The double inverted pendulum with real mass distribution stabilization*

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Abstract— **The stabilization of the real laboratory model of the underacuated double inverted pendulum having the actuator placed between its links and a realizable mass distribution adjustment is presented here. More specifically, this laboratory model is to be stabilized at its upper equilibrium with both links being stretched along a single line and pendulum achieving the maximal possible length. This paper presents various methods to design the stabilizing feedback for the double inverted pendulum actuated between its links and then performs optimization of the model with respect to masses distributions to minimize the controller torques. The results are then presented both in simulations and a simple laboratory experiment.**

I. INTRODUCTION

The main contribution of this paper consists in the real-time implementation study for the various stabilization methods of the double inverted pendulum with a single actuator placed between its links. More specifically, the respective laboratory model is to be stabilized at its upper equilibrium with both links being stretched along a single line and pendulum achieving the maximal possible length. Furthermore, the laboratory model provides realizable mass distribution adjustment used for the actuator torque minimization.

The underactuated double inverted pendulum is one of the simplest underactuated mechanical systems consisting of two rigid links and one actuator, which is either placed between the links, or at the contact point with supporting surface. While both actuator placements allow double pendulum stabilization in upper position both with overlapping and upright links, the placement between links allow a simple movement resembling a human walk as well. In the latter case, alternative names, like Acrobot, Compass gait walker or biped, are often used. To control the Acrobot walking, the partial exact feedback linearization method was developed along with rather special and complex procedures of the residual nonlinearity adjustement [1], [2], [3], [4], [5], [6]. The reason for that complexity was that the nonlinearity influence can not be neglected when nontrivial walking-like trajectory is to be tracked. Yet, for a local stabilization at an equilibrium, a linear feedback based on approximate linearization model might be sufficient while global stabilization still may need a more sophisticated treatment.

Even though the double pendulum with a single actuator placed between the links is the classical mechanical system with many applications, it can also serve as a simple test bed for verifications of feedback control algorithms for underactuated walking robots. Consider a different posture of the inverted pendulum, specifically, the free unconnected ends of each link touch the ground and the mutually connected ends equipped with an actuator are above the ground. Indeed the Acrobot is perhaps the simplest underactuated mechanical system capable to move (at least theoretically) in a way resembling a human walk.

In spite of the fact that the Spong's papers on the Acrobot feedback linearization [7], [8], [9] were published almost thirty years ago, the Acrobot control still belongs in active research field. In [10] the swing-up and stabilization problem for the Acrobot is solved via the stable manifold method for optimal control, which numerically solves Hamilton-Jacobi equations. The posture control problem of a two-link free flying Acrobot with nonzero initial angular momentum is studied in [11]. An optimal control system design for the Acrobot using inverse linear quadratic design method is proposed in [12].

Due to the nonlinearities of the respective Acrobot model, partial exact feedback linearization, fuzzy based control or neural networks based control methods (to name a few) have to be used to analyze them and to design the respective controllers at its whole range of movement except stabilization. The exact feedback linearization of the Acrobot was presented first in [7], [8], where two application examples of partial feedback linearization applied to the Acrobot were presented. The first one, called as the collocated linearization, is based on the output equation related to the actuated angle, whereas the second one, called as the non-collocated linearization, is based on the output equation related to the underactuated angle. In [13] a special normal form of the Acrobot with tree dimension linear part was developed based on a smooth change coordinates and feedback.

However, from practical point of view, an additional movable joint serving as a knee equipped with an actuator has to be added into each link in order to bend the link during the step. Otherwise the Acrobot is able to walk in simulations only. This mechanical system is in the literature usually called as the 4-link. Adding a movable torso to the 4-link or to the Acrobot forms a walking mechanism similar to a human with an ability of underactuated walking. The 4-link or the Acrobot with torso walking control is based on the Acrobot walking control via the embedding method of the Acrobot into the more complicated walking mechanism, see [14], [15], [16]. Therefore the Acrobot control definitely worth study. Further research on Acrobot control can be summarized as follows. In [1], [2], [3] a partial feedback linearization of order 3 of the Acrobot was introduced. The

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partial feedback linearization is based on the same idea as the linearization in [8], [13] with different change of coordinates. Moreover, in [3], an asymptotical stabilization of the system was obtained. In [17], [18], [14] the approach from [3] was extended, nevertheless, the results of the Acrobot walking control were demonstrated in simulations. A simple model of an underactuated walking-like mechanical system was built in our laboratory, see [4], [5], [6] for description and simple experiments.

Aim of the current paper is to present the very first results of verifying some previous results using the real laboratory robot. To do so, this paper presents various methods to design the stabilizing feedback for the double inverted pendulum actuated between its link and then performs optimization of the model with respect to masses distributions to minimize the controller torques. Indeed, by virtue of the partial exact feedback linearization and corresponding linearized coordinates, yet another control method to stabilize the double inverted pendulum in its upper position is proposed in the current paper. Its ability to stabilize the double inverted pendulum with real mass distribution at the upper equilibrium with links in upright position is demonstrated here in simulations and a numerical optimization of the mechanical parameters is performed to minimize the required torque of the actuator. The results are then presented both in simulations and a simple laboratory experiment.

The paper is organized as follows. The next section contains preliminary results including modeling, feedback linearization together with linearized model in upper equilibrium and brief description of the laboratory model. The third section deals with parameters optimization in order to perform the simulation of stabilization in upper equilibrium whereas the fourth section presents simulations and simple experiment. The final section conclude the paper.

II. PRELIMINARY RESULTS

In this section a preliminary results including the Acrobot model, its linearization in upper equilibrium, the partial feedback linearization and brief description of the laboratory model are presented.

A. Dynamical model of the Acrobot

The well-known Euler-Lagrangian approach, see [19], [20] will be used here. First, define Lagrangian $\mathscr{L}(q,\dot{q})$ given by the difference between kinetic $\mathscr{K}$ and potential $\mathscr{P}$ energy of the modelled mechanical system

$$\mathscr{L}(q,\dot{q}) = \mathscr{K} - \mathscr{P}. \tag{1}$$

A general set of differential equations describing the time evolution of the Acrobot is obtained as follows. Let the underactuated angle at the pivot point be denoted as q_1, then the Euler-Lagrange equations give

$$\begin{bmatrix} \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{q}_1} - \frac{\partial \mathscr{L}}{\partial q_1} \\ \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{q}_2} - \frac{\partial \mathscr{L}}{\partial q_2} \end{bmatrix} = u = \begin{bmatrix} 0 \\ \tau_2 \end{bmatrix}, \tag{2}$$

where u stands for the vector of the external controlled forces. System (2) is the so-called underactuated mechanical system having the degree of the underactuation equal to one. Equation (2) leads to the dynamical equation in the form

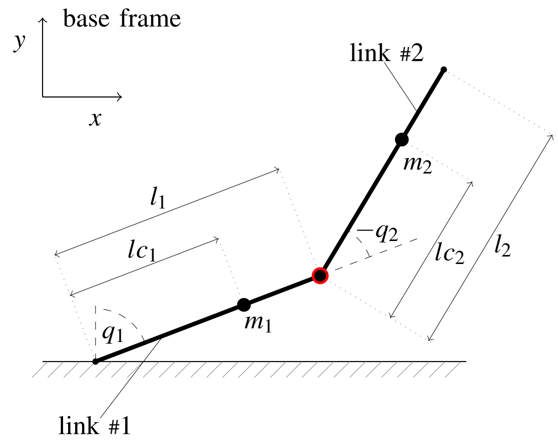

Fig. 1. Geometry of Acrobot

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u, \tag{3}$$

where $D(q)$ is the inertia matrix, $D(q) = D(q)^{\mathrm{T}} > 0$, matrix $C(q,\dot{q})$ contains Coriolis and centrifugal terms, vector $G(q)$ contains gravity terms and u stands for the vector of external forces.

For the simplicity, friction is not considered here. Nevertheless, the links are considered with mass distributed over the length of the link according to the real laboratory model. To do so, the whole mass is placed in the center of mass of the corresponding link and the mass distribution over the link is represented by its moment of inertia. See Figure 1 for links length specifications.

The following material parameter equations were introduced in [19]

$$\begin{aligned} \theta_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_{1zz}, \quad \theta_2 = m_2 l_{c2}^2 + I_{2zz}, \\ \theta_3 &= m_2 l_1 l_{c2}, \quad \theta_4 = m_1 l_{c1} + m_2 l_1, \quad \theta_5 = m_2 l_{c2}, \end{aligned} \tag{4}$$

where m_1, m_2 is the mass of the link #1, #2, respectively, l_1, l_2 is length of the link #1, #2, respectively, l_{c1}, l_{c2} is the distance to the center of mass of the link #1, #2, respectively, I_{1zz}, I_{2zz} is the moment of inertia about center of mass of the link #1, #2, respectively, g is gravity acceleration, q_1 is the angle that link #1 makes with the vertical, q_2 is the angle that link #2 makes with the link #1, τ_2 is torque applied at the joint between links #1 and #2.

The matrices $D(q)$, $C(q,\dot{q})$ and vector $G(q)$ from dynamical equation (3) with material parameters $\theta_{1,2,3,4,5}$ (4) are defined as follows

$$D(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos q_2 & \theta_2 + \theta_3 \cos q_2 \\ \theta_2 + \theta_3 \cos q_2 & \theta_2 \end{bmatrix}, \tag{5}$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3 \sin q_2 \dot{q}_2 & -(\dot{q}_1 + \dot{q}_2)\theta_3 \sin q_2 \\ \theta_3 \sin q_2 \dot{q}_1 & 0 \end{bmatrix}, \tag{6}$$

$$G(q) = \begin{bmatrix} -\theta_4 g \sin q_1 - \theta_5 g \sin(q_1 + q_2) \\ -\theta_5 g \sin(q_1 + q_2) \end{bmatrix}. \tag{7}$$

The continuous part of the movement is considered in the modeling procedure only because the impact is not taken into the account during the Acrobot stabilization in the upper equilibrium.

B. The linearization of the Acrobot

Consider dynamical model of the Acrobot (3). System matrices in upper equilibrium $q_1 = \pi/2$, $q_2 = 0$, $\dot{q}_1 = \dot{q}_2 = 0$ have the following form

$$\tilde{D}(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 & \theta_2 + \theta_3 \\ \theta_2 + \theta_3 & \theta_2 \end{bmatrix}, \tag{8}$$

$$\tilde{C}(q,\dot{q}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{9}$$

$$\tilde{G}(q) = \begin{bmatrix} -\theta_4 g - \theta_5 g \\ -\theta_5 g \end{bmatrix}, \tag{10}$$

so that the respective approximate linearization is given by the quartuple (A, B, C, D)

$$A = \begin{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \tilde{G}/\tilde{D} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}, B = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} / \tilde{D} \end{bmatrix} \tag{11}$$

$$C = \mathrm{diag}(1,1,1,1), \quad D = [0,0,0,0]', \tag{12}$$

having the state vector $[x_1, x_2, x_3, x_4]' = [q_1, q_2, \dot{q}_1, \dot{q}_2]'$. The feedback has the following form $\tau_2 = K_1 q_1 + K_2 \dot{q}_1 + K_3 q_2 + K_4 \dot{q}_2$, with gains given by the Linear-Quadratic Regulator approach.

C. Partial feedback linearization of the Acrobot

The **partial exact feedback linearization** method is based on a system transformation into a new system of coordinates that displays linear dependence between some auxiliary output and new (virtual) input [21]. In [22] it was shown that if the generalized momentum conjugates to the cyclic variable is not conserved (as it is the case of Acrobot) then there exists a set of outputs that defines a one-dimensional exponentially stable zero dynamics. That means that it is possible to find a function $\bar{y}(q,\dot{q})$ with relative degree 3 that transforms the original 4 dimensional system (3) by a local coordinate transformation $z = T(q,\dot{q})$ into the new input/output linear system which has 3 dimensional state plus the unobservable nonlinear dynamics of dimension 1. For details see [3].

In the case of the Acrobot there are two independent functions with relative degree 3 which transform the original system into the desired partial linearized form with one dimensional zero dynamics,[1] namely

$$\begin{aligned} \sigma &= \frac{\partial \mathscr{L}}{\partial \dot{q}_1} = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)\dot{q}_1 + \\ &\qquad (\theta_2 + \theta_3 \cos q_2)\dot{q}_2, \end{aligned} \tag{13}$$

$$\begin{aligned} p &= q_1 + \frac{q_2}{2} + \frac{2\theta_2 - \theta_1 - \theta_2}{\sqrt{(\theta_1+\theta_2)^2 - 4\theta_3^2}} \arctan \\ &\qquad \left(\sqrt{\frac{\theta_1 + \theta_2 - 2\theta_3}{\theta_1 + \theta_2 + 2\theta_3}} \tan \frac{q_2}{2} \right). \end{aligned} \tag{14}$$

In [3] it was shown that using the set of functions with maximal relative degree, the following transformation

$$\xi = \mathscr{T}(q,\dot{q}): \quad \xi_1 = p, \quad \xi_2 = \sigma, \quad \xi_3 = \dot{\sigma}, \quad \xi_4 = \ddot{\sigma} \tag{15}$$

can be defined. Notice, that by (13,14) and some straightforward but laborious computations the following relation holds

$$\dot{p} = d_{11}(q_2)^{-1}\sigma, \tag{16}$$

where $d_{11}(q_2) = (\theta_1 + \theta_2 + 2\theta_3 \cos q_2)$ is the corresponding element of the inertia matrix D in (3). Applying (15), (16) to (3) we obtain the Acrobot's dynamics in partial exact linearized form

$$\begin{aligned} \dot{\xi}_1 &= d_{11}(q_2)^{-1}\xi_2, \quad \dot{\xi}_2 = \xi_3, \quad \dot{\xi}_3 = \xi_4, \\ \dot{\xi}_4 &= \alpha(q,\dot{q})\tau_2 + \beta(q,\dot{q}) = w \end{aligned} \tag{17}$$

with the new coordinates ξ and the input w being well defined wherever $\alpha(q,\dot{q})^{-1} \neq 0$. The new input w is a virtual input in ξ coordinates. As a result of this, it is necessary to recompute the virtual input w in ξ coordinates to the real input u in $q, \dot{q}$ coordinates before applying the virtual input on the real model of the Acrobot.

1) Asymptotical stabilization of the Acrobot: Assume that an open loop control $w^r(t)$, generates a suitable reference trajectory in partial exact linearized coordinates (17). In other words, our task is to track the following reference system

$$\dot{\xi}_1^r = d_{11}^{-1}(q_2^r)\xi_2^r, \quad \dot{\xi}_2^r = \xi_3^r, \quad \dot{\xi}_3^r = \xi_4^r, \quad \dot{\xi}_4^r = w^r. \tag{18}$$

Denoting $e := \xi - \xi^r$ and subtracting (18) from (17) one obtains

$$\begin{aligned} \dot{e}_1 &= d_{11}^{-1}(\phi_2(\xi_1,\xi_3))\xi_2 - d_{11}^{-1}(\phi_2(\xi_1^r,\xi_3^r))\xi_2^r, \\ \dot{e}_2 &= e_3, \quad \dot{e}_3 = e_4, \quad \dot{e}_4 = w - w^r. \end{aligned} \tag{19}$$

Straightforward computations based on the Taylor expansions give

$$\begin{aligned} \dot{e}_1 &= \mu_2(t)e_2 + \mu_1(t)e_1 + \mu_3(t)e_3 + o(e), \\ \dot{e}_2 &= e_3, \quad \dot{e}_3 = e_4, \quad \dot{e}_4 = w - w^r, \end{aligned} \tag{20}$$

where $\mu_1(t), \mu_2(t), \mu_3(t)$ are known smooth time functions and are bounded from bellow and above. For details and definitions of functions $\mu_1(t)$, $\mu_2(t)$, $\mu_3(t)$ see [3].

[1] Actually, by (3) $\dot{\sigma} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{q}_1} = \frac{\partial \mathscr{L}}{\partial q_1}$ and therefore by the definition of the Lagrangian $\dot{\sigma} = -\frac{\partial V(q)}{\partial q_1}$ as $D(q) \equiv D(q_2)$ by definition of the inertia matrix. In other words, $\dot{\sigma}$ has relative degree 2, i.e. σ has the relative degree 3. Moreover, by the straightforward differentiation it holds $\dot{p} = d_{11}(q_2)^{-1}\sigma$, i.e. $\dot{p}$ has relative degree 2, i.e. p should have relative degree 3 as well.

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Using these properties [17], [3] provide feedback to stabilize the above error dynamics. In [17] it was shown that for the the open-loop continuous time-varying linear system

$$\dot{e} = A(t)e + B(w - w^r), \tag{21}$$

where

$$A(t) = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{22}$$

the tracking problem consists in finding the state-feedback controller

$$w - w^r = Ke, \quad K = \begin{pmatrix} K_1 & K_2 & K_3 & K_4 \end{pmatrix}, \tag{23}$$

producing the following closed-loop system

$$\dot{e} = (A + BK)e = \begin{pmatrix} \mu_1(t) & \mu_2(t) & \mu_3(t) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ K_1 & K_2 & K_3 & K_4 \end{pmatrix} e. \tag{24}$$

In the particular case of stabilization in the upper equilibrium feedback (23) with an amplifying parameter Θ has the following form

$$w = \Theta^3 K_1 \xi_1 + \Theta^3 K_2 \xi_2 + \Theta^2 K_3 \xi_3 + \Theta^3 K_4 \xi_4, \tag{25}$$

because $w_r, \xi_1^r, \xi_2^r, \xi_3^r, \xi_4^r = 0$ for stabilization in the equilibrium. Further, let $K_1 < 0$ and $K_{2,3,4}$ are such that the polynomial $\lambda^3 + K_4\lambda^2 + K_3\lambda + K_2$ is Hurwitz. Then there exist $\Theta > 0$ such that system (24) is stabilized. For details, see [3].

D. Linearization in linearized coordinates

The classical linearization method in upper equilibrium could also be applied on the Acrobot's dynamics in partial exact linearized form (17) which is almost linear except the first line. The therm $d_{11}(q_2)$ is defined in (5) and its linearized form in upper equilibrium is straightforward $d_{11}^{lin} = \theta_1 + \theta_2 + 2\theta_3$ and results in following linearized system in the linearized coordinates (17)

$$\tilde{\tilde{A}} = \begin{pmatrix} 0 & 1/d_{11}^{lin} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\tilde{B}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{26}$$

$$\tilde{\tilde{C}} = \mathrm{diag}(1,1,1,1), \quad \tilde{\tilde{D}} = [0,0,0,0]' \tag{27}$$

according to the state vector $[x_1, x_2, x_3, x_4]' = [\xi_1, \xi_2, \xi_3, \xi_4]'$. The feedback in ξ coordinates is following $w = K_1\xi_1 + K_2\xi_2 + K_3\xi_3 + K_4\xi_4$ with gains $K_{1,2,3,4}$ given by the Linear-Quadratic Regulator approach.

TABLE I

PARAMETERS OF THE DOUBLE PENDULUM MODEL

l_1, l_2	length of each link	0.27	$[m]$
l_{c1}	center of gravity of upper link	0.2	$[m]$
l_{c2}	center of gravity of bottom link	0.12	$[m]$
m_1	mass of upper link	0.29	$[kg]$
m_2	mass of bottom link	0.14	$[kg]$
I_1	inertia of upper link	0.01	$[m^2 Kg]$
I_2	inertia of bottom link	0.007	$[m^2 Kg]$

E. Description of the real model

A simple model of the underactuated walking-like mechanical system was built and developed in our laboratory, see Fig. 2 for a brief idea. [4] provides a detailed description with simple leg movement experiment. Sensors of angular positions, velocities and input current measurements for DC motors are used. In contrast to simulation only, the real measured signals are corrupted with noise and therefore its filtering is, indeed, necessary. To do so, in [5] the Extended Kalman Filter was proposed and off-line verified using data obtained during a simple movement of one leg. In [6] the EKF was implemented using operations available on a low cost hardware and successfully verified in an application of on-line data processing.

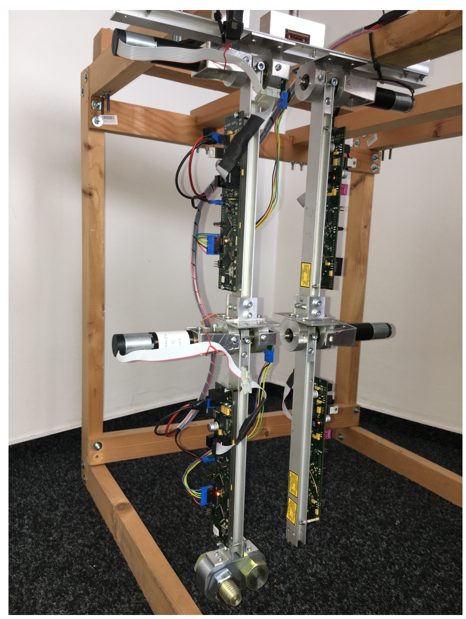

Fig. 2. Real four link walking robot. The left leg is equipped with an additional weight such that the controllability condition is fulfilled.

Physical quantities that describe the model of the robot leg together with its values are summed up in Table I. The values were either measured, especially length or mass of a link, or calculated especially center of mass or inertia of each link.

III. PENDULUM MODEL PARAMETERS TUNING

The focus in this section is on model parameters tuning such that the following controllability condition is fulfilled

$$\theta_5(\theta_1 + \theta_3) \neq \theta_4(\theta_2 + \theta_3), \tag{28}$$

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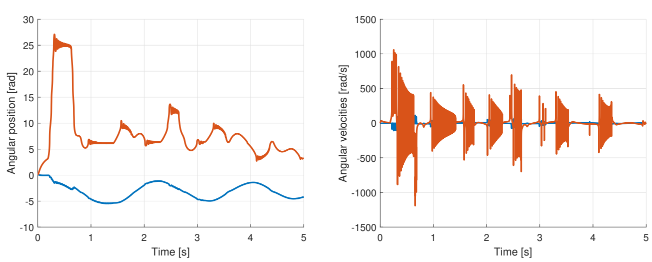
where $\theta_{1,2,3,4,5}$ are given by (4). To do so, a numerical optimization of the mechanical parameter is performed such that (28) is fulfilled and the required torque of the attached actuator is minimal such that the actuator is capable to stabilize the real Acrobot model in the upper equilibrium even after small deflection.

The obtained mechanical parameters given in Table I do not fulfill the controllability condition for the double pendulum (28) resulting in simulations of inverted double pendulum stabilization in the upper equilibrium as depicted in Figure 3. Whereas in the case of the inverted double pendulum with unrealistic mass displacement, i.e. the almost massless links with the mass placed in the ends of each link, the controllability condition (28) is fulfilled and therefore its stabilization in the upper equilibrium is sufficient, see Figure 4.

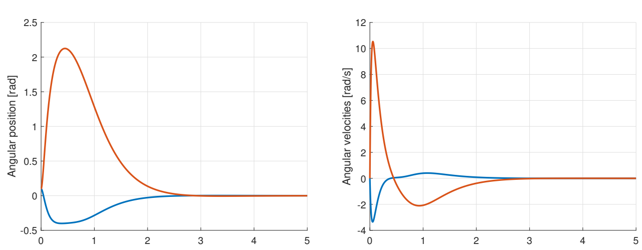
Due to the fact that the real model and its control is taken into the account, the opportunities to any change of the mechanical parameters are limited to putting an additional mass on the pendulum link in a specific position. *FMINCON* solver was used to find the optimal value of mass and its placement on the link with respect to minimal required torque to stabilize the pendulum model in the upper equilibrium even after small deflection from the equilibrium. The course of stabilization with added mass $m = 0.5\,\mathrm{Kg}$ to the end of the upper link is depicted in Figure 5.

In Figures 3,4,5 the blue courses correspond in (a) to angular positions q_1 and in (b) to angular velocities $\dot{q}_1$ whereas the red courses correspond in (a) to angular positions q_2 and in (b) to angular velocities $\dot{q}_2$.



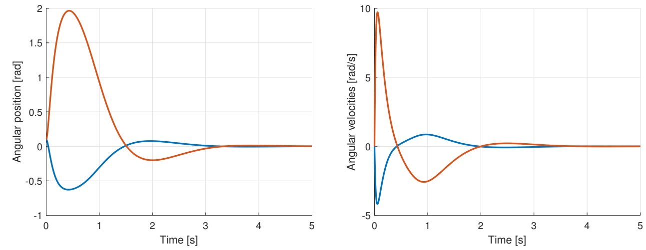
(a) (b)

Fig. 3. Stabilization response for (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ for double inverted pendulum with real mass distribution according to the real laboratory model.



(a) (b)

Fig. 4. Stabilization response for (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ with unreal mass distribution, i.e. massless links with mass in the end of the link.



(a) (b)

Fig. 5. Stabilization response for (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ for double inverted pendulum with real mass distribution according to the real laboratory model and added mass $m = 0.5\,\mathrm{Kg}$ in the end of the upper link.

IV. SIMULATIONS AND EXPERIMENTS

In this section simulation and experimental results of the Acrobot control in the upward position are presented.

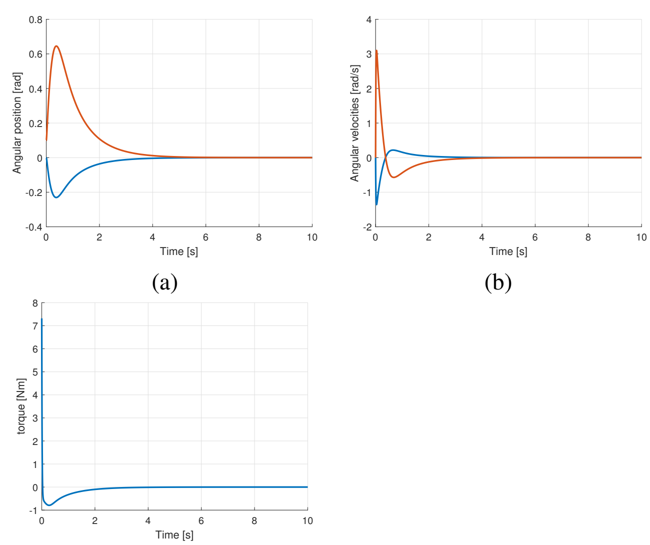
A. *Simulations*

several methods are applied and compared

The proposed feedback control methods were applied on the Acrobot with additional mass to stabilize it in the upper equilibrium with an initial error in angular position q_2.

The feedback gains for the Acrobot linearization based controller and controller based on linearization in linearized coordinates were established using the Linear-Quadratic Regulator (LQR) design in Matlab with focus on aplied torque less than $1\,\mathrm{Nm}$. The controller gains for the partial exact feedback linearization based feedback were established using *fminsearch* function in Matlab.

In the simulations of feedback control, the mechanical parameters from Table I together with the additional mass in the end of the upper link were used. The results are depicted in Figures 6,7,8



(a) (b)

(c)

Fig. 6. Simulations: Stabilization response (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ and (c) torque for the control approach based on the Acrobot linearization and the Linear Quadratic feedback design.

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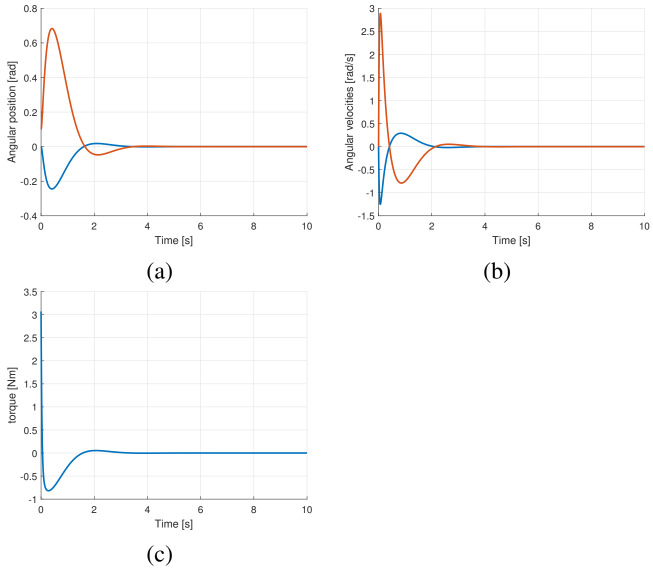

Fig. 7. Simulations: Stabilization response (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ and (c) torque for the control approach based on the partial feedback linearization.

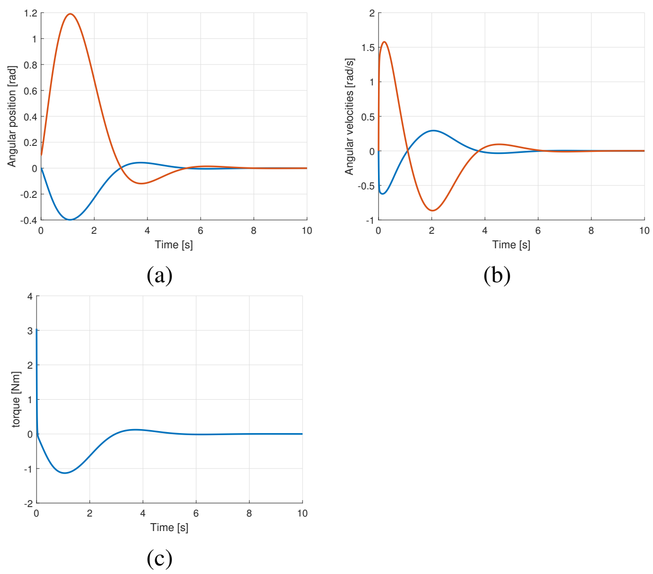

Fig. 8. Simulations: Stabilization response (a) angular positions $q_{1,2}$, (b) angular velocities $\dot{q}_{1,2}$ and (c) torque for the control approach based on the linearization in linearized coordinates and the Linear Quadratic feedback design.

B. Experiments

The feedback control method based on the linearized form of the Acrobot in the upper equilibrium with added mass according previous section was also verified in the real application of stabilization of the Acrobot featuring as the inverted double pendulum. The courses of angular positions q_1 and q_2 are depicted in Figure 9(a), (b).

Certain stabilization in the upper equilibrium was achieved[2]. Nevertheless, improvement of signals filtering and/or DC motor controller tuning is necessary before performing another experiments.

[2]Take into the account the values of unit in the y-axis

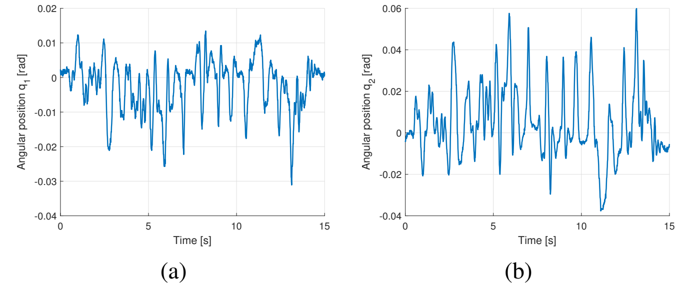

Fig. 9. Experiment: Stabilization response for angular positions (a) q_1 and (b) q_2.

V. CONCLUSIONS

Stabilization of the Acrobot with added mass in the end of the upper link in the upper equilibrium via three feedback methods was succsesfully demonstrated in simulations for the Acrobot model with real parameters as well for the real laboratory model in an real-time experiment.

Nevertheless, according to the experiment results, improvement of the DC motor controller tuning together with the signal noise filtering is necessary before ongoing experiments.

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