Physics 5A, Fall 2017 Homework Set 11

APF Ch 3: 3.4, 3.11, 3.18

KK: Ch 3: 3.15; Ch 6: 6.1

S 11.1 Consider the damped oscillator

$$0 = \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x,\tag{1}$$

with the initial condition $x(0) = x_0$ and $v(0) = v_0$.

(a) If the oscillator is lightly damped the solution to the differential equation is

$$x(t) = Ae^{-\gamma t/2}\cos(\omega t + \phi). \tag{2}$$

What is A and $\tan \phi$? Express your answer in terms of x_0 and v_0 .

(b) If the oscillator is over-damped, the solution is

$$x(t) = e^{-\gamma t/2} \left(A e^{\kappa t} + B e^{-\kappa t} \right), \tag{3}$$

where $\kappa^2 = \gamma^2/4 - \omega_0^2$. What is A and B? Express your answer in terms of x_0 and v_0 .

(c) If the oscillator is critically damped, the solution is

$$x(t) = (A + Bt)e^{-\gamma t/2}. (4)$$

What is A and B now?