Adaptive robust nonlinear control: a sliding mode approach

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Objectives of this class of <u>nonlinear</u> control ?

- Robustness versus uncertainties/perturbations
- Finite time convergence towards the objectives

Features of this class of control ?

- Discontinuous control law
- For standart sliding mode (*first order*) : chattering effect, robustness
- For high order sliding mode : accuracy, finite time convergence, robustness

This class of controllers has been studied

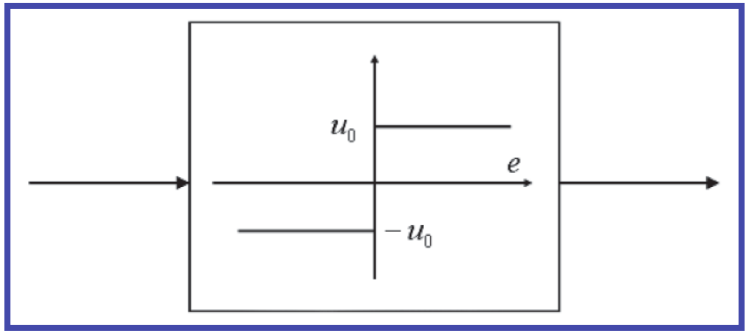
- from 40's in the former USSR
- intensively since 20 years

Sliding mode as a phenomenon may appear in a dynamic system governed by ordinary differential equations with discontinuous right hand sides.

The term « **sliding mode** » first appeared in the context of relay systems.

→ It may happen that the control as a function of the system state switches at high (theoretically infinite) frequency

→ This motion is referred to as « sliding mode ».



Second order nonlinear system (uncertain)

$$\boxed{\begin{array}{l} \ddot{x} + a_2\dot{x} + a_1 x = u \\ a_1, a_2, M, c \;\;\text{-}\;\; const \end{array}}$$

Sliding variable (Control objective) $s = cx + \dot{x}$

Control input $u = -M sign(s)$

M, control gain, sufficiently large

$$s\dot{s} \leq -\eta|s|, \quad \eta > 0$$

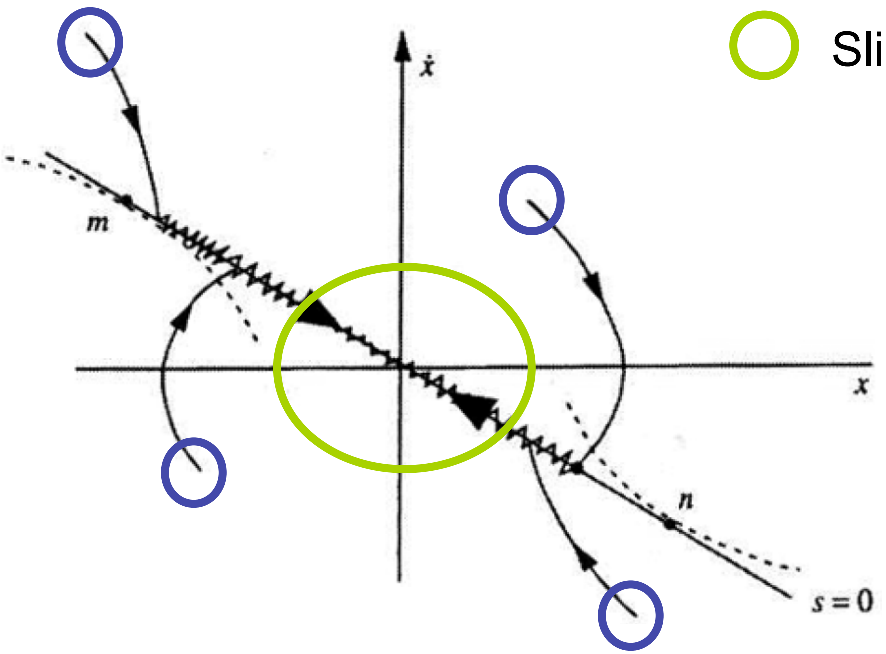
In a finite time ($t > t_1$), one has $s = 0$

Once the sliding surface, one has

$$\dot{x} + cx = 0 \Rightarrow \boxed{x(t) = x(t_1)e^{-c(t-t_1)}}$$

Convergence in a finite time

Sliding motion

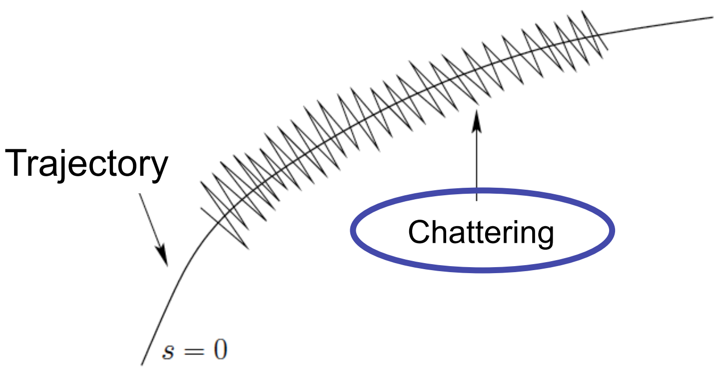


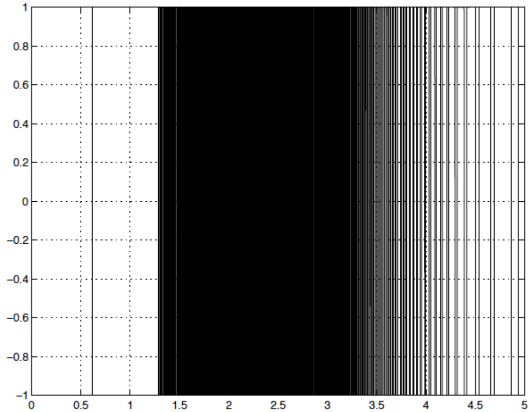
• The motion on the sliding surface depends neither on the plant parameters, nor the disturbances/uncertainties.

• This so-called « invariance » property looks promising for designing feedback control for the dynamic plants operating under uncertainty conditions.

Real implementation

- **Finite frequency** of the control commutation → it is not possible to reach *exactly* the sliding surface → notion of **real sliding mode**.





***Chattering* effect** : oscillations around the sliding surface (inducing commutation of the control input) → dangerous for the actuators.

How to decrease ?

- **Modification of « sign »-function (approximation)**
- **Use of observers (as a filter)**
- **Concept of *High Order Sliding Mode Control* (since mid'90)**

Standart sliding mode control.

Objective

$$s = 0$$

in a finite time

⬇

The discontinuous function *sign(s)* is acting on the control input (relative degree of s = 1)

High sliding mode control.

Objective

$$s = \dot{s} = \cdots = s^{(r-1)} = 0$$

in a finite time

⬇

The discontinuous function *sign(s)* can act on the control input high order time derivative

- **Concept of *Adaptive Sliding Mode Control* (very recent - 2010)**

Principle. The gain is *dynamically* adapted w.r.t. the uncertainties/perturbations magnitude, through the detection of establishement / loss of sliding mode.

→ The tuning of the gain is made *online*

→ The gain is reduced : reduction of *chattering*

→ No knowledge required on the uncertainties/pertubations bounds

→ Simplification of identification

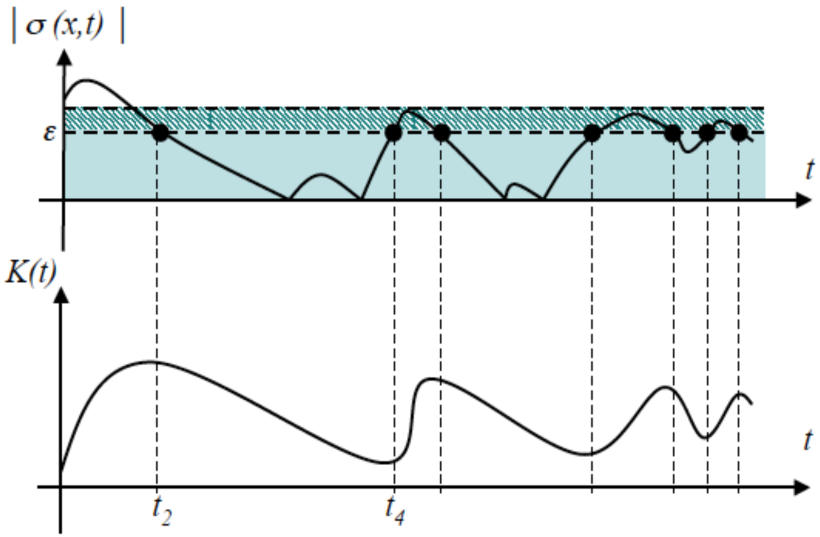
→ Great interest by a practical point-of-view.

[**Plestan *et al.***, *Int. J. Control*, 2010.] Adaptive sliding mode control

- The gain is adapted *w.r.t.* establishment/loss of real sliding mode
- **The proof is based on Lyapunov approach**

$$\dot{x} = f(x) + g(x) \cdot u \quad (1) \quad \Rightarrow \quad \boxed{\dot{\sigma} = \Psi(x,t) + \Gamma(x,t) \cdot u} \quad (2)$$

$$|\Psi| \leq \Psi_M, \quad 0 < \Gamma_m \leq \Gamma \leq \Gamma_M$$



$$u = -K \cdot \mathrm{sign}(\sigma(x,t)) \quad (3)$$

$$(4) \quad \dot{K} = \begin{cases} \bar{K} \cdot |\sigma(x,t)| \cdot \mathrm{sign}(|\sigma(x,t)| - \epsilon) & \text{if } K > \mu \\ \mu & \text{if } K \leq \mu \end{cases}$$

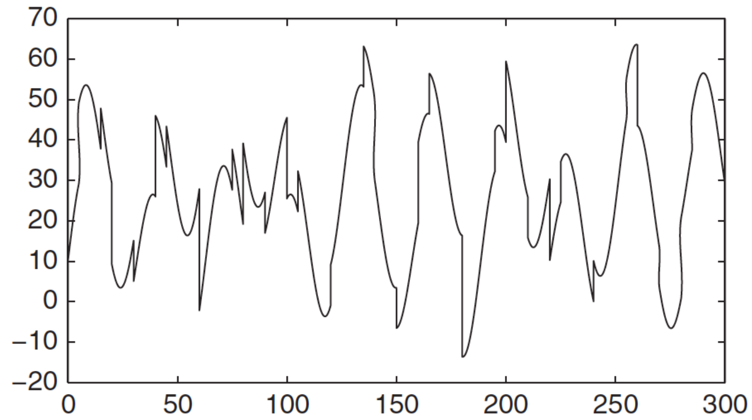
Theorem *Given the nonlinear uncertain system (1) with the sliding variable* $\sigma(x,t)$ *dynamics (2) controlled by* (3) *and* (4) *, there exists a finite time* $t_F > 0$ *so that a real sliding mode is established for all* $t \geq t_F$, i.e. $|\sigma(x,t)| < \delta$ *for* $t \geq t_F$, *with*

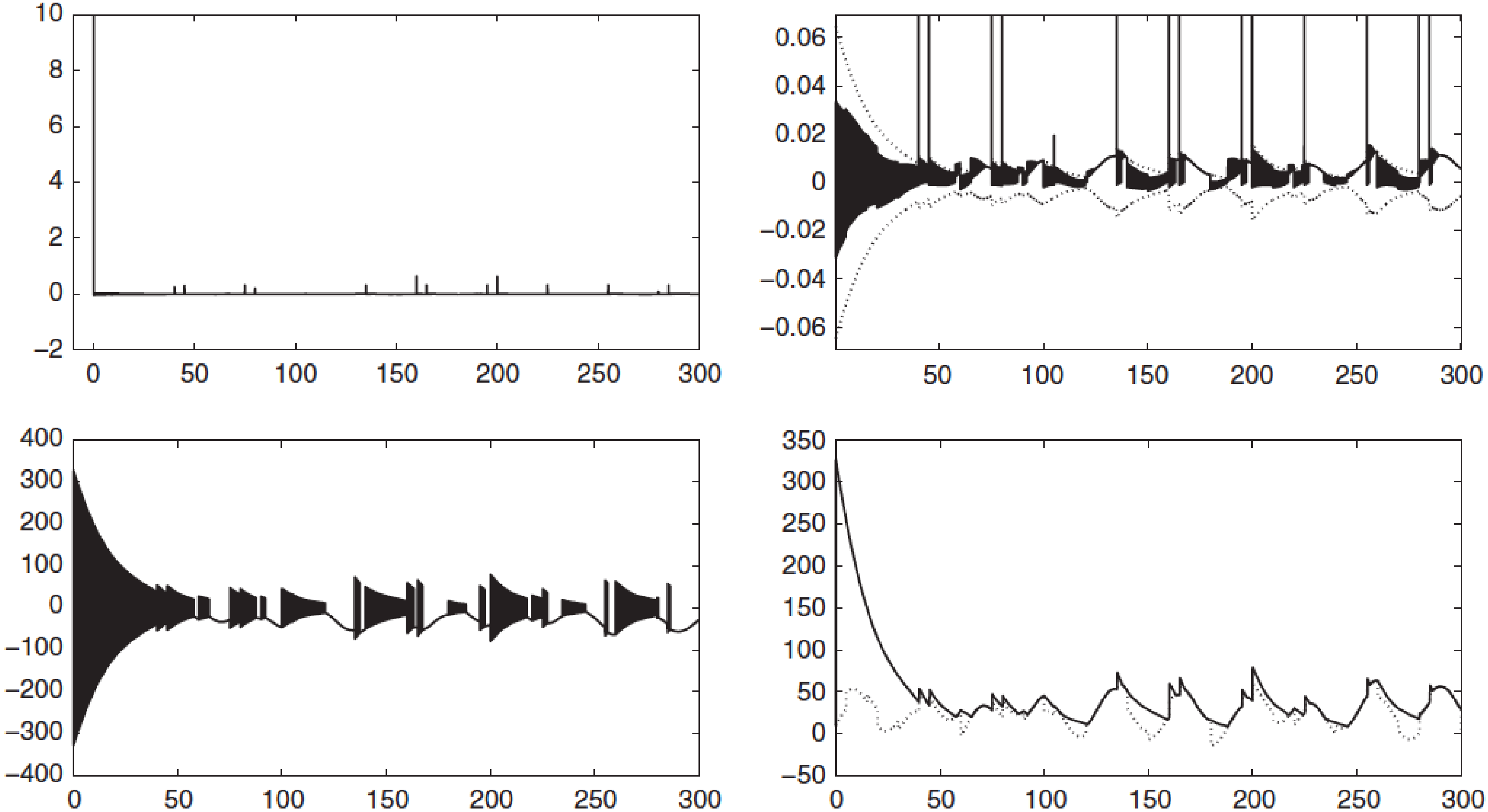
$$\delta = \sqrt{\epsilon^2 + \frac{\Psi_M^2}{\bar{K}\Gamma_m}}.$$

$\square$

Example.

$$\boxed{\dot{\sigma} = \Psi(t) + u}$$

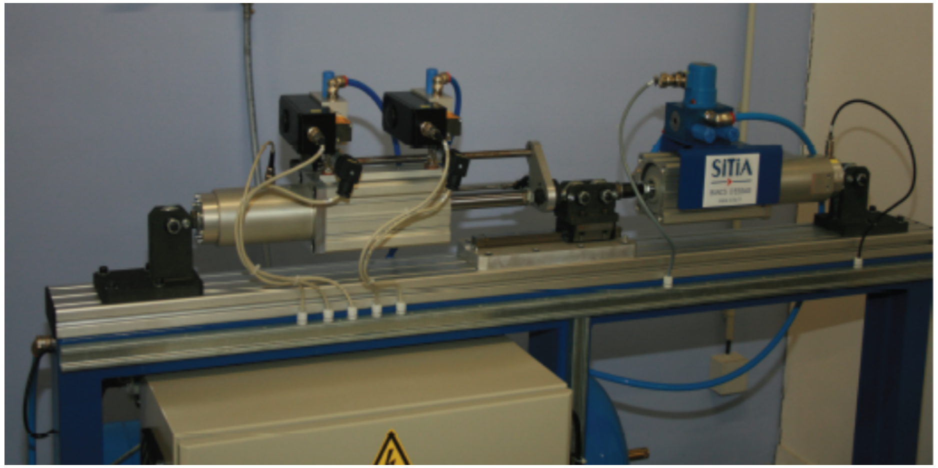




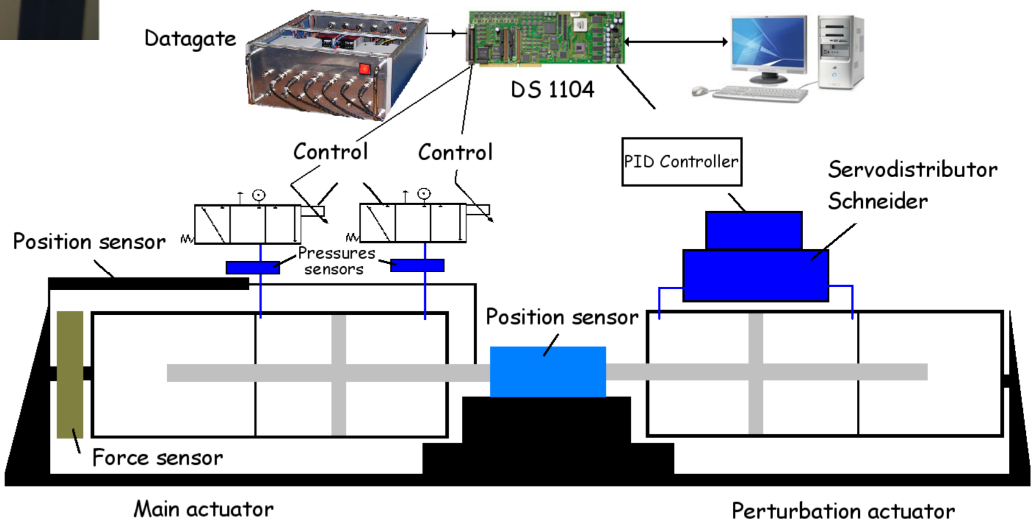
Control algorithm 2 in Equations (15), (16). Top-left. Sliding variable σ versus time (s). Top-right. Zoom on sliding variable σ (—), $\epsilon(t)$ and $-\epsilon(t)$ $(\cdots)$ versus time (s). Bottom-left. Control input u versus time (s). Bottom-right. Gain $K(t)$ (—) and perturbation $\Psi(t)$ $(\cdots)$ versus time (s).

Application to electropneumatic actuator control (IRCCyN)

[**Plestan *et al.***, *Control Engineering Practice*, 2012.]



Multivariable case:
position and average pressures control.



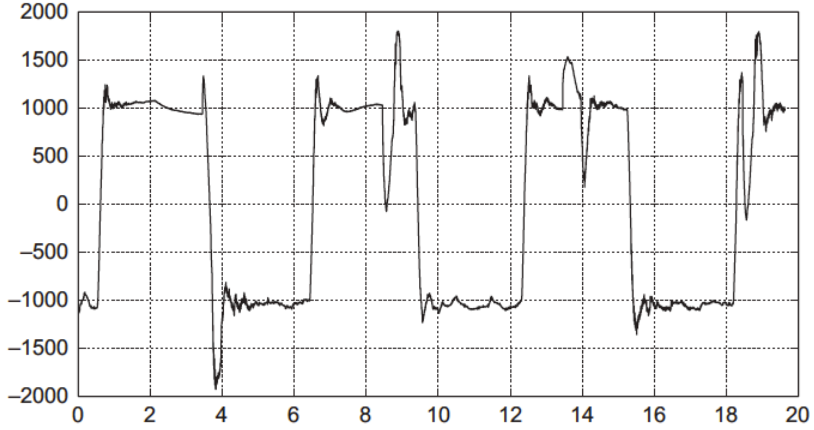
$$\dot{x} = f_n(x) + g_n(x)u \qquad x = [p_P \ p_N \ v_y \ y]^T, \ u = [u_P \ u_N]^T,$$

$$f_n(x) = \begin{bmatrix} \frac{krT}{V_P(y)}\left[\varphi_P - \frac{S}{rT} p_P v_y\right] \\ \frac{krT}{V_N(y)}\left[\varphi_N + \frac{S}{rT} p_N v_y\right] \\ \frac{1}{M}[S p_P - S p_N - b v_y - F] \\ v_y \end{bmatrix}, \quad g_n(x) = \begin{bmatrix} \frac{krT}{V_P(y)}\psi_P & 0 \\ 0 & \frac{krT}{V_N(y)}\psi_N \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

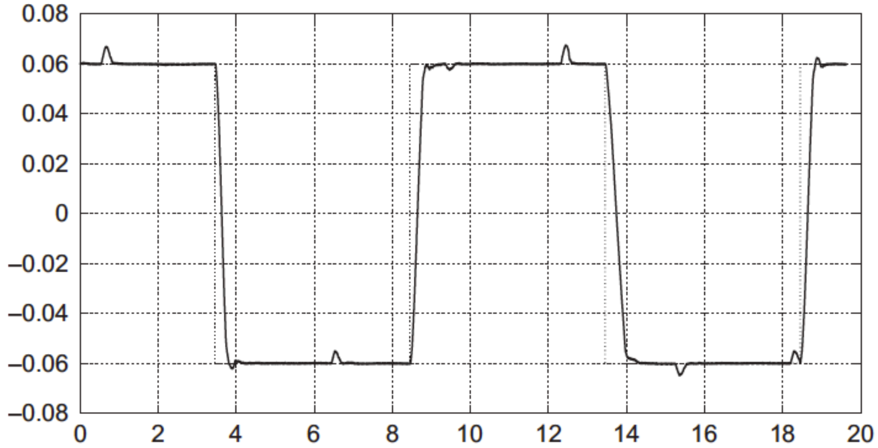
$$\sigma_1(x,t) = \dot{v}_y + \lambda_v \cdot v_y + \lambda_y \cdot (y - y_d)$$

$$\sigma_2(x,t) = (p_P + p_N)/2 - p_d$$

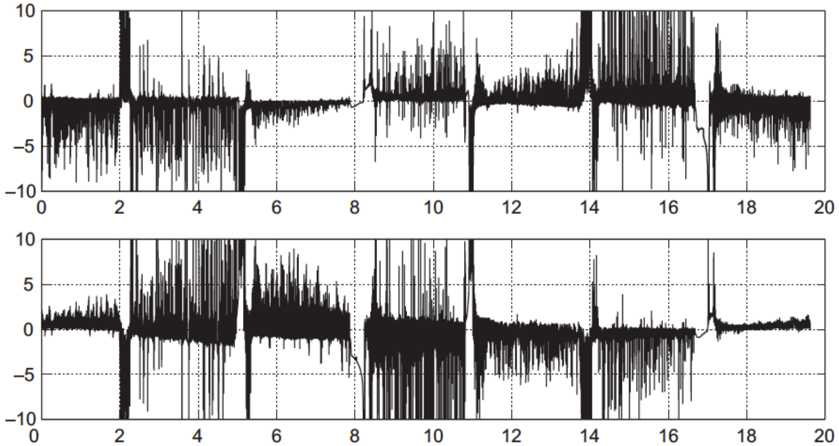
$$\boxed{\begin{bmatrix} \dot{\sigma}_1 \\ \dot{\sigma}_2 \end{bmatrix} = \overline{\Psi}(x,t) + \overline{\Gamma}(x) \cdot \begin{bmatrix} u_P \\ u_N \end{bmatrix}}$$



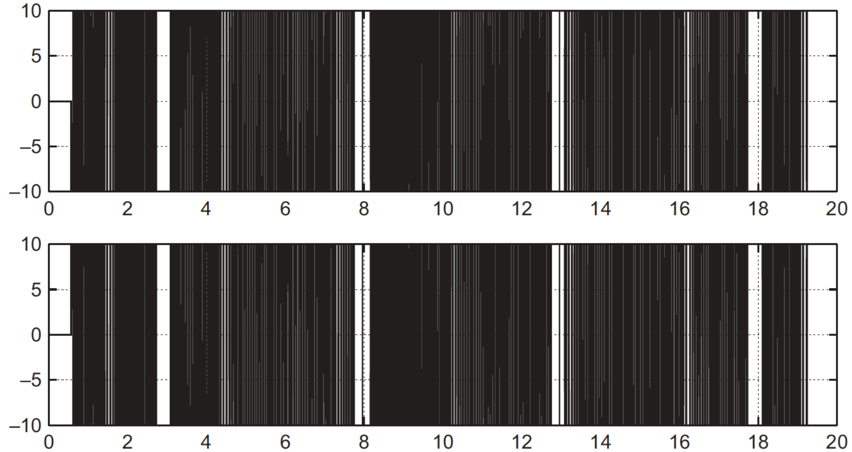
Perturbation



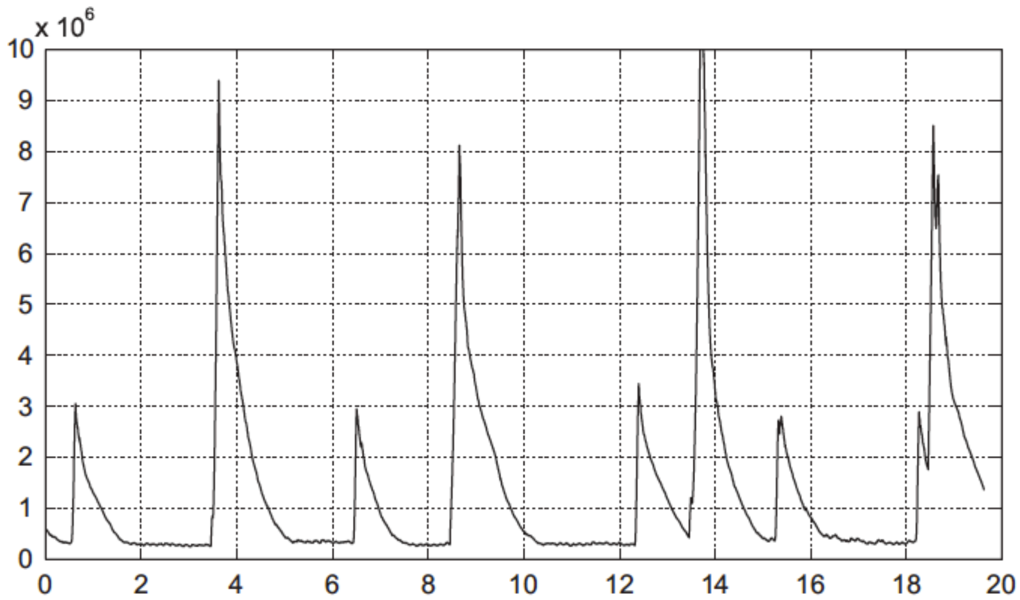
Position



Control input (adapted gain)



Control input (not adapted gain)



Control adapted gain

➡ **Extension of this approach to second order sliding mode control**

Objective. Design of a robust second order sliding mode controller

- Proof based on Lyapunov analysis
- Uncertainties/perturbations bounded, but unknown bounds.
- Establishment in a finite time of a real 2nd order sliding mode.

Super-Twisting algorithm [Levant, 1993].

- One of the most powerful 2nd order sliding mode controller
- Relative degree = 1 / no use of time derivative of sliding variable (output feedback → it allows time differentiation)
- The sliding variable and its time derivative go to 0 in a finite time
- The gain is tuned for the *worst* case → it is increasing the chattering *effect*.
- Identification/estimation of the bounds : not easy …

⇨ Is it possible to have time-varying gains s.t. knowledge of uncertainties bounds is not required ?

Dynamics of the sliding variable.

$$\dot{\sigma} = \varphi(x,t) + \underbrace{\left(1 + \frac{\Delta b(x,t)}{b_0(x,t)}\right)}_{g(x,t)} \omega$$

A solution [Shtessel *et al.*, Automatica, 2012]

$$\omega = -\alpha|\sigma|^{1/2}\mathrm{sign}(\sigma) + v, \quad \dot{v} = -\frac{\beta}{2}\mathrm{sign}(\sigma)$$

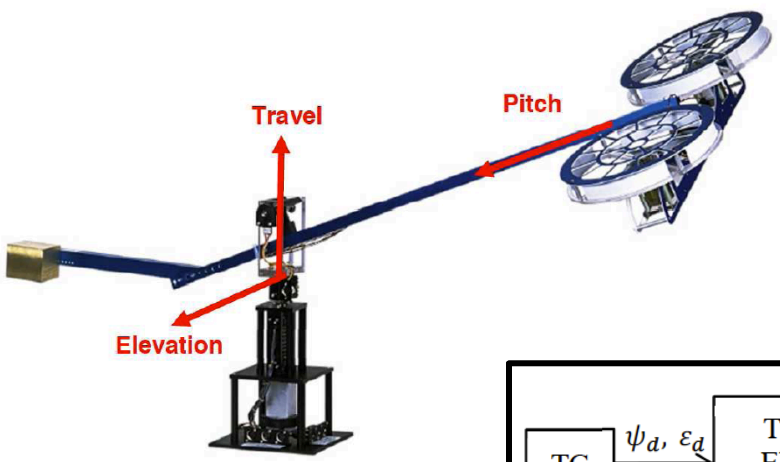
with adaptive gains $\alpha = \alpha(\sigma, \dot{\sigma}, t), \quad \beta = \beta(\sigma, \dot{\sigma}, t)$

$$\dot{\alpha} = \begin{cases} \omega_1 \sqrt{\frac{\gamma_1}{2}} \mathrm{sign}(|\sigma| - \mu), & \text{if } \alpha > \alpha_m \\ \eta, & \text{if } \alpha \leq \alpha_m \end{cases}$$
$$\beta = 2\varepsilon\alpha$$

Proof of finite time convergence based on Lyapunov analysis.

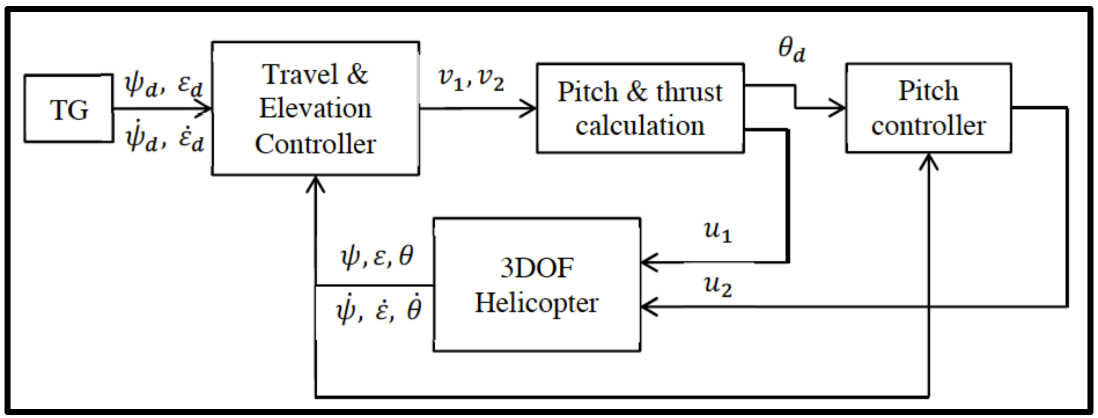
Application to experimental systems

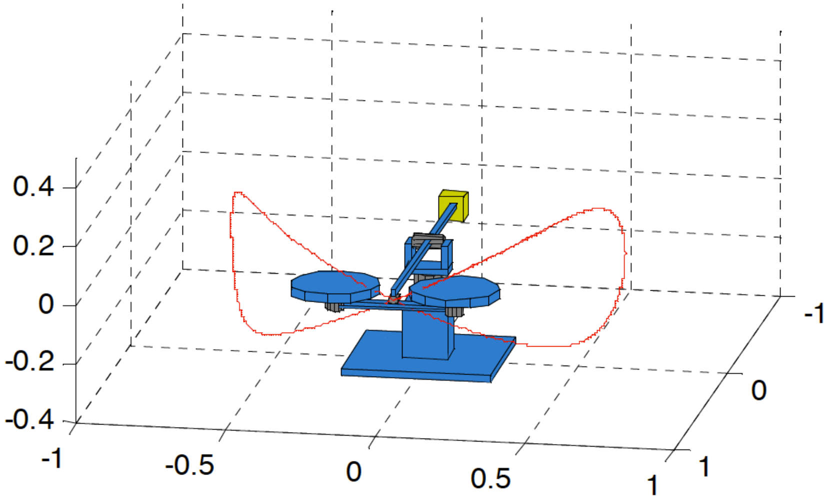
- Electropneumatic actuator [Shtessel *et al.*, Automatica, 2012]
- 3D Helicopter [Plestan *et al.*, IEEE CDC, 2012]

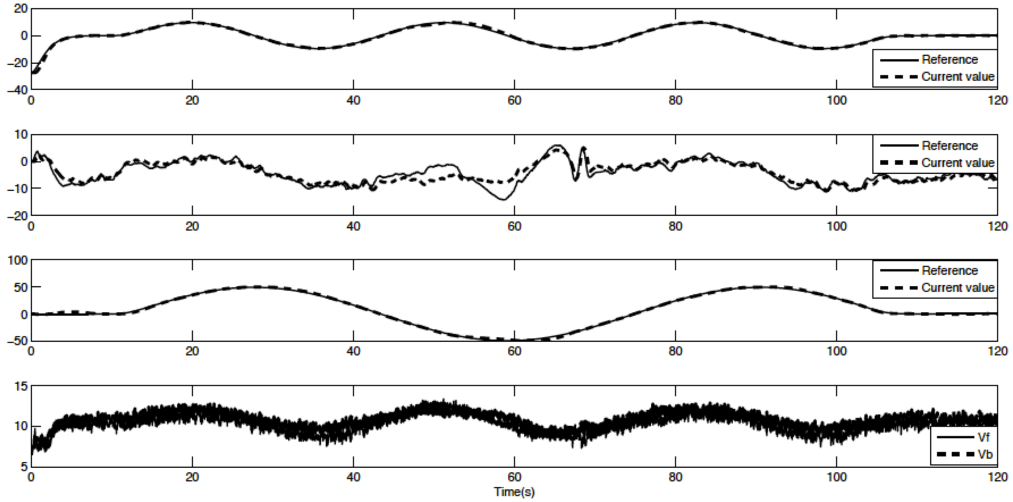


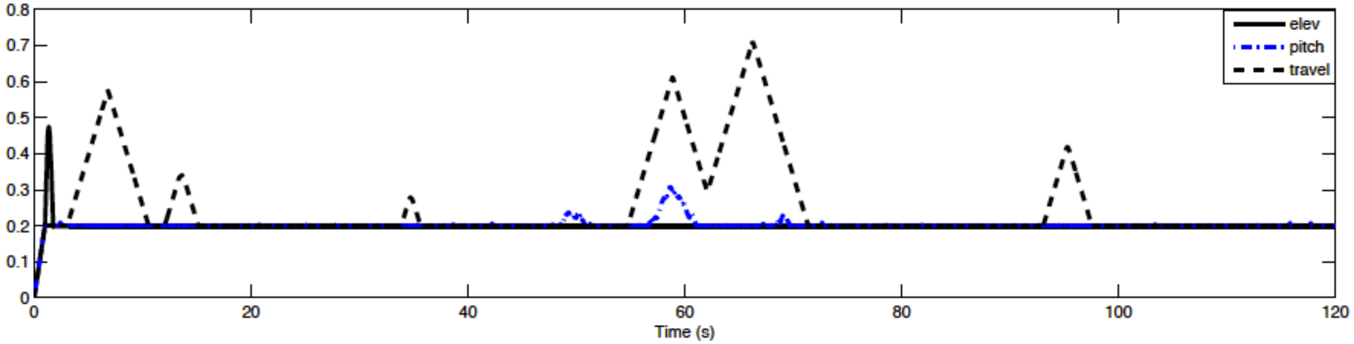
$$
\begin{aligned}
J_\varepsilon \ddot{\varepsilon} &= g(M_h L_a - M_w L_w)\cos\varepsilon + L_a \cos\theta\, u_1 \\
J_\theta \ddot{\theta} &= L_h\, u_2 \\
J_\psi \ddot{\psi} &= L_a \cos\varepsilon \sin\theta\, u_1
\end{aligned}
$$

$$
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_f + F_b \\ F_f - F_b \end{bmatrix}
$$



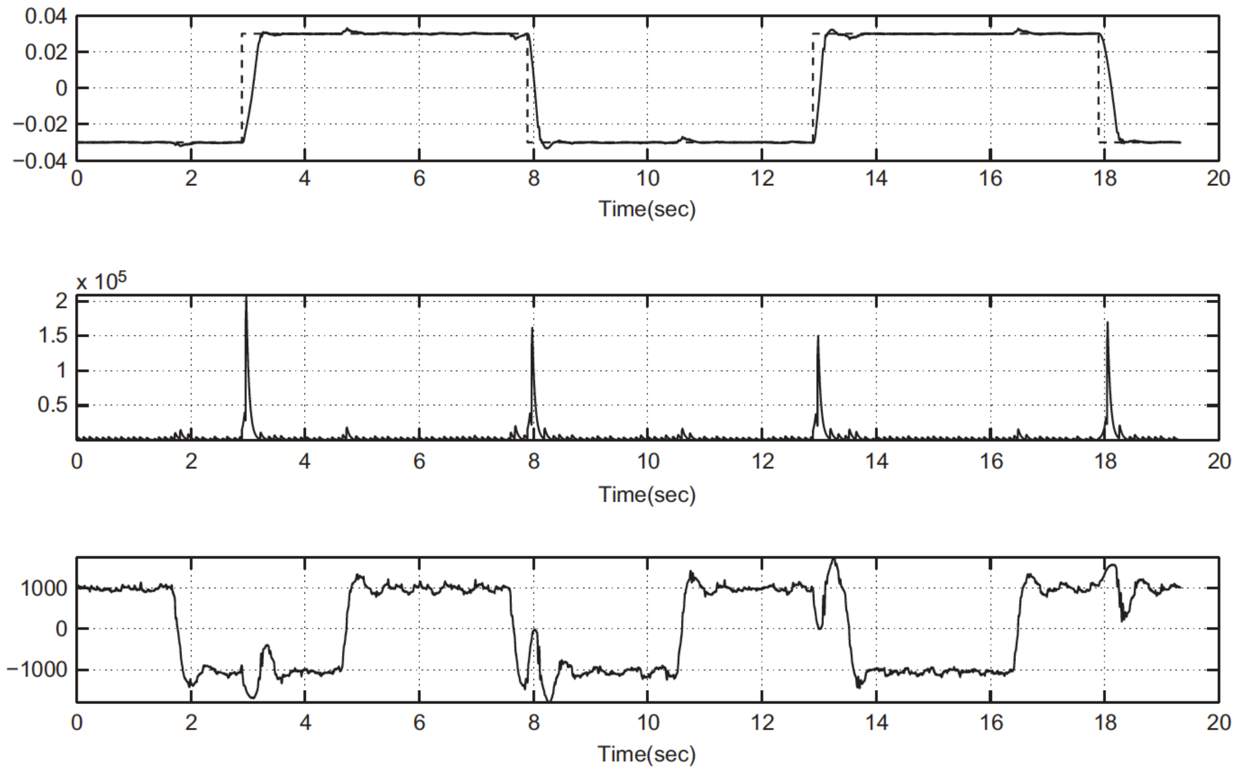

0.4
0.2
0
-0.2
-0.4
-1
-0.5
0
0.5
1
-1
0
1


Reference
Current value
Reference
Current value
Reference
Current value
Vf
Vb
Time(s)


elev
pitch
travel
Time (s)

Other results (in the same frame).

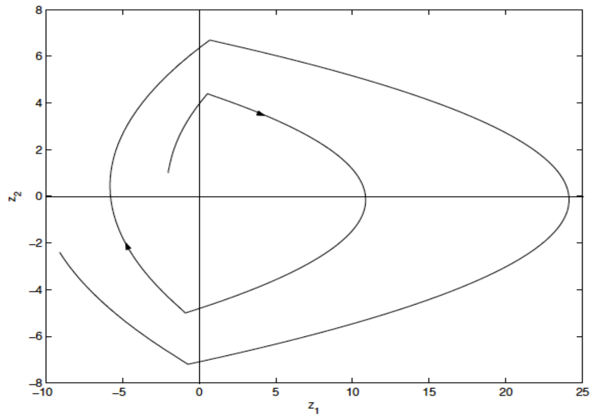
- **Adaptive *twisting* algorithm** [Bartolini *et al.*, *IMA J. Math. Control. Inf.*, 2012] and experimental results on pneumatic actuators [Taleb *et al.*, *Cont. Eng. Pract.*, 2012]
  - Can be applied to system with relative degrees 1 or 2
  - Requires the sliding variable and its time derivative.

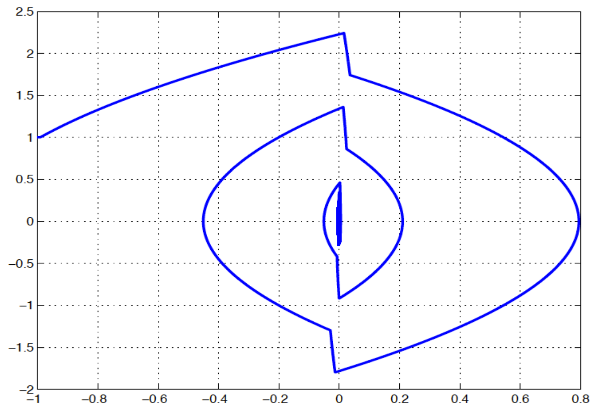


• **Second order sliding mode output feedback control** [Plestan *et al.*, *Automatica*, 2010] with impulsive/adaptive gain [Estrada *et al.*, *IEEE CDC*, 2012]

- This controller only requires the output for stabilization of a double perturbed integrator,
- To achieve the stabilization, the controller has to change its gain during a sampling period at a specific time (depending only on the sliding variable),
- **Advantages**: acceleration of the convergence, use of only the output (sliding variable), usable for relative degrees 1 or 2 (not the case of *supertwisting*)

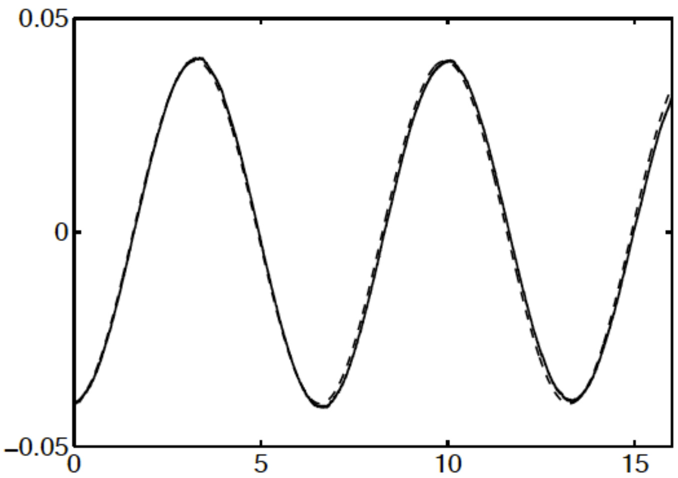
$$\begin{aligned}\dot{z}_1(t) &= z_2(t)\\ \dot{z}_2(t) &= a(\cdot) - b(\cdot)\; K\, \mathrm{sign}(z_1(t - d(t)))\end{aligned}$$

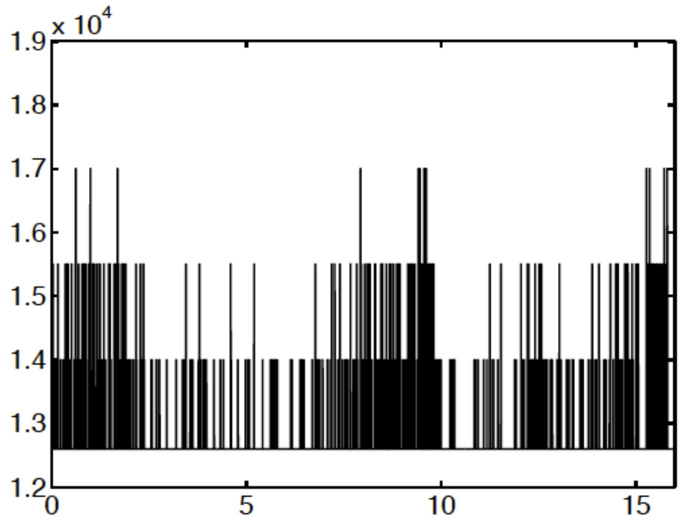




Other results (in the same frame).

- **Second order sliding mode output feedback control** with adaptive gain [Estrada *et al.*, soumis à *ECC*, 2013].
    - The magnitude of the impulsive gain can be adapted in case of establishment/loss of the second order sliding mode
    - Adaptation: knowledge of the perturbations/uncertainties bounds is not required.
    - ***First attempt of experimental validation****: electropneumatic actuator [Estrada* et al., soumis à *J. Franklin Inst*., 2012]





- **Generalization of the impulsive gain approach** [Glumineau *et al.*, *IFAC WC*, 2011; Shtessel *et al.*, *IEEE CDC*, 2012]

Other results (in the same frame).

- **High order sliding mode control** with adaptive gain [Taleb *et al.*, soumis à *ECC*, 2013].
  - Based on the concept of Integral Sliding Mode (nominal control)
  - Adaptation: knowledge of the perturbations/uncertainties bounds is not required.