Physics 5A, Fall 2017
Homework Set 12

APF Ch 3: 3.14

APF Ch 4: 4.4, 4.5, 4.8, 4.10, 4.12, 4.13, 4.14

S 11.1 Consider the driven oscillator analyzed in class,

$$\frac{F_0}{m}\cos\omega t = \frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x, \tag{1}$$

when $\gamma << 2\omega_0$. As done in class, take the solution of this differential equation to be $x(t) = A(\omega)\cos(\omega t - \phi)$ where

$$\begin{aligned} A(\omega) &= \frac{F/m}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \omega^2\gamma^2}}, \\ \phi &= \tan^{-1}\left(\frac{\omega\gamma}{\omega_0^2 - \omega^2}\right). \end{aligned} \tag{2}$$

(a) Show that $A(\omega)$ is maximum when $\omega = \omega_m$ where

$$\omega_m = \omega_0\sqrt{1 - \frac{1}{2Q^2}}, \tag{3}$$

and Q is the quality.

(b) Show that

$$A(\omega_m) \approx \frac{QF}{m\omega_0^2}. \tag{4}$$

(c) If $A_m/2 = A(\omega_m \pm \Delta\omega)$, then show that

$$\Delta\omega = \frac{\sqrt{3}\omega_0}{Q}. \tag{5}$$

(d) Finally, show that

$$\frac{d\phi}{d\omega}\Big|_{\omega_0} \approx \frac{2Q}{\omega_0}. \tag{6}$$