DATA 605 : Week 14 Discussion - Sequence & Series

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Chapter 8 (Sequence & Series) Section 8 Exercise 13

Show that the Taylor series for $f(x) = e^x$, as given in Key Idea 32, is equal to $f(x)$ by applying Theorem 77; that is show $\lim\limits_{n\to\infty} R_n(x) = 0$.

Solution

Per theorem 76, $|R_n(x)| \leq \frac{max|f^{n+1}(z)|}{(n+1)!}|x^{n+1}|$.

Derivative of e^x is e^x, so $|R_n(x)| \leq \frac{e^z}{(n+1)!}|x^{n+1}|$.

For any x, $\lim\limits_{n\to\infty} \frac{e^z x^{n+1}}{(n+1)!} = 0$. That means that $\lim\limits_{n\to\infty} R_n(x) = 0$.

Per theorem 77, $f(x) = \sum\limits_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$.

Setting $c = 0$, $f(x) = \sum\limits_{n=0}^{\infty} \frac{e^0}{n!}(x-0)^n = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!} = e^x$, per Key Idea 32.