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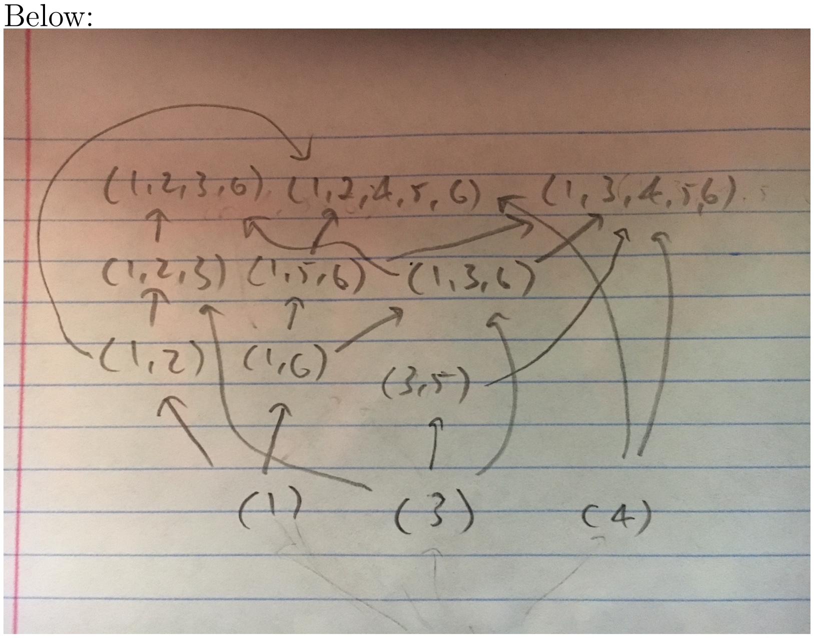
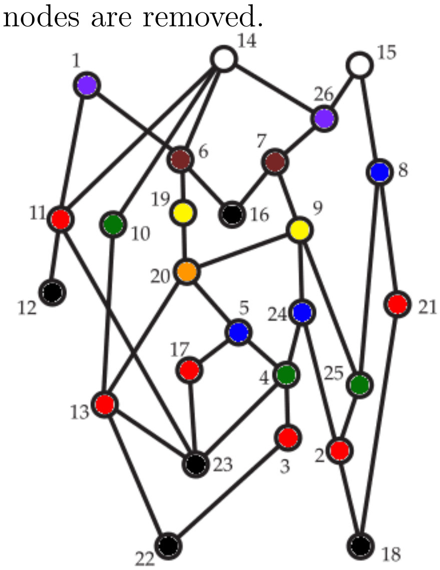
Math 3012 Combinatorics

3. That is not a partial order, because it does not contain the point (5,5), which fails the reflexive test. It passes the transitive and antisymmetric tests.

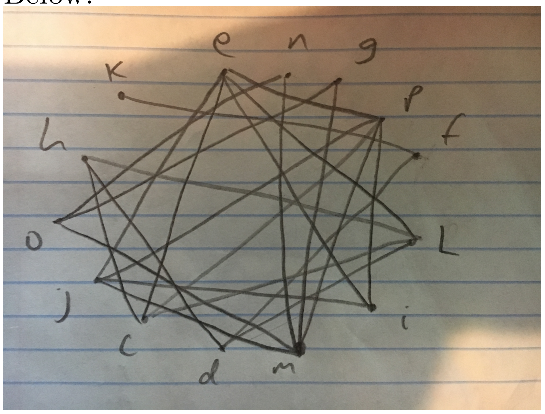
5. Below:



9. The maximum height and maximum chain is 9. The set {22,3,4,5,20,9,7,26,14} contains that height. In the following diagram, black is 1, red is 2, green is 3, blue is 4, orange is 5, yellow is 6, brown is 7, purple is 8, white is 9. These each correspond to their respective antichains. For example, {12,22,23,16,18} is an antichain because it corresponds to all the minimal elements colored in by black. Next, the red {11,13,17,3,2,21} is an antichain because it corresponds to the minimal elements existing when all black nodes are removed.



13. Below:



15. Does not exist because when dissolved into individual down-sets, then there exist sets that are not subsets of each other; there exist 2 + 2s.

18. The intervals ordered by the location of the left endpoint would be f, b, g, d, h, n, c, j, l, a, e, k, m, i, o. Using the first fit algorithm, these will go into chains if they are comparable. The first chain C_1 will have f. As b is incomparable with f, it will go into C_2, and because g is incomparable with any of the others, it will go into C_3. As d is comparable with f, then it goes into C_1. Continuing for the rest of the intervals, we have:

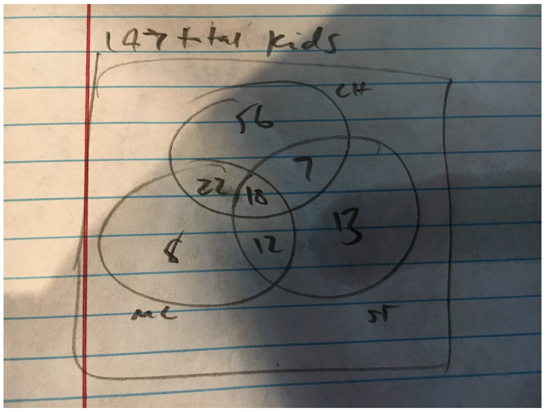
$$C_1 = \{\text{f,d,j,k}\}$$

$$C_2 = \{\text{b,h,c,l,e,m,o}\}$$

$$C_3 = \{\text{g,n,a,i}\}$$

The width of this interval order is 3, and it is partitioned thus by Dilworth's theorem into 3 chains. A possible antichain is {f,b,g}. ($w = 3$)

1. 11 students do not like a flavor. This is because there are 147 students total; 18 like all three flavors, which means that they can be subtracted out of the sets of students who only like two flavors. Thus, $30 - 18 = 12$ mint chip and strawberry, $40 - 18 = 22$ chocolate and mint chip, and $25 - 18 = 7$ chocolate and strawberry. These three sets can then be subtracted out of the sets of students who like only one flavor. Then, all the sets can be added to represent the students who like the three flavors, and this result from 147 is 11 students.



5. This can be represented by a Venn diagram with edges 3, 8 and 25. First, the numbers divisible by the three are floor(1000/3) = 333, floor(1000/8) = 125, floor(1000/25) = 40. However, adding just these overcounts the numbers divisible by two of the above numbers. Multiplying two numbers above gives the LCM of both, and so there are 41 divisible by 3 and 8, 13 by 25 and 3 and 5 by 8 and 25. This again overcounts the numbers divisible by all three, of which there is one. Thus 333 + 125 + 40 - 41 - 13 - 5 + 1 = 440 items that are divisible by some of those factors in [1000]. Thus, 1000 - 440 = 560 numbers are divisible by none of them.

9. By inclusion/exclusion, then because he ate 15 Tuesdays, and we need to avoid overcounting, then according to $N_0 = \sum_{S\subset(1,2,N)} (-1)^{|S|} N(S)$ then $15 - 11\binom{6}{1} + 9\binom{6}{2} - 6\binom{6}{3} + 4\binom{6}{4} - 4\binom{6}{5} + 1\binom{6}{6} = 1$, meaning he ate alone only once.

11.
 - It does not because f(2) = 2. It satisfies P_3 either because no f(j) = 3. Other satisfied properties are P_5 and P_7 for the same reasons.
 - g(i) = i while $1 \leq i \leq 7$. At i = 8, then g(i) can equal anything but 8, so a case g(8) = 3 fulfills this requirement.
 - It is impossible to satisfy no properties here, because 9 is not within the domain and thus no h(9) will equal 9. The property will be satisfied by it not existing.

19. $\sum_{n=0}^{9} (-1)^n \binom{9}{n} (9-n)! = 9! \times 1 - 8! \times 9 + 7! \times 36 - 6! \times 84 + 5! \times 126 - 4! \times 126 + 3! \times 84 - 2! \times 36 + 1! \times 9 - 0! \times 1 = 133496$

25. Because the inclusion exclusion formula is $\phi(n) = n(1 - 1/p_1)(1 - 1/p_2) \times \ldots \times (1 - 1/p_k)$ where p_k refers to the prime factors of n then $\phi(1625190883965792) = 1625190883965792(1-1/2)(1-1/3)(1-1/11)(1-1/13)(1-1/23)(1-1/181) = 4.32e+14$

1. (a) $\sum_{n=0}^{4} \binom{4}{n} x^n$

   (b) $1 + x + x^2 + x^3 + x^4 + x^7$

   (c) $\sum_{n=0}^{5} n x^{n+2}$

(d) $\sum_{n=0}^{6} x^n$

(e) $3 + x^3 - 4x^4 + 7x^5$

(f) $x^4 + 2x^5 - 3x^6 + x^8$

3. (a) $a_n = \binom{10}{n}$

(b) $a_n = 1$ if $n \bmod 4 = 0$ else $a_n = 0$

(c)

(d) $a_n = 2$ if $0 \leq n \leq 3$ else $a_n = 0$

(e) $a_n = 2$ for $n = 2, 3$ else $a_n = 0$

(f) $a_n = 4^n$

(g) $a_n = (-1)^n 4^n$

(h) $a_n = \binom{n-2}{3} : n \geq 5$ else $a_n = 0$

(i)

(j)

5. To account for the gold and white balloons, the generating functions $(x + x^2 + ...)(x + x^2 + ...)$ can be used, and thus $1 + x + x^2$ refers to the two blue balloons because there can be at most 2. Together, this makes $\frac{x^2(1 + x + x^2)}{(1 - x)^2} = x^2(1 + \frac{3x}{(1 - x)^2}) = x^2(1 - \frac{3}{1 - x} + \frac{3}{(1 - x)^2}) = x^2 - 3x^2 - 3x^3 - 3x^5 + ... = x^2 + \sum_{n=1}^{\infty} 3nx^{n+2}$ based on the two Taylor series of each individual one. For 10 balloons, this becomes 3(10-2) = 24 balloons.

13. (a) Because the first candy must be at least two, then its generating function starts at $x^{0+2} = x^2$ and because the second one must be even, then only even powers x^{2n} are available, and because at most six of the third candy are allowed, then that becomes finite. Thus $(x^2+x^3+x^4+...)(1+x^2+x^4+...)(1+x+x^2+x^3+x^4+x^5+x^6) = \frac{x^2(1 - x^7)(1 + 4x + 10x^2)}{(1 - x^2)(1 - x)^2}$

(b) For that, the coefficient must be over 400, in order to account for each student. The first term that does this is the x^13 term, which has a coefficient of 413 and thus each bag has 13 candies.

21. (a) $\sum_{n=0}^{\infty} \frac{7^n x^n}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{3^n x^(n + 2)}{n!}$

(c) $\sum_{n=0}^{\infty}(-1)^n x^n$

(d) $\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$