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Astron-84

Written Assignment Week 4

1. From the 4 equations given,  $\Delta t' = \gamma \Delta t$ ,  $v = (GM/R)^{1/2}$ ,  $P = (2\pi R/v)$  and  $\gamma = 1/\sqrt{1 - v^2/c^2}$ , then  $v = 2\pi R/P * ((G * M * P^2)/(4 * \pi^2))^{1/3}$ , giving values of 3072 m/s for an observer on Earth's surface (relative to the core of the Earth, with a period of 24 hours) and 2870 m/s for a satellite orbiting with a period of 12 hours. Thus, we can calculate the time dilation at the Earth's surface to be (assuming  $\Delta t$  at the Earth's core to be 1 s) approximately 1.000000000524 s, and for a satellite to be 1.000000000833 s. This gives a time dilation of approximately 0.000308999493 microseconds.
2. For the earth,  $(1 - (2 * G * M)/(R * c^2))^{1/2}$  gives 1.000000000695 s, and for a satellite at  $2.661 * 10^7$  m, 1.000000000167 s. This gives a difference of 0.00052834204 microseconds.
3. The larger effect is given by general relativity, in the second question. The signs of the two are switched as well; for the second problem, the signs were negative because the Earth had greater time dilation than the satellite, but for the first problem, the signs were positive because the satellite had greater time dilation.
4. For the speed of light times the time dilation, the first question's results would yield a positional error of 92635.718 meters, and the second's results would yield 158392.96 meters.
5. If the time dilation is  $\Delta t'$ , then because positional error  $E = c * \Delta t'$ , then 4.8768 m divided by  $c = 16.27$  nanoseconds.
6. Special relativity relies on the velocity proportional to the kinetic energy  $K = (1/2)mv^2$ . General relativity lies on the velocity proportional to the potential energy  $U = (-GMm/r)$ . Thus, one measures the energy relative to the intrinsic velocity (positive) and the other measures the energy relative to a spot infinitely far away, where the gravitational potential energy is 0. Thus, because of the differences in frames of reference, the general relativistic relationship would have the opposite sign from the special relativistic one.