Math 126 **Name:** ____________________

Summer 2018

Final

8/9/2018

Time Limit: 110 Minutes

This exam contains 6 questions. Total of points is 85.

For this exam, you are allowed a handwritten cheat sheet consisting of one side of an 8 1/2 by 11 piece of paper.

The last sheet of paper is intentionally blank so that you may use it to finish your solutions *if clearly marked where the first part of your solution stops.*

Grade Table (for instructor use only)

Question	Points	Score
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
Total:	85	

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1. (10 points) Suppose that $u(x,t)$ solves

$$\begin{aligned} u_t - ku_{xx} &= f(x,t) \qquad && 0 < x < \infty, t > 0 \\ u(x,0) &= \phi(x), && \infty < x < \infty \end{aligned}$$

where $\phi(x)$ is even and $f(-x,t) = f(x,t)$. Show that $u(-x,t) = u(x,t)$.

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2. (15 points) Solve

$$\begin{aligned} \Delta u = 0 \qquad & 0 \le x, y \le \pi \\ u(x,0) = u(x,\pi) = 0 \qquad & 0 \le x \le \pi \\ u(0,y) = 0, u(\pi,y) = \sin y \qquad & 0 \le y \le \pi \end{aligned}$$

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3. (15 points) (a) (8 points) Use the forward difference scheme discussed in class to solve

$$\begin{aligned} u_t &= u_{xx} && 0 < x < 6 \\ u(0,t) &= 1, u(6,t) = 0 && t > 0 \\ u(x,0) &= 0 && 0 < x < 6 \end{aligned}$$

with $\Delta t = \Delta x = 1$ until time $t = 4$. Express your answer in the form of a grid like we did in class and be clear to describe your scheme, as well as your notation. Also, for consistency, take $u(0,0) = 1$.

(b) (2 points) Is this scheme stable?

(c) (5 points) If the scheme is stable, then how do you know? If the scheme is not stable, then what could you do to fix it?

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4. (15 points) Use Fourier transform techniques to solve

$$\begin{aligned} u_t + cu_x &= 0 \qquad -\infty < x < \infty, t > 0 \\ u(x,0) &= \phi(x). \end{aligned}$$

(Hint: Instead of computing the inverse Fourier transform directly, you may want to simply recognize your function as the Fourier transform of something using the function and rule tables.)

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5. (15 points) Find the Green's function for $D = [0, 1]$ in one dimension. (Hint: First find a function $v(x)$ such that $v_{xx} = \delta(x)$, then adjust to fit the other conditions.

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6. (15 points) Use the method of reflection to solve the wave equation on the half-plane $\{y > 0\}$ with Neumann boundary condition, i.e.

$$\begin{aligned} u_{tt} &= c^2(u_{xx} + u_{yy}) && -\infty < x < \infty, y > 0, t > 0 \\ u_y(x, 0, t) &= 0 && -\infty < x < \infty, t > 0 \\ u(x, y, 0) = 0, u_t(x, y, 0) &= \psi(x, y) && -\infty < x < \infty, y > 0. \end{aligned}$$

In your answer, you may use any statements about even and/or odd solutions without justification provided they are true. Also, feel free to leave your solution in terms of the source function, provided you say what that is.

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