# F20 PHYSICS 137B: Final Review

December 9, 2020

My best recommendations for preparation: review the homework problems and the midterm (make sure you can do all the problems), look at say TBP for old quantum exams and try to do those. Make sure to really try to solve the problems using only the textbook and your notes, and not just be content with knowing how to do the problems "in principle." If you hit a wall or stumbling block, don't jump right to the solution; spend some time wrestling with it – this is what it means to practice physics problem-solving!

## 1 Identical particles

Key points:

- Bosons, fermions, and distinguishable particles differ under the *symmetrization postulate*
- Bosonic wavefunctions must be symmetric under the exchange of any 2 particles, fermionic wavefunctions must be antisymmetric under the exchange of any 2 particles, while distinguishable wavefunctions need not satisfy any kind of symmetrization
- Bosons have integer spins, fermions have half-integer spins
- Remember that wavefunctions usually have a spatial part and a spin part – the overall wavefunction needs to satisfy the symmetrization postulate, but each individual part can differ
- Implication: bosons can occupy the same state, fermions cannot

Review question:

- What is the ground state energy of 5 particles in a 3D harmonic oscillator if they are distinguishable, bosons, or fermions with spin 1/2?

## 2 Time-independent perturbation theory

Key points:

- Have the formulas handy for (non-degenerate) first-order energy corrections, first-order wavefunction corrections, and second-order energy corrections
- Remind yourself of some basic results from 137A, e.g. computing expectation values, the effect of raising/lowering operators on QHO states etc.
- Degenerate perturbation theory: remind yourself what it means to "diagonalize the degenerate subspace" to find "good states" – you are finding the degenerate states that separate under the effect of the perturbation
- Simplest application is hydrogen atom: relativistic correction and spin-orbit coupling to get fine-structure corrections

- Zeeman effect and the 3 different regimes of field strength
- Recall how to compute hyperfine structure corrections due to spin-spin coupling

Review questions:

- Compute the first and second-order corrections to the energy of the $n = 2$ eigenstate of the 1D SHO under the perturbation $H' = \lambda x^2$.
- Compute the corrections to the energy of the 2nd excited energy level ($E = 3\hbar\omega$) in the 2D SHO under the perturbation $H' = \lambda x_1^2 x_2^2$.

## 3 Variational principle

Key points:

- The ground state energy is always the lowest possible energy achievable for any state in a given system
- Any other state must have higher energy, and therefore, choose an ansatz wavefunction with some set of variable parameters, compute the energy expectation value, and then extremize with respect to the parameters
- Most mistakes arise due to algebraic or arithmetic errors

Review question:

- Use a gaussian trial wavefunction to obtain an estimate of the hydrogen ground state energy.

## 4 Time-dependent perturbation theory

Key points:

- Main idea is to rewrite the SE into a set of coupled differential equation for the *coefficients* of the energy eigenbasis expansion
- The differential equation can then be solved iteratively by substituting lower order solutions to get higher order solutions
- Important cases to know: constant (piecewise) and sinusoidal perturbations
- Review selection rules for dipole transitions and dipole radiation
- Recall Fermi's Golden Rule for determining transition rates – you will need to know how to get density of states for bosons and fermions, as well as how to average over an arbitrary probability distribution (e.g. incoming/outgoing angles, photon frequencies)

Review questions:

- Obtain the probability of transitions at $t > t_0$ for a 3-level system that starts in its ground state and whose basic time evolution is interrupted by a potential $\lambda t^2$ between $0 < t < t_0$.
- Compute the density of states for a fermion gas and a photon gas.

## 5 Scattering

- Recall the concept of the scattering amplitude – in particular, how it is related to the differential cross-section, and how it is defined via approximation (in what region is it valid?)
- Keep the formula for the differential cross-section in terms of partial waves handy – it is the most useful technique in terms of approximating via series expansion because it is always valid
- The Born approximation has a couple variants of the usual formula, depending on which regime you are in – you should know these (spherical, low energy etc.)
- In principle, you should know how to obtain the Born approximation in terms of Green's functions and the integral Schrodinger equation

Review questions:

- Flip through Griffiths and take a stab at a couple of the problems – there aren't many analytically doable scattering problems, and we did most of them on the homework

## 6 WKB

- Know how to obtain the WKB wavefunction between turning points (e.g. a single well)
- Recall how to use the quantization condition of the WKB momentum to find the approximate energies of the system – you will be given the phase shift, if needed

Review questions:

- Griffiths 9.16, 9.18, you can also try the longer 9.17 (except part (a))