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A RE-DETERMINATION OF THE TRICHROMATIC COEFFICIENTS OF THE SPECTRAL COLOURS

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ABSTRACT—Using a new trichromatic colorimeter a series of colour matches through the spectrum has been made by ten observers. The results have been averaged and a mean set of trichromatic coefficients for the spectral colours derived. These results are compared with previous determinations made by König and Abney. The variations in the coefficients that have been found amongst the ten observers must, as a consequence of a new method of basing the trichromatic units, be attributed to variations in the process of reception, but their magnitude appears to be of a small order. On the other hand, there are big differences in the amount of the macular pigment in different eyes and probably some variation in its dominant hue. These variations have been investigated by matches on a standard white, results for 36 observers being given in the paper and a mean value determined. This value, combined with the mean spectral coefficients, has been used to compute an average locus for the spectral colours in the colour triangle, with white at the centre.

Other points discussed in the paper include the technique of colour matching, the range of intensity over which matches remained valid, and variations of luminosity.

INTRODUCTION

IN a previous paper* a colorimeter was described which had been designed and built by the author to carry out some researches on colour vision in the Technical Optics Department of the Imperial College. The present paper describes the first results that have been obtained from observations made by a number of observers, all trichromats. The main results are concerned with the determination of the trichromatic coefficients of the spectral colours, that is to say, the proportions of

* W. D. Wright, "A Trichromatic Colorimeter with Spectral Primaries," *Trans. Opt. Soc.* **29** 1927–8 (225).

three selected primaries, red, green, and blue, which are required to match the colours ranging through the spectrum. These values determine the positions of the spectral colours in the colour triangle and when combined with relevant luminosity data they may be used to compute the spectral mixtures curves and the sensation curves. A set of sensation curves has been derived, but they are provisional only, as satisfactory luminosity values are not yet available.

The value of the results is two-fold: first, in the realms of physiology the information is fundamental to the study of colour vision and must underlie and be explained by any theories dealing with the subject; secondly, the science of colour-measurement or colorimetry depends for its basic principles on the colour triangle, the locus of the spectral colours in that triangle, and on the sensation curves. This is the ground that is being covered by the research.

The figures at present available in this field have been derived from experiments made either by König and Dieterici or by Abney. Both sets of results have been criticised on technical grounds, and the paucity in the number of observers has further diminished their value. Attention may be called to a paper by Guild* and a monograph by Troland†, both of whom refer to the urgent need for the accurate re-determination of the three-colour mixture data.

Theoretical Considerations

As the customary method of calculating the results has not been followed, it will be necessary to outline in brief the theoretical side of the problem. It is an experimental fact that, with some few exceptions, any colour can be matched by a mixture of three colours, which we call the primaries. Or, making no exceptions, every colour can be uniquely represented by an equation of the type

$$C = \alpha R + \beta G + \gamma B$$

where α, β, and γ are coefficients, positive or negative, representing certain proportions of the red, green, and blue primaries respectively. If α, β, and γ are all increased in the same ratio, the colour represented by the equation will not have changed in quality, but there will simply be more of it, that is, its intensity is increased. It is therefore quite legitimate and involves no loss of generality, if we arbitrarily decide to make $\alpha + \beta + \gamma = 1$. This restriction, in effect, separates the two variables, luminosity and colour, so that each can be dealt with individually. When the coefficients add up to unity the colour equation is referred to as a unit trichromatic equation and the amount of colour represented by the equation is one trichromatic unit (one T. unit). The conception of such a unit is extremely useful. It fulfils the ordinary arithmetical laws; for example, if we mix one T. unit of a colour C_1 with one T. unit of another colour C_2, then we have 2 T. units of some

* J. Guild, "A Critical Survey of Modern Developments in the Theory and Technique of Colorimetry and Allied Sciences," *Proc. Opt. Convention*, 1926 (61).

† L. T. Troland, "The Present Status of Visual Science," *Washington Nat. Acad. of Science, Bull. Nat. Res. Council*, vol. 5, Part 2, No. 27, 1922.

third colour C_3 and so on. The elaboration of this process is clearly explained in a paper by Guild*.

To determine the units of the primaries it is customary to match white by a mixture of the three primaries of the colorimeter in use and to correct the readings as given on the instrument scale by suitable factors, so that for this match all three shall be equal. These factors are then applied throughout any series of matches. This method has some underlying theoretical justification, since it may be considered that the proportions of the white match have equal colouring values as, when mixed, they produce a colourless sensation. But, in practice, it is really immaterial on what basis the units are founded, so long as they are consistent throughout.

A simple example of the method may be useful. Suppose that the intensities of the primaries in the colorimeter are controlled by variation of sector openings and that these angular apertures for the white match are 30° for red, 15° for green, and 60° for blue. To correct these readings so that they are in equal proportions, a factor of 2 must be applied to the red and 4 to the green and these factors must be used in any subsequent colour match. Thus, if for a new colour the instrument readings are 40° for red, 10° for green, and 20° for blue, the proportions, in the units agreed upon, are

red 80 green 40 blue 20

giving the trichromatic coefficients as

$$\text{red } 80/140 = \cdot 571, \quad \text{green } 40/140 = \cdot 286, \quad \text{blue } 20/140 = \cdot 143$$

from which we have the unit equation for the colour as

$$C = \cdot 571R + \cdot 286G + \cdot 143B.$$

The function of the white match in this example has been to provide two factors, one for the red readings and another for the green. Any other basis by which two such factors can be determined will be equally valid and provide an equally consistent series of results. Suppose, for instance, that we match a monochromatic orange plus the necessary small amount of desaturating blue and that the sector readings are 10° for red, 30° for green, and 6° for blue. We can agree to make the amounts of red and green in this match equal, so providing a factor 3 for the red. In the same way we can match a monochromatic blue-green, desaturated in this case with sufficient of the red primary for it to be matched, the instrument now reading, say, 3° for red, 30° for green, and 15° for blue. From this match we obtain a factor 2 for the blue, if we agree to call the amounts of green and blue in this colour equal. Applying these two factors to all future matches we obtain a series of results on a system as consistent as before and one that can be compared with the first by suitable mathematical transformations. The advantage of basing the units on matches of monochromatic colours becomes evident when we have to compare the results for two or more observers.

* J. Guild, "The Geometrical Solution of Colour Mixture Problems," *Trans. Opt. Soc.* **26** 1924–5 (139.)

Consider the case of an observer who has matched white and based his units on that match and has then made a series of matches to determine the coefficients of a number of the spectral colours. These can be plotted in the colour triangle with white at the centre. (The properties of the colour triangle are fully described in the paper by Guild quoted previously.) Now consider the effect of a yellow filter placed in front of the observer's eye if he repeats his observations, again basing his units on a match with white. The quality of the white will be affected by the filter, becoming a more or less saturated yellow according to the filter used, but the quality of the spectral colours will remain unchanged since they are monochromatic radiations. Thus the relation between the white and the spectral colours cannot remain the same as in the first series of observations; therefore, since the white will again be plotted in the same position as before, that is, at the centre of the triangle, it follows that the spectral colours will have a new locus.

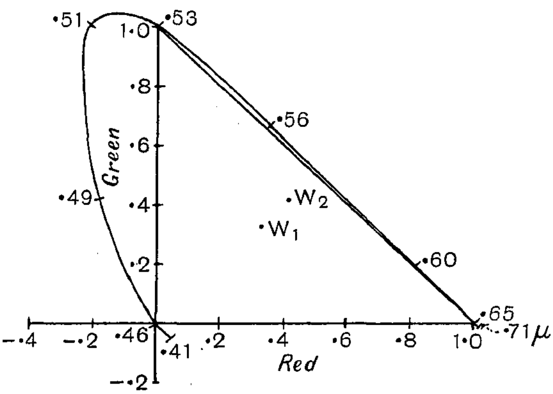

Fig. 1

If, however, the same two sets of observations are worked out by basing the units on matches with two monochromatic radiations, as suggested above, the position becomes reversed. Since the quality of neither the spectral colours nor the "unit-colours" is affected by the yellow filter, their relative positions in the colour triangle will be unchanged, with the result that the spectral locus will be the same in both cases. As the white, on the other hand, is modified by the filter, it will take up a new position in the triangle. The effect is shown in Fig. 1, where the continuous curve represents the spectral locus in the colour triangle, and W_1 and W_2 are the positions of the white point without and with the filter respectively. The distance between W_1 and W_2 is a measure of the yellowness, or saturation, of the filter and, for varying degrees of saturation, the locus of W_2 will be a straight line passing through W_1. The intersection of this line with the spectral locus gives the dominant hue of the filter. These pieces of information can be immediately deduced from the results, provided they are expressed in units based on spectral colours; on the white basis the facts are indicated by a general shift of the spectral locus, which illustrates the true state of affairs in a very indirect and obscure fashion.

The value of this discussion for comparing the results of several observers becomes apparent when we consider the constitution of the human eye. Between the cornea of the eye and the retina there exist a number of absorbent materials the most potent of which is the macula lutea, a yellow pigment of quite small extent, generally just exceeding the fovea centralis in area. Whilst the other absorbent materials are relatively transparent and unlikely to vary to any great extent from one person to another, the yellow pigment may have a very pronounced colour and may vary greatly in amount amongst different observers. Thus, although there may be variations in the receptor processes in the eye and in the nervous conductions to the brain, these cannot be investigated until the variations in the macular pigment are first isolated. This can be done by working with units on a spectral basis, for any variation in the density of the pigment will then show itself by a variation in the position of the white point in the colour triangle along a straight line. A position off this line may indicate a variation in dominant hue of the pigment, whilst differences in the coefficients of the spectral colours will imply variations in the physiological processes of reception and perception. However, although a variation in dominant hue may be indicated by the results, it does not necessarily follow that such variation is the result of a change in the constitution of the pigment, for it is possible that an increase in the density or the thickness of the pigment would produce a change in its dominant hue as well as in its saturation. This phenomenon does occur with some coloured liquids, but from the character of its absorption curve it appears probable that in the case of the macular pigment the effect, if any, would be small.

The Experimental Arrangements

For a description of the apparatus used, reference should be made to the paper by the author already quoted in the introduction. In using the instrument two persons are required, one for observing and the other for noting down the readings when matches have been made. The second experimenter is also responsible for setting new colours to be matched and for making any other experimental adjustments that may be required. The observer is thus solely concerned with colour matching, a fact that tends to create the right mental atmosphere, an important consideration in work such as this, in which the temperament of the observer plays a big part in the accuracy and reliability of his observations. Whilst no indication was obtained that the state of adaptation of the observer affected any match, it was thought desirable to allow the observer five or ten minutes' dark adaptation before commencing a series of observations; this at any rate gave time for the observer to settle down and for after-images to disperse to some extent.

The mean wavelengths of the primaries used in the experiments (spectral primaries are employed in the instrument) were ·650, ·530, and ·460 μ. The choice was an entirely practical one, the deciding factors being a desire to make the negative coefficients as small as possible and to have sufficient intensity of the blue primary. The results can, however, be transformed to any other set of primaries that may be desired, provided their position in the colour triangle is known. The widths of spectrum included in the pencils of light reaching the eye were $\cdot 0060\,\mu$ for the red primary, $\cdot 0035\,\mu$ in the green, and $\cdot 0016\,\mu$ in the blue. The corresponding width of any other part of the spectrum can be approximately interpolated from these figures. Whilst, with this relatively big range of wavelength, the beams cannot be considered strictly monochromatic, it is apparent that the colour of the integral light cannot differ greatly from the colour corresponding to the mean wavelength of the beam, since the light in the two halves of the band will mix to give a colour very nearly the same as that of the central pencil. A rough calculation based on the sensation curves for white light showed that the correction required amounted to about $\cdot 0002\,\mu$, even in the part of the spectrum of greatest hue sensitivity (about $\cdot 580\,\mu$). Any error introduced from this cause can, therefore, be safely neglected, since even in this region the hue sensitivity is only about $\cdot 0010\,\mu$.

A square field of view, divided horizontally into two equal rectangles, has been used, the whole subtending approximately 2° at the eye. This ensures that the fovea alone is involved in the colour matching and so avoids complications which might arise from the varying structure of the retina; it further ensures that, in general, the image falls entirely within the bounds of the yellow spot. A 2° field is in line with the standard suggested by Guild and with the standard used in flicker photometry. The effect of a light surround field, as used in flicker photometry, was also investigated, a surround field of about 10° angular subtense being provided.

As it was much to be desired that some luminosity measurements should be made in the course of the work, a form of flicker photometer was set up, by means of which the relative intensities of the three primaries could be measured. The arrangement consisted simply of an illuminated white sector which rotated so that the primary and the rotating sector came into view alternately. When the correct speed of rotation was obtained, the sensation of flicker could just be made to disappear by suitably varying the intensity of the primary. Repetition of the experiment for all three primaries gave their relative intensities. Further, by measuring the intensity of the illumination on the sector with a holophane lumeter, the approximate brightness level at which matches were being made was determined. Allowance had to be made here for the size of exit pupil used in the colorimeter.

To investigate the variation in a colour match, if any, with change of intensity level, a method was used in which a sector rotated so as to intersect both halves of the field. In this way the intensity of the two fields was reduced equally.

The white to which the results relate had an energy distribution similar to that of a black body radiating at 5000° K.* This was obtained with a 1000-watt lamp standardised at the National Physical Laboratory, the light from which, after passing through a standard liquid filter, fell on a white magnesium oxide screen.

THE TECHNIQUE OF COLOUR MATCHING

One or two notes on the technique adopted in making the observations may be usefully inserted here. In making the first match of a colour, practice and experience seem to be essential to attaining the desired result in a reasonable time.

* See Appendix, p. 158.

It was found impossible to formulate satisfactory rules for the guidance of fresh observers and the difficulty in such cases was largely overcome by a more practised observer making a rough match first. In repeating a match (each colour was matched three times to obtain a mean result) it is only necessary to upset it sufficiently for the match to be perceptibly wrong; to do more than this is a waste of time without any compensating gain in accuracy. Moreover, provided the first match is a reasonably good one, it is easier and appears legitimate to upset and readjust one primary at a time. This not only saves time, but also simplifies matters for the observer and so decreases the mental strain involved. If it is found that a good match cannot be made without adjusting all three primaries together, such an adjustment can always be made. A precaution, which probably prevented a number of bad matches being passed off as satisfactory, consisted in asking the observer to look away from the field of view for a few seconds when he thought he had made a match and then, after a short rest, to see if he was still satisfied with the match. The general practice of occasionally looking away for a second or two rather than steadfastly staring at the field is to be recommended for comfortable and accurate work. The time which is taken over a match depends largely on the temperament of the observer and the amount of practice he has had. It is best if possible to strike a happy mean between rapidity and care. Neither the very rapid nor the over-conscientious type of observer is likely to make consistent settings and the latter is bound to suffer from exhaustion in a short time. With practice it should usually be possible to make a satisfactory match in about a minute or a minute and a half.

The Results

Coefficient curves for the spectrum have been obtained for ten trichromats in all, each observer making about thirty matches through the spectrum. In order to determine the average values it was found more convenient to plot the coefficients for each primary against the wavelength of the spectral colours as a separate curve than to represent each colour as a point in the colour triangle. The difficulty, if this were done, arises in determining the mean curve; for not only has the average shape to be found, but the wavelength scales on the spectral loci must also be averaged. This complication can be overcome by plotting the coefficient curves separately.

In Fig. 2 the ten sets of results obtained are shown superposed; the coefficients are plotted as ordinates against a horizontal wavelength scale*. The wavelength of each primary is given by the point where its own curve has the value unity and the other two curves are zero, whilst the points at which the red and green curves and the green and blue curves intersect, namely at S_1 and S_2, indicate the two spectral colours, of wavelengths ·5825 and ·4940 μ, on which the units were based.

In addition to obtaining a mean set of curves from Fig. 2, the significance of the variations that occur between different sets needs to be investigated. In this connection it must be borne in mind that each set of curves must necessarily be

* It should be noted that in Figs. 2, 3, 4, and 5 the wavelength scale is not uniform, having been directly transposed from an arbitrary instrumental scale.

somewhat similar in shape by reason of the fixed points through which all the curves must pass. Thus the variations that occur may be of greater importance than would at first sight appear probable.

The part played by observational and instrumental errors in producing the differences has first to be discussed. Generally, the observers were new to the art of three-colour matching, but in all cases some time was spent on practice observations. The settings of the more accurate observers could usually be guaranteed to within 1 or 2 per cent. for the red or green primaries and 3 or 4 per cent. for the blue primary, provided the amount of the primary required in the match

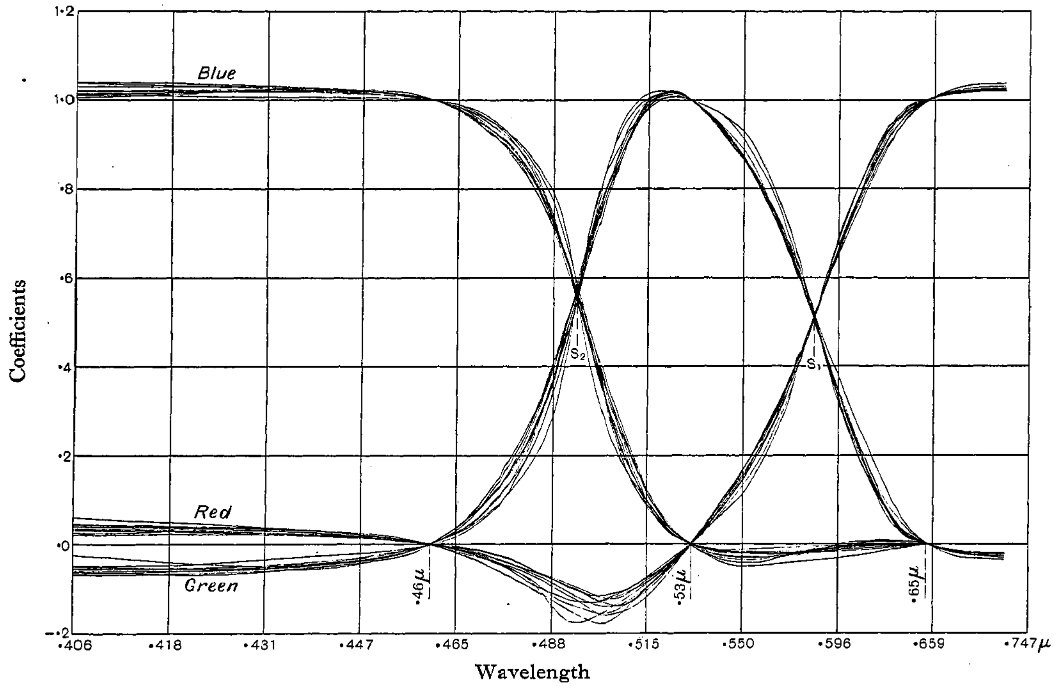

Fig. 2. Superposed coefficient curves of 10 observers

was reasonably high. The effect of these percentage errors on the derived trichromatic coefficients can be very easily determined for a given case and this is the only safe course to adopt. It is impossible to lay down general estimates of the reliability of the coefficients, quite apart from the variation of hue sensitivity over the colour field.

The chief sources of instrumental error are three in number: the inaccuracy of the wedges and their calibration; error of wavelength setting and calibration; and change in the colour temperature of the pointolite lamp. The first of these was reduced to a minimum by frequent calibration of the wedges; the second was almost certainly negligible since the wavelength scale was checked as a matter of routine with a mercury lamp before any series of results was attempted; whilst the possibility of the third was rendered remote by running the lamp off the batteries instead of the mains and by introducing a voltmeter and variable resistance into the circuit. The magnitude of the first source of error, namely that of the wedges, is the only one that need be seriously considered. The intensity of light transmitted could be measured to an accuracy of about 1 or $1\frac{1}{2}$ per cent. with the wedges that were in use. This is an error larger than one would wish, as it is of almost the same magnitude as the observational error of an experienced observer, but, in view of the big differences in matches made by different observers, its ultimate significance is small.

Considering now a concrete case, it is possible that between ·65 and ·53 μ the blue, which is only present in the match in small quantities, may be determined only to within 20 per cent. of its true value. If we take the coefficients of S_1 as

$$\cdot 507 \text{ red} \qquad \cdot 507 \text{ green} \qquad - \cdot 015 \text{ blue},$$

then a 20 per cent. change in the blue gives the new coefficients as

$$\cdot 509 \text{ red} \qquad \cdot 509 \text{ green} \qquad - \cdot 018 \text{ blue}.$$

Again, between ·53 and ·46 μ, the red may be matched to within 10 per cent. An error of this magnitude on the coefficients of S_2, which we may take as

$$- \cdot 130 \text{ red} \qquad \cdot 565 \text{ green} \qquad \cdot 565 \text{ blue},$$

will give a new value of

$$- \cdot 144 \text{ red} \qquad \cdot 572 \text{ green} \qquad \cdot 572 \text{ blue}.$$

Judging from these figures it appears that the variations that occur between ·65 and ·53 μ cannot be explained as observational errors. I am inclined to think, in fact, that there are genuine differences in the amount of blue required in the matches, but the question is a very open one. Over the blue-green portion of the spectrum the variations appear still more marked and cannot, I think, be wholly due to observational error, despite the fact that the observations were generally remarked upon as being more difficult in this region than elsewhere. The evidence for actual variation in the receptor or more central processes certainly appears strongest here. Variation in macular or any other pigment is, of course, eliminated by the system of units used. From the blue primary to the end of the spectrum the variations are surprisingly small and of no great significance.

The mean set of curves obtained from Fig. 2 is shown in Fig. 3. Of the qualitative features of these curves, the rather sudden bend in the negative portion of the red curve is of interest and the position of this minimum may have some significance. Any physical or physiological interpretation of such features is, however, hardly likely to be obtained until the data are converted into terms of "sensations." The method of representing the results that is used here is useful for practical purposes, but of no special value theoretically.

In Figs. 4 and 5 the results obtained by König and by Abney are shown; their results have been re-calculated for the same primaries and on the same unit-basis, so that they are directly comparable. The possibility of doing this further illustrates the value of the new method of calculating the coefficients, for the specification of the white used was not known in either case. The general agreement of the curves with each other and with Fig. 3 is remarkable: some difference is shown

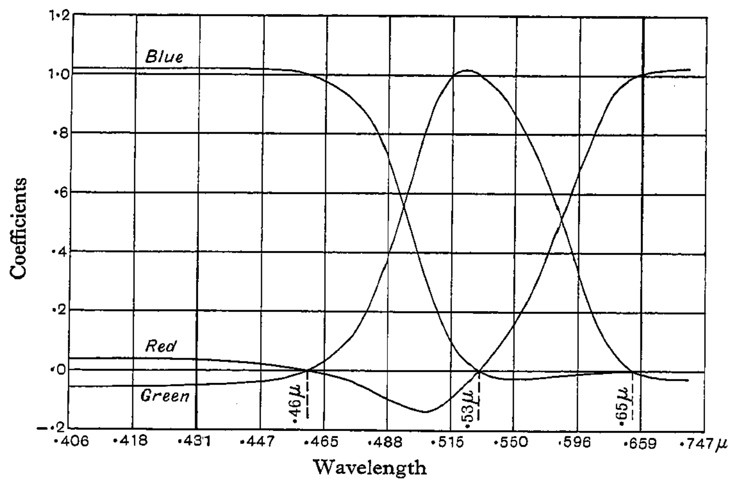

Fig. 3. Mean coefficient curves from results of 10 observers

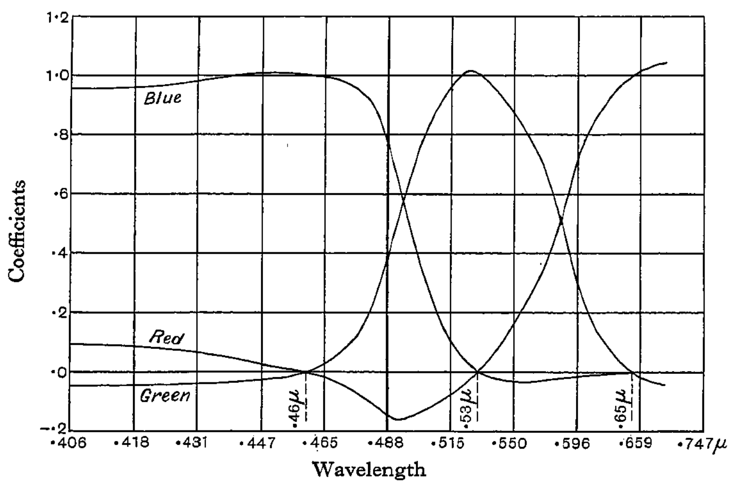

Fig. 4. Coefficient curves re-calculated from König's results

in the dip in the red curve, and the violet coefficients are definitely at variance. Some such variations were anticipated, as low intensity and stray light were bound to affect both Abney's and König's work, especially at the shorter wavelengths.

The data of Fig. 3 are represented in Fig. 6 by the curve lying outside the colour triangle. This is the spectral locus, the scale marked out along the line giving the

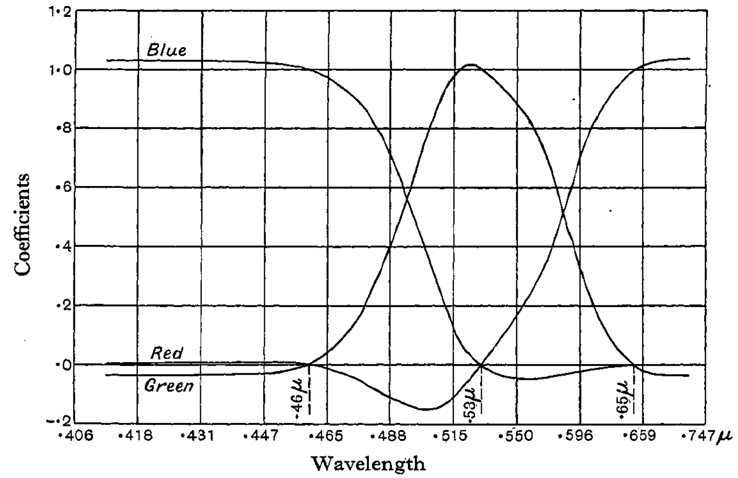

Fig. 5. Coefficient curves re-calculated from Abney's results

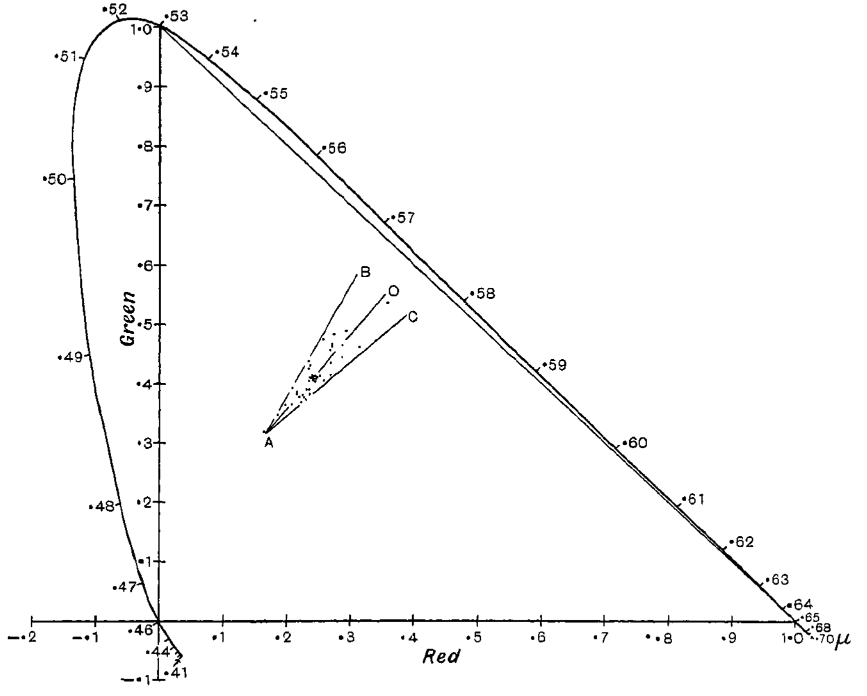

Fig. 6. Colour triangle showing mean spectral locus of 10 observers and white-points of 36 observers

wavelength in microns. The right-angled triangle used here is in practice more satisfactory than the customary equilateral triangle, as rectangular co-ordinates can be utilised. Such an arrangement has been discussed and used before by Ives and Guild. The abscissae represent the red coefficients and the ordinates the green, whilst the values for the blue are given as the differences from unity of the sums of the red and green coefficients. The blue primary is thus represented by the origin.

Inside the triangle I have plotted the "white-points" of thirty-six observers. In all, forty-six observers have been examined, but the results of ten have been discarded, three of them being in some way anomalous and seven showing marked inaccuracy and unreliability. Before the scattering of the points is interpreted, the magnitude of the probable observational error must be investigated.

The effect of lack of practice was diminished as far as possible by making use of the precautions mentioned in the section on the technique of matching. Judging from the accuracy with which the observations were repeated, it appears that the red and green coefficients can be relied on to within 4 per cent. and the blue to within 8 per cent. This estimate is on the safe side, but, even assuming these figures, the effect on the coefficients is not great. If we take as an average for the white coefficients the values

$$\cdot 250 \text{ red} \qquad \cdot 400 \text{ green} \qquad \cdot 350 \text{ blue}$$

and work out the effect of a 4 per cent. change on the red we find the new coefficients

$$\cdot 258\, R \qquad \cdot 396\, G \qquad \cdot 346\, B.$$

A 4 per cent. change in the green gives

$$\cdot 246\, R \qquad \cdot 410\, G \qquad \cdot 344\, B.$$

An 8 per cent. change in the blue gives

$$\cdot 243\, R \qquad \cdot 389\, G \qquad \cdot 368\, B.$$

These variations are small and cannot account for the differences seen in Fig. 6.

In the theoretical section it was shown that, if variations in the amount of the yellow pigment occurred, the white points would lie on a straight line. Therefore, if amount of pigment were the sole source of variation, we should expect the points we have obtained to lie, within the limits of experimental error, on some mean line such as AO. From the investigation of the observational error given above, added to the fact that the inaccuracy in the blue can play practically no part in making the points lie off AO, there seems little doubt that the failure of the points to lie on the line is the result of some further phenomenon. This conclusion is supported by other considerations. A simple explanation is to assume that the pigment not only varies in saturation, but also in dominant hue. If such were the case the white points would lie in a cone with its apex at the white point of an observer with no pigment, and to test this theory the radial lines AB and AC have been added to Fig. 6, A being taken as the "no-pigment" point. The position of this point is confirmed by some extra-foveal observations that were made. The grouping of the points undoubtedly suggests a conical arrangement, for the scattering at the upper end is much greater than at the lower, in spite of the fact that there are considerably more points in the narrower than in the wider part. If the scattering were accidental, if, for example, it were due to observational error, then the scattering should be greatest where the observations are most numerous. Such, however, is not the case. The conclusion has thus been reached, and is put forward here, that the major source of variation among normal trichromats is due to the macular pigment varying greatly in density and to a less extent in dominant hue. It may be further remarked that the variations represented in Fig. 6 are not insignificant quantities, but are surprisingly big. They must, in fact, seriously affect any system of colour measurement which does not make allowance for such differences.

Attention has frequently been drawn, notably by Maxwell, Helmholtz, König, and v. Kries, to the effect that the yellow pigment must have on colour matching, but the subject has generally been dealt with in a qualitative manner, with little indication how colour measurements were affected. More recently, R. H. Sinden* has discussed the problem from a consideration of complementary colours. He also concluded that the pigment was responsible for the main differences found and showed how, for the eight observers he examined, their "white points" lay approximately on a straight line when plotted in the colour triangle. Sachs† made a post-mortem examination of the pigments of nine human eyes and the absorption curves he obtained appear to support the possibility of a change in dominant hue as well as in saturation. Variation of hue would also explain many of the so-called anomalous matches made on the Nagel anomaloscope.

From the points plotted in Fig. 6, a point in a mean position has been selected to provide data for a "normal" eye. Clearly, the number of observations obtained, whilst considerable, was not sufficient to warrant the determination of the mean by any statistical method such as a frequency curve. The method of determination was therefore as follows. *AO* was drawn so that an equal number of points lay on either side of it; the position along that line was then found at which there were about an equal number of points above and below it. The coefficients for the point fixed in this way and marked with a small cross in Fig. 6 were

·243 red ·410 green ·347 blue.

Having regard to the end to which it is hoped this work, when combined with results of a similar nature obtained by other workers, will lead, namely, to the standardisation of a "normal" eye for colour matching, when this stage is reached it will be theoretically immaterial whether the units are based on a white match or on matches with two spectral colours. For, since only those observers who are normal, or groups of observers whose mean is normal, will be of use in colour measurement, individual variations will not enter into the problem and the particular advantage of the spectral method in demonstrating such variations in a simple manner will no longer be called for. Further, in many cases it will be more

* R. H. Sinden, "Studies based on the Spectral Complementaries," *J.O.S.A.* **7** 1923 (1123).

† M. Sachs, "Ueber die specifische Lichtabsorption des gelben Fleckes der Netzhaut," *Archiv f. d. ges. Physiologie,* **50** 1891 (574).

practicable to match the standard white than spectral colours and in any case this involves only one match instead of two. For these reasons the coefficients for the spectral colours have been re-calculated so that the mean white occurs at the mid-point of the colour triangle. The results expressed in this form are given in Table I and Fig. 7.

Table I

Wavelength (μ)	Trichromatic coefficients		
	Red (·65 μ)	Green (·53 μ)	Blue (·46 μ)
·41	·051	− ·047	·996
·43	·045	− ·043	·998
·45	·021	− ·024	1·003
·46	·000	·000	1·000
·47	− ·031	·057	·974
·48	− ·094	·182	·912
·49	− ·170	·420	·750
·50	− ·233	·772	·461
·51	− ·207	1·002	·205
·52	− ·111	1·049	·062
·53	·000	1·000	·000
·54	·123	·901	− ·024
·55	·232	·797	− ·029
·56	·354	·676	− ·030
·57	·480	·543	− ·023
·58	·604	·414	− ·018
·59	·720	·290	− ·010
·60	·811	·196	− ·007
·61	·881	·124	− ·005
·62	·929	·075	− ·004
·63	·966	·037	− ·003
·64	·988	·014	− ·002
·65	1·000	·000	·000
·68	1·011	− ·011	—
·70	1·015	− ·015	—
Standard white (5000° K.)	·333	·333	·333
König's white	·301	·278	·421
Abney's white	·429	·337	·234

It was seen in Figs. 3, 4, and 5 how the coefficient curves of König and Abney were in very fair agreement with the present results. In Fig. 7 the positions of König's and Abney's white points are shown, the new coefficients being calculated from their positions in their own triangles by suitable arithmetical transformations; they are seen to differ widely from each other and from the mean point just determined. This explains the apparent disagreement that existed between the two sets of coefficients when based on their white matches.

If we neglect the minor variations that were found in the coefficient curves, then, to determine whether an observer has a normal colour matching eye, matches on white and two spectral colours would have to be made. It must be realised that a person with a normal luminosity curve will not necessarily be normal in colour matching, nor will an observer with an abnormal luminosity curve necessarily be abnormal in colour matching. For, taking the second case first, an observer may be abnormal as regards luminosity through one of his primary sensations being reduced and yet have a normal amount of pigment. In such a case he will be normal in colour matching, since a diminution in one of the primary sensations does not, apart from reducing the sensitivity of an observer, affect his colour matching. On the other hand a person might have a normal luminosity curve but an abnormal amount of pigment, for the effect of the latter on his luminosity curve could conceivably be neutralised by his sensations being suitably modified. From this we can conclude that the standard luminosity curve has not indirectly standardised the normal amount of pigment and thence the normal colour matching eye. This conclusion is in no sense an idle one, for it would be somewhat of a paradox if two "normal" eyes existed.

Whilst discussing questions of luminosity, it may be worth while to give the results of a few measurements that were made to investigate luminosity variations

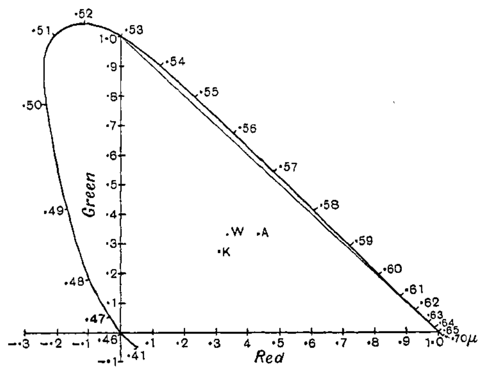

Fig. 7. Locus of spectral colours in colour triangle, with mean white at centre. *W*, Standard white (5000° K.); *K*, König's white; *A*, Abney's white

amongst different observers. The results differ from other measures that have been made before in that they rule out the effect of the yellow pigment. The method by which this was attained consisted in determining the relative luminosities of the amounts of the primaries required in the matches on the two standard spectral colours, and can best be demonstrated by an example. Suppose that an observer matched one of the standard spectral colours, say the blue-green, and that the amounts of green and blue required were 12 and 18 respectively, as measured on the arbitrary colorimeter scale. Now suppose that in the luminosity measurements 4 units of green matched 20 units of blue; then, measured in luminosity units, the proportions of green and blue in the blue-green match become 60 : 18. Suppose, now, that the observer had his yellow pigment increased and that this had the effect of diminishing the transmission of the green by a factor a and the blue by a factor b. The proportions of green and blue now required in the colour match will be $12/a$ and $18/b$ measured in the arbitrary instrument units, and in the luminosity match $4/a$ units of green will be required to match $20/b$ units of blue.

Then in luminosity units the proportions of green and blue in the match are still given as 60 : 18. Thus, luminosity measures made in this way will be unaffected by pigment differences and variations found must be due to physiological differences in the process of reception, presumably to be attributed to modifications in the sensation curves.

Table II

Observer	Relative luminosity of primaries		
	Red	Green	Blue
A	57·2	100·0	6·5
B	70·8	100·0	4·8
C	58·8	100·0	6·0
D	56·8	100·0	4·6
E	84·1	100·0	4·6
F	73·6	100·0	4·7
G	64·8	100·0	5·1

The results are given in Table II and were obtained by making colour matches of a monochromatic orange and a monochromatic blue-green; the three primaries were then matched in luminosity against a white rotating sector (see section on experimental arrangements). Calculations similar to the above then gave the relative luminosities of the three primaries, but, in order that the results might be comparable with one another, the values were adjusted so that the value for green was always 100·0. The amounts by which the results vary are seen to be very great, particularly for red. This demonstrates that the luminosity differences that have been found before were not solely, nor even largely, due to pigment variations.

With regard to the experimental arrangements for the luminosity matches, these were not carried out under strictly standard conditions. The field size was correct and an illuminated surround was used, but the intensity level of the matches was low. The illumination on the retina, in fact, was of the order of 10 photons only, instead of 120 as is recommended. Further, no observer known to have a normal luminosity curve was available. These results cannot, therefore, be correctly correlated with standard luminosity data, but for comparison amongst themselves they supply the necessary information. Incidentally, if the reasoning given above relative to the normal amount of pigment and the normal luminosity curve is correct, it follows that the luminosity curve has standardised the combined effects of the true relative luminosities of the spectral colours and the absorption factors due to the pigment, without standardising either set of values separately. Thus results such as those given in Table II, which express absolute luminosities, cannot at present be obtained for a standard eye, simply because there is no true luminosity standard. This fact is of no consequence in ordinary intensity measurements, but it is likely to cause considerable difficulty when the problem of standardising the sensation curves arises.

Nevertheless, it appeared that some interest would attach to those sensation curves that could be obtained with the available data, even though not referring to a standard eye, so the curves shown in Fig. 8 were calculated relative to a white-light spectrum. For this purpose the luminosity values in Table II were averaged and, as primaries, the hypothetical colours

$$\begin{array}{ll} \text{Red:} & 1{\cdot}015\,R - {\cdot}015\,G \\ \text{Green} & -{\cdot}900\,R + 2{\cdot}000\,G - {\cdot}100\,B \\ \text{Blue:} & \qquad\qquad -{\cdot}050\,G + 1{\cdot}050\,B, \end{array}$$

expressed in the co-ordinates of Fig. 7, were used. These gave an all-positive set of coefficients for the spectral colours. The actual shape of the sensation curves depends on the choice of the primaries; for instance, the rise in the red curve at the violet end of the spectrum can be removed by a modification in the selection.

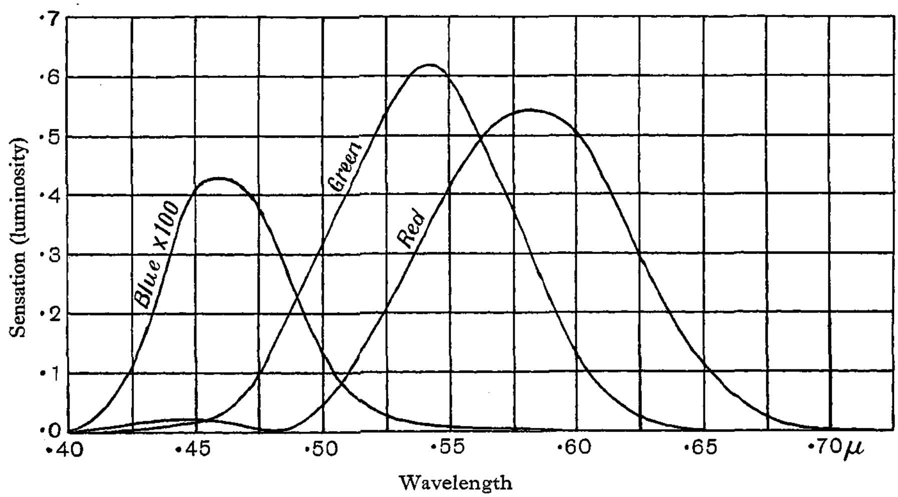

Fig. 8. Sensation (luminosity) curves for white-light spectrum (arbitrary sensation primaries)

Two further series of experiments, subsidiary to the main work, were carried out. The first consisted in testing the value of an illuminated surround in the field of view, by making twenty matches in succession, both with and without the surround, and then comparing the deviations of the readings in the two cases. Several sets of results were obtained, but these were inconclusive. However, as the accuracy of repetition was high in both cases, the mechanical manipulation of the wedges introduced a source of error of the same order as the differences that were being measured. As yet, therefore, no decision for or against a surround can be given. Another experiment which was made consisted in reducing the intensity of both halves of the field equally by means of a rotating sector until the match no longer held. Care was, of course, taken to ensure that the failure of a match was not the result of errors in the sector. Such might have occurred had the aperture become very small. The result of these experiments was to show that matches held over a very wide range and did not, in fact, begin to fail until the illumination had been decreased to about 1 or $\frac{1}{2}$ photon. The intensity at which colour matches were made in the ordinary work varied enormously, but an average estimate would give a value of about 250 photons.

There are three points, I think, which may well be emphasised in concluding this paper. First, it has become abundantly clear that, until data for a "normal" eye have become standardised, the scope of colorimetric science will be strictly limited. This applies to all forms of colorimetry and not merely to the trichromatic system. The second point that needs to be made is in the nature of a caution. The experimental facts that have been given and the discussion of the theory in general give the impression that colorimetry is an exact science following simple mathematical and physical laws. So far as can be judged at present this appears to be the case, but, when one considers the complexity of the physiological mechanism involved, it is only natural to anticipate correspondingly complex results. I should not, therefore, be surprised if subsequent research shows some modifying influences to exist; but if such is the case it is safe to predict that any eccentricities will be of a small order and unlikely to affect the value of practical colorimetry. The third point relates to future research. There are two directions in which, both from the theoretical and practical point of view, information is most urgently needed. First, a filter is wanted by which pigment variations can be eliminated; and when obtained, the magnitude of the residual variations that still remain will have to be investigated. Secondly, determinations of sensitivity must be made, both for saturation, when the hue remains constant, and for hue, for given degrees of saturation. Data of this nature are essential before the question of tolerances in colour measurement can be dealt with.

In my last paper I acknowledged with appreciation the help I had received from Dr Martin, Mr Colborne, and the Medical Research Council. In the present instance I have to add the name of Mr M. E. Sions, as his assistance in the experimental and observational work has been of great value and his enthusiasm in the search for recruits to make observations, a rather thankless task, was most gratifying. I have also to tender my very best thanks to all those who provided the observations, especially to those who spent several hours making a complete set of observations through the spectrum.

APPENDIX

During the discussion of the paper it was pointed out by Mr Guild that the standard white specified by the National Physical Laboratory did not, as stated, provide a source with a colour-temperature of 5000° K., but one with a colour-temperature more nearly 4800° K. To avoid confusion, the complete specification is given in this appendix.

The illuminant used is a gas-filled lamp, operated under specified conditions, used in conjunction with a filter consisting of two layers, each 1 centimetre thick, of selectively absorbent liquids. A subsidiary filter of the same type, but of different concentrations of the constituents, is employed in the process of rating the gas-filled lamps. This subsidiary filter is placed between a vacuum lamp, operated at a colour-temperature of 2360° K., and a Lummer-Brodhun photometer. On the other side of the photometer is placed the gas-filled lamp and the latter is adjusted in voltage until a colour-match is obtained in the photometer. In this way the gas-filled lamp is rated to a definite colour-temperature, although the actual value of this temperature does not appear in the specification. This procedure has been considered best owing to the uncertainties at present existing as to the temperature scale above 2500° K. The colour-temperature actually obtained in this way is at present believed to be very close to 2900° K.

The lamp, as thus rated, is used in conjunction with the "white light" filter, to provide a standard illuminant for coloured materials and a standard "white" for unit evaluation. The energy distribution departs appreciably from that of a black body, but the integral colour matches that of a black body at a temperature of approximately 4800° K. A colour-temperature of 5000° K. was originally aimed at, but after the filter had been developed it was found necessary to reduce the working temperature of the gas-filled lamp from that first contemplated, owing to rapidity of ageing.

The filters used have the following specification: Each filter consists of two solutions *A* and *B* contained in colourless glass cells having two compartments each 1·000 cm. thick.

White light filter

Solution *A*:	Cupric sulphate	2·30 gm.
	Ammonia (density ·90)	230 cc.
	Distilled water to	1000 cc.
Solution *B*:	Cupric sulphate	15·0 gm.
	Cobalt sulphate	15·0 gm.
	Distilled water to	1000 cc.

Auxiliary filter for rating lamps

Solution *A*:	Cupric sulphate	1·41 gm.
	Ammonia (density ·90)	141 cc.
	Distilled water to	1000 cc.
Solution *B*:	Cupric sulphate	11·0 gm.
	Cobalt sulphate	8·5 gm.
	Distilled water to	1000 cc.

Full particulars as to the making up, storing, and use of these filters are contained in the "Instructions for Use and Maintenance of the Guild Trichromatic Colorimeter, H. 93" issued by Messrs Adam Hilger, Ltd.

DISCUSSION

Mr J. Guild (partly communicated): I think Mr Wright is to be congratulated on an exceedingly valuable contribution to the subject of colour vision and colorimetrics. His conclusion that the evaluation of the primaries from two colour matches with monochromatic light instead of, as is usual, from a single match with white gives results in which the differences encountered between different observers are very much diminished, although not completely removed, is in accordance with our experience at the National Physical Laboratory. When Sinden's paper on complementary colours first suggested the possibility that the major differences between "ordinary" observers were due to variations in the colouring of the macula affecting the energy distribution reaching the sensitive elements, in the case of stimuli spread over the spectrum, and not to any large extent to variations in the response curves of the sensations themselves, I was attracted by the possibility of using the method of evaluation of units, which Mr Wright has employed, for general colorimetric purposes. The practical disadvantages, for general colorimetry, outweigh the advantages however. The method requires that two standard monochromatic radiations of suitable wavelength shall be readily available in any laboratory in which colorimetry is being done. Further, the evaluation of the units involves two measurements in each of which desaturation of the test field is required instead of a single match involving positive quantities of all three primaries. While these disadvantages are relatively insignificant in a research laboratory they involve a serious complication of routine technique for general colorimetric work. I concluded therefore, as Mr Wright also concludes, that the traditional method of evaluation of units from a white match is preferable for general purposes.

In this connection it may be well to point out that the white used by Mr Wright, which is the National Physical Laboratory standard white, is not correctly described as having an energy distribution similar to that of a black body at 5000° K.*

Of the results obtained by Mr Wright in this work, one of the most interesting and significant is the distribution of the "white" points in Fig. 6. It is difficult to see any way of avoiding his conclusion that this proves the major differences to be due to various degrees of macular pigmentation. There would, of course, be no reason for even trying to avoid this conclusion but for the fact that many physiologists fail to find any appreciable pigmentation in the living eye, and hold that the development of the yellow spot is a post-mortem effect. Mr Wright's result certainly adds a very weighty contribution to the circumstantial evidence, already considerable, in favour of the existence of pigmentation in the living eye even though this may be much more pronounced after death.

Most of the incidental results of this work, relating to the technique of colour matching, are in agreement with our experience at the National Physical Laboratory. In particular may be noted the fact, very important from the practical point of view, that the match, with the field size used, is not dependent on field brightness. I have verified this point carefully on many occasions, and if precautions are taken

* See Appendix, p. 158.

to eliminate misleading effects which may arise at very low or very high illuminations due to the large tolerances which exist under those conditions, it can be demonstrated that the match is quite independent of field brightness.

The only point of technique on which I disagree with Mr Wright is when he advises disturbing only one primary at a time when repeating observations. There is a great temptation to do this and in nine cases out of ten it may make no difference to the result. We have found, however, that the possibility of obtaining a systematic error running through the whole of a series of observations is greater if the plan is adopted than if all three primaries are disturbed before each matching.

Mr Wright is not himself satisfied with the luminosity measurements in this paper. It is very difficult indeed to obtain these constants by flicker photometer measurements, owing to the difficulty of obtaining the necessary high field brightness. With the big colour differences involved in comparing the primaries the results are very sensitive to departures from the standard field brightness.

The best procedure, in my opinion, is to obtain three filters, red, green, and blue, and calculate from their spectrophotometric curves and the visibility curve their relative transmission for light from a specified source. Then if these filters are successively matched in the colorimeter, using the source in question, the luminosity coefficients of the primaries can be calculated from the readings. This process requires the visibility curve to be known for each observer if the object of the work is to analyse the properties of individual observers. On the other hand, if the object is to work out the mixture curves for the mean of a number of observers, one may take the standard visibility curve as applying to them. By this method, applying the standard visibility data to the mean of the colour match data for seven observers, we have obtained a very consistent set of mixture curves which we employ provisionally at the Laboratory pending the extension of the work to a larger group.

There is only one point of criticism with regard to the presentation of the results which I wish to offer. I wish Mr Wright had given his final results (Table I) in terms of the primaries red ($\cdot 70\,\mu$), green ($\cdot 5465\,\mu$), and blue ($\cdot 436\,\mu$). As he points out, data given for any set of primaries can be converted to any other, but the conversion takes a little time, and it would facilitate comparison of results if each worker converted his own data to a common basis.

Now the primaries I have mentioned are quite as arbitrary as those used by Mr Wright. The point about them is, however, that they have for some time been adopted as standards for colorimetric purposes by the National Physical Laboratory, because of the practical convenience in standardising colorimeters without the aid of spectroscopic apparatus. They are the only set of primaries which has as yet been put forward for such purposes, and are in use by various colorimetrists in industrial laboratories.

Prof. L. C. Martin: May I express my congratulations and thanks to Mr Wright? It is clearly important that the physical specification of the standard white should be added to the paper. It would be helpful if we could obtain not only the relevant details of the method of standardisation by the National Physical Laboratory but also the transmission data of the filters and some approximation to the energy distribution.

It has been a great pleasure to watch the progress of Mr Wright's work, which has given our resumption of research in colour vision a valuable start. It seems to me that the great value of Mr Wright's work lies in the way in which he distinguishes between colour vision variations due merely to differences in macular pigmentation and those due apparently to some differences in the receptor processes. These results are of the greatest importance both practically and theoretically.

How far is it possible to train observers to make extra-foveal matches? It has seemed to me that valuable evidence might be obtained regarding the theories in the paper if the results of such tests could be made sufficiently reliable.

Apparently only a limited class of observers can hope to correct their colour vision by the use of filters, and the hope of standardising a "normal eye" is not so bright as formerly seemed to be the case.

Dr R. J. Lythgoe (communicated): Mr Wright's research is unquestionably the most thorough which has yet been undertaken in this subject. With his complete mastery of the physical side of the problem he has combined physiological sympathies; a combination of qualities which is rare and the absence of which has invalidated an enormous amount of well-meaning work. Mr Wright can express quantitatively the variation in macular pigmentation amongst his observers. Has he ever considered that this pigment may vary in amount in different regions of one and the same retina? This suggestion is not an idle speculation. It was found by Cobb that the difference threshold (Fechner's fraction) may be twice as great for one half of the test-field as for the other. It has also been pointed out to me repeatedly that workers using a Bausch and Lomb colorimeter quite often give wildly different readings for the position of equality of the two fields. Since this is particularly noticeable when using blue dyes, it seems likely that the explanation is to be found in regional variations of the macular pigment. It would be interesting to know whether Mr Wright's results would be influenced by reversing his fields. Selective absorption of light is known to occur not only at the fovea but also in the lens, especially in elderly people. Some factor such as this might account for the fact that the "white" points do not lie on a straight line and would obviate the necessity of assuming that the macular pigment varies in dominant hue as well as in saturation. Finally I should like to criticise the practice of taking readings with a partially dark-adapted eye. There is no condition in physiological optics more variable and more difficult to define than the degree of dark-adaptation. In view of the amount of work which has been done by Cobb, Ives, and Martin on the effects of light surrounds, it is a pity that Mr Wright did not always use a controlled light-surround rather than an uncontrollable condition of dark-adaptation. It is also known, empirically, that readings with the dark-adapted eye are subjectively more difficult.

Mr Wright: I must thank Mr Guild, Prof. Martin, and Dr Lythgoe for their kind appreciation of my work. It is gratifying to find that Mr Guild is able to confirm, from work carried out at the National Physical Laboratory, most of the points that have been brought out in the paper.

So far as the actual existence of the yellow pigmentation is concerned, my own experience leaves little room for doubt on the point. Cases occur in which the colouring is very faint or sometimes, apparently, absent altogether and this may go some way to explaining the fact that some physiologists fail to find any appreciable pigmentation in the living eye.

I note that Mr Guild disagrees with the method of repeating observations by disturbing one drum at a time. I cannot see adequate theoretical grounds why the results should be biased (the amount would, in any case, be very small), but I concluded that, as the matching would be simplified, the observer was less likely to be fatigued and his sensitivity would accordingly be greater. This, I expected, would more than outweigh any systematic error, if such existed, but I made no actual tests. In view, however, of Mr Guild's experiments this conclusion is apparently not correct. It may nevertheless be found, when the question of tolerances has been investigated, that the simpler method will satisfy the demands made in colour measurement.

It is interesting to see that it has been found possible to obtain a consistent set of mixture curves by using the mean data of seven observers. I take it that the observers have been selected to have a mean luminosity curve very similar to the standard curve and to give an average "white" point close to the average of a large number of "white" points. The difficulty in making such a selection is the difficulty I anticipated in obtaining a standard set of sensation curves, but it is, evidently, not insuperable.

As I was unaware of the primaries which the National Physical Laboratory had adopted as standard, I was perforce unable to express my results in terms of them. I can quite realise the practical advantages of using readily available spectral lines as primaries, but on the other hand there would be some advantages in the use of spectral primaries which could be actually embodied in a colorimeter. It is unfortunate that the mercury lamp does not possess any bright red line, as in that case an ideal colorimeter could be designed using the lamp as the source. As it is, with ordinary sources, $\cdot 436\,\mu$ is rather far down the spectrum to give sufficient intensity for its satisfactory use as a primary in a colorimeter and, further, $\cdot 5465\,\mu$ being somewhat towards the yellow-green makes the area of the colour field lying outside the colour triangle rather bigger than could be desired. It is a difficult point to decide which is the better compromise; personally I should like to see standardised three primaries which could, if desired, be actually used as primaries in a colorimeter, in which case no arithmetical transformations would be necessary.

As an appendix on the specification of the standard white has been added to the paper, there is no call for any further comment here.

In reply to Prof. Martin, extra-foveal matches certainly seem likely to afford valuable results. Those observations that were made in this way were hardly accurate or extensive enough to give in the paper. With practice it should be possible to obtain results of greater accuracy and some observers, notably those with large amounts of pigment, found the blue primary more sensitive in the extra-foveal than in the foveal region.

Dr Lythgoe's criticisms have to be faced, but they have mostly, I think, been investigated during the work. That the pigment might vary in amount in different parts of the fovea was realised and at least two cases were encountered in which the field did not appear uniform all over, due to this cause. The results in a particular case might be affected slightly by reversing the fields, but the results of a number of observers are hardly likely to be affected systematically.

The fact that the points lie off the line does not appear to be due to variations in the hue of other absorbing media, such as the crystalline lens, as in this case the points would lie in a parallel band and not in a cone.

The problems of adaptation and a light surround were not investigated extensively, but in no case was the state of adaptation or the surround found to affect the matches. Bearing in mind that all the observing was done at the fovea this result is not surprising, and to my mind the only effect that one could reasonably expect would be a modification in the accuracy of the settings or, in other words, an increase in the sensitivity. This may eventually prove to be the case, but the colorimeter used was not sufficiently delicate to indicate the changes. My personal opinion is that a surround field is not practicable in colorimetry. The intensity of the matching field varies so greatly from one match to another, that what would be just the right intensity for one match would be altogether too bright for another. And I think that to regulate the surround intensity according to the field intensity would defeat the end in view, namely, the definite control of observing conditions.

In conclusion I have to apologise for making an error in the paper which is not, I hope, of fundamental importance. It concerns the luminosity measurements and the sensation curves that were derived from them. By a mistake on my part the curves obtained were not, in effect, derived from the mean results of Table II, but the result is really of little consequence as the figures which were used are likely to be as near the true luminosity data as those obtained by experiment. In neither case could the curves be regarded as standard; they were given to indicate the general type of result to be expected.