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#### **HCAP Standardization, Normalization, and Thresholding**

(With adjustment for race/ethnicity)

**Objective**. This document continues analyses of US Health and Retirement Study Harmonized Cognitive Assessment Protocol (HCAP) data.

**Sample**. The HRS/HCAP sample was stratified into a norming sample and an excluded-from-norming sample (described elsewhere). The norming sample included 1787 persons, the excluded sample 1560 persons.

Measures and procedures. Estimates of cognitive performance, including factor score estimates (plausible values and expected a posteriori scores) assessing memory, executive, language, visuospatial, and global cognitive performance (described elsewhere) were standardized and normalized with respect to the effects seen in the normative sample attributable to age, sex, education level, and race/ethnicity. The age effect was modeled using restricted cubic splines, whereas all others predictors were treated as linear predictors. (We evaluated and rejected alternative parameterizations for education, including a priori defined categorization, restricted cubic splines, and multiple linear spline functions). Adjustment for sociodemographics was accomplished with linear regression. Residuals were normalized to a T score distribution and used as a demographically-adjusted measure of cognitive performance. The resulting scores have a mean of 50 and standard deviation of 10 in a hypothetical population with distribution of similar to our norming sample. As T scores, any participant scoring below 36 has a level of performance about 1.5 standard deviations below what would be expected for someone of their .

**Results**. We find that 7 % of the norming sample is impaired on a global composite of performance on the HCAP tests (GCP[PV]), and that 30 % of those excluded from the norming sample are so impaired. Overall in the HRS/HCAP sample, 18 % are impaired on this measure of cognition. This prevalence in the norming sample is determined by where we have decided to place the threshold, and the prevalence in the non-norming sample a reflection of how well our selections out of the norming sample identify persons with cognitive impairment. If we take performance on two or more sub-domains (memory, executive functioning, language and fluency, visuospatial, orientation<sup>1</sup>) as a indicator of possible dementia, we have 7 % of the norming sample falling into this category, 26 % of those excluded from the norming sample are in this category, and 16 % are impaired on this measure of cognition in the HRS/HCAP overall. The resulting normalized and standardized scores are only correlated with to the extent that these factors are correlated with membership in the norming sample. Within the norming sample, the normalized and standardized scores are not correlated with these participant characteristics. **Conclusions**. We recommend using normalized factor scores, standardized with respect to sociodemographics, and derived from single domain models. For most applications scores derived as plausible values are preferrable to a posteriori scores.

<sup>&</sup>lt;sup>1</sup>Orientation was not standardized and normalized, due to extremely skewed distribution. We use just count of 10 orientation to time and place questions as the measure of orientation. Cut-offs for impairment are defined in the norming sample.

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## Note on language

Derived scores are Standardized, adjusted, normalized scores

**Normalized** - taking the rank-based percentile normalization transformation (Blom transformation)

of HCAP normative sample scores

Adjusted - taking the residuals from regressions of normalized scores on demographics

**Standardized** - scaling residuals according to the standard error of estimate from the regression model

I may be inconsistent but wanted to get it down one place at least.

#### 1 Source data

Making use of three data files:

Using the factor score estimates from the CFA-HCAP workflow, including both Bayesian plausible values and expected a posteriori (EAP) factor score estimates.

**Important Note:** The norming sample has been modified as provided by Ryan McCammon. It has been modified to exclude persons with a MMSE score of less than 19, or missing. An additional 0 are excluded.

Using an HRS sample data file compiled in the A1 report (February 2019)

Using a data file with HCAP participant's level of education provided by Ryan.

Using weights  $hcap2016\_weight\_20201222.dta$  Using a data file with self-reported memory worsening (PD102) from the HRS 2016 Core

## 2 Normalizing, adjusting, and standardizing cognition scores

The estimated factor scores, from the factor analysis work, are on an arbitrary scale. We desired to produce scores that adjusted for the effect of demographic variables in the norming sample and were on a more interpretable scale.

We accomplish this by placing the scores on a T-score metric. A T-score metric has a mean of 50 and standard deviation of 10. T-score metrics are often used in health research settings.

We use a regression adjustment procedure to account for the effect of demographic variables on test scores.

We use a rank-based normalizing transformation to accomplish two goals. First, the transformation limits the possibility of obtaining out-of-range values from the adjustment procedure. Second, the normalizing transformation makes it easy to identify persons falling below a fixed threshold (in our case, 1.5 SD, or a T score of 35) below the mean in the norming sample.

These are the steps in generating standardized scores:

- 1. **Raw score is rank-normalized** Each factor score estimate is subject to a rank-based normalizing transformation (Blom transformation) within the sample of persons selected for generating norms (the norming sample).
  - Model back-translation of rank-normalized and raw scores in norming sample. I regress the Blom-transformed score back on to the original factor score estimate using a restricted cubic spline with four knots. This regression model is used to generate Blom transformed factor scores in the norming sample and the non-norming sample and any future observation(s). This step is necessary because the Blom transformation is dependent upon the sample in which it is derived. By generating this regression model, I can produce Blom-transformed scores in any future sample that reflect the distribution of scores observed in the norming sample, regardless of the distribution of scores in the new sample. Using restricted cubic splines allows a flexible curve shape and typically very high predictive accuracy (e.g., for memory plausible values the model r-squared is 0.9995, both have means of 0 and standard deviations of 0.999, and the plausible value range is from -3.42 to +3.42 and the predicted range is from -3.48 to 3.30)
  - The reason for the Blom transformation is: In the next step I will be regressing (Blom-transformed) observed score performance on demographic variables. The normalizing transformation helps make sure that the predicted values from this regression model do not lead to implausible values
- 2. **Regression adjustment**. Regress each Blom-transformed factor score (separately) on age, sex, "race/ethnicity," and educational attainment.
  - **Age** is modeled as a continuous predictor using restricted cubic splines with knots at 70, 78, 86, 94 (on a range of 65-103). These knots were chosen ad hoc using an empirical process, and fall at the 25th, 60th, 88th, and 99th percentiles of hcapage.
    - The somewhat unusual choice of knot locations is driven by the cross-sectional re-

lationship between age and cognitive test score. The shape is distinctly hockey-stick-shaped relationship where a nearly linear performance-age relationship is seen through most of the age range (older people performing worse) but then the direction shifts and older people perform better. Consider the relationship for general cognition:

This effect is likely caused by the retention of only the most cognitively-intact persons among the oldest-old following our exclusions from the norming sample. The knot choice is meant to get more parameters estimated in the region where the age-performance relationship is more dynamic.

- Sex is modeled as male and female using a dummy variable
- Race and ethnicity is coarsely modeled with two dummy variables, one indicating Black or African-American, the other Hispanic ethnicity.
- Education is included as a continuous predcitor (0-17)
  - I compared different ways for controlling for education
    - \* A continuous variable (0-17)
    - \* A categorical predictor identifying the following groups defined in terms of years of completed schooling: 0 \1-8 \9-11 \12 \13-15 \16 \17 and higher.
    - \* A restricted cubic spline with 4 knots placed at default locations
    - \* A set of models including two linear splines with knots placed from 4 to 15 years
  - I regressed the estimated GCP (EAP), GCP (PV), MEM (EAP), LFL (EAP), vdori1, vdvis1, and h1rmseotal on each of the above representations of education. For all except vdori1 and h1rmseotal the model with the lowest BIC was the continuous linear function of number of years of education. Orientation favored two linear splines with a knot at 13 years of education, and the MMSE preferred the restricted cubic splines.
  - Based on the predominance of evidence, I decided to keep education as a continuous predictor.
- Main effects and two-way interactions are included. The only two-way interaction that is not included is black\*hisp, because in sample there are no persons both Black and Hispanic.
- 3. **Compute an** *expected score* for every combination of age, sex, education level, and race/ethnicity, using the results of the regression model.
- 4. **Compute an** *adjusted score* for each person as their observed score minus their expected score given age, sex, education level, and race/ethnicity.
- 5. **Compute an** *adjusted, standardized* score as their observed minus expected score, all divided by the *standard error of estimate* from the regression model, which is the overall sample standard deviation of the raw score multiplied by  $\sqrt{1-R^2}$  where  $R^2$  is the r-squared from the adjustment model in the norming sample.

- 6. **Compute a** *adjusted*, *standardized*, *and scaled* score as their *adjusted*, *standardized* score multiplied by 10, plus 50, and rounded to the nearest integer. This places the standardized score on a roughly T-score metric.
  - Rounding We round all factor scores after transformation to the nearest whole number, which provides two digits precision.
- 7. **The 7.5th percentile** for a T score is a value of 35.6. Since we are rounding to the nearest whole number, a T-score scaled factor score of 36 or higher will be considered above threshold, and a factor score of 35 or below will be considered below threshold.

<sup>&</sup>lt;sup>2</sup>This should be justified given standard error of measurement for factor scores.

# 2.1 Key to various scores and tests used in normalization and standardization

Domain	Description
Memory	Memory is a factor score estimated from delayed recall and recognition tasks of episodic memory (10 word delayed recall, 3 word delayed recall, Logical Memory II, story recall (EBMT), 10 word recognition and Logical Memory recognition).
Executive functioning	Executive functioning is a factor score estimated from attention and speed tasks, set shifting tasks, and logical reasoning tasks, including Standard Progressive Matrices, HRS number series, trail making (part A & B), Symbol Digit Modalities Test, Backwards spelling, Backwards counting, and letter cancellation.
Language, fluency	Language, fluency is a factor score estimated from animal naming, object naming (two objects from TICS), two objects from MMSE, objects from the CSI-D, sentence writing, and read and follow command.
Orientation	Orientation is not a factor score, but is the observed performance on 10 orientation to time and place items from the Mini-Mental State Examination. For ease of interpretation the observed score is placed on a T-score metric and standardized in the HCAP normative sample. No Bayesian plausible values are estimated for this score.
Visuospatial	Visuospatial is not a factor score, but is the observed performance on a constructional praxis (immediate) task. For ease of interpretation the observed score is placed on a T-score metric and standardized in the HCAP normative sample. No Bayesian plausible values are estimated for this score.
General cognitive performance	The GCP (General cognitive performance) score is a second order factor score estimate derived from a model with first-order factors for orientation, memory, executive functioning, language/fluency, and visuospatial functioning.

Source		Source	Type of
estimate	Domain	model	estimate
gmemm1	Memory	Second order	PV
memm1		Single factor	PV
gmem		Second order	EAP
mem		Single factor	EAP
gexfm1	Executive Fxn	Second order	PV
exfm1		Single factor	PV
gexf		Second order	EAP
exf		Single factor	EAP
glflm1	Language, fluency	Second order	PV
lflm1		Single factor	PV
glfl		Second order	EAP
IfI		Single factor	EAP
vdori1	Orientation	Sum of correct responses	NA
vdvis1	Visuospatial	CERAD constructional praxis	NA
gcpm1	Global	Second order	PV
gcp		Second order	EAP
h1rmsetotal		MMSE total score	NA

Notes: EAP, Expected a posteriori; NA, Not applicable; PV, Bayesian plausible value.

I will append a T to the front of a source estimate (e.g., Tgmemm1) to indicate the T-score (mean 50, sd 10) standardized and normalized estimate. These variables have been rank normalized, adjusted for demographics, and standardized on a T-score metric (in the normative sample).

I will append a IMPAIRED to the front of a source estimate (e.g., IMPAIRED\_gmemm1) to identify dummy variables that indicate if a person has scored less than 36 on the normalized, adjusted, and standardized estimate.

## 2.2 Some statistics in the normative sample

. su `Tlist´	<pre>if normexcl==0</pre>				
Variable	Obs	Mean	Std. Dev.	Min	Max
Tgmemm1	1,787	49.99236	9.943624	17.29197	82.47731
Tmemm1	1,787	49.88824	9.986352	12.20103	82.92719
Tgmem	1,787	49.81238	9.903523	14.31529	87.08157
Tmem	1,787	49.8853	9.913377	15.51777	81.9619
Tgexfm1	1,787	49.83914	9.949604	12.98327	92.3077
Texfm1	1,786	49.78591	9.903515	15.52633	83.24976
Tgexf	1,787	49.76783	9.870728	12.82936	88.95788
Texf	1,786	49.78573	9.86297	13.62295	88.55615
Tglflm1	1,787	49.65112	9.936679	13.96208	89.29607
Tlflm1	1,787	49.85247	9.972104	16.32137	81.43411
Tglfl	1,787	49.73899	9.915508	16.63568	87.59777
Tlfl	1,787	49.89143	9.922824	17.63156	80.80674
Tvdvis1	1,784	49.60828	10.02776	12.38522	82.22475
Tgcpm1	1,787	49.72299	9.915295	18.03143	82.99464
Tgcp	1,787	49.73131	9.911813	15.53	88.90277
Th1rmsetotal	1,787	49.78532	9.945405	17.58554	83.12962

## 2.3 Some statistics in both samples

	norme	excl=0	norme	excl=1
Estimate	Mean	SD	Mean	SD
Tgmemm1	50.0	(9.9)	42.6	(13.5)
Tmemm1	50.0	(10.0)	44.1	(12.4)
Tgmem	50.0	(9.8)	41.6	(14.0)
Tmem	50.0	(9.8)	43.4	(13.0)
Tgexfm1	50.0	(9.8)	41.2	(13.2)
Texfm1	50.0	(9.8)	42.2	(12.1)
Tgexf	50.0	(9.9)	40.6	(13.6)
Texf	50.0	(9.9)	41.7	(12.2)
Tglflm1	50.0	(9.8)	41.3	(14.4)
Tlflm1	50.0	(9.9)	45.5	(12.1)
Tglfl	50.0	(9.8)	39.9	(15.5)
Tlfl	50.0	(9.9)	44.2	(12.5)
Tvdvis1	50.0	(9.6)	46.1	(12.3)
Tgcpm1	50.0	(9.8)	41.3	(14.5)
Tgcp	50.0	(9.8)	39.8	(15.4)
Th1rmsetotal	50.0	(9.8)	41.6	(16.7)

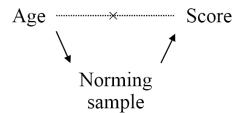
2.3.1 Impairment

Percent impaired (having normalized, adjusted, standardized score less than 36)

	normexcl=0	normexcl=1	All HCAP
Estimate	%	%	%
IMPAIRED_gmemm1	7.7	28.6	16.2
IMPAIRED_memm1	7.1	24.7	14.2
IMPAIRED_gmem	6.4	30.9	16.3
IMPAIRED_mem	7.3	27.3	15.4
IMPAIRED_gexfm1	6.1	30.0	15.8
IMPAIRED_exfm1	6.3	27.5	14.9
IMPAIRED_gexf	6.5	30.8	16.4
IMPAIRED_exf	6.8	28.3	15.5
IMPAIRED_glflm1	7.0	30.0	16.4
IMPAIRED_lflm1	8.2	21.1	13.4
IMPAIRED_glfl	7.4	32.8	17.7
IMPAIRED_1f1	8.6	22.9	14.4
IMPAIRED_vdvis1	8.0	17.7	11.9
IMPAIRED_gcpm1	6.4	30.2	16.1
IMPAIRED_gcp	7.0	33.3	17.7
IMPAIRED_h1rmsetotal	8.1	30.7	17.3

#### 2.3.2 But are scores related to age?

The normalized, adjusted, and standardized scores is – by construction – not be related to age in the normative sample. This is a consequence of the adjustment procedure. However, the normalized, adjusted and standardized scores may be related to age in the sample excluded from norming, and the HCAP overall. This is because the some correlation of age and test score is caused by the association of age and inclusion in the norming sample. The association between age and test score is indirect, not direct, as a consequence of our design.



Consider that the Spearman correlation of age (limiting to age 65-90) and inclusion in the norming sample is 0.28, and the Spearman correlation of being included in the norming sample and the GCP (plausible value) score (before adjustment, gcpm1) is -0.42, then we expect a correlation of the age (and other demographic factor-) adjusted GCP score and age to be no more than  $0.28 \times -0.42 = -0.12$ . We actually observe -0.12.

The implication to all this is, if the correlation of the normalized, adjusted, and standardized test scores with age in the overall HCAP sample is lower than what would be desired, the way to increase this is to increase the magnitude of the correlation of age and being included in the norming sample, or increase the magnitude of the correlation of being included in the norming sample and cognitive test performance.

The table below contains means across age group and Spearman rank correlations of the T score and age, but limiting to ages 65 to 90. In parenthesis, I show the Spearman correlation of the unadjusted score with age (also limiting to ages 65-90).

	normexcl=0	normexcl=1	All HCAP
Estimate	Mean SD	Mean SD	Mean SD
Tgmemm1			
1	50 (10)	45 (13)	49 (11)
2	50 (10)	45 (12)	48 (11)
3	50 (10)	43 (14)	47 (12)
4	50 (10)	39 (13)	44 (13)
5	50 (11)	40 (13)	43 (13)
6	51 (7)	40 (15)	41 (14)
7	()	44 (17)	44 (17)
	$r_s = -0.01(-0.23)$	$r_s = -0.17(-0.32)$	$r_s = -0.16(-0.35)$

Tmemm1

```
1
                50 (10)
                                  45 (12)
                                                   49 (11)
2
                50 (10)
                                  46 (11)
                                                   48
                                                       (10)
3
                50
                                  45 (11)
                    (10)
                                                   48
                                                       (11)
4
                49 (10)
                                                   45 (12)
                                  42 (12)
5
                                  42 (11)
                50 (10)
                                                   45 (11)
6
                52 (11)
                                  42 (15)
                                                   44 (15)
7
                                  47 (19)
                                                   47 (19)
                     ()
               r_s = 0.01(-0.22) r_s = -0.13(-0.30) r_s = -0.13(-0.33)
Tgmem
                50 (10)
                                  44 (13)
1
                                                   49 (11)
2
                                  44 (12)
                50 (9)
                                                   48
                                                       (11)
3
                50 (10)
                                  42 (13)
                                                   47
                                                       (12)
4
                49 (10)
                                  38 (14)
                                                   43 (14)
5
                                                   42 (14)
                51
                    (11)
                                  39 (13)
6
                51 (9)
                                  38 (16)
                                                   40 (16)
7
                     ()
                                  44 (19)
                                                   44 (19)
              r_s = 0.00(-0.26) r_s = -0.17(-0.33) r_s = -0.16(-0.37)
Tmem
1
                50
                    (10)
                                  46 (12)
                                                   49
                                                       (11)
2
                50 (10)
                                  46 (12)
                                                   48 (10)
3
                                                   48 (11)
                50 (10)
                                  44 (12)
4
                49 (10)
                                  40 (13)
                                                   44 (13)
5
                51
                                  40 (12)
                                                   43 (13)
                    (12)
6
                51 (10)
                                  41 (15)
                                                   42 (15)
7
                                  45 (19)
                                                   45 (19)
                    ()
              r_s = -0.00(-0.23) r_s = -0.17(-0.32) r_s = -0.15(-0.34)
Tgexfm1
1
                50 (10)
                                  43 (12)
                                                   49 (11)
2
                50 (10)
                                  44 (13)
                                                   48
                                                       (11)
3
                51 (10)
                                  41 (13)
                                                   47
                                                       (13)
4
                                  39 (13)
                50 (9)
                                                   44
                                                       (13)
5
                                  40 (13)
                50 (8)
                                                   43
                                                       (13)
6
                50
                    (7)
                                  37 (13)
                                                   39 (13)
                     ()
                                  38 (15)
                                                   38 (15)
              r_s = 0.02(-0.28) r_s = -0.11(-0.31) r_s = -0.14(-0.38)
Texfm1
                50 (10)
                                  44 (11)
                                                   49 (11)
1
2
                50 (10)
                                  44 (12)
                                                   48
                                                       (11)
3
                50 (10)
                                  42 (13)
                                                   47
                                                       (12)
4
                50 (9)
                                  41 (11)
                                                   45
                                                       (11)
5
                50
                    (8)
                                  41 (12)
                                                   44 (11)
```

```
44 (15)
                50 (9)
6
                                               40 (13)
7
                    ()
                                                44 (15)
              r_s = 0.00(-0.27) r_s = -0.09(-0.29) r_s = -0.13(-0.37)
Tgexf
1
                50 (10)
                                 42 (12)
                                                  48 (11)
2
                50 (10)
                                 43 (13)
                                                  48 (11)
3
                50 (10)
                                 41 (14)
                                                  46 (13)
4
                50 (9)
                                 38 (13)
                                                  43 (13)
5
                                 39 (13)
                50 (8)
                                                  42 (13)
6
                50 (8)
                                 37 (15)
                                                  39 (15)
7
                    ()
                                 41 (15)
                                                  41 (15)
              r_s = 0.01(-0.30) r_s = -0.11(-0.31) r_s = -0.15(-0.39)
Texf
1
                50 (10)
                                 43 (12)
                                                  48 (11)
2
                50 (10)
                                 44 (13)
                                                  48 (11)
3
                50 (10)
                                 41 (13)
                                                  46 (12)
4
                50 (9)
                                40 (12)
                                                  44 (12)
5
                50 (8)
                                 41 (12)
                                                  44 (11)
                                 39 (13)
6
                50 (8)
                                                40 (13)
                                 45 (11)
7
                    ()
                                                  45 (11)
              r_s = 0.01(-0.30) r_s = -0.08(-0.30) r_s = -0.13(-0.39)
Tglflm1
                50 (10)
                                 43 (13)
1
                                                  48 (11)
2
                50 (10)
                                 44 (13)
                                                  48 (11)
3
                50 (10)
                                 42 (14)
                                                  46 (13)
4
                50 (10)
                                 39 (15)
                                                  44 (14)
 5
                                 40 (15)
                51 (9)
                                                  43 (14)
6
                                 38 (17)
                49 (7)
                                                  39 (16)
                    ()
                                 41 (17)
                                                  41 (17)
              r_s = 0.03(-0.24) r_s = -0.11(-0.28) r_s = -0.12(-0.34)
Tlflm1
                50 (10)
                                 46 (12)
                                                  49 (11)
1
2
                50 (9)
                                 46 (11)
                                                  48 (10)
3
                51 (10)
                                 47 (11)
                                                  49 (11)
4
                50 (10)
                                44 (12)
                                                  46 (12)
5
                50 (9)
                                 44 (12)
                                                  46 (11)
6
                50 (9)
                                 43 (14)
                                                44 (14)
7
                                 50 (16)
                                                  50 (16)
                    ()
              r_s = 0.02(-0.10) r_s = -0.06(-0.15) r_s = -0.07(-0.18)
```

Tglfl

```
1
                50 (10)
                                  42 (14)
                                                   48 (11)
2
                50 (9)
                                  43 (14)
                                                   48
                                                       (11)
3
                50 (10)
                                  41 (15)
                                                   46 (14)
4
                                                   42 (15)
                50 (10)
                                  36 (16)
5
                                  37 (15)
                51 (9)
                                                   41 (15)
6
                50 (8)
                                  36 (18)
                                                   38 (18)
7
                                  41 (20)
                    ()
                                                   41 (20)
              r_s = 0.01(-0.28) r_s = -0.14(-0.31) r_s = -0.15(-0.38)
Tlfl
                50 (10)
1
                                  45 (12)
                                                   49 (11)
2
                50 (9)
                                  45 (12)
                                                   48
                                                       (11)
3
                50 (10)
                                  45 (12)
                                                   48
                                                       (11)
4
                50 (10)
                                 42 (12)
                                                   46 (12)
                                 43 (12)
5
                                                   45 (12)
                51 (8)
6
                50
                                  42 (14)
                    (8)
                                                   43 (13)
7
                    ()
                                  48 (15)
                                                   48 (15)
              r_s = 0.00(-0.16) r_s = -0.06(-0.19) r_s = -0.09(-0.26)
Tvdvis1
1
                50 (10)
                                  47 (11)
                                                   49
                                                       (10)
2
                                  47 (10)
                51 (9)
                                                   50 (10)
3
                                  47 (13)
                50 (10)
                                                   48 (12)
4
                49 (11)
                                 44 (12)
                                                   47 (12)
5
                                 44 (13)
                52 (10)
                                                   46 (13)
6
                48 (10)
                                 45 (14)
                                                  45 (14)
7
                                 48 (14)
                                                   48 (14)
                    ()
              r_s = 0.00(-0.09) r_s = -0.08(-0.10) r_s = -0.08(-0.15)
Tgcpm1
                50 (10)
                                  43 (13)
1
                                                   48 (11)
2
                50 (10)
                                  44 (12)
                                                   48 (11)
3
                50 (10)
                                 42 (14)
                                                   46 (13)
4
                                                   43 (14)
                50 (9)
                                  38 (15)
5
                                  39 (15)
                50 (9)
                                                   43 (14)
6
                50 (7)
                                  38 (18)
                                                   40 (17)
 7
                                  42 (21)
                                                   42 (21)
                    ()
              r_s = 0.03(-0.26) r_s = -0.12(-0.30) r_s = -0.12(-0.36)
Tgcp
                50 (10)
                                  42 (13)
                                                   48 (11)
1
2
                50 (9)
                                  43 (14)
                                                   48 (11)
3
                50 (10)
                                  40 (15)
                                                   46
                                                       (14)
4
                50
                                                   42 (15)
                    (10)
                                  36 (16)
5
                51 (9)
                                  37 (15)
                                                   41 (15)
```

Note: table means reflect weights

#### 2.3.3 But are scores related to education?

The normalized, adjusted, and standardized scores should not be related to education in the normative sample, but may be in the non-normative sample and HCAP overall. As with age discussed in the previous section, the correlation of education and test score, in the overall HCAP sample, some correlation of education and test score derives indirectly from the association of education and inclusion in the norming sample.

Consider that the Spearman correlation of education (schlyrs) and inclusion in the norming sample is -0.20, and the Spearman correlation of being included in the norming sample and the GCP (plausible value) score (before adjustment, gcpm1) is -0.44, then we expect a correlation of the education (and other demographic factor) adjusted GCP score and education to be no more than  $-0.20 \times -0.44 = 0.09$ . We actually observe 0.06.

The table below contains means across education groups and Spearman rank correlations of the T score and educational attainment. In parenthesis, I show the Spearman correlation of the unadjusted score with education.

	normexcl=	=0 norm	nexcl=1	All F	ICAP
Estimate	Mean SD	Mean	SD	Mean	SD
Tgmemm1					
schlyrs 0-8	50 (10	) 43	(12)	45	(12)
schlyrs 9-11	49 (10	) 43	(11)	45	(11)
schlyrs 12	50 (10	) 43	(13)	47	(12)
schlyrs 13-15	50 (9)	42	(14)	47	(12)
schlyrs 16	50 (10	) 41	(13)	47	(11)
schlyrs 17up	51 (10	) 41	(17)	48	(13)
	$r_s = 0.01(0$	$(42)   r_s = -$	0.04(0.29)	$r_s = 0.$	04(0.40)
Tmemm1					
schlyrs 0-8	53 (11	) 47	(14)	49	(13)
schlyrs 9-11	49 (10	•	(10)	47	(10)
schlyrs 12	50 (10	•	(11)	47	(11)
schlyrs 13-15	50 (10	•	(13)	47	(12)
schlyrs 16	50 (10	•	(13)	48	(11)
schlyrs 17up	51 (9)	43	(14)	49	(11)
	$r_s = 0.03(0$	$(35)  r_s = -$	0.07(0.24)	$r_s = 0.$	04(0.34)
Tgmem					
schlyrs 0-8	51 (10	) 44	(13)	46	(13)
schlyrs 9-11	48 ( 9)	•	(11)	45	(11)
schlyrs 12	50 (10		(13)	47	(12)
schlyrs 13-15	50 (10	, ) 41	(15)	46	(13)
schlyrs 16	50 (10	•	(13)	47	(12)
schlyrs 17up	51 (10	•	(17)	48	(13)
	$r_s = 0.02(0$	$(46)  r_s = -1$	0.07(0.31)	$r_s = 0.$	05(0.43)

```
Tmem
 schlyrs 0-8
                          (10)
                                                          48
                                                               (13)
                     52
                                       46
                                           (13)
 schlyrs 9-11
                     48
                          (9)
                                       44
                                           (11)
                                                          46
                                                               (10)
 schlyrs 12
                     50
                          (10)
                                       44
                                           (12)
                                                          47
                                                               (12)
 schlyrs 13-15
                     50
                          (10)
                                       43
                                           (14)
                                                          47
                                                               (12)
 schlyrs 16
                     50
                          (10)
                                       41
                                            (13)
                                                          47
                                                               (12)
                     51
 schlyrs 17up
                          (10)
                                       42
                                           (15)
                                                          48
                                                               (12)
                 r_s = 0.02(0.38)
                                   r_s = -0.07(0.25)
                                                       r_s = 0.03(0.36)
Tgexfm1
 schlyrs 0-8
                     48
                          (11)
                                       40
                                           (12)
                                                          43
                                                               (12)
 schlyrs 9-11
                          (11)
                                       42
                                           (11)
                                                          44
                                                               (12)
                     48
                                                          47
 schlyrs 12
                     51
                          (10)
                                       42
                                           (13)
                                                               (12)
                     49
                                       41
                                            (14)
                                                          46
                                                               (12)
 schlyrs 13-15
                          (9)
 schlyrs 16
                     50
                          (10)
                                       41
                                           (13)
                                                          47
                                                               (12)
 schlyrs 17up
                     50
                          (9)
                                       40
                                           (16)
                                                          47
                                                               (13)
                 r_s = 0.02(0.47)
                                   r_s = -0.00(0.40)
                                                       r_s = 0.08(0.47)
Texfm1
 schlyrs 0-8
                                       44
                     48
                          (10)
                                           (12)
                                                          46
                                                               (12)
 schlyrs 9-11
                     48
                          (10)
                                       42
                                           (12)
                                                          45
                                                               (11)
 schlyrs 12
                     51
                          (10)
                                       43
                                           (12)
                                                          48
                                                               (11)
 schlyrs 13-15
                     49
                          (10)
                                       42
                                           (12)
                                                          47
                                                               (11)
                                                               (11)
 schlyrs 16
                     50
                          (9)
                                       40
                                           (12)
                                                          47
 schlyrs 17up
                     50
                          (10)
                                       39
                                           (14)
                                                          47
                                                               (12)
                                   r_s = -0.06(0.40)
                 r_s = 0.02(0.43)
                                                       r_s = 0.05(0.45)
Tgexf
 schlyrs 0-8
                     48
                          (11)
                                       42
                                           (12)
                                                          44
                                                               (12)
 schlyrs 9-11
                     48
                          (10)
                                       41
                                           (12)
                                                          44
                                                               (12)
 schlyrs 12
                     51
                          (10)
                                       41
                                            (13)
                                                          47
                                                               (13)
 schlyrs 13-15
                     49
                                       40
                                           (14)
                                                          46
                          (10)
                                                               (12)
 schlyrs 16
                     50
                          (9)
                                       39
                                           (13)
                                                          47
                                                               (12)
 schlyrs 17up
                     50
                                       38
                                           (17)
                                                          47
                                                               (13)
                          (9)
                 r_s = 0.03(0.49)
                                   r_s = -0.04(0.41)
                                                       r_s = 0.07(0.48)
Texf
 schlyrs 0-8
                     48
                          (11)
                                       44
                                           (11)
                                                          45
                                                               (11)
 schlyrs 9-11
                     48
                          (10)
                                       42
                                           (11)
                                                          44
                                                               (11)
 schlyrs 12
                     51
                          (10)
                                       43
                                           (12)
                                                          47
                                                               (12)
 schlyrs 13-15
                     49
                                       41
                                                          46
                          (10)
                                            (12)
                                                               (11)
 schlyrs 16
                     50
                          (10)
                                       40
                                           (13)
                                                          47
                                                               (12)
 schlyrs 17up
                     50
                          (9)
                                       39
                                           (14)
                                                          47
                                                               (12)
                 r_s = 0.03(0.47)
                                   r_s = -0.07(0.42)
                                                      r_s = 0.05(0.47)
```

```
Tglflm1
                                                          44
 schlyrs 0-8
                    49
                          (11)
                                      41
                                           (14)
                                                               (13)
 schlyrs 9-11
                     48
                          (9)
                                      41
                                           (13)
                                                          44
                                                               (12)
                                                          47
 schlyrs 12
                    51
                          (10)
                                      42
                                           (14)
                                                               (12)
 schlyrs 13-15
                    50
                          (10)
                                      41
                                           (15)
                                                          47
                                                               (12)
 schlyrs 16
                    50
                          (10)
                                      41
                                           (14)
                                                          47
                                                               (12)
                                      39
 schlyrs 17up
                    50
                          (10)
                                           (18)
                                                          47
                                                               (13)
                 r_s = 0.03(0.45)
                                   r_s = -0.02(0.34)
                                                      r_s = 0.06(0.43)
Tlflm1
 schlyrs 0-8
                    51
                          (11)
                                      47
                                           (12)
                                                          48
                                                               (12)
 schlyrs 9-11
                    50
                          (11)
                                           (11)
                                                          48
                                                               (11)
                                      46
                                                          48
 schlyrs 12
                    50
                          (10)
                                      46
                                           (11)
                                                               (11)
                    49
                                      46
                                                          48
 schlyrs 13-15
                          (9)
                                           (13)
                                                               (11)
 schlyrs 16
                     50
                          (10)
                                      45
                                           (12)
                                                          48
                                                               (11)
 schlyrs 17up
                    51
                          (10)
                                      42
                                           (13)
                                                          48
                                                               (12)
                 r_s = 0.03(0.28)
                                   r_s = -0.06(0.20)
                                                      r_s = 0.02(0.28)
Tglfl
 schlyrs 0-8
                                      42
                                                          44
                    50
                          (11)
                                           (14)
                                                               (13)
 schlyrs 9-11
                     48
                          (9)
                                      40
                                           (13)
                                                          44
                                                               (12)
 schlyrs 12
                     51
                          (10)
                                      41
                                           (15)
                                                          46
                                                               (13)
 schlyrs 13-15
                    49
                          (10)
                                      39
                                           (17)
                                                          46
                                                               (13)
 schlyrs 16
                    50
                          (10)
                                      39
                                           (15)
                                                          46
                                                               (12)
 schlyrs 17up
                    51
                          (10)
                                      37
                                           (20)
                                                          47
                                                               (15)
                                   r_s = -0.03(0.38)
                 r_s = 0.02(0.51)
                                                      r_s = 0.07(0.48)
Tlfl
 schlyrs 0-8
                    51
                          (10)
                                      45
                                           (11)
                                                          47
                                                               (11)
 schlyrs 9-11
                     49
                          (9)
                                      45
                                           (11)
                                                          46
                                                               (11)
 schlyrs 12
                    50
                          (10)
                                      45
                                           (11)
                                                          48
                                                               (11)
 schlyrs 13-15
                    50
                                      44
                                                          48
                          (9)
                                           (14)
                                                               (12)
 schlyrs 16
                    50
                          (10)
                                      44
                                           (12)
                                                          48
                                                               (11)
 schlyrs 17up
                    51
                          (11)
                                      41
                                           (16)
                                                          48
                                                              (13)
                 r_s = 0.03(0.37)
                                   r_s = -0.02(0.30)
                                                      r_s = 0.05(0.38)
Tvdvis1
 schlyrs 0-8
                    50
                          (10)
                                      49
                                           (13)
                                                          49
                                                               (12)
 schlyrs 9-11
                    50
                          (11)
                                      44
                                           (12)
                                                          47
                                                               (12)
 schlyrs 12
                    50
                          (10)
                                      47
                                           (12)
                                                          49
                                                               (11)
 schlyrs 13-15
                    50
                          (10)
                                      46
                                                          48
                                           (12)
                                                               (11)
 schlyrs 16
                    50
                          (9)
                                      46
                                           (10)
                                                          49
                                                               (9)
 schlyrs 17up
                    50
                         (8)
                                      44
                                           (14)
                                                          49
                                                               (10)
                 r_s = 0.00(0.32)
                                   r_s = -0.01(0.30) r_s = 0.03(0.34)
```

```
Tgcpm1
 schlyrs 0-8
                    49
                         (11)
                                     43
                                         (16)
                                                        45 (15)
 schlyrs 9-11
                    48
                         (10)
                                     41
                                          (12)
                                                        44
                                                            (12)
 schlyrs 12
                    51
                                     42
                                                        47
                         (10)
                                         (14)
                                                             (12)
 schlyrs 13-15
                    50
                         (10)
                                     41
                                          (15)
                                                        47
                                                             (12)
 schlyrs 16
                    50
                                                        47
                         (9)
                                     40
                                          (13)
                                                             (12)
 schlyrs 17up
                    50
                         (10)
                                     39
                                         (18)
                                                        47
                                                             (13)
                 r_s = 0.02(0.47)
                                 r_s = -0.03(0.35) r_s = 0.06(0.45)
Tgcp
 schlyrs 0-8
                    50
                         (11)
                                     42
                                          (14)
                                                        44
                                                            (13)
 schlyrs 9-11
                         (9)
                                         (13)
                                                        43
                    48
                                     40
                                                            (12)
 schlyrs 12
                    51
                                                        46
                         (10)
                                     41
                                          (15)
                                                            (13)
 schlyrs 13-15
                    49
                         (10)
                                     39
                                          (16)
                                                        46
                                                            (13)
 schlyrs 16
                    50
                         (9)
                                     39
                                                        46
                                          (15)
                                                            (12)
 schlyrs 17up
                    51
                         (9)
                                     37
                                          (19)
                                                        47
                                                            (14)
                 r_s = 0.03(0.51) r_s = -0.04(0.38)
                                                    r_s = 0.07(0.48)
Th1rmsetotal
 schlyrs 0-8
                    47
                         (14)
                                     42
                                          (18)
                                                        44
                                                            (17)
 schlyrs 9-11
                    49
                         (11)
                                     41
                                          (14)
                                                        44
                                                            (13)
 schlyrs 12
                    51
                                     42
                                         (17)
                                                        47
                         (10)
                                                            (14)
 schlyrs 13-15
                    51
                         (10)
                                     42
                                         (18)
                                                        48
                                                            (14)
 schlyrs 16
                    51
                         (9)
                                     40
                                         (15)
                                                        47
                                                            (12)
 schlyrs 17up
                    49
                                         (18)
                                                            (13)
                        (9)
                                     40
                                                        46
                 r_s = 0.00(0.33)
                                   r_s = 0.02(0.32)
                                                    r_s = 0.07(0.37)
```

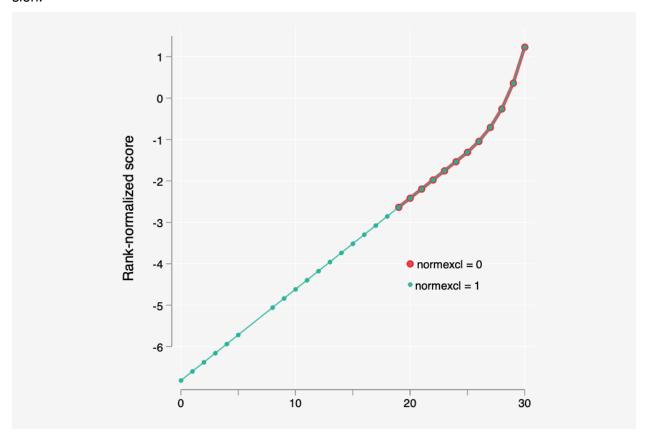
Note: Means (SD) reflect weights

#### 3 Procedure illustrated

I will illustrate the normalization-adjustment-standardization steps with pictures. For the sake of illustration, I will use the MMSE score as the test score.

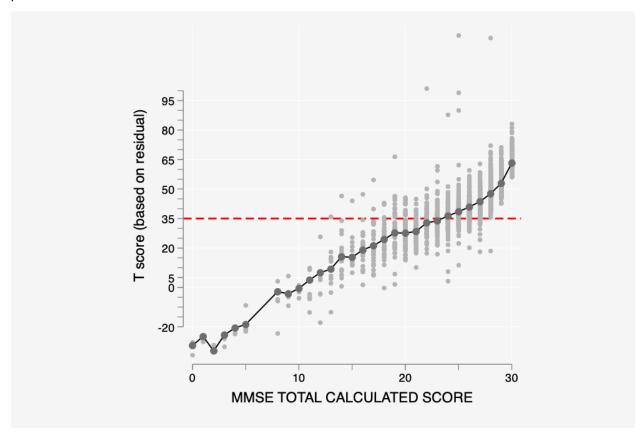
#### Rank-normalized scores vs "raw" scores

This first picture illustrates the modeled relationship between raw scores (x-axis) and rank-normalized scores (y-axis). The red dots illustrate the modeled relationship applied in the normative sample. The green dots illustrate the modeled relationship applied in the non-norm sample. Note that the relationship between the two scores is defined in the norm sample, using linear regression with restricted cubic splines based on the "raw" score. The first spline (and last) spline in a restricted cubic spline is a linear function, so the out-of-range values with respect to the norming sample (MMSE scores between 0 and 10) are related to the Blom-transformed metric using linear regression.

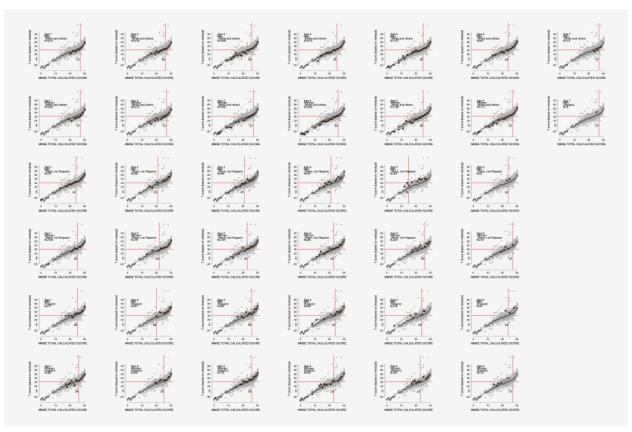


#### Adjusted and standardized rank-normalized scores vs "raw" scores

The next picture illustrates the scatter of expected T-scores for the MMSE after adjusting for demongraphics (and their two-way interactions) based on model results obtained from the normative sample. The overall mean relationship is shown with connected dots. Each dot on the plot represets a particular expected value for a given combination of MMSE total score and demographic profile.



Adjusted and standardized rank-normalized scores vs "raw" scores by demongraphic group Here is a plot that shows the raw and adjusted scores within major demongraphic groups. Also illustrated and labeled is the cut-point on the "raw" score variable that corresponds to the threshold defined on the adjusted, standardized and normalized test score. NB the effect of education is fixed at 12 years of schooling. This version is too small to get any details, but you can view a high-resolution copy at this link: https://s3.amazonaws.com/hrshcap/explanatory\_figure\_10k.png.



#### 4 How to generate scores in a new sample

- 1. Administer the HCAP according to the procedures and scoring rules used in the HRS/HCAP.
- 2. Collect sociodemographic information including age at testing, sex (only male/female), whether or not the examinee identifies as Black or African American, whether or not the examinee identifies as Hispanic, and the number of years of schooling the examinee has obtained. The collection of this information should also conform to the procedures used in HRS to maximize comparability.
- 3. Generate factor scores for processed HCAP data. I use Mplus (version 8.1, Muthen & Muthen, Los Angeles, CA) to generate these factor scores. The factor scores are either means or random draws from a posterior distribution of plausible scores given the previously estimated model in HRS/HCAP. (It may be possible to work out an open source solution to estimating these scores if Mplus is not available.) Snippets of this procedure are contained in the code prepared for the HRS/HCAP factor analysis manuscript, but are not currently ready for production. Code for this step will run from Stata, and use a wrapper function to call Mplus, and therefore will not require specialized knowledge of Mplus. Mplus is only used as a "factor scoring machine".
- 4. Normalization of factor scores. Factor scores were normalized in the norming sample using a rank-based transformation (the Blom transformation). We have linear functions based on restricted cubic splines that can accomplish the same rescaling in a new sample without making direct use of the HRS/HCAP norming sample. These transformations are available as production-ready Stata code.
- 5. Standardization of factor scores using the regression adjustment results obtained in the HRS/HCAP norming sample. This step involves subtracting a predicted performance score from an observed performance score, and scaling according to the standard error of estimate in the norming sample. This step is available as production-ready Stata code.

# 5 Appendix 1 - Number of observations by sex, age, race/ethnicity and education

#### White and others, men

Education			Age g	group at HO	CAP		
level	[65-70)	[70-75)	[75-80)	[80-85)	[85-90)	[90-95)	[95-100)
schlyrs 0-8	1	7	12	7	8	3	
schlyrs 9-11	11	8	27	13	13	3	
schlyrs 12	52	66	93	73	35	11	1
schlyrs 13-15	58	49	54	27	19	7	2
schlyrs 16	34	32	40	33	9	9	3
schlyrs 17up	42	35	47	34	25	10	1

#### White and others, women

Education			Age g	group at HO	CAP		
level	[65-70)	[70-75)	[75-80)	[80-85)	[85-90)	[90-95)	[95-100)
schlyrs 0-8	4	4	11	9	7	6	4
schlyrs 9-11	19	28	42	32	20	10	1
schlyrs 12	90	111	124	105	63	31	9
schlyrs 13-15	87	75	80	61	35	19	3
schlyrs 16	44	36	34	27	10	14	2
schlyrs 17up	58	40	38	24	21	4	2

## Black or African American, men

Education level	[65-70)		Age group [75-80)		[85-90)	[90-95)
schlyrs 0-8	6	2	3	8	4	
schlyrs 9-11	7	7	6	3	5	
schlyrs 12	18	11	14	15	1	2
schlyrs 13-15	18	4	7	2		
schlyrs 16	9	3		3	1	
schlyrs 17up	5	5		2		

#### Black or African American, women

Education			Age group at HCAP				
level	[65-70)	[70-75)	[75-80)	[80-85)	[85-90)	[90-95)	[95-100)
schlyrs 0-8	6	3	7	9	5	1	2
schlyrs 9-11	21	10	19	10	2	1	1
schlyrs 12	33	24	23	21	6	3	
schlyrs 13-15	27	19	18	16	4	1	
schlyrs 16	16	8	9	3	2	1	
schlyrs 17up	8	2	8	6	1		

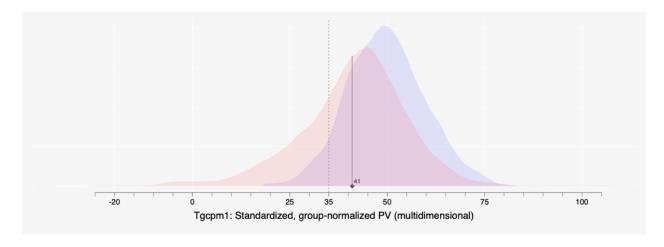
# Hispanic, men

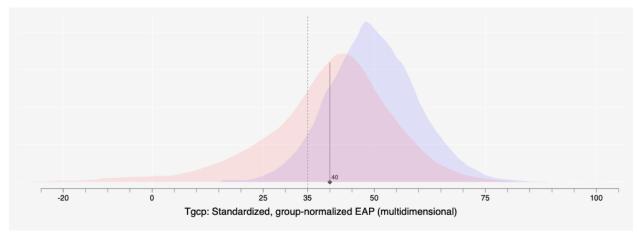
Education	Age group at HCAP						
level	[65-70)	[70-75)	[75-80)	[80-85)	[85-90)	[90-95)	[95-100)
schlyrs 0-8	19	7	16	7	4	2	1
schlyrs 9-11	4	5	4	3	2		
schlyrs 12	7	4	6	3	1	1	
schlyrs 13-15	18	3	7	1			
schlyrs 16	4	2	3	1	1		
schlyrs 17up	4		1	1			

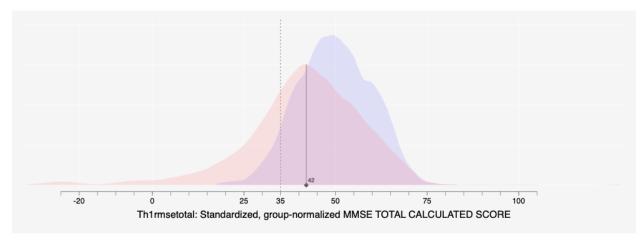
## Hispanic, women

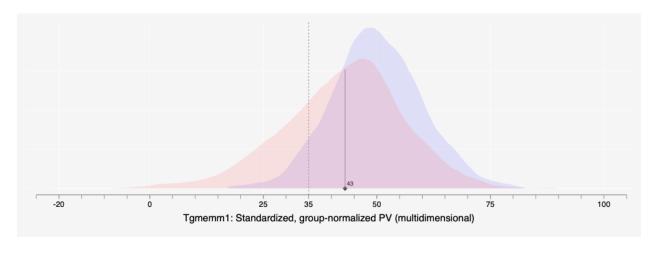
Education	Age group at HCAP						
level	[65-70)	[70-75)	[75-80)	[80-85)	[85-90)	[90-95)	[95-100)
schlyrs 0-8	23	16	34	12	9	5	1
schlyrs 9-11	13	2	13	6			
schlyrs 12	20	12	7	6	4		
schlyrs 13-15	9	8	4	3	1		
schlyrs 16	4	1		2			
schlyrs 17up	3	1	1				

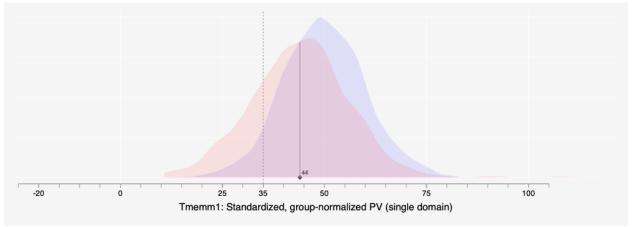
# 6 Appendix 2 - Density comparisons by norming sample inclusion

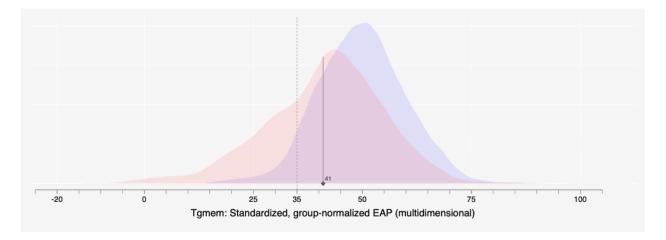


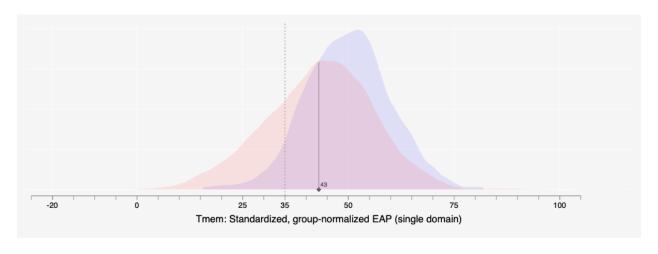


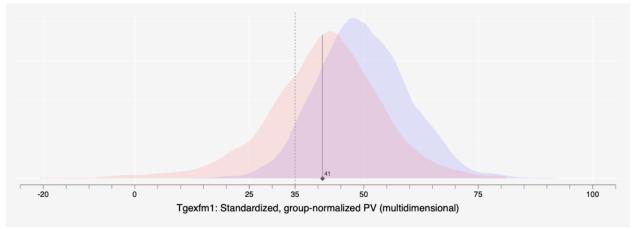


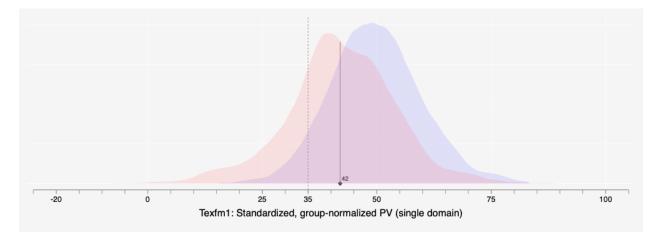


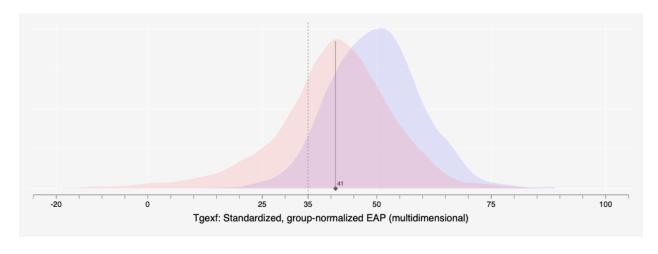


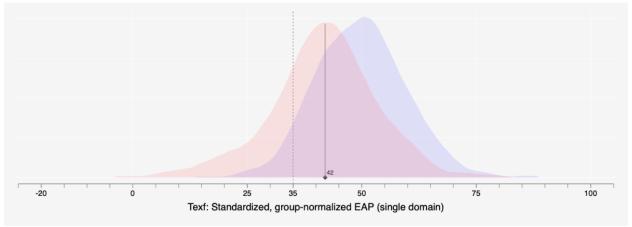


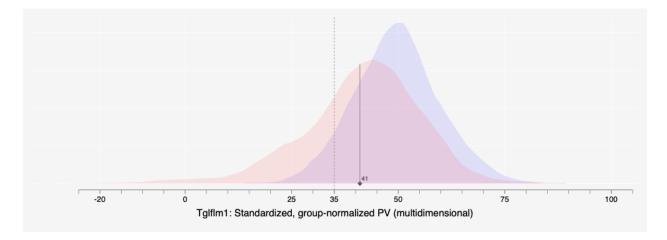


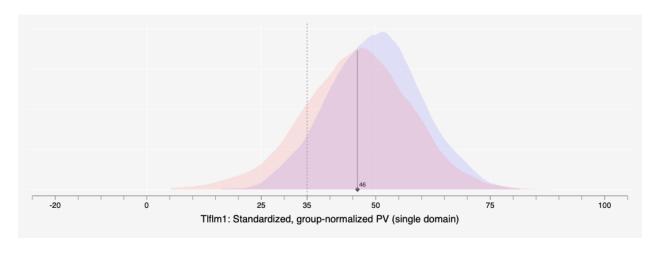


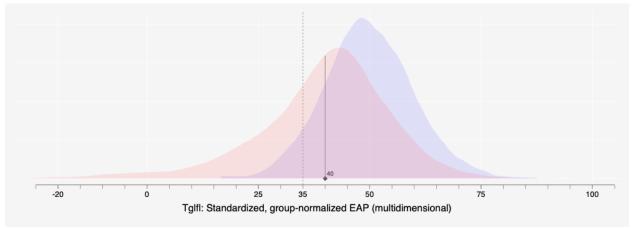


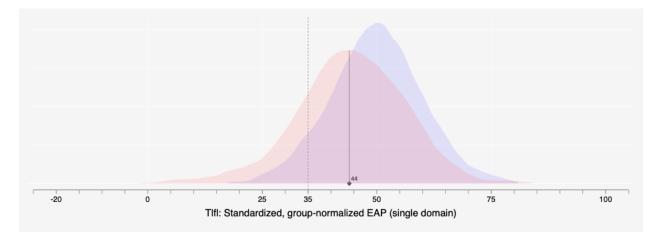


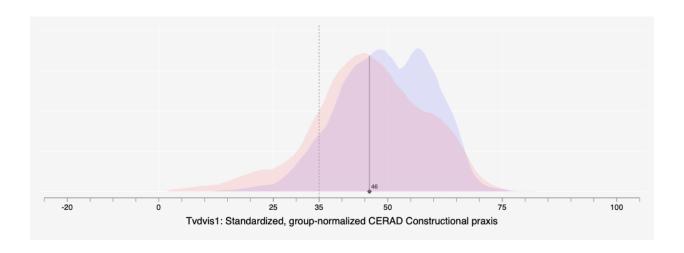












## 7 Appendix 3 - Recommended scores

I recommend to users the single domain scores. Either the expected *a posteriori* or plausible values are suitable for use.

Domain	EAP	PV
Memory	mem	memm1
	Tmem	Tmemm1
	IMPAIRED_mem	IMPAIRED_memm1
Executive	exf	exfm1
function	Texf	Texfm1
	<pre>IMPAIRED_exf</pre>	<pre>IMPAIRED_exfm1</pre>
Language and	lfl	lflm1
fluency	Tlfl	Tlflm1
	<pre>IMPAIRED_lf1</pre>	<pre>IMPAIRED_lflm1</pre>
Orientation	vdori1	NA
Visuospatial	vdvis1	NA
	Tvdvis1	NA
	<pre>IMPAIRED_vdvis1</pre>	NA
General	gcp	gvpm1
cognitive	Tgcp	Tgcpm1
performance	IMPAIRED_gcp	IMPAIRED_gcpm1

#### 7.1 When to use EAP vs PV

When we estimate a factor score, there is some level of imprecision in that estimate. The imprecision is determined by the number of items used in the factor score, the strength of the correlation between the items and the underlying factor, and the distribution of difficulty levels of the items. Factors with more items, items with strong relationships with the underlying factor, and many and widely dispersed difficulty levels will have less imprecision than factors with only a few items with weak relationships with the underlying factor and coarse and skewed responses. If a factor is measured by all continuous indicators, imprecision is constant across the level of the latent trait. But if a factor is measured with at least one categorical indicator, imprecision will vary across the level of the latent trait, generally being higher at the extremes.

When we generate factor score estimates as plausible values, each person's score is a draw from the posterior distribution of their factor score estimate, which is determined by the level of imprecision of the factor score. These are analogous to imputations in multiple imputation. In fact, it might be desireable to use multiple plausible values generated for each participant as if they were multiply imputed values in a data analysis.

If we were to take a large number of draws from the posterior for each participant, and then compute the mean of each persons' plausible values - that mean would approach the expected *a posteriori* estimate we obtain for each person.

I recommend using plausible values (or multiple plausible values) in any cirmumstance where population-level parameter estimation and inference is desired. Use of EAP estimates in such circumstances is anti-conservative and may result in biased low standard errors in inflated type-l error levels. In some situations, such as descriptive analysis<sup>3</sup>, or in a high-stakes decision making procedure (e.g., selecting participants for a module or sub-study) the EAP estimates would be preferable.

# 7.2 Why single domain scores of individual domains are preferred to scores derived from multidimensional models

Specific domain factor scores, when derived from a model that only includes more that one latent trait (e.g., a general trait and specific domains), reflect performance in general and on the specific trait. Specific domain factor scores when generated from an item set that only includes items assessing the specific domain are blind to performance on the other domains. It is hard to imagine a situation where the estimates deriving from multidomain models would be preferable to single domain models.

The main purpose of multidomain model-derived factor scores was to explore ways of harmonizing cross-national data where some of the items may differ. This is ongoing work.

#### **Demonstration**

Based on the above comments, when we examine the different kinds of scores, we should see:

- plausible values return smaller effect sizes and larger P-values than EAP scores
- Multidomain derived scores return larger effect sizes than single domain scores.

Let's use the contrast between those excluded from the norming sample to test these predictions. The table below contains z statistics (estimate divided by standard error) for the contrast of normexcld=1 vs normexcld=0 on various flavours of factor scores.

	Single	domain	Multiple domain EAP PV		
	EAP	PV	EAP	PV	
MEM	-16.9	-15.4			
EXF	-21.7	-20.2	-23.1	-21.6	
LFL	-14.6	-11.0	-22.8	-19.5	

The table demonstrates the predictions are borne out, and reinforces that **plausible value single domain scores** are the preferred scaling, being the theroretically most appropriate and empirically most conservative set of scores.

<sup>&</sup>lt;sup>3</sup>Especially describing the limits of resolution of the factor score. EAP estimates will retain floors, ceilings, and discontinuities in measurement due to coarse or sparsely distributed difficulty parameters, whereas Bayesian plausible values will invariably return smoothed and normally distributed factor score estimates. It is important to examine both to be sure that the factor has items that measure underlying ability meaningfully in the region relevant to the research question.

8	APPENDIX 4: Parameter estimates and technical results from regression models

# 8.1 gmemm1

Source	SS	df	MS	Number of obs	=	28365049
				F(24, 28365024)	>	99999.00
Model	7900290.32	24	329178.764	Prob > F	=	0.0000
Residual	19649753.5	28365024	.69274588	R-squared	=	0.2868
				Adj R-squared	=	0.2868
Total	27550043.8	28365048	.971267308	Root MSE	=	.83231

Pgmemm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	1031974	.0003102	-332.64	0.000	1038055	1025894
spage2	.4913836	.0019127	256.90	0.000	.4876348	.4951325
spage3	-1.636708	.0082767	-197.75	0.000	-1.65293	-1.620486
female	-2.063781	.0078499	-262.90	0.000	-2.079167	-2.048395
black	-3.271924	.0147776	-221.41	0.000	-3.300888	-3.242961
hisp	1849698	.0167045	-11.07	0.000	2177101	1522296
-				0.000		
schlyrs	0968879	.0014693	-65.94	0.000	0997676	0940081
c.spage1#c.female	.0298274	.0001067	279.53	0.000	.0296183	.0300366
c.spage1#c.black	.0368744	.0002015	183.04	0.000	.0364796	.0372692
c.spage1#c.hisp	.0075602	.0002359	32.05	0.000	.0070978	.0080226
c.spage1#c.schlyrs	.0034891	.000021	165.88	0.000	.0034478	.0035303
c.spage2#c.female	1889607	.0006614	-285.70	0.000	190257	1876643
c.spage2#c.black	2238609	.001198	-186.87	0.000	2262088	2215129
c.spage2#c.hisp	0758625	.0019915	-38.09	0.000	0797658	0719591
c.spage2#c.schlyrs	0268863	.0001358	-198.02	0.000	0271524	0266201
c.spage3#c.female	.7871967	.0028214	279.01	0.000	.7816669	.7927265
c.spage3#c.black	.8178906	.0044121	185.38	0.000	.8092431	.826538
c.spage3#c.hisp	1900114	.0123682	-15.36	0.000	2142525	1657703
c.spage3#c.schlyrs	.0774886	.0006127	126.47	0.000	.0762878	.0786895
c.female#c.black	.0974763	.0012324	79.09	0.000	.0950608	.0998918
c.female#c.hisp	. 1843918	.001367	134.89	0.000	.1817126	.187071
c.female#c.schlyrs	.0132103	.0001253	105.40	0.000	.0129646	.0134559
c.schlyrs#c.black	.0006465	.0002428	2.66	0.008	.0001707	.0011224
c.schlyrs#c.hisp	0585017	.0001699	-344.39	0.000	0588346	0581688
_cons	5.39931	.0217487	248.26	0.000	5.356684	5.441937

### 8.2 memm1

Source	SS	df	MS	Number of obs		
				F(24, 28365024)	>	99999.00
Model	5676277.06	24	236511.544	Prob > F	=	0.0000
Residual	22416015.3	28365024	.790269568	R-squared	=	0.2021
				Adj R-squared	=	0.2021
Total	28092292.3	28365048	.990384092	Root MSE	=	.88897

Pmemm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	0834737	.0003314	-251.92	0.000	0841231	0828242
spage2	.513895	.0020429	251.55	0.000	.509891	.5178991
spage3	-2.196252	.0088401	-248.44	0.000	-2.213578	-2.178926
female	-1.97525	.0083843	-235.59	0.000	-1.991683	-1.958817
black	-1.272782	.0157835	-80.64	0.000	-1.303717	-1.241847
hisp	1.676438	.0178416	93.96	0.000	1.641469	1.711407
schlyrs	0141698	.0015693	-9.03	0.000	0172455	011094
c.spage1#c.female	.0342602	.000114	300.61	0.000	.0340368	.0344835
c.spage1#c.black	.0184939	.0002152	85.95	0.000	.0180722	.0189156
c.spage1#c.hisp	0175313	.000252	-69.58	0.000	0180252	0170375
c.spage1#c.schlyrs	.0021513	.0000225	95.76	0.000	.0021072	.0021953
c.spage2#c.female	2078073	.0007064	-294.17	0.000	2091919	2064228
c.spage2#c.black	237975	.0012795	-185.99	0.000	2404828	2354672
c.spage2#c.hisp	.0785884	.0021271	36.95	0.000	.0744194	.0827575
c.spage2#c.schlyrs	0285424	.000145	-196.82	0.000	0288266	0282581
c.spage3#c.female	.8656184	.0030134	287.25	0.000	.8597122	.8715246
c.spage3#c.black	1.020663	.0047124	216.59	0.000	1.011427	1.0299
c.spage3#c.hisp	9765704	.0132101	-73.93	0.000	-1.002462	9506791
c.spage3#c.schlyrs	.1222347	.0006544	186.79	0.000	.1209521	.1235172
c.female#c.black	.2918041	.0013163	221.69	0.000	.2892242	.2943839
c.female#c.hisp	0517383	.00146	-35.44	0.000	0545998	0488767
c.female#c.schlyrs	0104487	.0001339	-78.05	0.000	010711	0101863
c.schlyrs#c.black	0413511	.0002593	-159.46	0.000	0418593	0408428
c.schlyrs#c.hisp	0500944	.0001814	-276.10	0.000	05045	0497388
_cons	4.061674	.0232292	174.85	0.000	4.016146	4.107202

### 8.3 gmem

Source	SS	df	MS	Number of obs		
Model	9334976.6	24	388957.358	F(24, 28365024) Prob > F	<i>&gt;</i>	
Residual	17900834.2	28365024	.631088278		=	
Total	27235810.8	28365048	.96018913		=	0.3427 .79441

Pgmem_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	120461	.0002961	-406.81	0.000	1210413	1198806
spage2	.6556234	.0018256	359.13	0.000	.6520452	.6592015
spage3	-2.493959	.0078998	-315.70	0.000	-2.509442	-2.478476
female	-1.356235	.0074925	-181.01	0.000	-1.37092	-1.34155
black	-3.259475	.0141046	-231.09	0.000	-3.28712	-3.231831
hisp	3322354	.0159438	-20.84	0.000	3634847	3009861
schlyrs	1721454	.0014024	-122.75	0.000	174894	1693968
SCHIYIS	1721434	.0014024	-122.75	0.000	174034	1093900
c.spage1#c.female	.0198536	.0001018	194.94	0.000	.019654	.0200532
c.spage1#c.black	.0382031	.0001923	198.69	0.000	.0378263	.03858
c.spage1#c.hisp	.0103286	.0002252	45.87	0.000	.0098873	.0107699
c.spage1#c.schlyrs	.0046898	.0000201	233.60	0.000	.0046504	.0047291
c.spage2#c.female	1500977	.0006313	-237.77	0.000	1513349	1488604
c.spage2#c.black	2864608	.0011434	-250.53	0.000	2887019	2842197
c.spage2#c.hisp	0455015	.0019008	-23.94	0.000	0492271	0417759
c.spage2#c.schlyrs	0404283	.0001296	-311.96	0.000	0406823	0401743
c.spage3#c.female	.6700106	.0026929	248.81	0.000	.6647326	.6752886
c.spage3#c.black	1.084879	.0042111	257.62	0.000	1.076625	1.093133
c.spage3#c.hisp	4076282	.0118049	-34.53	0.000	4307654	384491
c.spage3#c.schlyrs	.1480778	.0005848	253.22	0.000	.1469316	.1492239
c.female#c.black	.1311229	.0011763	111.47	0.000	.1288175	.1334284
c.female#c.hisp	.1070127	.0013047	82.02	0.000	.1044555	.1095699
c.female#c.schlyrs	.0205817	.0001196	172.04	0.000	.0203472	.0208162
c.schlyrs#c.black	0080748	.0002317	-34.85	0.000	008529	0076206
c.schlyrs#c.hisp	0643847	.0001621	-397.11	0.000	0647025	0640669
_cons	6.467142	.0207583	311.55	0.000	6.426457	6.507828

### 8.4 mem

Source	SS	df	MS	Number of obs		
				F(24, 28365024)	>	99999.00
Model	6739517.56	24	280813.232	Prob > F	=	0.0000
Residual	20558064.2	28365024	.724768088	R-squared	=	0.2469
				Adj R-squared	=	0.2469
Total	27297581.8	28365048	.962366846	Root MSE	=	.85133

Pmem_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	1199879	.0003173	-378.12	0.000	1206099	119366
spage2	.6391496	.0019564	326.69	0.000	.6353151	.6429841
spage3	-2.40761	.0084658	-284.39	0.000	-2.424203	-2.391018
female	-1.155198	.0080293	-143.87	0.000	-1.170935	-1.139461
black	-2.540253	.0151153	-168.06	0.000	-2.569878	-2.510627
hisp	2371293	.0170862	-13.88	0.000	2706177	2036408
schlyrs	2462246	.0015029	-163.84	0.000	2491702	2432791
c.spage1#c.female	.0166599	.0001091	152.64	0.000	.016446	.0168738
c.spage1#c.black	.0322208	.0002061	156.37	0.000	.031817	.0326247
c.spage1#c.hisp	.0090163	.0002413	37.37	0.000	.0085433	.0094892
c.spage1#c.schlyrs	.0053353	.0000215	247.98	0.000	.0052931	.0053775
c.spage2#c.female	1310511	.0006765	-193.71	0.000	1323771	1297252
c.spage2#c.black	3113285	.0012253	-254.07	0.000	3137302	3089269
c.spage2#c.hisp	0331562	.002037	-16.28	0.000	0371488	0291637
c.spage2#c.schlyrs	0396184	.0001389	-285.27	0.000	0398906	0393462
c.spage3#c.female	.6139799	.0028858	212.76	0.000	.6083237	.619636
c.spage3#c.black	1.20124	.0045129	266.18	0.000	1.192395	1.210085
c.spage3#c.hisp	5742524	.0126508	-45.39	0.000	5990475	5494573
c.spage3#c.schlyrs	.1423717	.0006267	227.18	0.000	.1411434	.1435999
c.female#c.black	.1764663	.0012606	139.99	0.000	.1739957	.178937
c.female#c.hisp	.1350902	.0013982	96.62	0.000	.1323497	.1378306
c.female#c.schlyrs	.0259471	.0001282	202.39	0.000	.0256959	.0261984
c.schlyrs#c.black	0173489	.0002483	-69.86	0.000	0178356	0168621
c.schlyrs#c.hisp	0605713	.0001738	-348.61	0.000	0609118	0602307
_cons	6.720366	.0222457	302.10	0.000	6.676765	6.763967

# 8.5 gexfm1

Source	SS	df	MS	Number of obs		28365049
				F(24, 28365024)	>	99999.00
Model	11002479.1	24	458436.63	Prob > F	=	0.0000
Residual	16212210.8	28365024	.571556393	R-squared	=	0.4043
				Adj R-squared	=	0.4043
Total	27214689.9	28365048	.959444523	Root MSE	=	.75601

Pgexfm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	0408112	.0002818	-144.82	0.000	0413635	0402588
spage2	.0982091	.0017374	56.53	0.000	.0948039	.1016142
spage3	18457	.0075179	-24.55	0.000	1993049	1698351
female	-1.533899	.0071303	-215.12	0.000	-1.547874	-1.519923
black	-4.753945	.0134229	-354.17	0.000	-4.780253	-4.727636
hisp	.5797154	.0151732	38.21	0.000	.5499765	.6094543
schlyrs	.3430804	.0013346	257.07	0.000	.3404646	.3456961
c.spage1#c.female	.0270902	.0000969	279.50	0.000	.0269002	.0272801
c.spage1#c.black	.0477834	.000183	261.14	0.000	.0474248	.0481421
c.spage1#c.hisp	0072419	.0002143	-33.80	0.000	0076619	0068219
c.spage1#c.schlyrs	0026127	.0000191	-136.75	0.000	0026501	0025752
c.spage2#c.female	1521661	.0006008	-253.28	0.000	1533436	1509886
c.spage2#c.black	2135324	.0010881	-196.23	0.000	2156652	2113997
c.spage2#c.hisp	.0280186	.001809	15.49	0.000	.0244731	.0315642
c.spage2#c.schlyrs	.002593	.0001233	21.02	0.000	.0023513	.0028347
c.spage3#c.female	.5669732	.0025627	221.24	0.000	.5619503	.571996
c.spage3#c.black	.6040192	.0040076	150.72	0.000	.5961644	.6118739
c.spage3#c.hisp	2883583	.0112343	-25.67	0.000	3103772	2663394
c.spage3#c.schlyrs	0198882	.0005565	-35.74	0.000	0209789	0187974
c.female#c.black	.0471912	.0011194	42.16	0.000	.0449972	.0493853
c.female#c.hisp	0073411	.0012417	-5.91	0.000	0097747	0049075
c.female#c.schlyrs	0126809	.0001138	-111.38	0.000	012904	0124577
c.schlyrs#c.black	.0361442	.0002205	163.89	0.000	.035712	.0365764
c.schlyrs#c.hisp	0426134	.0001543	-276.18	0.000	0429159	042311
_cons	.9839422	.019755	49.81	0.000	.9452232	1.022661

### 8.6 exfm1

Source	SS	df	MS	Number of obs		
				F(24, 28351355)	>	99999.00
Model	9502663	24	395944.292	Prob > F	=	0.0000
Residual	17526847.4	28351355	.618201403	R-squared	=	0.3516
				Adj R-squared	=	0.3516
Total	27029510.4	28351379	.953375511	Root MSE	=	.78626

1						
Pexfm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	0505826	.000294	-172.05	0.000	0511588	0500063
spage2	.1551794	.0018134	85.57	0.000	.1516252	.1587336
spage3	663201	.0078396	-84.60	0.000	6785664	6478356
female	7804782	.0074163	-105.24	0.000	7950139	7659426
black	-4.502196	.0139609	-322.49	0.000	-4.529559	-4.474833
hisp	1.853086	.0157842	117.40	0.000	1.82215	1.884023
schlyrs	.2126255	.0013919	152.76	0.000	.2098974	.2153537
c.spage1#c.female	.0171473	.0001008	170.04	0.000	.0169496	.0173449
c.spage1#c.black	.0414537	.0001908	217.27	0.000	.0410798	.0418277
c.spage1#c.hisp	0244312	.0002229	-109.60	0.000	0248681	0239942
c.spage1#c.schlyrs	0008588	.0000199	-43.09	0.000	0008978	0008197
c.spage2#c.female	0668816	.0006251	-106.99	0.000	0681068	0656564
c.spage2#c.black	2012664	.0011351	-177.31	0.000	2034913	1990416
c.spage2#c.hisp	.1098918	.0018815	58.41	0.000	.106204	.1135795
c.spage2#c.schlyrs	0080828	.0001287	-62.81	0.000	008335	0078306
c.spage3#c.female	.0871636	.002666	32.69	0.000	.0819383	.0923889
c.spage3#c.black	.7395861	.0041758	177.11	0.000	.7314017	.7477706
c.spage3#c.hisp	7150899	.0116841	-61.20	0.000	7379902	6921895
c.spage3#c.schlyrs	.0495614	.0005801	85.43	0.000	.0484244	.0506985
c.female#c.black	.1537353	.0011687	131.55	0.000	.1514448	.1560259
c.female#c.hisp	0799054	.0012916	-61.87	0.000	0824369	077374
c.female#c.schlyrs	0212018	.0001186	-178.76	0.000	0214343	0209694
c.schlyrs#c.black	.0473748	.0002347	201.84	0.000	.0469148	.0478349
c.schlyrs#c.hisp	0484552	.0001605	-301.96	0.000	0487698	0481407
_cons	1.797081	.020605	87.22	0.000	1.756696	1.837466

8.7 gexf

Source	SS	df	MS	Number of obs		28365049
			400044 000	F(24, 28365024)		99999.00
Model	11976278.9	24	499011.623	Prob > F	=	0.0000
Residual	15592795.4	28365024	.549719097	R-squared	=	0.4344
				Adj R-squared	=	0.4344
Total	27569074.3	28365048	.971938222	Root MSE	=	.74143

Pgexf_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	0625662	.0002764	-226.39	0.000	0631078	0620245
spage2	.241432	.0017039	141.70	0.000	.2380926	.2447715
spage3	-1.049644	.0073729	-142.36	0.000	-1.064095	-1.035194
female	-1.065188	.0069928	-152.33	0.000	-1.078894	-1.051482
black	-5.251982	.013164	-398.97	0.000	-5.277783	-5.226181
			-1.41	0.159	0501444	.0081861
hisp	0209791	.0148805				
schlyrs	.2292907	.0013089	175.18	0.000	.2267254	.231856
c.spage1#c.female	.0201808	.0000951	212.31	0.000	.0199945	.0203671
c.spage1#c.black	.0554846	.0001795	309.19	0.000	.0551329	.0558363
c.spage1#c.hisp	.0008509	.0002101	4.05	0.000	.000439	.0012628
c.spage1#c.schlyrs	0008446	.0000187	-45.07	0.000	0008813	0008079
c.spage2#c.female	1126206	.0005892	-191.15	0.000	1137754	1114659
c.spage2#c.black	2646592	.0010672	-248.00	0.000	2667508	2625676
c.spage2#c.hisp	007068	.0017741	-3.98	0.000	0105451	0035909
c.spage2#c.schlyrs	0103863	.000121	-85.87	0.000	0106234	0101492
c.spage3#c.female	.4055802	.0025133	161.37	0.000	.4006542	.4105062
c.spage3#c.black	.9044079	.0039303	230.11	0.000	.8967047	.9121112
c.spage3#c.hisp	2429699	.0110176	-22.05	0.000	264564	2213757
c.spage3#c.schlyrs	.0546653	.0005458	100.16	0.000	.0535956	.055735
c.female#c.black	.0949485	.0010978	86.49	0.000	.0927968	.0971002
c.female#c.hisp	0454941	.0012177	-37.36	0.000	0478808	0431074
c.female#c.schlyrs	0145117	.0001117	-129.97	0.000	0147305	0142929
c.schlyrs#c.black	.0293755	.0002163	135.82	0.000	.0289516	.0297994
c.schlyrs#c.hisp	0419418	.0001513	-277.17	0.000	0422384	0416452
_cons	2.421745	.0193739	125.00	0.000	2.383773	2.459717

8.8 exf

Source		SS	df		MS	Number of			51380
Model	1119	31050.2	24	46587	7.092	F(24, 283 Prob > F	=		99.00 0000
Residual			351355		31827	R-squared			4062
	100			.0700		Adj R-squ			4061
Total	2752	29343.8 283	51379	.9710	05461	Root MSE	=		75936
Pexf_	blom	Coef.	Std	. Err.	t	P> t	[95%	Conf.	Interval]
sr	age1	0501831	.00	02839	-176.73	3 0.000	0507	7396	0496266
sr	age2	.1421182	.00	17514	81.19	0.000	.1386	8856	.1455508
-	age3	7371062	.00	75715	-97.3	5 0.000	7519	9459	7222664
-	emale	9092469	.00	71626	-126.94	4 0.000	9232	2854	8952085
ŀ	lack	-5.539214		34834	-410.83		-5.565		-5.512787
_	hisp	0012951		52443	-0.08		0311		.0285832
act	lyrs	.2856882		13443	212.5		.2830		.288323
BCI	iryrs	. 2000002	00	10440	212.02	2 0.000	.2000	,000	. 200323
c.spage1#c.fe	emale	.0188531	.00	00974	193.58	0.000	.0186	622	.019044
c.spage1#c.h	olack	.0574992	2 .00	01843	312.0	5 0.000	.057	7138	.0578603
c.spage1#c.	hisp	0011769	.00	02153	-5.4	7 0.000	0015	5989	000755
c.spage1#c.sch	nlyrs	0018081	.00	00192	-93.93	3 0.000	0018	3458	0017704
c.spage2#c.fe	emale	0952632	2 .00	06037	-157.79	9 0.000	0964	1465	0940799
c.spage2#c.l	olack	2767469	.00	10963	-252.43	3 0.000	2788	3956	2745981
c.spage2#c.	hisp	0000458	.00	18172	-0.03	3 0.980	0036	3074	.0035158
c.spage2#c.sch	nlyrs	0035431	.00	01243	-28.5	1 0.000	0037	7867	0032995
c.spage3#c.fe	emale	.3051345	.00	25748	118.5	0.000	.300	8800	.3101811
c.spage3#c.l	olack	.9826382	2 .0	04033	243.6	5 0.000	.9747	7337	.9905427
c.spage3#c.	hisp	1848035	.01	12844	-16.38	0.000	2069	9206	1626865
c.spage3#c.sch	llyrs	.0363499	.00	05603	64.88	0.000	.0352	2517	.0374481
c.female#c.h	olack	.1095416	.00	11287	97.0	5 0.000	.1073	3294	.1117538
c.female#c.	hisp	0733102	2 .00	12474	-58.7	7 0.000	075	755	0708653
c.female#c.sch	llyrs	0205477	.00	01145	-179.38	0.000	0207	7722	0203231
c.schlyrs#c.t	olack	.0401819	.00	02267	177.26	0.000	.0397	7376	.0406262
c.schlyrs#c.	hisp	0323893	.0	00155	-208.99	9 0.000	0326	8931	0320856
-	cons	1.714832	.01	99002	86.1	7 0.000	1.675	828	1.753835

# 8.9 glflm1

Source	SS	df	MS	Number of obs		28365049
				F(24, 28365024)	>	99999.00
Model	8846983.63	24	368624.318	Prob > F	=	0.0000
Residual	18254935.4	28365024	.643572005	R-squared	=	0.3264
				Adj R-squared	=	0.3264
Total	27101919	28365048	.955468822	Root MSE	=	.80223

Pglflm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	1054098	.000299	-352.51	0.000	1059959	1048238
spage2	.4040249	.0018436	219.15	0.000	.4004116	.4076382
spage3	-1.316438	.0079775	-165.02	0.000	-1.332073	-1.300802
female	-1.113778	.0075662	-147.20	0.000	-1.128608	-1.098949
black	-4.868154	.0142434	-341.78	0.000	-4.896071	-4.840238
hisp	2.164125	.0161007	134.41	0.000	2.132569	2.195682
schlyrs	0879259	.0014162	-62.09	0.000	0907015	0851502
c.spage1#c.female	.0221873	.0001028	215.73	0.000	.0219857	.0223889
c.spage1#c.black	.0559207	.0001942	288.00	0.000	.0555401	.0563012
c.spage1#c.hisp	02393	.0002274	-105.24	0.000	0243757	0234843
c.spage1#c.schlyrs	.0035789	.0000203	176.53	0.000	.0035392	.0036187
c.spage2#c.female	1234317	.0006375	-193.62	0.000	1246812	1221822
c.spage2#c.black	3053356	.0011547	-264.44	0.000	3075987	3030725
c.spage2#c.hisp	.1301727	.0019196	67.81	0.000	.1264105	.133935
c.spage2#c.schlyrs	0237378	.0001309	-181.39	0.000	0239943	0234813
c.spage3#c.female	.544327	.0027194	200.16	0.000	.5389971	.5496569
c.spage3#c.black	1.032707	.0042526	242.84	0.000	1.024372	1.041042
c.spage3#c.hisp	7835758	.0119211	-65.73	0.000	8069408	7602109
c.spage3#c.schlyrs	.0699214	.0005905	118.40	0.000	.0687639	.0710788
c.female#c.black	.0163297	.0011879	13.75	0.000	.0140016	.0186579
c.female#c.hisp	2267412	.0013176	-172.09	0.000	2293236	2241589
c.female#c.schlyrs	0153869	.0001208	-127.37	0.000	0156236	0151501
c.schlyrs#c.black	.0165878	.000234	70.88	0.000	.0161292	.0170465
c.schlyrs#c.hisp	0502598	.0001637	-306.97	0.000	0505807	0499389
_cons	5.389931	.0209626	257.12	0.000	5.348845	5.431017

### 8.10 lflm1

Source		SS		df	MS		Number of obs		2836	
Model Residual		3487.07 78493.7	28365		134020.295 .873558002		F(24, 2836502 Prob > F R-squared	4) > = =	0.0	0000
 Total	2799	94980.8	28365	048	.986953407		Adj R-squared Root MSE	=		1149 3464
Plflm1_	blom		Coef.	Std.	Err.	t	P> t	[95%	Conf.	Inte

	Γ					
Plflm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	107895	.0003484	-309.70	0.000	1085778	1072122
spage2	.5083149	.0021479	236.66	0.000	.5041051	.5125246
spage3	-2.068015	.0092943	-222.50	0.000	-2.086231	-2.049798
female	-1.548043	.0088151	-175.61	0.000	-1.56532	-1.530766
black	-1.709069	.0165944	-102.99	0.000	-1.741593	-1.676544
hisp	1.413332	.0187583	75.34	0.000	1.376567	1.450098
schlyrs	2888694	.0016499	-175.08	0.000	2921032	2856356
Builyis	.2000034	.0010433	170.00	0.000	.2021002	.2000000
c.spage1#c.female	.0218721	.0001198	182.54	0.000	.0216373	.022107
c.spage1#c.black	.0233829	.0002262	103.36	0.000	.0229395	.0238263
c.spage1#c.hisp	0046855	.0002649	-17.69	0.000	0052048	0041663
c.spage1#c.schlyrs	.0055924	.0000236	236.77	0.000	.0055461	.0056387
c.spage2#c.female	1456416	.0007427	-196.09	0.000	1470973	1441859
c.spage2#c.black	1385531	.0013453	-102.99	0.000	1411897	1359164
c.spage2#c.hisp	0196856	.0022364	-8.80	0.000	0240688	0153024
c.spage2#c.schlyrs	0296052	.0001525	-194.17	0.000	029904	0293063
c.spage3#c.female	.6477903	.0031682	204.46	0.000	.6415806	.6539999
c.spage3#c.black	.4989451	.0049545	100.71	0.000	.4892345	.5086558
c.spage3#c.hisp	.0325743	.0138888	2.35	0.019	.0053528	.0597958
c.spage3#c.schlyrs	.1154655	.000688	167.82	0.000	.114117	.116814
c.female#c.black	1941089	.0013839	-140.26	0.000	1968213	1913964
c.female#c.hisp	.1401454	.001535	91.30	0.000	.1371368	. 143154
c.female#c.schlyrs	.0132834	.0001407	94.38	0.000	.0130075	.0135593
c.schlyrs#c.black	0171989	.0002726	-63.08	0.000	0177332	0166645
c.schlyrs#c.hisp	0991172	.0001908	-519.61	0.000	0994911	0987434
_cons	6.226239	.0244226	254.94	0.000	6.178372	6.274107

8.11 glfl

c.schlyrs#c.hisp

\_cons

-.0634951

5.867085

Source		SS	df		MS		Number of o			
Model	1165	55174.7	24	48563	32.277		Prob > F	=		0000
Residual	15	5459442 283	65024	.5450	17764		R-squared	-	0.	4298
							Adj R-squar			4298
Total	2711	14616.6 283	65048	.9559	16472	1	Root MSE	-	. 7	3825
Pglfl_	blom	Coef.	Sto	l. Err.	-	t	P> t	[95%	Conf.	Interval]
sp	age1	1155555	.00	002752	-419.	93	0.000	116	0949	1150162
sp	age2	.6130749		16965	361.		0.000		7497	.6164001
	age3	-2.306798		73413	-314.		0.000	-2.32		-2.292409
	emale	-1.387136		069628	-199.		0.000	-1.40		-1.37349
ł	olack	-4.275594		131076	-326.		0.000	-4.30		-4.249904
1	hisp	5032181		148167	-33.		0.000	532		4741779
scr	ılyrs	0873129	.00	013032	-67.	00	0.000	089	8672	0847586
c.spage1#c.fe	emale	.0226533	.00	000946	239.	35	0.000	.022	4678	.0228388
c.spage1#c.h	olack	.0480693	.00	001787	269.	02	0.000	.047	7191	.0484195
c.spage1#c.	hisp	.0127669	.00	002092	61.	01	0.000	.012	3568	.0131771
c.spage1#c.sch	nlyrs	.0038681	.00	000187	207.	33	0.000	.003	8316	.0039047
c.spage2#c.fe	emale	1620402	.00	05867	-276.	21	0.000	1	6319	1608903
c.spage2#c.h	olack	2528096	.00	10626	-237.	92	0.000	254	8922	250727
c.spage2#c.	hisp	0745357	.00	17665	-42.	19	0.000	077	9979	0710735
c.spage2#c.sch	llyrs	0374724	.00	001204	-311.	15	0.000	037	7085	0372364
c.spage3#c.fe	emale	.6777792	.00	25025	270.	84	0.000	.672	8743	.6826841
c.spage3#c.h	olack	.8769273	.00	39135	224.	80	0.000	.86	9257	.8845975
c.spage3#c.	hisp	1366647	.01	109704	-12.	46	0.000	158	1663	1151631
c.spage3#c.sch	lyrs	.137413	.00	005434	252.	85	0.000	.136	3478	.1384781
c.female#c.b	olack	.0224229	.00	10931	20.	51	0.000	.020	2804	.0245653
c.female#c.	hisp	.0368015	.00	12125	30.	35	0.000	.034	4251	.0391779
c.female#c.sch	nlyrs	.0017613	.00	001112	15.	84	0.000	.001	5434	.0019792
c.schlyrs#c.h	olack	.0052001	.00	002154	24.	15	0.000	.004	7781	.0056222

.0001507 -421.41 0.000

304.14 0.000

.0192909

-.0637904

5.829275

-.0631998

5.904894

8.12 IfI

Source		SS	df	MS	Number of		65049
Model	5784	1253.24	24 2410	010.552	F(24, 283 Prob > F	•	99.00
Residual		29452.3 2836		590475	R-squared	i = 0	.2095
Total	2761	13705.5 2836	5048 .97	351168	Adj R-sqı Root MSE		.2095 87726
Plfl_	blom	Coef.	Std. Err	·. t	t P> t	[95% Conf	. Interval]
sp	age1	1536011	.000327	′ -469.7	74 0.000	154242	1529602
	age2	.8223383	.002016			.818387	.8262896
-	age3	-2.906114	.0087237			-2.923212	-2.889016
	male lack	9505854 -2.684374	.0082739			9668019 -2.714902	9343689 -2.653847
	hisp	-1.504869	.0135750			-1.539377	-1.47036
	lyrs	4848041	.0015486			4878394	4817688
c.spage1#c.fe	male	.0171649	.0001125	152.6	0.000	.0169445	.0173853
c.spage1#c.b	lack	.0325793	.0002123	153.4	14 0.000	.0321631	.0329954
c.spage1#c.	hisp	.0340818	.0002486	137.0	0.000	.0335944	.0345691
c.spage1#c.sch	lyrs	.0090404	.0000222	407.7	77 0.000	.0089969	.0090838
c.spage2#c.fe	male	1461474	.0006971	-209.6	0.000	1475137	1447811
c.spage2#c.b	lack	1415904	.0012627	-112.1	0.000	1440652	1391156
c.spage2#c.	hisp	2108412	.0020991	-100.4	0.000	2149554	2067271
c.spage2#c.sch	lyrs	0573263	.0001431	-400.5	0.000	0576068	0570458
c.spage3#c.fe	male	.6072431	.0029737	204.2	20 0.000	.6014146	.6130715
c.spage3#c.b	lack	.3770192	.0046503	81.0	0.000	.3679047	.3861337
c.spage3#c.	hisp	.5248574	.0130361	40.2	26 0.000	.4993071	.5504077
c.spage3#c.sch	lyrs	.1953794	.0006458	302.5	0.000	.1941137	.1966451
c.female#c.b	lack	1639605	.001299	-126.2	0.000	1665064	1614146
c.female#c.	hisp	0069198	.0014408	-4.8	0.000	0097437	0040959
c.female#c.sch	lyrs	0051282	.0001321	-38.8	0.000	0053872	0048693
c.schlyrs#c.b	lack	0051796	.0002559	-20.2	0.000	0056812	0046781
c.schlyrs#c.	hisp	0800688	.000179	-447.2	20 0.000	0804197	0797178
	cons	8.883129	.0229233	387.5	0.000	8.838201	8.928058

8.13 vdvis1

	Source		SS		df	MS		Number of obs		2834	
	Model Residual		3355.42 13071.8	28345		150556.476 .635476045		F(24, 2834579 Prob > F R-squared	2) > = =	0.0	9.00 0000 1671
_	Total		26427.2			.762949537		Adj R-squared Root MSE		0.	1671 9717
_	Pvdvis1_	blom	C	oef.	Std.	Err.	t	P> t	[95%	Conf.	Inte

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Pvdvis1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	040513	.0002972	-136.32	0.000	0410955	0399305
spage2	.2739882	.0018325	149.52	0.000	.2703966	.2775798
spage3	7356267	.0079297	-92.77	0.000	7511686	7200849
female	920054	.0075208	-122.33	0.000	9347945	9053135
black	-1.261813	.0141787	-88.99	0.000	-1.289603	-1.234023
hisp	.4425547	.0159993	27.66	0.000	.4111966	.4739128
schlyrs	0204586	.0014076	-14.53	0.000	0232175	0176997
schiyis	0204566	.0014076	-14.55	0.000	0232175	0176997
c.spage1#c.female	.0095893	.0001022	93.80	0.000	.0093889	.0097896
c.spage1#c.black	.0056738	.0001934	29.33	0.000	.0052947	.0060529
c.spage1#c.hisp	0089268	.0002259	-39.51	0.000	0093696	0084839
c.spage1#c.schlyrs	.0017165	.0000202	85.18	0.000	.001677	.001756
c.spage2#c.female	0676531	.0006337	-106.76	0.000	0688951	0664111
c.spage2#c.black	.1099702	.0011491	95.70	0.000	.1077181	.1122224
c.spage2#c.hisp	.1150039	.0019075	60.29	0.000	.1112653	.1187425
c.spage2#c.schlyrs	0182956	.0001301	-140.64	0.000	0185506	0180406
c.spage3#c.female	.1014087	.0027032	37.51	0.000	.0961106	.1067067
c.spage3#c.black	6210063	.0042298	-146.82	0.000	6292965	6127161
c.spage3#c.hisp	7470169	.0118461	-63.06	0.000	7702347	723799
c.spage3#c.schlyrs	.0514886	.000587	87.71	0.000	.050338	.0526391
c.female#c.black	0254295	.0011811	-21.53	0.000	0277444	0231145
c.female#c.hisp	.0155488	.0013093	11.88	0.000	.0129827	.0181149
c.female#c.schlyrs	.0126261	.0001201	105.15	0.000	.0123907	.0128614
c.schlyrs#c.black	.0228329	.0002327	98.12	0.000	.0223768	.0232889
c.schlyrs#c.hisp	.0109985	.0001627	67.60	0.000	.0106796	.0113174
_cons	1.694443	.0208349	81.33	0.000	1.653607	1.735278

8.14 gcpm1

Source	SS	df	MS	Number of obs		28365049
				F(24, 28365024)	>	99999.00
Model	9990264.99	24	416261.041	Prob > F	=	0.0000
Residual	17317855.5	28365024	.610535547	R-squared	=	0.3658
				Adj R-squared	=	0.3658
Total	27308120.4	28365048	.962738383	Root MSE	=	.78137

Pgcpm1_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
		0000010				
spage1	0810651	.0002912	-278.34	0.000	081636	0804943
spage2	.2892764	.0017956	161.10	0.000	. 285757	.2927957
spage3	8936572	.0077701	-115.01	0.000	9088863	8784282
female	4334176	.0073694	-58.81	0.000	4478614	4189738
black	-4.378705	.013873	-315.63	0.000	-4.405896	-4.351515
hisp	2.336685	.015682	149.00		2.305949	2.367421
schlyrs	.0445203	.0013794	32.28	0.000	.0418169	.0472238
c.spage1#c.female	.0109097	.0001002	108.91	0.000	.0107134	.0111061
c.spage1#c.black	.0470365	.0001891	248.71	0.000	.0466659	.0474072
c.spage1#c.hisp	0284917	.0002215	-128.65	0.000	0289257	0280576
c.spage1#c.schlyrs	.0016884	.0000197	85.50	0.000	.0016497	.0017271
c.spage2#c.female	0873775	.0006209	-140.72	0.000	0885945	0861605
c.spage2#c.black	2264965	.0011246	-201.39	0.000	2287007	2242922
c.spage2#c.hisp	.2300752	.0018696	123.06	0.000	.2264107	.2337396
c.spage2#c.schlyrs	0154071	.0001275	-120.87	0.000	0156569	0151572
c.spage3#c.female	.4533211	.0026487	171.15	0.000	.4481298	.4585124
c.spage3#c.black	.7048116	.004142	170.16	0.000	. 6966935	.7129298
c.spage3#c.hisp	-1.582442	.0116111	-136.29	0.000	-1.605199	-1.559684
c.spage3#c.schlyrs	.0382811	.0005752	66.55	0.000	.0371537	.0394084
c.female#c.black	044085	.001157	-38.10	0.000	0463526	0418174
c.female#c.hisp	156757	.0012833	-122.15	0.000	1592722	1542418
c.female#c.schlyrs	0077451	.0001177	-65.82	0.000	0079758	0075145
c.schlyrs#c.black	.0233819	.0002279	102.58	0.000	.0229351	.0238286
c.schlyrs#c.hisp	0497206	.0001595	-311.78	0.000	0500331	049408
_cons	3.715046	.0204175	181.95	0.000	3.675029	3.755064

8.15 gcp

	Source	SS	df	MS	Number of obs		
_	Model	11887183.2	24	495299.302	F(24, 28365024) Prob > F	> =	
	Residual	15293477.8			R-squared	=	
-	Total	27180661.1	28365048	.958244846	Adj R-squared Root MSE	=	0.4373 .73428
	10001	2.10000111	20000010		11000 1102		

Pgcp_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	1061269	.0002737	-387.75	0.000	1066633	1055905
spage2	.5664878	.0016874	335.71	0.000	.5631805	.5697951
spage3	-2.160239	.0073018	-295.85	0.000	-2.17455	-2.145927
female	-1.353748	.0069253	-195.48	0.000	-1.367321	-1.340174
black	-4.366507	.013037	-334.93	0.000	-4.392059	-4.340955
hisp	3940637	.014737	-26.74	0.000	4229476	3651797
schlyrs	0277726	.0012962	-21.43	0.000	0303132	0252321
<b>j</b>						
c.spage1#c.female	.0220659	.0000941	234.40	0.000	.0218814	.0222504
c.spage1#c.black	.0489213	.0001777	275.27	0.000	.0485729	.0492696
c.spage1#c.hisp	.010399	.0002081	49.97	0.000	.0099911	.0108069
c.spage1#c.schlyrs	.003009	.0000186	162.15	0.000	.0029726	.0030454
c.spage2#c.female	1555431	.0005835	-266.57	0.000	1566868	1543995
c.spage2#c.black	2580044	.0010569	-244.12	0.000	2600758	255933
c.spage2#c.hisp	0567184	.001757	-32.28	0.000	060162	0532748
c.spage2#c.schlyrs	0337061	.0001198	-281.39	0.000	0339408	0334713
c.spage3#c.female	.6508438	.0024891	261.48	0.000	.6459653	.6557222
c.spage3#c.black	.906269	.0038924	232.83	0.000	.89864	.9138979
c.spage3#c.hisp	2150439	.0109114	-19.71	0.000	2364298	193658
c.spage3#c.schlyrs	.1258863	.0005405	232.90	0.000	.1248269	.1269457
c.female#c.black	.0415605	.0010872	38.23	0.000	.0394295	.0436914
c.female#c.hisp	.0341505	.001206	28.32	0.000	.0317868	.0365141
c.female#c.schlyrs	.0023326	.0001106	21.10	0.000	.0021159	.0025493
c.schlyrs#c.black	.0059763	.0002142	27.90	0.000	.0055565	.0063961
c.schlyrs#c.hisp	0607339	.0001499	-405.27	0.000	0610276	0604401
_cons	5.222462	.019187	272.19	0.000	5.184857	5.260068

#### 8.16 h1rmsetotal

Source	SS	df	MS	Number of obs		
				F(24, 28365024)	>	99999.00
Model	4838901.27	24	201620.886	Prob > F	=	0.0000
Residual	18314169.1	28365024	.645660271	R-squared	=	0.2090
				Adj R-squared	=	0.2090
Total	23153070.4	28365048	.816253523	Root MSE	=	.80353

Ph1rmsetotal_blom	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
spage1	1110934	.0002995	-370.92	0.000	1116804	1105064
spage2	.5111354	.0018466	276.80	0.000	.5075162	.5147546
spage3	-2.226211	.0079904	-278.61	0.000	-2.241872	-2.21055
female	-2.699219	.0075785	-356.17	0.000	-2.714072	-2.684365
black	-3.221603	.0142665	-225.82	0.000	-3.249565	-3.193641
hisp	-2.406037	.0161268	-149.19	0.000	-2.437645	-2.374429
schlyrs	2358848	.0014185	-166.29	0.000	238665	2331047
c.spage1#c.female	.0426963	.000103	414.47	0.000	.0424944	.0428982
c.spage1#c.black	.0256938	.0001945	132.11	0.000	.0253126	.0260749
c.spage1#c.hisp	.0334116	.0002277	146.70	0.000	.0329652	.033858
c.spage1#c.schlyrs	.004905	.0000203	241.55	0.000	.0048652	.0049448
c.spage2#c.female	2696516	.0006385	-422.30	0.000	2709031	2684001
c.spage2#c.black	1029399	.0011565	-89.01	0.000	1052067	1006731
c.spage2#c.hisp	.0475146	.0019227	24.71	0.000	.0437462	.0512829
c.spage2#c.schlyrs	0286959	.0001311	-218.92	0.000	0289529	028439
c.spage3#c.female	1.302694	.0027238	478.26	0.000	1.297356	1.308033
c.spage3#c.black	.5401369	.0042595	126.81	0.000	.5317885	.5484854
c.spage3#c.hisp	938091	.0119404	-78.56	0.000	9614938	9146882
c.spage3#c.schlyrs	.1189913	.0005915	201.17	0.000	.117832	.1201506
c.female#c.black	.0362123	.0011898	30.44	0.000	.0338804	.0385443
c.female#c.hisp	2061054	.0013197	-156.18	0.000	2086919	2035188
c.female#c.schlyrs	0019314	.000121	-15.96	0.000	0021685	0016942
c.schlyrs#c.black	.0614217	.0002344	262.04	0.000	.0609623	.0618811
c.schlyrs#c.hisp	0022363	.000164	-13.64	0.000	0025577	0019149
_cons	6.381657	.0209966	303.94	0.000	6.340505	6.42281