F20 PHYSICS 137B: HW 10

Due November 13 at 11:59 pm

November 3, 2020

1 Griffiths problems

Do the following problems from Griffiths: 10.8, 10.10

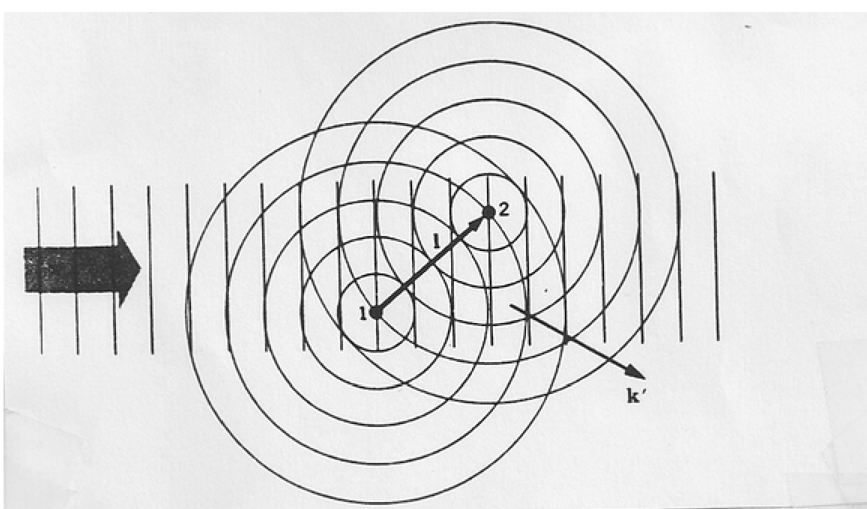
2 Other problems

2.1

A molecule of a homonuclear diatomic gas may roughly be regarded as composed of two identical spherically symmetric scattering centers separated by a (vectorial) distance $\mathbf{l}$. If the scattering amplitude for a certain kind of particle directed against the atom is known, what scattering cross section is measured for the molecules of the gas? Contrast the atomic and molecular cross-sections, and neglect effects of multiple scattering, that is, of particles which bounce back and forth between the two centers.

Hint: The calculation is in two parts: first the calculation of σ for a molecule in a particular spatial orientation and then the average over all orientations. The first thing to note is the difference in phase between the waves arriving at the two scattering centers: compared to the phase at the center of the molecule, that at $\mathbf{l}$ is $\frac{1}{2}\mathbf{k}\cdot\mathbf{l}$ early, while that at 2 is equally late. Thus the wave at the counter is:

$$\psi_s = e^{\frac{i}{2}\mathbf{k}\cdot\mathbf{l}}\left[\frac{e^{ikR_1}}{R_1}f(\theta)\right] + e^{-\frac{i}{2}\mathbf{k}\cdot\mathbf{l}}\left[\frac{e^{ikR_2}}{R_2}f(\theta)\right] \tag{2.1}$$



The denominators can be set equal to r without error, while in the exponents:

$$kR_1 = kr - \frac{1}{2}\mathbf{k}'\cdot\mathbf{l}, \qquad kR_2 = kr + \frac{1}{2}\mathbf{k}'\cdot\mathbf{l} \tag{2.2}$$

Thus,

$$\psi_s = \frac{e^{ikr}}{r}\left(e^{\frac{i}{2}(\mathbf{k}\cdot\mathbf{l}-\mathbf{k}'\cdot\mathbf{l})} + e^{-\frac{i}{2}(\mathbf{k}\cdot\mathbf{l}-\mathbf{k}'\cdot\mathbf{l})} f(\theta)\right) \tag{2.3}$$

$$= 2\frac{e^{ikr}}{r}\cos\left[\frac{1}{2}\left(\mathbf{k}-\mathbf{k}'\right)\cdot\mathbf{l}\right] f(\theta), \tag{2.4}$$

so that

$$\frac{d\sigma}{d\Omega} = 4\cos^2\left[\frac{1}{2}\left(\mathbf{k}-\mathbf{k}'\right)\cdot\mathbf{l}\right] |f(\theta)|^2. \tag{2.5}$$

The vector $\mathbf{K} \equiv \mathbf{k} - \mathbf{k}'$ is shown below: You will need to compute the length and average over the

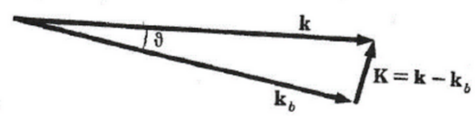


FIGURE 8.9
Relation between $\mathbf{k}$, $\mathbf{k}_b$, and $\mathbf{K}$.

directions of $\mathbf{l}$.
You may find the following useful:

$$f(\theta) = -\frac{1}{4\pi}\frac{2\mu}{\hbar^2}\int e^{-i\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}')\psi(\mathbf{r}')d^3\mathbf{r}'. \tag{2.6}$$