Trend Modeling

Jared Fisher

Lecture 1b

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- Memo: Friday lab sections consist of three parts: a worksheet (usually), a pre-recorded video from the GSIs, and live Q&A on Friday.
- Reminder: please respond to the time zone and project topic polls on Piazza. Project datasets will be chosen and published next week in conjunction with your project

Waitlist and Concurrent Students

- The waitlist is moving slower than in past semesters.

Accomodations and Schedule Conflicts

- Please let me know of any conflicts or accomodations (religious, DSP, or otherwise) as soon as possible.

Recap

Our Modeling Approach

- For time series Y_t

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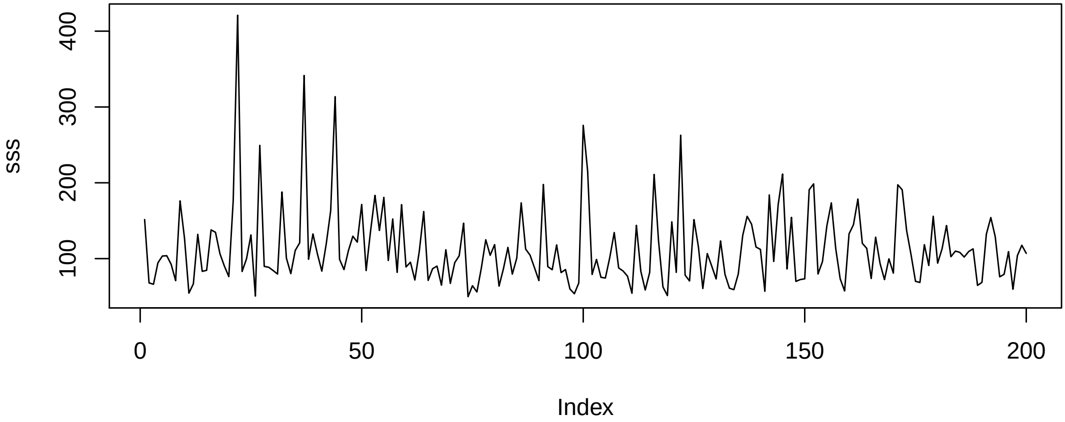
- More generally:

$$Y_t = signal(t) + X_t$$

- Where X_t is a **stationary process**

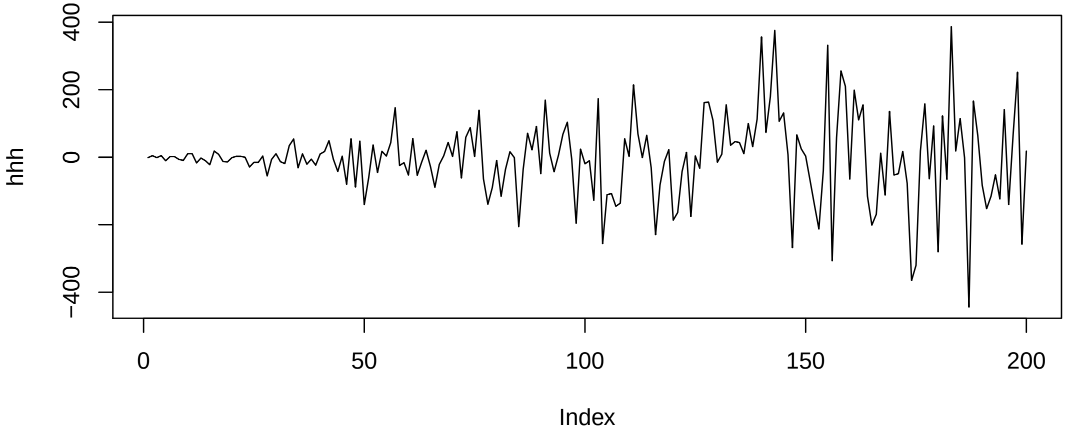
Stationary (But Not White Noise)

```r
sss = rt(200,4,90)
plot(sss,type='l')
```

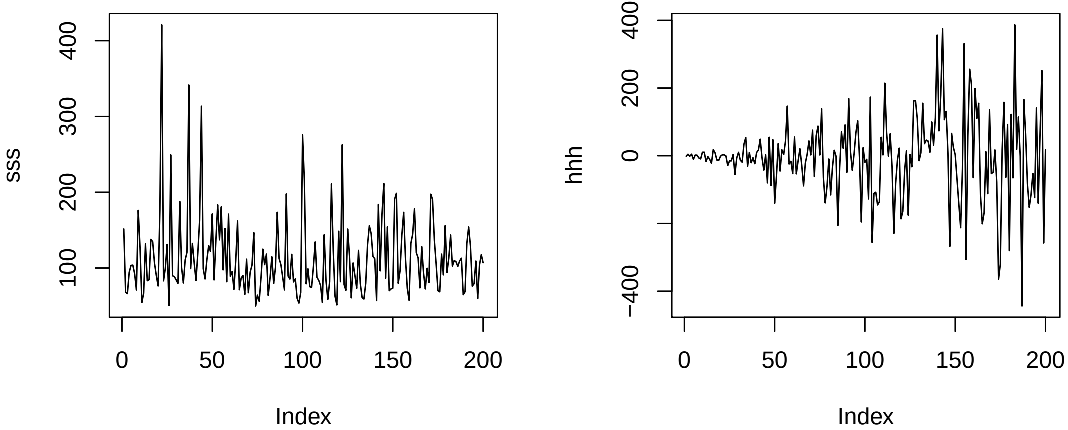


Not Stationary - Heteroskedastic

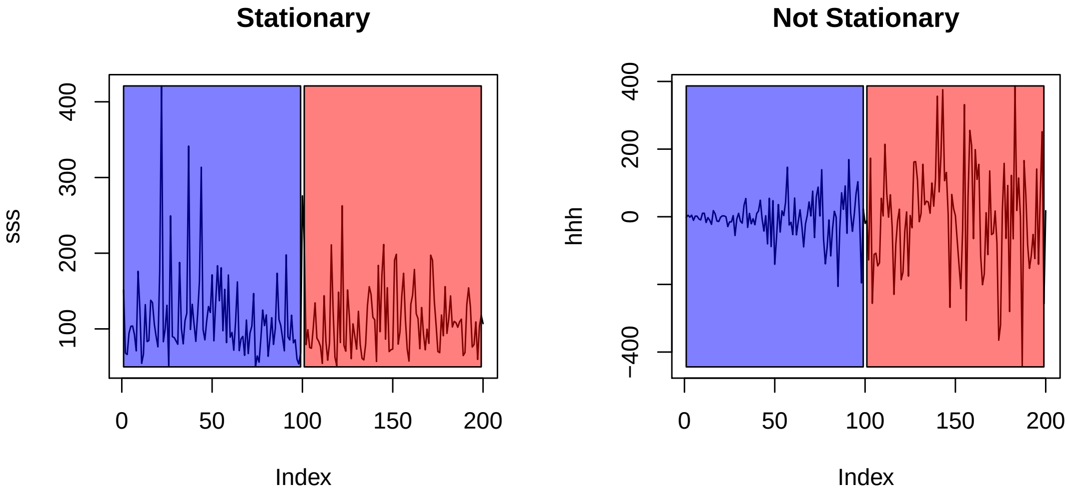
```r
hhh = rnorm(200,0,1:200)
plot(hhh,type='l')
```



Side by Side


sss
100
200
300
400
0
50
100
150
200
Index
hhh
−400
0
200
400
0
50
100
150
200
Index

Side by Side


Stationary
sss
100
200
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400
0
50
100
150
200
Index
Not Stationary
hhh
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200
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100
150
200
Index

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- $\Rightarrow \hat{X}_t = y_t - \widehat{signal}(t)$
- We'll build our signal function "$signal(t)$" such that the assumption that X_t is stationary is reasonably appropriate.
- In other words, we will model the signal such that $\hat{X}_t$ look like come from a stationary process.

Trend

Decomposing a time series into *signal*(t) and noise

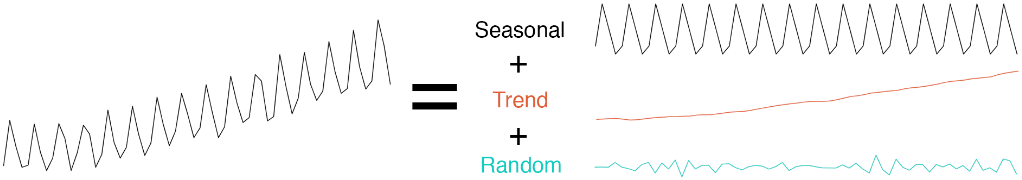

Figure 1: From anomaly.io

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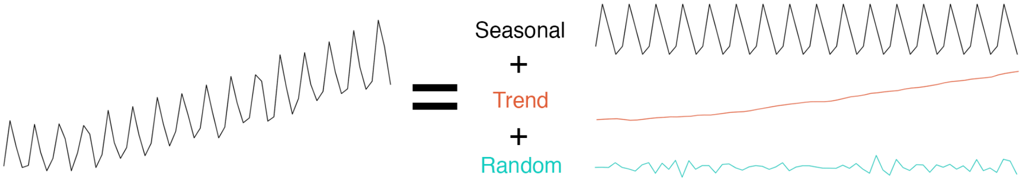

Figure 1: From anomaly.io

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- We usually decompose the signal into a trend component "m_t" and a seasonal component "s_t":

$$signal(t) = m_t + s_t$$
$$\Rightarrow Y_t = m_t + s_t + X_t$$

This Unit: “Pursuing Stationarity”

- Today we will focus on the trend component.

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- Let's briefly look at the course calendar on bCourses: https://bcourses.berkeley.edu/calendar#view_name=month&view_start=2021-02-01

Trend Models

- As we'll look at seasonal effects later, our model today is

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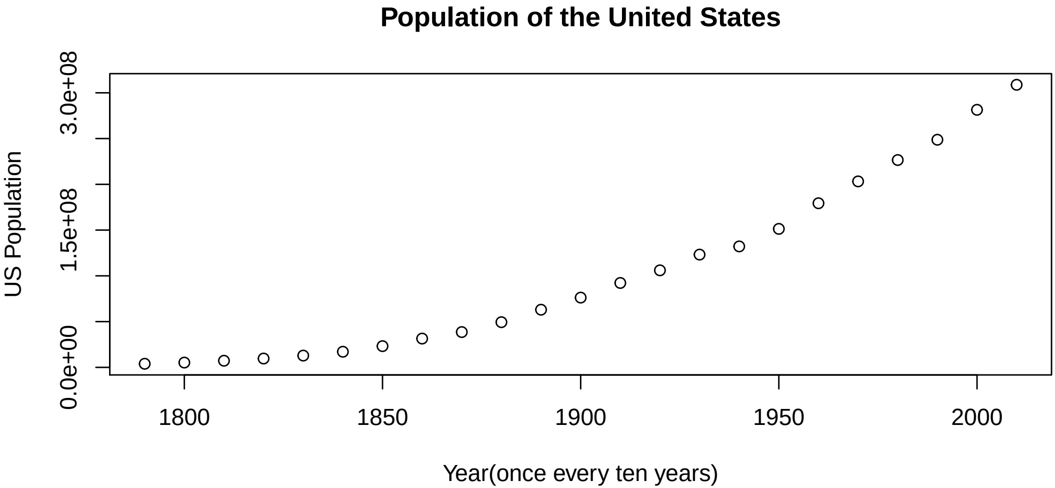
$$Y_t = m_t + X_t$$

- m_t is the trend
- X_t is as stationary process, perhaps white noise
- **Idea:** Model then remove the trend, so that data exhibits steady behavior over time, i.e. looks stationary. Then exploit dependence structure for estimation and prediction.

Example



Population of the United States
US Population
0.0e+00
1.5e+08
3.0e+08
1800
1850
1900
1950
2000
Year(once every ten years)

First Idea: Estimate trend $\hat{m}_t$.

If $\hat{m}_t \approx m_t$, then the residuals

$$y_t - \hat{m}_t \approx y_t - m_t = X_t$$

will have no trend over time.

Methods for estimating the trend:

- Parametric form for m_t, e.g., fit a polynomial with least squares

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- Smoothing/Filtering - remove noise by averaging (lectures 3b and 4a)

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Methods for estimating the trend:

- Parametric form for m_t, e.g., fit a polynomial with least squares
- Smoothing/Filtering - remove noise by averaging (lectures 3b and 4a)
- Other nonparametric methods - e.g. isotonic models, etc.

Parametric Trend Estimation

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- $f_j(t)$ can be any function: t, t^2, $log(t)$, $t * log(t)$, etc.

Least Squares

- Estimate the β parameters with least squares:

$$\hat{\beta} = \arg\min_{\beta} \sum_t (Y_t - \beta_0 - \beta_1 f_1(t) - \ldots - \beta_p f_p(t))^2$$

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- Lab this week will cover least squares, especially for those not familiar with it!

Example: Quadratic Curve/Parabola

- Quadratic trend line: $m_t = \alpha + \beta t + \gamma t^2$

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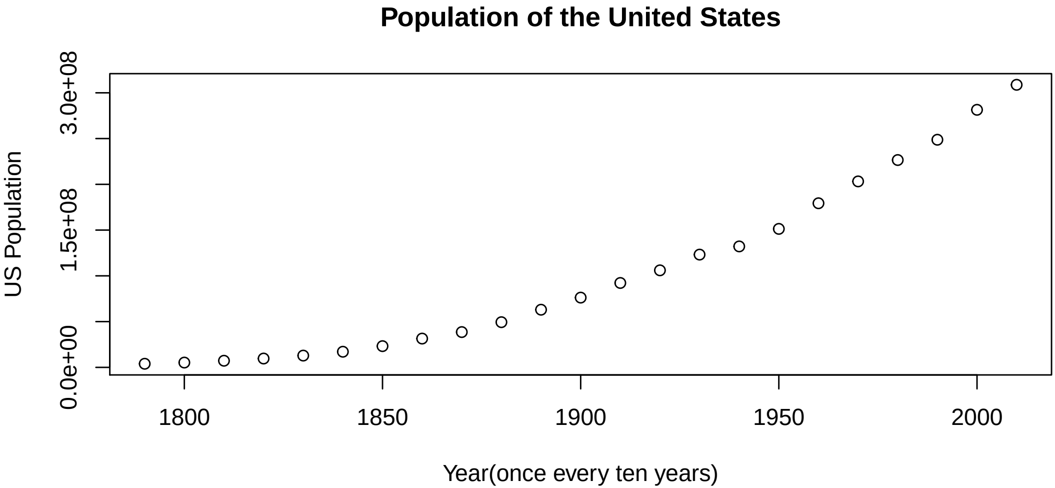
- Quadratic trend line: $m_t = \alpha + \beta t + \gamma t^2$
- Fit parameters with least squares

$$(\hat{\alpha}, \hat{\beta}, \hat{\gamma}) = \operatorname{argmin} \sum_t \left(Y_t - [\alpha + \beta t + \gamma t^2] \right)^2$$

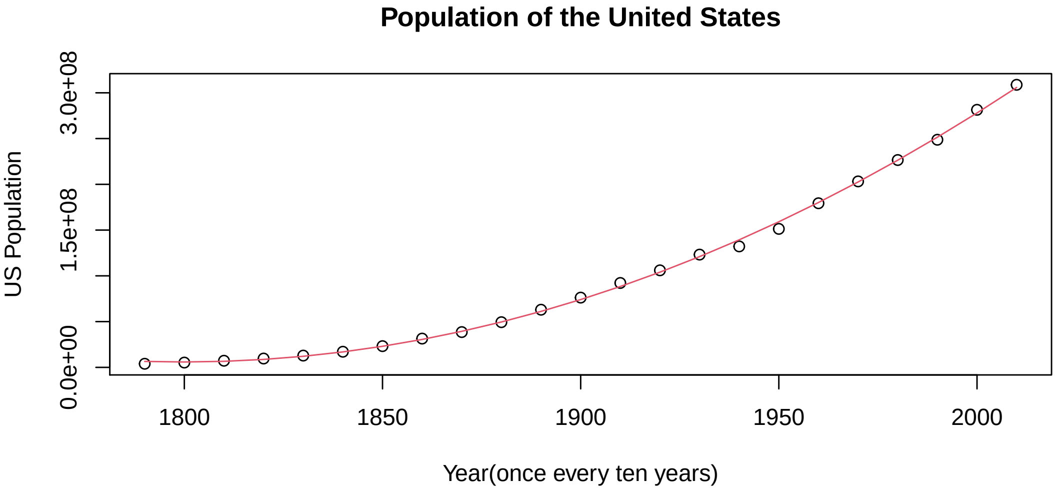
then

$$\hat{m}_t = \hat{\alpha} + \hat{\beta} t + \hat{\gamma} t^2$$

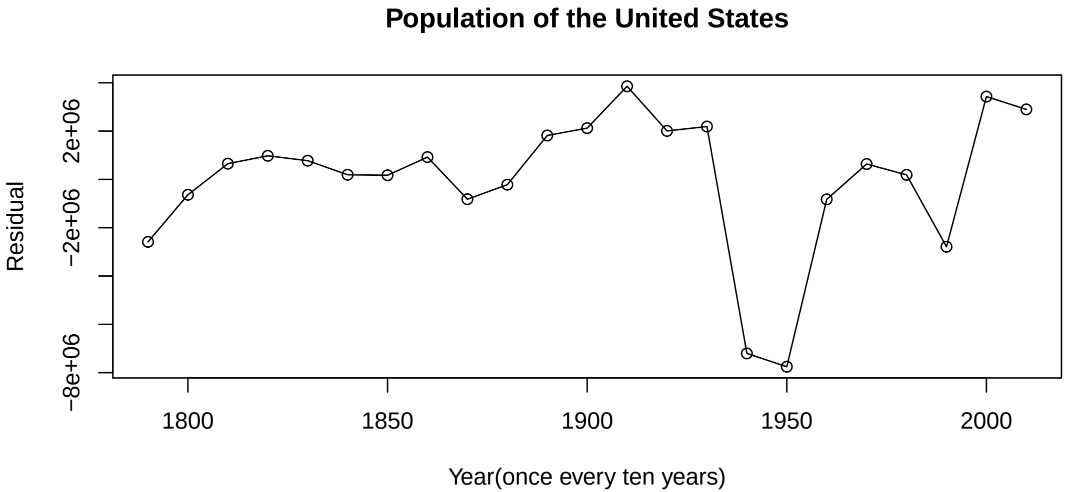
Example - US Population



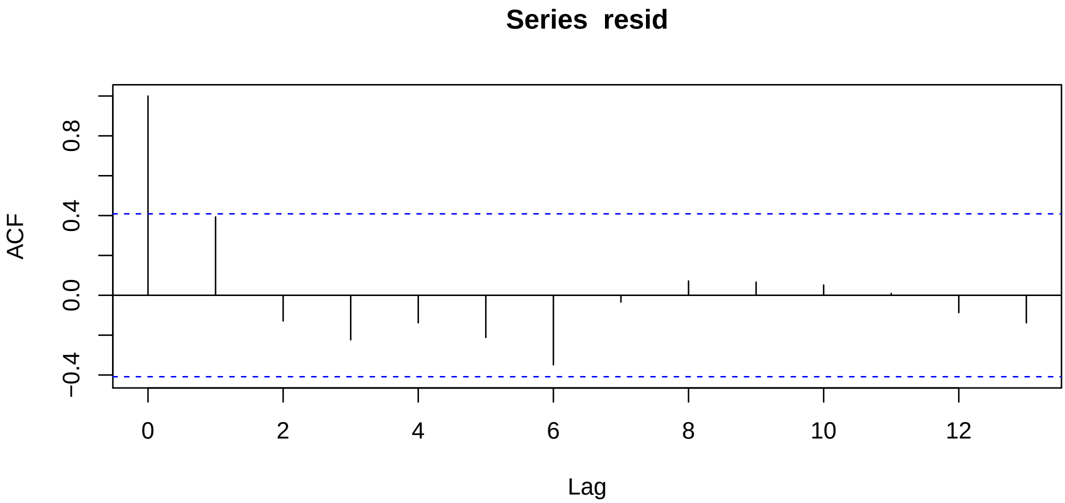
Example - with Trend



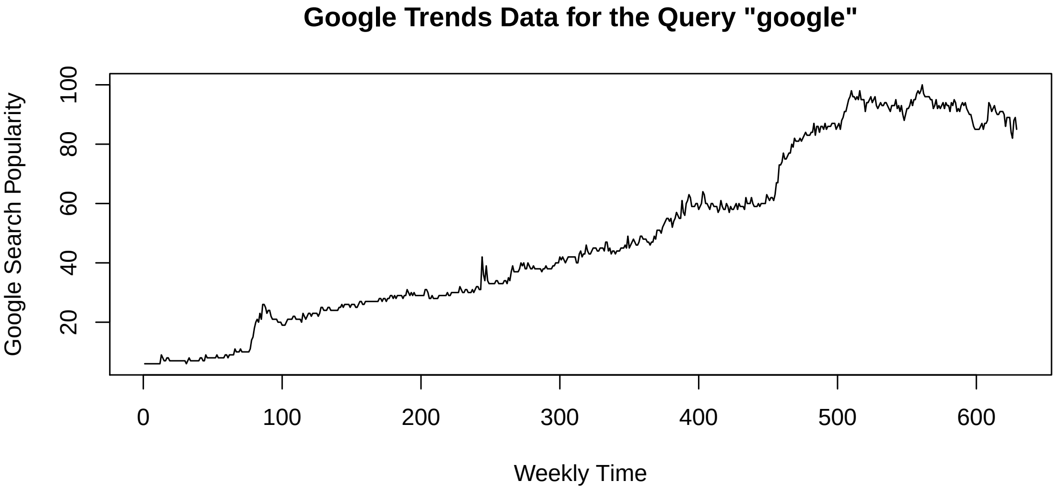
Example - Residuals



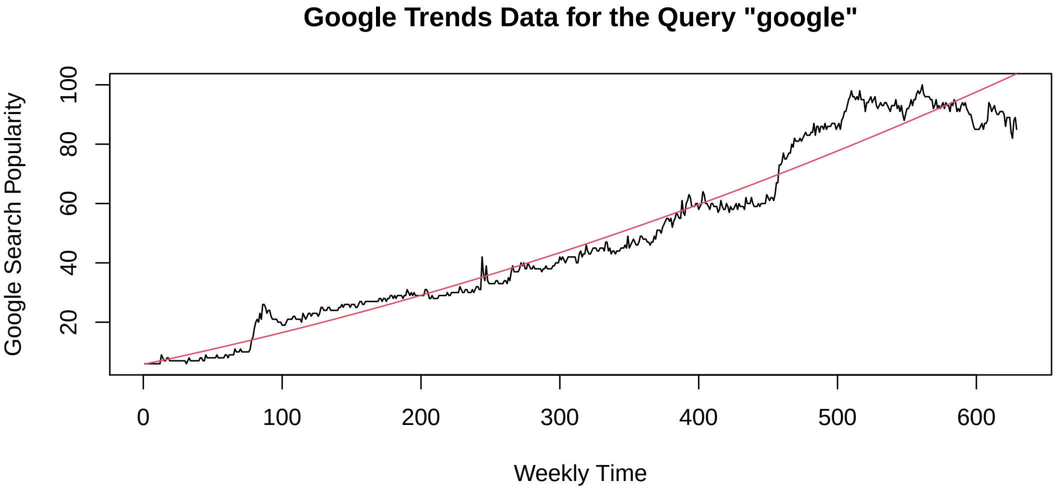
Example - ACF Correlogram of Residuals: $\hat{X}_t$ is plausibly white noise



Example - Googling "Google"



Example - with Trend



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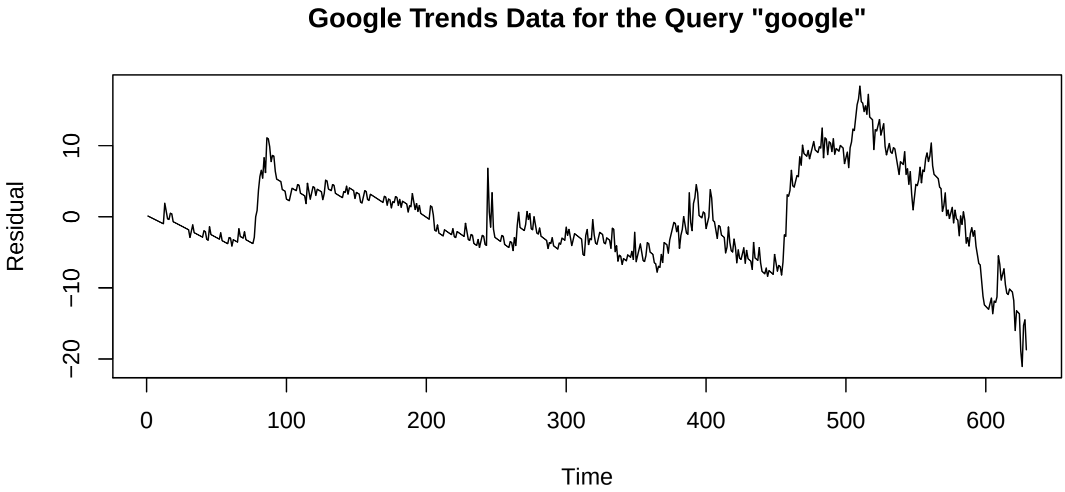
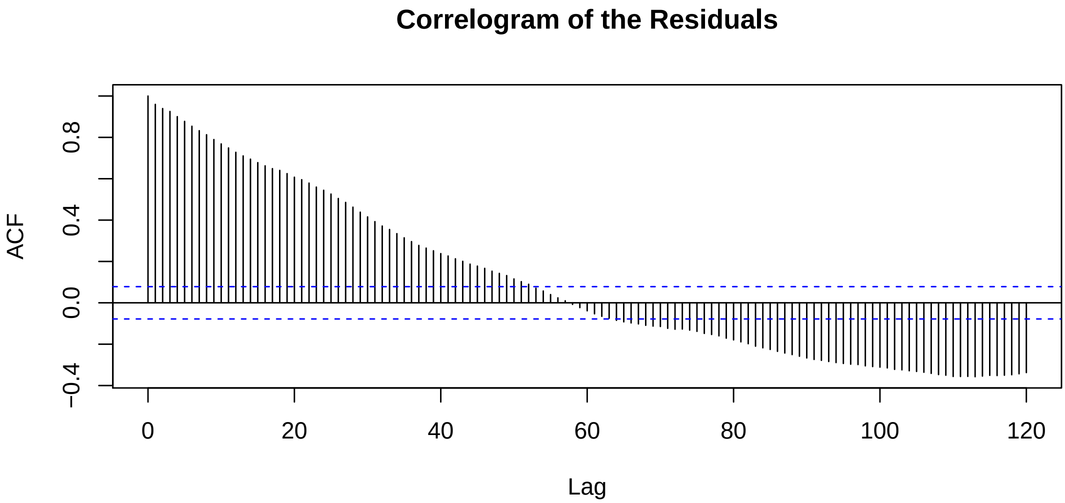


Example - ACF Correlogram of Residuals



Parametric Trend Estimation

Effectively deterministic in our model: better make sure it's plausible!

Advantages

- Gives very accurate estimates when model assumptions are correct.

Disadvantages:

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Disadvantages:

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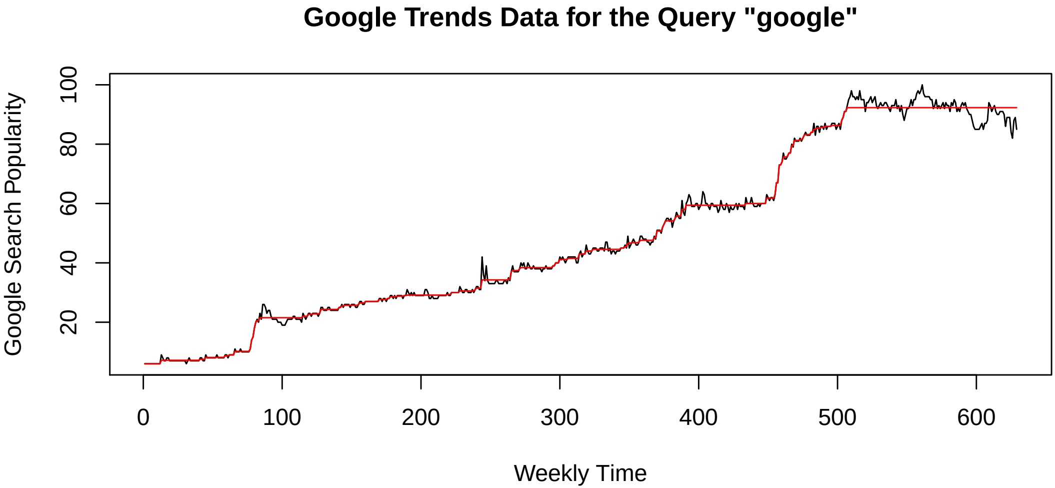
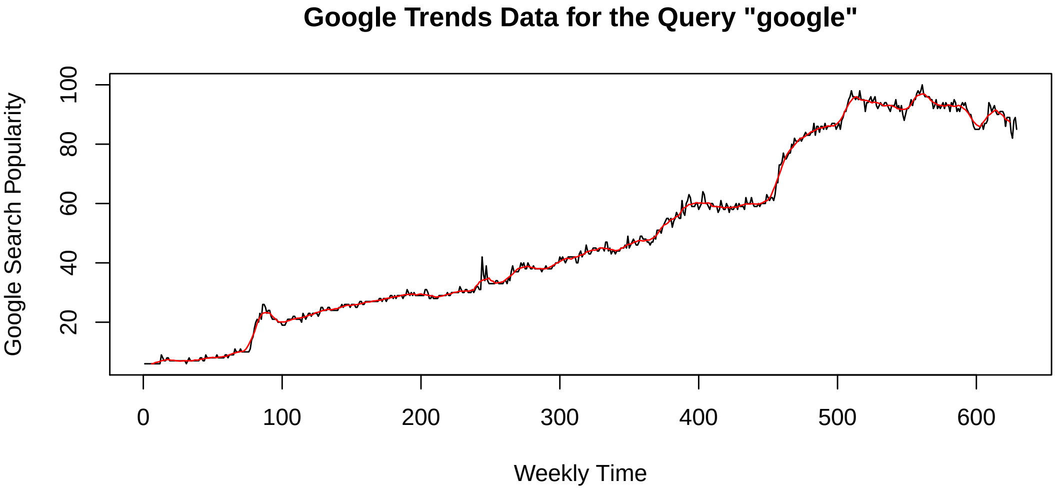
- Parametric form might be unrealistic in practice.

Other Methods

Google + Isotonic



Google + Two-sided Smoothing



Original Motivation:

Model and subtract the trend, so that the new series (the residuals) are steady over time ("reasonably stationary")

Further Reading

https://anomaly.io/seasonal-trend-decomposition-in-r/index.html