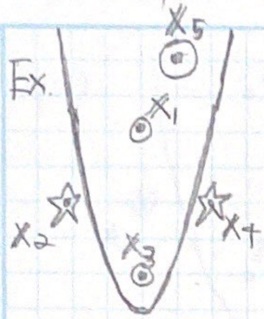
* 2-D 문제 ⇒ 5차원(R^5)에서만 선형분리(linearly Separable)가능한 경우 *

Ex.



⇒ $\{x_1, x_3, x_5\}$와 $\{x_2, x_4\}$를

. 선형(linearly)적으로 separation이 불가능하다

o Quadratic decision functions & boundaries,

we get N=5 and $d=2 < 5=N$.

o Linear separation in R^5 is possible.

선형

Nw (선-차원 : 2, 차원수 2)

Nw=6

dnew = Nw − 1 = 5

* Nonlinear decision function & boundaries 곡선으로 이루어진 차원.

실함수 기저

과선형

예 Five - 2-D patterns (regularly distributed)

패턴의 갯수가 차원보다 클 때

* 차원: 2차원, 패턴갯수: 4개

The total linear dichotomies

pairwise, 차원

$$2\sum_{i=0}^{2}\binom{5-1}{i} = 2\left[\binom{4}{0}+\binom{4}{1}+\binom{4}{2}\right] = 2\left(\frac{4!}{4!0!}+\frac{4!}{3!1!}+\frac{4!}{2!2!}\right)$$

$$=2(1+4+6)=22$$

* If we decide to use general quadratic decision functions, the dimensionality of the patterns increases from 2 to 5.

$$d(x) = w_1 \underset{①}{x_1^2} + w_2 \underset{②}{x_1 x_2} + w_3 \underset{③}{x_2^2} + w_4 \underset{④}{x_1} + w_5 \underset{⑤}{x_2} + b$$

2차원 기존 / 5차 변화된 환경

The number of non-constant terms

* 상수가 아닌 항의 갯수 (2개 ⇒ 5개로)

이 경우에 Linear dichotomies의 수는 22개에서

$D(5,5) = 2^5 = 32$ 개로 바뀐다.

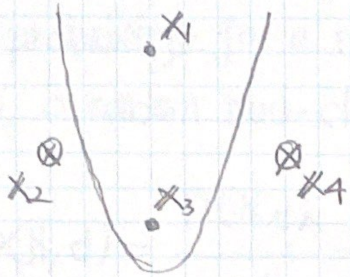
$D(N,d)$

= 2차원에서 5차원으로 변형되었으니, 차원은 5차원이고 패턴은 5개주어

Dichotomies(L.D.)의 갯수는, (패턴의 갯수가 차원보다 작으므로 $N=5 \leq d=5$),

* $2^N = 2^5 = 32$ 가 된다.

예2)



2D problem linearly separable only in $\mathbb{R}^5$.

- $\{x_1, x_3\}$, $\{x_2, x_4\}$ classes cannot be separated linearly.
- Using quadratic decision functions & boundaries

$$d(x) = w_1 x_1^2 + w_2 x_1 x_2 + w_3 x_2^2 + w_4 x_2 + w_5 x_1 + b$$

we get 5 basis in $d(x)$.

- $d=5$, $N=4$. $\Rightarrow$ Pattern # $<$ Dimension : $4 < 5$

  (N: # of Patterns)

- The total # of dichotomies $\triangleq D(4,5) = 2^4 = 16$.

  (The total # of two-class grouping : 2^N, N : the number of patterns.

- Four 2-D patterns (Regularly distributed)

$$2\sum_{i=0}^{2}\binom{4-1}{i} = 2\left[\binom{3}{0}+\binom{3}{1}+\binom{3}{2}\right] = 2(1+3+3) = 14.$$

$$P(4,2) = \frac{D(4,2)}{2^4} = \frac{14}{16} = \frac{7}{8} = 0.875.$$

The Pr. for a random dichotomy to be linearly implementable is 0.875.

* The probability for a random dichotomy to be linearly implementabl
(i.e, a random two-class grouping of the pattern set)
is

$$P(N,d) = \frac{D(N,d)}{2^N} = \begin{cases} 2\sum_{i=0}^{d}\binom{N-1}{i} \Big/ 2^N = 2^{1-N}\sum_{i=0}^{d}\binom{N-1}{i}, & N>d \\ 2^N \Big/ 2^N = 1, & N \leq d \end{cases}$$

* Consequently, if $N<d$, each two disjoint pattern classes are linearly separable in the d-dimensional space

* 예그의 경우 : (2-D problem linearly separable only in $\mathbb{R}^5$.)

d : dimension $= 2$

N : Number of Patterns $= 4$

$d_{new} = 2d+1 = 5.$

$$* \binom{N}{0}+\binom{N}{1}+\binom{N}{2}+\cdots+\binom{N}{N-1}+\binom{N}{N} = 2^N$$

# of patterns

$$* \sum_{i=0}^{d}\binom{2(d+1)-1}{i} = \sum_{i=0}^{d}\binom{2d+1}{i}$$

$$= \binom{2d+1}{0}+\cdots+\binom{2d+1}{d}$$

$$= 2^{d+1} - \left[\binom{2d+1}{d+1}+\binom{2d+1}{d+2}+\cdots+\binom{2d+1}{2d+1}\right]$$

$$\frac{(2d+1)!}{(d+1)!(d!)}+\frac{(2d+1)!}{(d+2)!(d-1)!}+\cdots+\frac{(2d+1)!}{(2d+1)!}$$

$$\sum_{k=0}^{r}\binom{n+k}{k} = \binom{n+r+1}{r}$$

$N=2d+2$ 의 경우.

$$P(2d+2,d) = \frac{D(2d+2,d)}{2^{2d+2}}$$

$$= \frac{2\sum_{i=0}^{d}\binom{2d+1}{i}}{2^{2d+2}}$$

?

$$= 2^{-(2d+1)}\boxed{\sum_{i=0}^{d}\binom{2d+1}{i}} = 2^{-(2d+1)}\boxed{\frac{2^{2d+1}}{2}} = \frac{1}{2}.$$

- 주어진 pattern의 갯수가 $2(d+1)$보다 적으면,

  주어진 pattern을 2개로 묶는 $(d+1)$개의 decision 함수를 찾을 확률 $\frac{1}{2}$

- $a = \frac{N}{d+1}$, $\lim_{d \to \infty} p(a(d+1), d) = 1$, $0 \le a \le 2$.

  The dichotomization capacity of generalized decision functions defined by $(d+1)$ parameters : $2(d+1)$.

- The dichotomization capacity of a general quadratic surface in R^n is $(n+1)(n+2)$.