you can find, very different from "monochromatic" colours, which—as object colours—are all black. A surface that scatters only a "single" wavelength necessarily scatters no radiant power at all! (This may need some reflection.)*

From a formal perspective the Newtonian spectrum and the edge colours (either series will do) contain exactly the same information. In practice Goethe was right in his critique that the spectrum is indeed a "spectre", a ghost, in that you cannot see it. Indeed, the better the spectrum (better means higher resolution) the less visible it becomes. Power spectra of infinite resolution do not exist in reality, as mathematical analysis shows. That Newton saw a clear spectrum was due to two factors, he used direct sunlight, the most intense source available at the time and his resolution was lousy. It is different with the edge colours, they can actually be produced and seen, you don't need a dark room, or anything special for that.

From a pragmatic perspective, any one of the two edge colour series is the most convenient tool in colorimetric calculations. Why is that? It is because spectral reflectance factors are between zero and one. This implies that the brightest colours of a given hue (or dominant wavelength) are of a very specific type: The reflectance factor is either zero or one, with no more than two transitions in the visual range. This was proven by Erwin Schrödinger in 1920, but it was already used by Wilmhelm Ostwald to construct his colour atlas, which dominated colour science in continental Europe for the first half of the twentieth century, and it was implicitly already used by Goethe and Schopenhauer a century earlier.

Flower colours from the warm edge colour series are very common. The basis is white, due to scattering by water droplets (=cells) in vacuum (= interstitial air). The colours derive from absorption by a filter of carotenoids in front of the white "canvas".

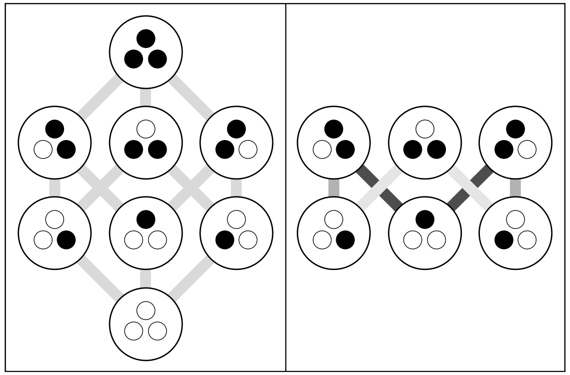
This means that the most vivid object colours are either edge colours, or sums or difference of edge colours. For instance the colours of the red poppy, the marigold, the dandelion and the white daisy are all warm edge colours, with reflectance spectra that are all or nothing with a single transition.

A single transition splits daylight into two parts, one part in the warm, the other in the cool edge colour series. Thus possible pairs are black–white, red–cyan and yellow– blue, but there is really a continuous series of such pairs. Schopenhauer noticed that red–turquoise and yellow– blue are special parts, because both members are exceptionally vivid. Other choices lead to parts of which one member is a "shade" (blackish variety of a vivid colour), the other a "tint" (whitish variety of some vivid colour). The vivid colours are somehow special.

Schopenhauer noticed this phenomenologically, only a century later Ostwald (▷ p.85) understood the reason: the most vivid colours have mutually complementary transition wavelengths. If you slightly perturb these wavelengths, keeping the dominant wavelength constant, you either attenuate the colour ("add black" to it), or you add white to it. In either case the colour becomes less vivid. This is a crucial insight.

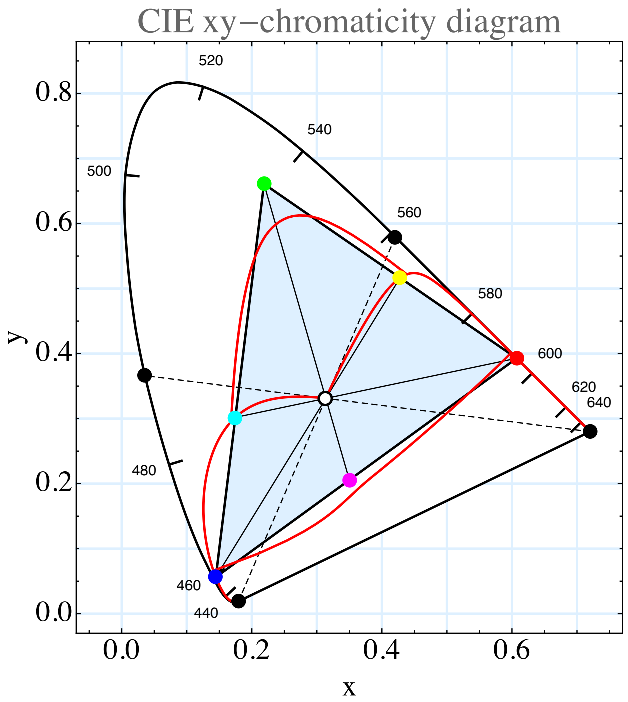
Thus the best red-turquoise occurs for a transition at the complementary wavelength of the ultraviolet spectrum limit (that is 560.97nm)), whereas the best yellow-blue occurs for a transition at the complementary wavelength of the infrared spectrum limit (that is 486.83nm). These loci are very close to the boundaries of the red, green and blue parts (482.65nm and 565.43nm).

This implies that the best turquoise is very close to the union of the blue and green parts, the best yellow is very close to the union of the red and green parts and the best purple is very close to the union of the blue and red parts Indeed the differences are small enough that you may safely ignore them in practice. It is easy enough to check this with explicit calculations.

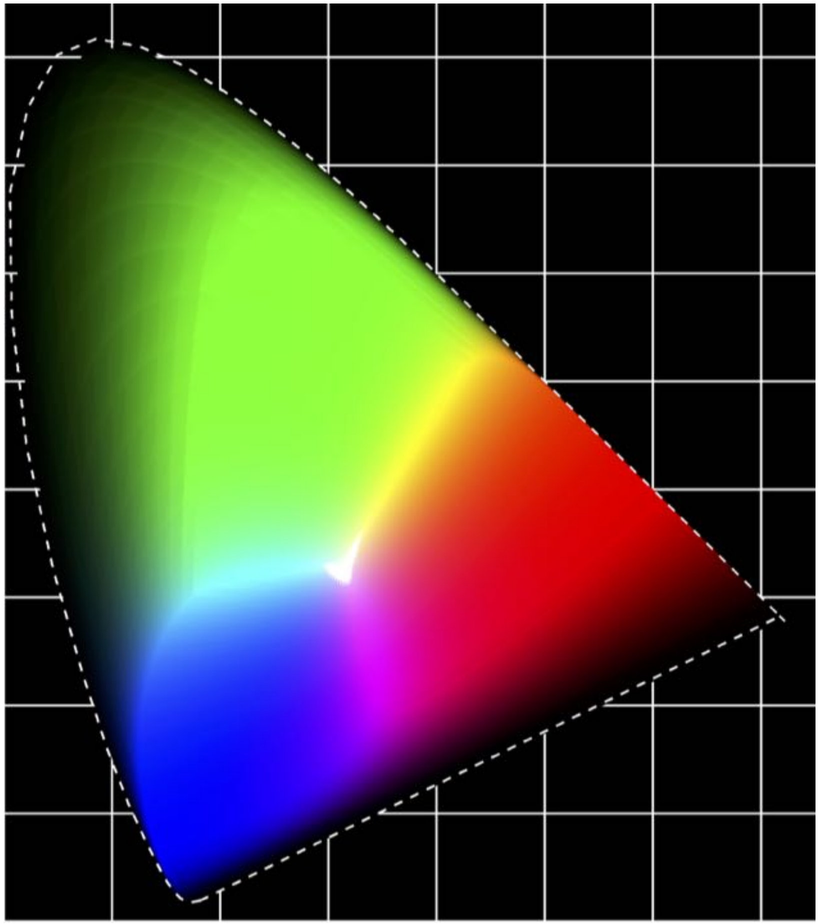


At left the Hasse diagram of the superset of the set of three elements. It has eight members, these include the full set and the empty set. Leave out the latter two and you obtain a periodic sequence. This is the "colour circle". This formal structure shows ***how the colour circle derives from the spectrum*** *and why purple is a natural and equal part of the colour circle, although it does not occur in the spectrum. I feel the idea is really Schopenhauer's, though Goethe was the first to point out that purple is extra-spectral and Helmholtz (perhaps preceded by Maxwell) cleared up the colorimetric relations.*

The daylight spectrum gives rise to a tight structure in the CIE xy–chromaticity diagram. This structure is not merely due to the colour of average daylight, it really depends upon its spectrum. A source with the same colour, but a different spectrum—there exist infinitely of those—would yield a different structure.



This figure summarises the basic structure due to average daylight. There are two mutually distinct, but practically coinciding structures. One is the RGB*–triangle in which I have also indicated the complementaries of the parts, turquoise, purple and yellow. The other structure is closely related to the Schopenhauer bisections, indicated by the complementaries of the spectrum limits. The red curves are the Schopenhauer/Ostwald most vivid colours. From either spectrum limit there runs a curve towards the white point, these are the Goethe edge colour families. These approximate the blue-turquoise and red-yellow semi-edges of the* RGB*-triangle. The additional red curves are the best colours as defined by Ostwald, one closely approximates the turquoise–green–yellow part of the* RGB*–triangle, the other the (complementary) blue–purple–red part. With some scrutiny you will see that the two intertwining structures do not fit perfectly but that the fit is good enough to ignore the differences. Why? Who knows? My guess is that it is the result of the evolution of* homo erectus *under the influence of average daylight much like the* CIE D65.

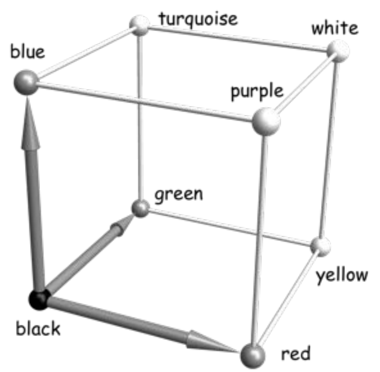


Here is the CIE *xy–chromaticity "painted" with the brightest object colours for any chromaticity. It looks rather different from what you will see in the mainstream literature, but this rendering is correct! The main difference is that all monochromatic, or spectral colours are black. The most vivid colours lie along the loci marked as the red curves in the previous picture. Notice the marked presence of the Goethe edge colour loci and the loop in the green due to the Ostwald "semichromes". Mainstream colorimetry misses such structures because it treats average daylight as nothing special, merely incidental. Notice also that the* RGB*–triangle captures "most of the action".*

This is obvious when you view objects illuminated with an artificial "daylight" source. A mercury lamp makes a good example. In that respect, one has to grant that Goethe was right to insist on the importance of daylight (or sunlight). With such a sourse the world looks very different. *Homo erectus* is—through evolution—tuned to the average daylight structure.

Here is a statement that will irritate the hard-core scientist greatly: I consider average daylight to be *part of our visual system* because both the physiology and the psychogenesis of visual awareness developed together in this ecological context. This is like cursing in church, but I mean it.

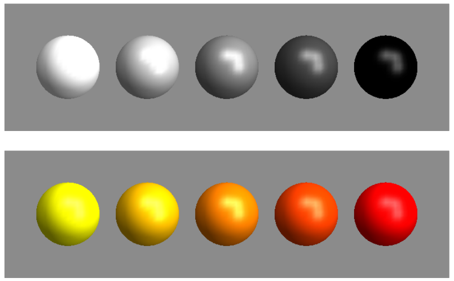
It is no accident that red, green and blue are phenomenologically important, they are indeed "natural parts" of daylight. The superset parts consisting of the eight colours red, green, blue, yellow, purple, turquoise, black and white form a natural skeleton of the realm of colours, they are the vertices of the RGB–cube.



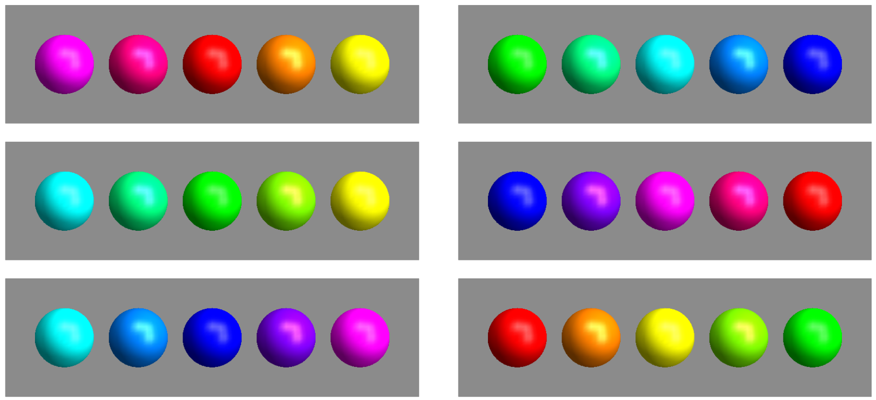
This is the RGB*–cube. Its vertices are the members of the* $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$*–superset. It is good exercise to find the grey–axis and the colour circle in this representation. It beats the* CIE *xy–diagram big time in yielding an intuitive lever on the structure of the realm of colours.*
The spectrum is the sequence KRYGCBK *(*K *for black), the colour circle is* YGCBMRY.

They also form an apt skeleton for the phenomenological structure. Why? It seems obvious to me. It is a matter of ecological optics. But "ecological optics" is often confused with physics. It isn't. Such a view takes physiology and psychogenesis as "given"—if these topics are considered worth attention at all. But that is simply wrong. They are tightly interconnected.

The "achromatic colours"' are the body–diagonal black–white, whereas the closed polygonal arc yellow–green–turquoise–blue–purple–red and yellow again exhausts the realm of hues. The intermediates are—colorimetrically speaking—linear interpolates. It would be surprising to find a new quale on a tract of linear interpolation. I can think of only two examples, these are grey and orange.



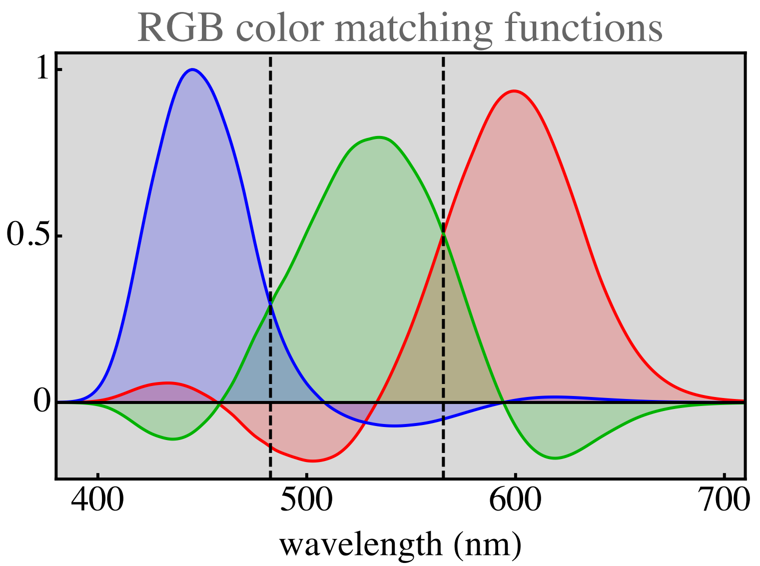
These scales look reasonable to me, I see obvious gradation. In the case of the greys, that depends upon the intermediaries, because a single grey looks qualitatively distinct from either white or black. How is that with the orange? Judge for yourself.



Study these sequences. Are the endpoints qualitatively different from each other? Is the mid-point qualitatively different from either end? People, even cultures, appear to differ on such issues.

Grey is the more interesting case. greys are "between black and white", that is to say, both “black” and “white” are evidently not “grey”, but all greys are qualitatively the same, only quantitatively different. I easily accept greys as “between black and white”, but I also immediately spot the difference between a grey and a black or white. This packs an important problem of phenomenology in a nutshell. Anyway, the greys are natural interpolants between black and white, I mean *phenomenologically*!

How is that with orange? Orange is “mid-way” between yellow and red. I can indeed see it that way, like grey between black and white. Yet orange is very close to being an individual quale. I feel that I need the word.

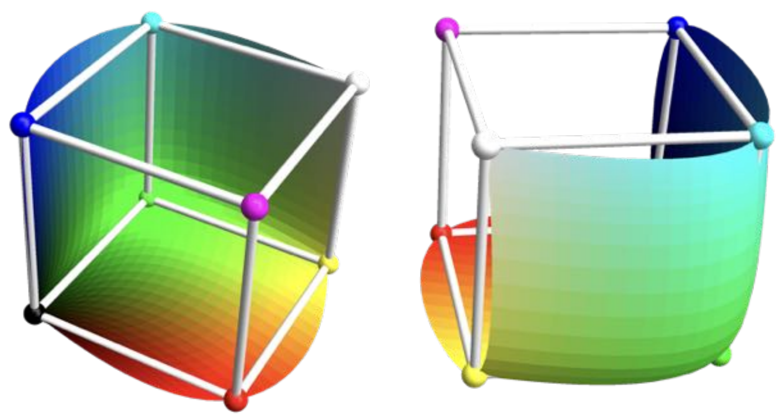


The colour matching functions that yield $\{r, g, b\}$ coordinates. The dashed lines indicate the boundaries between the red, green and blue parts. It is especially interesting to compare these to the cone action spectra (page 17). Both fit the psychophysical facts equally well. The action spectra fit the retinal physiology. The RGB*-colour matching functions fit the phenomenology. Of course, various brain functions are in between.*

For many people the position of turquoise, as an individual quale between blue and green, is questionable. To me it is not, but there is no right or wrong here. I do feel that yellow has a stronger position between red and green, than turquoise has between green and blue. Thus maybe I share some of the feeling with those who are not prepared to accept turquoise as an individual quale at all.

Something similar plays with the location of purple between blue and red. I really feel it is an individual quale, but many people are satisfied with “blue–reds”, just as many people naturally accept “blue–greens”.

There obviously is no science of the matter, there cannot be. All we (the old boys *homo sapiens* club, I mean) can do is “compare notes”.



In the RGB*–system the Schrödinger colour solid is very close to a “rounded cube”. Notice how the* RGB*–crate fits snugly inside it. (This interesting structure is mostly lost when you see the Schrödinger colour solid in the* CIE–XYZ *space (page 26).) Of course, this is only one half of the surface of the colour solid, but it is easy enough to imagine the other half, since the colour solid is a centrally symmetric body.*

As a final remark, I notice that the colour matching functions can also be set up to yield the $\{r, g, b\}$ values instead of the $\{X, Y, Z\}$ values. It is interesting to compare the resulting curves to the cone action spectra. They are much nicer in several ways. Yet they are fully equivalent to each other from a mere colorimetric perspective.

This also applies to the various other configurations. The Schrödinger colour solid appears as a "slightly rounded cube".* The chromaticity diagram can be taken as an equilateral triangle. Things look nice and symmetric, because the parts of white are treated on the same footing.

This choice is a matter of taste or a matter of convenience, up to you.

There are also some disadvantages. For instance, the fact that green is spectrally in between red and blue is not explicitly visible. This will have practical implications, as I will discuss later.

⋆ ⋆ ⋆

*I do not imply such a thing as Meinong's infamous "square circle" here, an object of which it is true that there is no such object. The Schrödinger colour solid simply "looks like" a slightly rounded cube to me. Try for yourself, you may feel different.

Metamerism explained

METAMERISM is often considered a spooky phenomenon. It is often thought that the explanation of metamism involves a esoteric kind of math. Not so. The problem with metamerism for the intuition is that it involves a space that does not appear "spatial" to most of us. Metamerism in "real" space is usually considered trivial, yet we're talking about essentially equivalent geometrical effects.

Three dimensional objects (a potato, a human face, and so forth) look different from different viewpoints. This happens because the field of view is two-dimensional, the dimension of distance to the eye being lost. Nobody is particularly surprised that a face appears differently when seen *en–face* as when seen *en–profil*. Two points may appear superimposed (at the same place) in the view, whereas they are really miles apart. A change of viewpoint will cure that.

For instance, in a portrait the tip of the nose may accidentally meet the contour of the cheek. This is generally considered an unfortunate coincidence, because possibly ambiguous. A professional photographer will notice that and change the viewpoint. A step to one side will bring the tip of the nose inside a cheek, a step towards the other side will let the tip of the nose stick out of the face. Thus the singular view can be resolved into either of two closeby generic views. One has a choice of qualitatively distinct views of the same face. That is the effect of metamerism in the perception of solid shape.

The same phenomenon occurs in colour. Colours do not live in a mere three-dimensional space, like shapes, but in an infinitely dimensional space. We sample a three-dimensional projection of an infinitely dimensional object. Small wonder that all views, obtained by changing the projection, look different from each other!

How does one change the projection? Simple enough, one changes the illuminant, say switch from sunlight at noon to sunlight at sunset. An object colour is formally specified by a spectral reflectance factor illuminated by standard daylight. This yields a "normal view" of the object colour. Changing the illuminant scales any of the infinitely spectral reflectances by the radiance of the illuminant. This has an effect that is similar to changing the viewing direction in the case of shapes (▷ p.89).

By adjusting the illuminant you get to see different projections of the object colour.

In the old days of B&W photography, the "colour space" was only one dimensional. The photographer would change the illuminant of the film by placing filters of coloured glass in front of the lens. The freckles in the face of a redhead were removed with a red filter or emphasised by a blue filter. The photographer thus used metamerism as a tool. Extreme cases occur in using infrared illumination, then speckles completely "disappear".

An extreme case of metamerism. The left side was photographed in "standard" illumination, the right side in infrared illumination. In the infrared the freckles are lost. Thus metamerism can have a huge influence on how things look.

In order to "judge the colour" of an object, you need to view it "in good light", which means diffuse noon daylight. Then the freckles are obvious. Infrared is "bad light".

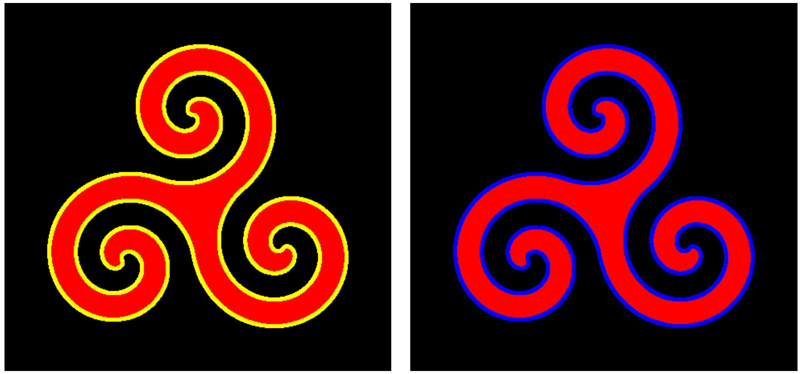
⋆ ⋆ ⋆

Phenomenological Intermezzo

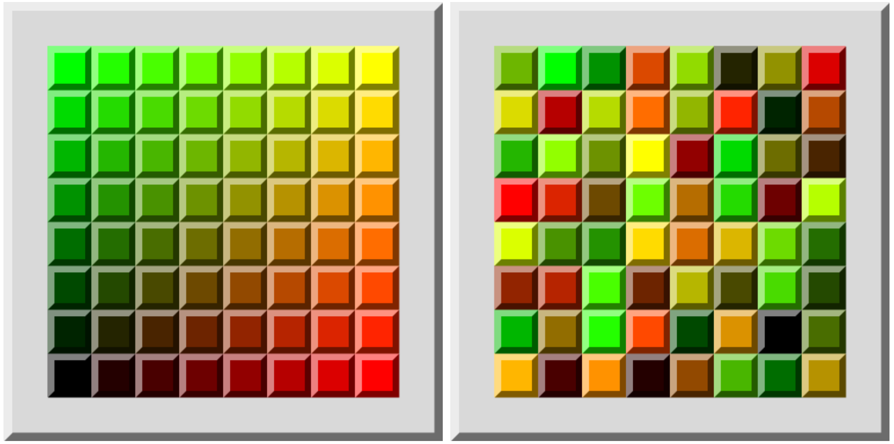
COLORIMETRY only indicates whether two beams of radiation can be discriminated. The RGB-cube colorimetry, based on average daylight, describes object colour in relation to a white chalk standard. Including the heuristic I suggested, it purports to say something about how things *look*. This may, or may not work out in practice. It depends.

It is likely to work out as expected if the setting is right. For instance, a coloured patch on an average grey background with both white and black references, will typically look as predicted. When the references are lacking, or the spatial distribution is manipulated, all kinds of things may happen, some expected, some surprising.

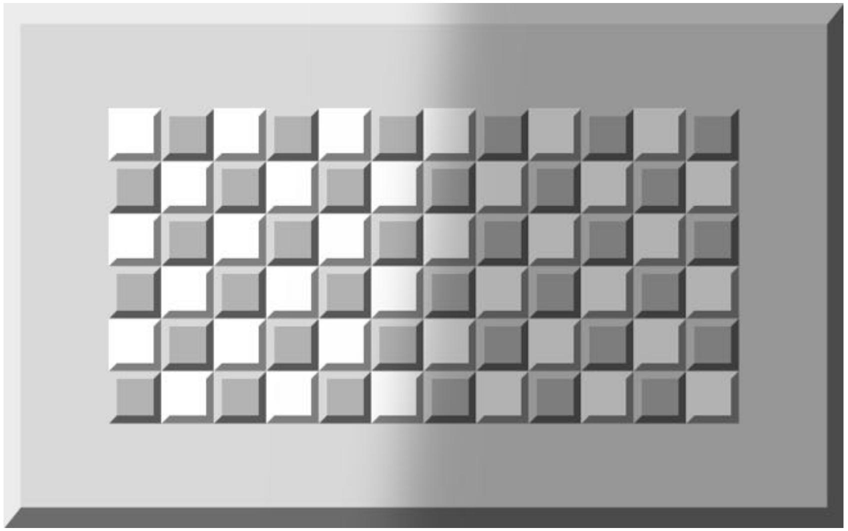
In this book I will typically assume the generic setting sketched above. But in order to show a few of the (naively perhaps) unexpected interactions I just show a few examples that are selected to make you think twice.



Here the reds of the Celtic triskelion are colorimetrically identical, I varied only their outlines. Notice that the reds look wildly different. They seem to partake of the colour of the boundary, the so called effect of "assimilation". Is assimilation an "explanation"? Not by a long shot! It is merely a name for a phenomenologically striking "effect". Numerous such effects have been described. As a painter you have to know about them.



Notice how the colours change as you shuffle the tiles. Yes, same set of tiles at both sides, only shuffled. Try to identify identical tiles at left and right, compare their "colours".



The light tiles on the "shadowed" side are the same grey as the dark tiles on the "illuminated" side! Would you believe it? Yet it is true. Honest!

These examples show that we really need the simplified setting I described earlier in order to judge colours. In even slightly more complicated settings the colorimetric coordinates have limited power in predicting what you will see. I showed only a few examples here, but I could easily write a large book about it with numerous examples that are just as striking.

Some people draw the conclusion that colorimetry is therefore useless. I do not agree with that. Of course, one should keep such effects in mind. The painter learns how to recognise and to use them.

From the biological perspective we relate segregation and recognition to awareness and reflective thought. Awareness is guts and intuition. Reflective thought is rational mind and language.

Most probably the earliest hominids did not have a spoken language. They must have been in the situation of the gorillas, chimps and so forth of today. Gorillas do not have a spoken language, but I wouldn't say they "have no language" as is usually said. I have no doubt that they communicate and they speak to themselves. For want of a term, let me pose that they speak the "Language of the Ancients", or LOA (I made up the term, no need to Google it).

I think some dialect of the LOA is used by dogs too. Maybe all vertebrates? But where to stop? Are there any sentient beings that are only machines? How about plants? It is only too easy to be caught in the anthropocentric delusion.

Most scientists would disagree with me on all counts. But I feel that I speak the LOA all the time, to myself and to others. Most people do too, but don't even notice. But without emphatic communication (the LOA) society would collapse. I feel pretty sure on that. Notice how easy it is to make yourself "understood" in countries where you don't speak the language, at least, with regards to the basic issues, eating, drinking and getting somewhere.

You even communicate okay with your dog. Do you and your dog have the same reflective or discursive thoughts then? No, the dog has none, you have too many. The reflective thoughts you have play little role in the actual communication with your dog–friend.

If you do not believe that you speak the LOA too, tune your TV to a movie and switch off the sound (and the subtitles if present). Forget your intellect, you won't need it. Listen with the guts. You need a movie that is directed at the heart, rather than the head. There are plenty of those, usually not high art (though some are!). Avoid movies aimed at hard-core intellectuals, those are "conceptual–art" (nowadays the official "high art"), which depends on conceptual thinking and linguistics. What you want is art–art. Now your ears are no good (no spoken–or written–language), so you're forced to rely on the LOA. If the director was any good, you will have no problem to follow what goes on, not only the action, but also the intentions and reactions of the protagonists will be immediately clear to you. You may miss the plot—if intricate—and everything that takes place on the longer time scale. The LOA is only concerned with the *now*, that is the presence of the present, including the presence of the past and the presence of the future. In the arts (not only movies) the LOA is the *lingua franca*.

The LOA is based on habit and type,* not concepts. It is fully pre–conceptual, with the gorillas (or probably hominoids) perhaps proto–conceptual.

The LOA deals only with the here and now, although the now is a construction that involves retentions and protentions. If there are "rounded off" objects, these are happenings (think of a handshake).† Happenings are types that are of the nature of actions, though they contain what might be called "deeds", "objects", "perceptions" and "qualities". Objects do not exist in LOA, except as aspects of types. Moreover, objects are not defined through property lists. There is no way to think "a tomato is red" in LOA. A hand-sized–round–firm–tasty–red may be part of a type. Such clusters bridge modalities and even ontologies. An itch-scratch is a feeling–perception–action cluster.

Say there is a faint lemon smell in the air. It may be unnoticed, except for a taste (Kant's phantasmic self-stimulation) and an anticipation of hand-sized–round–yellow, that is itself unnoticed, but may pop up in awareness at any moment because psychogenesis has entered an anticipatory mode.

Here is an example from praxis that may clarify the issues involved. Suppose you crush the skull of some animal, physically looking much like you,

*On habit and type read David Hume, Immanuel Kant, Jakob von Uexküll and Edmund Husserl.

†If I say "think of ..." you might use that hint trying to obtain a feeling for what the Language of the Ancients might be. I cannot "translate" that into a language of speech.

because it is obviously an intruder, possibly after your females,[*] and you see a stream of green goo oozing out.

This may well elicit a feeling of alienation. Not because you reason in reflective thought: "*crushing a skull results in a stream of blood, which is red, not green. This is an odd case, an alien being, or a wizard perhaps*". No, you don't have "red" and "green" as concepts. You merely have habits and types. "Bleeding" is a type that induces an expectation of red (not the word, an awareness), an instance of Kant's phantasmic self–stimulation.

Now your system[†] starts to build a new type (like in von Uexküll"s famous example of the toad eating worms). A next event of this type will strengthen it, a third time definitely clinch it. You have acquired a new type, a certain type of alien that, when you crush their skull, bleed greenly. Serves them well!

Next time you crush an alien's head you immediately "recognise" the particular blood colour. You have gained a habit that automatically puts you on the right foot, so to speak. Habits imply automatic anticipations.

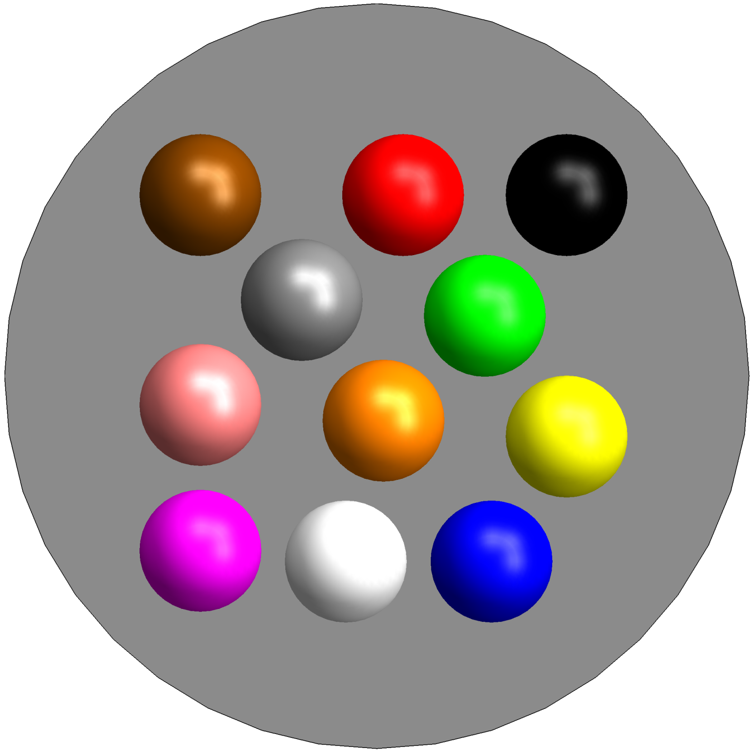
In the LOA there are no words for colours, there are no words at all. We all speak this language, just as well as the gorillas and the chimps. The language communicates awareness, to yourself and others, not concepts. It remains on the gut level. It is the level of concrete actuality. For the visual artist it is reality. For the scientist it is a primitive level below the conceptual level that has nothing to do with reality.

Concepts are useful, but not specific. Conceptual (that is linguistic) thought helps in premeditated action. Awareness is specific and immediate. It is decisive in a bar fight, then the guts (immediate action), not the brain (premeditated action), wins.

In my view one should speak both the Language of the Ancients and whatever language you speak at home or at the laboratory. In the latter case colour words like "red" or "green" are useful. In the former case they are irrelevant. In the visual arts proper only the LOA counts. This is quite different in the conceptual art of today, which seems often just a bit too conceptual.

[*] We're talking early times, so this would only be the natural thing to do.

[†] What I really mean when I say "system" is a body–mind–environment complex. When I say "your" system, I do not imply that you have something we moderns often call "self".



These are the eleven basic colour categories suggested by Berlin & Kay[a]*. To have a mental model of maybe a dozen or less colours, in no particular sorting order, may actually serve one well. They certainly look very distinct to me, they make good labels. I noticed that many (colour–)naive people do not know the sequence of colours in the rainbow, so maybe this isn't that rare either.*

[a] Berlin, B., Kay, P., 1969. Basic Color Terms: Their Universality and Evolution. University of California Press, Berkeley; see also Robert M. Boynton (1989). *Eleven Colors That Are Almost Never Confused.* In SPIE Proc. #1077 — Human Vision, Visual Processing, and Digital Display, pp. 322–332, Bellingham, WA, 1989. SPIE.

Thus I feel the numerous investigations of the number of colour terms in various languages are very interesting indeed,[‡] but have nothing to do with our

[‡] Starting with Brent Berlin and Paul Kay's "Basic Colour Terms: Their Universality and

question of “How many colours are there?”. Concepts simply have nothing to do with types, which are nothing more but mere “sedimented experience” (Husserl).

Anyway, one answer to the question “How many colours are there?” is

SOLUTION #2 [Cultural]

DEPENDENT ON CULTURE THERE ARE 2–11 COLOURS

Of course, “colours” stand for “basic colour terms” here.

As an add-on, let me say that I do in no way suggest that language (as we know and use it) is irrelevant. However, I would prefer to use “culture” instead of “language”. Of course, culture and language are almost identical, but conceptually, it is culture that is the driving force of a kind of essentially Lamarckian evolution. It is not the anatomy/physiology that evolves here, it is the mind, the primordial urges. I would not be shocked if that could be shown to change awareness. From such a perspective the Neanderthaler would indeed have enjoyed a quite different awareness from ours. I am hardly prepared to work this out in terms of the experience of colours though. Too complicated for my bird brain! Maybe you can do better.

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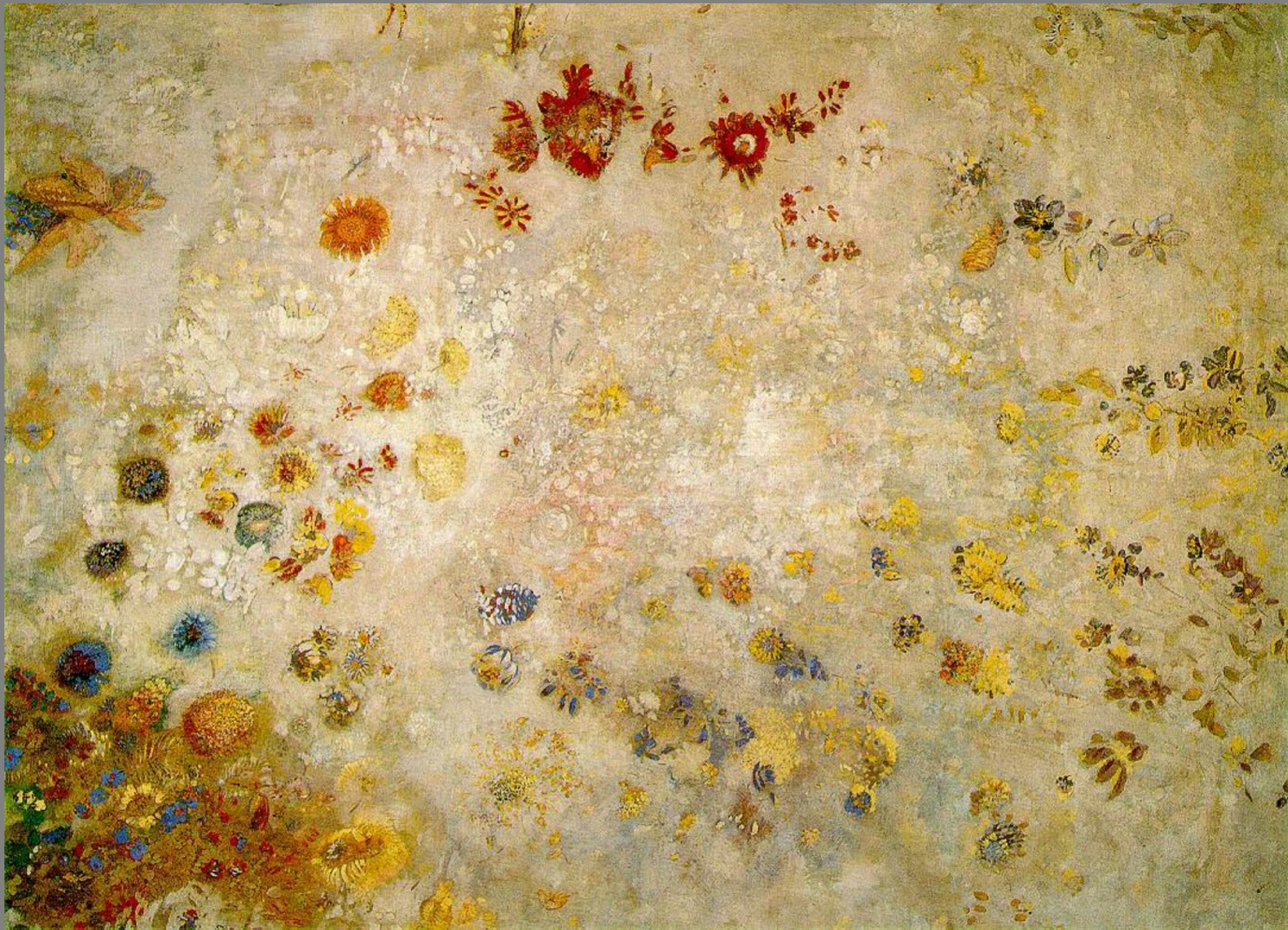
Evolution” of 1969.



Colorimetry in the wild

COLORIMETRY is nothing more than the description of the discriminability of radiant spectra. Just simple linear algebra and a tabular representation of the "colour matching functions" are all you need. The colour matching functions were established psychophysically, a major chore. Then the CIE did a lot of averaging and smoothing. The fact that most instances of *homo sapiens* are mutually equivalent makes that this work has lasting value. We don't have to do tedious psychophysics anymore, we simply download the tables. Instant psychophysics, so to speak.

Thank you CIE!

With respect to the "daylight" needed to emulate the effects of evolution we have to depend on measurements of ecological radiometry. The CIE D65 average daylight table is probably as good as any.

Again, thank you CIE!

This allows one to set up the fundamental colorimetric structures as they pertain to *Homo erectus*. That involves average daylight and its consequences. It can be summed up in the structure of the Parts of White and the RGB–cube. You'll never need more.

So far, so good. After that, you're all alone, out in the cold. The standard tables are nice, but you will have to deal with all kinds of non-obvious problems. In this chapter I collect a number of such issues that occur over and over again.

They are of a variety of kinds, methods, principles, facts, data. Some of the methods are just of practical interest. Many of the facts only apply to "natural" things. Science considers such facts only incidental and of no generic interest. That is surely correct.

However, "correct" does not imply *relevant*. Relevant is what *drives evolution*. It is that kind of fact I will stress.

I find it hard to put some natural order on the topics. So this chapter is a kind of potpourri. But don't mistake: it is important. Especially if you want to develop things for yourself.

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The "claim" of the RGB–cube

THE $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$ CRATE claims the largest volume of any inscribed crate of the colour solid. The colour solid is a smooth convex body, it has almost anywhere a convex boundary surface. The RGB–cube is a polyhedron, with flat surfaces. Thus the crate will necessarily have a smaller volume than the colour solid.

For instance, by way of comparison, the inscribed cube of the unit sphere claims only $(\frac{2}{\sqrt{3}})^3/(\frac{4\pi}{3}) = \frac{2}{\pi\sqrt{3}} = 0.3675\ldots$ of the volume of the sphere. Compare that with a numerical calculation, which reveals that the $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$–crate claims 0.63…of the volume of the colour solid. This implies that the colour solid is much closer to a cube than to a sphere. It is far less "inflated".

Okay, so what? What does it mean in practice?

At the boundary of the colour solid the spectra are all or none, with at most two transitions. Such spectra are very rare indeed. Real spectra do have smooth transitions and are nowhere truly zero or one. Thus colours near the boundary are very rare. (The main exceptions are colours close to black.) The upshot is that the volume ratio is not very informative.

If you test a database of "natural" spectra, you find that only a few percent of them lead to colours outside the crate. Moreover, those that do are "only barely" outside. In practice, we compute the $\{r, g, b\}$ coordinates and clip them to zero or one. Only a few percent needs to be clipped and if so doesn't differ by more than 0.05 from zero or one.

This is highly important in practice, as in the evolution of *Homo erectus*, just as it is irrelevant to colorimetry proper. In this book I take the side of praxis, that is to say, biology.

For all practical purposes the $\{\mathbf{R}, \mathbf{G}, \mathbf{B}\}$–crate exhausts the set of object colours. Something one should keep in mind.

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Sets of radiant power spectra

ONE OFTEN HAS TO DEAL with sets of radiant power spectra. Think of the "daylight spectrum" throughout the day, or the set of thermal spectra. One often wants to use such a set as a basis from which to generate arbitrary instances.

The standard way to do so is to use principal components analysis (PCA, ▷ p.86). This is—in general—wrong. Why? Because PCA is a linear method, which implies that the generated instances may well have *negative* spectral power over certain wavelength ranges. Of course, that cannot be! Hardly surprising, people know that. The standard solution is to clip the negative parts. Is that a solution? Not at all.

It means that the method is wrong. Correct *methods* possibly yield inaccurate answers, but should never yield impossible answers, that should be impossible if the method is correct to begin with.

In this case the solution is simple enough. The non-informative prior distribution for a non-negative quantity is hyperbolic, that is uniform on a logarithmic scale. This at once suggests how to handle the problem, that is by homomorphic filtering (▷ p.84). We transform to the logarithmic domain, do our PCA and transform back. This solves the problem in one go and moreover treats the data in a dimensionless manner.

A problem that occurs in doing this is that the radiant power may vanish in certain wavelengths ranges. That would throw the logarithm to minus infinity. One way to handle that is to set a limit to the minimum. But to do that, one needs to understand the physics and the methods used to acquire the data.*

In studying the distribution of spectral radiant power at some spectral location, it is preferable to do it in the physical domain (▷ p.84). In many cases one notices that histograms of the distribution in the physical domain are approximately normal.

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*Another way to deal with such problems is to use more robust statistics. For instance, the median does not have problems with the infinities, whereas the mean chokes on it.

Sets of spectral reflectance factors

ONE OFTEN HAS DATABASES of spectral reflectances and wants to setup methods to generate novel instances with the same statistical structure. The standard solution is PCA. There are numerous examples of that in the literature. This obviously yields many results that have reflectance factors that are negative or larger than one, both impossible. Clipping is the standard cure. Of course, that is no solution at all. This problem is more intricate than for the case of radiant source spectra.

Doing PCA on reflectance factors makes little sense because the domain has a very non-linear relation to the underlying physical parameters. For the setting I use in the book the relevant physical parameter is the ratio of the absorption cross-section to the scattering cross-section K/S.

The relation to the reflectance factor is given by the Kubelka-Munk theory (▷ p.83), it is known as the "spectral signature". One has

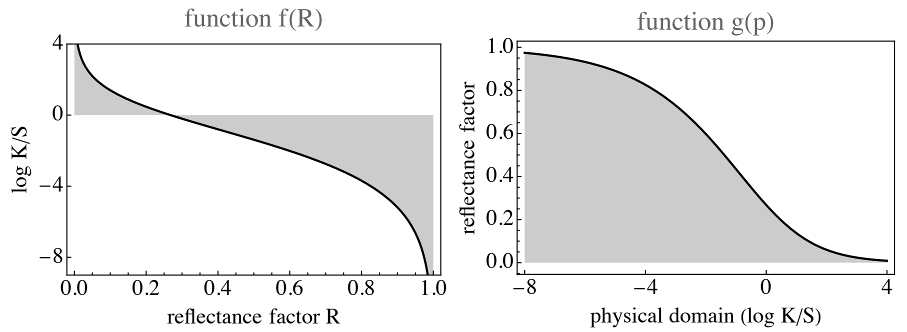
$$\frac{K}{S} = \frac{(1-R_\infty)^2}{2R_\infty},$$

where R_∞ is the reflectance factor of a thick (opaque) layer. The parameter K/S is non-negative, thus one needs to consider it on a logarithmic scale in order to enable linear methods (such as PCA) to make sense.

Hence I consider the two functions

$$\begin{aligned} f(R) &= \log\left(\frac{(1-R)^2}{2R}\right), \\ \text{and} & \\ g(p) &= 1 + \mathrm{e}^p - \sqrt{2\mathrm{e}^p + \mathrm{e}^{2p}}, \end{aligned}$$

which are each other's mutual inverses.



The Log–Kubelka–Munk–function f and its inverse g.

The parameter $p = \log(K/S)$ lives in the "physical domain" whereas the (spectral) reflectance factor R_∞ is immediately observed. Notice that $f : \mathbb{I} \mapsto \mathbb{R}$ (f maps the unit interval to the real numbers $(-\infty, +\infty)$), whereas $g : \mathbb{R} \mapsto \mathbb{I}$ (g maps the real numbers $(-\infty, +\infty)$ to the unit interval).

In practice one needs to take care of reflectance factors identically equal to zero or one, which may be due to limited experimental accuracy, but usually result from running beyond the (instrumental) limits. In the latter (common) case the zero and one values act essentially as garbage bins and the corresponding data should be deleted. How to handle this depends on your understanding of the measurement process.

In studying the distribution of values at some spectral location, it is preferable to do so in the physical domain. In many cases one notices that histograms of the reflectance factor have complex shapes, are usually often multimodal, whereas the distribution in the physical domain is approximately normal.

Multimodality in the physical domain is likely to be meaningful, whereas bimodality in the reflectance domain most likely is not.

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Different sources and phenomenology

ALTHOUGH *Homo erectus* EVOLVED under natural daylight, its vision had to be able to cope with variations over the day, due to meteorological circumstances, as influenced by treetops filtering sunlight, and so forth.

Phenomenologically, one observes that white chalk looks like white chalk in almost any natural setting. One way to capture this is to simply normalise the colorimetric $\{r, g, b\}$–coordinates such that the white chalk always maps on $\{1, 1, 1\}$. It is like applying different gain settings on the colour channels.

This is essentially what digital cameras do when you set the white balance to "automatic"—as most people do, perhaps unknowingly because it is a factory preset. I guess evolution converged on much the same method.

This is similar to a suggestion by Johannes von Kries (1853–1928; Helmholtz's best known pupil) in 1905.[*] However, Von Kries applied the so called "Coefficients Law" to the retinal receptor activity (indeed, the only logical choice for a physiologist), where I apply it to the red, green and blue parts here, which makes more sense to the phenomenologist.

How efficient this is depends upon how you count. It is not hard to think of cases where the method would be less than useful. However, as I will show later in the book, it would almost perfectly serve *Homo erectus* under virtually any natural setting.

Exceptions are "artificial sources". For us that would imply, neon lights, low pressure sodium discharges, mercury light and so forth. For *Homo erectus* it would imply fire light, that is to say, thermal radiation of very low temperature, say $1000°\mathrm{K}$.

For such a low temperature thermal source only the red channel is active, the blue channel is not activated at all, whereas the green channel may show some activity, although more than an order of magnitude less than that in the red channel. Thus vision becomes almost "monochromatic".

In such cases the normalisation will yield very bad results, for instance one would have to increase the gain factor by so many orders of magnitude in the

[*] Johannes von Kries (1905). *Die Gesichtsempfindungen*. Handbuch der Physiologie der Menschen, Hrsg. von W. Nagel, dritter Band, Physiologie der Sinne.

blue channel, that one would obtain merely noise.

In these cases an alternative strategy might be expected to work much better. One sets the gain factor to the same value in all channels, such that the largest $\{r, g, b\}$–coordinate for the white chalk is normalised to 1.

I'm pretty sure that's what evolution put us up with. We should probably be happy with that. Indeed "probably" because evolution tends to converge on what serves us best *on the average*. Where it didn't work, our ancestors died for that—to our eventual advantage (speaking "on the average"). Individual life is irrelevant, dying because of a failure is an asset to the evolutionary process. Thank you ancestors!

As I will show later in the book, this works remarkably well in accounting for the phenomenology.

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RGB and electronic displays

MOST ELECTRONIC DISPLAYS use mixtures of three luminous sources,* which are blended per pixel according to the local $\{r, g, b\}$–coordinate values.

It only makes sense that the colours of the three sources have evolved so as to closely mimic those of the parts of white for human vision under natural daylight. Indeed, the industry has largely achieved this, although mainly by trial and error, driven by customer appreciation of quality and price. The qualitative appreciation by the bulk of customers drives the technology towards an implementation of the parts of white.

However, just using $\{r, g, b\}$–coordinates to drive the displays is likely to lead to disappointment. This is due to the fact that the software typically applies a nonlinear transformation, the so-called "gamma-correction" (▷ p.82).† When displaying RGB–data one needs to take this into account.

Consider the "average grey", which should be a grey that looks (I'm using eye–measure here!) mid-way between black and white. One might expect the $\{r, g, b\}$–coordinates to be something like $\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$. However, such a grey looks far too light. Remember that an "average grey card" as used by photographers has a reflectance factor of about 20%.

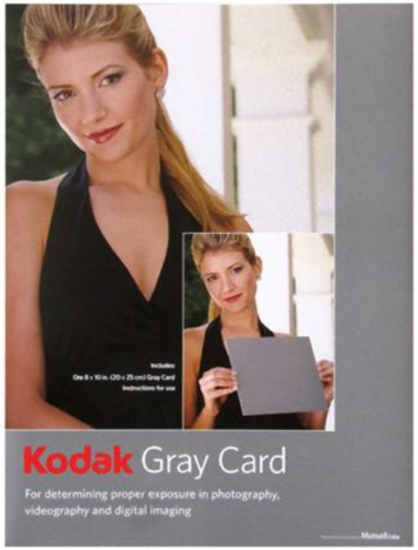
If you drive the display with a (fairly typical, but there are numerous variations) RGBCOLOR[0.5,0.5,0.5],‡ one obtains a satisfactory "average grey". However, the $\{r, g, b\}$–coordinates will be $\{0.218, 0.218, 0.218\}$, because the gamma correction applies a power of 2.2 (the conventional gamma) and $0.5^{2.2} = 0.217636\ldots$, so the command effectively specifies the grey card reflectance. In order to show $\{r, g, b\} = \{0.5, 0.5, 0.5\}$ one should use the

*I refrain from listing the various technical solutions that make this possible. In practice the implementation may make a difference. I assume that the reader who is aware of such facts will know how to deal with the various choices.

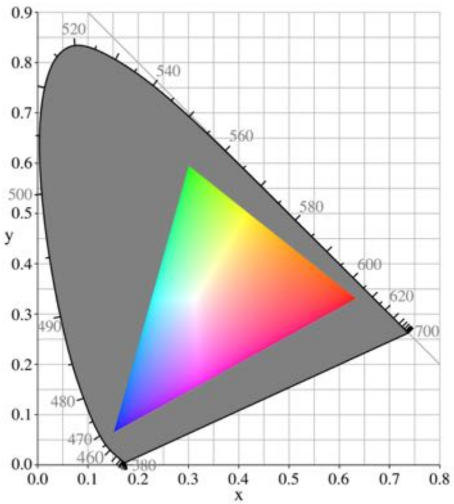
†I will not go into the history and various other considerations concerning the gamma-correction here, I simply take it as "given" and suggest how to deal with it.

‡This is the command I would use in Mathematica. In Processing I would use COLOR(127), and so forth.

command RGBCOLOR[0.73,0.73,0.73], because $0.72674\ldots^{2.2} = 0.5$. This would yield a fairly light grey, certainly not an “average grey”.



This is the famous Kodak grey card, used by numerous photographers. Apparently, its reflectance factor is supposed to be 18%, implying a gamma of 2.47, somewhat higher than the current standard of 2.2.



This is the gamut of the S–RGB standard for electronic displays. The RGB–triangle is similar to the optimum I described earlier.

Most electronic displays used three bytes* to specify a colour. Thus one has $2^8 = 256$ steps, which makes a total number of colours of $256^3 = 16\,777\,216$. One often reads this as an answer to “How many colours are there?”:

SOLUTION #3 [Display technology]

THERE ARE 16 777 216 RGB-COLOURS (REALLY!)

Of course, this is nonsense. It is used by the industry in order to sell their display units. Indeed, it is a step-up from the early computer colour(!) displays, that used a palette of 256 colours. Perhaps of some interest to notice that this didn’t even look so bad!

These are the “dominant colours” of the Kodak test image, as returned by Mathematica. These 25 colours would be a sufficient palette to paint a satisfactory copy.

As an example I use a well known Kodak test image. It has been designed to display a full colour gamut and is used to test, or calibrate your printer. The original uses the 16 777 216 colours palette. When I ask Mathematica for a list of “dominant colours” it returns a palette of 25 ($1.5\,10^{-4}\%$!).

A quite reasonable rendering can be made with half of that. I used this for illustration because the difference shows in print. With a 256 colours palette you wouldn’t be able to spot the difference with the original.

*Actually, usually four by including an “opacity” channel. Four bytes fit neatly in a 32-bit word, so this is convenient.

At top a Kodak test image in its full glory (16 777 216 rgb-colours! The bottom image has been quantised to only 12(!) colours (I used dithering and Wu's algorithm). Most images can easily be captured in such a "limited palette". With a 256 colours palette you wouldn't be able to spot the difference.

Varying the physical setting

THE SIMPLE SETTING I described earlier fails to apply to many—perhaps most—settings in real life. Here are some consequences.

Gold is not Lambertian. (The "Great Torc" from the English Snettisham Hoard, 1st century BC.*)*

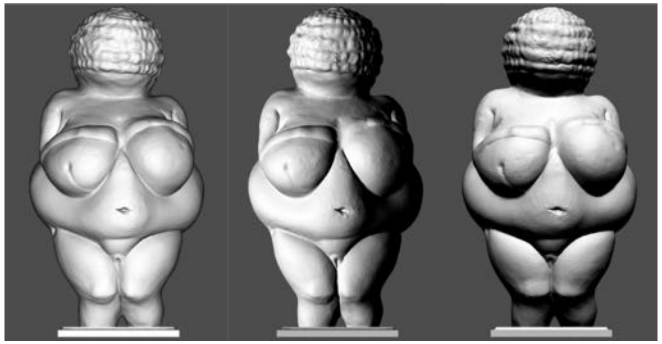
Non-Lambertian objects are common in our artificial environment, somewhat less common "in the wild". Notice that the gilded ornamental neck–ring (20cm diameter, weight 1kg!) is not "yellow", in fact, no colour from the RGB–cube really fits it. It "looks metallic".

The black lacquer of the Rolls Royce really "has no colour". You see the colours of the environment, it acts like a mirror. Just imagine how it would look without the reflections ("black"?). Again, it has no place in the RGB–cube.

Black lacquer has no colour of its own: in the image (as it would be on your retina!) some parts of the "black lacquer" are rendered pure white, others pure black. Yet other parts have the blue of the sky. So what do you see? I would say: "black lacquer". Does that make sense?

The same objects (fish hanging to dry) with illumination from the back (top) and the front (bottom).

The drying fish are close to being Lambertian, but they're not opaque, they are translucent, as becomes evident when illuminated from behind. What colour do they have? To me they look chalky white in frontal illumination and easily find a place in the RGB–cube. In the case of illumination from the back they look yellowish–brown perhaps, but they don't really fit the RGB–cube because they have an aperture colour character to them. They are somewhere between objects and something "aerial".



This is a white Lambertian object illuminated with uniform directional beams. Notice the grey tones.

Lambertian white objects ("chalk") are not necessarily a dead white.

In the cavity the light source is different from outside it.
In most scenes there are distinct compartments, each with its own (scaled) version of "colour space".

In the example of the Venus of Willendorff statuette* you notice a whole spectrum of grey tones that depend upon the direction of illumination. Thus the surface of the statuette does not map on the white point of the RGB–cube, but it maps on the full body diagonal, running from black to white. The grey

*The artist lived about 28 000 years ago in what is now Austria.

tones depend upon the local surface attitude of the surface, so called "shading". This is what makes sculpture in marble effective, the sculptor really "paints" with his chisel and drill.

The glint on the weapon is not white. It lies far outside the RGB*–cube. This is common with non-Lambertian surfaces. Your visual awareness "knows" that, it does present you with a glint that is outside your colour space.*

In many scenes the effective light source is different in different parts of the scene. You really need a different RGB–cube for each part! Your vision easily takes that in its stride though. In most scenes there are distinct compartments, each with its own (scaled) version of "colour space". Visual awareness does that, but photography cannot. So called HDR ("high dynamic range") imaging is just a kludge with results that usually look in doubtful taste.

With reflecting objects in a scene there are always local patches in the visual field that are evidently outside the RGB-cube. Such reflections are usually added by the painter after the painting is in a final state, they are finishing touches that do not belong to the local colour gamut. In real life your vision treats them as singularities too.

These remarks only show you the tip of the iceberg. I could easily write a handsome volume on such deviations from the simple colorimetric setting. That I simply ignore them in this book does in no way imply that they are not important, they are!

The conclusion that colorimetry is therefore useless is also inappropriate though. It is certainly necessary to describe the physics of the umwelt. However, the merkwelt differs in many respects from the physical umwelt. This is the topic of research in Experimental Phenomenology. I would say the best researchers are the visual artists. The contributions by Vision Science are relatively meagre.

All this goes to say that the topic of this book is a very focussed one. However, in many respects I leave the safe ground of mainstream colorimetry. In fact, hard-core colorimetrists are likely to frown on it.

Telemaco Signorini (1835–1901), Una via di Ravenna. *The white reference is different at the sunny and at the shaded side of the street. In fact the nature of the light field is very different!*

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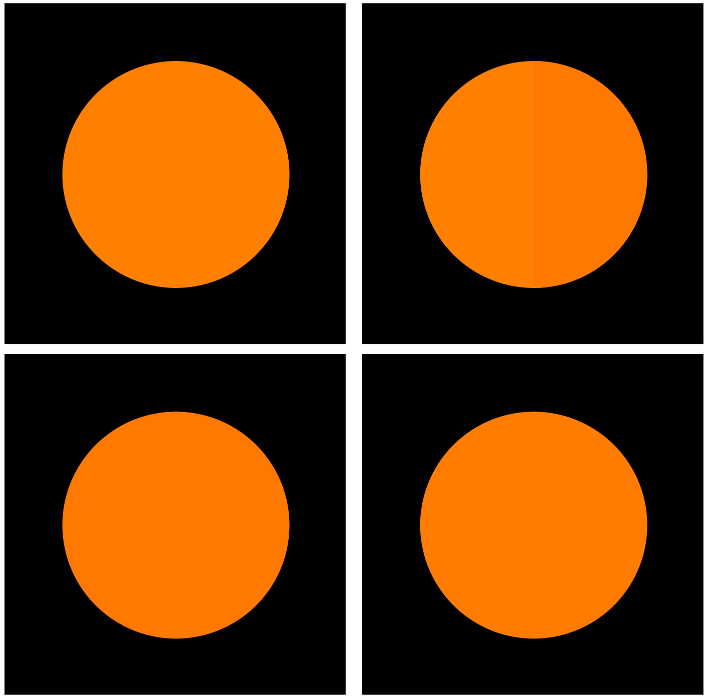
How many colours are there empirically?

One way to get at an answer to the question "How many colours are there?" is to simply count them! Of course, that does not yield any conceptual insights. However, it renders the facts explicit.

So far, the answers from colorimetry are very far from reality. That is because of the definition of "different colours". When are two colours mutually different?



Two different colours that can easily be discriminated. In this picture the difference is almost ten times the threshold. In this experiment you need to decide which disk has the two colours at either side of the bisector. Notice that the other three disks are uniform, they are all different colours. (The bicoloured disk is at top right if you didn't notice, look again.)

The standard method is based on JNDs, that are Just Noticeable Differences.* Of course, it depends a lot on the circumstances whether a difference will be noticeable! The usual method is to depend upon the detection of a common border between two abutting uniform areas.

Does such a number make sense? Well, if you need to paint out a scar on your car body, probably yes. In that case you're not interested in colour at all! All you are interested in is whether the scar is hidden.

Thresholds for detecting the sharp boundary between two uniform fields of slightly different intensity suggest that you can discriminate at least a two-hundred intensities. So one guesses (raising this number to the cube power) that the total number of colours might be in the ten-million range. Psychophysics indeed yields numbers in that general ballpark. More recent estimates are somewhat lower. I will take these as the answer from psychophysics:

SOLUTION#4 [Psychophysics]

THERE ARE 2 000 000 TO 10 000 000 DISCRIMINABLE COLOURS

A more practical number was found by Linhares, Pinto, and Nascimento (2008, the number of discernible colours in natural scenes), it is three-quarters of a million.

SOLUTION#5 [Psychophysics+Ecology]

THERE ARE ABOUT 689 734 DISCRIMINABLE COLOURS

These authors find that only about 31% of the colour solid is claimed by colours from the natural environment. Notice that this is considerably less than the fraction claimed by the RGB–colours. Indeed, one expects the RGB–cube to be only partly filled. This is an interesting estimate of the number of discriminable colours from a biological perspective. I like this study a lot because the biological (or ecological) perspective is so rarely taken in science.

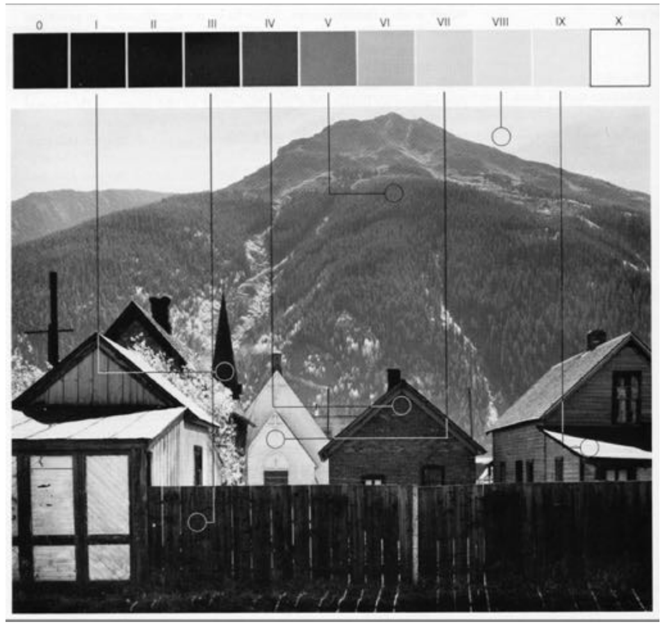
*The classical reference is: Macadam, D. L. (1942). *Visual Sensitivities to Color Differences in Daylight*. Journal of the Optical Society of America, 32(5), 247.

Whether these numbers are of much relevance to the evolution of human vision is another matter. Notice that being “discriminable” involves intense scrutiny. So the number may be *much* lower in circumstances where you have to act fast. From a biological perspective, the number of colours relevant in a bar fight might be many orders of magnitude lower than this.

Now suppose you have a colour in mind and you look for a suitable paint. Then such numbers are outrageous.

The Pantone system offers about two-thousand colours, more than you’ll ever need.

Pastels are not mixed (like paints), but used as is. Most artists are happy with a hundred pieces.



Ansel Adams recognised ten grey values. This is overkill for most photographers.

The PanTone colour samples are obviously overkill, but contain about two-thousand samples. The most luxurious pastels boxes contain about two-hundred pieces. Most photographers consider Ansel Adam’s 10-step grey scale extreme. Painters agree, they use 2, 3, 5 or (at most) 7-step grey scales.

From such numbers you guestimate that the smallest relevant distance in the RGB–cube is about 0.15. That implies a dozen grey levels (a body diagonal), forty hues (a polygonal arc of six edges long), about twenty members in an edge colour family (three edge lengths in series), about forty to fifty colours in a planar section and about three-hundred colours in total (the volume of the cube). These are numbers that approximate what general praxis indicates. There are apparently about four to twelve independent steps in any RGB–coordinate.

So the answer to the question “How many colours are there?” from the perspective of artistic praxis is

SOLUTION#6 [Praxis]

THERE ARE BETWEEN A HUNDRED AND A THOUSAND COLORS

These colours are not only "discriminable", they are immediately seen as mutually distinct. Having a colour in mind gives you a good chance to find it in a box of pastels.

Notice that this number is about three orders of magnitude lower than the number of discriminable colours in the hunter–gatherer's Umwelt.

In real life the distribution of mutually distinct colours is most likely uneven, so these are only ballpark values. What is really needed is a bean count![*] That is not a simple task and likely to take considerable effort. It is not even clear how to go about it.

So how to make a start? Our[†] idea was to use a "colour picker". I'll explain that in the next section.

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Just for kicks, consider what colours the Cro–Magnons had to hand. Roughly these:

Earth pigments.

They did not have a pastel box of two-hundred colours. They had mainly blacks (a range of manganese oxides), whites (kaolin clay) and earth pigments, mainly yellow and red ochres. The ochre pigments are applied in Lascaux, for instance:

Lascaux painting of a dun horse (equine). This is the early Magdalenian, about 17 000 *years ago.*

Yellow ochre (limonite) is a hydrated iron hydroxide, red ochre (hematite) is an anhydrous iron oxide. Ochre painting is very ancient, Pleistocene burials with red ochre date as early as 40 000 years ago.

So the ancient palettes were not very rich. This is no hindrance to the artist. However, it does not allow us to make reliable guesses as to early colour vision.

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[*]Dictionary meaning of BEAN COUNTER: a person, typically an accountant or bureaucrat, perceived as placing excessive emphasis on controlling expenditure and budgets. ("their bean counters will switch to a new way of calculating GDP").

[†]Andrea van Doorn, Karl Gegenfurtner and myself.

Colour Synthesizers

COLOUR "PICKERS" are a practical way to navigate the RGB–cube. It is a type of navigation that is not all too intuitive because it is a navigation in three dimensions, whereas humans are much better in navigation in one and two dimensions.

Colour pickers are used in many software packages and many people are familiar with them. Most of these tools are pretty bad from an interface designer's perspective. There are few colour pickers in general use that do not violate some of the fundamental rules of user interface design.

Designing an effective colour picker is apparently not simple, given the fact that the industry does not too well. (That there is no money in colour pickers might play a role here.)

We did extensive research on colour picker ergonomy. Here are the findings that are of relevance here. They apply to colour pickers of any design that is not just too bad:

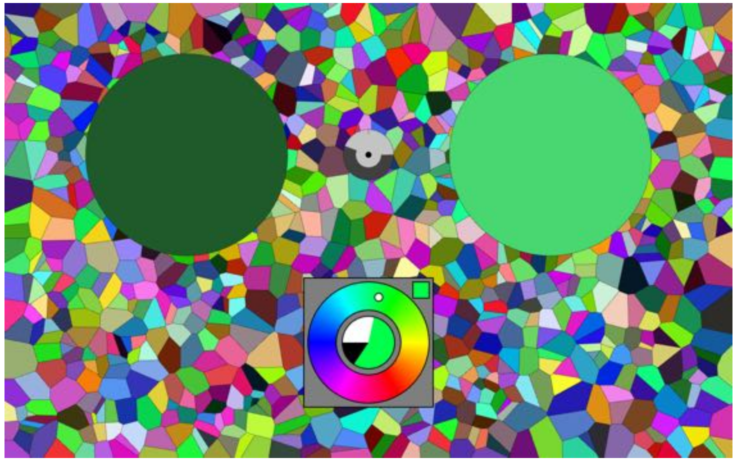
- users zoom in fast, within about 20s they're in the general ball park;
- after 20–30s all users enter an errant mode. They do not make further progress to their target but perform some randown walk in the general neighbourhood;
- the final accuracy is rarely better than 10% of full scale in each of the RGB–channels.*

Such results are already interesting, they imply that there may be at most a thousand colours ($1/0.1^3 = 10^3$).

We prefer the term "colour synthesizer" over "colour picker", because that is how we use it. Think of it as a musical synthesizer for colour.

*Because of the gamma correction Weber's Law does not apply here. Moreover, Weber's Law applies only at a given adaptation level. In real settings there is some overall adaptation level and some parts of the scene simply drown in the darks, some light parts in the lights. This is much like photography with a dynamic range of (typically) about $1:100$ to $1:1000$. Visual awareness does better, because it shifts the range as you look around in the scene.

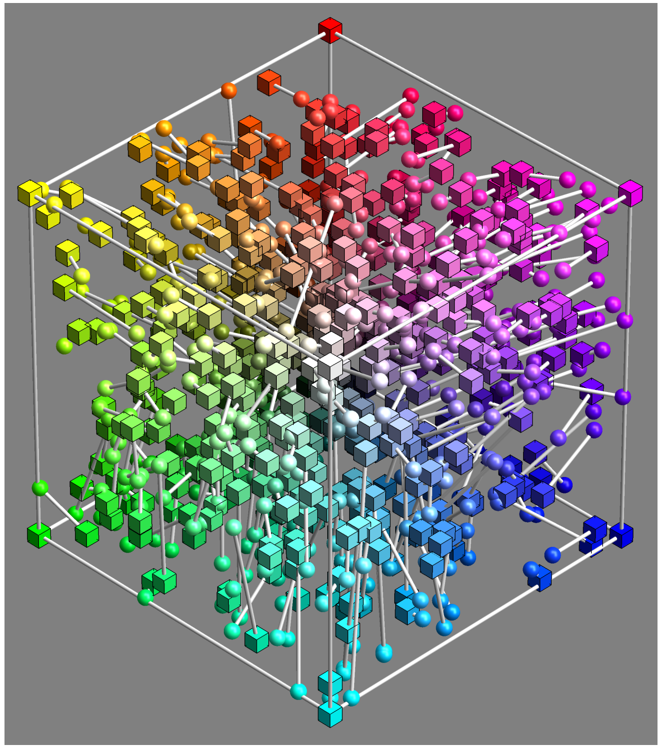
The colour picker used in the bean count. All interaction is through the keyboard. The user controls hue, colour, white and black content. The interface is modal (its main weak property), the mode is indicated in the square at top right.



The fiducial colour is displayed in the disk at left. The participant has to synthesize it in the disk at right. This implies looking back and forth all the time, by design, the two colours are never looked at simultaneously. The colour synthesizer is at the lower centre, above it is a timer. The participant is allotted 45s for a setting, but most use less than that. Notice the background, a random gamut from the RGB–cube, fixing the "adaptation level". The background is refreshed every second. This is a poor man's way to mimic vision in practice, in a controlled manner.

Sampling the RGB–cube

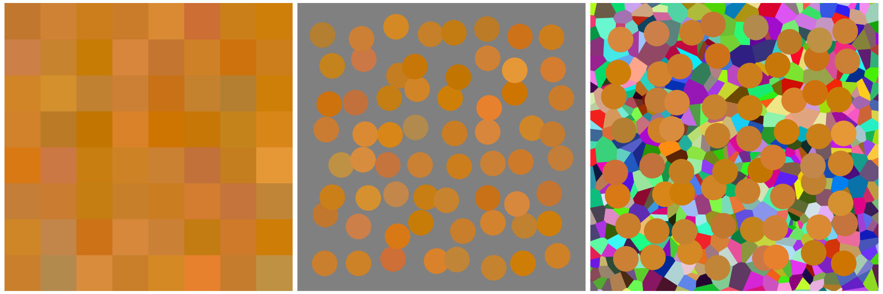
FOR A USEFUL BEAN COUNT we let a group of fifty naive participants do sixty settings each, yielding a sample of three-thousand settings. We summarise these in terms of covariance ellipsoids on a regular $10 \times 10 \times 10$–lattice covering the RGB–cube.



Here are 500 of the 3000 settings. The fiducial colour shown as a box, the synthesized colour as a sphere. Notice the size of the discrepancies.

The synthesized colours apparently spread fairly widely about the fiducial. This becomes obvious when you scrutinise a selection of settings for the same fiducial, especially when the results are plotted so as to subtend common edges. Removing the edges make them look more similar. Adding a textured background makes them even more similar. This graphically indicates the irrelevance of the psychophysical JNDs for real life.

If the Neanderthal is required to act, that asks for an obvious *Merkmal*, not a mere JND. It is not all that different with us modern *Home sapiens*, at least, the participants in the bean count experiment were all smart university students that did their best to perform the task.



Here are 64 actual settings for a light brown (almost orange) fiducial. The left representation has hard-edge boundaries, thus the variations are very obvious. At centre I left out the edges and the differences are perhaps a little less obvious. At right I added the context and you see that the settings are actually very similar.

From the images you see that the "grain" in the RGB–cube is far from being uniform and isotropic. The data is clean enough that specific counts are possible along curves, over surfaces and in volumes.

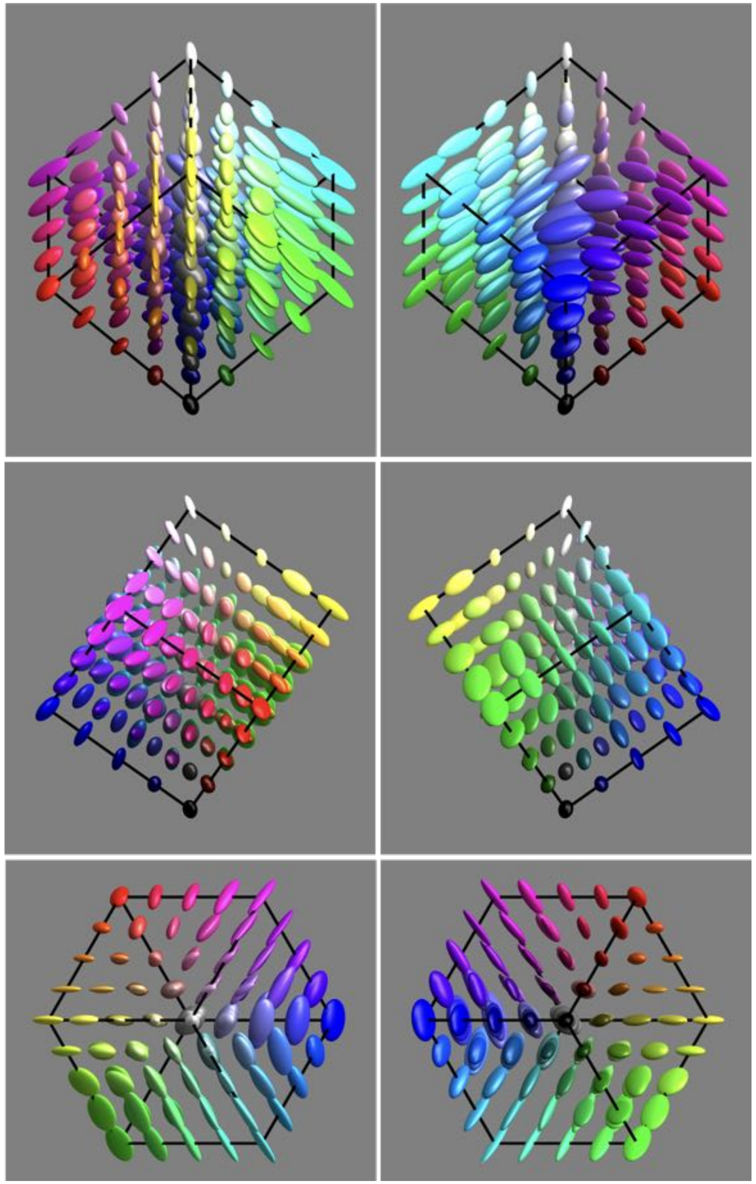
The overall summary is the bean count over the volume of the RGB–cube. The result is

SOLUTION#7 [Bean counting]

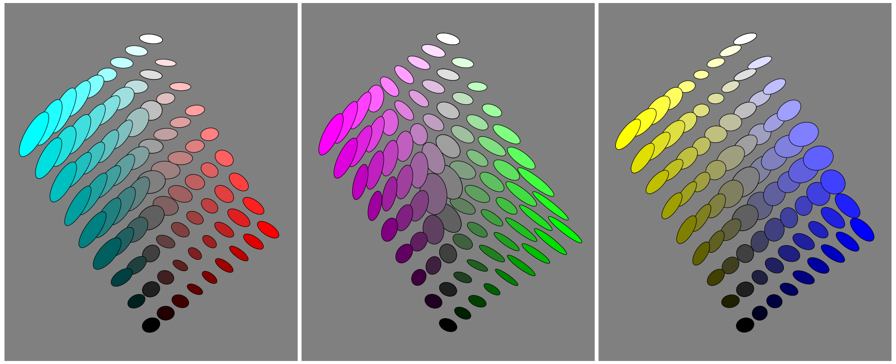
THERE ARE A LITTLE OVER A THOUSAND COLOURS

This is a number that appears to fit praxis.

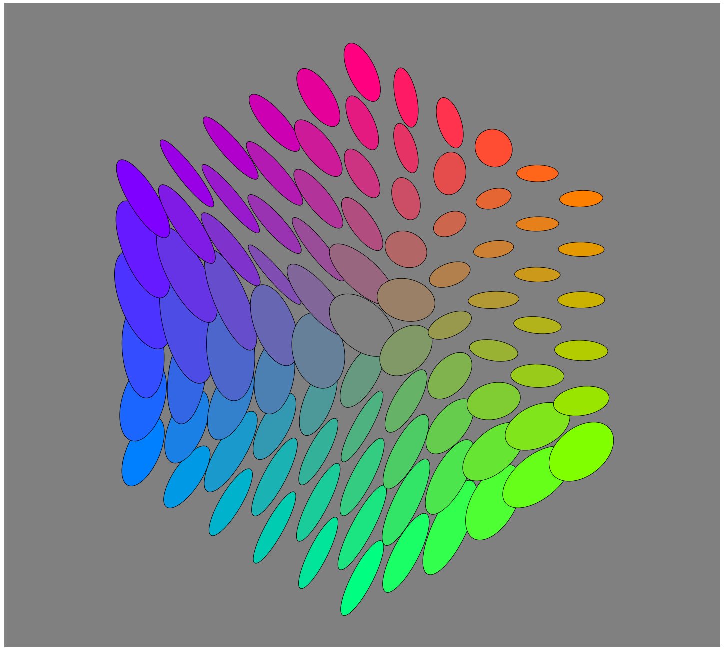
Because I ignore the obvious structure over the RGB–cube, this number is only a very coarse ball park figure. I won't go into nitty–gritty details though.



These are confusion ellipsoids fitted to the data. These allow a precise bean count to be made.



Some planar sections. Notice that the distribution of "grain" is far from being uniform or isotropic.



Another planar section, this one at right angles to the black-white body diagonal.

Issues of metamerism

METAMERISM is what causes objects to look different under different illuminants. It is easy enough to construct examples that are rather alarming. One may set up cases where a white and a black object switch colour under a change of illuminant! There is no doubt that metamerism can—in principle—completely throw the recognition of objects on the basis of their colours completely awry (▷ p.89).

Thus one reads accounts (especially by philosophers) that colour vision is really impossible and colour is basically just arbitrary “mental paint”.

Don’t you believe it! I think such ideas make very little sense. The point is that the astounding examples are based on spectra that *will never occur in nature*. Few examples of metamerism are quoted from outside the laboratory, perhaps the best known being a gem-stone Alexandrite. Fine-quality alexandrite has a green to turquoise colour in daylight, changing to a red to purplish–red colour in incandescent light. But “fine-quality alexandrite” is quite rare. Few people have actually witnessed the effect.

In discussing the importance of metamerism one should limit possible illuminant and reflectance spectra to what actually occurs “in the wild”. Only then can one just whether metamerism may be considered a problem to the hunter–gatherer life.

The problem with this is that any example one uses will evoke the objection that they are not “general”. Indeed, a completely general approach must admit the singular examples. The “general case” prohibits *natural vision*, which is a “singular case” that tends to be pervasive in real life. Vision implies biology, not physics, or colorimetry.

What I will do here is consider some databases that at least fit the generic hunter–gatherer umwelt. I’ll try to broaden the scope later.

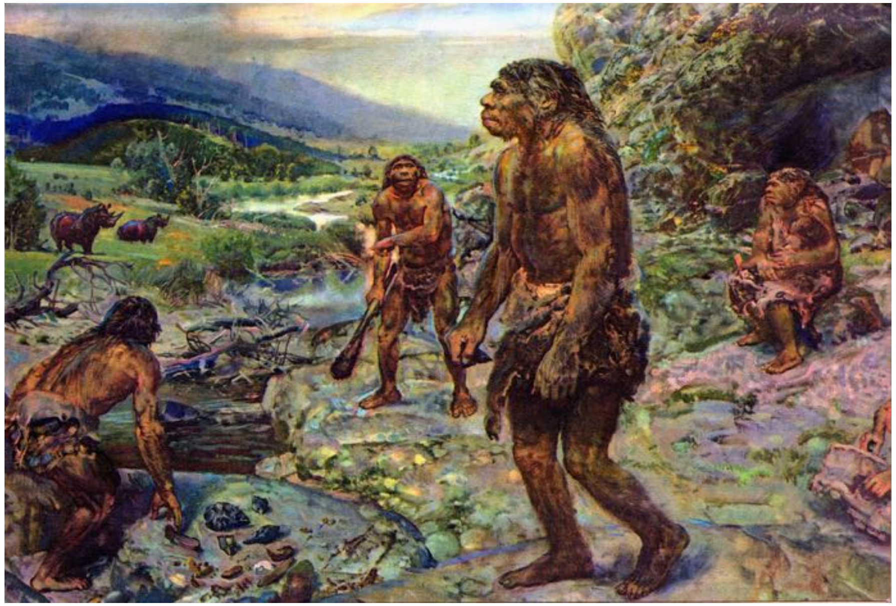
Notice that one needs to set up two realms of entities, one of illuminants and one of reflectances, or “things”.

I propose to use two well known databases in the public domain, one of varieties of daylight, the other of a few hundred wild flowers.

⋆ ⋆ ⋆

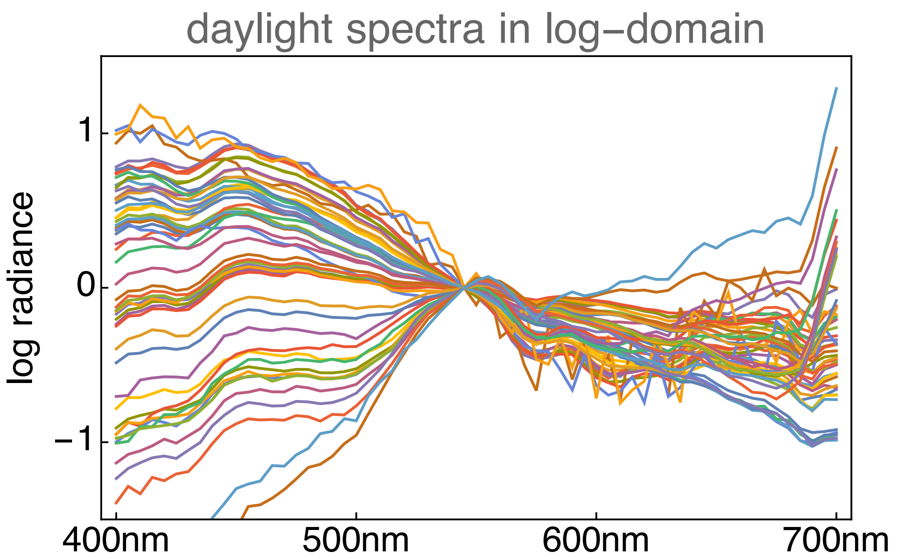
Sampling daylight

AVERAGE DAYLIGHT is a convenience offered by the CIE, but, of course, the illuminant is never “average daylight”, like there are few people of exactly the average weight or length. The database I use has samples of the daylight spectrum at different hours of the day and in different meteorological conditions. It covers blue sky, sun, cloud, overcast sky and even light reflected from green trees. It should cover the range of illuminant spectra of the hunter–gatherer of the North.



Zdenek Burian (1950), “The Neanderthal encampment”. In this phantasy the umwelt of the Neanderthaler is quite colourful. It probably was to them, most probably their visual awareness was like ours, although their reflective thoughts must have been quite different.

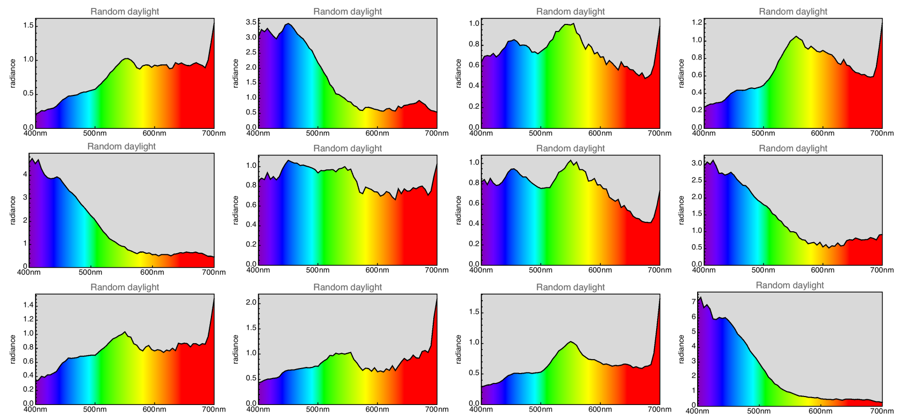
I use a small database (▷ p.87) of daylight spectra measured in Finland. It lists samples from 390–1070nm in 4nm intervals. The datebase consists of three parts. There are 15 samples of a $BaSO_4$ surface,* illuminated by cloudless sky, half cloudy and overcast sky at different times of the day. There are 22 samples of the sky reflected in a mirror, again different times of day and meteorological conditions. Finally, there are 15 samples of radiation scattered from a Spruce tree.



Normalized daylight spectra in the logarithmic domain. The normalization arbitrarily sets the radiance in the middle of the spectral range to a fixed value. This shows the full database.

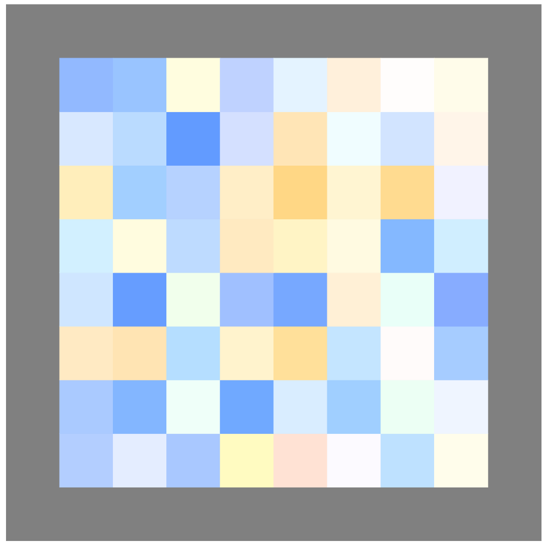
A PCA in the logarithmic domain reveals that 9 components account for 95% of the variance. The distribution of the coefficients of the projection of the database on the components is roughly multinormal, yielding a very simple method to synthesize arbitrarily many instances of daylight with the same statistics as the database.

*Pressed barium-sulfate (the mineral barite, a pigment also used by early man) powder is often used as a white reflectance standard.



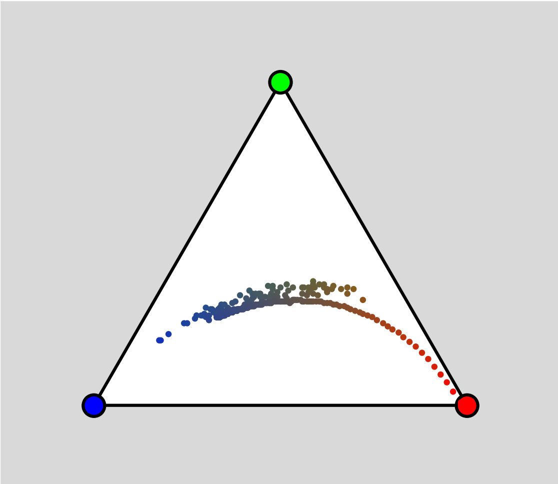
Synthetic daylights with the same statistics as the database items.

These daylights have quite a variety of colours, one spots blue sky, sunset and tree–greens.



The "colours of daylight". Here you can see the colours of synthesized daylight because I switched off the Von Kries coefficient law.

In the chromaticity diagram the daylight colours are seen to cluster on the locus of thermal radiation, although there are minor deviations away from that.



Here are random daylights represented in the RGB*–colour triangle. The curve is the Planckean locus.*

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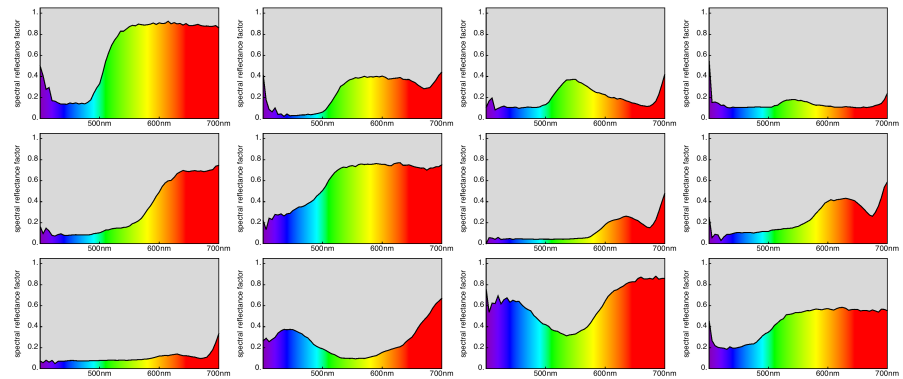
Sampling spectral reflectance factors

SPECTRAL REFLECTANCE FACTORS are physical properties of objects. So what objects to take? I decided to use a database of flower colours.* This covers vegetation, although bark and foliage might be underrepresented. One would probably like to add colours of rocks and earth and colours of blood, meat, fat, bone, feathers and fur. The colours of the sky probably don't count, as you don't have to fight them and can't eat them. Together that would about cover the hunter–gatherer Umwelt. However, here I'll stick to the flower colours. I will consider extensions later.



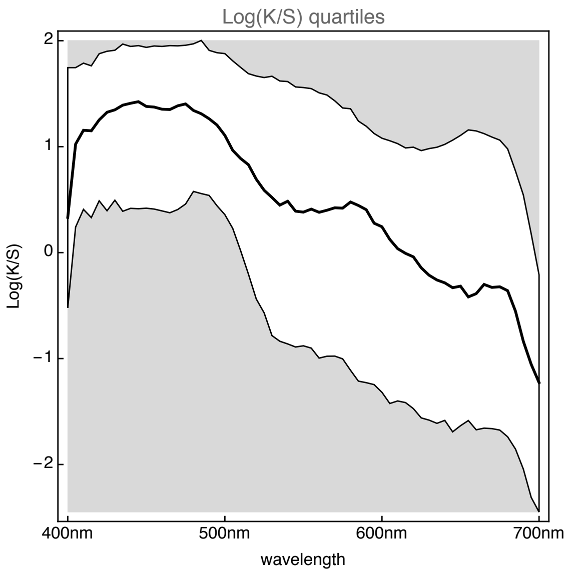
Some examples from the Finnish flower database.

The database (▷ p.87) has 218 samples of flower colours, measured from 400–700nm in 5nm intervals.

The first action is to map the spectral reflectance factors to the physical domain. I find that the histogram is wavelength dependent and unimodal at every wavelength, although not quite normally distributed.

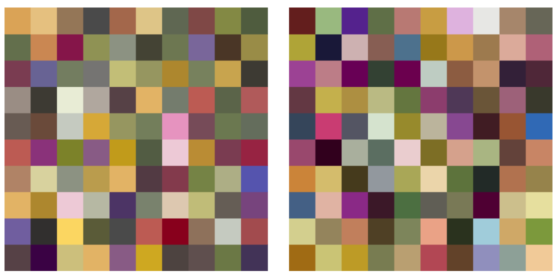
*Parkkinen, J., Jaaskelainen, T. and Kuittinen, M.: "Spectral representation of color images,"IEEE 9th International Conference on Pattern Recognition, Rome, Italy, 14–17 November, 1988, Vol. 2, pp. 933–935.

In order to explain over 95% of the variance one needs 19 PCs. The coefficients of the projections of the database on the PC–basis are roughly multinormally distributed. So a simple algorithm suffices to generate as many instances with the database statistics as needed.

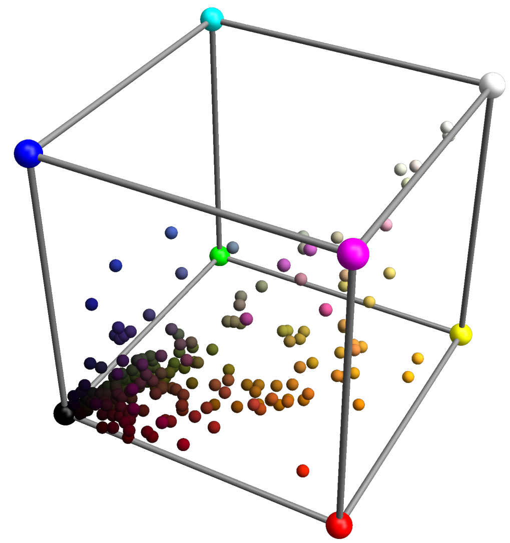


The quartiles of the $\log(K/S)$ *values are wavelength dependent. Overall, vegetation reflects more of the long wavelengths.*
A graph like this is revealing in many respects, though I haven't encountered it in the main stream literature thus far.

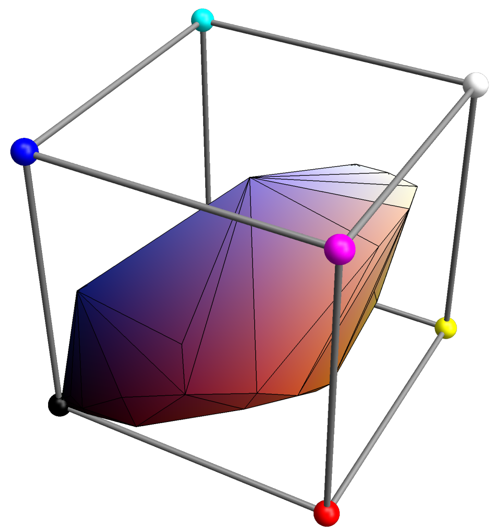
These synthesized spectral reflectances cannot be distinguished from actual database items by eye. To have a random spectral reflectance factor generator is a great asset in studying the effects of metamerism.



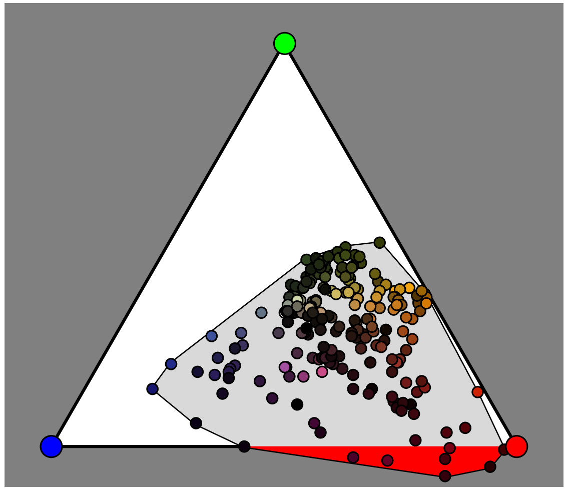
At left the colours of Finnish flowers database for the standard average daylight illuminant. At right an equal number of synthesized instances.



The Finnish flowers database mapped in the RGB*–cube for the standard average daylight illuminant.*
Notice that only a fraction of the RGB*–cube volume is claimed by the convex hull of the samples.*



The convex hull is a fairly generous estimate of the volume claimed by the Finnish flowers database. Its volume is only 20.5% of the volume of the RGB*–cube.*



The Finnish flowers database mapped in the RGB*–chromaticity triangle for the standard average daylight illuminant. Notice a few "outliers": for 3% of the colours there is a (very minor) negative coordinate. There are no colours with a coordinate value exceeding one.*

The plot of all database items in the RGB–triangle reveals that a few instances map outside the "claim" of the RGB–cube. It involves about 3% very dark (almost black) colours. In terms of the convex hull, 8.5% of the area sticks out of the triangle. Also notice that the convex hull claims only 49% of the area of the triangle. There are virtually no really vivid green or blue flowers, most lie in the red–yellow–purple sub-triangle.

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Metamerism in the wild

METAMERISM is what happens when colours become mixed up under a change of illuminant. Here is what I mean:

Consider four colours in the RGB–cube that subtend a generic (non–empty) tetrahedron under average daylight (under average daylight we have the "real colours" for all practical purposes). Now change the illuminant. The colours will shift, but (hopefully) Von Kries comes to the rescue and the shifts will be minor. If the orientation of the tetrahedron changes we may say that the relation between the colours has been destroyed, "due to metamerism". If we do this for all combinations of illuminant reflectance spectra in our database, we can figure out how far (on the average, etcetera) two colours have to be apart in order to always stay apart.

In order to do such computations conveniently, one needs to set up algorithms that will let you construct:

representatives given RGB–coordinates, find a spectral reflectance that will generate this colour under the standard illuminant. The representative should come from the same statistical environment as the database instances;

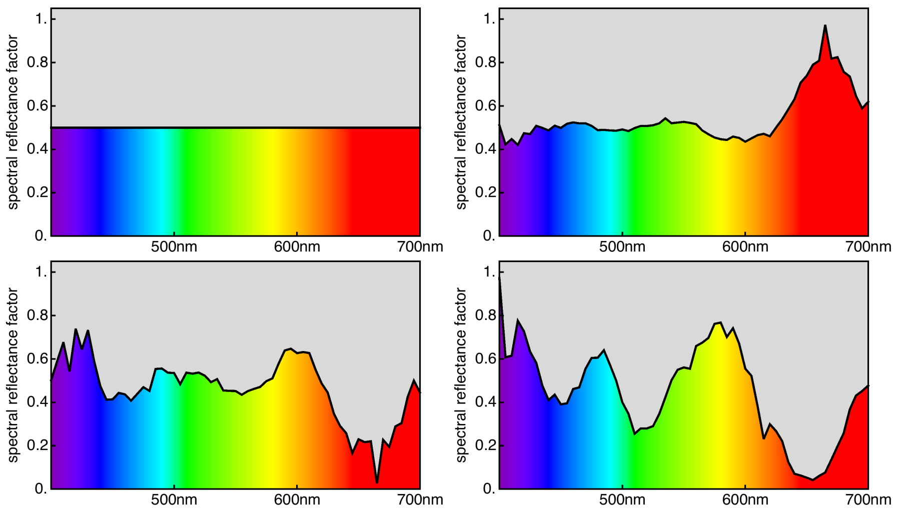
metamers given a spectral reflectance generate arbitrarily many different spectral reflectances that yield the same RGB–colour—so called metamers.

It is easy enough to do such constructions with elementary linear algebra. The fact that we can generate instances on call is also very convenient

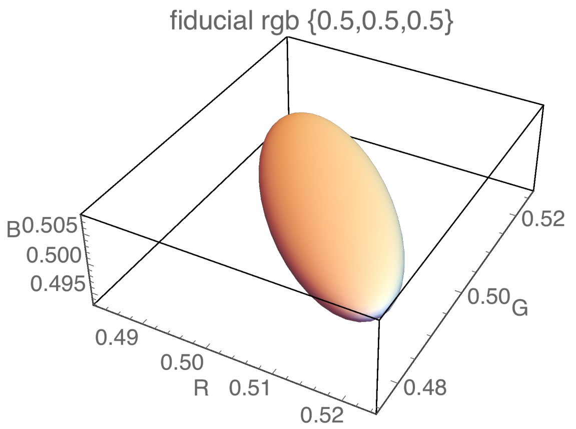
In order to determine the "grainsize" at any location in the RGB–cube I first generate a representative. Then I generate a dozen (say) metamers of it. I find the colours of all these under random illuminants with the statistical structure of daylight, say another dozen. This yields a cloud of over a hundred colours, of which I find the covariance ellipsoid. This is the required "grain".

This immediately yields an estimate of the number of colours that can reliably (that is: under any illumination) be discriminated. It is simply the volume of the RGB–cube divided by the volume of the ellipsoid. There are obviously numerous possible refinements, but they are indeed no more than that: *refine-*

ments. Here I stick to the big picture.



A representative of $\{r, g, b\} = \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$ (top left) and three metamers. These reflectance spectra have exactly the same colour under the standard illuminant. However, the spectra are so different that it is hardly surprising that they will look different when the illuminant is changed.



The confusion ellipsoid ("grain") for the $\{r, g, b\} = \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$ colour. Of course, such ellipsoids are generally different at different locations in the RGB–cube.

The size of the grain depends crucially on the sets of illuminants and reflectance spectra. In the "general case" the grain-size would be a fair fraction of the size of the RGB–cube!

This is the deciding importance of the biological perspective. It is entirely decisive with respect to the evolution of the visual system. There is no way to arrive at this conclusion from the contexts of psychophysics, colorimetry, physiology or physics. The answer from biology is clear enough:

SOLUTION#8 [Ethology/Ecology]

THERE ARE AT MOST A LITTLE OVER A THOUSAND COLOURS

This is a number that we also arrived at by way of the bean count. It is hardly a coincidence. The visual system is only as good as it is necessary to provide an effective user interface.

Here is a final consideration: What about metamers *not* in the wild? In my coarse grained optics spectra are 68–dimensional vectors. The CIE colour matching functions span a three-dimensional space, its null-space is thus 65–dimensional. Any spectrum from the null space maps on the origin, that is on *black* of colour space. This means that any spectrum implies a 65–dimensional space of spectra that have the same colour. This is the metamer that belongs to that colour.

No metameric black (except the origin) can be physically realised. This is because radiant power cannot be negative. For spectral reflectances the values have to be in the range $(0, 1)$. Not withstanding such constraints, the metamers tend to be large sets that contain all kinds of weird functions. (I show a striking example in the technical annex.)

It are really the *ecological constraints* that cut the metamers down to size, most spectra being highly unlikely instances in terms of the ecological statistics. The relevant instances claim only a small corner of the metamer. That is why the question "How many colours are there?" can only be answered in a biological context.

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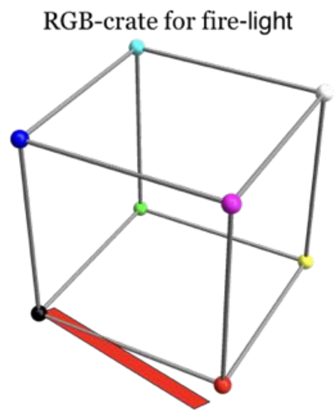
Fire light

FIRE LIGHT is the illumination as it might have been in a Neanderthal cave. It is low temperature thermal radiation (page 13). Humans have controlled fire for about a million years, whereas even gorillas have no control over fire.

The candle light spectrum is completely outside the range of natural daylight(s). This will almost certainly have serious consequences for colour vision. It is a singular case, as it was for the Neanderthals. The cave umwelt must have been completely distinct from the daytime umwelt. This renders it a most interesting special case.



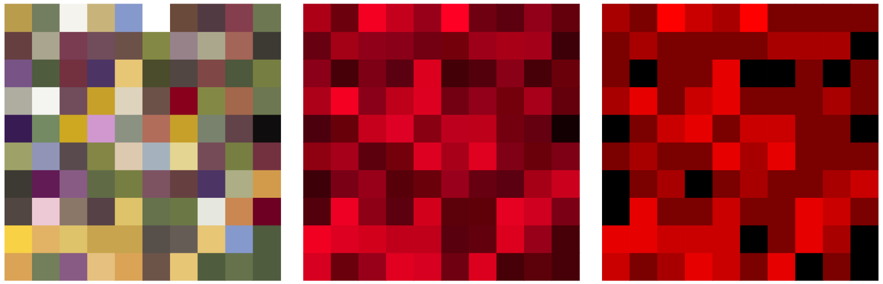
This is the RGB–crate for fire light, that is thermal radiation of $1000°$K. The crate is so anisotropic that the Von Kries coefficients law is inappropriate. I simply scale all colour channels by the same factor, so as to make the largest one equal to 1. Apparently the fire light only effects the red channel, there is some (but very little) activity in the green channel, virtually none in the blue channel. Because of this scaling, white paper looks red, not white (as when the coefficients law were in effect.)

Fire light predominantly excites the red channel, so the Von Kries coefficients law is useless. Scaling all channels by the same factor yields a very anisotropic crate, with a volume of only 0.041^3 (as different from 1^3 for the full cube). The edge ratios are 1:0.085:0.0009 instead of 1:1:1 for the cube. The illuminant colour is $\{1.000, -0.127, 0.014\}$, thus the crate sticks out of the RGB–cube (the red is redder than the red part of white).

Here is the colourful Kodak scene in firelight. Only four tones in red remain.

With a resolution of about 0.1 per channel one would have about less than a dozen levels in the red, an all or none signal in the green and a silent blue channel.

How many distinct colours there really are depends upon metamerism. Using the same method as before I find that there are only four colours left, all varieties of red.

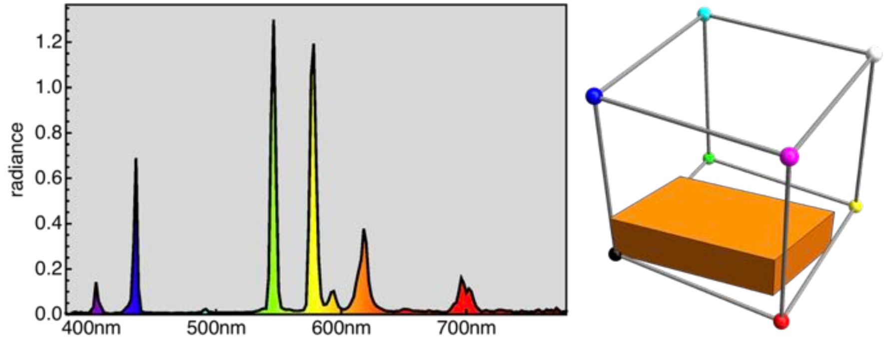


At left the colours of flowers under normal daylight. At centre the same flowers under candle illumination. At right the same flowers under candle light, coarse-grained according to the metamerism grain estimate. Could you ever trace a certain flower? No chance!

A STRANGE ASIDE that is completely outside the Neanderthals scope, may yet be of interest as an illustrative example. A mercury source has a weird spectrum, composed of somewhat broadened mercury lines.

The crate has aspect ratios 1:0.64:0.23, thus is really trichromatic, not monochromatic like the fire light. A metamerism calculation reveals a resolution of 5, 3 and 2 steps in the red, green and blue. Thus the source allows of thirty mutually distinct colours.

It would not be easy to trace a particular colour, known from its “normal” colour, under mercury illumination. This is probably the main reason why most people hate mercury illumination although it is very economical to run and allows trichromatic vision. The “colours are off”. The colours allow reasonable segregation on the basis of colour, although not so good as daylight (the crate claims only 12.7% of the RGB–crate). With at most thirty colour labels, the recognition possibilities are quite bad: notice how many of the flowers map on the same label!



At left the spectrum of the mercury source, at right its crate.

At left the colours of flowers under normal daylight. At centre the same flowers under mercury illumination. At right the same flowers under mercury illumination, coarse-grained according to the metamerism grain estimate.

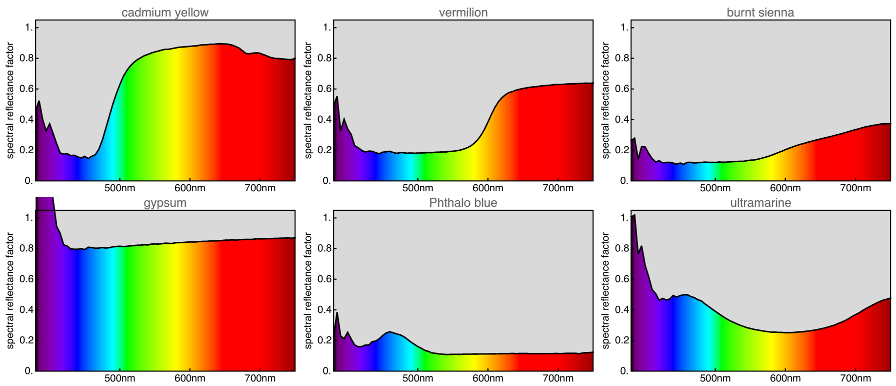
Thus we have good reasons to hate mercury sources.* For the Neanderthals it would have been a significant step up over fire light though.

*Mercury-vapor lamps are gas discharges due to electric arcs through vaporised mercury. The arc discharge is housed in a small fused quartz arc tube mounted within a larger borosilicate glass bulb. Typically a phosphor coating is used for some “colour correction” through fluorescence. There are strong emissions in the ultraviolet. The strongest lines in the visual are 405nm, 436nm, 546nm and 578nm. The phosphor (if present) adds to the red end.

Additional databases

THE FINNISH FLOWERS DATABASE is perhaps overly focussed? The only way to find out is to consider a few—hopefully very different—alternatives. Here I considered a few, just the cases I could download for free from the Internet.[*]

One interesting database, because so different from the flowers, involves powdered mineral (mostly) pigments, the colorants that where (for the most part) used by artists for centuries, some for millennia or more. It is the FORS–database from an Italian group. It has 55 spectral reflectances of powdered pigments, measured from 360.169–1000.94nm.

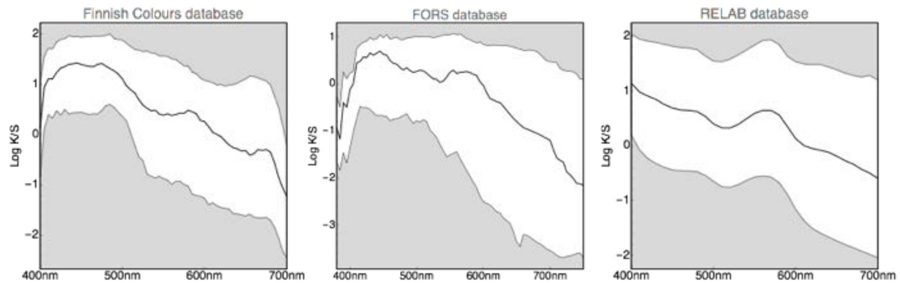


Some instances from the FORS powdered pigments database.

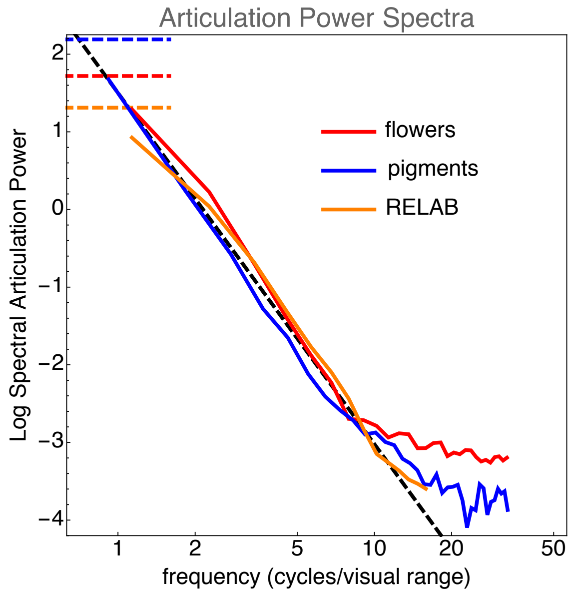
This database may be considered to represent the (extreme!) earth and mineral colours. From a cursory investigation by eye it appears that this database hardly expands the flowers database and a statistical analysis corroborates that.

*There are databases for real professionals that are expensive, but I am a retired professor with not that much money to burn. It is amazing how people dare to ask money for data that were probably gathered on the taxpayer's expense, whereas academics like me offer all their fruits for free. It is not too different from the publishing's racket.

The RELAB database (▷ p.87) is unfocussed and huge (114 120 samples!), but somehow I don't feel at ease with it. It was distilled from hyper-spectral images (page 87), so many samples derive from a single image, thus all kinds of calibration issues will not average out. Anyway, it is interesting to see that it does not really contribute a novel perspective either.



There appears to be a consensus on overall spectral trend and variance over very different databases.



The articulation spectrum of the spectral reflectance factor. Here are results for the Finnish flowers, the powdered pigments and the RELAB databases. The dashed curve has a fairly steep (common!) slope (about -4) in this log–log-representation. The DC–levels are slightly different. This yields an estimate of the articulation autocorrelation function.

Notice that we obtain good estimates of overall spectral trends and the variance of $\log(K/S)$ at various spectral locations. The estimates of the articulation power spectra also agree very well. The spectra I show were obtained using a Hann window. Perhaps unfortunately, the length of the visual spectrum is very short, about a decade in frequency.

Spectral databases of meat, fat and bone are hard to find, but incidental spectra of skin, blood, melanins and carotenes are easy to find. They also fit easily into the Finnish flowers statistics.

More generally, one would like to consider biological pigments (or "biochromes"), because these were doubtless of the greatest interest to the hunter–gatherers. I exclude structural colours here, as these typically are so far outside the generic gamuts, that any visual system is likely to label them "outliers". Biological evolution made sure of that.

The most important biochromes are of one of three general families:

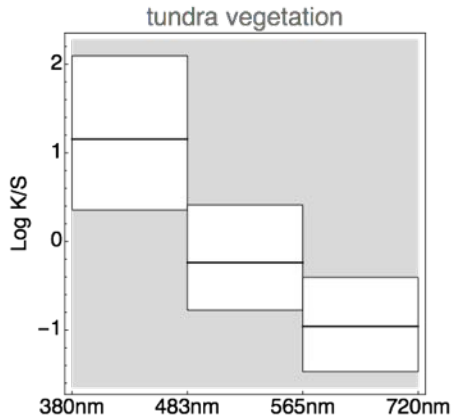
heme/porphyrin–based chlorophyll, hemocyanin, hemoglobin, myoglobin
carotenoids hematochromes, carotenes, xanthophylls
varia melanins, urochromes, flavonoids

A short investigation reveals, again, that this hardly reveals a need to adjust the statistics.

All considering, the recipe I gave for the "generic case" holds good. Minor adjustments are very unlikely to change the overall conclusions anyway.

HERE IS A NOTION THAT MAY SURPRISE YOU. Electronic images (say ".png" files) are arrays of pixels. For the case of colour images, each pixel is a list of three coordinates in the RGB–cube. Typical pictures will have of the order of a thousand ("icons") to tens of millions (quality photography) of such pixels.

Remember the "parts of white" and conclude that each pixel is actually a coarsely sampled spectrum. Thus any image is a huge (say a million samples) database of the radiant power in the scene in front of a camera at the moment of exposure. Although the databases are typically huge, each instance is only a spectrum that covers the visual range in three bins. Calibration data are usually lacking, but the fact that they live together in an image suggests that all these three–bin spectra come—calibration-wise—from a single context.



An RGB–image is a huge spectral database. Here are spectral trend and variation (compare page 65). Instead of the articulation spectrum we have to make do with the covariance structure. The eigenvalues of the covariance matrix are in the ratios 73:18:9, where 73% of the variance is taken by $R+G+B$, 18% by $B-(R+G)/2$ and 9% by $R-G$. Images are indeed very informative!

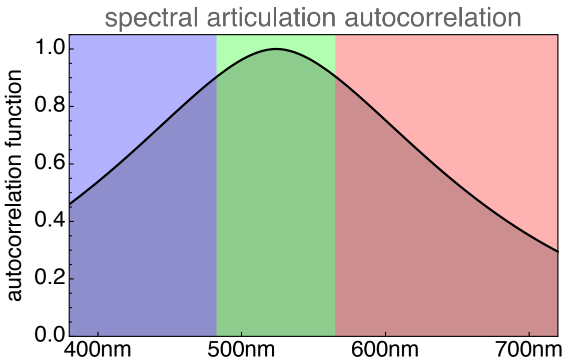
Moreover, if the calibration is really off, the image is bound to look bad. Images due to standard in-camera JPEG–processing probably apply a von Kries scaling ("automatic white balance") and a gamma transformation of about $\gamma = 2.2$. Of course, "raw" files are even better, if you can get them (you can if you photograph yourself!). If you make the images yourself, no one keeps you from having a calibration target in the image.

This source of data should not be despised. After all, the fine-grained spectral articulations of the radiance are lost to the human visual system anyway. The low spectral resolution implies that metamerism cannot be studied in single images (but, of course, it can in images of the same scene taken under different illuminants), but a variety of very useful overall statistical measures can easily (and cheaply!) be derived. Examples are estimates of spectral slope and curvature, variability in the three channels and so forth.

In order to use RGB–images I clip the coordinate at both sides, because all kinds of debris from the imaging pipeline accumulates there and I remove the gamma nonlinearity. Next I apply the $\log(K/S)$–transform and study the pixel statistics in the physical domain.

I found this to be an excellent "poor man's way" to study the optical umwelt statistics.*

Although obviously limited because of various unknown calibration factors, images are a great way to investigate various generic environments. From the RGB statistics one readily derives the parameters needed to construct a random spectral reflectance generator that is likely to yield very reasonable results. It allows one to study the effects of metamerism without access to a (terribly complicated and expensive to acquire and to maintain!) hyper-spectral imaging device. The results are not likely to be greatly different either.



I constructed this autocorrelation function for the tundra image by fitting the standard form on the pre–whitened (▷ p.89) pixels in the physical domain. The function is not too different from what I estimate from spectral databases, apparently a three-bin spectrum (pixel!) is fine. The (normalised) covariance matrix of the tundra image is

$$\begin{bmatrix} 100 & 71 & 37 \\ 71 & 71 & 40 \\ 37 & 40 & 53 \end{bmatrix}.$$

*For some details see Jan Koenderink and Andrea van Doorn, *Colors of the Sublunar*, *i*–Perception September–October 2017, 1–30.

This is because the spectra to be expected live in a very limited domain. All that is needed to emulate databases of the natural environment are a few basic parameters. These are about as well specified by image pixels as by high resolution spectral databases. This is hardly surprising, for why are we happy with RGB–photographic renderings? Yes, because they look good to us!

Some experimentation then reveals that the effects of metamerism are virtually irrelevant in the wild. Even with the coarse–graining due to metamerism there are still plenty of colours left for use as labels to keep things apart under even wild variations of daylight.

Thus the conclusion that metamerism renders colour unfit to see the world, but are at most a charming mental paint are so pessimistic as to be essentially *wrong*. In the wild, metamerism plays no role. The Neanderthaler is not bothered by it, but is right in assuming that "things have colours", just as they "have shapes" or "have weights".

At left the tundra image, at centre the tundra image coarse–grained, so as to be robust against wide variations of daylight. At right an image with wildly perturbed pixels, it is often suggested that metamerism run wild could do this to you, then "colours are meaningless". In fact, the centre image is hard to distinguish from the actual image, although it uses only a fraction of less than 10^{-5} of the possible pixel values. The conclusion is that the effects of metamerism are virtually irrelevant in the wild.

I guess the general public of today is likely to agree with that.

⋆ ⋆ ⋆

Generic spectra of the hunter–gatherer's Umwelt

SO FAR I have discussed results obtained from three rather specific databases. Is it possible to capture the statistics of the natural environment in generic terms? Sure, I believe this to be possible. I'll spend a short discussion on it.

One somehow needs to come up with generic instances of "typical" illuminants and spectral reflectances. Here "typical" implies that the instances have similar statistics as one finds in a large collection of specific samples. The latter are large or small databases, as the case may be, in every case focussed in some way or other.

Ways to summarise the statistics are shapes of histograms and fall off of power spectral articulation spectra, or— equivalently—the spectral articulation autocorrelation function.

There are also some meaningful means, for instance, the illuminants should cluster on the average daylight spectrum. The overall mean spectral reflectance factor should be like the Kodak grey Card 20% reflectance.

Variances of various quantities can also be estimated in an overall manner.

The natural domain for these estimations is log–spectral radiance for the illuminants and the log–Kubelka-Munk K/S for the spectral reflectance factors.

Perhaps the most important statistics involves the articulation of the spectra. What immediately meets the eye when looking at hundreds of natural spectra is how relatively smooth they are. Large amplitude modulations take place over wide spectral regions, whereas the amplitudes of articulations that play on short spectral stretches are small. This is seen in the power spectra of the articulation of radiant power spectra.

From a fundamental physics perspective, the statistics are likely to be translation invariant in the wavelength domain. That is because the relevant physical processes do not change over the—very narrow—visual band. Different processes appear in the ultraviolet (ionisation, atomic electronic transitions) and in the infrared (molecular vibrations and rotations). The visual band basically reflects chemical processes.

The fall off with articulation frequency varies only little from database to database and tends to be near an inverse fourth power. The articulation autocorrelation function is very broad, of the order of half of the width of the visual range.

An apt phenomenological* model would be as follows. Reflection spectra are essentially noise, due to many unknown, mutually independent processes. Suppose any single effect has a spectral footprint given by a Laplace distribution, that is to say, it is localised and its effects falls off exponentially to either side, thus (notice that $\mathrm{d}x/\tau$ is dimensionless)

$$L(x,\tau)\,\mathrm{d}x = \frac{e^{-\frac{|x|}{\tau}}}{2}\,\frac{\mathrm{d}x}{\tau},$$

where τ is some characteristic spectral extent. The width at half-height is $1.386\ldots\tau$. The Fourier transform is a Cauchy distribution

$$(\mathcal{F}L)(x,\tau) = \frac{1}{\sqrt{2\pi}\,(1+f^2\tau^2)},$$

which can be used as a filter for Gaussian white noise to generate random instances. This generates a random signal with autocorrelation function (the Laplace distribution convolved with itself)

$$R(x,\tau) = e^{-\frac{|x|}{\tau}}(1+\frac{|x|}{\tau}),$$

which has a width at half height of $3.357\ldots\tau$. Its spectrum is (notice that $\tau\mathrm{d}f$ is dimensionless)

$$(\mathcal{F}R)(x,\tau)\,\mathrm{d}f = \frac{\sqrt{\frac{8}{\pi}}}{(1+f^2\tau^2)^2}\,\tau\mathrm{d}f,$$

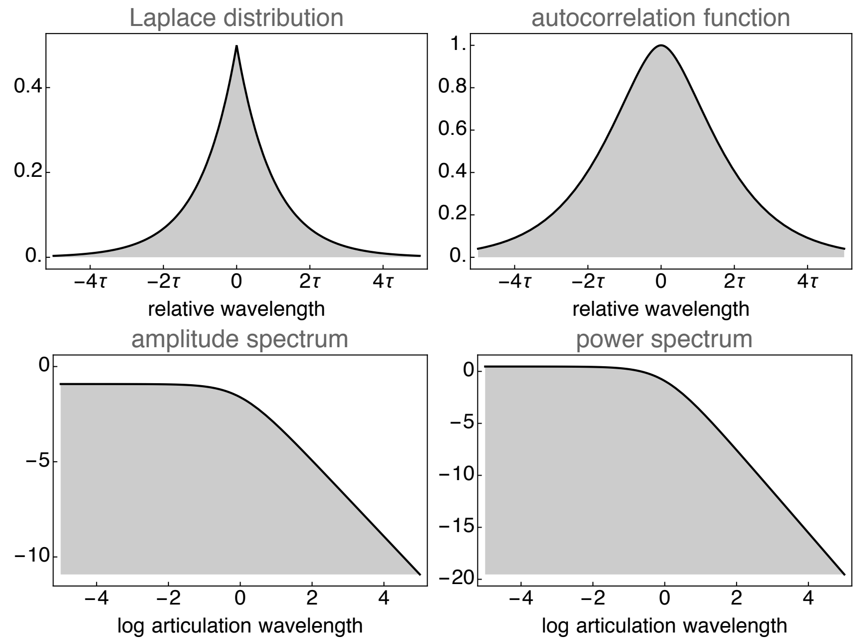
which falls off as f^{-4}, just as estimated from the databases.

So for a start, all one needs to do is estimate the parameter τ. An apt value is about 110nm. If that estimate holds roughly true, it means that the spectral

*"Phenomenological" in the sense of classical physics.

resolution of human vision approximately matches the autocorrelation width of the spectral articulations of natural spectra.

This immediately yields the simplest model for spectral reflectance factors. One does the inverse $\log(K/S)$ transform on a Gaussian process with a broad autocorrelation function. One also needs to set the mean and the standard deviation to reasonable values.

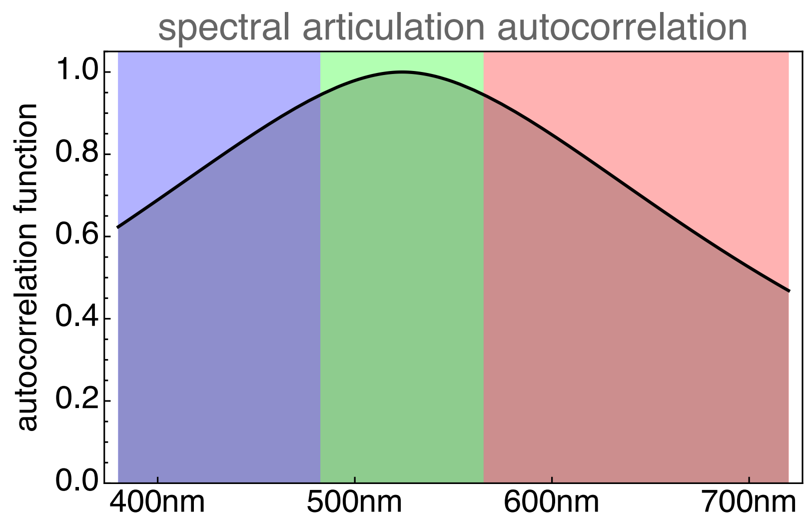


The various functions involved in the modelling of the generic articulation spectrum. (Top row linear–linear, bottom row log–log plots.)

This works very well indeed. The simple model is a random spectral reflectance factor generator that yields instances that appear immediately "natural". Of course, one needs to check a number of statistical measures to back this up. There is no reason to emulate particular databases, one wants to generate instances that will pass a check of "belonging" (albeit perhaps marginally) for all the databases one can find.

The model can easily be "tuned" to do even better. For instance, in most databases one notices a mean overall spectral slope. Such observations can easily be incorporated in the model without essential changes.

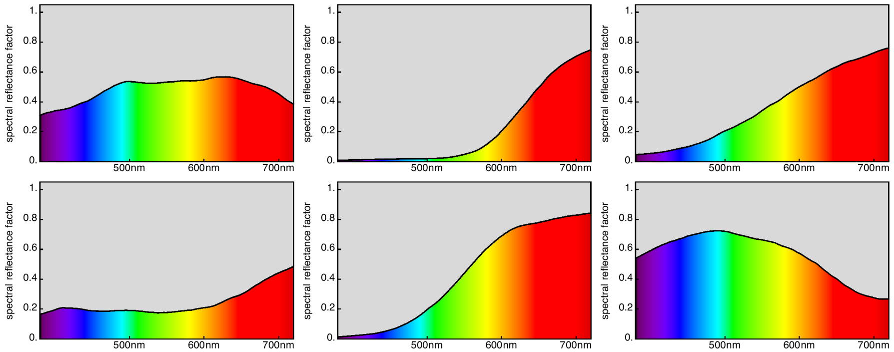
Estimates can be obtained from spectral databases, but also from RGB–images. The model applies very well in all cases I studied, with essentially the same (or at least very similar) parameters.



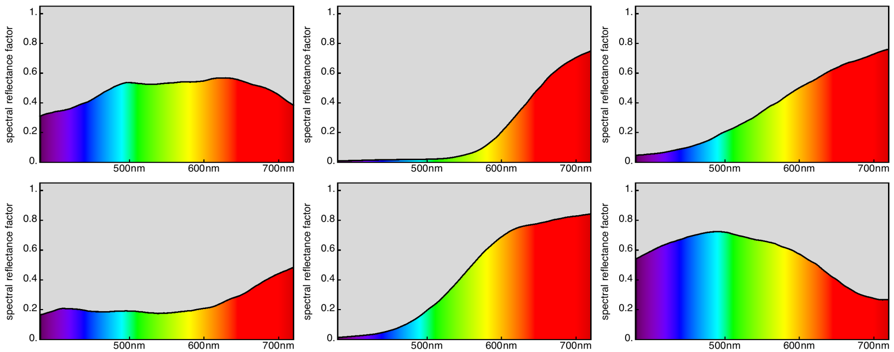
This is an estimate of the spectral articulation autocorrelation function, arbitrarily centred. (Compare page 67.)

Constructing random illuminants is simpler. In principle one needs smooth functions that remain in the general neighbourhood of daylight. The physical causes are various: time of day, weather, scattering from the environment. The simplest model perturbs a thermal spectrum with smooth random functions in the physical domain. Such functions are easily generated as the sum of the Legendre polynomials of order $1 \ldots 4$ (say) with random coefficients drawn from normal distributions with standard deviations in the ratio $1 : 1/2 : 1/3 : 1/4$ (say). Such a simple method yield results that closely resemble the daylight spectra I discussed earlier.

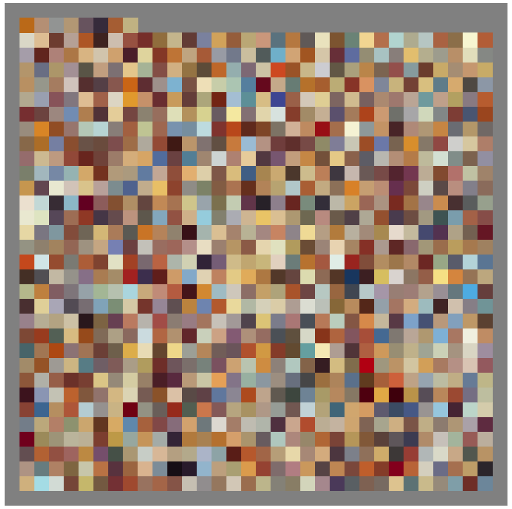
The chromaticities of such illuminants cluster about the standard white and—not unexpectedly—about the Planckean locus. However, they also scatter away from that locus in the green–purple dimension.



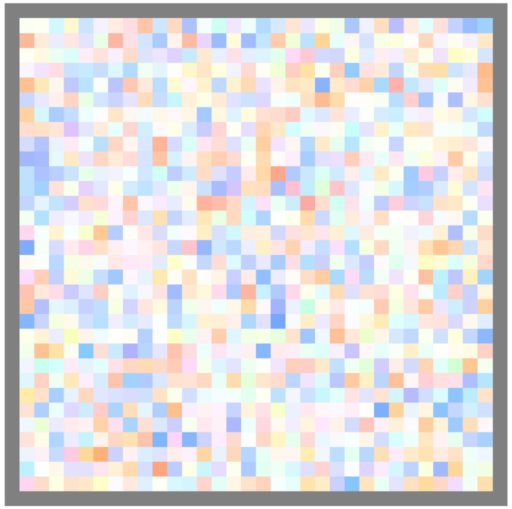
Here are some synthetic random spectral reflectance factors. One readily generates a hundred a second on a regular laptop. These spectra have statistics that are very similar to those of the relevant databases. One checks the overall power spectrum, spectral slope and variance. Converted to RGB*–colors for a standard illuminant, one checks the histograms of the* RGB*–coordinate values and the distributions in the* RGB*–cube and the* RGB*-chromaticity triangle.*



These are some synthetic random illuminant spectra. One readily generates a thousands a second on a regular laptop. These spectra have statistics that are similar to those of the relevant databases. On the average they are a 6500°K *thermal radiator spectrum. The deviations are mainly in overall slope and curvature.*

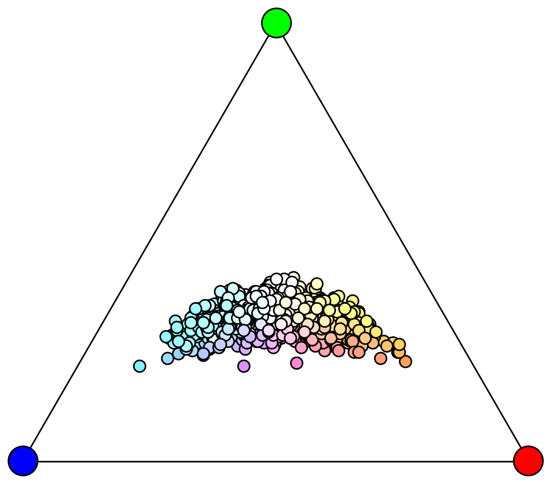


Here are a thousand synthesised spectral reflectance factors under normal illumination.



Here are a thousand synthetic illuminants (I switched off the von Kries coefficients law for the occasion).

It is very hard to say whether such a model captures the “actual” distribution, simply because there is no way one could make a comparison. It might be thought that the main variation would be in the nature of daylight during an “average day”. The CIE even provides a pair of principal components to capture that.



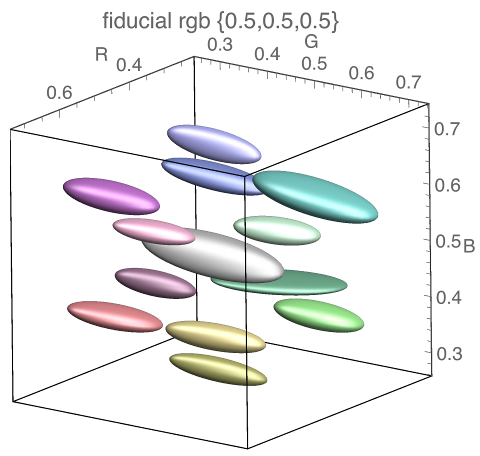
Chromaticities for a set of one hundred random illuminants in the general neighbourhood of a $6500°\mathrm{K}$ *thermal radiator.*

However, there are many more causes, different from mere meteorological, that influence the illuminant spectrum in the grasp space of a hunter–gatherer. In the grasp space the spectrum is highly influenced by scattering of radiation from the environment. So soil, rock and vegetation cause the effective illuminant to be different from mere sky-light.

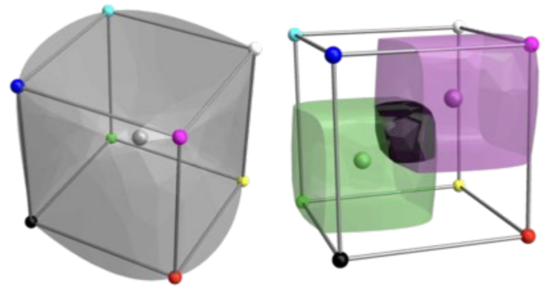
The simple model proposed here is probably a good starting point, although a number of obvious “tunings” might be used to refine it. That is a major task and not likely to change the final conclusions very much.

With these two models one conveniently studies the influence of metamerism on the grain of human colour vision with respect to recognition.

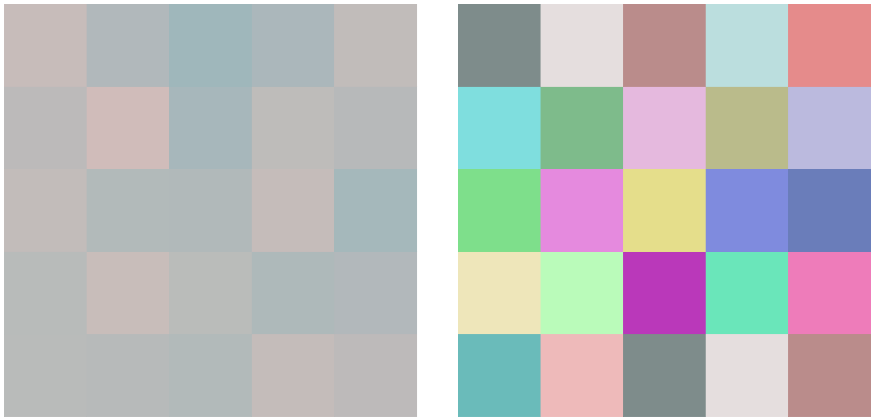
The results are not essentially different from what I found using the Finnish databases. That makes sense, because the statistical structure has been set up to mimic these (and various other databases). However, it is far more convenient to do experiments with such synthetic data.



This is a field of local grains (estimated from only 32^2 *samples per point), in the neighbourhood of* $r = 0.5, g = 0.5, b = 0.5$, *for a synthetic ecology. Here the grains are plotted at three times their actual size, for the sake of clarity. Apparently there occur regional differences, thus a more detailed analysis seems indicated. (Notice that this shows only part of the RGB–cube!)*



I constructed a really bad world (page 89), so bad that the ecological probability of finding yourself there is zero. In such a freaky, but possible world, human colour vision is worse than useless. In the image at left the grey volume has colours that are not categorically different from average grey, in the sense that there exist (freaky!) illuminants and reflectances that makes them all look equal. In the image at right I show this for a green and a purple. Since the volumes overlap, there exist colours that—for appropriate (freaky) reflectances and illuminants look either like the green or the purple. In such a bad world colours are not “properties of a objects”. The Neanderthaler would be unable to grok that and would be better off to ignore colour as a property altogether. Fortunately, the hunter–gatherer ecology is not like that. Not by a long shot! Our roots, remember? Unconstrained colorimetry (the CIE’s pride) leads you far astray.



At left colours that might not be different under generic ecological conditions, at right colours that would not necessarily be different in the bad–ass world I illustrated in the previous figure. The difference is huge *(infinite if you want!). The ecologically okay case on the left one might believe . . . , but only because of our common hunter–gatherer roots. But the bad–ass example—never! That looks simply not reasonable, it defeats intuitive human understanding. This teaches you that what is "possible" need not be "reasonable" (= generic in the biological sense).*

An investigation of the grain structure in the neighborhood of the centre, or even the full RGB–can be done in seconds and great precision because one has metamers to spare. In the example one sees that the grains are not isotropic, the axes have lengths in (roughly) the ratios $10:3:2$, they are elongated along the green–purplishRed axis.[*] Thus the resolution is best in brightness (along the grey axis) and worst in the green–purplishRed. There are also noticeable spatial non–uniformities in grain-size. So this type of analysis yields a wealth of interesting data. These are specific for the combination of visual pigments (the physiology), the significant objects and the illuminants (the ecology). Here the constraints reflect the hunter–gatherer life-style (the ethology). It is a *biological* analysis.

[*]Not surprisingly, the results are not essentially different for the actual databases.

No doubt it is feasible to develop such methods to apply to the ecological niches of the modern Western urban environment. The methods discussed here have obvious potential applications.

It is of some interest to notice that this analysis depends crucially on the ethology/ecology, it is indeed a truly *biological* analysis. In the general case, preferred by physics, colorimetry and (odd enough!) most of vision science, one considers *all possible worlds*. Here "possible" implies anything admitted by physics.

It is not hard to construct possible worlds in which human colour vision is essentially useless (▷ p.89). The number of categorically distinct colours then shrinks from about a thousand ("ecologically valid") to just *one* (meaning *no discriminatory power at all*). However, such bad–ass worlds contain strange illuminants, strange spectral reflectances and—even more unlikely—spooky correlations between these. Modern technology is not even up to the task of constructing such worlds in hardware, although this should be possible "in principle". The ecological probability of ever finding yourself in such a freak world is virtually zero.

Good thing too! For this has to do with a very important issue, namely whether colours may be considered *properties of objects*. The "official" answer is an obvious *no*! I prove that in the annex. But the biologically relevant answer is *yes, why not?*" Indeed, there appear to be no ecologically valid cases for which this fails.[†]

So, in practice, the (potential) existence of bad possible worlds renders the "general case" fully irrelevant to the understanding of human colour vision, or biological vision in general. That is an important insight, not often suggested in textbooks on colorimetry. Yet it is exactly why evolution equipped us with colour vision in the first place!

⋆ ⋆ ⋆

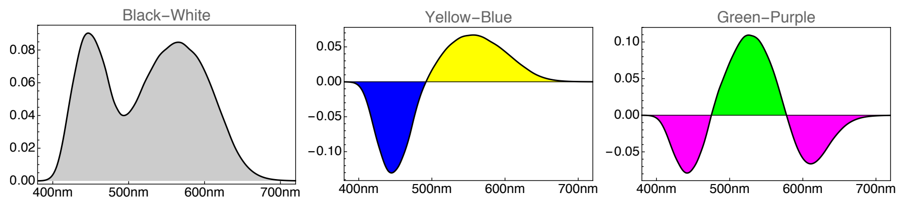
[†]Alexandrite perhaps? Perhaps. But, if so, you can't eat it and probably never saw it. That should convince the Neanderthaler of the irrelevance of the argument.

The anisotropy of the RGB–cube

THE NOTION OF THE PARTS OF WHITE puts equal emphasis on all three parts. Indeed, in Schopenhauer's discussion it is not really relevant that green is *spectrally* between red and blue. From the perspective of physics that cannot be. *Red is spectrally more remote from blue than from green.* Thus the physicist expects the red to be more strongly correlated with the green than with the blue. This is a very generic argument.

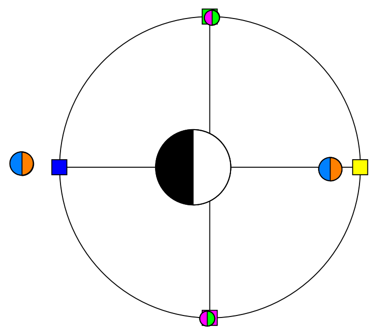
There are also incidental arguments to expect anisotropies. The illuminants are mainly distributed about the locus of thermal radiations, thus the yellow–turquoise direction must be special.

Such anisotropies show up in the metamerism studies and in the covariance ellipsoids collected in the bean count operation as well (lower–right figure on page 55). One almost universally finds a characteristic pattern in the covariance ellipsoids of RGB data. Simply computing the covariance matrix for the RGB coordinates of the pixels of some photograph—almost any picture—will show that pattern (compare page 67).



The "opponent channels" are simple linear combinations of the RGB colour matching functions. They measure the level, slope and curvature of the spectrum. Thus they are mutually almost uncorrelated, unlike the RGB channels themselves, which are strongly correlated because of the width of the spectral articulation autocorrelation function.
The black–white opponency may puzzle you. It may help to remember that you have black afterimages from white and white after images from black. Moreover, the average of black and white is the same as the average of yellow and blue, or of green and purple, namely the centre of the RGB–cube.

Perhaps surprisingly, this was also proposed in a purely phenomenological account by Ewald Hering[*] in the second half of the nineteenth century. Hering suggested that colours can be understood in terms of three mutually independent systems of *Gegenfarben* (usually translated as "opponent processes"). The processes would be black–white, blue–yellow and red–green. Hering's strongest argument was that you cannot even imagine a "yellowish blue" or a "bluish yellow" and so forth, In the past this was misunderstood as being based on *four* basic colours (yellow, green, blue, red) instead of three (red, green, blue). *Vierfarbentheorie* instead of *Dreifarbentheorie*! It took Schrödinger to put an end to this nonsensical discussion (mainly between students of Helmholtz and those of Hering).[†]



This shows a stereographic projection of the space of mean eigenvectors of the Finnish flowers database. (The circle bounds one hemisphere, because the eigenvectors are only orientations—not directions—the two hemispheres are mutually congruent.) They are very close to $\{1,1,1\}/\sqrt{3}$ *(black–white),* $\{1,0,-1\}/\sqrt{2}$ *(red–blue) and* $\{-1,2,-1\}\sqrt{6}$ *(green–purple; the sizes reflect the eigenvalues). This is entirely typical for spectral databases and images alike.*

The opponent processes can be formalised in terms of the RGB colour matching functions, by transforming $\{R, G, B\}$ into the linear combinations $R+G+B$, $R-G$ and $2G-(R+B)$. These channels carry signals that are hardly mutually correlated, unlike the red, green and blue parts of white, which are strongly correlated. Almost any spectral database of "natural" objects and any image of "natural" scenes will reflect this general pattern. It is a major fact of ecology.

[*]Ewald Hering (1878), *Zur Lehre vom Lichtsinne*

[†]Schrödinger (1925), *Über das Verhältnis der Vierfarben- zur Dreifarben–theorie*, Stz.ber. Akad. Wiss. (Wien) Mathematisch-Naturwissenschaftliche Klasse 134, 471–490.

The stereographical representation of the eigenvectors[*] of the Finnish flowers data base illustrates that very well. You will find essentially the same pattern for almost any database or image. It is an immediate reflection of the spectral autocorrelation typical for the natural environment. Small wonder that the phenomenology of colour vision reflects this ecological invariant.

Here phenomenology hits on an ecologically important fact that remained unrecognised in the "hard" colour science community. The Hering *Gegenfarben* are indeed a necessary addendum to the Schopenhauer *parts of white*. They are very much related though, because, like the parts of white, the opponent colours depend on the daylight anchoring. In phenomenology all colours are "related", they require the RGB–cube as defined by the white point due to the present illuminant. Of course, it only works when the relations are *manifest*, that is to say, optically present in the field of view.[†]

⋆ ⋆ ⋆

[*]Let the eigenvectors be $\mathbf{e}_1 = \{1,1,1\}/\sqrt{3}$ (black–white), $\mathbf{e}_2 = \{1,0,-1\}/\sqrt{2}$ (red–green) and $\mathbf{e}_3 = \{-1,2,-1\}\sqrt{6}$ (green–purple), an arbitrary direction the combination $\mathbf{c} = u\mathbf{e}_1 + v\mathbf{e}_2 + w\mathbf{e}_3$ with $u^2+v^2+w^2=1$, then the stereographic projection has the planar coordinates $\{\xi,\eta\} = \{v,w\}/(1+u)$. The size of the symbol in the projection can be used to indicate the eigenvalue. This is a very useful plot to characterise the nature of a gamut, although I have never seen it used in the literature.

[†]Don't forget that the full field of view will usually divide into partitions that each have their own anchoring. The most common example of this is the partitioning into directly illuminated and shadowed parts in a sunlit scene. The demarcation is generally known as "the bed–bug–line" by visual artists. ("Terminator" is the term to use in polite company.)

RGB–spectra all the way

SPECTRA are physical objects found in the environment. However, the human umwelt only knows colours, psychophysically represented—stripped from their qualitative character—as colorimetric RGB–coordinates. Photographic images make do with RGB–pixels, which can be regarded as physical records, or as physical structures sampled by the visual system. Although we are faced with ontologically distinct objects, I venture to say that they are formally not all that different.

First of all, the colorimetric RGB–coordinates may well be considered *spectra*, the only difference with "real spectra" being spectral resolution. Thus RGB–coordinates are nothing but three–bin spectra (see page 26). Having only three bins does not render them "less spectra", indeed, the "real spectra" used in this book have only 68–bins and cutting that in half wouldn't make a difference. There are no infinitesimally determined spectra, that is even formally impossible. Power spectra are *necessarily* of finite resolution. Three bins isn't that bad either, it allows you to judge *level*, *slope* and *curvature*.

If the colorimetric coordinates are really spectra, then the colours are apparently spectra in qualitative guise: *"we see spectra" in a sense that is not at essentially different from "seeing shapes"!*

Likewise the images are (spatially ordered) databases of spectra too, again three–bin spectra. In this case there is no problem with qualia, the pixels really *are* spectra in the most obvious sense.

Thus we actually have spectra all the way: in the physical environment, the umwelt, the images and visual awareness.

In my view this renders colour science much more transparent than the way the CIE would have it. It also suggests that the RGB–colorimetric coordinates are to be preferred in generic contexts as yielding the simplest, natural setting. The CIE setting does not recognise daylight (or sunlight) as proper part of vision. Many of Goethe's polemics against Newton apply here.

Of course, there may be good reasons to use different representations in various contexts, but it asks for some rational excuse.

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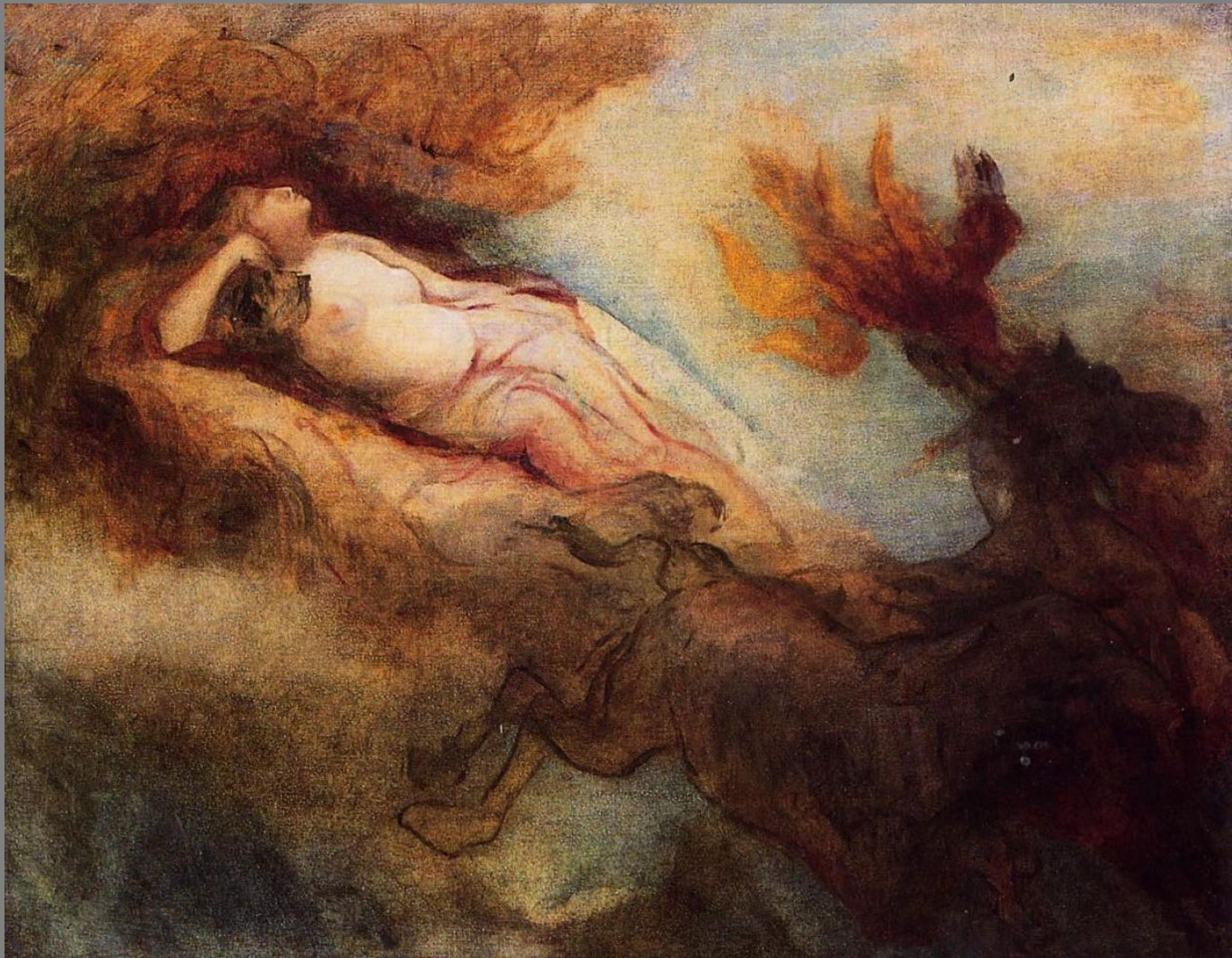
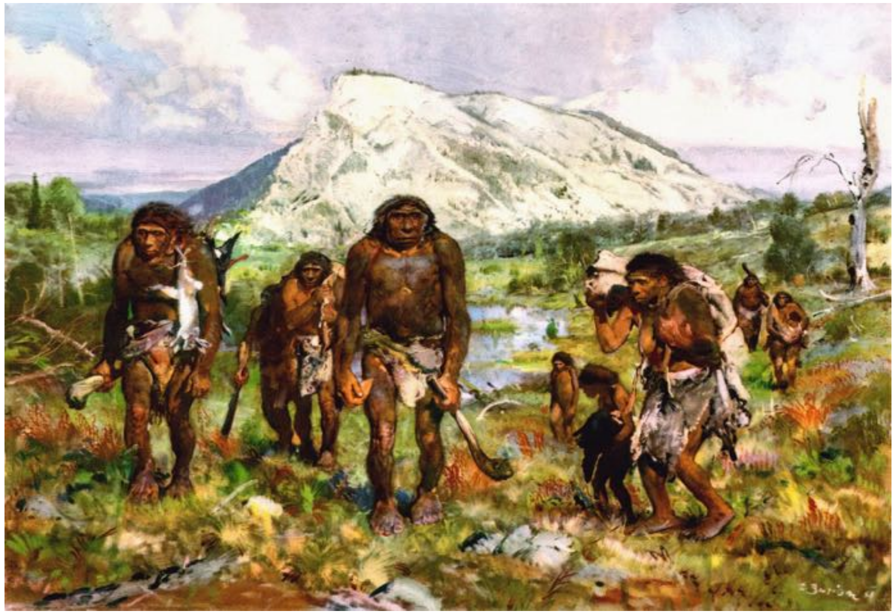


So how many colours are there?

IS THERE A “FINAL ANSWER?” In biology there cannot be, but we can certainly say something reasonable about the matter. First of all, the question “How many colours are there?” is a question that can only find an answer in a biological context. As I said upfront, colour vision serves essentially two objectives:

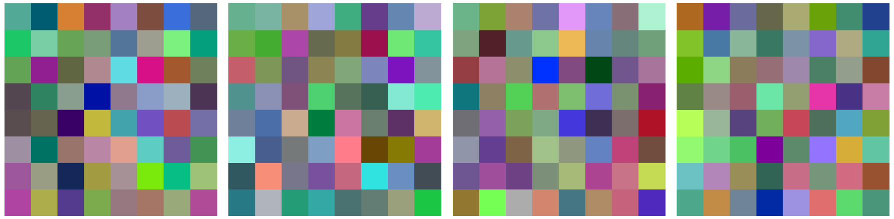


Zdenek Burian (1958), “Moving to new hunting grounds”. These Neanderthals live in social groups, like gorillas, led by an alpha–male. Different from gorillas, they carry foods for on the road, perhaps they carry fire and they carry tools. One blow with such a club will smash your skull, they understood the power of leverage. They no doubt mutually communicated and thought for themselves, although perhaps not in spoken language. Their “sedimented experience” must have approximated a proto-conceptual level.

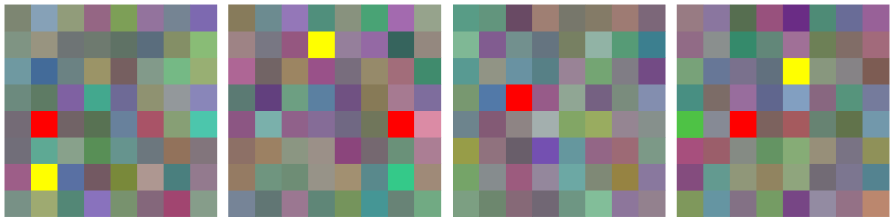
SEGMENTATION of the the visual field into optically distinct segments;

RECOGNITION of segments in the visual field on the basis of optical cues.

Both are important.



Segregation *makes you see the “array of squares”. The geometry is perfectly clear and identical in the four images. It is fully irrelevant what the colours are, as long as any two neighbours are sufficiently distinct. (Notice various instances where smallish differences almost lead to “grouping” of squares unto elongated rectangles and so forth!) You don’t even have to notice the actual colours, you’ll never “miss” any particular colour. All awareness presents you with is “a multicoloured lattice of squares”*



Recognition *implies a unique label. Here the red and yellow squares are easily traced in each image, although they occur at random locations. (In the third image you sorely “miss” the yellow square, where might it be? Hiding behind the red one? Left the arena?) No way you will ever mix up the two either. One square is “red” the other is “yellow”, these are rock-solid labels.*

Both objectives are served by an increase in the dimension of colour space. If there is some given number of discriminable steps per dimension, say m then adding an additional dimension yields a boost in discriminability by a factor of m. That is evidently desirable (m being at least ten or so) up till the point where the system is "good enough". For humans the dimension is three, many mammals doing fine with two, so there is unlikely to be a strong pressure to increase it.

Notice that increasing discriminability boosts segmentation greatly since the number of discriminable colours is m^3. If m is estimated from psychophysics (about two hundred) one arrives at the so often quoted numbers of millions of colours, numbers that are biologically meaningless.

Praxis and bean counting suggest a number of about a thousand, implying $m \approx 10$, *much* lower than the psychophysical JNDs suggest. So discrimination in praxis, or action, is much lower than what the physiology offers. Visual awareness wastes resolution by three to four orders of magnitude! Why is that?

The answer is to be found with the requirements for *recognition*. Robust recognition requires colour to be a label that will not be confused with other labels when the light changes. Recognition is limited by the effects of metamerism.

At this point one needs to bring the nature of the umwelt into the equation. The effects of metamerism depend crucially on the set of relevant objects and the set of relevant illuminants. That implies that one needs ecological statistics tuned to the umwelt and lifestyle of the agent. In this case savanna and tundras sustaining a hunter–gatherer life.

Reasonable estimates on this basis immediately yield numbers that fit the bean counts. Apparently the system wastes some fine physiology (very useful in segregation) because it is irrelevant to recognition tasks.

It would seem that *much of segregation happens in pre-awareness*. Visual awareness is simply presented with the result, which is a kind of skeleton spatio–temporal structure. On the other hand the *recognition related labels appear as qualia in awareness* because they have to make their way into the system of habits and types, Husserl's "sedimented experience". In *Homo sapiens* they often make it into reflective thought and even receive some linguistic labels.

Reflective thought and linguistic labels are in no way required to conduct a successful life as an animal or a hunter–gatherer though. Then "red" is part of types such as "blood" and many others, not necessarily differentiated from additional (not even visual) parts of the type.

Psychogenesis of awareness starts from a current "situational awareness" (the term often used in psychology) that is only a pre-awareness or vague anticipation that lets psychogenesis ignore much and hunt for a focal, very vaguely defined, family of objects or processes. This system is of vital importance to cut down on the complexities of the umwelt. It is ultimately based on feelings, smells, sounds, . . . , that are perhaps never presented in awareness. All that is in the mind are the presentations. The Kantian phantasmic self–stimulations are of this type.

Becoming aware is a very different process from *being aware*.

No doubt, most of the brain action deals with becoming aware and with the automatic control of action. This involves numerous modal and multimodal structures in the blackboards (continuously overwritten by new structure distilled from what goes on at the sensitive body surfaces) that never make it to qualia or even presentations. Such structures play a role in the (involuntary) planning and guidance of action patterns and in building anticipations that guide the psychogenesis of awareness without appearing in awareness as such. Structures are meaningless in themselves, although they play a key role in efficacious actions.

Most of animal actions are meaningless in this sense—that is to say, they simply "happen", involuntarily—although they may enter awareness after the fact and acquire a meaning then.

Thus stimulation of the sensitive body surfaces need not make it into presentations at all in order to be effective. Perhaps most of the brain subserves actions guided by such stimulations that are fully involuntary. The *segmentation* possibilities offered by the visual system may be crucial in that. These processes, or really their results, sometimes enter awareness after the fact. For instance they are useful in monitoring the present space–time envelope that is somehow present as a kind of skeleton in the back of the mind.

Being aware involves the construction of presentations that are the concrete actuality, the manifest NOW. Its elements are user interface templates that result from controlled hallucinations. They are meaningful *because* constructed. Remember Vico: VERUM FACTUM. All you can ever *know* (in the sense of "possess" perhaps) is what you *made* yourself. For meaning cannot be "found" or "computed", but has to be *imposed*.

The visual presentations crucially involve the *recognition* system, albeit in a pre–conceptual (perhaps proto–conceptual) and pre–linguistic mode.

The qualia in the colour domain, the "redness" of red, the "blueness" of blue and so forth, are user interface elements, a kind of adaptable templates. They result from their role in voluntary efficacious actions, acquired over a life on actions fired up by primordial urges and the constraints offered by the environment. Eventually they may acquire a role in communication and become frozen in symbols—at that time they are colourless in the presentational sense.

These considerations result from an understanding of humans as animals (first of all vertebrates, then mammals, then primates, finally *Homo sapiens*).

This is not a perspective commonly taken in psychology. One really would like to have a human ethology, but even that would not suffice. One would also need a heuristic to link the ethology to the *Innenwelt* ("inner world"), a term proposed by Jakob von Uexküll.

Perhaps unfortunately, such a link between ethology and phenomenology is sadly wanting.

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Early Humans and Computer Vision/Graphics

MOST LIKELY THE NEANDERTHALS did not use computers or electronic displays. So why be interested in their vision? Well, first of all, *our* vision is pretty much *their* vision. For, indeed, our vision is deeply rooted in the needs of the hunter–gather umwelt. We don't look so much like the Neanderthals, but probably more like Cro-Magnon, who appeared about when the Neanderthals died out. The ecology must have been pretty similar, it makes not much of a difference to the arguments in this book.

Any user–interface, whether a Palaeolithic Cro–Magnon cave painting of 40 000 years ago, or a fancy OLED–display of tomorrow should acknowledge that. Indeed, as I showed before, displays all converge on the optimum RGB–space of human vision.

CV (Computer Vision) has emulated many of the methods apparently used in the psychogenesis of human visual awareness. In some cases CV does better than human vision, typically by using technology that is not available to biological systems. However, it is probably fair to say that biological systems are on the whole far more robust and adaptable than robotic systems. CV has still much to learn from the early man's vision.

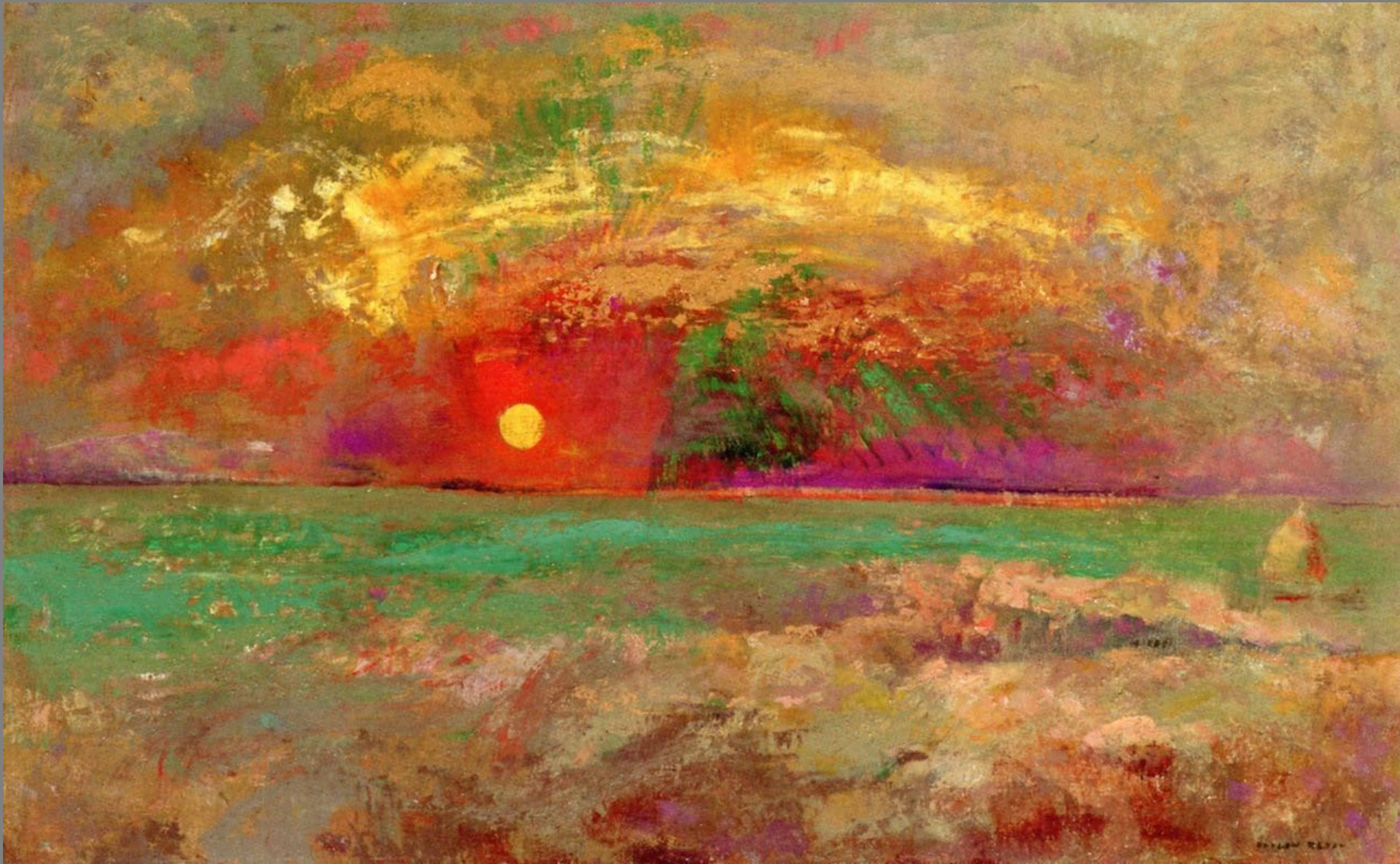
CG (Computer Graphics) is in a different ball park, because its products are for consumption by human observers. It is the human optical interface that needs to be addressed, not the "pixel peeper". Thus CG *has* to take the quirks of human vision into account, or it will fail.

It is commonly believed that it is the task of CG to emulate photography, which means it has to emulate the physics of the environment and of the imaging of the environment. This is probably a bad idea.

Good realistic paintings tend to look (not only much nicer, but) more *real* than even the best CG. Painters achieve that on the guts, burning almost no computer power and coming up with far fewer "pixels". But the painter's "pixels" (read: degrees of freedom) count for much more than the CG pixels.

There is much to gain from a close study of hunter–gatherer–style visual awareness, which is not too different from the painter's visual awareness.

Cro-Magnon hunter of the upper Palaeolithic, by Zdenek Burian. The Cro-Magnons were the first early anatomically modern humans (early Homo sapiens sapiens*). They lived in the European Upper Palaeolithic, about 50 000 years ago. They may have been the painters of the Lascaux cave. Of course, such a painting as this is a romantic interpretation. There so much we don't know for sure.*



Some technical details

THIS BOOK was written with a summer course in mind. For the convenience of the students I add some pointers to technical details and to literature that might fill the (numerous) lacunæ in this far too short text. It is in no way a scholarly bibliography. I concentrate on a few points that might need detailing and on texts that might increase understanding of the basic concepts, the relevant formal tools and useful data sources. I keep it short.

The sources I recommend maybe in languages that you don't fancy. I'm pretty sure that you will be able to find translations for any of them. I hate translations (they usually manage to get the meaning wrong), so I don't read them if I can avoid that. The unfortunate consequence is that I am in no position to suggest "the better" translations. Thus read for yourself and see whether the text somehow gives you good vibes. Even very inaccurate translations may trigger good ideas in you, like we see wonderful visions in the clouds.

You probably know this better than I do, but don't forget the Internet! Any term in the book will lead to additional material if you Google it. First try for possible wiki's. This is likely to solve most of your immediate problems in reading the text.

Remember that the Internet does not necessarily distribute correct, complete and unbiased concepts or facts. Think for yourself and always compare multiple (independent!) sources.

And yes, don't believe books either, nor academic papers, nor experts, don't believe *anything* or *anyone* but your own best judgment. On the other hand, take notice of anything you can find or hear, especially if it is different from your own thoughts. You may be deluded about anything!

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Okay, so here are a few leads that you might find useful.

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ALTHOUGH there does not exist any causal relation between wavelengths and "seen colours" (quale), it is often useful to be able to have some make-belief relation. For instance, this happens frequently if you need to visualise something. I'm obviously guilty of using such a method at various places in this book.

Since you need just a heuristic, methods are up for grabs. One possibility is to look up wavelengths quoted for "absolute yellow" and so forth and do some interpolation. There are algorithms, due to Andrew Young (see `https://aty.sdsu.edu/explain/optics/rendering.html`) and to Dan Bruton (see `http://www.physics.sfasu.edu/astro/color/spectra.html`). Enjoy!

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SHOWING object colours (as "chips", say), is also a common requirement in visualisation. Usually you have RGB–coordinates. In order to visualise the coordinates you first clip them to the unit interval (if you're stuck with $0 \ldots 255$ you know what to do) and raise the coordinates to the power $1/\gamma$, where you can assume a value $\gamma = 2.2$. This yields good results.

Of course, be wary of clipping! If you find that you clip more than the flimsiest fluff, it is time to look for the cause and fix it in a well considered manner. Never clip so as to hide nonsense!

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THERMAL RADIATION is described by Planck's formula. This looks impressive: the spectral radiance of a body for wavelength λ at absolute temperature T is given by

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1},$$

where k_B is the Boltzmann constant, h is the Planck constant, and c is the speed of light in vacuum. Of course, you can rewrite this for frequencies, or photon energies and numbers.

This yields a unimodal distribution, the position of the maximum being given by Wien's displacement law

$$\lambda_{\max} = \frac{b}{T}, \text{where} \quad b = 2.898\ldots 10^6 \text{nm K},$$

(the analytic solution involves the Lambert W function and yields little insight).

The total radiant energy by the Stefan–Boltzmann law is

$$\sigma = \frac{2k_B^4 \pi^5}{15c^2h^3}T^4.$$

. If you are curious how anyone ever thought of that, you're in for a physics course. I suggest:

- Richard Feynman, Leighton, R., & Sands, M. (1963–1965). *The Feynman lectures on physics*, (Library of Congress Catalog Card No. 63–20717), Reading, Mass: Addison-Wesley Pub. Co.

Thermal radiation is random noise, the correlation length is roughly a wavelength. Reading

- Axel Donges (1998). *The coherence length of black-body radiation*, Eur. J. Phys. 19, 245–249.

will make you understand this.

Not that it is important in my context, but it proves that the Newtonian spectrum is far from being a superposition of monochromatic components, as is often assumed thoughtlessly by non-physicists.

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ECOLOGICAL OPTICS is a huge and (necessarily) ill defined topic. One book that is a must-read is

- Marcel Minnaert (1954). *The nature of light & colour in the open air*, Dover.

I'm sure you will read Minnaert with pleasure.

A journal that regularly carries interesting material is Applied Optics. Much recommended to scan issues every year.

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KUBELKA–MUNK analysis is an approximate (and very practical!) solution for the diffusion of radiation in stratified media. It is extensively used in industrial applications (papers, paints and so forth). In this book I only allude to the tip of the iceberg.

It is not hard to get up to steam. Read

- Kubelka, P., & Munk, P. K. F. (1931). *An article on optics of paint layers*, Zeitschrift für technische Physik, 12, 593–609.

Summarily, consider a medium characterised through two physical constants (defined for a particular wavelength of monochromatic radiation):

- K is the Absorption Coefficient, that is the fraction of radiant power absorbed per unit layer thickness
- S is the Scattering Coefficient, that is the fraction of radiant scattered backwards per unit thickness

One only considers variation in the dimension orthogonal to the layer, say the X–direction. All radiant power fluxes are assumed to be diffuse. The layer is irradiated by a radiant flux I_0, the scattered flux is I. The reflectance factor is $R = I/I_0$. The reflectance factor depends upon the thickness of the layer and the reflectance of the substrate. For an infinite thick layer (a half–space) one has R_∞, which is independent of the substrate.

Inside the medium one distinguishes a flux i_T in the incident direction and a flux i_R in the opposite direction. For an infinitesimal layer of thickness $\mathrm{d}x$ one has the balance equations

$$\begin{aligned} -\mathrm{d}i_T &= -(S+K)i_T\mathrm{d}x + i_R\,\mathrm{d}x, \\ \mathrm{d}i_R &= -(S+K)i_R\mathrm{d}x + i_T\,\mathrm{d}x, \end{aligned}$$

These first order coupled differential equations are easily solved using hyperbolic functions. One has:

$$R_\infty = 1 + \frac{K}{S} - \sqrt{\left(\frac{K}{S}\right)^2 - 2\frac{K}{S}},$$

from which we obtain

$$\frac{K}{S} = \frac{(R_\infty - 1)^2}{2R_\infty},$$

the "spectral signature" used in the text. It is apparently a material property.

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HOMOMORPHIC FILTERING is a simple, but very important and effective technique. The paper that popularised the method (I think) was:

- Oppenheim, A., Schafer, R., & Stockham, T. (1968). *Nonlinear filtering of multiplied and convolved signals*, Proceedings of the IEEE, 56, 1264–1291.

The essential idea is simple enough. If your "natural operation" is multiplication, whereas your methods require addition, you simply move to the logarithmic domain. That is "homomorphic filtering", the actually filtering is done in an appropriate domain, whereas you work in another, conventional (but not quite appropriate) domain.

What is essential here is to know the appropriate domain. This requires good horse sense, as is often found with old-timers in (classical) physics.

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THE PHYSICAL DOMAIN is the natural setting for non–negative physical parameters. This is very nicely explained by Edwin Jaynes in

- Edwin Jaynes (1968). *Prior probabilities*, IEEE Transactions on Systems Science and Cybernetics, 4, 227–241.

I use the idea all the time in this book. Finding natural domains is similar to setting up good Bayes priors, the original ideas were due to Jeffreys. A sign that the domain is "natural" tends to be that relevant samples are normally distributed.

An instance that finds frequent application is that of positive physical parameters. Here the natural domain is the logarithmic domain, the non–informative prior being hyperbolic:

$$P(x) \propto \frac{\mathrm{d}x}{x} = \mathrm{d}\log x,$$

thus lacking a "unit" magnitude, as it should be.

For more complicated cases, like the spectral reflectance factor, one needs to apply some understanding of the relevant physics, such as Kubelka–Munk analysis.

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ETHOLOGY is a subfield of biology that was established with the Nobel Prize in Physiology or Medicine 1973 awarded to Konrad Lorenz, Niko Tinbergen and Karl von Frisch. In reality I would say that the actual "Father of Ethology" is Jakob von Uexküll. About von Uexküll read

- Bernhard Hassenstein (2001). *JAKOB VON UEXKÜLL (1864-1944)*, In: Jahn, Ilse und Schmitt, Michael (Hrsg): Darwin & Co. Eine Geschichte der Biologie in Portraits. Band II. München: Verlag C.H.Beck.

Von Uexküll himself is fun to read:

- Jakob von Uexküll (1921). *Umwelt und Innenwelt der Tiere*, 2. Aufl., Berlin: Springer.
- Jakob von Uexküll (1928). *Theoretische Biologie*, 2. Aufl., Berlin: Springer.
- Jakob von Uexküll, & Kriszat, G. (1956). *Streifzge durch die Umwelten von Tieren und Menschen, und: Bedeutungslehre*, Reinbeck: Rowohlt.
- Jakob von Uexküll (1957). *Nie geschaute Welten*, München.: Paul List Verlag.

In order to understand the impact of evolution and ethology the best source is Rupert Riedl. It makes a good start and (especially the second book listed here) makes a good read too:

- Rupert Riedl (1978). *Order in Living Organisms: A Systems Analysis of Evolution*, New!York: Wiley.
- Rupert Riedl (1984). *Biology of Knowledge: The Evolutionary Basis of Reason*, Chichester: John Wiley & Sons.

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THE EVOLUTION of human colour has generated a huge literature. I list a modern text and one on a historical issue. Use both to make your way into the topic by way of the Internet:

- Jeremy Nathans (1999). *The Evolution and Physiology of Human Color Vision: Insights from Molecular Genetic Studies of Visual Pigments*,

Neuron, Vol. 24, 299–312.
- Jeremy Kargon (2014). *The Logic of Color: Theory and Graphics in Christine Ladd-Franklin's Explanation of Color Vision*, Leonardo, Volume 47, Number, 151–157.

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ON PHYSIOLOGY David Hubel's book has become a classic "read first".
- David Hubel (1995). *Eye, brain, and vision*, New York: Scientific American Library.

The literature is beyond human comprehension. If you are a "general reader" the Internet makes a good start. A modern textbook is
- Dale Purves, G.J.Augustine (2011). *Neuroscience*, Oxford University Press.

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ON PSYCHOPHYSICS there is also too much to mention. If you are a "general reader" I would recommend two classics:
- Hermann von Helmholtz (1867). *Handbuch der physiologischen Optik*, Leipzig: Leopold Voss.
- Ewald Hering (1920). *Grundzüge der Lehre vom Lichtsinn*. Berlin: Julius Springer.

A modern textbook is
- Stephen Palmer (1999). Vision Science, Photons to Phenomenology. MIT Press.

In order to get an idea of todays "psychophysics" scan through the issues of recent years in Perception, *i*–Perception, Journal of Vision, Vision Research and The Journal of the Optical Society of America A.

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COLORIMETRY has generated a literature that tends to be dry and technical. Anyway, you need to get familiar with the essentials. The best source I know is Bouma, a more modern text is Koenderink (yes, that's me). Both books stress intuitive understanding over formalities or technicalities.
- Pieter Johannes Bouma (1948). *Physical aspects of colour: an introduction to the scientific study of colour stimuli and colour sensations*, Eindhoven: Philips Gloeilampenfabrieken (Philips Industries) Technical and Scientific Literature Dept..
- Jan Koenderink (2010). *Color for the sciences*, Cambridge, MA: MIT Press.

You really do not need more to start doing work yourself. If you need psychophysical details there is a mountain of literature, if you need technical information you need to mine yet another mountain. It is not particularly easy to find things, it will take you considerable effort and time.

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WILHELM OSTWALD is a must–read. His colour theories dominated colour science in continental Europe from the end of the first to the conclusion of the second world–war.

After his retirement Ostwald started research on colour vision. He wrote thirty books and three hundred papers on it. These will probably do for you:
- Wilhelm Ostwald (1916). *Die Farbenfibel*, Leipzig: Unesma.
- Wilhelm Ostwald (1917). *Der Farbatlas*, Leipzig: Unesma.

It is best to see the actual books (paper, not files) in a library, because Ostwald hand–painted samples personally and had them pasted in the books. I own a copy of one of his books and the colours are as pure and beautiful as when he painted them a century ago!
Remember what I said about appropriate domains!

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THE THEORY OF THE COLOUR SOLID is fundamental in RGB–colour space. Erwin Schrödinger (of quantummechanical fame) was the first to work it out in formal detail. His paper is easy and fun to read.
- Erwin Schrödinger (1920). *Theorie der Pigmente von größter Leuchtkraft*, Annalen der Physik, 4(62), 603–622.

If you want to follow this up, read my book (mentioned above). The upshot can be stated simply enough: The colours that are on the boundary of the colour solid (so called “optimal colours) are of the form

$$\int_{\lambda_1}^{\lambda_2} \mathbf{C}^\mathsf{T}\mathbf{d65},$$

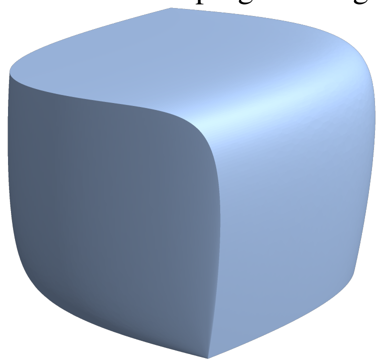
If $\lambda_1 > \lambda_2$ one takes the complement of the optimal colour for the swapped wavelengths. Any object colour is a uniquely scaled optimal colour, with scaling factor less than unity.

All optimal colours are edge colours or the sum or difference of two edge colours. It is even sufficient to stick to just one family, say the warm edge colours. This shows that the Goethe edge colours contain the same information as the Newtonian spectrum. (Only in a more convenient and useful form.)

The most colourful of the optimal colours are the “full colours”, colours that are as far from the “achromatic axis” (scaled daylight color) as possible. These are of one of the forms

$$\begin{array}{ll} (a) & \int_{\mathrm{UV}}^{\lambda} \mathbf{C}^\mathsf{T}\mathbf{d65} \\ (b) & \int_{\lambda}^{\mathrm{IR}} \mathbf{C}^\mathsf{T}\mathbf{d65} \\ (c) & \int_{\lambda}^{\overline{\lambda}} \mathbf{C}^\mathsf{T}\mathbf{d65} \\ (d) & \int_{\mathrm{UV}}^{\lambda} \mathbf{C}^\mathsf{T}\mathbf{d65} + \int_{\overline{\lambda}}^{\mathrm{IR}} \mathbf{C}^\mathsf{T}\mathbf{d65} \end{array}$$

with the following meaning:
a cold edge colours, that are blues and turuoises;
b warm edge colours, that are reds and yellows;
c band pass colours, that are various greens;
d band gap colours, that are various purples.
All these are either edge colours or sums or differences of a pair of edge colours. A very simple method to compute the colour solid is to generate a hundred thousand random optimal colours (just generate two random transition wavelengths and compute the RGB–coordinates) and find the convex hull. It takes almost no programming and this is what I get:



Doesn’t it look much like a “rounded cube”?

Only one family, say the warm edge colours, suffices to construct them all (inclusive the optimal colours). This is *amazing*! Indeed, the whole colorimetric structure is contained in the warm edge colour series. This is a mere curve but it wraps the daylight spectrum and the fundamental responses curves (ecology and physiology) in a single package that allows you to derive the colour solid, the full colours, the RGB–cube, the colour circle and so forth. Goethe showed good horse sense when he concentrated on the *Kantenfarben*, although he could not foresee the full impact of that idea and, indeed, hated mathematics and formal methods.

The warm edge colours are a complete package that fully implement the Newtonian spectrum plus the colour matching functions and the daylight structure.

You might find it instructive to write a colorimetry library that does the most diverse things, only taking the warm edge colour curve (as a table) as the basic data. This makes for a good exercise. If you succeed, you needn’t take no crap from anybody on the topic of colorimetry!

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PRINCIPAL COMPONENT ANALYSIS in colour science has generated some literature, though not too much. (It is possible to read all there is, but do you want to?)

I list two examples.

- Tzeng, D. -Y., & Berns, R. S. (2005). *A review of principal component analysis and its applications to color technology*, Color Research & Application, 30, 84–98.
- Fairman, H. S., & Brill, M. H. (2004). *The principal components of reflectance*, Color Research & Application, 29, 104–110.

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HYPERSPECTRAL IMAGES are great, but very hard to get. Just try the Internet and you will find examples, the literature is not large.

Here is the paper I mention in the text:

- João Manuel Maciel Linhares, Paulo Daniel Pinto, and Sérgio Miguel Cardoso Nascimento (2008). *The number of discernible colors in natural scenes*, J. Opt. Soc. Am. A/Vol. 25, No. 12/, 2918–2924.

It is a very good paper I think. It has implications for my arguments in this book. I recommend that you read it.

I love hyperspectral imaging in principle, much as I hate the technique in practice. Fortunately, for the study of colours in the wild RGB–images (all you need is a decent digital camera!) are likely to satisfy most needs.

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DATA BASES, especially spectral databases are essential to test my arguments. You may want to download some databases for your own experiments.

On the Finnish databases see:

- Parkkinen, J., Jaaskelainen, T. and Kuittinen, M. (1988). *Spectral representation of color images*, IEEE 9th International Conference on Pattern Recognition, Rome, Italy, 14–17 November, 1988, Vol. 2, pp. 933–935.

On the pigments database see:

- Tiziana Cavaleria, Annamaria Giovagnolia & Marco Nervoa (2013). *Pigments and mixtures identification by Visible Reflectance Spectroscopy*, Procedia Chemistry 8, 45–54.

On the database from hyperspectral images see:

- http://capbone.com/spectral-reflectance-database/

Most spectral databases are used for research in remote sensing and focus on the infrared. There are actually not that many databases in the visual band of the electromagnetic spectrum. There are even fewer databases that focus on the material you are interested in.

Consider using images as substitutes for spectral databases. The techniques of mining images has hardly been researched though. Here is an example of work by myself:

- Jan Koenderink and Andrea van Doorn (2017). Colours of the sublunar. *i*–Perception (Sept/Oct 2017, 1–30).

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GOD'S EYE, is a term for the belief that visual objects are caused by physical objects and look exactly like these. Yes, that is a very strange idea. Yet the majority of scientists is a strong believer.

It implies that there is a way the world is and a way vision can be said to be veridical. Your vision is veridical to the degree that it is like the way God's Eye sees it.

Even believers in God's Eye may grant that different animals have different visual awareness. They usually assume that humans come closest to veridical (animals being—well—animals), so the belief often comes with an anthropocentric attitude.

Anyway, the God's Eye notion is utterly at odds with biology.

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THE PARTS OF WHITE, are a set of three elements: RED, GREEN and BLUE, say $\{r, g, b\}$. The superset thus contains eight elements, it is

$$\{\{\}, \{r\}, \{g\}\,\{b\}, \{r,g\}\,\{r,b\}, \{g,b\}, \{r,g,b\}\},$$

which can also be written

$$\{k, r, g, b, y, c, m, w\},$$

where "k" stands for BLACK, "w" for WHITE, "y" for YELLOW, "c" for TURQUOISE (cyan) and "m" for PURPLE (magenta). These may be called the "cardinal colours", because any colour on the surface of the RGB–cube can be obtained by linear combination of these.

A good way to keep the cardinal colour gamut in mind is as the configuration of the I CHING trigrams:

where I put black and white (the empty set and the full set) at the centre as a YIN–YANG symbol.

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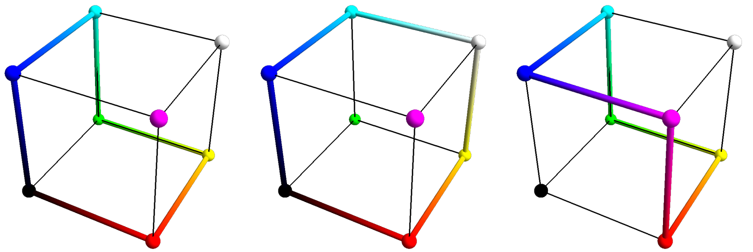
THERE ARE VARIOUS SPECIAL LOCI, on the RGB–cube. It is good to be aware of that, since it allows you to see the RGB–cube in terms of landmarks and familar structures instead of a mere formal "unit cube" $\mathbb{I}^3$.

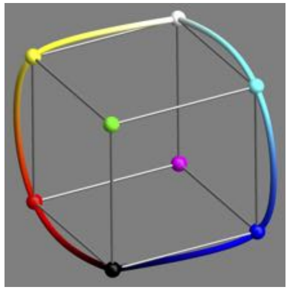
Of course, you are already familiar with the meaning of the vertices. Here I indicate some special curves—or, in this case "edge progressions"'. Here I have drawn three of those:



At **left** you have the colours of the Newtonian spectrum, it is the edge progrssion black, blue, turquoise, green, yellow, red and back to black again.

At **centre** you have the Goethe edge colours. The warm edge colour family is the edge progression that runs from from black, red, yellow to white, whereas the cool edge colour family is the edge progression that runs from from white, turquoise, blue to black.

Finally, at **right** I drew the colour circle (or rather, colour hexagon, a nonplanar, closed polygonal arc), that runs from yellow, over green, turquoise, blue, purple and red back to yellow again.



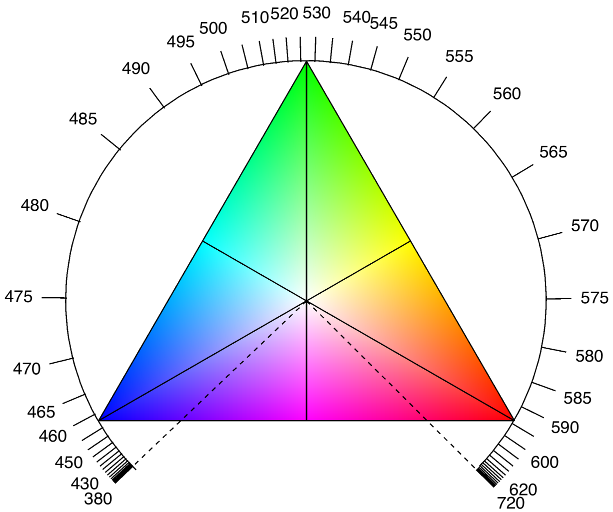
This shows the (actual, thus curvilinear) edge colour families with the RGB*–cube. Notice how closely the edge–colour curve "hugs" the cube. It meets the cube at the vertices of black, white and the cardinal colours with the exception of green and purple.*

Of course, this is by no means a complete account. For instance, one might also set up a similar story for the cube faces and various face progressions. I'll leave that up to you (my textbook on colorimetry has details).

What is especially useful here is that the cube structure is a true image of the structure of the Schrödinger color solid. Understanding the structure of the RGB–cube is most helpful in understanding this fundamental object of the colorimetry of object colours. Of course, the discrete objects then have to be understood in terms of differential geometry, thus edge progressions become space–curves and so forth.

⋆

THE EXTENT OF THE "GAP OF PURPLES", that is the "extra–spectral" part of the colour circle, is perhaps best seen in the RGB–colour triangle, that is the RGB–chromaticity diagram, the plane $R + G + B = 1$:



Here I added a dominant wavelength scale, so you can see the extent of the Newtonian spectrum. The gap of purples occurs on the blue–red edge of the RGB–triangle. As seen from the centre (achromatic, say white) it subtends about a right angle in this representation, thus a quarter of the full colour circle.

The red and blue vertices of the triangle have dominant wavelengths that are well inside the spectral limits of the infrared and ultraviolet (themselves not very well defined).

⋆

PRE–WHITENING is used here mostly as normalisation, I do not imply the removal of correlations. In order to determine the spectral autocorrelation function, I do the following:

- the image is flattened into a list of RGB–values;
- extreme values are removed by simply discarding pixels of which one of the coordinates has an extreme value zero or one;
- the RGB–coordinates are mapped into the physical domain;
- the histograms are studied. Obvious multi–modality suggests that the image be segmented first. Usually there will be peaks at the extremes due to various technical causes. I usually set a quantile limit and retain only pixels with coordinate values in all channels lying in the range. This might typically involve a 5–95% quantile range. Resulting histograms should be roughly normal. At this point I have discarded quite a few pixels, but there are plenty anyway;
- for each channel I subtract the mean and divide by the standard deviation (this is the actual "pre–whitening");
- compute the covariance matrix;
- average over the main and side diagonals;
- normalise by dividing by the mean main diagonal.

These are values of the autocorrelation function spaced by the width of the mid (green) "part of white". Fit the standard form $R(x, \tau) = e^{-\frac{|x|}{\tau}}(1 + \frac{|x|}{\tau})$ to these values and you're done! (Of course, it only makes good sense to check on the quality of the fit!) The result of all this work is an estimate of the autocorrelation length τ.

⋆

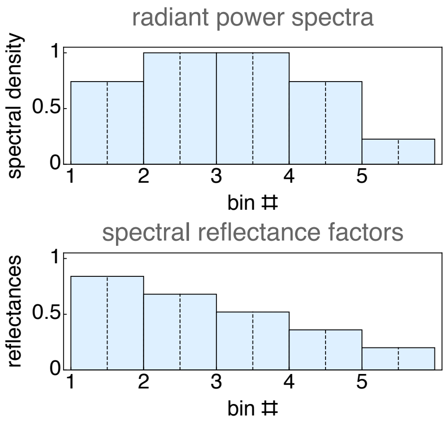
BAD possible worlds *exist*! That is to say, it is possible to construct models of freaky worlds that do not violate laws of physics. That is not the same as there being a finite probability for you to ever stumble into one! So don't get too excited, this is a theoretical exercise.

In order to obtain a feeling for this, there is no better way than to construct some freaky worlds yourself. In order for a world to be bad, the illuminants and spectral reflectances need to be highly articulated. Such highly articulated spectra would be emission line spectra for illuminants and—for instance—rare earths glass powders for objects. More importantly, the illuminant and reflectance spectra should be mutually highly correlated (or anti-correlated). I know of no examples of that in nature, but it is not impossible in principle.

Here is a simple way to construct the worst possible scenario. I start with our standard sampled system, 380–720nm in 5nm steps.

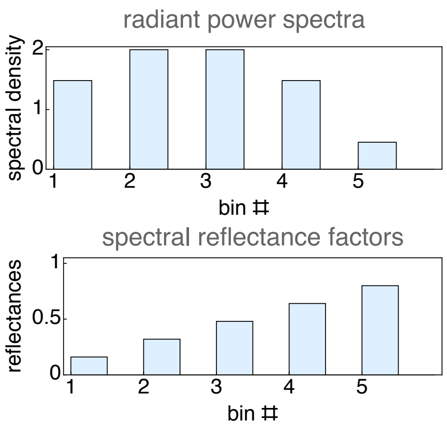
So far so good. Now I am going to prepare for some magic, by setting up what I will refer to as "spectrum–folding". I will *fold two spectra in one*.

First I split each bin into two equal halves, and simply put equal values in the two bins. This changes nothing, you don't even see the split (imagine it!). A small wavelength range might look like this:



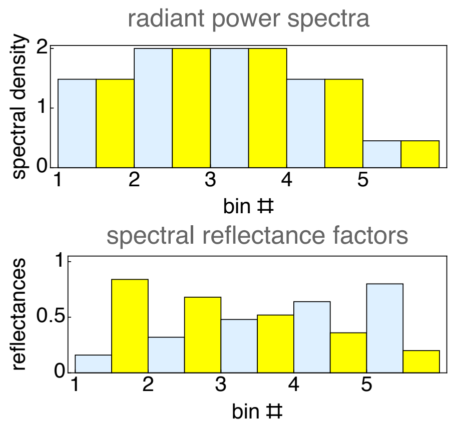
This is the standard situation we used all the time. Corresponding bins of the radiant power spectrum of the illuminant and the spectral reflectance factor have to be bin-wise multiplied in order to find the contribution to the RGB*–coordinates.*

Next I *use* the split of the bins and I assign only the first half of each bin. I double the values of the spectral radiance in order to hold the spectral flux in each bin the same, like this:



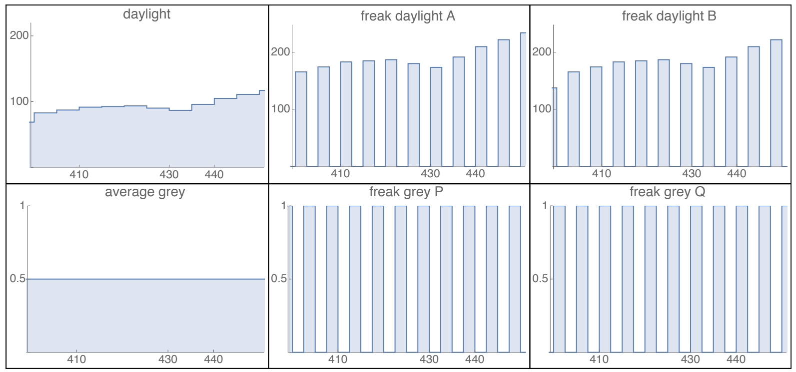
Here the bin-widths have been cut in half and the spctral radiances doubled. The situation is effectively identical to the previous case.

This changes nothing at all, so what have I achieved here? Well, now the second halves of the bins are available! I can use them to fill with another, fully independent illuminant and another, fully independent spectral reflectance.



Here I use the second halves of the bins to "fold in" another pair of illuminant (here taken equal) and spectral reflectance (here taken complementary)! (The bluish and yellowish parts are fully independent.)

The bluish half-bins and the yellowish half-bins do not at all interact! This is the crux, for it allows me to introduce illuminants that pick out one of the two reflectances, or reflectances that only "feel" one of the two illuminants.



So now I can use this "spectrum–folding" trick to define *freaks*, the magic begins! I construct two freaky "daylights" A and B and two freaky "average greys" P and Q.

Compare the phases of the articulations. It indeed allows me to perform magic, for you can easily check the following facts:

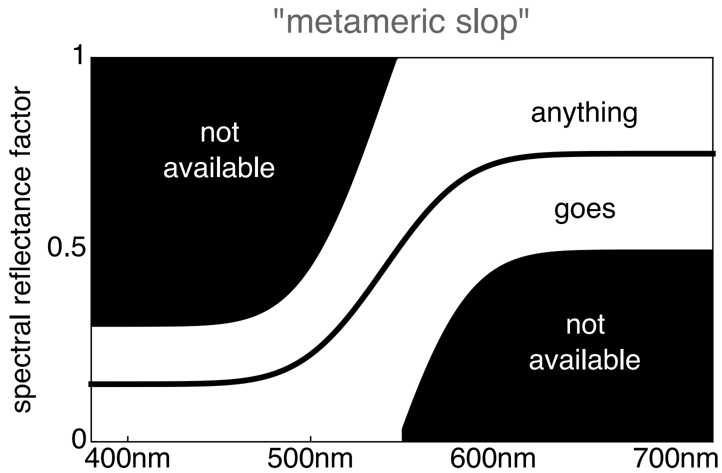
- the average grey surface looks the same (average grey) under normal daylight and either freak daylight, so you would never guess that the freak daylights are anything "special";
- under normal daylight illuminations the "normal" grey surface and the two freaky grey surfaces look the same. Again, you would never suspect something fishy going on;
- but, surprise! the freaky surface P looks black under freaky daylight A, but white under freaky daylight B;
- no end to surprises: the freaky surface Q looks white under freaky daylight A, but black under freaky daylight B!

In this world a surface may appear grey, white or black under illuminants that appear to be the same. White and black surfaces may switch their colours for no apparent reason.

Thus "spectrum-folding" is an admittedly mean, but very powerful trick! It really allows you to play "magic", yet it does not violate the constraints imposed by physics. All radiant power spectra are non-negative and all spectral reflectances are in the range zero to one. Moreover, this is by no means the limit of what you can do, because you can fold arbitrarily many illuminants and reflectances into the bins, simply subdivide them further. This allows magic that will surely astonish any audience! Philosophers will, no doubt, conclude that colours are nothing but mental paint. But not so, the situations is not essentially different from the fact that you cannot see the back of the head of a person you look in the face and who is amazed by that?

You may find it instructive to show that it is easy to construct surfaces that are grey under daylight, but may show *any specified colour* under the freaky illuminant A (say). So really anything goes, colour vision is less than useless in this freaky world, call it KOENDERINK'S JUNGLE.(Everyone should have a right to one's favourite jungle, I feel. I'm proud of mine.) This case is so bad it couldn't be worse. Apparently I hit on a (perhaps not unique) "worst case".

It is a good exercise to see that it is not really true that "anything goes". It is indeed true for the average grey spectrum (reflectance factor one-half at any wavelength), but it is not quite true in general. The colours on the boundary of Schrödinger's colour solid (thus the cardinal colours of the RGB–cube) do not admit of metamers. You might find it instructive to study the details, that is a good start for a thorough investigation of metamerism. Here is an example:



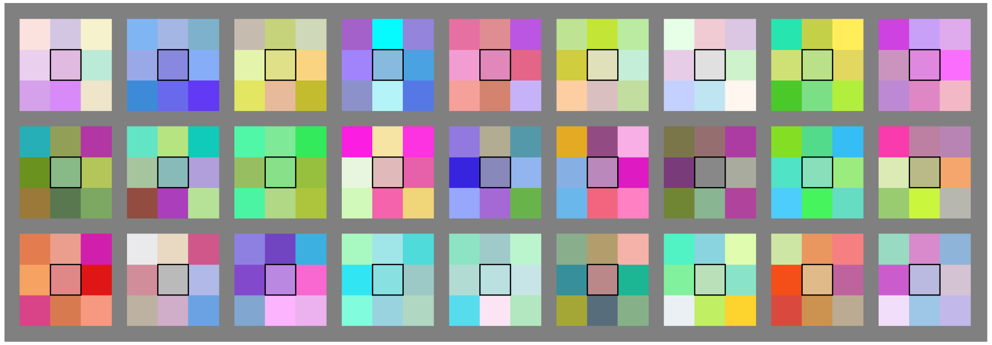
You may draw any graph that avoids the no–go areas (runs only inside the white area) and you will be able to construct a surface that looks identical to the colour of the drawn spectral reflectance curve under natural daylight, but will appear very different under one of the freaky daylights. The closer the spectral reflectance is to either zero or one, the lesser the available "slop".

Thus "not anything goes". That is expected, because the Schrödinger optimal colours have no metamers. For instance, the original spectrum (drawn curve) yields a warm colour under normal daylight. It is apparently not possible to turn it into a cool colour.

Not that such constraints help much to render the jungle palatable. It remains true that *any* colour seen under some freak daylight may turn into average grey under normal daylight. That should for most of us be enough to feel that it renders the world unfit for human consumption.

It is an easy exercise to compute volumetric regions in the RGB–cube that contain colours that will be confused with the colour of any specified reflectance spectrum, for some illuminant. This problem—if you think it over—turns out to be identical to the problem of *finding the Schrödinger colour solid*. In the case of the average gray you actually obtain the colour solid itself: "anything goes!". This is a good exercise, much recommended.

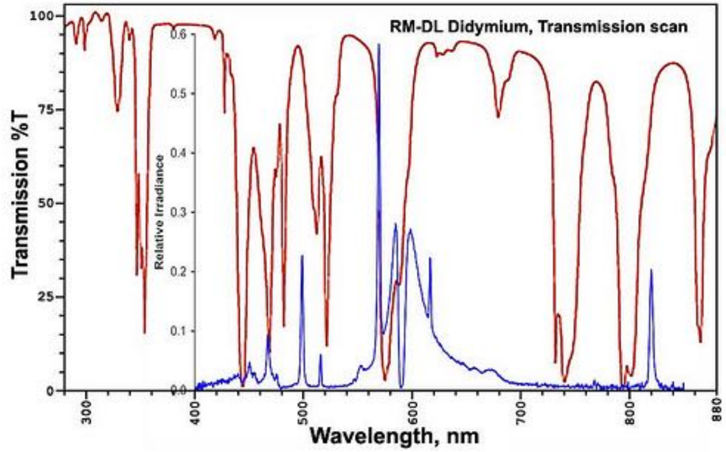
Here are examples of possible surface samples under freak daylight A, that would look like the central square under natural daylight:



Notice that all these arrays would look uniform under natural daylight!

KOENDERINK'S JUNGLE is a very unlikely world. The spectral articulations are strange enough (although there is some leeway in defining them, perhaps a random scramble appears "more natural"). What really makes the probability be essentially zero is the correlation between the reflectance factors and the illuminants. The articulations must be due to *different physical processes*, so there is no possible reason for correlation except pure chance.

Here is an example of very highly articulated spectra generated by different processes: The blue line is the light pollution relative brightness due to sodium outdoor lights, while the red line is the transmission curve for a didymium[*] glass filter. This is a combination used by astronomers to beat light pollution. As you see, the curves don't correlate too well, any coincidence between peaks and pits is there by chance. That is the typical case. Highly articulated spectra (here emission and absorption spectra) certainly exist, but they are highly unlikely to mutually correlate much. Anyway even the spectra shown here will hardly occur "in the wild".



Thus the freaky example is a mere formal plaything. I am not ready to draw strong conclusions from such a "possibility"—but "possible" it is: I know, because I made it, VERUM FACTUM!

It simply proves—if you didn't know that already—that what is *possible* will not necessarily become *actual*. It illustrates the importance of taking account of the *actual*.

This is a biological perspective, physicists think differently nowadays.

⋆

THE simplest way to compute the Schrödinger colour solid changes as computers get faster and math packages smarter. I use Mathematica for most of my work, that is because I do a lot of formal mathematics next to numerical calculations. For the latter, MatLab, or Python packages might well beat Mathematica in raw speed (I'm not really interested in checking that).

The fastest method right now seems to be a random algorithm. I generate a pair of indices in the wavelength table and generate a spectral reflectance that

[*] Didymium is a mixture of the elements praseodymium and neodymium. It has been used in glass workshops since the mid nineteenth century. It is used in safety glasses for glassblowing and blacksmithing. It is effective in blocking the 589nm sodium emission. I used it a lot when doing flame photometry in an atomic physics research lab in the sixties. There are other uses, some perhaps unexpected. For instance, during WW–I it was used to transmit Morse Code across battlefields. Not exactly an application the Neanderthaler would have been waiting for.

toggles between zero and one, flipping at these indices. I use the sorting order of the indices to decide on the initial phase. I compute the RGB–coordinates in the usual way.

Finding such a random Schrödinger optimal colour (for that's what they are) is very fast. No problem to compute a hundred thousand of them. Next I find the convex hull of this random set, the result is an excellent approximation of the colour solid.

There are some advantages to that, for I can query Mathematica for various properties, such as volume, or surface area, I can use the data-structure in various geometrical computations, such as intersections or unions, moreover, I can use it for purposes of graphics—all that for no additional effort.

In the past I used much neater, differential geometrical methods. Neat indeed, but a lot of work.

If you want to do quick and easy experimentation, the method I sketch here will save you time and effort.

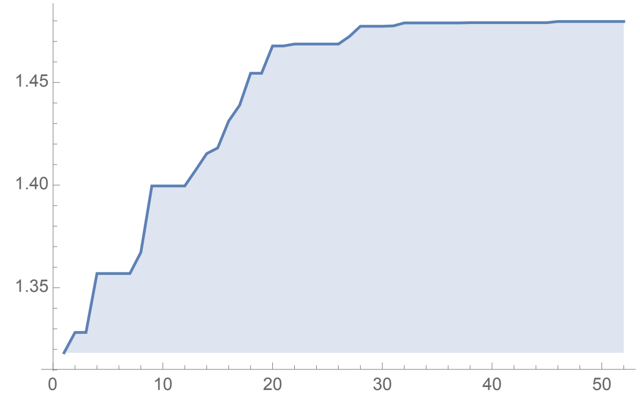
Of course, the old fashioned methods have their charms too. For instance, they force you to understand the geometry at the level of blood and bones. From a pessimist's perspective, they force you to implement what you get for free in the modern math packages. I'm an old timer, so I tend to think that the newer packages are appreciated best if you suffered the old ways first (it wasn't that bad either and it bred character).

⋆

FINDING the cut loci for the parts of white implies serious computation. The only way to solve the problem is by exhaustive search. You need to compute the crate volume for the parts defined by two arbitrary cut loci and search for the largest one. In order to find the crate volume you need to find the colours of the parts and compute their determinant. For a 341-step wavelength table (380–720nm at 1nm intervals, say) that implies about sixty-thousand of such computations.

A more convenient way is again a random algorithm. The "harmony search" algorithm* is a "soft computing" algorithm that is especially simple to use. You search through random instances and slightly perturb the most promising ones. This lets you soon crawl to a 'good enough" solution (about 1nm accuracy is all you need).

I got this in less than a second and a half:



This graph shows crate volume as a function of the iteration index. Apparently, the algorithm was already "done" after a second, about thirty iterations. Here I computed 64 cases at each iteration, thus I did only 3% of the full chore.

The speedup may not excite you though. We're only talking seconds on a small laptop. Whether this takes the fun out of it, or rather excites it, is a decision I leave up to you.

⋆

THE spectrum is most naturally parameterised by photon energy from the perspective of physics, but in this book I use the conventional wavelength measure. It is the convention preferred by non-physicists. A certain photon energy is labeled as the wavelength in nanometers in vacuum. The "visual range" subtends, very roughly, 380–720nm. Of course, the "ultraviolet" and "infrared" limits are only vaguely defined.

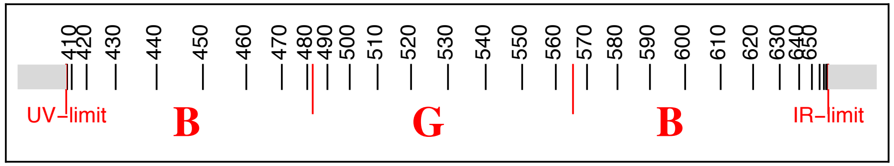
The wavelength parameter has reached almost mystical meaning in various fields. Thus a philosopher may mention "seeing by wavelength" and be understood. It would take a long historical exegesis to explain this, it would

*Geem, Zong Woo, Joong Hoon Kim, and G. V. Loganathan (2001). A new heuristic optimization algorithm: harmony search. *Simulation* **76.2**, pp. 60-68.

naturally end with Newton's original experiments.

But you don't "see by wavelength", in fact, wavelength has very little, if any, to do with colour vision. It is merely used as a convenient parameter in computations. Plotting spectral distributions in wavelength terms thus leads to a representation that only makes full sense to the professional eye.

A better scale is the arc–length along a Goethean edge spectrum. At least, it has an immediate relation to vision. The arc-length should then be measured in the Schopenhauer parts of white representation, the RGB–space. The edge colours are located on a curvilinear connection of the black and white points, that has a total length of about 3.15, just a little over 3. (Remember that the KBCW edge progression has total length 3.) You obtain this gauge:



A spectrum gauge on the basis of arc-length along a, edge colours locus. It is a finite intervals between the UV *and* IR*-limits. The "parts of white" almost claim equal parts in this representation, so it appears intuitively "natural".*

The infinite extent of the physical spectrum is compressed in a finite interval between the ultraviolet and infrared limits. Notice that the spectral extents of the parts of white have almost identical widths in this representation.

It would be far more intuitive to plot all spectral distributions on such a natural scale. It is a simple enough matter to adjust the colour matching functions to work in this gauge.

I do not do that in this book, because people are "not yet ready for it". I wonder whether they'll ever be. We're already waiting for over a century.

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The Clootcrans Press is a selfpublishing initiative of Jan Koenderink. Notice that the publisher takes no responsibility for the contents, except that he gave it an honest try—as he always does. Since his books are free you should have no reason to complain.

The "Clootcrans" appears on the front page of Simon Stevin's (Brugge, 1548–1620, Den Haag) *De Beghinselen der Weeghconst*, published 1586 at Christoffel Plantijn's Press at Leyden in one volume with *De Weeghdaet, De Beghinselen des Waterwichts*, and a *Anhang*. In 1605 there appeared a supplement *Byvough der Weeghconst* in the *Wisconstige Gedachtenissen*. The text reads "*Wonder en is gheen wonder*". The figure gives an intuitive "eye measure" proof of the parallelogram of forces.

The key argument is

> *de cloten sullen uyt haer selven een eeuwich roersel maken, t'welck valsch is.*

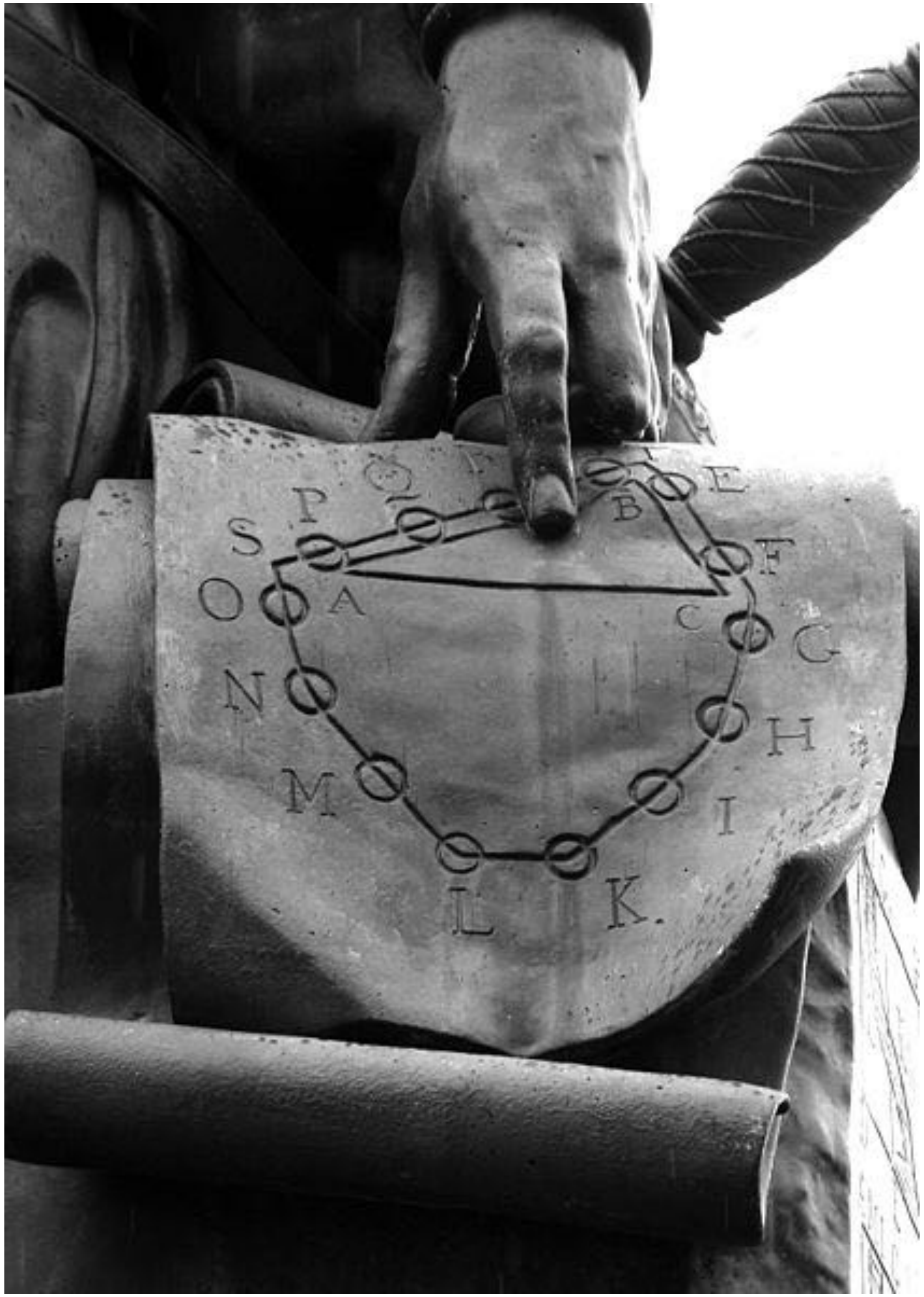
Simon Stevin was a Dutch genius, not only a mathematician, but also an engineer with remarkable horse sense. I consider his "clootcrans bewijs" one of the jewels of sixteenth century science. It is "natural philosophy" at its best.

Traiectum is the city of Utrecht in the Netherlands where I live. Traiectum was a *castrum* on the frontier of the Roman Empire in *Germania Inferior*.

The fortress was built of wood, it measured a little over a hundred by a hundred meters and held five hundred troops, a *cohors quingenaria* — the *II Hispanorum peditata*, a standing non-citizen infantry corps of the Imperial Roman army.

The region was designated *frontier* by the Emperor Claudius. It must have been much like what you see in the familiar fifties western movies. The castellum was destroyed by the Franks before 270, although there remained Roman presence up to the fourth century.

The fort's exact location is the current Utrecht city centre, where I live close. My Alma mater takes her name *Rheno–Traiectina* or *Universitas Ultraiectina* from this.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