Two simple examples of periodograms

1. Let $y_t = 5\cos(2\pi(0.2)t) + 3\sin(2\pi(0.4)t)$, $t = 0, 1, \ldots, 49$. Plot the periodogram.

   Hint: For this exercise, you should see if you can calculate the periodogram from `fft`*. If you get stuck, You might find the function* `periodogram()` *in the TSA package or the* `pgram()` *function from in class helpful.*

2. Let $x_t = 5\cos(2\pi(0.23)t) + 3\sin(2\pi(0.37)t)$, $t = 0, 1, \ldots, 49$. Plot the periodogram for $\omega_j = 1/50, \ldots, 25/50$.

3. What is the difference between the two?

R Practice

The dataset *wages* (in the R package `TSA`) contains monthly values of the average hourly wages (in dollars) for workers in the U.S. apparel and textile products industry for July 1981 through June 1987. Load it into your workspace with the command `data(wages, package = 'TSA')`.

4. Make a time series plot of the data.

5. Smooth the data using the filter $a_{-2} = a_{-1} = a_0 = a_1 = a_2 = 1/5$. Plot the data and the smoothed version in the same graph.

6. Smooth the data using exponential smoothing. Plot the data and the smoothed version in the same graph. You'll need to choose the value of the smoothing rate parameter as well as how many discrete data points to include in the filter.

7. Use least squares to fit a linear time trend to this time series. Plot the data and the fitted values in the same graph. Make a time series plot and an ACF plot of the residuals. Do the residuals look like white noise?

8. Use least squares to fit a quadratic time trend to the wages time series. Plot the data and the fitted values in the same graph. Make a time series plot and an ACF plot of the residuals. Do the residuals look like white noise?

9. Generate the forecast for July 1987 based on

   1. fitting a linear trend;
   2. fitting a quadratic time trend.

   Are these forecasts the same?

Bias/Variance Tradeoff and Smoothing

Let (y_t) be a time series. Consider the simple averaging filter with parameter q,

$$\hat{m}_t = \frac{1}{2q+1} \sum_{j=-q}^{q} y_{t-j}. \tag{1}$$

Suppose the true data generating process of (y_t) is $y_t = m_t + w_t$, where $m_t = \beta_0 + \beta_1 t + \beta_2 t^2$ and (w_t) is zero-mean white noise with variance σ^2.

We view $\hat{m}_t$ as an estimator of m_t.

10. Compute the bias of $\hat{m}_t$ with respect to m_t. How does the bias change as q increases? (Hint: Recall that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.)

11. Compute the variance of $\hat{m}_t$. How does the variance change as q increases?

12. Find the MSE of $\hat{m}_t$.

13. As σ^2 increases, how does the optimal q change? (For this problem, the optimal q is defined to be the value of q that minimizes the MSE.)

14. As $|\beta_2|$ increases, how does the optimal q change?