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The General Linear Model as Structural Equation Modeling

James M. Graham
Western Washington University

Statistical procedures based on the general linear model (GLM) share much in common with one another, both conceptually and practically. The use of structural equation modeling path diagrams as tools for teaching the GLM as a body of connected statistical procedures is presented. A heuristic data set is used to demonstrate a variety of univariate and multivariate statistics as structural models. Implications for analytic strategies and education are discussed.

Keywords: *structural equation modeling; general linear model; statistics; multiple regression*

The vast majority of parametric statistical procedures in common use are part of the general linear model (GLM), including the *t* test, analysis of variance (ANOVA), multiple regression, descriptive discriminant analysis (DDA), multivariate analysis of variance (MANOVA), canonical correlation analysis (CCA), and structural equation modeling (SEM). Moreover, these procedures are hierarchical, in that some procedures are special cases of others. All classical univariate techniques, for example, are special cases of multiple regression (Cohen, 1968). In addition, all univariate and multivariate GLM procedures are special cases of canonical correlation analysis (Knapp, 1978). Finally, all GLM procedures (both univariate and multivariate) are special cases of SEM (Bagozzi, Fornell, & Larcker, 1981; Fan, 1997).

All statistical procedures based on the GLM share a number of characteristics. As stated by Thompson (1998), all GLM procedures are

> least squares procedures that implicitly or explicitly (a) use least squares weights . . . to optimize explained variance and minimize model error variance, (b) focus on latent synthetic variables . . . created by applying the weights . . . to scores on measured/observed variables . . . and (c) yield variance-accounted-for effect sizes analogous to r^2. (p. 5)

Although these weights, latent variables, and effect sizes may have different names across GLM procedures, they are all analogous.

Despite the fact that all GLM procedures are interrelated and that the same interpretive procedures can be used for all GLM procedures, statistics are commonly taught to undergraduate and graduate students outside of a GLM framework. In this approach, each statistical technique is taught as a stand-alone procedure, and students must learn how to approach and interpret each statistical procedure anew. Taught in this manner, multivariate statistical procedures can seem overly complex and intimidating to students, who lack an overall framework in which to couch a new procedure.

This article seeks to demonstrate the use of path diagrams such as those used in SEM as a tool in teaching GLM statistics. It is suggested here that the use of path diagrams throughout statistics courses has the benefit of (a) providing a consistent framework from which to evaluate GLM procedures, (b) providing a standardized interpretive strategy across GLM procedures, and (c) making SEM procedures more accessible to students. First, the common characteristics of all GLM procedures are described. Next, a series of heuristic examples is used to demonstrate path diagram models of correlations, ANOVAs, multiple regression, DDA, CCA, and by extension, MANOVAs. Finally, a general interpretive strategy based on the GLM framework is discussed.

It should be noted that SEM is more than just a statistical tool. By using the proper causal and theoretical assumptions, SEM can be used as a methodological tool, not just to determine probability as is done in statistical procedures (Kline, 2004). As a result, this discussion of SEM as a statistical tool is a gross oversimplification. Nonetheless, causal modeling is beyond the scope of this work, which instead focuses on the statistical (versus methodological) applications of SEM.

Common Characteristics of the General Linear Model

The GLM is perhaps best understood as it pertains to multiple regression, a univariate procedure in which the characteristics of the GLM are made explicit. In multiple regression, one or more independent, or predictor, variables are used to "predict" a single dependent, or criterion, variable. Whereas many are first introduced to these procedures using continuous variables, dichotomous categorical variables can also be used as predictors or the criterion (Kerlinger & Pedhazur, 1973). A series of additive and multiplicative weights is applied to the independent variables. These weights are typically referred to as *b* weights when applied to unstandardized scores and β weights when applied to scores in standardized, or *z* score, form. The weights are used to create a synthetic variable "y-hat," or $\hat{y}$. The weights are such that the differences between $\hat{y}$ and the dependent variable are as small as possible. In essence, the variance of the dependent variable is partitioned into the variance shared with the independent variables (the variance of $\hat{y}$) and into the variance not shared with the independent variables (error variance). The proportion of the variance in the dependent variable shared with the independent variables to the total variance in the dependent variable is typically called R^2, a commonly used measure of overall effect size. R^2 is a variance-accounted-for effect size, created by squaring the correlation between $\hat{y}$ and the dependent variable y.

In addition to weights, structure coefficients can be computed for all GLM procedures. Structure coefficients can be defined simply as a correlation between a latent variable and a measured variable. In the case of multiple regression, the structure coefficients would merely be the correlations between the measured independent variables and the latent variable $\hat{y}$ (Courville & Thompson, 2001; Thompson & Borrello, 1985). Structure coefficients are of special interest in interpreting statistical results, as the least squares weights are not always indicative of the relationship between the independent variables and the latent variable of interest. In short, the weights are not always equivalent to a correlation. Thus, in multiple regression, it is possible to have a weight of zero between an independent and dependent variable and also have a near perfect correlation between the two variables (Thompson, 1998). By examining only the weights, one might come to the conclusion that the independent variable in question is of no use in predicting the dependent variable. In actuality, that independent variable may be an excellent predictor of the dependent variable, although other independent variables in the model share the same variance with the dependent variable as does the independent variable in question. The importance of examining structure coefficients in addition to weights in GLM procedures has been described in the case of multiple regression (Cooley & Lohnes, 1971; Thompson & Borrello, 1985) as well as with CCA (Thompson, 1984) and confirmatory factor analysis (Graham, Guthrie, & Thompson, 2003; Thompson, 1997).

The characteristics of GLM procedures are not, however, as explicit in all statistical procedures as they are in the case of multiple regression. As a result, many students learning these methods fail to recognize that the same general procedure is occurring, with weights being used to create latent variables that are the focus of the procedure, and with resulting variance-accounted-for effect sizes and structure coefficients.

Heuristic Examples

To demonstrate the use of SEM path diagrams for evaluating GLM procedures, a heuristic data set was created, the variance/covariance matrix of which is presented in Table 1. All initial analyses were run using SPSS Version 13.0; all SEM analyses used Amos Version 5 (Arbuckle, 2003). The variables labeled Con_1 through Con_3 are continuous variables that will be used as independent variables in the analyses that follow. The variables labeled DV_1 and DV_2 are continuous variables that are used as the dependent variables of interest. The variables labeled Cat_1 and Cat_2 are orthogonal categorical variables with two groups each, coded as -1 and $+1$. The variable labeled $Cat_1 \times Cat_2$ is the product of Cat_1 and Cat_2, signifying the interaction between Cat_1 and Cat_2. Dichotomous categorical variables are used in these examples because it allows group membership to be represented by a single contrast variable. These same procedures could be used with categorical variables with more than 2 values, although it would require a number of contrast variables equal to the ANOVA

TABLE 1
Heuristic Data Variance/Covariance Matrix

Variable	Con_1	Con_2	Con_3	Cat_1	Cat_2	$Cat_1 \times Cat_2$	DV_1	DV_2
Con_1	8.121							
Con_2	7.639	82.656						
Con_3	7.255	10.979	155.172					
Cat_1	.537	1.662	.892	1.003				
Cat_2	−.007	2.164	−3.902	0.000	1.003			
$Cat_1 \times Cat_2$	−.174	.519	.362	0.000	0.000	1.003		
DV_1	8.206	34.082	25.008	2.634	1.045	1.373	59.655	
DV_2	3.141	15.216	8.104	.408	−.387	.324	11.178	22.302
Mean	14.28	18.13	40.89	0.00	0.00	0.00	33.63	24.43
SD	2.85	9.10	12.46	1.00	1.00	1.00	7.72	4.72

Note: $N = 288$.

degrees of freedom (see O'Grady & Medoff, 1988; Wolf & Cartwright, 1974, for a discussion of dummy-coded categorical variables in regression). For all Except Amos analyses save for CCA, unweighted least squares was used as the method of estimation. The analysis of the CCA model used maximum likelihood.

Multiple Regression

To demonstrate multiple regression as part of the GLM, the variable DV_1 was regressed on variables Con_1, Con_2, and Con_3. The results of the regression analysis as performed by SPSS are shown in Table 2. As shown here, the independent variables accounted for 32.3% of the variance in the dependent variable.

The initial SEM path diagram for this model is shown in Figure 1. In all of the following figures, the input model is shown at the top of the figure; any fixed parameters added to the input model are unstandardized estimates. The lower part of each figure shows the output model, with standardized coefficients. As is the case in most path diagrams, variables that are directly measured or observed are shown as squares, whereas synthetic or latent variables (in this case, e_1, which is the error term) are shown as circles. A double-ended arrow denotes a correlation, whereas a single-ended arrow represents a regression path. Figure 1 shows that the three independent variables are correlated with one another and are used to predict the dependent variable. The path between the error term and the dependent variable is initially set at 1. This is arbitrary and is used to identify the model. Note also that there are no error terms associated with the independent variables; GLM procedures assume that the independent variables are error free. The estimated model with standardized output is shown in the lower portion of Figure 1. As seen here, the numbers on the paths from the independent variables to the dependent variable are equal to the β weights from the SPSS analysis in

TABLE 2
SPSS Multiple Regression Analysis Results

Variable	b	SE	β	t	p	r_s
Con_1	.590	.141	.218	4.187	< .01	.656
Con_2	.343	.043	.404	7.903	< .01	.854
Con_3	.109	.031	.176	3.531	< .01	.457

Note: Dependent variable = DV_1. $R^2 = .323$ [$F(3, 284) = 45.187$, $p < .001$].

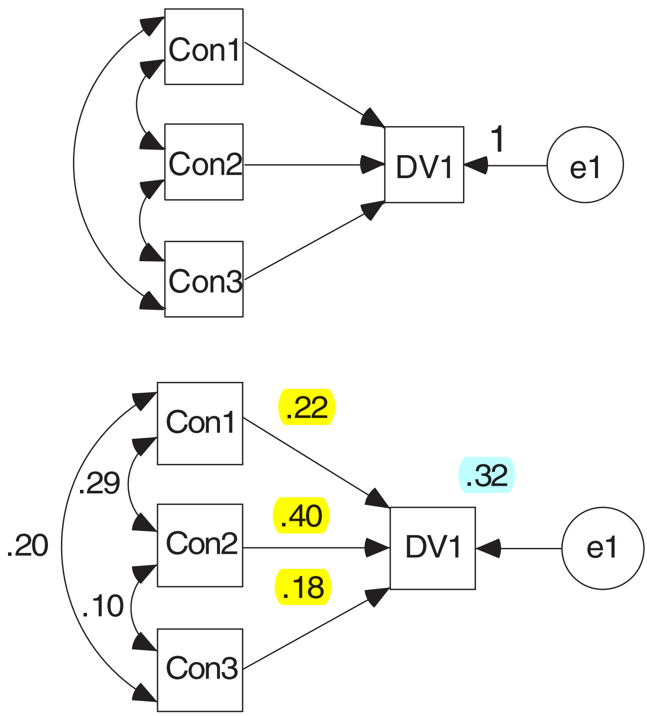

FIGURE 1. *Multiple regression input and output path diagrams.*

Table 2. Furthermore, the number above and to the right of the dependent variable is the R^2, which is the overall model variance-accounted-for effect size. In addition, the numbers on the double-ended arrows between the independent variables are equal to the bivariate correlations between those variables.

Although this model provides all of the information that is provided by the SPSS analysis, it is also possible to gain further information by making the latent variable $\hat{y}$ explicit in the model. The input model and standardized output

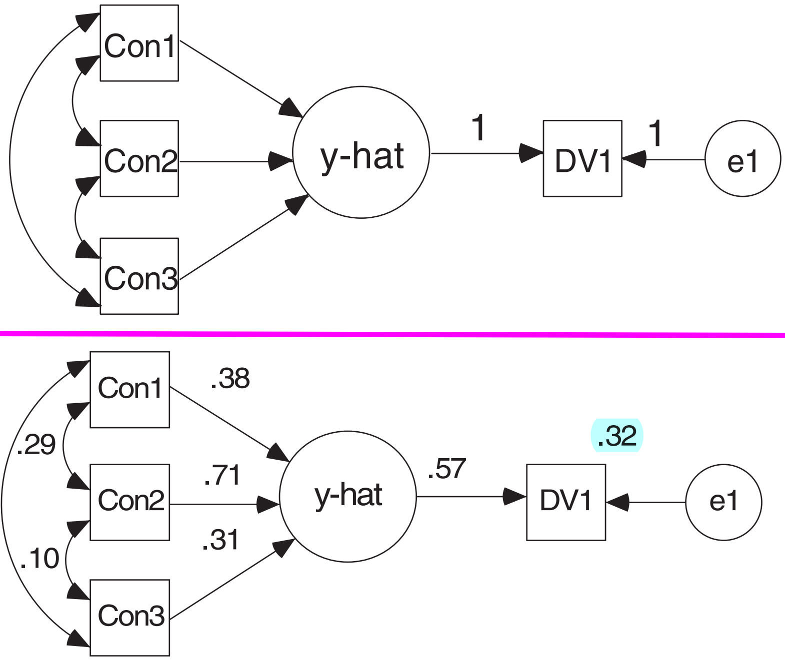

FIGURE 2. *Multiple regression with explicit predicted value input and output path diagrams.*

for this model are shown in Figure 2. As shown here, the R^2 value remains the same. To obtain the β weights, however, it is now necessary to multiply the weight on the path from the independent variable to $\hat{y}$ by the path from $\hat{y}$ to the dependent variable. In the case of Con_1, for example, $(.38)*(.57) = .22 = \beta$. Note that the models shown in Figure 2 and Figure 1 are identical, save that Figure 2 makes the latent variable of interest, $\hat{y}$, explicit. This can be useful because Amos can also estimate the implicit correlations between all of the variables in the model by selecting "standardized estimates" and "all implied moments" as output. Thus, Amos can supply the correlations between the independent variables and the latent variable $\hat{y}$. These structure coefficients are an essential piece of information in interpreting regression results (Thompson & Borrello, 1985) and are not supplied by SPSS without saving the predicted value of the dependent variable as a variable and running separate correlational analyses between these scores and the independent variables.

Two-Way Analysis of Variance

Because all GLM univariate statistics are special cases of multiple regression (Knapp, 1978), it stands to reason that the models of all other univariate

TABLE 3
SPSS Two-Way Analysis of Variance Results

Source	SS	*df*	MS	*F*	*p*	η^2	p. eta
Cat_1	1984.5	1	1984.5	39.454	$< .001$	.122	0.1159
Cat_2	312.5	1	312.5	6.213	.013	.021	0.0182
$Cat_1 \times Cat_2$	539.0	1	539.0	10.716	.001	.036	0.0314
Error	14285.0	284	50.3				
Total	17121.0	287					

Note: Dependent variable = DV_1. $R^2 = .166$. SS = sum of squares; MS = mean square.

procedures can be derived from the regression path diagram. To demonstrate a two-way ANOVA as a special case of regression, a 2 × 2 ANOVA was first run using SPSS. The effects of Cat_1, Cat_2, and the interaction between those variables were tested on DV_1. The SPSS output for this analysis is presented in Table 3.

Figure 3 shows how these results are replicated as an SEM model. As seen here, the model is identical to that shown in Figure 1, save for the names of the variables and that the independent variables are uncorrelated with one another. This is because this ANOVA uses a balanced design (has an equal number of participants in each cell); therefore, each of the main effects and the interaction effect are orthogonal, or uncorrelated, with one another. Were the design unbalanced, the model could be modified to allow the independent variables to correlate with one another. As seen in the standardized output portion of Figure 3, the values on the paths from the independent variable to the dependent variables are equivalent to β weights in multiple regression; however, in the ANOVA, the weights are referred to as eta, or η. Whereas R^2 is often the effect size of interest in regression, the amount of variance in the dependent variable explained individually by each of the independent variables is typically the effect sizes of interest in ANOVA procedures. These values are obtained by simply squaring the standardized regression weights, resulting in a separate η^2 for each of the main effects and the interaction effect. As can be seen in this example, the squared regression weights are equal to the η^2 statistics reported by SPSS. Note also that the R^2 equivalent shown in Figure 3 is equal to the R^2 reported by SPSS in Table 3. Because the independent variables are uncorrelated, the regression weights are equal to the structure coefficients. If an unbalanced design were being used, with correlated main and interaction effects, the regression model shown in Figure 2 could be used to obtain the structure coefficients, in the same manner as with regression.

If categorical variables with more than two possible values are used, it is necessary to construct a number of contrast variables equal to the degrees of freedom for each ANOVA effect. First, a model predicting the dependent variable

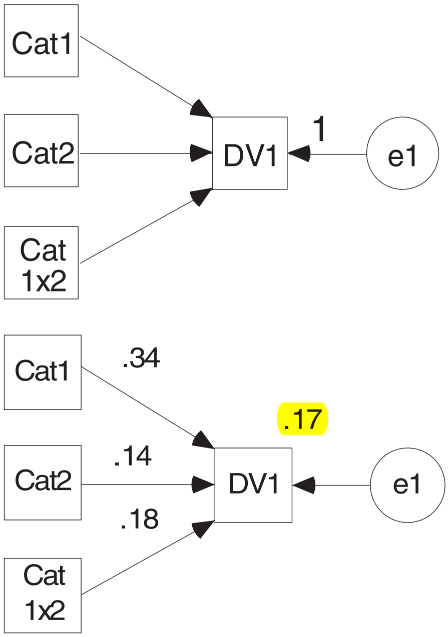

FIGURE 3. *Two-way analysis of variance input and output path diagrams.*

with all contrast variables is run, and the resulting R^2, called here R^2_{tot}, is noted. A second model is then calculated, predicting the dependent variable with all of the contrast variables, save for those contrast variables pertaining to main effect "A." If the R^2 from this second model is called R^2_{tot-A}, then the ANOVA partial eta-squared for main effect A can be calculated using the following equation:

$$\eta^2_{partial} = 1 - [(1 - R^2_{tot})/(1 - R^2_{tot-A})]. \qquad (1)$$

This process can then be repeated for the other main and interaction effects.

Bivariate Correlation

As an additional univariate example, consider the bivariate correlation between two continuous variables: Con_1 and DV_1. Using SPSS, an r of .373 was obtained. The simplest way to model a correlation in Amos is to connect two measured variables with a double-ended arrow; however, this does not make the correlation's position in the GLM explicit. The model shown in Figure 4 is a representation of a bivariate correlation as regression. As can be seen here, the model is identical to the regression model, save that DV_1 is regressed on a

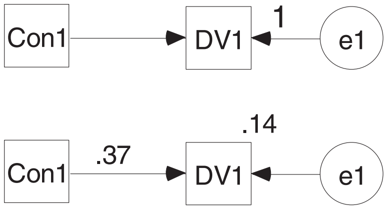

FIGURE 4. *Bivariate correlation as regression input and output path diagrams.*

single continuous independent variable. The regression path coefficient is equal to the correlation between the two variables, and the squared multiple correlation assigned to DV_1 is equal to the squared bivariate correlation. It should be noted here that because a correlation is bidirectional, identical results would be obtained if the model regressed Con_1 on DV_1. The bivariate correlation is an expression of the GLM at its simplest: the linear relationship between two continuous variables. In the case of a single independent variable, the standardized weight, structure coefficient, bivariate correlation, and square root of the variance-accounted-for effect size are all equivalent.

rs= 100%

Independent Samples t *Test*

An independent samples t test compares the mean of a variable in one group with the mean of the same variable in another group. Alternatively, an independent samples t test examines the same hypothesis as a one-way ANOVA with two groups. Although both the t test and the one-way ANOVA test the same hypothesis, they go about it in different ways: In the t test, a confidence interval is constructed about the mean difference, and in the ANOVA, the variance of the dependent variable is partitioned into the variance explained by group membership and the variance unexplained by group membership. Despite the difference in how they are calculated (and typically taught), these procedures are equivalent. In essence, these procedures examine the correlation between a dichotomous categorical variable and a continuous variable, also called a point-biserial correlation.

To examine these tests as part of the GLM, SPSS was used to conduct an independent samples t test comparing the mean of DV_1 between the two groups defined by Cat_1. The mean DV_1 score of group "1" emerged as statistically significantly higher than the mean DV_1 score of group "-1" [$t(286) = -6.123$, $p < .001$]. An ANOVA conducted using the same variables resulted in identical findings [$F(1, 286) = 37.497$, $p < .001$, $\eta^2 = .116$]. The equivalence of these two tests can be further seen when one considers that F values are equivalent to squared t-values, as is the case here ($37.497 = -6.123^2$). Alternatively, the

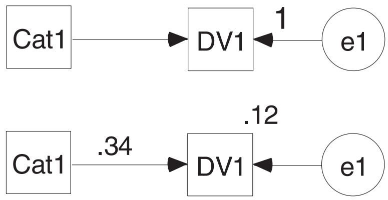

FIGURE 5. *Independent samples* t *test input and output path diagrams.*

point-biserial correlation between DV_1 and Cat_1 as calculated by SPSS was .34, which can be squared to arrive at the ANOVA η^2 of .116.

A demonstration of these equivalent tests as SEM path diagrams is shown in Figure 5. The input model is identical to that of the bivariate correlation shown in Figure 4, save that the independent variable is categorical rather than continuous. The square of the resulting path estimate shown in the output portion of Figure 4 is equivalent to the squared multiple correlation assigned to DV_1. Both of these estimates are equal to the eta-squared value obtained from the ANOVA results and the squared point-biserial correlation. A comparison of the Amos and SPSS analyses makes it clear that independent sample *t* tests, one-way ANOVAs, and point-biserial correlations are simply regression with a single categorical variable. As is the case with the bivariate correlation, the standardized weight, structure coefficient, bivariate correlation, and square root of the variance-accounted-for effect size are identical. If a one-way ANOVA had more than two levels, it would be necessary to use multiple contrast variables, as was discussed in the section on the factorial ANOVA; in this instance, the R^2 associated with the dependent variable would be equal to the ANOVA η^2.

Univariate General Linear Model

When depicted as structural models, all GLM analyses with a single dependent variable can be easily identified as special cases of the multiple regression structural model. The only characteristics that differ from analytic procedure to analytic procedure are the number and types of variables, and the aspects of the model that are focused on. The linkage between the univariate GLM procedures described by Cohen (1968) can be seen clearly when the procedures are modeled visually. Because the same procedure is being used for each analysis, one can see that it would be a simple matter to model discrete and continuous variables together. In fact, one can use discrete or continuous variables (or any combination of the two) as criterion or predictor variables in multiple regression (Kerlinger & Pedhazur, 1973; McNeil, Newman, & Kelly, 1996). Furthermore, it is clear that the interaction effect is not only the purview of the factorial ANOVA; interaction terms can be computed between any two variables (Aiken & West, 1991).

TABLE 4
SPSS Descriptive Discriminant Analysis Function Coefficients and Model Statistics

Wilks's Lambda	Canonical Correlation	Variable	Function Coefficient
.88	.34	DV_1	1.02
		DV_2	−.06

Note: Dependent variable = Cat_1.

Descriptive Discriminant Analysis

DDA is a procedure that describes the differences between multiple groups on multiple continuous descriptors. DDA is a use of discriminant analysis that is different from predictive discriminant analysis, whose purpose is to maximize correct classification of individuals based on the continuous descriptors. To demonstrate DDA as a GLM procedure, a DDA was run using SPSS, using DV_1 and DV_2 to describe the differences between the groups defined by Cat_1. The weights, called function coefficients in DDA, and a model summary are presented in Table 4.

Stevens (1972) has described DDA as multiple regression between two or more groups. This relationship can be easily seen in the DDA path diagram shown in Figure 6. The model is identical to that used in multiple regression, as shown in Figure 2 when the synthetic variable of interest (in regression, $\hat{y}$) is made explicit. In DDA, the latent variable of interest is called the discriminant function. The discriminant function is made explicit here because of the ways in which DDA results are typically reported. One can see here that the DDA model is identical to the multiple regression model, save that in DDA, the dependent variable is categorical rather than continuous. The results of this analysis show that the regression paths are identical to the standardized canonical discriminant function coefficients obtained in the SPSS analysis. Furthermore, the path from the discriminant function to Cat_1 is the canonical correlation from the SPSS analysis. The R^2 assigned to Cat_1 is equal to the squared canonical correlation from Table 4 (or, alternatively, $1 - \lambda$).

Whereas this example of DDA uses a dichotomous discrete variable, discrete variables with more than two categories can be used in DDA. Using more than two categories would require a more complex series of models and will be discussed as a special case of canonical correlational analyses.

Canonical Correlation Analysis

CCA is a statistical procedure that looks at the relationships between two sets of multiple variables. In essence, a CCA creates a series of latent variables, called canonical functions, for both sets of variables. First, one canonical function is created for each set of variables, so that the relationship between the

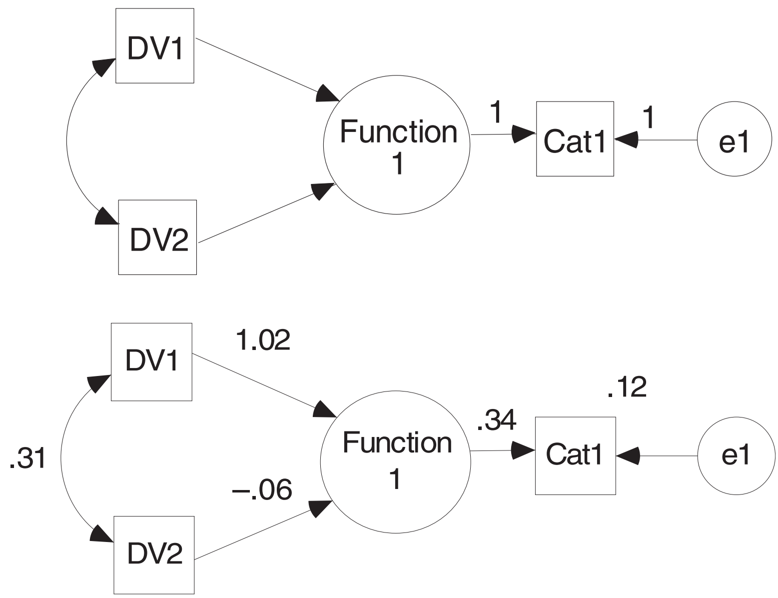

FIGURE 6. *Descriptive discriminant analysis input and output path diagrams.*

functions is as large as possible; the correlation between the canonical functions is called a canonical correlation. With the variance of the measured variables that remains, a second set of canonical functions is created, so that the second canonical functions are uncorrelated with the first canonical functions and the relationship between the two canonical functions from the second set is maximized. This process is repeated until *k* sets of functions have been created, where *k* is the number of measured variables in the smallest of the two sets of measured variables.

A CCA was performed using Con_1, Con_2, and Con_3 as one "left" set of variables, and DV_1 and DV_2 as the second "right" set of variables. The analysis was performed using the canonical correlation macro that is included with SPSS. The resultant weights, called standardized canonical coefficients in CCA, and structure coefficients are presented in Table 5, with a model summary in Table 6.

While actually running a CCA using SEM is more complex than the other models that have been discussed thus far, the logic of the CCA is best demonstrated in the conceptual path diagram shown in Figure 7. As seen here, the variables Con_1, Con_2, and Con_3 are combined with a series of regression weights to create a latent variable labeled "left function 1." Likewise, the variables DV_1 and DV_2 are combined with a different series of regression weights to create a latent variable labeled "right function 1." These weights are configured so that the correlation between the left and right function 1 is maximized. This same process is

TABLE 5
SPSS Canonical Correlation Analysis Standardized Canonical Coefficients and Structure Coefficients (in parentheses)

Set	Variable	Function 1	Function 2
Left	Con_1	−.37 (−.64)	−.43 (−.37)
	Con_2	−.73 (−.87)	.67 (.48)
	Con_3	−.29 (−.43)	−.71 (−.73)
Right	DV_1	−.81 (−.93)	−.66 (−.36)
	DV_2	−.38 (−.63)	.98 (.78)

TABLE 6
SPSS Canonical Correlation Analysis Function Statistics

		Test That Remaining Correlation = 0			
Function	Canonical Correlation	Wilks's Lambda	χ^2	*df*	*p*
1	.610	.626	132.9	6	< .001
2	.051	.997	.8	2	.686

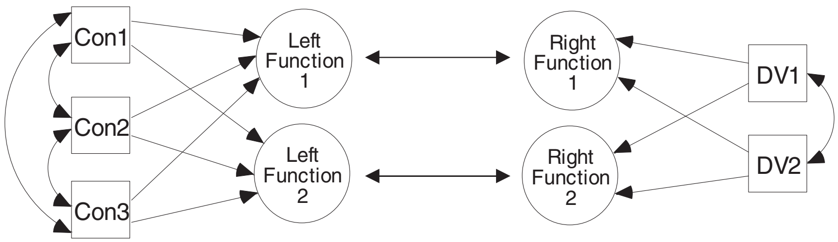

FIGURE 7. *Canonical correlation analysis path diagram.*

also repeated for the second set of canonical functions. Note also that the left functions 1 and 2 are uncorrelated with one another; likewise, the right functions 1 and 2 are uncorrelated with one another. Essentially, the CCA is simply two sets of regression equations, and the canonical correlation is the correlation between the two sets of latent variables created by those regression equations.

Although the conceptual model shown in Figure 7 is an accurate representation of the logic of CCA, it is an unidentified model and cannot be used directly. Previous research has suggested that the Multiple Indicators/Multiple Causes (MIMIC) model can be used to obtain the regression weights used by CCA (Bagozzi et al., 1981; Fan, 1997); however, this technique does not make available the full information obtained through CCA without additional hand calculations. After a discussion of the MIMIC model, it is suggested here that the weights obtained from the MIMIC model can be used to identify the conceptual model, which can then be used to compute the canonical correlations and cross loadings.

MIMIC model. The MIMIC model is a latent variable model in which the latent variable also has both multiple causes and multiple effects (MacCallum & Browne, 1993) and can be used to calculate the initial canonical functions used in a CCA. The following analyses use Amos 5.0, with the options "standardized output," "squared multiple correlations," and "all implied moments" checked in the Analysis Properties/Output menu. Note also that these analyses use maximum likelihood (rather than unweighted least squares) as the method of estimation, as the intent is to create latent variables that have a maximized correlation rather than latent variables that explain a maximal amount of variance in the measured variables.

The initial MIMIC model, shown in Figure 8, is run to create a series of estimates from the Con_1 through Con_3 variables to the first left canonical function. This model is easily recognizable as the regression model shown in Figure 2, save that the model in Figure 8 has two, not one, dependent variables. The standardized weights on the paths from the Con_1 through Con_3 variables to canonical function 1 are identical to the canonical weights shown in Table 5, save that the signs are reversed. This is the result of the measured variable that was chosen as the scaling variable and has no effect on the analyses.

Next, a second model is created, as shown in Figure 9. In this model, a second left canonical function has been added. In addition, the weights both to and from left canonical function 1 have not been left free to be estimated; rather, they have been set to equal the unstandardized weights derived from the first model. Amos allows paths to be fixed rather than estimated, but they must be fixed using the unstandardized estimates. These estimates can be obtained from either the unstandardized graphic output or the text output of Amos. To avoid undue inaccuracy as the result of rounding error, it is recommended that the unstandardized estimates be entered to a large number of decimal places. The following analyses use 6 decimal places, although researchers using this method for non-heuristic purposes may wish to use more. After the paths to and from left function 1 are fixed, the model is analyzed again. The results, shown in Figure 9, now include the canonical coefficients for the left function 2, which are the same as those shown in Table 5. If more functions were indicated, this process would then be repeated until the number of functions equaled the number of measured variables in the smallest variable set.

To estimate the canonical coefficients to the right functions, the process described above is repeated, with the direction of the effects reversed. The model used to estimate the weights from DV_1 and DV_2 to the first right canonical function is shown in Figure 10. The results of this analysis show the same canonical coefficients shown in Table 5.

To complete the series of MIMIC models, the paths to and from right function 1 are fixed to be equal to the unstandardized regression estimates found

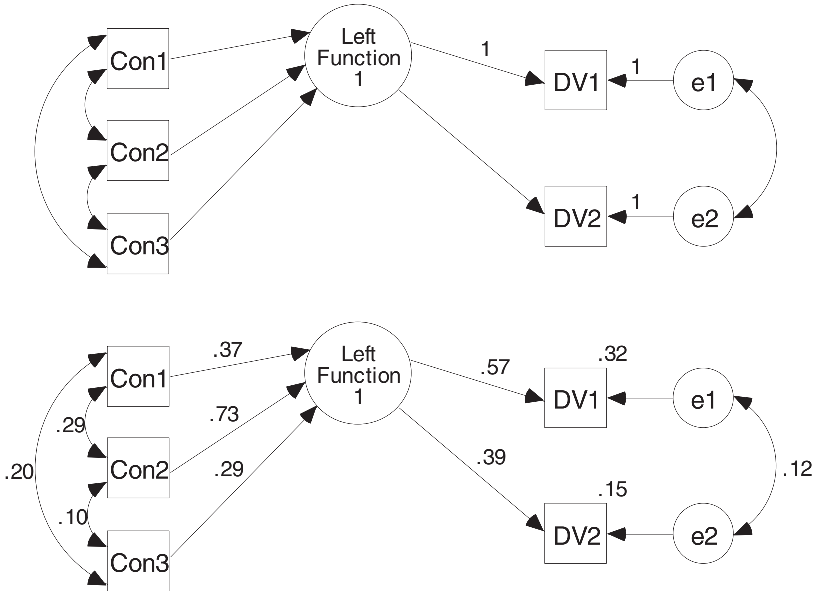

FIGURE 8. *Canonical correlation analysis Step 1 input and output path diagram.*

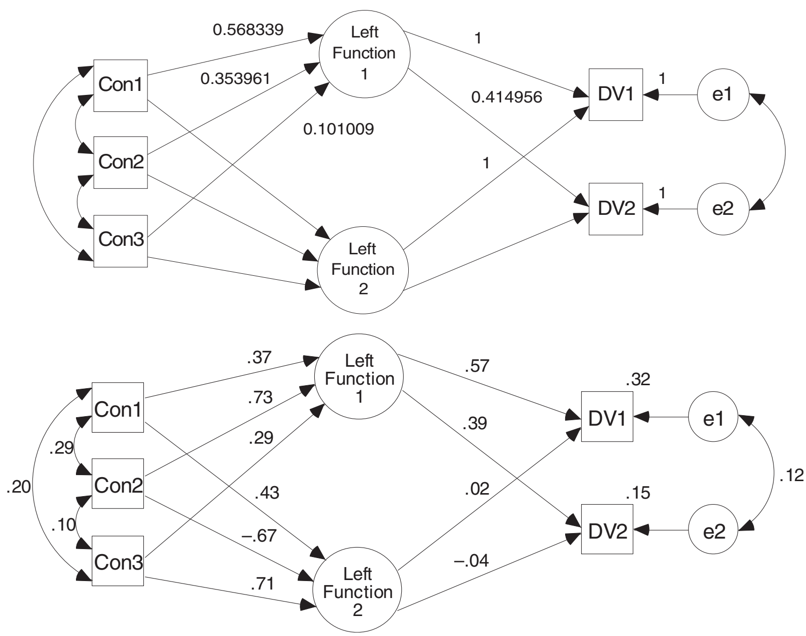

FIGURE 9. *Canonical correlation analysis Step 2 input and output path diagram.*

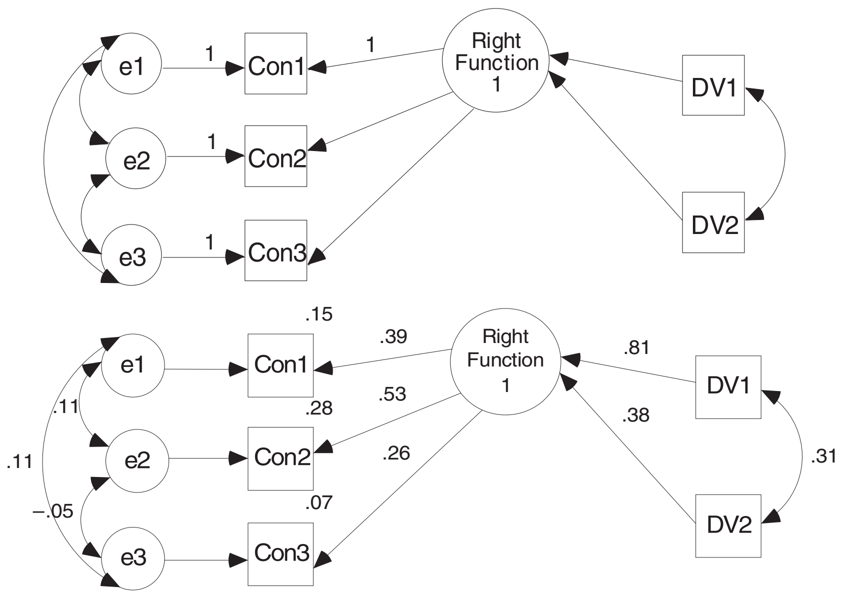

FIGURE 10. *Canonical correlation analysis Step 3 input and output path diagram.*

from the previous model. Right function 2 is then added to the model, with the associated paths from DV_1 and DV_2 and to Con_1 through Con_3. The resultant model, shown in Figure 11, reveals the final set of canonical coefficients found in Table 5.

CCA conceptual model. The use of the MIMIC model is advantageous over the use of SPSS in conducting CCAs in that Amos provides the standard errors of each of the canonical coefficients (Fan, 1997). This may be of use to researchers who wish to construct confidence intervals about the canonical coefficients or who wish to examine the statistical significance of individual paths; however, the MIMIC model does not supply the canonical correlations, nor does it supply the cross-correlations between measured variables and the canonical functions of the opposite set. The canonical coefficients derived from the MIMIC models can be applied to the previously described conceptual model to provide this information.

index coeff.

The model used as input for this final step is shown in Figure 12. In this model, all of the regression paths have been fixed to equal the appropriate unstandardized canonical coefficients derived from the MIMIC models. Note that the application of the conceptual model differs from the conceptual model itself in that the right and left canonical functions are not set to correlate. Amos does not allow correlations between two endogenous variables but will calculate the correlations between these variables as part of the “standardized estimates”

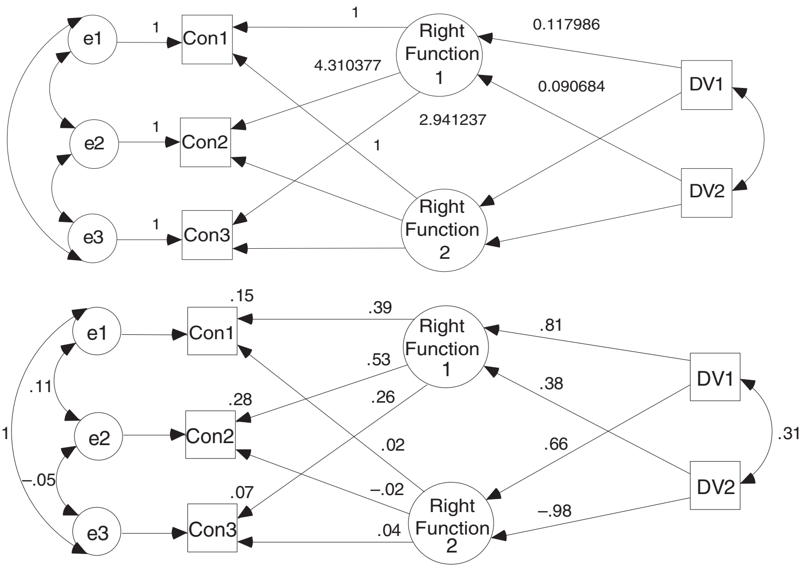

FIGURE 11. *Canonical correlation analysis Step 4 input and output path diagram.*

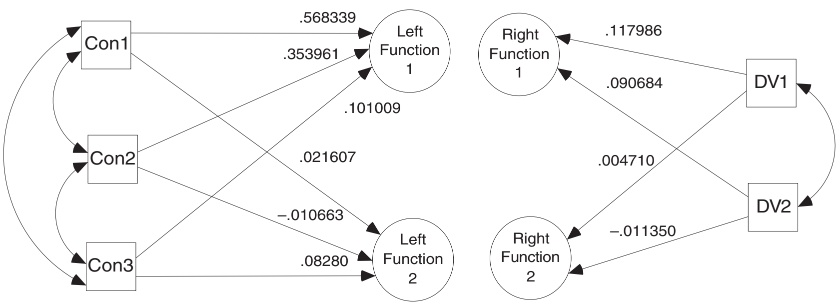

FIGURE 12. *Canonical correlation analysis Step 5 input path diagram.*
Note: Diagram omits correlations between set 1 and set 2 measured variables.

and "all implied moments" commands. It is also necessary to account for the fact that the variables from the left set and right set can be correlated with one another. Although this is not shown in Figure 12 (for the sake of avoiding visual confusion), it is necessary to include double-ended arrows between each of the measured variables on the left set and each of the measured variables on the right set in the input model. Once the model has been specified in this manner, it can be analyzed.

The text output from this analysis includes an implied correlation matrix between all of the measured and latent variables in the model. The correlation

TABLE 7
Canonical Correlation Analysis Correlation Matrix

Variable	1	2	3	4	5	6	7	8
1. Left 1	—							
2. Left 2	.00	—						
3. Right 1	.61	.00	—					
4. Right 2	.00	.05	.00	—				
5. Con_1	.64	.37	.39	.02	—			
6. Con_2	.87	−.48	.53	−.02	.29	—		
7. Con_3	.43	.73	.26	.04	.20	.10	—	
8. DV_1	.57	.02	.93	.36	.37	.49	.26	—
9. DV_2	.39	−.04	.63	−.77	.23	.35	.14	.31

matrix from this analysis has been reproduced in Table 7. As shown here, the correlation between right function 1 and left function 1 is equal to the first canonical correlation shown in Table 6. Likewise, the correlation between right function 2 and left function 2 is equal to the second canonical correlation. The correlation between the first and second sets of canonical functions is zero (or near-zero in the case of rounding error resulting from entering the unstandardized regression weights). The correlations between the measured variables of the left set and the canonical functions of the right set (and vice versa) are the cross-correlations that can be obtained from SPSS. Finally, the correlations between the measured variables of the left set and the canonical functions of the left set (also with right and right) are the structure coefficients (sometimes called canonical loadings by SPSS) shown in Table 5.

Although the process of deriving the results of CCA from SEM is rather involved, the conceptual model demonstrates that CCA is a part of the GLM. CCA applies weights (canonical function coefficients) to the measured variables to create a series of latent variables (canonical functions) that are the focus of analysis. Variance-accounted-for effect sizes (squared canonical correlations) and structure coefficients are also provided. To revisit DDA, in instances where multiple contrast variables are used to depict a single discrete variable, the canonical model could easily be adapted by using the multiple contrast variables as one set, with the continuous variables making up the second set.

Multivariate Analysis of Variance

The MANOVA is an extension of the ANOVA that examines the differences between multiple groups on multiple continuous dependent variables. When modeling ANOVA as a structural model, recall that the only difference was that the predictor variables in regression were replaced with dichotomous contrast variables. Because CCA can be conceptualized as multivariate regression, it stands to reason that MANOVA could be modeled as CCA between contrast variables on one side and continuous dependent variables on the other. This conceptualization of the MANOVA as a structural model differs from that offered by Bray and Maxwell (1985), who depicted MANOVA as a combination of multiple overlapping ANOVAs. Although the latter model provides a conceptual representation of the MANOVA, it cannot be used in SEM to provide MANOVA results.

Multiple contrast variables need to be used for an effect with more than two groups. As such, it is necessary to build multiple canonical models to estimate the effects of the MANOVA. The same procedures described in the section on ANOVA can be followed here. To reiterate, $k + 1$ canonical models must be estimated, where k is the number of main and interaction effects. First, all contrast variables are used as one set of variables in a CCA, whereas the other set of variables is made up of the dependent variables. The MIMIC models are built up, with the resulting canonical coefficients applied to the conceptual model. The resulting canonical correlations are then noted. The same procedure is then repeated with a second canonical model, this time omitting those contrast variables pertaining to a given effect.

If precisely the same hypotheses were being tested by MANOVA and the structural model of MANOVA, it would be possible to calculate the MANOVA eta-squared using Equation 1 and substituting the squared canonical correlations (R_c^2) for the squared multiple correlations (R^2). However, the multivariate tests reported by statistical packages such as SPSS are not tests of single functions, as they would be if one followed the procedures for the MANOVA structural model. Rather, most statistical packages report a test of all of the functions combined (Thompson, 1984, 1991). Because of the difference in focus, the structural model may be of particular interest to researchers wishing to examine the functions individually, rather than in an omnibus test. Regardless, provided the functions are uncorrelated with one another, as they are in CCA, the $\eta^2_{partial}$s can be summed across functions for an omnibus measure of effect.

Implications

Typically, in statistics education, different parametric procedures are taught in different manners, using different methods of calculation and interpretation. This often results in students seeing GLM statistics as separate procedures. The use of SEM path diagrams serves the function of underscoring the similarity between all GLM procedures and can help students to take a common interpretive approach to different procedures.

For example, all GLM procedures provide one or more overall variance-accounted-for statistics that are equivalent to the R^2 of multiple regression. In the case of a single dependent variable, only one variance-accounted-for effect size is provided; in the case of multiple dependent variables, multiple variance-accounted-for effect sizes are provided. If the overall model effect size(s) is judged to be sufficiently large by whatever means the researcher chooses to employ (effect-size magnitude, confidence intervals, statistical significance testing), then the interpretation of the results typically continues. If the overall model effect size is judged not to be sufficiently large, the researcher fails to reject the null hypothesis of the procedure, and the interpretive process is typically over. Put another way, if the overall effect size says that two sets of variables are not related ($R^2 = 0$, $R_c^2 = 0$, etc.), one is typically not interested in determining how much of a nonexistent relationship is contributed by each individual variable.

All GLM procedures produce standardized weights, which can be used to determine the influence of individual variables to the overall effect. The standardized weight of a variable can be judged relative to those of other variables in the model, or it can be judged against a predetermined criteria by invoking the standard errors of the weights (using confidence intervals or statistical significance testing). The fact that the commonly used names for these weights differ from procedure to procedure (beta weights in regression, eta in ANOVA, r in a correlation, discriminant function coefficients in DDA, canonical function coefficients in CCA) serves to draw artificial barriers between what are truly equivalent procedures.

If the overall model effect size(s) is sufficiently large, if the analysis contains more than two measured variables, and if the independent variables are correlated with one another, it is necessary to consider the structure coefficients in addition to the standardized weights. Whereas structure coefficients are provided by commonly used statistical software for some GLM procedures (e.g., CCA, DDA), in other cases, researchers must calculate the structure coefficients by other means (e.g., regression). Discussions of computing and interpreting structure coefficients for a variety of statistical procedures are available in the literature (Courville & Thompson, 2001; Graham et al., 2003; Thompson, 1990, 1997). The structure coefficient of a measured variable can be interpreted relative to the structure coefficients of other variables in the model and relative to the standardized weights of all variables in the model. Although more attention has been given recently to the use of structure coefficients in the available literature, the importance of structure coefficients in some GLM procedures has remained virtually ignored. For example, structure coefficients are neither calculated nor interpreted in common ANOVA practice; although this is not an issue when using a balanced design (in that case, the ηs and structure coefficients are equal), it may lead to misinterpretation of results if the group sample sizes are not equal.

Conclusion

The importance of the conceptual framework underlying the GLM cannot be overstated. The GLM applies to all classical parametric procedures and suggests two concepts often overlooked in the interpretation and reporting of results: structure coefficients and, to an increasingly lesser extent, effect sizes.

The GLM is a useful and flexible statistical tool. Despite the fact that all GLM procedures share common characteristics, many students (and researchers) fail to conceive of different parametric procedures as part of an overarching model. The use of the path diagrams used in SEM techniques can make these relationships explicit. Furthermore, familiarity with the use of SEM software and a conceptual understanding of path diagrams can ease students into the use of more complicated SEM procedures. If pervasiveness is any sign of importance, the GLM is extremely important; the GLM is an essential part of practically all statistical procedures in common use in the fields of psychology and education. A solid understanding of the concepts and implications of the GLM is therefore essential to the proper use and interpretation of GLM procedures.

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Author

JAMES M. GRAHAM, PhD, is an assistant professor in the Department of Psychology at Western Washington University, 516 High Street, Bellingham, WA 98225-9089; Jim.Graham@wwu.edu. His substantive research examines adaptive processes in romantic relationships. His methodological work focuses primarily on score reliability and addressing best practices in quantitative analysis.

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