KK Ch 3: 3.14, 3.20, 3.21, 3.23, 3.25

KK Ch 4: 4.4, 4.5, 4.15

S 3.1 In the sci-fi classic Ringworld, Larry Niven suggested that an advanced civilization would be capable of constructing a variation of a Dyson sphere where all the mass contained in a solar system (with exception of the star) was used to construct a circular ring that would encircle the star at a habitable orbital radius. It was then pointed out that this ring world would be unstable, and would eventually fall into the star. In this problem, we will show this.
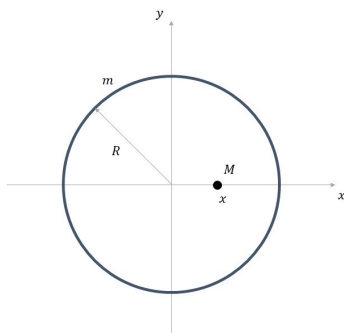


(a) Consider a thin ring with radius $R$ and mass $m$ spread uniformly on it (see above figure). A mass $M$ is placed at a distance $x$ from the center of the ring. Show that the gravitational force on $M$ is given by

$$F_x = \frac{GmM}{\pi} \int_0^\pi \frac{(R\cos\theta - x)}{(R^2 + x^2 - 2Rx\cos\theta)^{3/2}} d\theta, \tag{1}$$

(b) Unlike the spherical case, this integral cannot be expressed in terms of elementary functions (elliptic functions are needed instead). To proceed further, take $u = x/R < 1$, and expand the integrand of $F_x$ to a Taylor series about $u = 0$ and drop terms that are proportional to $u^n$ for $n > 1$ (or, in physics jargon, keeping terms linear in $u$). With this approximation, show that the force on $M$ is

$$F_x = \frac{1}{2}\frac{GmM}{R^3}x, \tag{2}$$

and is pointed away from the center of the ring.

(c) Argue—using physics—that the ring world in Larry Niven's book is unstable.