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Discussion # 3

Exercise 1 (Maximum singular value) Prove $\max_{\|u\|_2=1} \|Au\|_2 = \sigma_1(A)$, where $\sigma_1(A)$ is the maximum singular value of A.

Exercise 2 (Frobenius norm and Least Squares) Let $A \in \mathbb{R}^{m\times n}$, consider the optimization problem given by $\min_X \|AX - I_m\|_F$, where the variable is $X \in \mathbb{R}^{n\times m}$, I_m is the $m \times m$ identity matrix, and $\|\cdot\|_F$ is the Frobenius norm.

1. Show that the problem can be reduced to a number of ordinary least squares problems. How do you recover X?
2. Show that when A is full column rank, then the optimal solution is unique, and given by $X^* = (A^TA)^{-1}A^T$.

Exercise 3 (Null space) Let $A \in \mathbb{R}^{n\times m}$, prove that $N(AA^T) = N(A^T)$.

Exercise 4 (Frobenius norm and trace) Let $A \in \mathbb{S}^n_+$, be a symmetric, positive semidefinite matrix. Show that trace A and Frobenius norm, $\|A\|_F$, depend only on its eigenvalues, and express both in terms of the vector of eigenvalues.