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Lumped-Parameter Modeling: Enabling Real-Time Battery Management

Mehrdad Babazadeh[1] and W. Dhammika Widanage[2]

***Abstract*— Lumped parameter models simplify Li-ion battery understanding, design, management, and simulation, the key to their impact on advancing battery technology and safe, efficient operation. While these models might seem straightforward, precisely identifying their key parameters, especially in real-time scenarios, presents a significant challenge. Unlike their complex counterparts, these models are plagued by hidden dynamics and fast-changing intricacies, leading to data saturation and confusion over identification methods. This paper proposes an improved Electrothermal model for Li-ion batteries, applicable under diverse operating conditions. Our novel techniques and a modified recursive least squares algorithm boost the model's parameter identification robustness, eliminating the need for prior assumptions. We ensure method effectiveness through simulated testing.**

I. INTRODUCTION

Developing lumped parameter thermal models (LPTMs) for Li-ion batteries is crucial for Design Optimization, Real-time Monitoring and Control, Thermal Management, Fault Diagnosis, Prediction, and Battery Performance Simulation. These models provide a simplified yet effective representation of complex electrochemical and thermal processes within Li-ion batteries. They gain insights into the battery behavior under various operating conditions, enabling producers to optimize battery design parameters for improved performance, safety, and lifespan. They also can be integrated into battery management systems (BMSs) to monitor the battery's state of health (SoH), including its capacity, temperature, and remaining useful life. Lumped parameter thermal models capture the thermal behavior of batteries, enabling the prediction and analysis of temperature distribution and heat generation. This crucial information helps prevent overheating, a significant safety concern for Lithium-ion batteries. By analyzing deviations from the model's predicted behavior, engineers can diagnose issues early on and take corrective actions to prevent catastrophic failures.

Authors of [1] introduce a one-dimensional lumped thermal model for Li-ion batteries. The model considers heat generation within the cell due to irreversible reactions and heat dissipation through convection and radiation. The model successfully captures the battery's temperature response under various operating conditions. The next reference, [2] extends the one-dimensional model to a two-state model, incorporating a thermal resistance network to represent the battery's internal structure. The model provides a more detailed representation of heat transfer within the cell, improving its accuracy for predicting temperature distributions. [3] addresses the limitations of previous models that neglected radiative heat transfer. The authors incorporate radiative heat transfer into the LPTM, enhancing its accuracy in predicting battery temperature under conditions where radiative heat loss is significant. Authors in [4] focus on lumped thermal modeling of Li-ion battery packs with phase change materials and liquid cooling systems. The authors develop a comprehensive model that considers the thermal interactions between the battery cells, phase change materials, and liquid cooling systems, providing insights into the effectiveness of these thermal management strategies. [5] presents a lumped-parameter Electrothermal model for cylindrical Li-ion batteries that considers the battery's electrical and thermal behavior. The model is used to study the effect of different operating conditions on the battery's temperature distribution and performance. Authors in [6] propose a new formulation of the adaptive lumped parameter thermal model that considers radiative heat transfer in addition to convection heat transfer. The proposed model improves the accuracy of temperature predictions compared to conventional LPBTMs that only consider convection heat transfer. Regarding the identification methodology applied to the LPTMs, in [7], they address the challenges of parameter estimation of lumped thermal models. The authors propose a simplified refined instrumental variable (SRIVC) method, specifically tailored for Li-ion battery thermal models. The SRIVC method effectively estimates model parameters from experimental data, improving the accuracy of the model's predictions. Authors of [8] propose a fast identification method for thermal model parameters of Li-ion batteries based on the analysis of discharge temperature rise. The proposed method is accurate and efficient and can identify real-time thermal model parameters. The authors propose a fast identification method for thermal model parameters of lithium-ion batteries based on the analysis of discharge temperature rise. In [9], they present three lumped parameter thermal models for hard-cased Li-ion batteries and compare their accuracy. The proposed models are shown to be accurate and efficient, and they can be used to optimize the thermal management of Li-ion batteries. Some works have minimized a cost function including temperature and voltage error in Li-ion batteries while making LPTMs. One such paper is [10]. They discuss the different types of lumped parameter thermal models, their advantages and disadvantages, and their applications. They also discuss the different performance indices to evaluate the accuracy

[1]Mehrdad Babazadeh is a research fellow with the WMG group at the University of Warwick/UK and an Assistant Professor at the University of Zanjan/Iran mehrdad.babazadeh@warwick.ac.uk

[2]Dhammika Widanalag Widanage is an Associate Professor with the WMG group at the University of Warwick/UK Dhammika.Widanalage@warwick.ac.uk

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of these models, including temperature error and voltage error. [11] addresses an important issue with the fast varying parameter identification, the data saturation. They introduce a method called Limited Memory Recursive Least Squares Algorithm with Variable Forgetting Factor (VFF-LMRLS). The method tries to remove old data usage from the usual RLS method to make it possible to track the changes as is the case with the Li-ion batteries. LPTMs have already been studied in some limiting conditions of the battery state of charge (SOC), with particular excitation signals, etc. Most of the previous works focused on off-line (batch) identification to identify fixed unique values of parameters of the LPTMs which might not appropriately represent the time-varying parameters of a Li-ion battery. There was no guarantee that the identification yielded non-biased results with other initial conditions or different excitation signals that may occur during the normal operation. The following study addresses a critical challenge in real-time Li-ion battery modeling: enhancing the robustness of parameter identification under broader and more practical operating conditions. We focus on a combined Electrothermal lumped parameter model, aiming to improve the accuracy and reliability of extracted parameters across diverse operational scenarios. This work is organized to comprehensively address the challenges of robust parameter identification in the proposed model of a Li-ion battery. It starts by formulating an electrical model of the battery using a first-order RC equivalent circuit and developing a corresponding linear regression equation. This lays the foundation for capturing the electrical dynamic behavior. A separate thermal model is then established to account for the crucial thermal dynamics and their impact on parameter identification robustness. This section identifies the challenges associated with offline and online identification methods within the combined model context and critically evaluates various options for achieving robust parameter identification. It presents the optimal solution for effectively characterizing the combined Electrothermal model across diverse operating conditions.

A. Main Contribution

Existing research works either focus solely on ECMs or LPTMs individually or present the combined, offline model known as the Thermally Coupled Equivalent Circuit Model (TECM). Many lumped thermal parameter models also rely on pre-specified values for some parameters to identify others. [14] focuses on the design of the observers, where nonlinear two-way coupling between the electrical and thermal sub-models and nonlinear dependence of the model parameters on the system states have been addressed. The most common lumped-parameter equivalent circuit models used in lithium-ion battery energy storage applications were evaluated in [15]. The merits for comparison were modeling accuracy in terms of average root-mean-squared error for two sets of lithium-ion cells of different electrode chemistries, namely the LiFePO4 and LiNMC. The generality of each model structure was examined over a temperature range of 5–45 °C. The battery models' parameters and states were

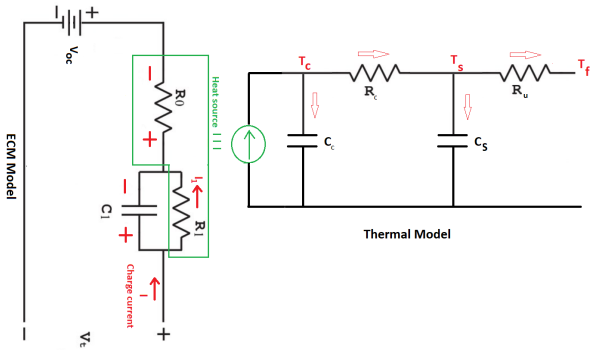

Fig. 1. TECM: First-order RC coupled to the thermal model via R_0 and R_1 as the heat-producing sources.

recursively estimated using a nonlinear system identification technique based on the dual-EKF algorithm. In [12] they assume a known thermal resistance R_u between the battery surface and the environment to find a heat-producing source, R_e. However, neither R_u can be accurately measured, nor does R_e correspond to a real battery parameter. Similarly, [13] assumes known core and surface thermal capacitance (C_c and C_s) to identify other parameters. However, accurately measuring or pre-calculating these "known" real-time values is impractical. Taking benefit of a new procedure, this paper overcomes limitations in existing methods by redesigning lumped parameter models to avoid requiring fixed initial settings. This new combined model approach provides a robust and reliable solution to previously outlined challenges. It improves the model's overall robustness and tackles the data saturation issue mentioned in [11].

II. Lumped Parameter Models of Li-ion Batteries

Lumped parameter models are important since they are relatively simple and computationally efficient. This makes them well-suited for real-time applications, such as battery management systems (BMS) and battery state estimation. Depending on the level of complexity, they can capture the essential dynamics of lithium-ion batteries and can be used to predict the behavior of lithium-ion batteries under a variety of operating conditions including different charge and discharge rates, temperatures, and states of charge.
See Fig. 1 which showcases the electrical and thermal sub-models coupled through specific parameters and interactions. In the present research work, an ECM is developed to supply the thermal model to make a TECM. Both models are identified during normal charge/discharge cycles. The main challenge is that both models deal with fast-varying parameters and non-modeled dynamics. In addition, the linear regression equations in both cases include derivative terms and offsets that make the identification procedure sensitive to the initial conditions and inputs. Therefore, the research looks to find a real-time, more robust solution. The following section begins with building the foundation of the study, an Equivalent Circuit Model (ECM).

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A. ECM-based regression equation

Fig. 1 depicts the electrical sub-model on the left side. Three key elements with unknown values are highlighted:

- The Ohmic Resistance (R_0): Represents the overall electrical resistance of the battery's internal components, including the electrodes, electrolytes, and separators. This parameter accounts for the energy losses due to Joule heating during charge and discharge cycles.
- Polarization Resistance (R_1): Shows the resistance due to various electrochemical processes, including charge transfer and diffusion. The rate of lithium-ion transport within a battery is heavily influenced by diffusion. This process governs the mobility of lithium ions through the electrolyte and electrodes, ultimately dictating the battery's performance. The polarization resistance encompasses the effects of diffusion along with other electrochemical processes.
- Polarization Capacitance (C_1): Gives the time-dependent capacitance, associated with the electrochemical reactions at the electrode surfaces. It accounts for the battery's ability to store and release energy over time.

Applying Kirchhoff's voltage law (kVL) in the Laplace domain, the transformation creates a fundamental relationship within the proposed model:

$$kvl :\Rightarrow V_t - V_{oc} - R_0 \cdot I - \frac{R_1 \cdot \frac{1}{C_1 \cdot s}}{R_1 + \frac{1}{C_1 \cdot s}} \cdot I = 0 \tag{1}$$

$$\Delta V = [V_t - V_{oc}] \Rightarrow \Delta V = [R_0 + \frac{R_1 \cdot \frac{1}{C_1 \cdot s}}{R_1 + \frac{1}{C_1 \cdot s}}] \cdot I \tag{2}$$

$$\Delta V = [\frac{R_0 \cdot R_1 \cdot C_1 \cdot s + (R_0 + R_1)}{R_1 \cdot C_1 \cdot s + 1}] \cdot I \tag{3}$$

$$R_1 \cdot C_1 \cdot s \cdot \Delta V = -\Delta V + [R_0 \cdot R_1 \cdot C_1] \cdot s \cdot I + (R_0 + R_1) \cdot I \tag{4}$$

$$y_v = s \cdot \Delta V = -\frac{1}{R_1 \cdot C_1} \cdot \Delta V + R_0 \cdot s \cdot I + \frac{(R_0 + R_1)}{R_1 \cdot C_1} \cdot I \tag{5}$$

where y_v is the output and ΔV represents the voltage drop on the RC part of the model. Transforming the ECM through the Laplace domain s and expressing it as a matrix product simplifies the analysis by yielding a linear regression equation of the form:

$$y_v = \phi^\top \cdot \theta = s \cdot \Delta V = \begin{bmatrix} \Delta V & s \cdot I & I \end{bmatrix} \times \begin{bmatrix} -\frac{1}{R_1 \cdot C_1} \\ R_0 \\ \frac{(R_0 + R_1)}{R_1 \cdot C_1} \end{bmatrix} \tag{6}$$

$$where \ \ \phi^\top = \begin{bmatrix} \Delta V & s \cdot I & I \end{bmatrix} \tag{7}$$

It is presumed, in this research, that the open circuit voltage (OCV) and battery current (I) in $[A]$ that represents the charge/discharge ($C - Rate$) are both available and known in real-time. An offline TECM that models the ohmic, polarization, and soc-distributed voltage losses. It assumes an Arrhenius-type temperature dependence of the parameters. The surface temperature is generated by an equivalent circuit model including one thermal resistor and one thermal capacitor. Leveraging the existing framework for the ECM, a comprehensive thermal model will be developed. This continuity, leveraging the same theoretical foundation and Laplace domain transformation, ensures consistency and simplifies the overall model construction process.

B. Thermal-based regression equation

The right side of Fig. 1 presents a simplified thermal model for the Li-ion battery, characterized by four unknown parameters. These parameters, representing crucial aspects of the battery's thermal behavior, will be the subject of further analysis and identification in subsequent sections. Existing studies established LPTMs for cylindrical batteries, using four parameters like the model shown in [12] and Inspired by the three-parameter model in [13]. Both applied the following thermal balance equation:

$$C_c \cdot \dot{T}_c = P + \frac{T_s - T_c}{R_c} \tag{8}$$

$$C_s \cdot \dot{T}_s = \frac{T_f - T_s}{R_u} + \frac{T_s - T_c}{R_c} \tag{9}$$

To represent the internal heat generation, the authors of above mentioned papers included an equivalent resistor, R_e, which dissipates power, ($P = R_e \cdot I^2$), into the LPTM. This internal power source serves as the driving force for heat transfer within the system. The study then focused on estimating the unknown resistance, R_e, alongside other key thermal parameters.

Recognizing the limitations of using a generic internal resistor, the present research proposes a more realistic approach for calculating heat dissipation. Instead of a single unknown parameter (R_e), the model focuses on the individual contributions of specific resistors within the ECM (R_0 and R_1) and their corresponding currents generating the Joule heating. It is noticed that another heating source due to the electrochemical reactions is not considered here. Having $P = R_0 \cdot I^2 + R_1 \cdot I_1^2$ offers several advantages:

- Physically meaningful: Each resistor directly relates to specific physical elements within the cell, enhancing the model's interpretability and accuracy.
- Improved predictive capability: By dynamically estimating R_0 and R_1 in real-time, the model can adapt to changing conditions and provide more precise thermal predictions.
- Stronger link between models: Integrating the ECM and LPTMs (TECM) through estimated parameters creates a more robust and cohesive system analysis.

This approach paves the way for a deeper understanding of the battery's thermal dynamics and ultimately contributes to improved performance.

$$\begin{aligned} y_T = s^2 \cdot T_s - s \cdot T_s(0) = \\ \frac{1}{C_c \cdot C_s \cdot R_c} \cdot P - \frac{1}{C_c \cdot C_s \cdot R_c \cdot R_u} \cdot (T_s - T_f) \\ -(\frac{C_c + C_s}{C_c \cdot C_s \cdot R_c}) \cdot (s \cdot T_s - T_s(0)) - \frac{1}{C_s \cdot R_u} \cdot s \cdot (T_s - T_f) \end{aligned} \tag{10}$$

where y_T is the output of the regression equation and $T_s(0)$ denotes the surface temperature T_s at time $t = 0$. For a clear presentation, the Laplace operator (s) has been omitted from the arguments, acknowledging its implicit presence. In this case, for instance, T_s represents $T_s(s)$. With the cooling system, regulating the steady-state surface temperature, the above equation might be the source for different forms of the regression equations mainly by including the ambient temperature T_f, a fixed value, in either of the other terms. T_f can cause ill-conditioning when the regression equation or the number of the regressors is set improperly. For instance, if $T_s(0) = T_f = 0$, the number of the regressors is reduced to three by mixing two regressors in one. We can rewrite the regression equation to incorporate T_f as a constant independent variable. A single regressor like T_f which is a constant component (sometimes called an intercept term or a bias term) allows the model to account for the mean value of the dependent variable when all independent variables are set to zero and some potential issues can arise, particularly in the context of recursive identification. The intercept term acts as a baseline value for the dependent variable,

representing the predicted outcome when all independent variables are zero. This additional parameter allows the model to capture unexplained systematic influence on the dependent variable.
Considering the outlined facts, a new arrangement is introduced assuming the system starts from a steady rest mode ($T_s(0) = T_f$) and $s \cdot T_f = 0$:

$$y_T = s^2 \cdot T_s = \frac{1}{C_c \cdot C_s \cdot R_c} \cdot P - \frac{1}{C_c \cdot C_s \cdot R_c \cdot R_u} \cdot (T_s - T_f) - (\frac{C_c + C_s}{C_c \cdot C_s \cdot R_c}) \cdot (s \cdot T_s - T_f) - \frac{1}{C_s \cdot R_u} \cdot s \cdot T_s \tag{11}$$

$$y_T = \phi^\top \cdot \theta = s^2 \cdot T_s = \begin{bmatrix} P & T_s & s \cdot T_s - T_f & s \cdot T_s \end{bmatrix} * \begin{bmatrix} \frac{1}{C_c \cdot C_s \cdot R_c} \\ -\frac{1}{C_c \cdot C_s \cdot R_c \cdot R_u} \\ -(\frac{C_c + C_s}{C_c \cdot C_s \cdot R_c}) \\ -\frac{1}{C_s \cdot R_u} \end{bmatrix} \tag{12}$$

$$where \;\; \phi^\top = \begin{bmatrix} P & T_s & s \cdot T_s - T_f & s \cdot T_s \end{bmatrix} \tag{13}$$

$$\theta = \begin{bmatrix} \frac{1}{C_c \cdot C_s \cdot R_c} \\ -\frac{1}{C_c \cdot C_s \cdot R_c \cdot R_u} \\ -(\frac{C_c + C_s}{C_c \cdot C_s \cdot R_c}) \\ -\frac{1}{C_s \cdot R_u} \end{bmatrix} = \begin{bmatrix} \alpha_T \\ \beta_T \\ \gamma_T \\ \mu_T \end{bmatrix} \tag{14}$$

During online identification, four key parameters (α_T, β_T, γ_T, and μ_T) are estimated. These serve as inputs to calculate the remaining four unknown thermal model parameters (C_c, C_s, R_c, and R_u).

III. Parameter identification and robustness

Linear systems are straightforward to identify regardless of starting conditions. While linear regression offers a straightforward approach, its low-order nature can limit its ability to accurately model complex, dynamic systems. This can lead the model to converge on suboptimal solutions (local minima) that deviate from the true underlying relationship. This happens because the simple model can't capture the system's true behavior. The optimization algorithm trying to fit the model gets trapped in an incorrect solution, leading to unrealistic outcomes. Additionally, if the solution and parameters heavily depend on starting conditions, it's another sign that the linear model isn't suitable for the complex system.
A typical optimization routine tends to find the first local minimum that might offer nonrealistic parameters of the equivalent model, for instance, negative capacitances, etc. This is why some constraints have to be applied to the least square algorithm to force it to find the best realistic solution. This is why some constraints in the form of penalties have to be applied to the nonlinear least square to empower it to find the best realistic solution.
Li-ion batteries exhibit rapid parameter changes during charge and discharge cycles, a phenomenon known as data saturation in system identification. This means gathering more data won't necessarily improve the accuracy of the model for several reasons: the current data might already effectively capture the system's behavior, the data might not reflect reality, or the algorithm might struggle to extract further information.
To prevent it, collecting more diverse data from different operating conditions, longer periods, using more powerful identification algorithms, or a forgetting factor may help. Another technique is to use a regularizer, a penalty term added to the objective function of the identification algorithm. The regularizer helps to prevent the identification algorithm from overfitting the data. This means that it will not produce a model that is too specific to the training data and that will not generalize well to new data. Because of the dependency on old data, usual identification methods such as Recursive Least Squares (RLS) and Adaptive Kalman Filter (AKF) may lose robustness even with small forgetting factors due to:

- impact of the non-modelled dynamics involved in the parameter identification. The simpler the model structure, the more difficult it is to identify the model due to the presence of non-modeled dynamics.
- Data saturation due to the fast time-varying nature of the battery.
- Methodology of continuous or discrete time identification.

The data saturation will adversely impact the robustness of the identification procedure of the fast-varying Li-ion battery. Even with a well-known VFF-LMRLS method that may work for higher-order ECMs in limited situations, it can be used neither for the first-order ECM nor the proposed thermal model identification. This is because of the high sensitivity of this method to non-modelled dynamics. Therefore, the following methodologies are evaluated to distinguish how they enhance the robustness of the parameter identification of the combined model:

- Arrangement of the Regression Equation
- To use a more accurate discretization technique
- Approximate initial conditions
- Data normalization in the identification procedure
- New configuration of RLS, inner loop, and normalization
- Regularization

A. Arrangement of the Regression Equation

While traditional approaches in the field often build thermal models by using regression equations with three or four parameters, regardless of the specific variables involved, this work recognizes the significant impact of regression equation design on the robustness of the results. The fixed value nature of T_f can negatively influence least-squares minimization and lead to inaccurate predictions.

B. To use a more accurate discretization technique

For the normal discrete-time linear time-invariant systems with sampling time T, the regression equation can be transformed to the z-domain by a stable method such as the backward method where $s = \frac{1-z^{-1}}{T}$. While a discrete-time regression approach offers simplicity, its limitations in accuracy and robustness, particularly for the initial parameter selection in the battery modeling cases, necessitate a more sophisticated approach. Therefore, this work utilizes the trapezoidal method, albeit slightly more complex, to achieve superior accuracy and robustness in parameter estimation. It will replace s with $\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}$, an oscillatory replacement for the derivative term. Therefore, it should be filtered using a derivative smoothing first-order dynamic filter with a coefficient k_v to control the cutoff frequency. The cutoff frequency of the filter should be set to be lower than the frequency of the noise. If the cutoff frequency is set too high, the filter will remove some of the useful information in the derivative signal. The filter has to be applied to both sides of the regression equation. Considering T_v as the sampling time of the ECM, k_v has to be at least $10 \times T_v$ to respect the sampling theorem. Otherwise, the filtered derivative term will oscillate. In the end, the new regression equation of the ECM where the open circuit voltage (V_{oc}), current (I), and terminal voltage (V_t) are already known values are given in the beneath:

$$s \cdot I \Rightarrow \frac{\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}}{k_v \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} + 1} \cdot I \tag{15}$$

$$y = \frac{\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}}{k_v \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} + 1} \cdot \Delta V \tag{16}$$

$$\phi^\top = \frac{1}{k_v \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} + 1} \begin{bmatrix} \Delta V & \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \cdot I & I \end{bmatrix} \tag{17}$$

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Method	Off-line identification Results								Comment
	α	β	γ	μ	Ru	Rc	Cc	Cs	
Lsqnonlin	5.09E-06	-1.00E-07	-0.000316	-0.000316	5.08E+01	75370	0.041957	62.186	
Lsqcurvefit	1.01E-04	-3.58E-07	-0.000711	-0.000713	2.81E+02	958.53	2.0785	4.9824	
Fmincon	5.09E-06	-1.00E-07	-0.000316	-0.000316	5.08E+01	75370	0.041957	62.186	
Genetic Algorithm	5.89E-05	-2.40E-05	-0.001345	-5.82E-06	2.45E+00	-3.46E-06	-70075	70098	unrealistic values
Paternsearch	1.95E-06	-9.54E-07	-9.54E-07	-9.54E-07	2.05E+00	-1.95E-06	-5.12E+05	5.12E+05	unrealistic values
Surrogateopt	2.00E-02	-9.49E-03	-8.95E-01	-1.06E+00	2.11E+00	2.53E+00	4.44E+01	4.46E-01	

Fig. 2. ECM: off-line identified parameters by using different algorithms

$$\theta(t) = \begin{bmatrix} -\frac{1}{R_1 \cdot C_1} \\ R_0 \\ \frac{(R_0+R_1)}{R_1 \cdot C_1} \end{bmatrix} = \begin{bmatrix} \alpha_v \\ \beta_v \\ \gamma_v \end{bmatrix} \tag{18}$$

A recursive identification technique constantly will update the parameter estimates, denoted as $\theta(t)$, at every time step, t. This approach ensures that the latest and most accurate parameter values are readily available throughout the analysis. Following the estimation of $\theta(t)$, the remaining unknown dependent parameters can be calculated based on established relationships or specific equations in the following:

$$R_0 = \beta_v \; ; \; R_1 = -(\frac{\gamma_v + \alpha_v \cdot \beta_v}{\alpha_v}) \; ; \; C_1 = \frac{1}{\gamma_v + \alpha_v \cdot \beta_v} \tag{19}$$

The same discretization method can be applied to the LPTM. Since the second-order derivative is needed to make the output, with a coefficient k_T to control the bandwidth, The following second-order filter will be applied:

$$y_T = (\frac{\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}}{k_T \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} + 1})^2 \cdot T_s = (\frac{1}{k_T \cdot \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} + 1})^2 \cdot \begin{bmatrix} P & T_s - T_f & (\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}) \cdot T_s - T_f & (\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}) \cdot (T_s - T_f) \end{bmatrix} \cdot \theta \tag{20}$$

Similarly, the time constant k_T has to be at least $10 \times T_T$ where T_T is the sampling time of the LPTM. Thereafter, other parameters can be calculated accordingly:

$$\begin{aligned} R_u &= -\frac{\alpha_T}{\beta_T} \; , \; C_s = \frac{\beta_T}{\alpha_T \cdot \mu_T} \; , \\ C_c &= -\frac{\beta_T + \gamma_T \cdot \mu_T}{\alpha_T \cdot \mu_T} \; , \; R_c = -\frac{\alpha_T \cdot \mu_T^2}{\beta_T \cdot (\beta_T + \gamma_T \cdot \mu_T)} \end{aligned} \tag{21}$$

C. Approximate initial conditions

To further evaluate the parameter estimation procedure, a Problem-Based Optimize Live Editor Task in Matlab has been used with different solvers to compare as shown in the table represented in Fig. 2. Among them, the solver Fmincon has been used to apply batch identification to the LPTM speeding up the convergence of the possible real-time estimations.

It is important to note that according to the time-varying nature of the system, it is expected that the overall estimated values of the offline identification differ from the online method even with a great convergence. Furthermore, offline parameter identification methods for Li-ion battery models offer a diverse toolbox but often yield inconsistent results. Some methods, like the Genetic Algorithm, are susceptible to randomness, potentially leading to unrealistic parameter estimates across different runs. This variability presents a significant challenge: how can we reliably estimate battery parameters regardless of the chosen method or initial conditions? To address this issue, robust identification methodologies are crucial. Such methodologies would ensure consistent and undependable parameter estimation, paving the way for more accurate and trustworthy models. Nevertheless, using $Fmincon$ will result in the following parameters of the LPTM: $[\alpha_T \; \beta_T \; \gamma_T \; \mu_T] = [5.0852e-06 \; -1.0007e-07 \; -0.00031644 \; -0.00031644]$. Thereafter, using (21) to (24), the model parameters can be calculated as: $[R_u \; R_c \; C_c \; C_s] = [50.8 \; 75370 \; 0.042 \; 62.186]$

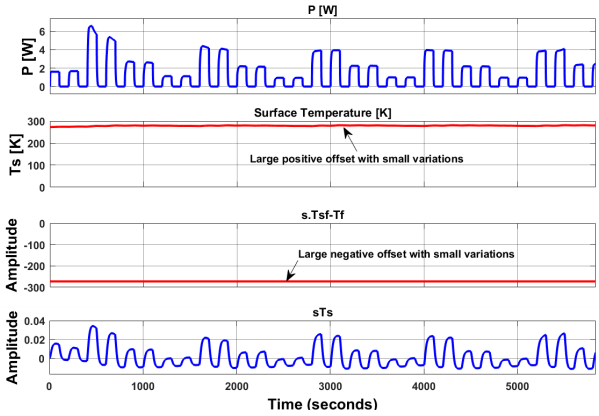

Fig. 3. Thermal model regressors before applying normalization.

D. Data normalization in the identification procedure

The regression equations in the LPTMs often involve a diverse mixture of regressors and outputs, from rapidly changing, higher-order derivatives to smoothly varying, steady-state values with substantially smaller magnitudes. Large differences in data scales can make it harder to accurately identify parameters, as the process becomes overly sensitive to minor variations. To address this challenge, this work proposes introducing new matrices, denoted as $A(t)$ which normalize the identification algorithm by bringing all regressors and outputs to a consistent scale. This normalization effectively enhances the robustness of the parameter identification process in time moment t_t as follows:

$$\phi^\top(t) = \begin{bmatrix} \phi_1(t) & \phi_2(t) & \phi_3(t) & \phi_4(t) \end{bmatrix} \tag{22}$$

$$A_x(t_t) = Max|\phi_x(t)| \; \forall \; 0 \le t \le t_t \; ; \; x = 1 \sim 4 \tag{23}$$

where regressors $\phi_1(t)$ to $\phi_4(t)$ are scalar and the maximum values used to make the matrix $A(t)$ improve over real-time.

$$A = A(t_t) = \begin{bmatrix} A_1(t_t) & 0 & 0 & 0 \\ 0 & A_2(t_t) & 0 & 0 \\ 0 & 0 & A_3(t_t) & 0 \\ 0 & 0 & 0 & A_4(t_t) \end{bmatrix}^{-1} \tag{24}$$

$$y = \phi^\top \cdot \theta = \phi^\top \cdot A \cdot A^{-1} \cdot \theta = (\phi^\top \cdot A) \cdot (A^{-1} \cdot \theta) \tag{25}$$

$$\theta_1 = A^{-1} \cdot \theta \tag{26}$$

$$y = (\phi^\top \cdot A) \cdot \theta_1 \Rightarrow y_x = \frac{y}{y_{max}} = (\phi^\top \cdot A) \cdot \frac{\theta_1}{y_{max}} \tag{27}$$

$$\theta_x = \frac{\theta_1}{y_{max}} \Rightarrow y_x = (\phi^\top \cdot A) \cdot \theta_x \tag{28}$$

where index t_t has been omitted from all variables and parameters for simplicity of presentation. By the above transformation, the absolute value of the normalized elements will be regulated to less than 1. Immediately after normalized parameters are estimated, the original parameters have to be calculated back, to be ready for the second stage calculation of the model parameters as:

$$\theta = A \cdot y_{max} \cdot \theta_x \tag{29}$$

Fig. 3 and Fig. 4 represent regressors before and after the normalization for the thermal model, respectively.

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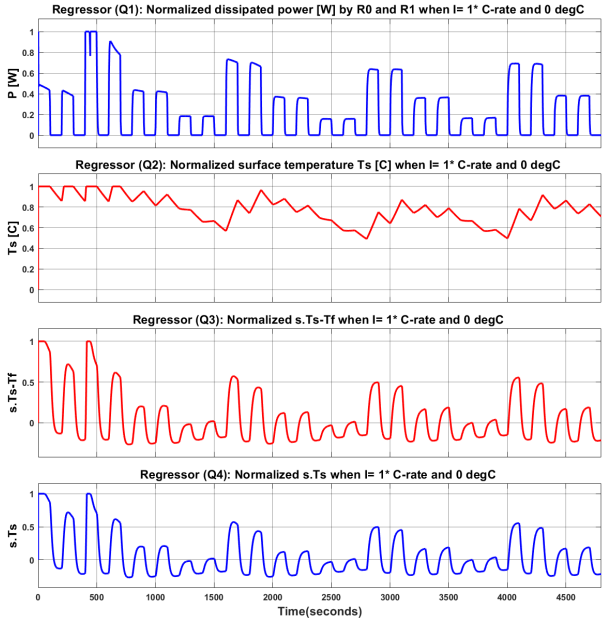

Fig. 4. Thermal model regressors after applying normalization.

E. New configuration of RLS, inner loop, and normalization

For linear time-invariant (LTI) or slowly time-varying systems, Recursive Least Squares (RLS) with a forgetting factor (exponential forgetting) can be a suitable approach. A smaller forgetting factor (λ) will make the algorithm more sensitive to the recent data, while a larger forgetting factor will make it more noise-resistant. Assigning the appropriate values to the covariance matrix (P) and λ will control the performance of the resultant parameter estimation. A large covariance matrix can lead to overfitting, where the model tracks noise and small fluctuations in the data instead of the underlying battery behavior, which results in unrealistic parameter values and poor generalization. RLS cannot track the changes appropriately in the LPTMs of the fast-time-varying Li-ion batteries with non-modelled dynamics. Therefore, matrix P is not the only tool to set to enhance tracking capability, and increasing P can cause instability. Different linear Recursive Least Square methods in Matlab (Forgetting Factor, Kalman Filet, Gradient, and normalized Gradient) were tested to detect the best stable method. None of them worked properly even the methodology mentioned in [6], Variable Forgetting Factor Limited Memory Recursive Least Squares (VFF-LMRLS) that was supposed to work for the second-order RC model, didn't work for the first-order RC model which is the model in the present study. A well-suited new methodology for fast time-varying parameters in the present study, Algorithm 1 represents a pseudo code of an improved RLS algorithm. It employs an exponential forgetting factor with RLS that controls the trade-off between tracking recent data and forgetting past data. In addition, the parameter estimates are further updated inside an inner iterative loop every two consecutive sampling times to predict the output more accurately. Choose inner loop parameters carefully to prevent unrealistic outcomes. Initial values and parameters for the ECM have been set as the covariance matrix $P_v(0) = 100 \times eye(3)$ and the initial parameters $\alpha_v = -0.015$, $\beta_v = 0.035$, and $\gamma_v = 0.005$. For the thermal model, the covariance matrix $P_T(0) = 0.1 \times eye(4)$ and the initial parameters $\alpha_T = 0.001$, $\beta_T = -0.001$, and $\gamma_T = -0.001$, and $\mu_T = -0.001$. In summary, Fig. 5 illustrates the effectiveness of the proposed techniques in enhancing the robustness of parameter identification. This figure indicates a higher core temperature than the surface temperature which is consistent with expected behavior. Although higher values in matrix P could yield a higher correlation in the whole range of the diagrams. However, it could affect the solver's smoothness while finding the parameters. Higher values could result in unacceptable negative values for the expected positive model parameters such as R_c, etc.

Data: Define initial values: (P_0, $\theta_x(0)$)
Set $t = 0$, covariance matrix ($P = P_0$), and $\theta_x = \theta_x(0)$, T, ϵ, and max of internal iteration (i_{max}), $\Delta\theta$, to improve θ, Min/Max forgetting factors($\lambda_{max}, \lambda_{min}$) and τ.
while *main loop of parameter identification* **do**
  Make normalized Regressors $\phi_x(t)$, normalized output $y_x(t)$ and matrix $A(t)$, and $y_{max}(t)$
  $\theta_x = A^{-1} \cdot \frac{\theta}{y_{max}}$
  $\phi_x^\top = \phi^\top \cdot A$
  $y_x = \frac{y}{y_{max}} = \phi_x^\top \cdot \theta_x$
  $i = 1$
  while ($i \leq i_{max}$ *and* $e_x \geq \epsilon$) **do**
    $i++$
    $\lambda = \lambda_{min} + (\lambda_{max} - \lambda_{min}) * e^{-\tau * |e_x|}$
    $K = P \cdot \phi_x \cdot (\phi_x^\top \cdot P \cdot \phi_x + \lambda)^{-1}$
    $P - K \cdot \phi_x^\top \cdot \frac{P}{\lambda} \rightarrow P$
    $e_x = y_x - \phi_x^\top \cdot \theta_x$
    $\theta_x + K \cdot e_x \rightarrow \theta_x$
    $\theta = A(t) \cdot y_{max}(t) \cdot \theta_x$
    while *exceeding limits apply Escape procedure* **do**
      $\theta + \Delta\theta \rightarrow \theta$
      $\theta_x = A^{-1} \cdot \frac{\theta}{y_{max}}$
    end
  end
  $t = t + T$
  Extract α, β,... from θ
  Calculate model parameters R, ...
end

Algorithm 1: Pseudo Code designated for the proposed recursive Least Square

Technique	Impact
Arrangement of the Linear Regression Equations.	High
Use of a more accurate discretization technique.	Low
Approximate initial conditions (offline ID)	Low
Data normalization in the identification procedure.	Highest
Choice of the cost functions	Low
Inner loop inside RLS	High
Escape procedure	High in weak excitation

Fig. 5. Effectiveness of different methodologies.

F. Regularization

This means making the equation work well not only on the data they were trained on but also on unseen data. In the presence of nonlinearities, parameter estimation within complex systems can pose significant challenges. To address this, incorporating well-defined constraints plays a crucial role in guiding the solver towards the optimal solution and accelerating parameter tracking. These constraints effectively reduce solution space by eliminating regions known to be irrelevant or infeasible. The constraints help guide the solver towards physically or theoretically plausible solutions, ultimately increasing the accuracy of parameter estimates. They

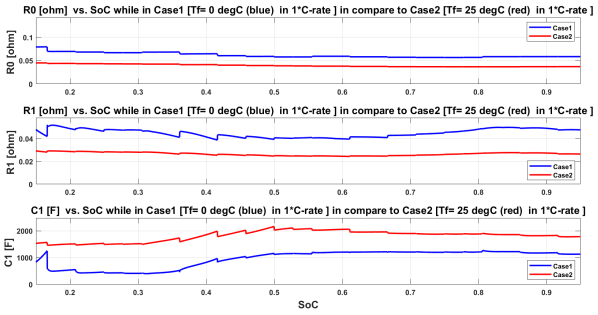

Fig. 6. ECM parameter estimations vs. SoC in $0°C$ and $25\ °C$.

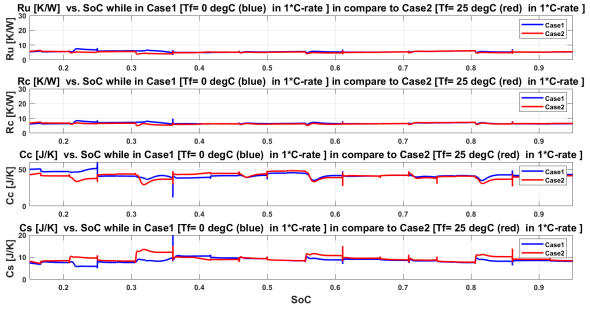

Fig. 7. ECM parameter estimations vs. time in $0°C$ and $25\ °C$.

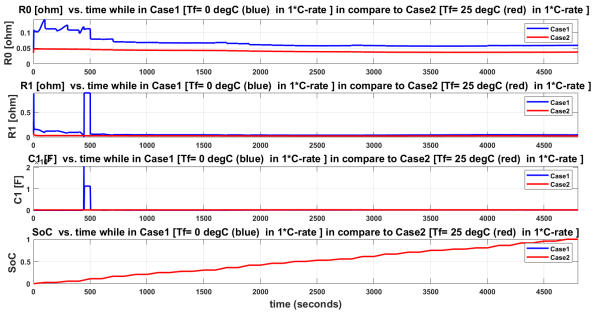

Fig. 8. Thermal model parameter estimations vs. SoC in $0°C$ and $25\ °C$.

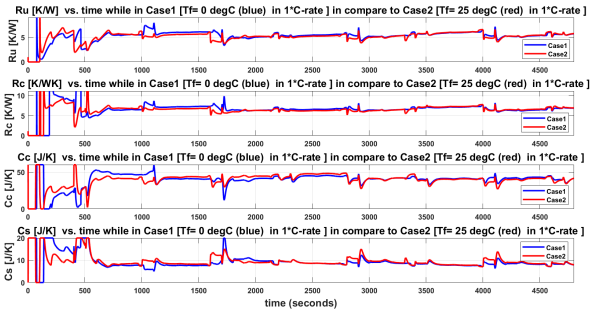

Fig. 9. Thermal model parameter estimations vs. time.

also improve the robustness of the parameter tracking in dynamic or noisy environments and prevent the solver from being misled by spurious data points. The constraints come from the relation between α_v, β_v, and γ_v with the parameters in the ECM (R_0,...) and from α_T, β_T, γ_T, and μ_T with R_c,... in LPTM. The constraints have to be applied to either offline or online estimations as follows:

$$\underline{ECM} : \alpha_v < 0\ ;\ \beta_v > 0\ ;\ \gamma_v > 0\ ;\ \gamma_v + \alpha_v \cdot \beta_v > 0 \tag{30}$$

$$\begin{aligned} &\underline{Thermalmodel} : \alpha_T > 0\ ;\ \beta_T < 0\ ;\ \gamma_T < 0 \\ &\gamma_T - \beta_T/(C_s \cdot \alpha_T) < -C_s \cdot \alpha_T \Rightarrow \beta_T + \gamma_T \cdot \mu_T > 0 \end{aligned} \tag{31}$$

Exceeding established limits for identified parameters, especially within the LPTM, poses a significant risk to its validity. In such cases, some of the calculated parameters (R_c,C_c,...) are likely to deviate from realistic values, though may result in an accurate prediction. This overstepping of boundaries can occur due to:

- Insufficient excitation: When the applied signal lacks persistence, the algorithm may struggle to fully explore the parameter space and settle on an inaccurate local or global minimum.
- Model inadequacy: An improperly defined model structure might not capture the true dynamics of the system, leading the algorithm to compensate by assigning unrealistic values to identified parameters.
- Rapid parameter variations: If the underlying system's parameters change faster than the identification process can adapt, the estimated values become outdated and lose their practical relevance.

Regardless of the event's trigger, diverse approaches can be deployed to handle its occurrence:

- Termination: This strategy halts the algorithm upon reaching parameter limits. While a straightforward approach, it leaves the algorithm in a potentially suboptimal state with inaccurate estimations. Re-initiating the algorithm under different circumstances might be necessary to break free from the erroneous state.
- Parameter Adjustment: This approach actively steers the algorithm out of the limits by modifying one or more parameters. While this can lead to an optimal solution, careful manipulation is crucial. The magnitude and direction of each parameter change significantly impact the new minimum's accuracy and escape velocity from the limiting region. To optimize the adjustments and expedite the escape process, consider utilizing the proposed inner loop outlined in subsection F. This allows for fine-tuning the new minimum after exceeding the original parameter boundaries.

The choice of excitation signal plays a crucial role in parameter estimation accuracy and sensitivity. While step-wise signals offer easy implementation, sinusoidal signals with frequencies higher than $1rad/s$ might outperform them. Because the higher frequency signal generates a larger derivative term. This amplification effectively reduces the sensitivity of the ECM regression equation to noise and improves the model's ability to capture relevant dynamics. However, designing a persistent excitation signal for the LPTM presents a distinct challenge. Although one of the regressors, Power (P) directly depends on the square of the input current (I^2), the remaining regressors are predominantly driven by the slow-changing surface temperature. Consequently, achieving efficient persistent excitation for the LPTM necessitates more sophisticated techniques, such as multi-sine excitation or frequency sweeps to stimulate the fast and slow dynamics. The core temperature can be estimated by designing an observer, which has not been provided in the present paper. Instead, the simulated battery core temperature using the following formula is given in Fig. 11 where the real-time estimation of parameters of the TECM are used. As shown, the excitation is turned off at $t = 5000s$. Thereafter, both T_s and T_c reduce to the ambient temperature at $T_f = 25°C$. The primary transient behavior of the simulated core temperature is related to the early stage of the parameter estimations which can be ignored.

$$T_c = \frac{R_c \cdot P + T_s + R_c \cdot C_c \cdot T_s(0)}{\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \cdot R_c \cdot C_c + 1} \tag{32}$$

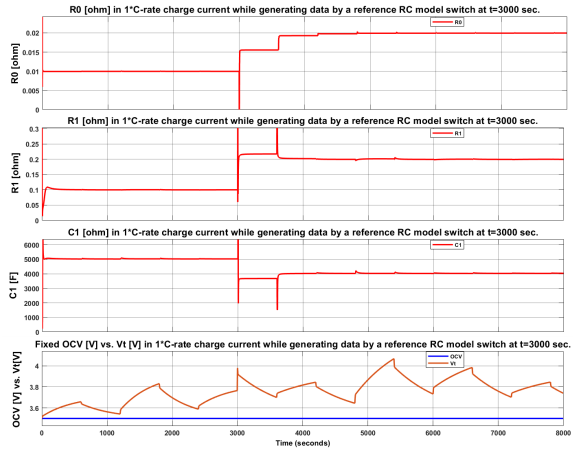

Fig. 10. ECM: Estimation results of a linear time-invariant RC model while changing parameters.

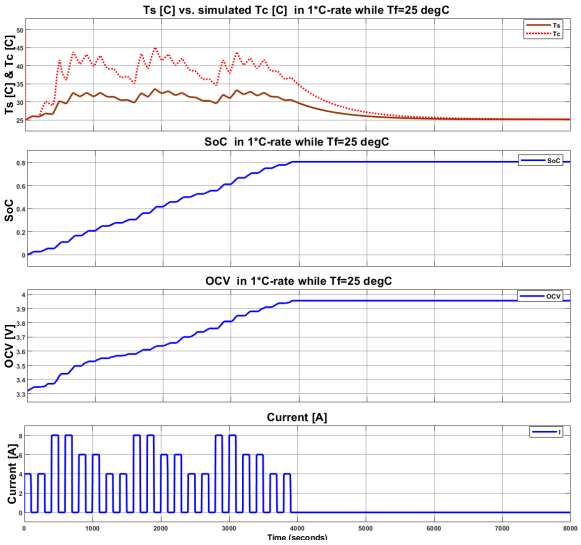

Fig. 11. Simulated core temperature vs. measured surface temperature with $\lambda = 1$ where a permanent rest mode starts at $t = 5000s$.

IV. Validation

To check the performance of the algorithm, a time-invariant linear RC circuit similar to the ECM has been used. At $t = 0$ the circuit starts with the initial model parameters $R_0 = 0.01$, $R_1 = 0.1$, and $C_1 = 5000F$. At $t = 3000s$, it switches to $R_0 = 0.02$, $R_1 = 0.2$, and $C_1 = 4000F$. Fig.10 represents the convergence of the estimated parameters to the correct values. This simulation has used the following initial parameters and values, where $eye(3)$ is a 3×3 unity matrix, $\lambda_{min} = 0.85$, $\lambda_{max} = 1$, $P = 100 \cdot eye(3)$, and $\tau = 200$ where excitation signal is of lower frequency to let the RC model represents exponential variations in response to the excitation signal. The initial parameters are $\alpha = -0.015$, $\beta = 0.035$, and $\gamma = 0.005$. The data is generated by the same structure as the model. With the linearity and the time-invariant nature, the solver tracks the changes rapidly. Meanwhile, the transient part can be controlled by selecting more appropriate values for P, τ, λ_{min}, and λ_{max}.

V. Conclusions

Moving beyond the limitations of existing methods, this paper offers a breakthrough in online parameter identification for Li-ion battery models. These models go beyond inherent simplicity by addressing the hidden complexities and dynamic intricacies that confound conventional identification methods, preventing them from being overwhelmed by data saturation and ambiguity. Applying a few proposed key points worked fine and the unknown parameters could be identified well. Instead, the lumped parameter thermal model was more sensitive to initial parameters and the excitation signal because of having higher-order derivative terms in the regression equation and the non-modeled dynamics. As the key contribution, introducing a novel formulation of an Electrothermal model for Li-ion batteries, applicable across diverse operational scenarios could mitigate the mentioned estimation issues. Equipped with important techniques, this formulation empowered real-time parameter identification without requiring presumptions about specific parameters. By eliminating the need for pre-assumptions, this method fostered a clearer understanding of the intricate interplay between electrical and thermal dynamics within the battery.

References

[1] M. Doyle, T. F. Fuller, and J. Newman, "A lumped parameter thermal model for lithium-ion batteries," Journal of the Electrochemical Society, vol. 140, no. 6, pp. 1526-1533, Jun. 1993.

[2] C. Forgez, M. Salomé, A. De Guiri, O. Gaillot, P. Legrand, "A Two-State Lumped Parameter Model for Lithium-Ion Batteries", (2003).

[3] Allafi, Walid, Cheng Zhang, Kotub Uddin, Daniel Worwood, Truong Quang Dinh, Pedro Ascencio Ormeno, Kang Li, and James Marco. "A lumped thermal model of lithium-ion battery cells considering radiative heat transfer." Applied Thermal Engineering 143 (2018): 472-481.

[4] Lebrouhi, Badr Eddine, B. Lamrani, M. Ouassaid, M. Abd-Lefdil, M. Maaroufi, and Tarik Kousksou. "Low-cost numerical lumped modeling of lithium-ion battery pack with phase change material and liquid cooling thermal management system." Journal of Energy Storage 54 (2022):

[5] "A lumped-parameter Electrothermal model for cylindrical batteries" by Alessandro Pesaran, Sreejithkumar S. Gowda, and Michael K. Ecker (2014)

[6] Dey, S.et. all, (2019). Model-based battery thermal fault diagnostics: Algorithms, analysis, and experiments. IEEE Transactions on Control Systems Technology, 27(2), 576-587. doi: 10.1109/TCST.2017.2776218

[7] "A Simplified Refined Instrumental Variable Method for Parameter Estimation of Li-ion Battery Thermal Models" by Sun, et al. (2017)

[8] Fengjiao Liu, Pengcheng Guan, Yanzhu Wang, Yuxuan Yang, and Jianqiu Li, "Fast identification method for thermal model parameters of Lithium-ion battery based on discharge temperature rise", (2021)

[9] "Optimization of the lumped parameter thermal model for hard-cased Li-ion batteries" by Pengcheng Guan, Yanzhu Wang, Jianqiu Li, and Fengjiao Liu (2020)

[10] "A comprehensive review of lumped parameter thermal models for Li-ion batteries" by Fengjiao Liu, Pengcheng Guan, Yanzhu Wang, and Jianqiu Li (2018)

[11] Peng, X., et al. (2022). Parameter identification of lithium-ion battery equivalent circuit model based on limited memory recursive least squares algorithm with variable forgetting factor. Journal of Physics: Conference Series, vol. 2246, no. 1, p. 012025. doi: 10.1088/1742-6596/2246/1/012025

[12] X. Lin, A. G. Stefanopoulou, H. E. Perez, J. B. Siegel, Y. Li, and R. D. Anderson, "Quadruple adaptive observer of the core temperature in cylindrical Li-ion batteries and their health monitoring," 2012 American Control Conference (ACC), Montreal, QC, Canada, 2012, pp. 578-583, doi: 10.1109/ACC.2012.

[13] X. Lin et al., "Online Parameterization of Lumped Thermal Dynamics in Cylindrical Lithium Ion Batteries for Core Temperature Estimation and Health Monitoring," in IEEE Transactions on Control Systems Technology, vol. 21, no. 5, pp. 1745-1755, Sept. 2013, doi: 10.1109/TCST.2012.

[14] Dong Zhang, Luis D. Couto, Saehong Park, Preet Gill, Scott J. Moura," Nonlinear State and Parameter Estimation for Li-ion Batteries with Thermal Coupling", IFAC-PapersOnLine, Volume 53, Issue 2, 2020, Pages 12479-12484,

[15] S. Nejad, et. all, "A systematic review of lumped-parameter equivalent circuit models for real-time estimation of lithium-ion battery states", Journal of Power Sources, 2016