

Homework 3

EECS/BioE C106A/206A
Introduction to Robotics

Due: September 22, 2020

Note: This is a programming assignment. You must fill in the provided `hw3.py` file with your solutions to the problems in this assignment. There will be no PDF submission for this problem set.

You will also need to use `kin_func_skeleton.py`, which you implemented as part of your Lab 3 prelab, and which you also use in Lab 3. To get started, you should place both `hw3.py` and `kin_func_skeleton.py` in the same directory, as `hw3.py` imports `kin_func_skeleton`.

Your deliverables for this assignment are:

1. `hw3.py` with your implementation of your solutions to this assignment.
2. `kin_func_skeleton.py` with your implementations from Lab 3 prelab.

Both the above files should be submitted together to the Gradescope assignment **Homework 3**. Your credit for this problem set will be awarded by the autograder.

Theory

For this problem set, you should recall the various formulas for finding the Twist corresponding to the type of joints that a robot has. We will consider the following types of joints:

1. **Prismatic Joints:** These are joints that implement a pure translation along an axis (a screw with infinite pitch $h = \infty$). The twist corresponding to such a joint is

$$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix}$$

where v is a unit vector in the direction of the translation.

2. **Revolute Joints:** These are joints that implement a pure rotation about some axis (a screw with zero pitch $h = 0$). These are the kinds of joints you saw in Lab 3 with the Baxter robot. The twist corresponding to such a joint is

$$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -\omega \times q \\ \omega \end{bmatrix}$$

where ω is a unit vector in the direction of the rotation axis, and q is a point through which the rotation axis passes, all as seen with the robot in its initial configuration.

3. **Screw Joints:** These are joints that implement a simultaneous rotation and translation along the same axis (a screw with nonzero finite pitch h). The twist corresponding to such a joint is

$$\xi = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} -\omega \times q + h\omega \\ \omega \end{bmatrix}$$

where ω is a unit vector in the direction of the screw axis, q is a point through which the axis passes, and h is the pitch of the screw.

In each of the above cases, the point q can be chosen as any point on the axis. Every such point will result in the same twist. However some points may be more convenient from a computational viewpoint than others (for instance, if the axis passes through the origin, we usually want to pick $q = 0$ to simplify calculations).

If a robot has n joints (or n "degrees of freedom") then to compute the forward kinematics map of the robot, we must start off by finding the twists ξ_1, \dots, ξ_n corresponding to each of the joints of the robot.

The "joint angles" $\theta_1, \dots, \theta_n$ specify the amount by which each joint on the robot is to be actuated. So for revolute and screw joints, θ is the angle through which we rotate the joint from its initial position. For prismatic joints, θ is the amount by which the joint is distended (the distance we translate by). The θ_i 's are what we get to control to move the robot to our desired location. Think of these as our control inputs, which we can pick to move the motors on the robot to a particular position.

Forward Kinematics is the problem of finding where the end effector of the robot ends up when we pick particular values of $\theta_1, \dots, \theta_n$. In particular, we fix a spatial frame S to be stationary, and attach a tool frame T rigidly to the end effector of the robot. Then solving the forward kinematics map of the robot amounts to finding $g_{ST}(\theta) \in SE(3)$ as a function of $\theta = (\theta_1, \dots, \theta_n)$. Once we have the twists ξ_1, \dots, ξ_n corresponding to each joint of the robot, we can use the *product of exponentials* formula to write down an expression for the forward kinematics map as

$$g_{ST}(\theta) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 \theta_2} \dots e^{\hat{\xi}_n \theta_n} g_{ST}(0)$$

where $g_{ST}(0) \in SE(3)$ is the pose of the tool frame with respect to the spatial frame when all joint angles are set to zero (the *initial configuration*). Usually, we get to pick the initial configuration, so we know what $g_{ST}(0)$ is (or we can easily find it).

Problem 1. Forward Kinematics for 3DOF Manipulators

Figure 1 shows a 3 degrees-of-freedom manipulator in its initial configuration. The robot has three revolute joints. Take $L1 = 1$. In this problem, you will implement the forward kinematics map of this manipulator. Do this by writing down

- (a) $g_{st}(0)$, the rigid pose corresponding to the initial configuration.
- (b) The twists ξ_1, ξ_2, ξ_3 corresponding to each of the three joints of the manipulator.
- (c) An expression for the forward kinematics map $g_{st}(\theta)$ where $\theta \in \mathbb{R}^3$ is the vector of joint angles $(\theta_1, \theta_2, \theta_3)$. You may leave your answer in terms of the exponentials of known matrices.

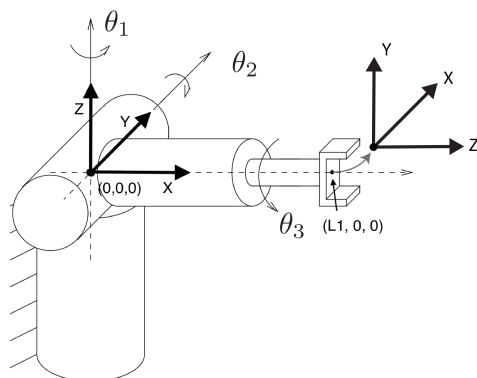


Figure 1: A three degree of freedom manipulator

Problem 2. Forward Kinematics with Screw Joints

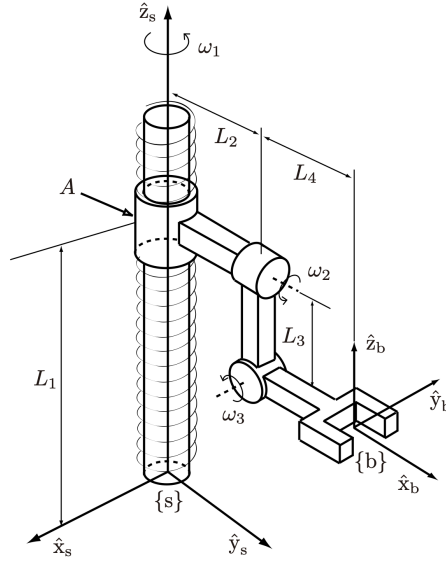


Figure 2: A 3 DOF manipulator.

Figure 2 shows the initial configuration of a robot arm whose first joint is a screw joint of pitch $h = 2$. The other two joints are revolute. The arm's link lengths are $L_1 = 10$, $L_2 = L_3 = 5$, and $L_4 = 3$. In this problem you will implement the forward kinematics map of this robot, in the following steps:

- Write down the 4×4 initial end effector configuration of the manipulator, $g_{sb}(0)$.
- Find the twists ξ_1, ξ_2, ξ_3 corresponding to each of the joints of the manipulator, and hence write down an expression for the forward kinematics map $g_{sb}(\theta)$. You may leave your answer in terms of the exponentials of known matrices.

Problem 3. Forward Kinematics for 6DOF Manipulators

Figure 3 shows the 6DOF Stanford arm in its initial configuration, with 5 revolute joints and one prismatic joint (joint 3). Take $l_0 = l_1 = 1$. You may also assume that in the initial configuration, q_w is a distance l_1 away from q_1 . Implement the forward kinematics map of this robot by finding:

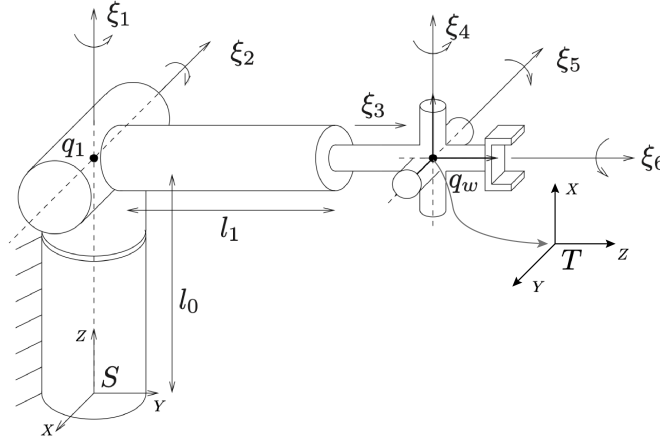


Figure 3: An idealized version of the Stanford arm.

- $g_{st}(0)$, the rigid pose corresponding to the initial configuration.
- The twists $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6$ corresponding to each of the six joints of the manipulator.
- An expression for the forward kinematics map $g_{st}(\theta)$ where $\theta \in \mathbb{R}^6$ is the vector of joint angles. You may leave your answer in terms of the exponentials of known matrices.