DATA 605 : Assignment Week 7

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1 : Let X1, X2, . . . , Xn be n mutually independent random variables, each of which is uniformly distributed on the integers from 1 to k. Let Y denote the minimum of the Xi's. Find the distribution of Y

Answer : Suppose that each Xi has k possibilities: 1, 2, …, k. Then, the total possible number of assignments for the entire collection of random variables X1, X2, …, Xn is

$$k^n$$

This will form the denominator for our probability distribution function.

Now, the number of ways of getting Y = 1 is

$$k^n - (k-1)^n / k^n$$

, since kn represents the total number of options and (k −1)n represents all of the options where none of the Xi's are equal to 1.

When X = 1:

$$P(X=1) = k^n - (k-1)^n / k^n$$

Similarly when X = 2:

$$P(X=2) = (k-2+1)^n - (k-2)^n / k^n$$

Also when X = 3:

$$P(X=3) = (k-3+1)^n - (k-3)^n / k^n$$

Proceeding in the same manner, we see that, in general, if Y = j then there are

$$P(X=j) = (k-j+1)^n - (k-j)^n / k^n$$

ways to assign X1, …, Xn so that the minimum value is j. Therefore, we should define m(j) to be

$$(k-j+1)^n - (k-j)^n / k^n$$

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2. Your organization owns a copier (future lawyers, etc.) or MRI (future doctors). This machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years. (Include the probability statements and R Code for each part.)

a. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a geometric. (Hint: the probability is equivalent to not failing during the first 8 years..)

b. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as an exponential

c. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a binomial. (Hint: 0 success in 8 years)

d. What is the probability that the machine will fail after 8 years?. Provide also the expected value and standard deviation. Model as a Poisson

Answer :

a. Model as a geometric Machine has a manufacturer's expected lifetime of 10 years. This means that we expect one failure every ten years

P(machine will fail after 10 years) = p=1/10 P(machine will not fail after 10 years) = q =1-p

Geometric Distribution :

$$P(X = n) = (1 - p)n - 1 * p$$

```
# probability that the machine will fail after 8 year
#probability_pgeom=((1-1/10)**(8-1))*.1
#probability_pgeom

n=8
probability_pgeom  <- 1-pgeom(n-1, 0.1)
probability_pgeom
```

```
## [1] 0.4304672
```

```
# expected value (np) 1/p
expected_value <-1/0.1
expected_value
```

```
## [1] 10
```

```
# standard deviation = sqrt(1-p/p^2) or sqrt(q/p^2)

standard_deviation <-sqrt(.9/.1^2)
standard_deviation
```

```
## [1] 9.486833
```

b. Model as a exponential

$$P(X >= 8) = e^-(k/u)$$

```
# probability that the machine will fail after 8 year
k=8
u=10
probability_pexpo  <- exp(-8/10)
probability_pexpo
```

```
## [1] 0.449329
```

```
# expected value 1/lambda (where lambda= 1/p)
expected_value <-1/0.1
expected_value
```

```
## [1] 10
```

```
# standard deviation = sqrt(1/lambda^2)

standard_deviation <-sqrt((1/.1)**2) # (1/1/10)
standard_deviation
```

```
## [1] 10
```

c. Model as a binomial.

$$P(X > 8) = 1 * p^x (1 - p)^n - x$$

```
# probability that the machine will fail after 8 year
n=8
x=0
probability_binomial  <- 1*(.1)^(0)*(1-.1)^(8-0)
probability_binomial
```

```
## [1] 0.4304672
```

```
# expected value np
expected_value <-8*.1
expected_value
```

```
## [1] 0.8
```

```
# standard deviation = sqrt(n*p*q)

standard_deviation <-sqrt(8*.1*.9)
standard_deviation
```

```
## [1] 0.8485281
```

d. Model as a Poisson

$$P(X > 8) = (\lambda^x * e^-\lambda)/x!$$

```
# probability that the machine will fail after 8 year
x=0
probability_poisson  <- .8^(8)*exp(-.8/8)
probability_poisson
```

```
## [1] 0.1518065
```

```
# expected value (lambda=np/t, t=1)
expected_value <-8*.1/1
expected_value
```

```
## [1] 0.8
```

```
# standard deviation = sqrt(lambda)

standard_deviation <-sqrt(.8)
standard_deviation
```

```
## [1] 0.8944272
```