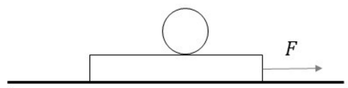
Physics 5A, Fall 2017
Homework Set 8

KK Ch 7: 7.23, 7.26, 7.37

KK Ch 8: 8.1, 8.3, 8.11

S 8.1 The figure to the right shows a disk with mass m on top of a board with mass M. The board rests on top of a frictionless table. A force F is applied to the board to the right. The disk rolls on top of the board without slipping.



(a) If a_m is the center of mass acceleration of the disk, a_M is the acceleration of the board, and α is the angular acceleration of the disk, then show that

$$a_m + R\alpha = a_M, \tag{1}$$

where R is the radius of the disk.

(b) What is a_m and a_M?

(c) If the coefficient of static friction between the disk and the board is μ_S, what is the maximum that a_m can be?

S 8.2 In this problem you will show that for a point particle,

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}, \tag{2}$$

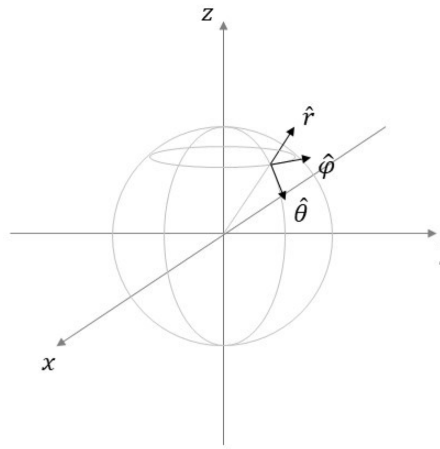
when $\dot{r} = 0$.

(a) The figure below shows the definition of $\hat{r}, \hat{\theta}, \hat{\phi}$ for spherical coordinates. Show that

$$\begin{aligned}
\hat{r} &= \sin\theta\cos\phi\hat{\imath} + \sin\theta\sin\phi\hat{\jmath} + \cos\theta\hat{k}, \\
\hat{\theta} &= \cos\theta\cos\phi\hat{\imath}\cos\theta\sin\phi\hat{\jmath} - \sin\theta\hat{k}, \\
\hat{\phi} &= -\sin\phi\hat{\imath} + \cos\phi\hat{\jmath}. \qquad (3)
\end{aligned}$$

(b) Show also that

$$\hat{r} \times \hat{\theta} = \hat{\phi}, \quad \hat{\phi} \times \hat{r} = \hat{\theta}, \quad \hat{\theta} \times \hat{\phi} = \hat{r}. \tag{4}$$



(c) Then using $\vec{r} = r\hat{r}$, show that when $\dot{r} = 0$,

$$\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}, \tag{5}$$

where

$$\omega = \dot{\theta}\hat{\phi} - \sin\theta\dot{\phi}\hat{\theta}. \tag{6}$$

(d) Finally, using part c,

$$\omega = \frac{1}{r^2}\vec{r} \times \frac{d\vec{r}}{dt}. \tag{7}$$

(Remember problem KK 1.13.)

S 8.3 Consider the yo-yo set up given in in KK 7.27, but the force $\vec{F}$ is applied at an angle θ from the horizontal. Assume that the yo-yo does not slide on the table.

(a) Find the horizontal acceleration, $a(\theta)$, of the yo-yo as a function of θ.

(b) For what θ_c is $a(\theta_c) = 0$?

(c) For what θ will the yo-yo move to the right? To the left?

Comments:

- For problem KK 7.23, unlike the Atwoods machine, you will need to determine hoe the length of tape connecting the mass to the disk changes with time.
- For problem KK 7.37, determine whether or not angular momentum and linear momentum is conserved in each case.
- For the supplemental problem, remember KK 1.17.