Midterm 1

EECS/BioE C106A/206A
Introduction to Robotics
Fall 2020

Issued: September 30, 2020, 8 PM
Due: October 1, 2020, 8 PM

Problem	**Max. Score**
Cross Products and Commutators	10
Planar Rigid Body Transforms	10
Forward Kinematics	10
Inverse Kinematics	15
Pokemon Search and Rescue	25
Total	70

Instructions

1. This exam is open-book, open-notes and open-internet.
2. The exam is, however, NOT open-peer, and any collaboration between students on the exam will be considered academic misconduct.
3. Please include all your work so that you may be eligible for partial credit.
4. You do not need to print out the exam or annotate on it. Feel free to write out your solutions the way you would for a homework assignment.
5. You have 24 hours to take this exam. You should submit your solutions to the Gradescope assignment by ***Thursday, October 1st, 8:00:00 PM PST.***

Problem 0. Honour Code

Copy down the following honour code in your submission and sign and date it. *Failure to do so will result in an automatic 0 on the exam.*

I swear on my honour that:

1. *I alone am taking this exam.*
2. *I will not have assistance from anyone while taking this exam.*
3. *I will not discuss this exam with anyone else until exam solutions have been released by course staff.*

Problem 1. Cross Products and Commutators (10 points)

(a) (5 points) You are given two skew symmetric matrices $\hat{\omega}_1$ and $\hat{\omega}_2$. If $\omega_3 = \omega_1 \times \omega_2$, show that $\hat{\omega}_3 = \hat{\omega}_1\hat{\omega}_2 - \hat{\omega}_2\hat{\omega}_1$.

Hint: It may help to recall the Jacobi identity for cross products:

$(a \times b) \times c = a \times (b \times c) - b \times (a \times c)$

(b) (5 points) You are given two twists $\hat{\xi}_i$ and $\hat{\xi}_2 \in \mathfrak{se}(3) \subset \mathbb{R}^{4\times 4}$ with twist coordinates v_1, ω_1 for ξ_1 and v_2, ω_2 for ξ_2. Compute $\hat{\xi}_1\hat{\xi}_2 - \hat{\xi}_2\hat{\xi}_1$. Is it a twist? If so then give its twist coordinates v_3, ω_3 in terms of the twist coordinates $v_1, \omega_1, v_2, \omega_2$.

Problem 2. Planar Rigid Body Transformations (10 points)

A transformation $g = (p, R) \in SE(2)$ consists of a translation $p \in \mathbb{R}^2$ and a 2×2 rotation matrix R. We represent this in homogeneous coordinates as a 3×3 matrix:

$$R = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \tag{1}$$

A twist $\hat{\xi} \in \mathfrak{se}(2)$ can be represented by a 3×3 matrix of the form:

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \quad \hat{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad \omega \in \mathbb{R}, v \in \mathbb{R}^2 \tag{2}$$

The twist coordinates for $\hat{\xi} \in \mathfrak{se}(2)$ have the form $\xi = (v, \omega) \in \mathbb{R}^3$. Note that v is a vector in the plane and ω is a scalar. A *unit* twist is defined similarly to the way it is in 3D: $\xi = (v, \omega)$ is a unit twist if $\omega = 0$ and $||v|| = 1$ (pure translation) or if $\omega = 1$ (general).

(a) (2 points) For a given Twist $\xi = (v, \omega) \in \mathfrak{se}(2)$, show the following about the powers of the matrix $\hat{\xi}$ for $n \geq 1$

$$\hat{\xi}^n = \begin{bmatrix} \hat{\omega}^n & \hat{\omega}^{n-1} v \\ 0 & 0 \end{bmatrix} \tag{3}$$

(b) Using the previous part, show that the exponential $e^{\hat{\xi}\theta}$ of a unit twist in $\mathfrak{se}(2)$ gives a rigid body transformation in $SE(2)$. Consider both the pure translation case, $\xi = (v, 0)$, and the general case, $\xi = (v, \omega), \omega \neq 0$. Do this by showing that:

(i) (3 points) When $\xi = (v, 0)$ with $||v|| = 1$ (pure translation), then

$$e^{\hat{\xi}\theta} = \begin{bmatrix} I & v\theta \\ 0 & 1 \end{bmatrix}$$

(ii) (5 points) When $\xi = (v, \omega)$ with $\omega = 1$ and arbitrary v, then

$$e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & ((I - e^{\hat{\omega}\theta})\hat{\omega} v) \\ 0 & 1 \end{bmatrix}$$

Problem 3. Forward Kinematics (10 points)

Figure 1 shows a robot in its initial configuration. The robot has three revolute and one prismatic joint (RRPR). Solve for the forward kinematics map of the manipulator by finding:

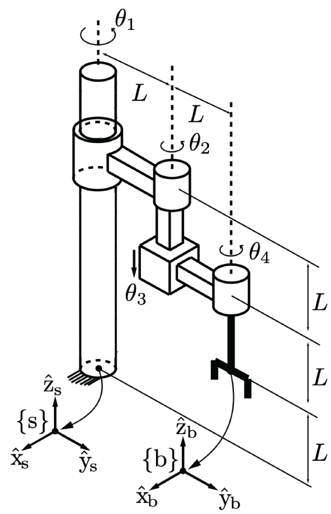

Figure 1: A four degree of freedom manipulator

(a) (2 points) The initial configuration $g_{sb}(0) \in SE(3)$ of the robot.

(b) (6 points) The twists $\xi_1, \xi_2, \xi_3, \xi_4$ corresponding to each joint of the robot.

(c) (2 points) An expression for the forward kinematics map $g_{sb}(\theta)$ in terms of the vector of joint angles $\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$. You may leave your answer in terms of the exponentials and products of known matrices.

Problem 4. Inverse Kinematics with a Twist (15 points)

(a) Figure 2 shows a 4 degrees of freedom manipulator with three revolute joints and one prismatic joint (joint 3); RRPR:

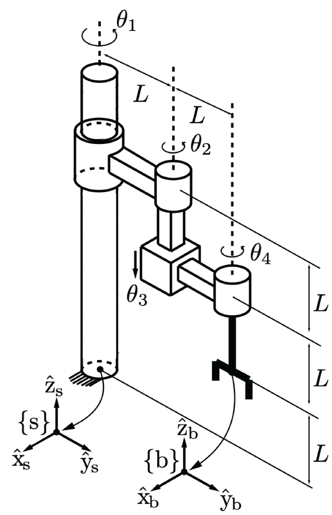

Figure 2: A four degree of freedom manipulator

(i) (2 points) Describe the workspace of the end effector. State the reachable orientations $\in SO(3)$ and the reachable positions $\in \mathbb{R}^3$ as seen from frame $\{s\}$. Ignore any self-collisions.

(ii) (7 points) Solve the inverse kinematics problem for this manipulator. Also indicate the maximum number of IK solutions.

(b) Figure 3 shows the same 4 degrees of freedom manipulator with joint 1 replaced by a screw joint of finite pitch h. Assume that $0 < \theta_1 < \infty$ and $0 < \theta_3 < \infty$ with the only restriction being that the origin of the end effector frame cannot go below the ground. In other words, we assume that the screw in joint 1 is of infinite height, and joint 3 is allowed to extend arbitrarily far so long as the end effector does not collide with the floor.

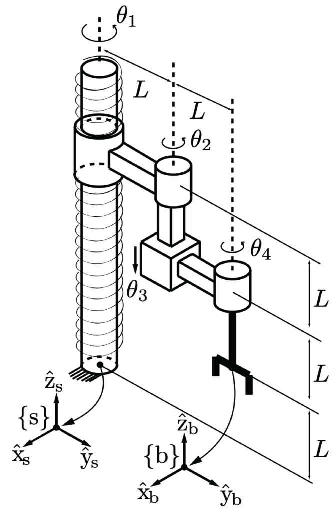

Figure 3: A four degree of freedom manipulator with a screw joint.

(i) (2 points) Describe the workspace of the end effector. State the reachable orientations $\in SO(3)$ and the reachable positions $\in \mathbb{R}^3$ as seen from frame $\{s\}$. Ignore any self-collisions.

(ii) (4 points) Explain how you might modify or use your solution to the previous problem (part (a) (ii)) to solve the inverse kinematics problem for this new manipulator. Indicate the maximum number of IK solutions.

Problem 5. Pokemon Search and Rescue (25 points)

Oh, no! Team Rocket tried to steal all your Pokemon! Thankfully, your Pokemon were able to escape, but in their panic they all ran into a large construction site. As it is too large (and potentially dangerous) for you to search the entire place for them yourself, you decide to utilize your mobile robots and ROS knowledge in order to rescue your team. After all, you can't become a Pokemon Master without your Pokemon!

You have a team of 3 mobile robots in this unknown space trying to locate a known set of n Pokemon while also creating a map of the environment. The robots will be able to recognize the Pokemon automatically and your Pokemon will recognize the robot as yours and will follow it until completion of the task. The robots have completed their task once all n Pokemon have been located and all robots have returned to the starting position (and thereby have led all Pokemon to safety). The robots should not return to the starting position until all Pokemon have been found. You may assume that every robot has perfect odometry and that each robot is equipped with a laser-scanner. You may further assume that all robots start from the same starting position, and that each robot has its own built-in controller included in its node.

(a) (7 points) Design a ROS based communication system for this setup. On the next two pages, we have provided two tables for you to complete. In the table titled "Nodes", we have provided a list of necessary ROS nodes for this system, along with a description of the functionality of each node. Your job is to design the system of topics and services that these nodes will use to communicate with each other.

First, create a list of topics and/or services your ROS system will need as well as a short description of each one, by filling in the table titled "Topics and Services". You may treat each robot as a separate ROS node all operating under a single master node. Then, **fill in the middle two columns of the "Nodes" table to specify what topics each node will publish and what topics each node will subscribe to. Here, you should also specify which nodes will run any services you choose to define, and which nodes will send requests to that service.**

Note that you do not need to worry about the intricacies of the mapping or localization algorithms. You may assume that each robot has perfect knowledge of its location relative to a common global reference frame, and that you have a black-box algorithm that can take in laser scan readings and locations from multiple robots and produce a map from that information. You should only design the communication system between the various nodes.

Please complete the following two tables. We have filled in a few to get you started.

Note 1: Please specify if something is either a topic or a service for the middle two columns of the Nodes table.

Note 2: You do not need to print out the table or annotate on it, although you may feel free to do so. You may make a copy of the table for yourself to fill out, so long as you ensure that all necessary information is present.

Nodes			
Name	**Topics Publishing to/Services Requested**	**Topics Subscribed to/Services Run**	**Description**
`robot1`			Robot 1's node
`robot2`			Robot 2's node
`robot3`			Robot 3's node
`task_master`			A node that: • takes raw sensor readings and current locations from each robot and uses them to build a map of the environment • sends a message to each robot telling them which locations to explore next • keeps track of the task's progress and alerts the robots when all Pokemon have been located and to return to the starting position • tells the robots when the task is fully complete and to shut down

Topics and Services		
Topic or Service	**Name**	**Description**
Topic	`robot1/scan`	The topic carrying laser scan data from robot 1.
Topic	`robot1/odom`	The odometry topic carrying the location of robot 1 relative to the fixed world frame.

(b) (3 points) Now, draw out an excerpt of a possible RQT graph of how your ROS nodes would communicate with each other. Your drawing does not need to look 100% accurate to how an actual RQT graph would look like, but please show us which topics and/or services will be used for communication between which nodes. Your diagram should include nodes `robot1`, `robot2`, and `task_master` as well as all necessary topics/services needed for communication between them (don't worry about `robot3` as the graph will quickly become very chaotic and hard to follow).

(c) Oh no, it looks like one of your Pokemon, Vulpix, got lost while following one of your robots back to you! Luckily, you were able to hack into the construction site's security cameras and locate your lost Vulpix. You decide to send two of your robots to retrieve your Pokemon to prevent her from getting lost again. Figure 4 depicts Vulpix' and the two robots' locations.

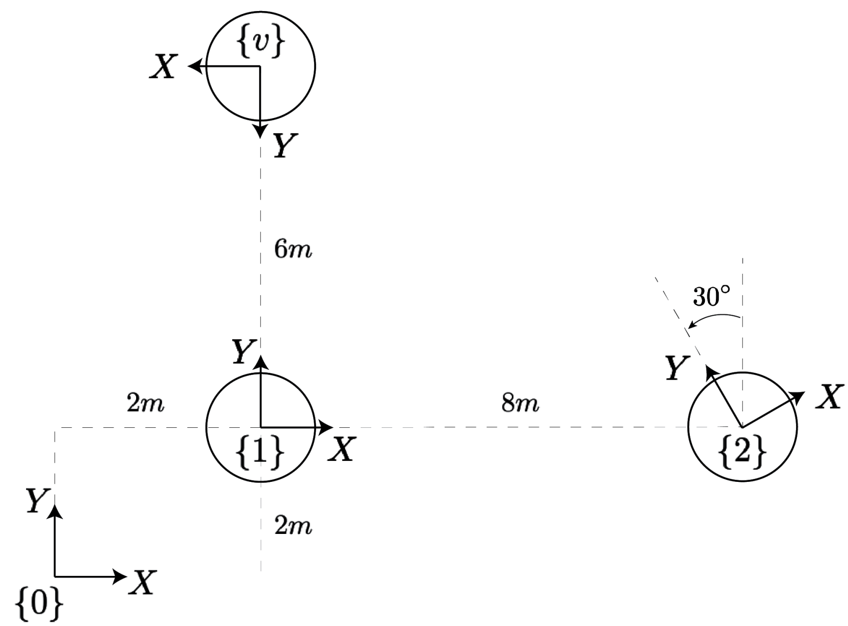

Figure 4: A top-view of the locations of various parts of the scene. Frames {1} and {2} are the reference frames of your two robots, and {v} is the reference frame of your Vulpix. {0} is the spatial reference frame. All Z-axes point directly out of plane of the paper.

(i) (5 points) Find the $SE(3)$ rigid-body transforms g_{1v} and g_{12}.

(ii) (2 points) Hence write down an expression for g_{2v}. You may leave your answer in terms of other known matrices.

(iii) (4 points) Find a set of exponential coordinates for g_{1v} by first describing the associated screw motion.

(d) (4 points) The mission continues uninterrupted for a while, but soon you grow suspicious that one or more of your robot's sensors may be failing. You decide to probe the relative transforms between your three robot's reference frames $\{1\}, \{2\}$ and $\{3\}$, as detected by their sensors. You receive the following transformations:

$$g_{12} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, g_{23} = \begin{bmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}, g_{13} = \begin{bmatrix} 0 & 1 & 0 & -9 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is this enough information to conclude that one or more of your robot's localization systems are supplying erroneous measurements?