Statistics 153 - Homework 3

Filters, Differencing, and Variance Stabilization

Due on **March 3rd**, 2021, 11:59 PM PST

Computer exercises:

The data for these exercises are on bCourses.

1. Recall the Google Trends "iPod" data. Using only differencing and/or variance stabilizing transforms, pursue stationarity.
   (a) (1 point) Plot the original time series
   (b) (2 points) Plot the plausibly-stationary series after you have performed differencing and/or variance stabilization. (The `diff` function in `R` might be helpful).
   (c) (1 point) Write the mathematical expression for your plausibly-stationary series. For example, $\nabla(log(Y_t)) = log(Y_t) - log(Y_{t-1})$, but you should see that
2. Recall the Google Trends "Basketball" data. Using only differencing and/or variance stabilizing transforms, pursue stationarity.
   (a) (1 point) Plot the original time series
   (b) (2 points) Plot the plausibly-stationary series after you have performed differencing and/or variance stabilization. (The `diff` function in `R` might be helpful).
   (c) (1 point) Write the mathematical expression for your plausibly-stationary series.

Theoretical exercises:

3. (2 points) While analyzing their annual sales data (liters of wine sold per year) for the past 30 years, a vineyard found that after taking four successive differences, the resulting data had a mean of 2458 liters and looked like a stationary process (e.g. white noise). If the actual data for the past four years were 2018 - 2134 liters, 2017 - 2403 liters, 2016 - 2076 liters, and 2015 - 2290 liters. What would be a reasonable forecast for sales in 2019? Explain.
4. (5 points) Consider the stochastic trend model

$$X_t = m_t + Z_t, \quad t = 1, \ldots, n$$

where Z_t is a white noise process (with variance σ_Z^2) and m_t is a stochastic trend which follows a random walk with drift model

$$m_t = \delta + m_{t-1} + W_t,$$

where W_t is another white noise process (with variance σ_W^2), independent of Z_t. Let $m_0 = 0$. (This is essentially the same as the example given in Lecture 8).

(a) Find the mean function $\mu(t) = \mathbb{E}(X_t)$, autocovariance function $\gamma(s,t) = \mathrm{Cov}(X_t, X_s)$, and autocorrelation function $\rho(s,t) = \gamma(s,t)/\sqrt{\gamma(t,t)\gamma(s,s)}$ of $\{X_t\}$. To this end, first show that the model can be written as $X_t = \delta t + \sum_{k=1}^{t} W_k + Z_t$.

(b) A time series process is denoted as *weakly stationary* if the mean function $\mu(t)$ is constant (i.e., does not depend on t) and the autocovariance function $\gamma(s,t)$ depends on s and t only through their difference $|s-t|$. Is the process X_t defined above weakly stationary? Explain.

(c) Show that $\rho(t-1,t) \to 1$ as $t \to \infty$. What is the implication of this result for observations with large t?

(d) Suggest a transformation to make the series X_t weakly stationary and prove that the transformed series is weakly stationary.

5. (5 points) Let (X_t) be a weakly stationary process with mean μ and autocovariance function $\gamma(k) = \mathrm{Cov}(X_t, X_{t+k})$. Consider a derived series (Y_t) defined as

$$Y_t = \sum_{j=a}^{b} c_j X_{t+j}, \tag{1}$$

where a and b are integers with $a \leq b$, and $(c_a, ..., c_b)$ are all fixed real numbers.

(a) Show that $\{Y_t\}$ is a weakly stationary series.

(b) Show that order k ($k \geq 1$) differencing (that is, $\nabla^k X_t$) can be put in the form of Equation (1). Identify the corresponding a, b, and $(c_a, ..., c_b)$.

(c) Recall that smoothing via simple averaging with parameter q corresponds to computing for each t

$$\frac{1}{2q+1} \sum_{j=-q}^{q} X_{t+j}.$$

Show that smoothing via simple averaging with parameter q can be put in the form of Equation (1). Identify the corresponding a, b, and $(c_a, ..., c_b)$.

(d) Is the kth differenced version of a weakly stationary process always weakly stationary? Is the smoothed (via simple averaging) version of a weakly stationary process always weakly stationary?