

# DATA 605 : Week 14 Discussion - Sequence & Series

Ramnivas Singh

12/05/2021

## Chapter 8 (Sequence & Series) Section 8 Exercise 13

Show that the Taylor series for  $f(x) = e^x$ , as given in Key Idea 32, is equal to  $f(x)$  by applying Theorem 77; that is show  $\lim_{n \rightarrow \infty} R_n(x) = 0$ .

### Solution

Per theorem 76,  $|R_n(x)| \leq \frac{\max|f^{n+1}(z)|}{(n+1)!} |x^{n+1}|$ .

Derivative of  $e^x$  is  $e^x$ , so  $|R_n(x)| \leq \frac{e^z}{(n+1)!} |x^{n+1}|$ .

For any  $x$ ,  $\lim_{n \rightarrow \infty} \frac{e^z x^{n+1}}{(n+1)!} = 0$ . That means that  $\lim_{n \rightarrow \infty} R_n(x) = 0$ .

Per theorem 77,  $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n$ .

Setting  $c = 0$ ,  $f(x) = \sum_{n=0}^{\infty} \frac{e^0}{n!} (x - 0)^n = \sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ , per Key Idea 32.