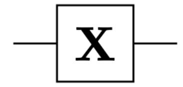
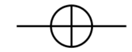
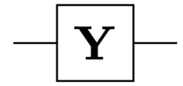
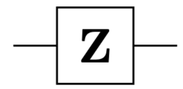
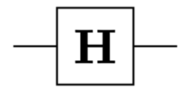
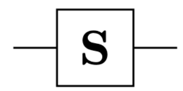
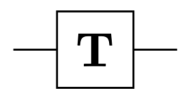
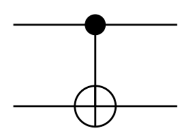
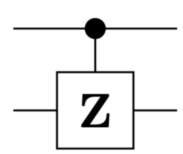
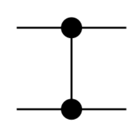
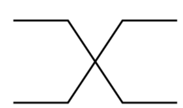
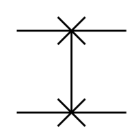
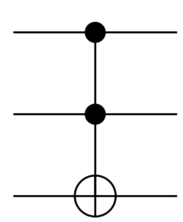
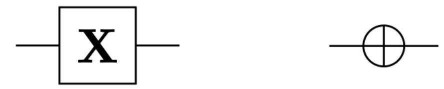
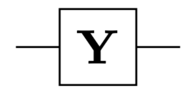
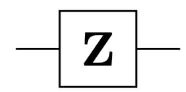
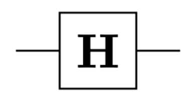
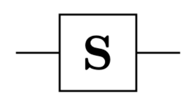
Quantum Circuits and Quantum Programs

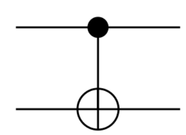
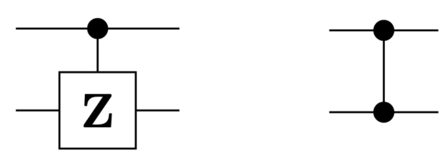
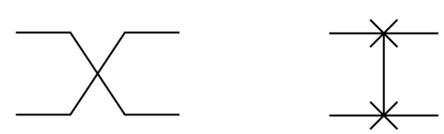
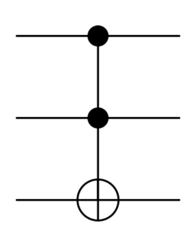
Robert Rand

Winter School on Quantum Computing at Emory

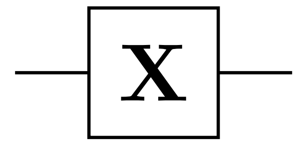
Operator	**Gate(s)**		**Matrix**
Pauli-X (X)			$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)			$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)			$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)			$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)			$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
$\pi/8$ (T)			$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP			$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

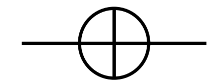
Operator	Gate(s)	Matrix
Pauli-X (X)		$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Pauli-Y (Y)		$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
Pauli-Z (Z)		$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Hadamard (H)		$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Phase (S, P)		$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

https://en.wikipedia.org/wiki/Quantum_logic_gate

Controlled Not (CNOT, CX)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
Controlled Z (CZ)		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
SWAP		$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)		$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

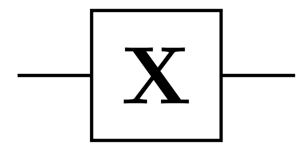
Pauli-X (X)

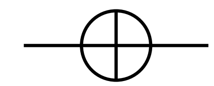




$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Pauli-X (X)

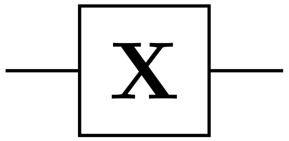


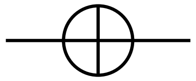


$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Pauli-X (X)

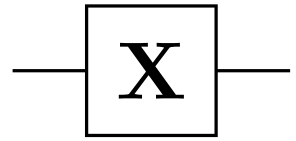


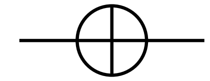


$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Pauli-X (X)





$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Pauli-X (X) $\quad -\boxed{\mathbf{X}}- \quad -\oplus- \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

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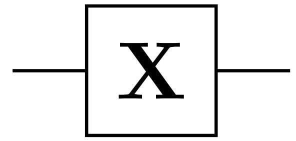
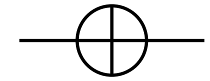
Pauli-X (X)





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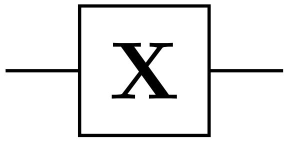
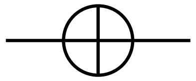
Pauli-X (X)





$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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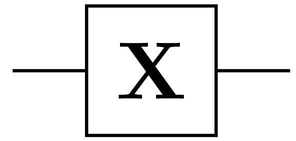
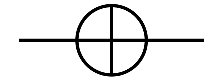
Pauli-X (X)



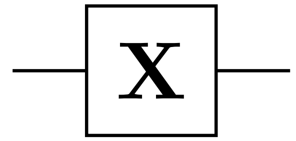


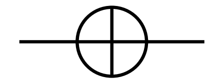
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \overset{|0\rangle}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Pauli-X (X)





$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{array}{c} |0\rangle \\ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{array} = \begin{array}{c} |1\rangle \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

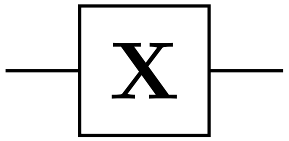
Pauli-X (X)

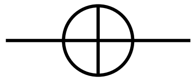
$$-\boxed{\mathbf{X}}- \qquad -\oplus- \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \overset{|0\rangle}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \overset{|1\rangle}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underset{|1\rangle}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Pauli-X (X)





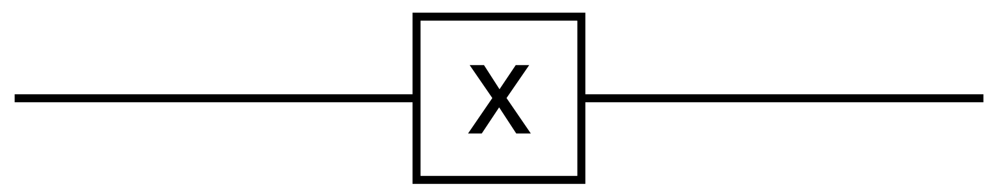
$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\overset{|0\rangle}{\begin{bmatrix}1\\ 0\end{bmatrix}} = \overset{|1\rangle}{\begin{bmatrix}0\\ 1\end{bmatrix}}$$

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\underset{|1\rangle}{\begin{bmatrix}0\\ 1\end{bmatrix}} = \underset{|0\rangle}{\begin{bmatrix}1\\ 0\end{bmatrix}}$$

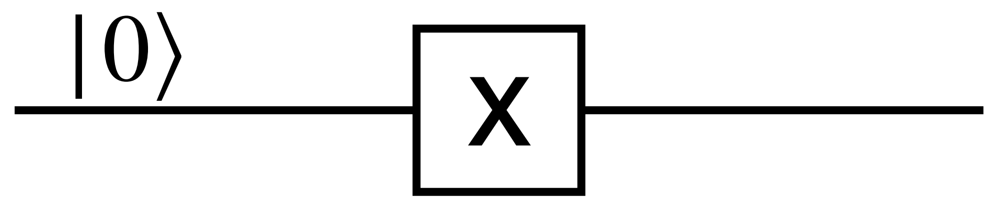
Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



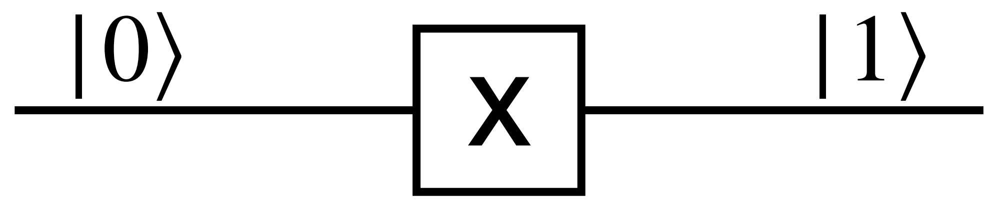
Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



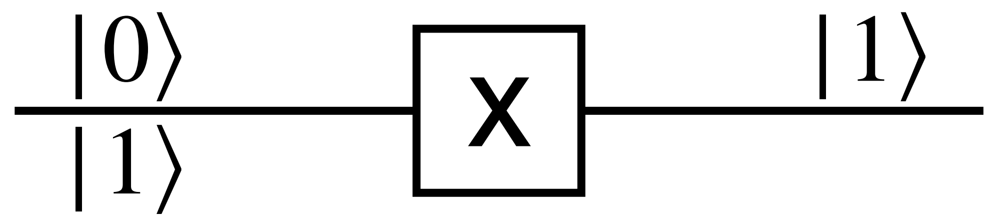
Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



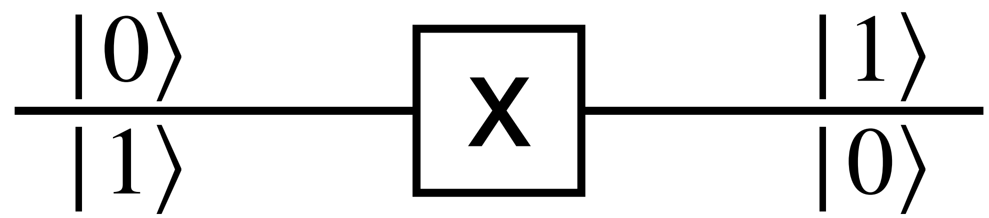
Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



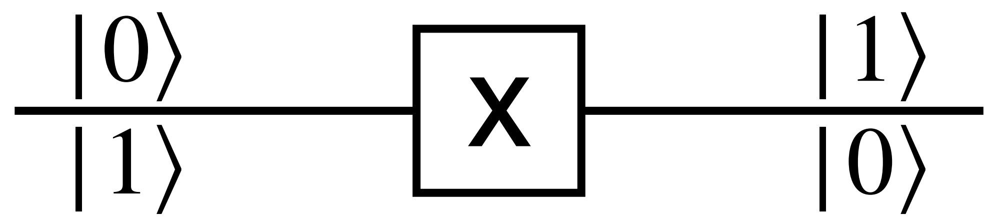
Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

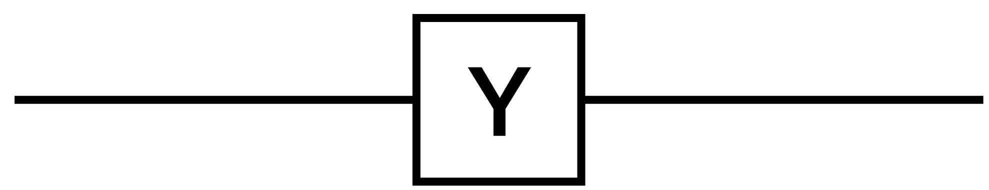


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

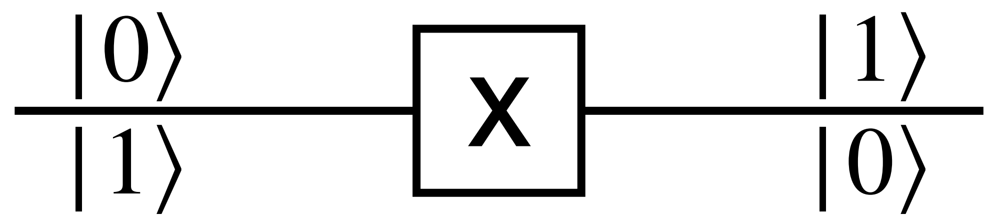


$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

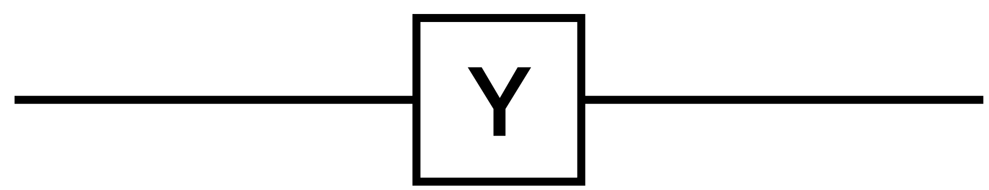


Pauli Gates

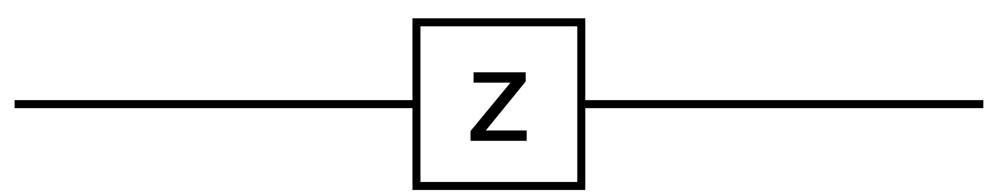
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

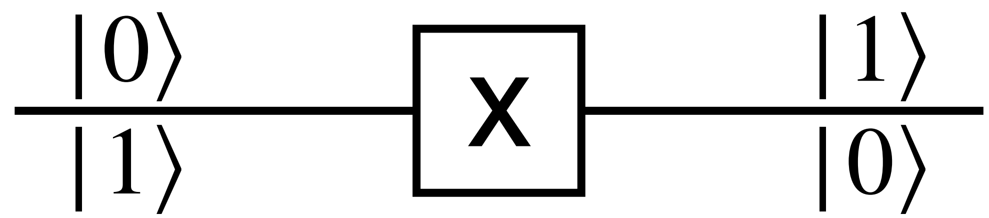


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

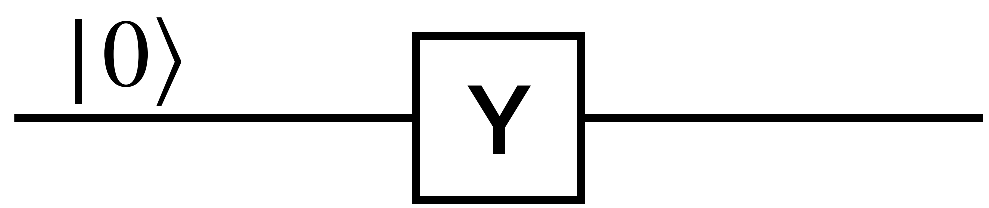


Pauli Gates

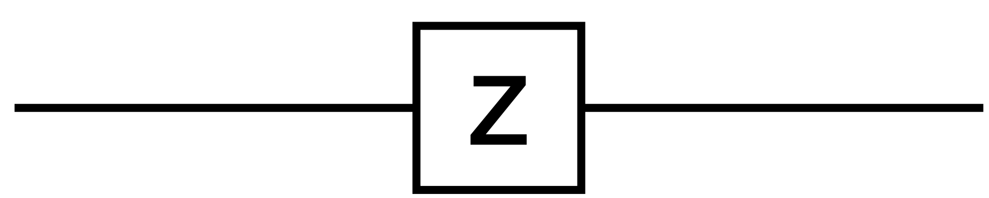
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

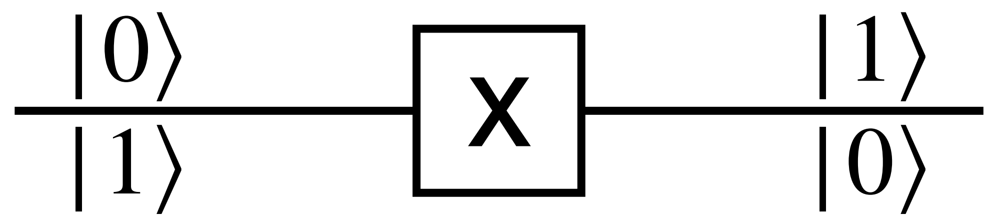


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

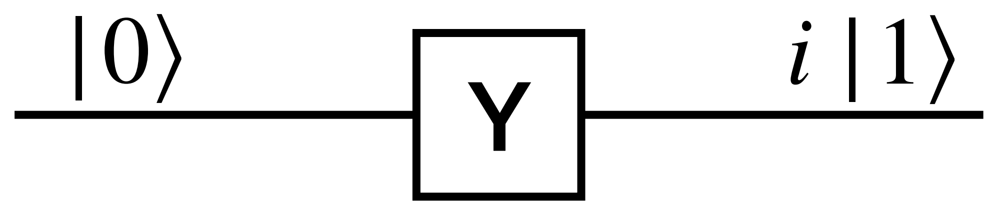


Pauli Gates

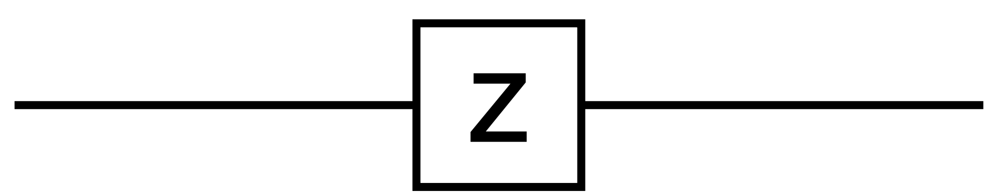
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

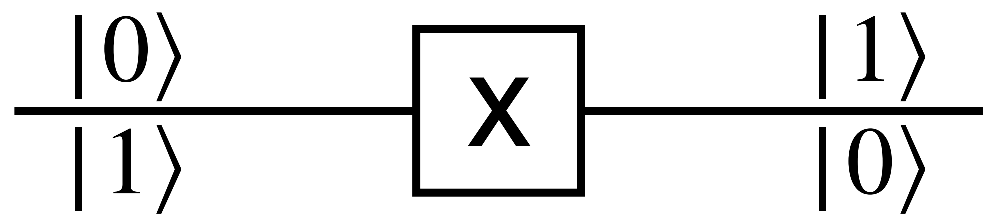


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

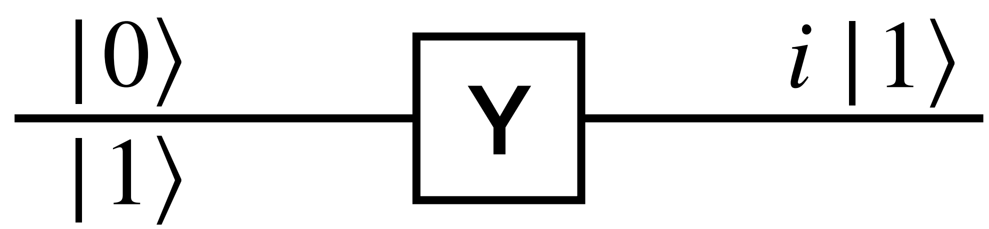


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



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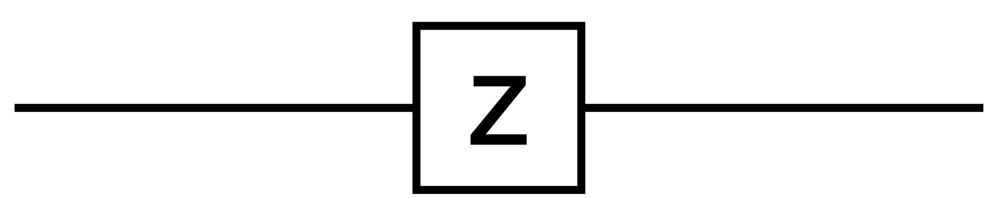
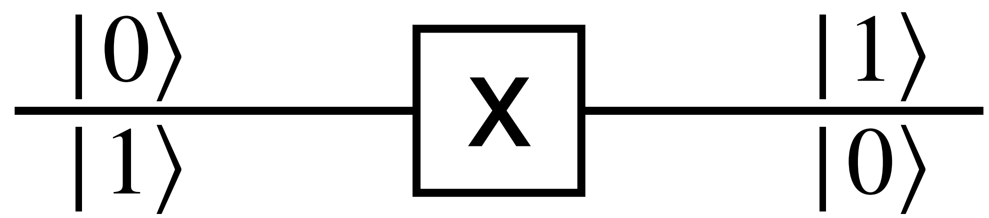


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



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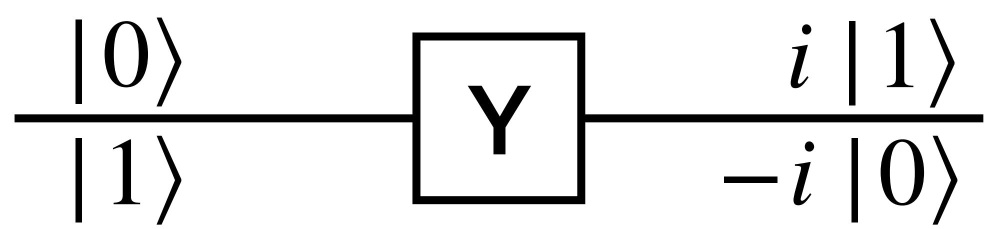

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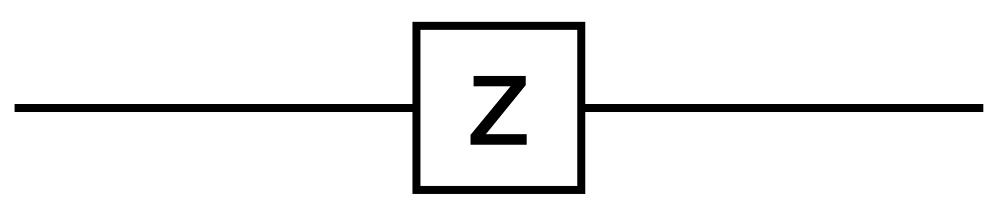

Pauli Gates

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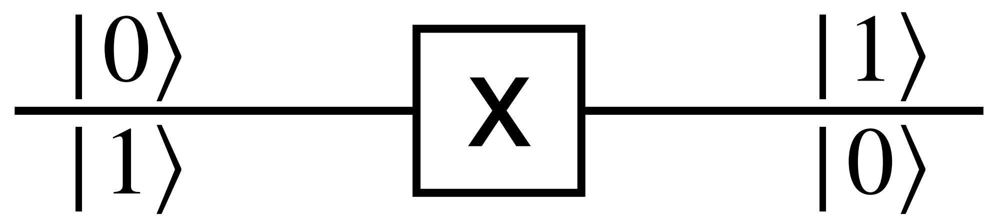

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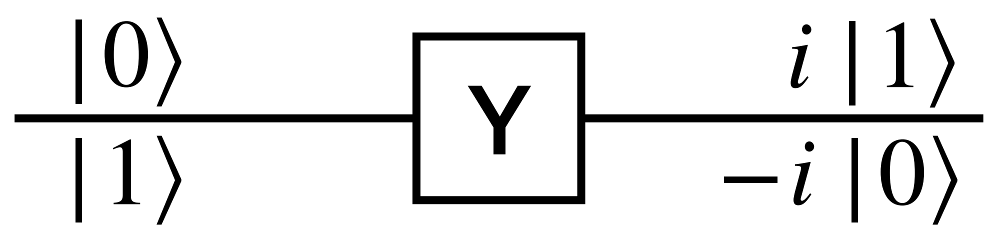

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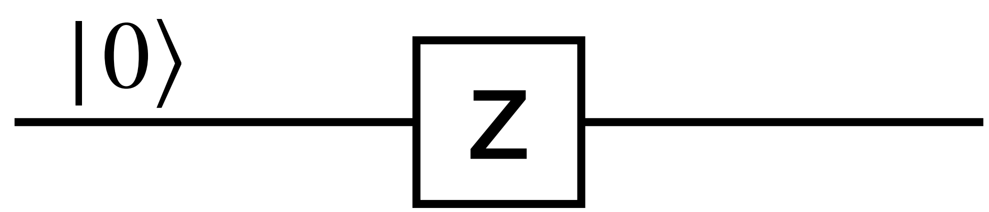

Pauli Gates

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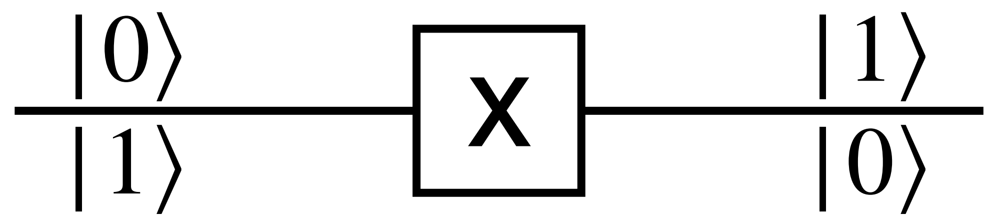

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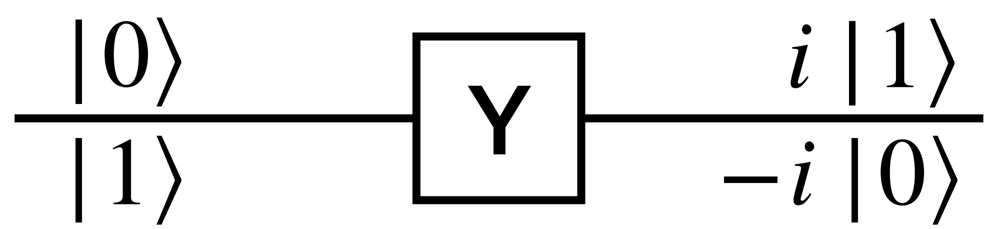

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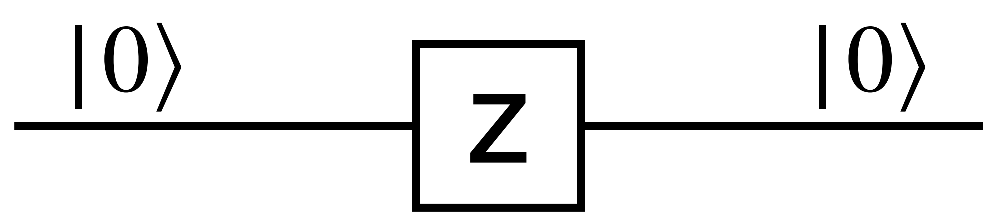
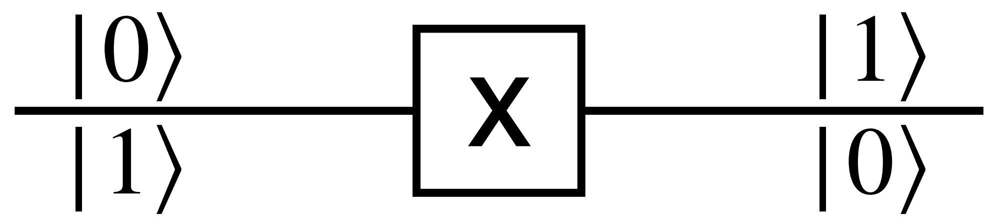


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



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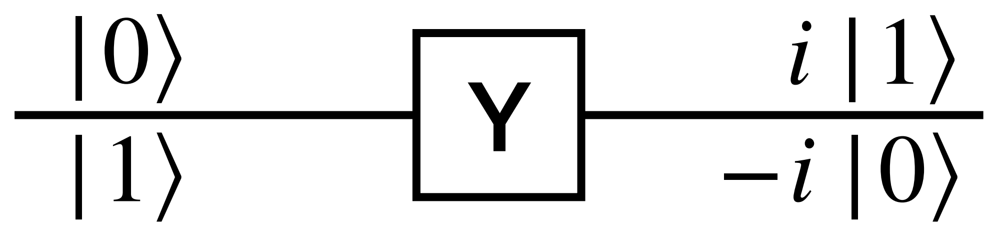

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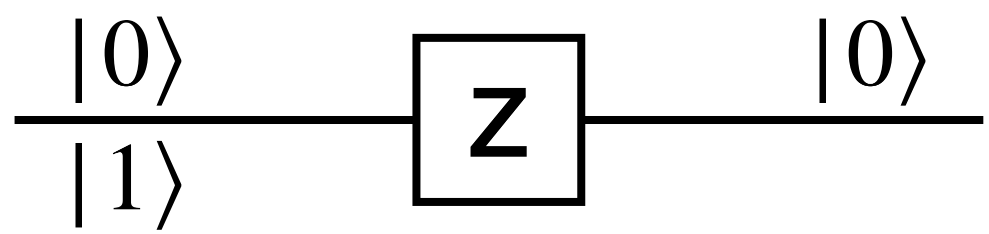
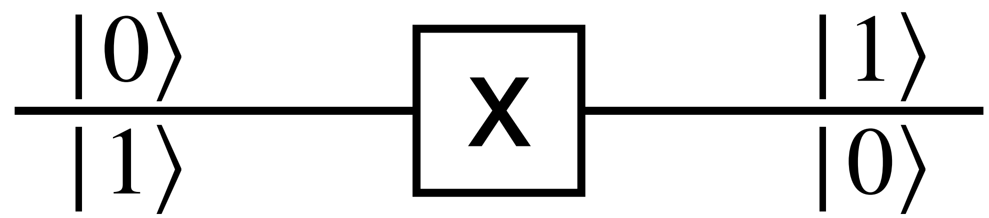
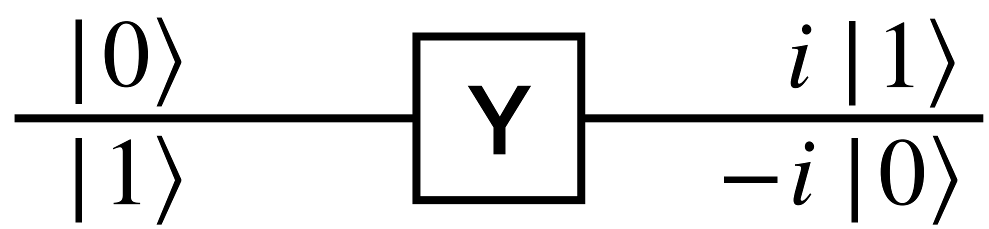


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



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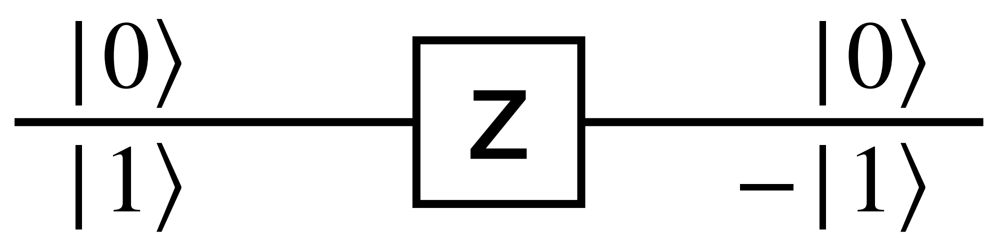

Pauli Gates

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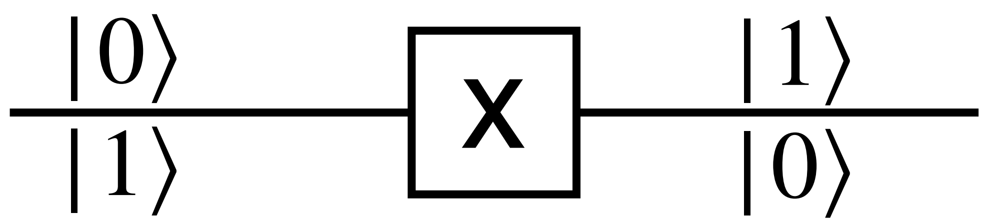

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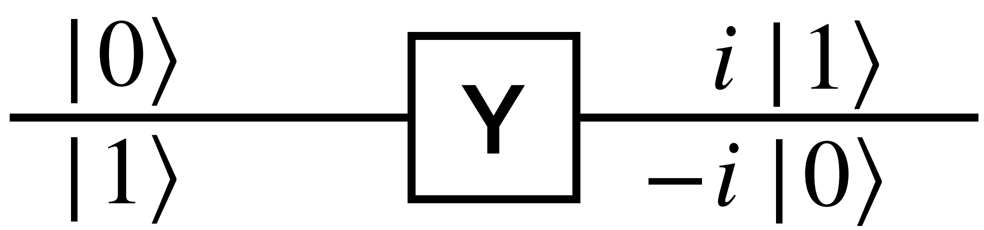

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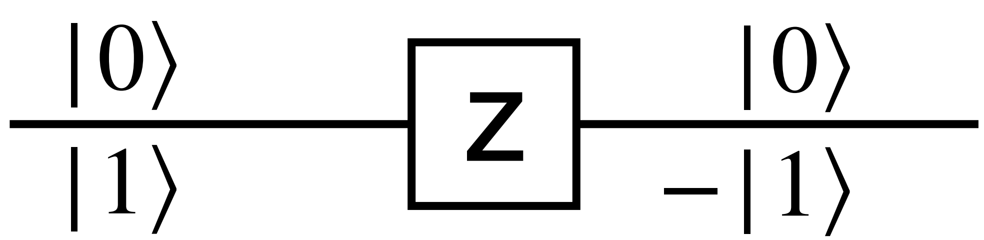

Pauli Gates

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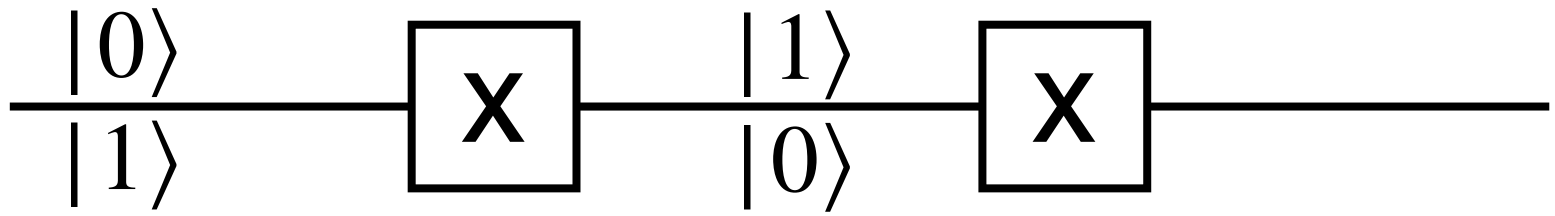
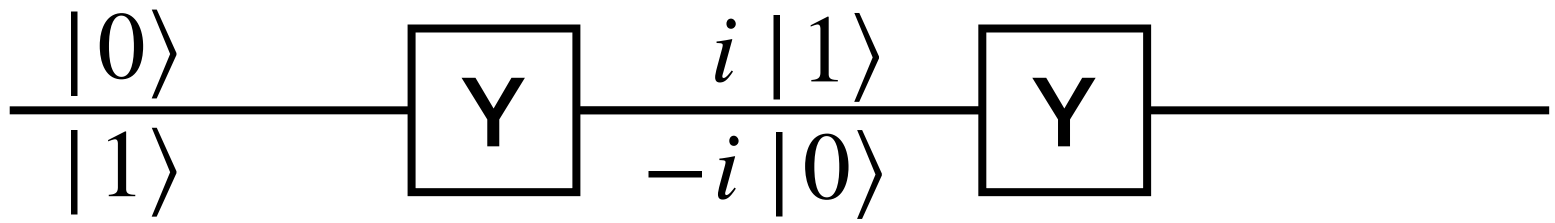
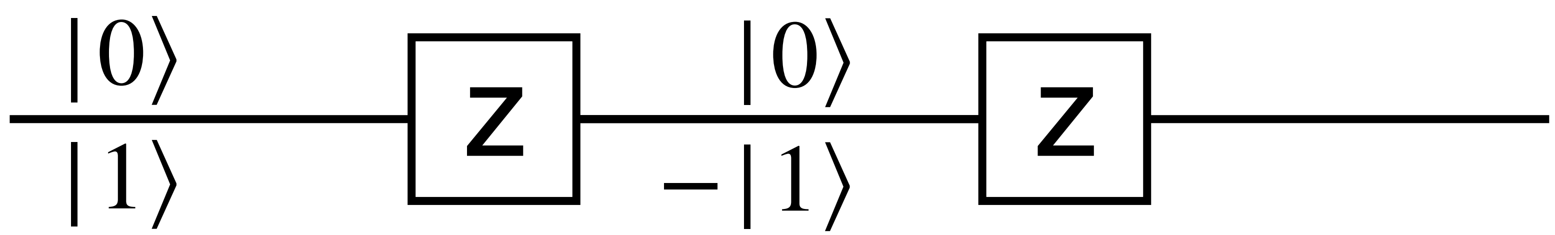


$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

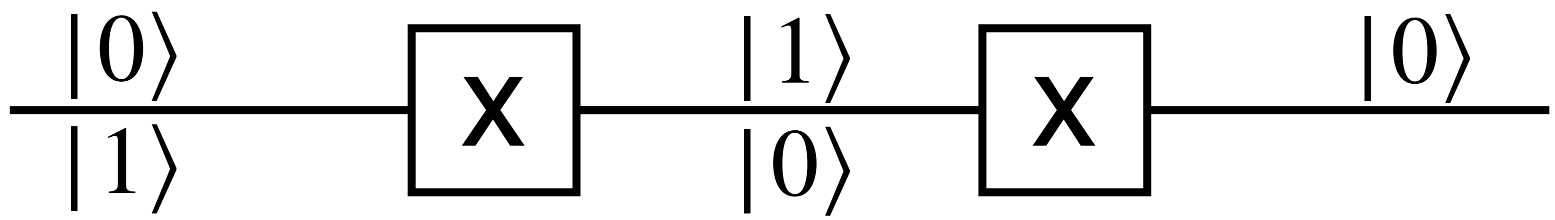


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

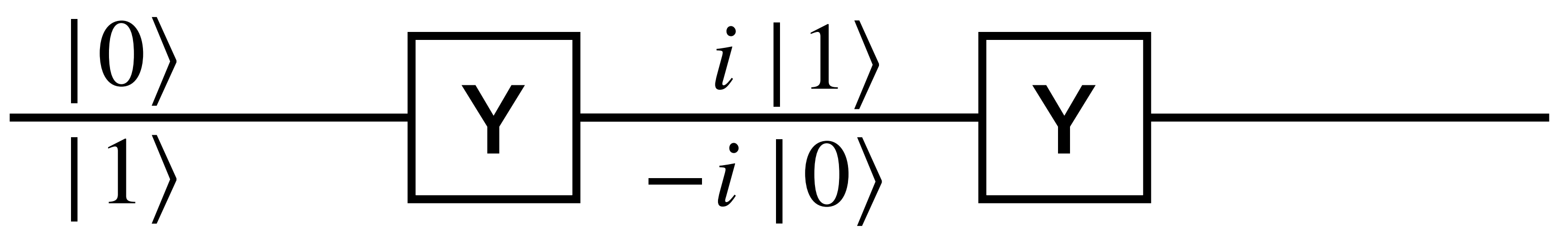


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



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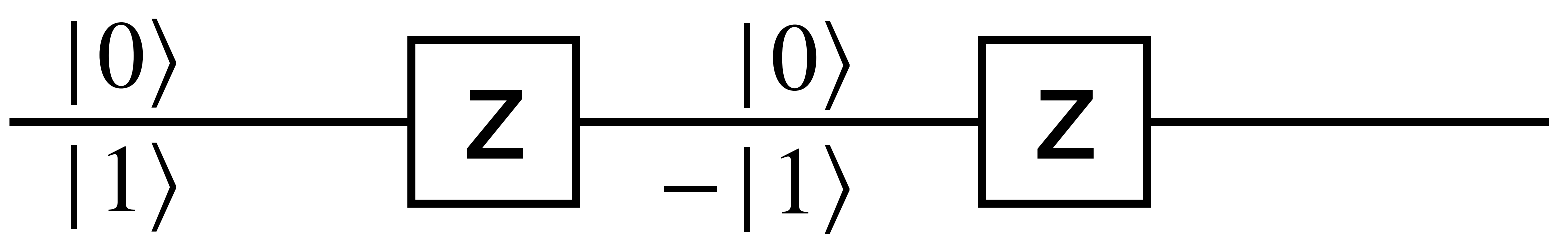

Pauli Gates

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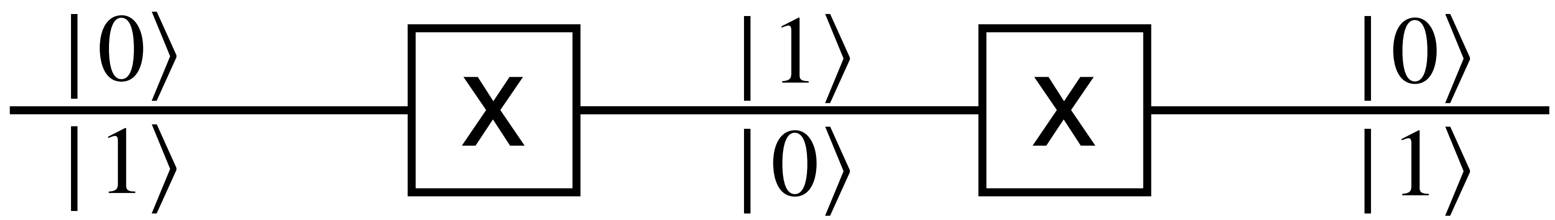
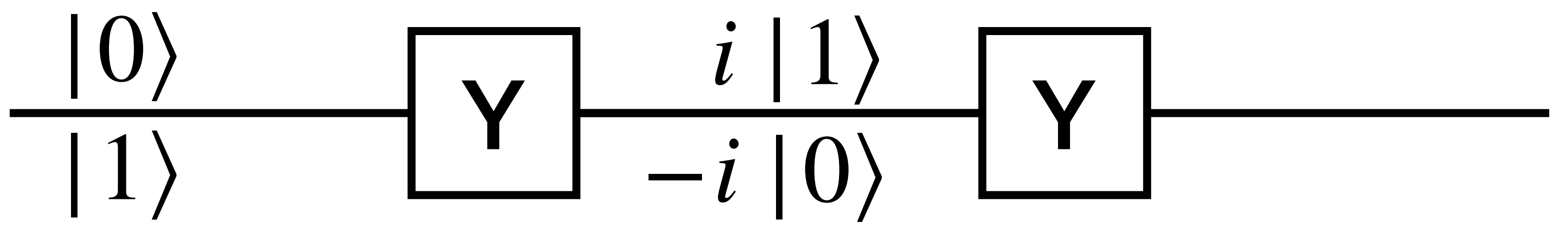


$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



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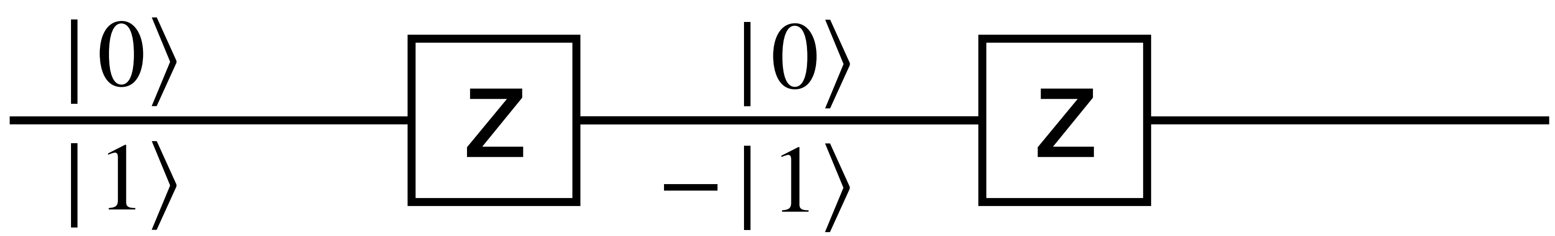

Pauli Gates

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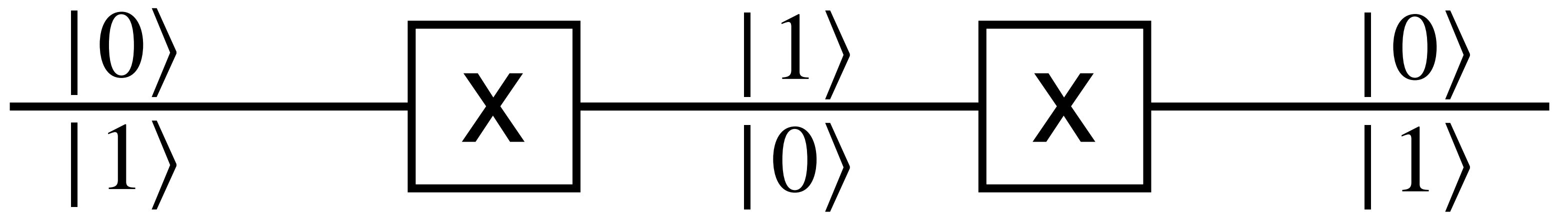

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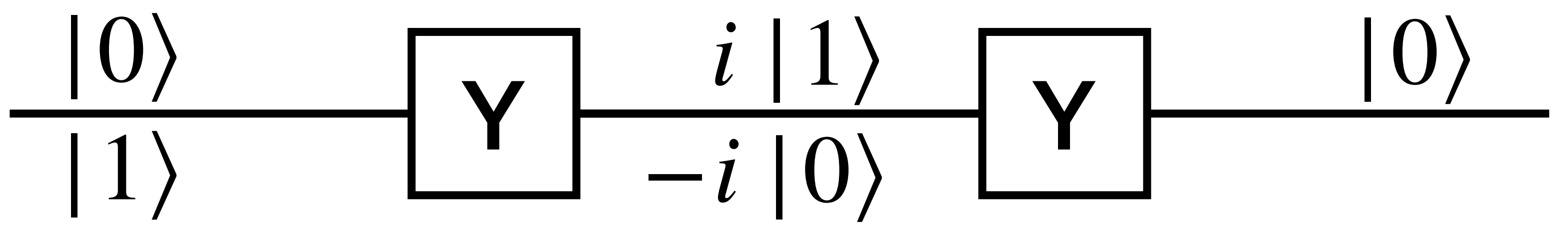

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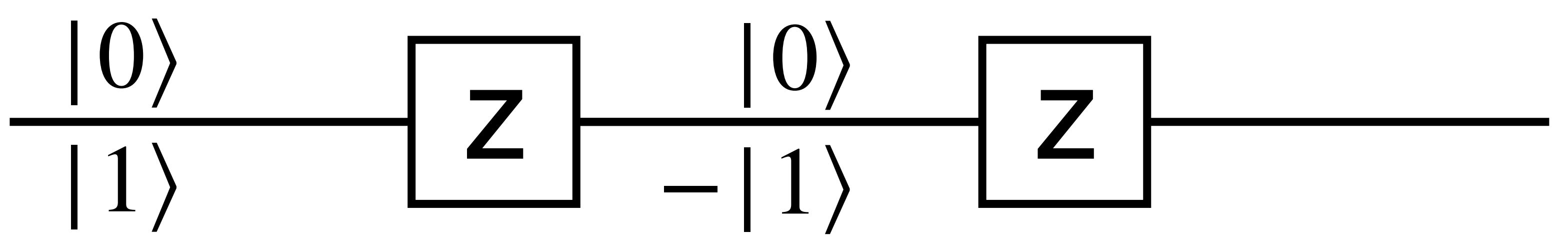
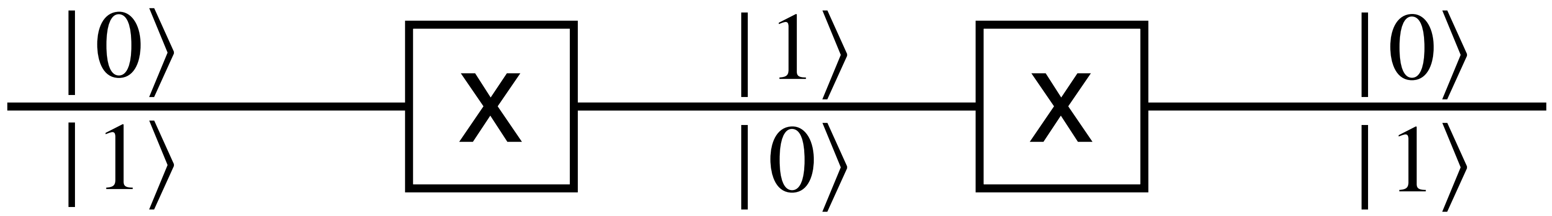
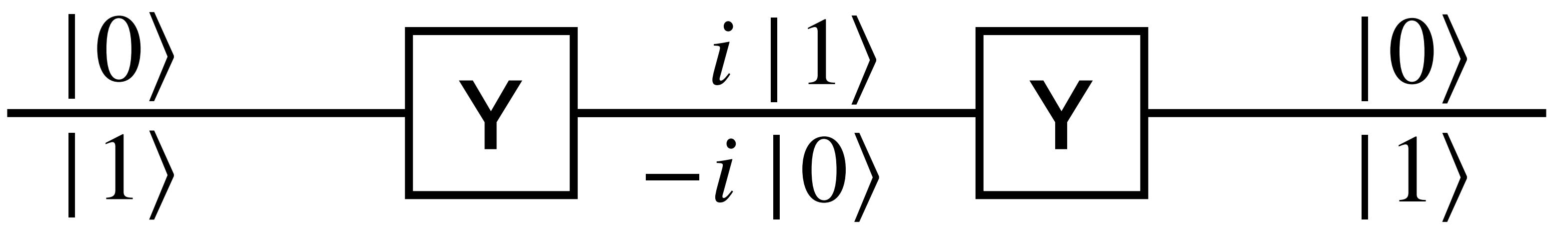


Pauli Gates

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



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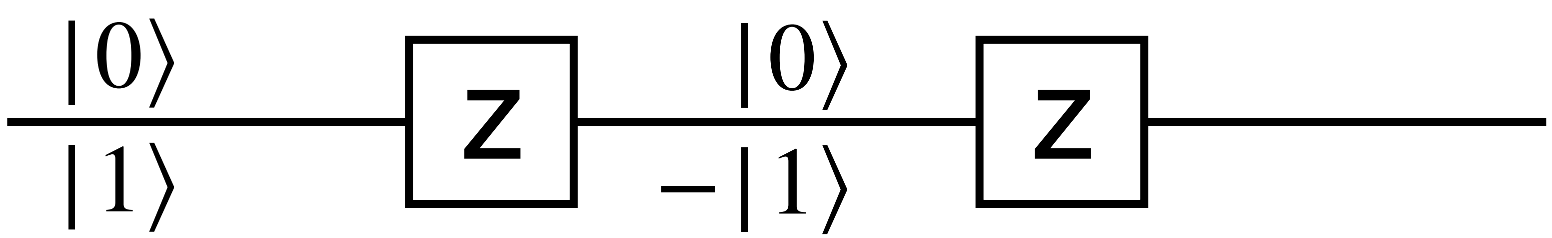
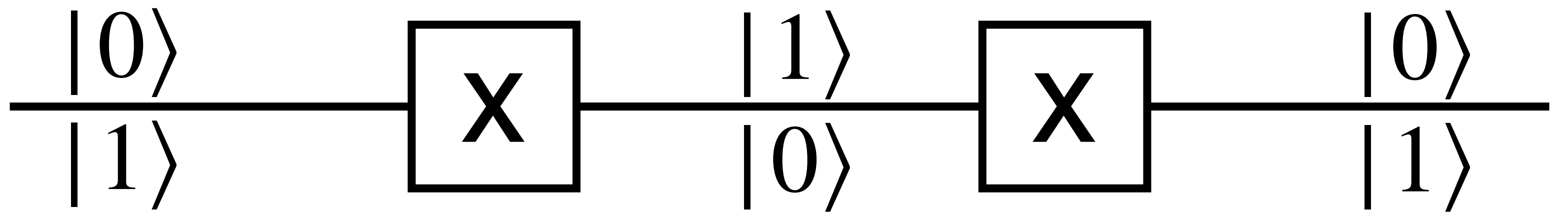
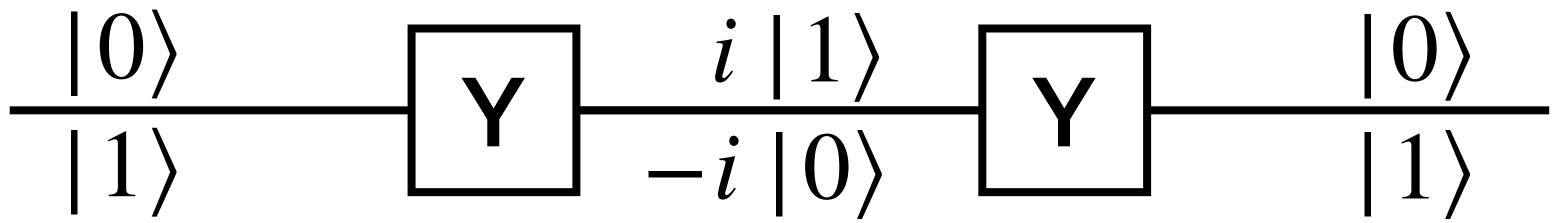


Pauli Gates

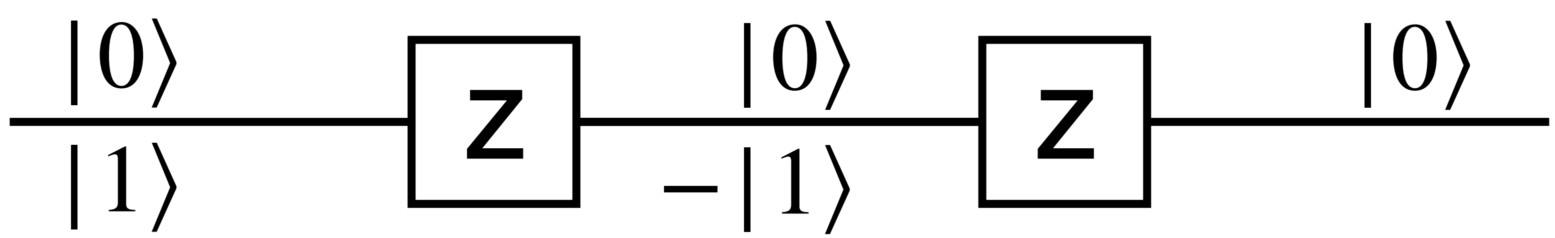
$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Pauli Gates

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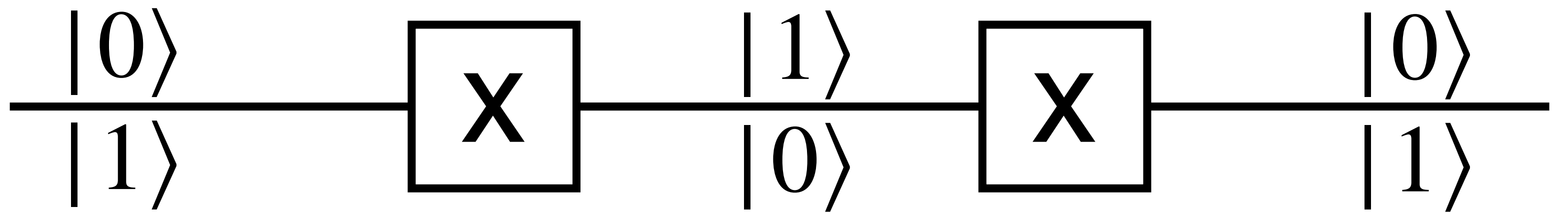

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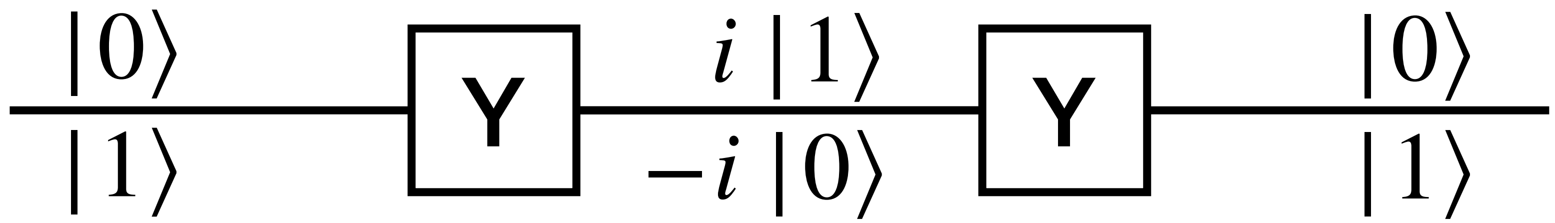
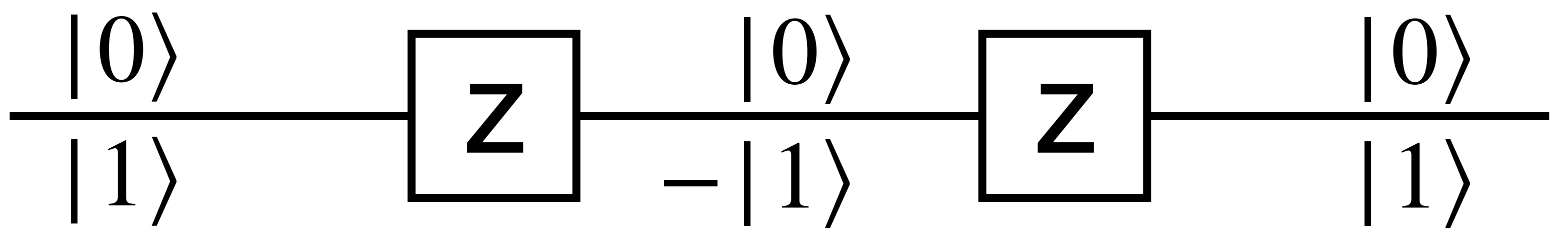
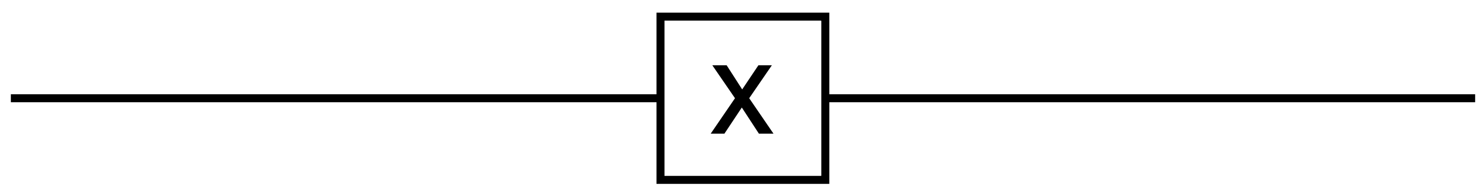


$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Pauli Gates



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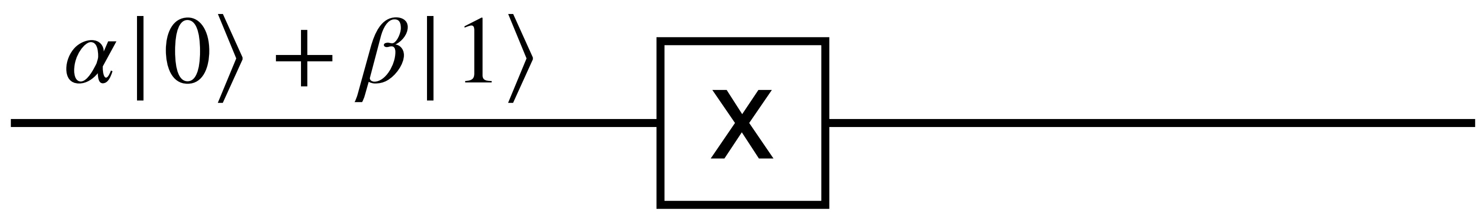
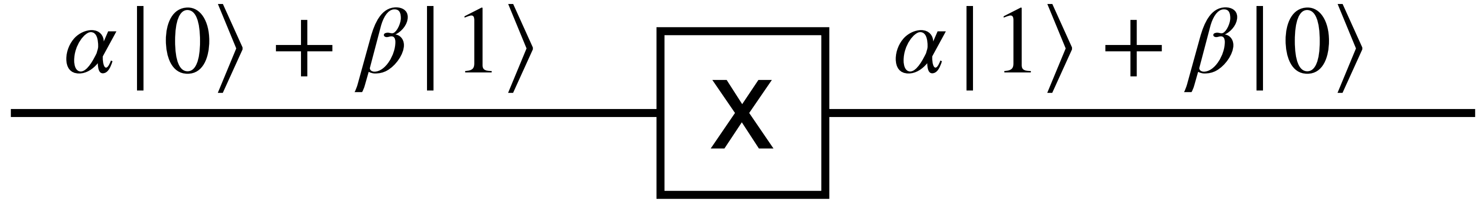
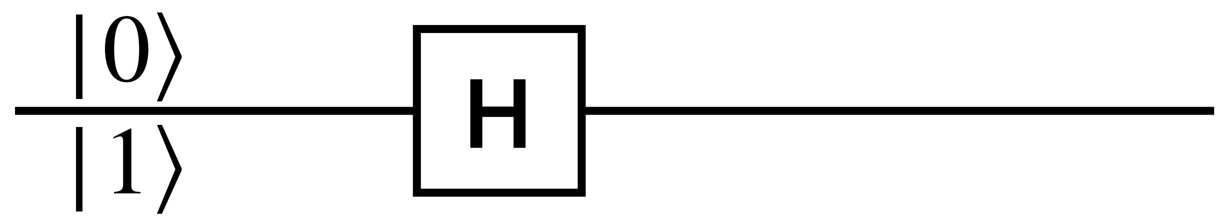


Pauli Gates



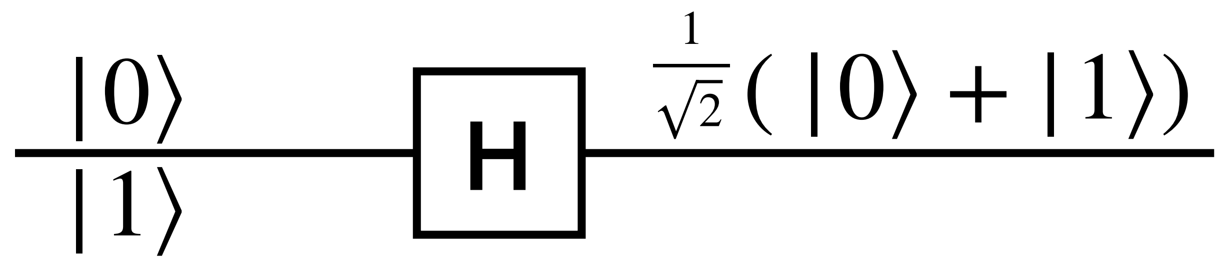
Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



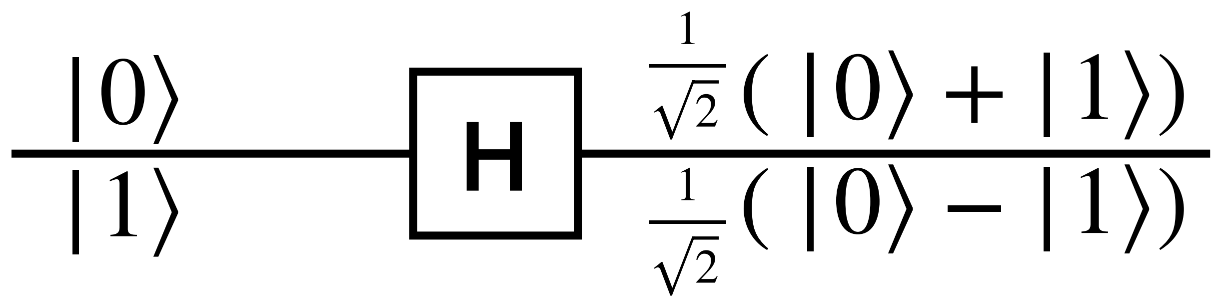
Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$|0\rangle \xrightarrow{\boxed{H}} \frac{1}{\sqrt{2}}(\,|0\rangle + |1\rangle) \equiv |+\rangle$$

$$|1\rangle \xrightarrow{\boxed{H}} \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle)$$

Clifford Gates

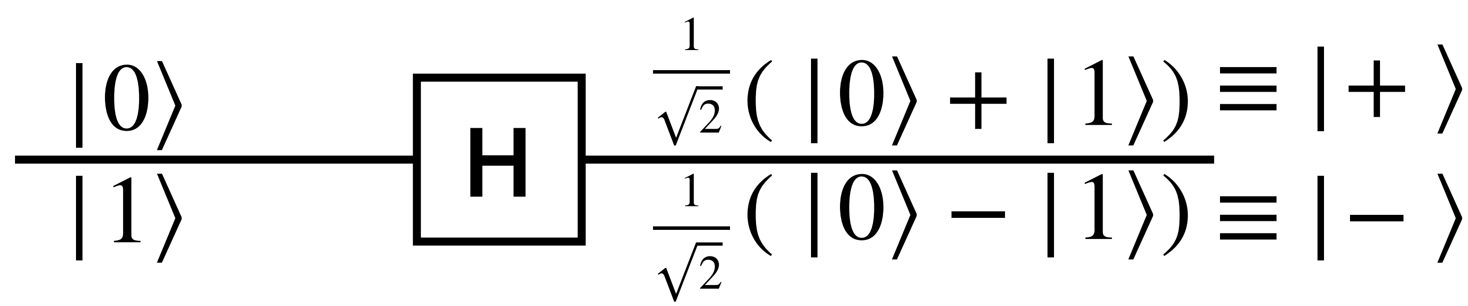
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$|0\rangle \xrightarrow{\boxed{H}} \frac{1}{\sqrt{2}}(\,|0\rangle + |1\rangle) \equiv |+\,\rangle$$

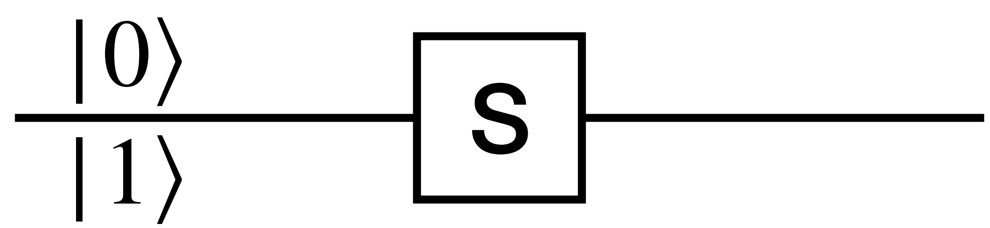
$$|1\rangle \xrightarrow{\boxed{H}} \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle) \equiv |-\,\rangle$$

Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

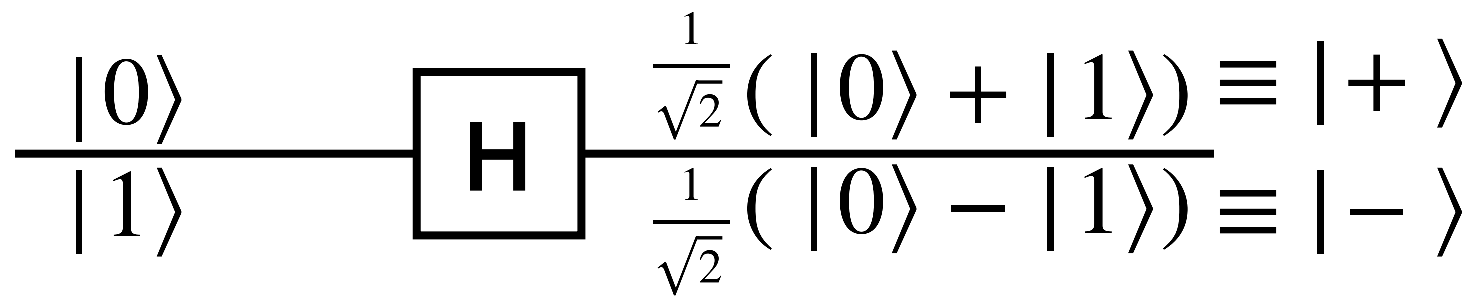


$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

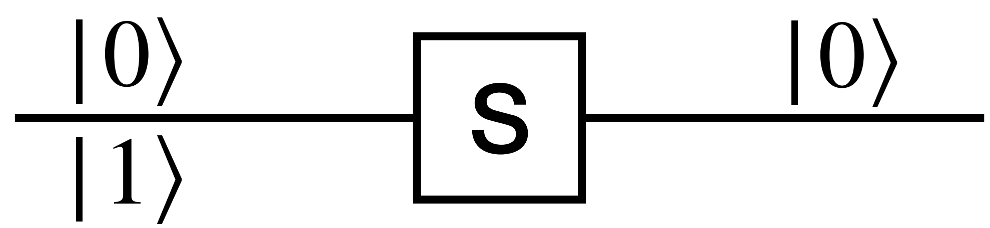


Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

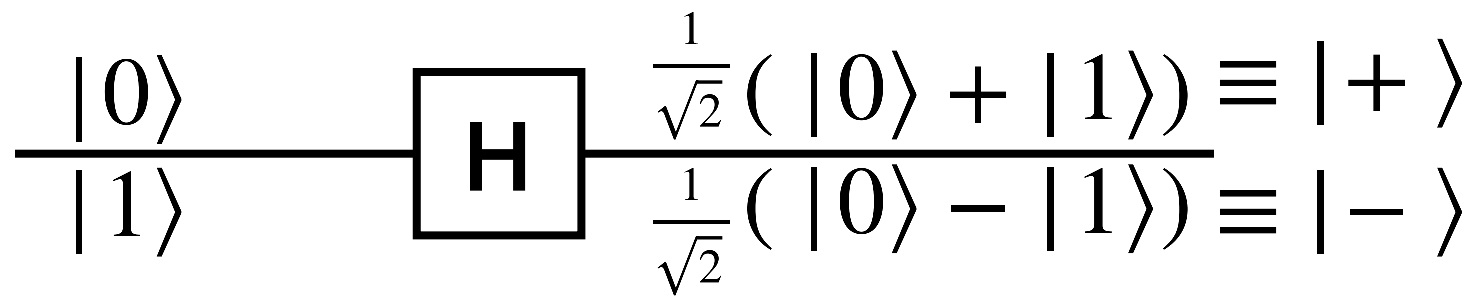


$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

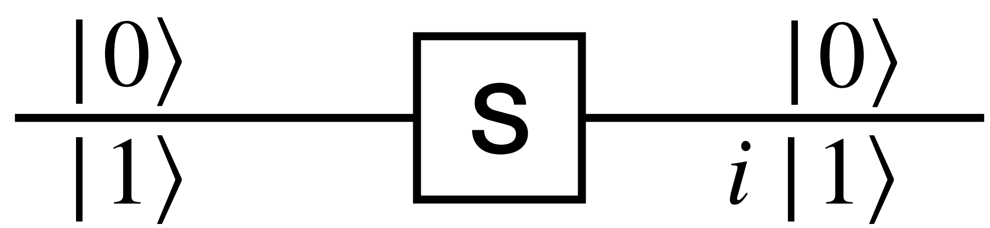


Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

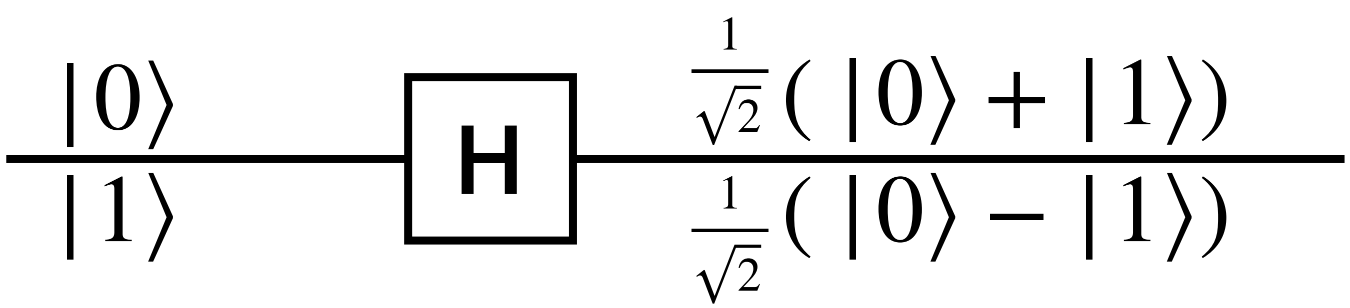


$$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$



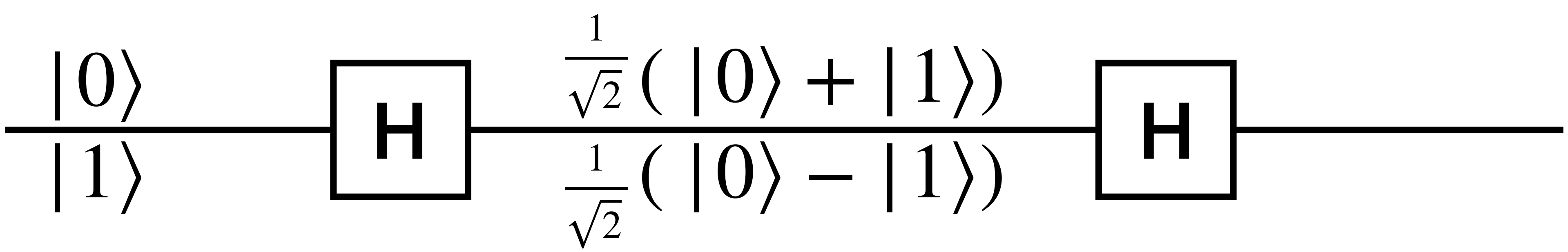
Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



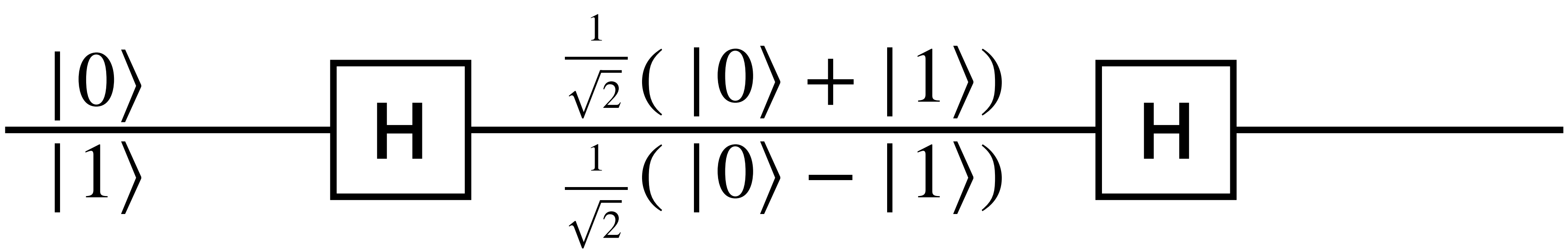
Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



Clifford Gates

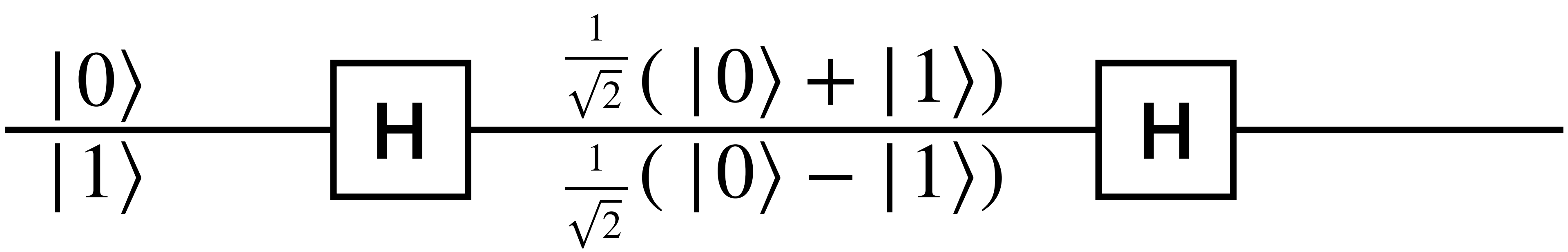
$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



$$\frac{1}{\sqrt{2}}\left(H|0\rangle + H|1\rangle\right)$$

Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

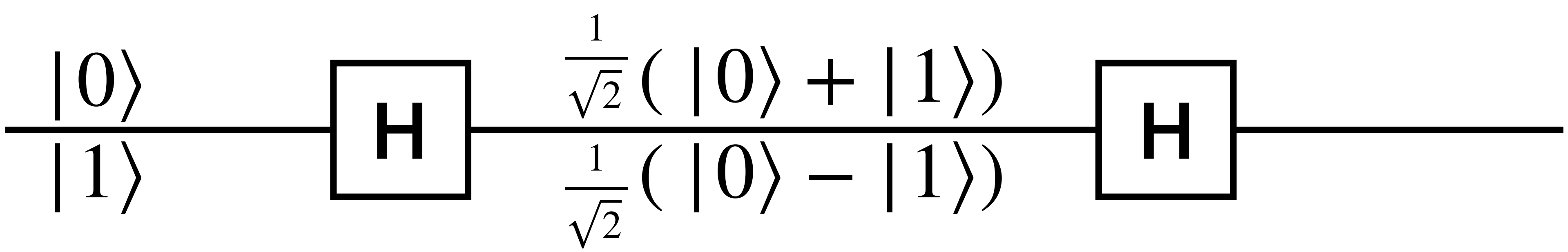


$$\frac{1}{\sqrt{2}}\left(H\left|0\right\rangle + H\left|1\right\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(\left|0\right\rangle + \left|1\right\rangle\right) + \frac{1}{\sqrt{2}}\left(\left|0\right\rangle - \left|1\right\rangle\right)\right)$$

Clifford Gates

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$



$$\frac{1}{\sqrt{2}}\left(H\,|0\rangle + H\,|1\rangle\right)$$

$$\frac{1}{2}\Big(\quad (\,|0\rangle + |1\rangle) + \quad (\,|0\rangle - |1\rangle)\Big)$$

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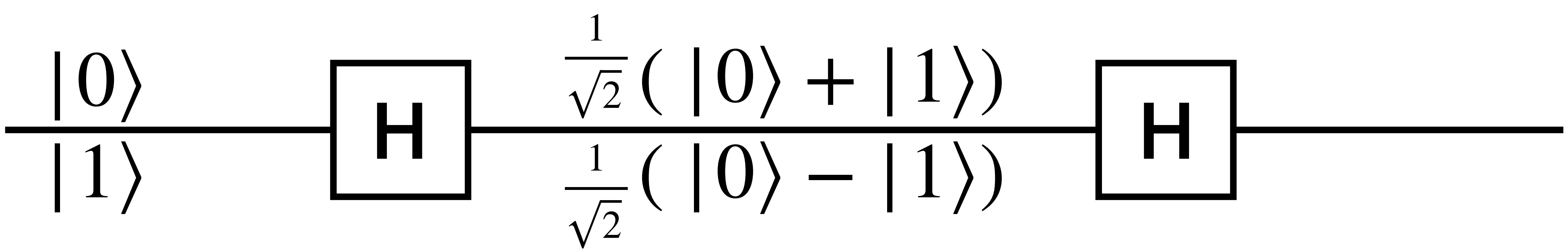

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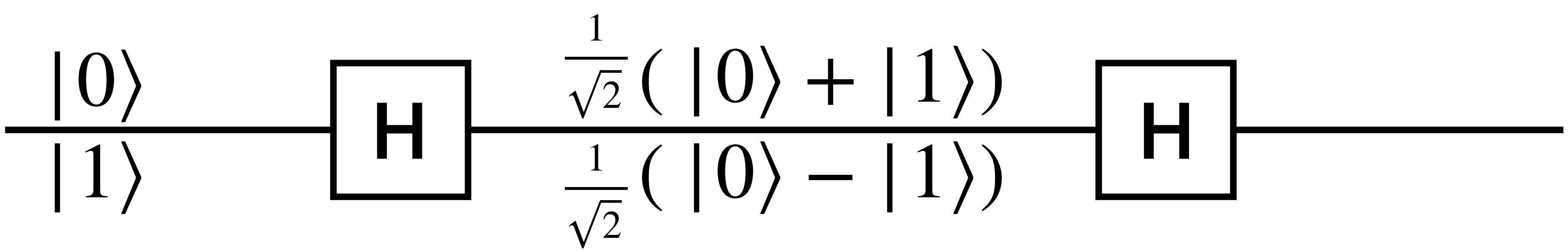

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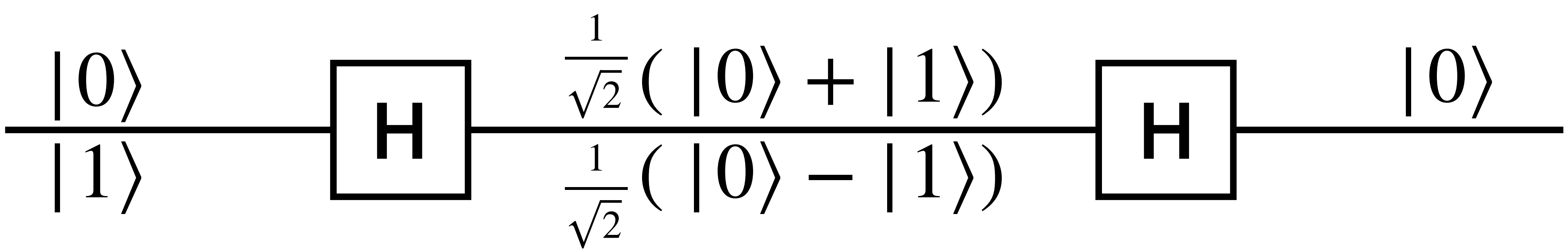

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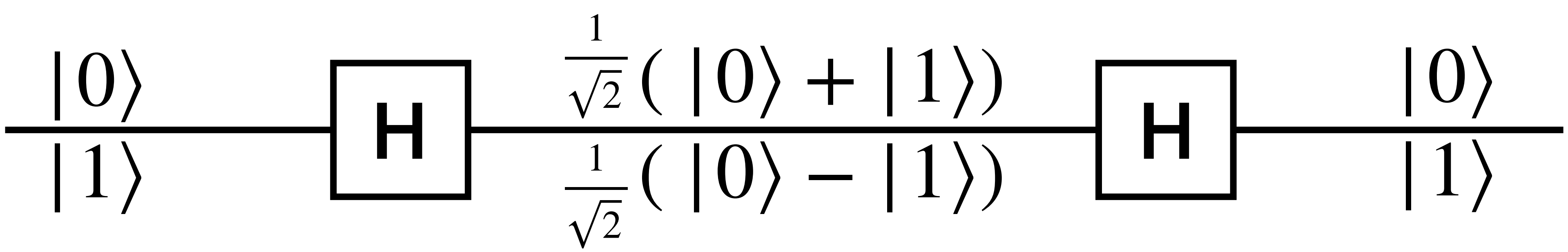
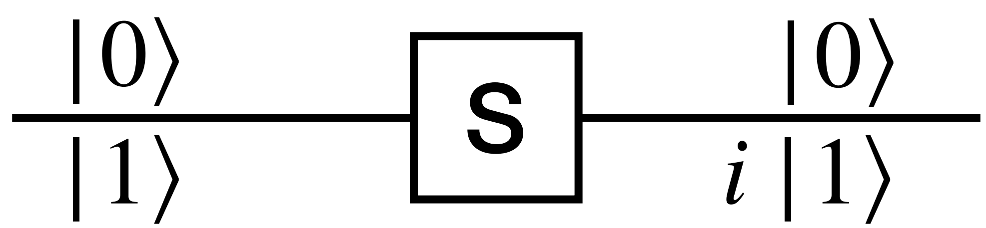


$$\frac{1}{\sqrt{2}}\left(H|0\rangle + H|1\rangle\right)$$

$$|0\rangle$$

Clifford Gates

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$



Clifford Gates

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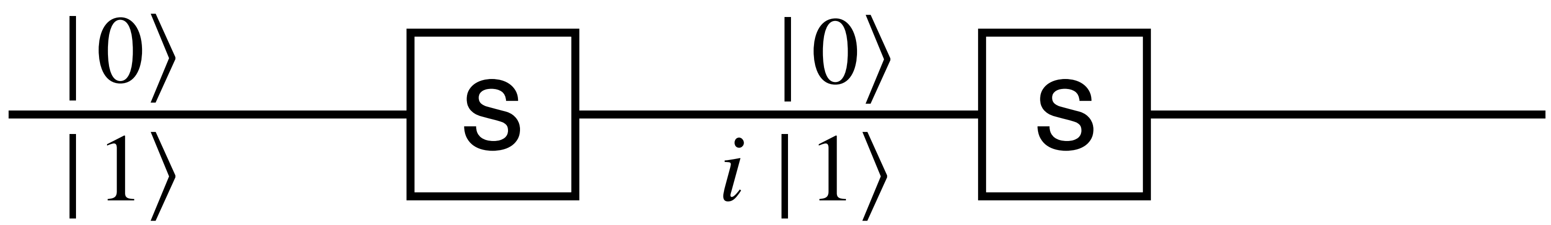

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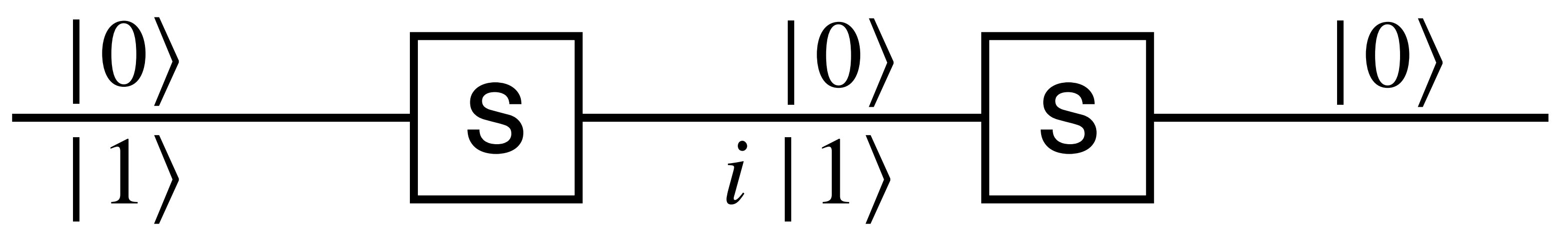

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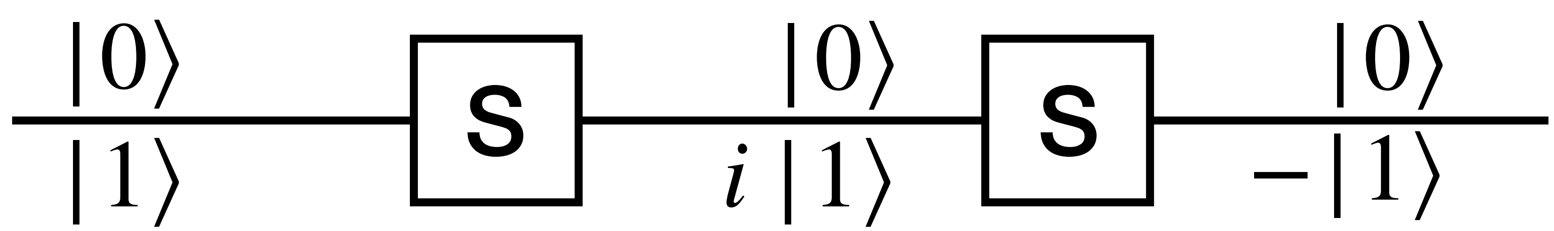

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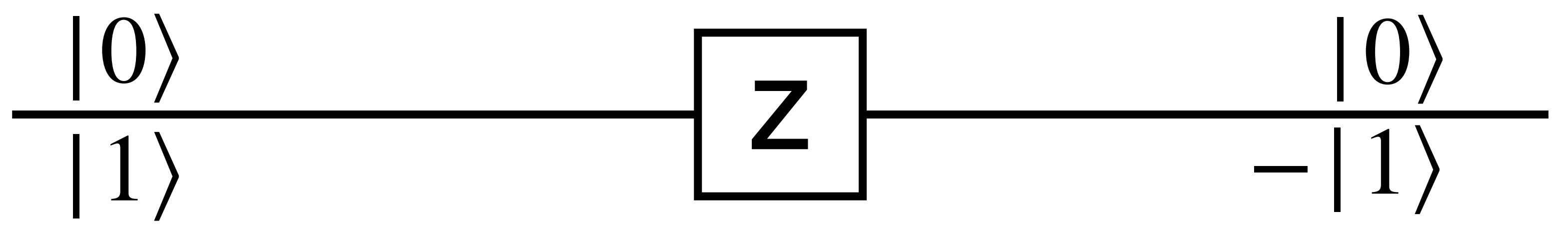
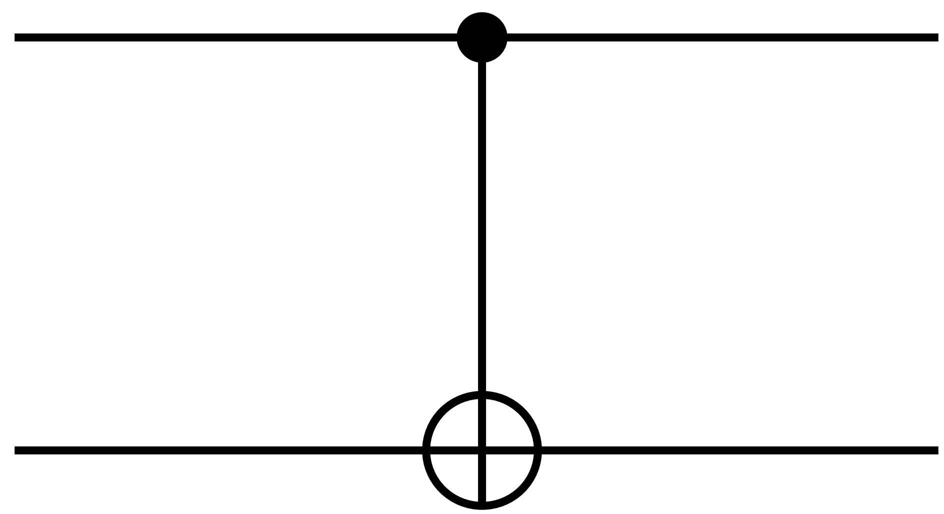


Clifford Gates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



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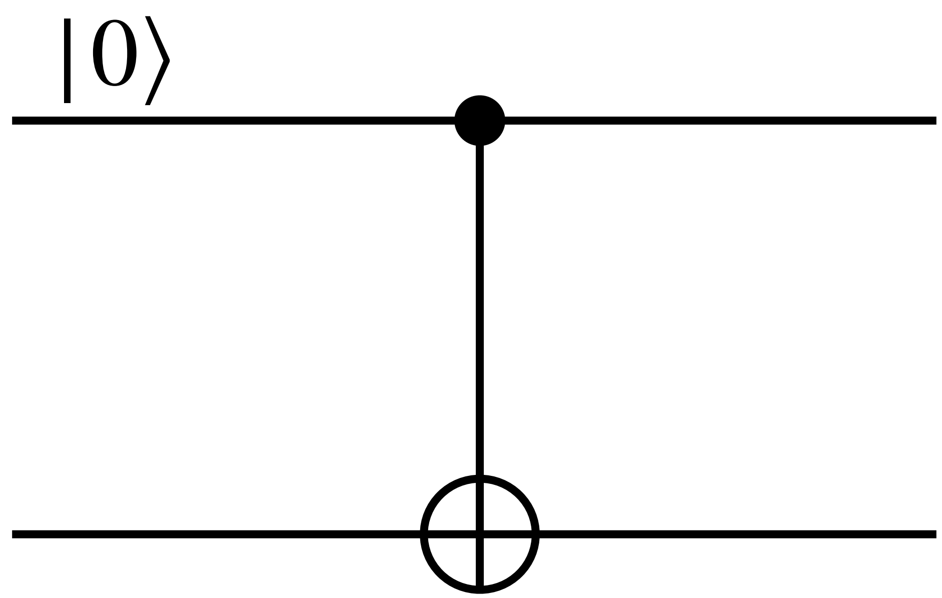
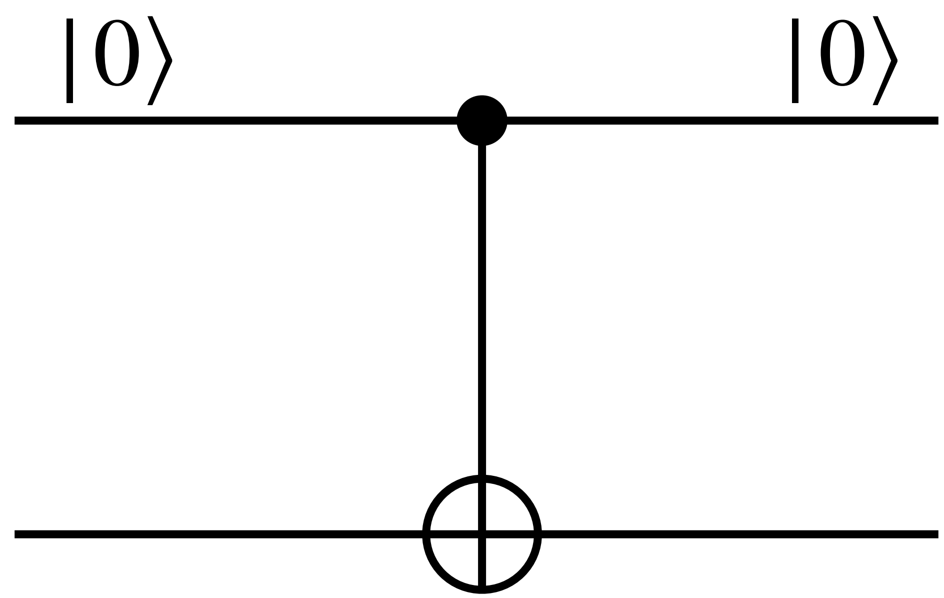


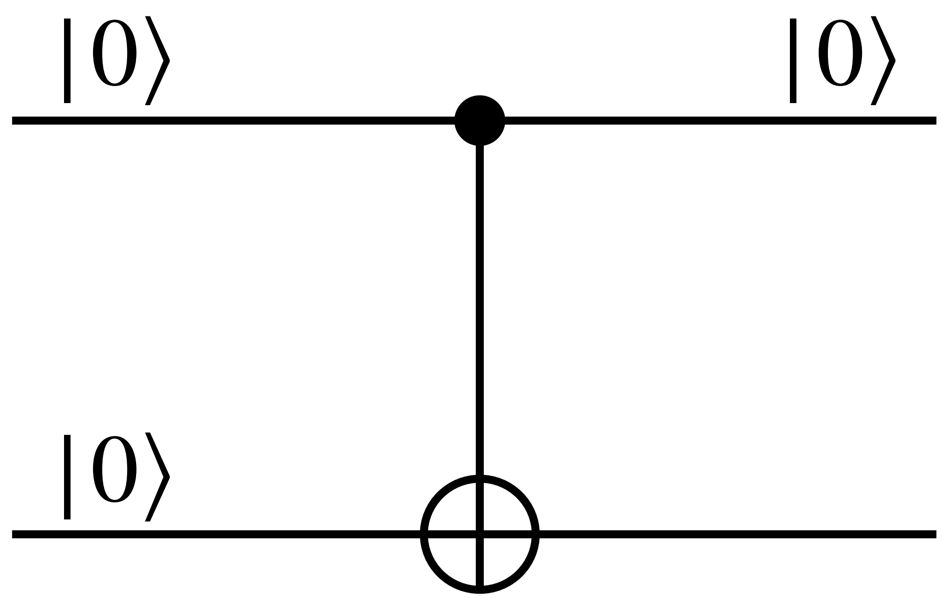
Clifford Gates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Clifford Gates

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



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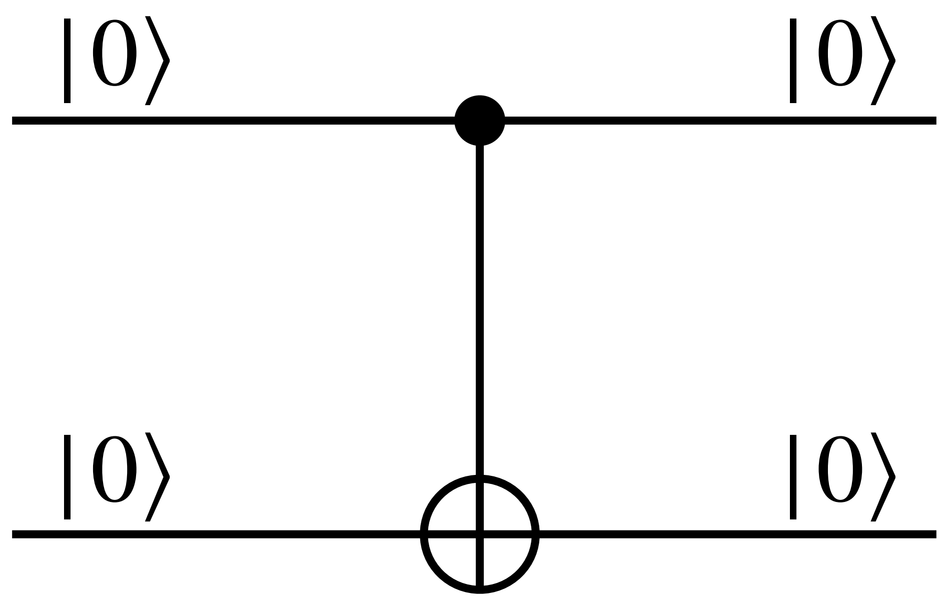

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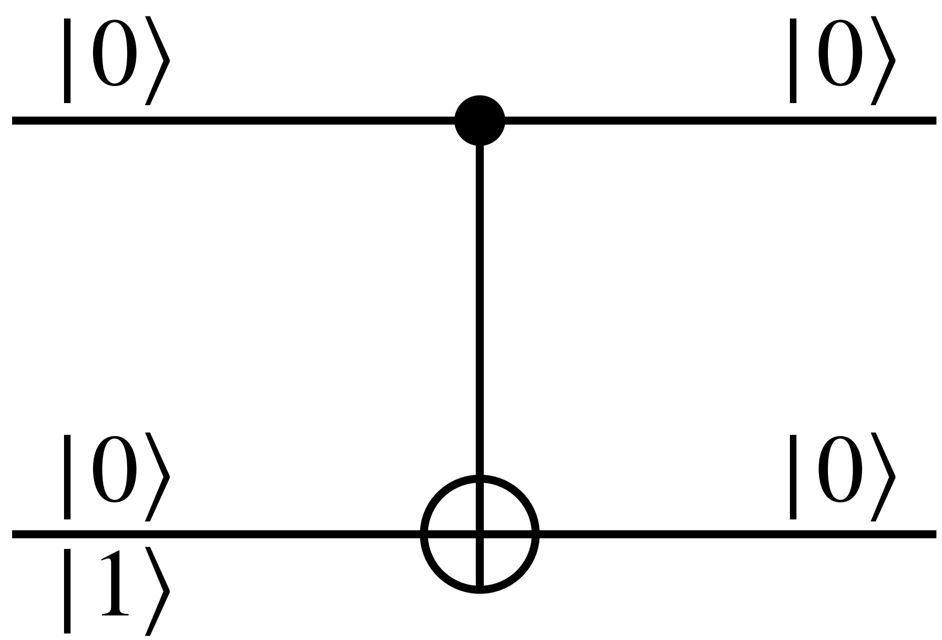

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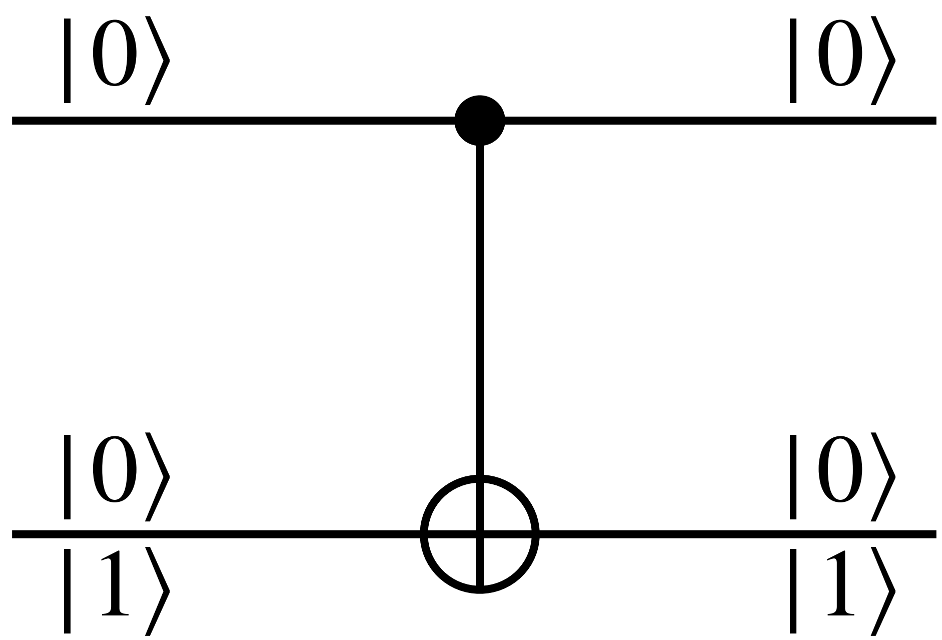

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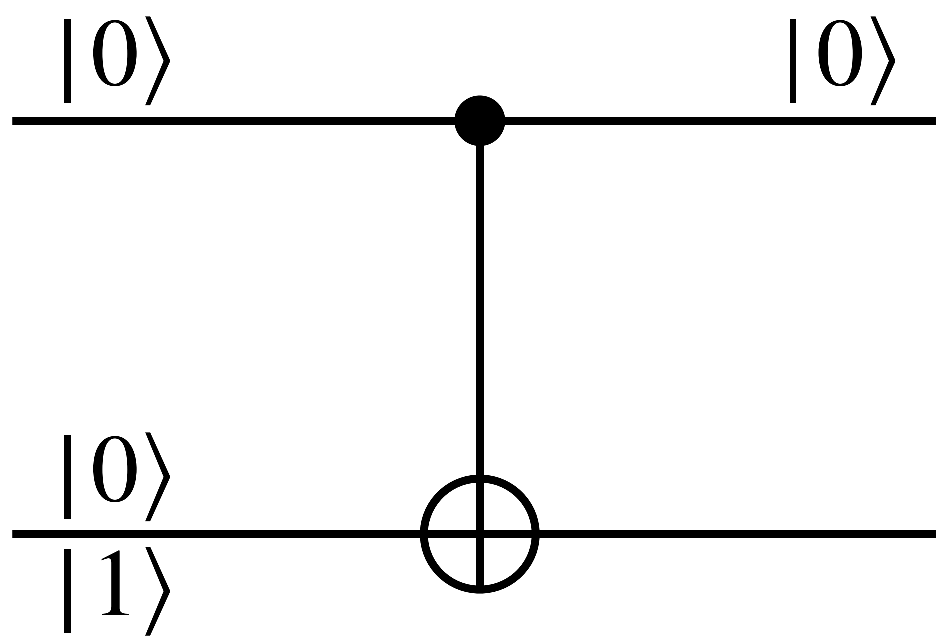

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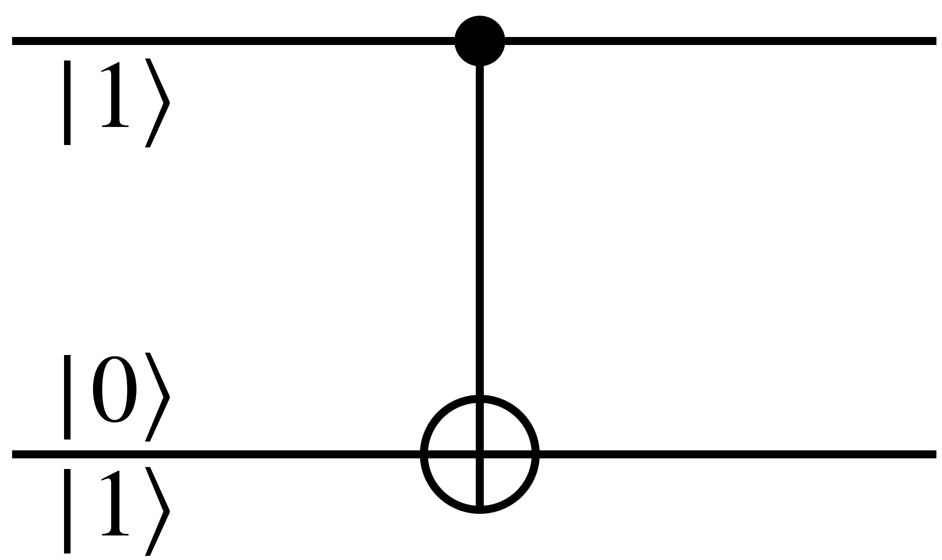

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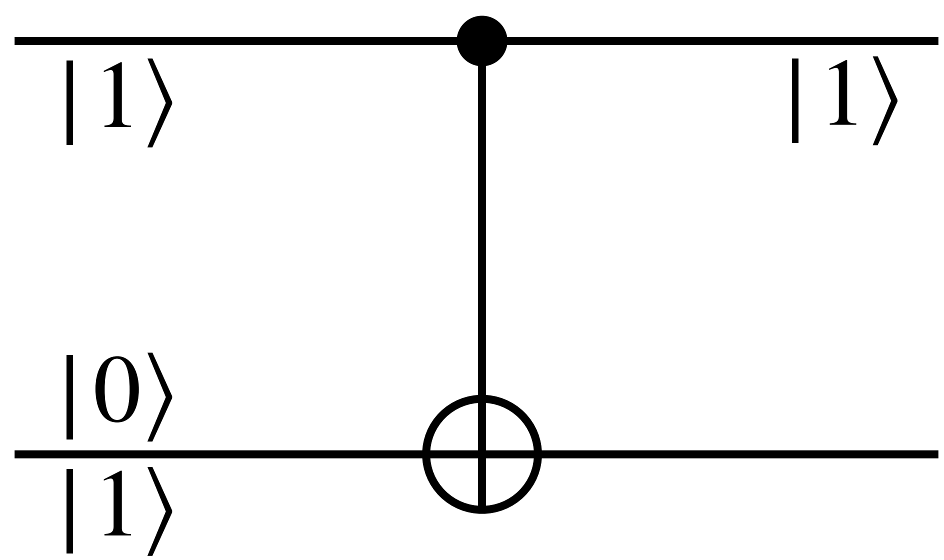

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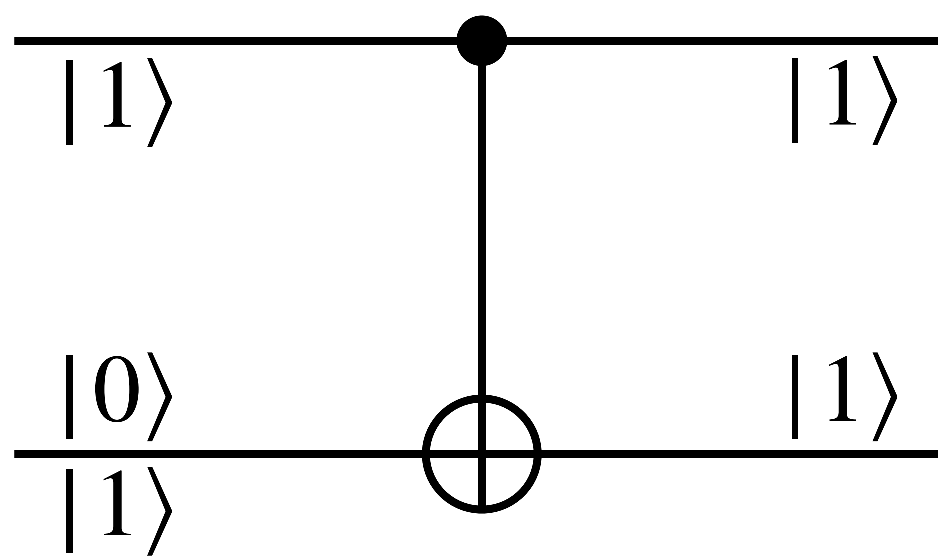

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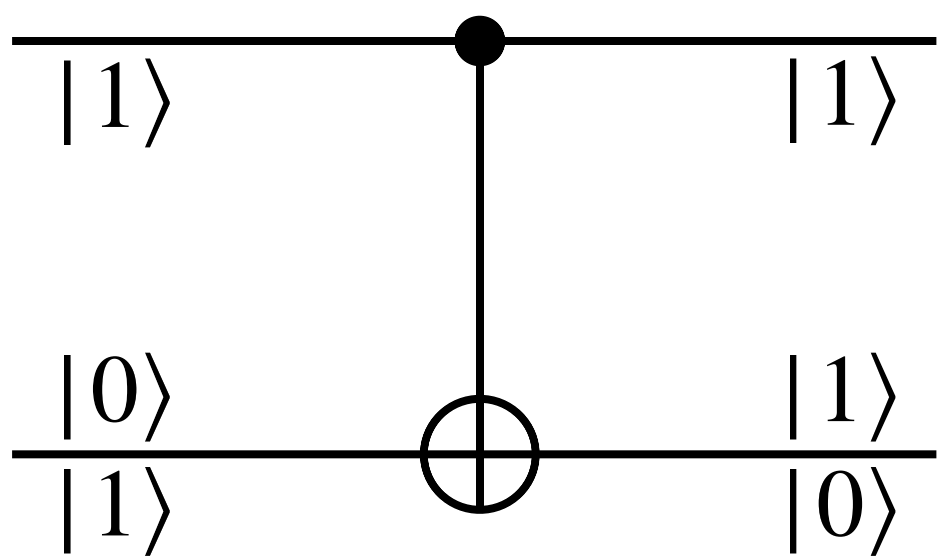
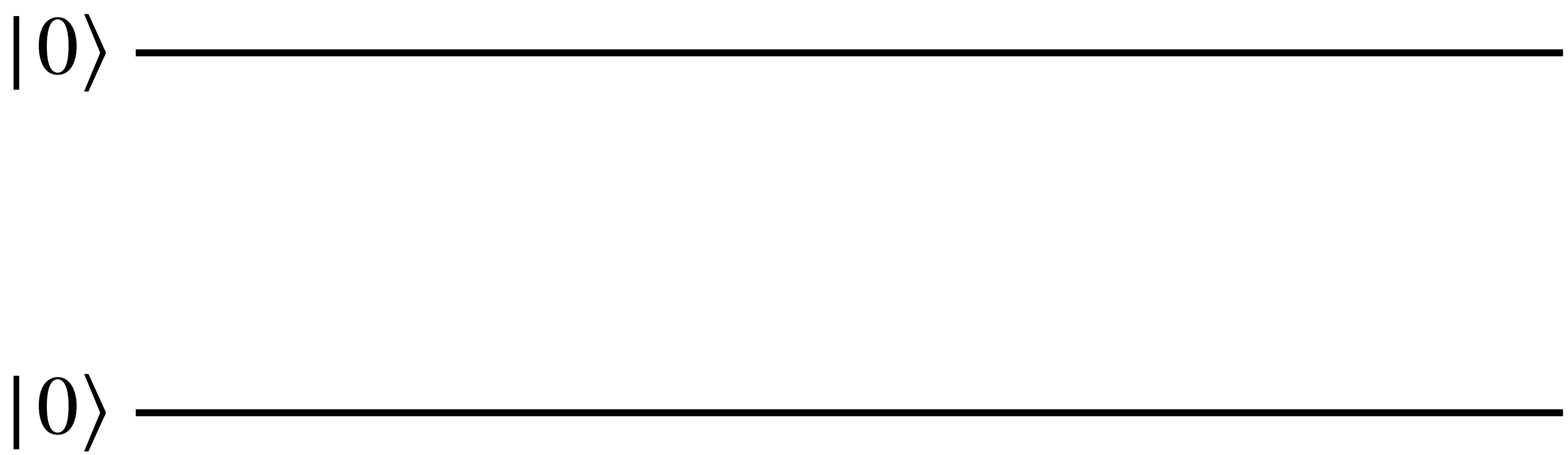


Bell Pair

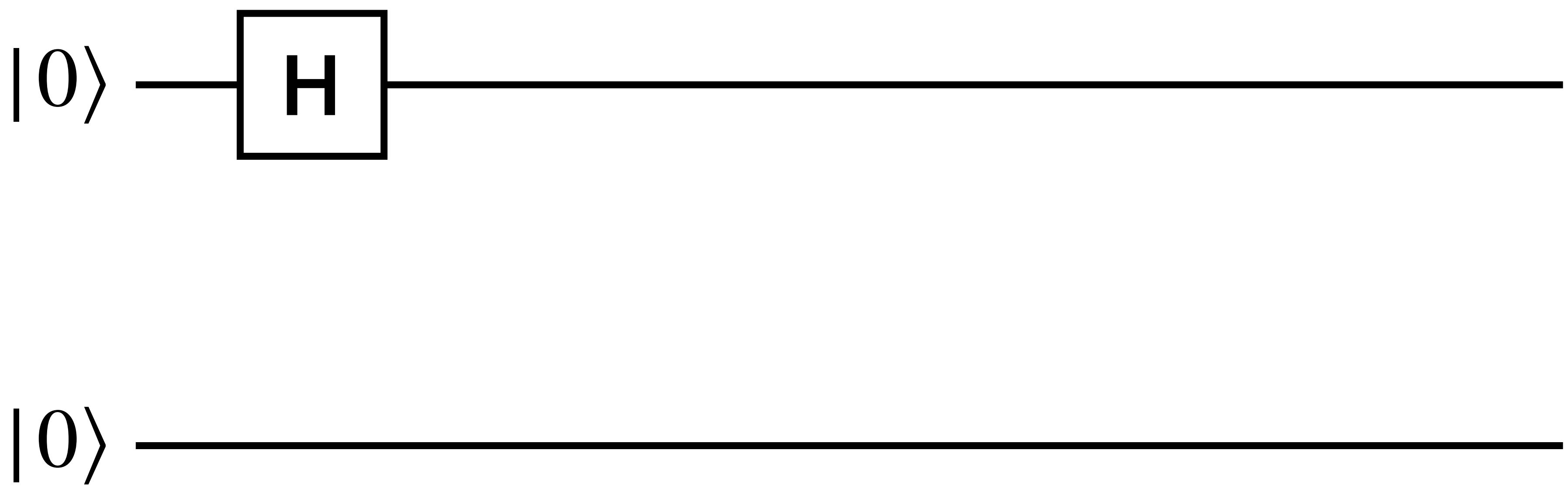
Bell Pair

$|0\rangle$

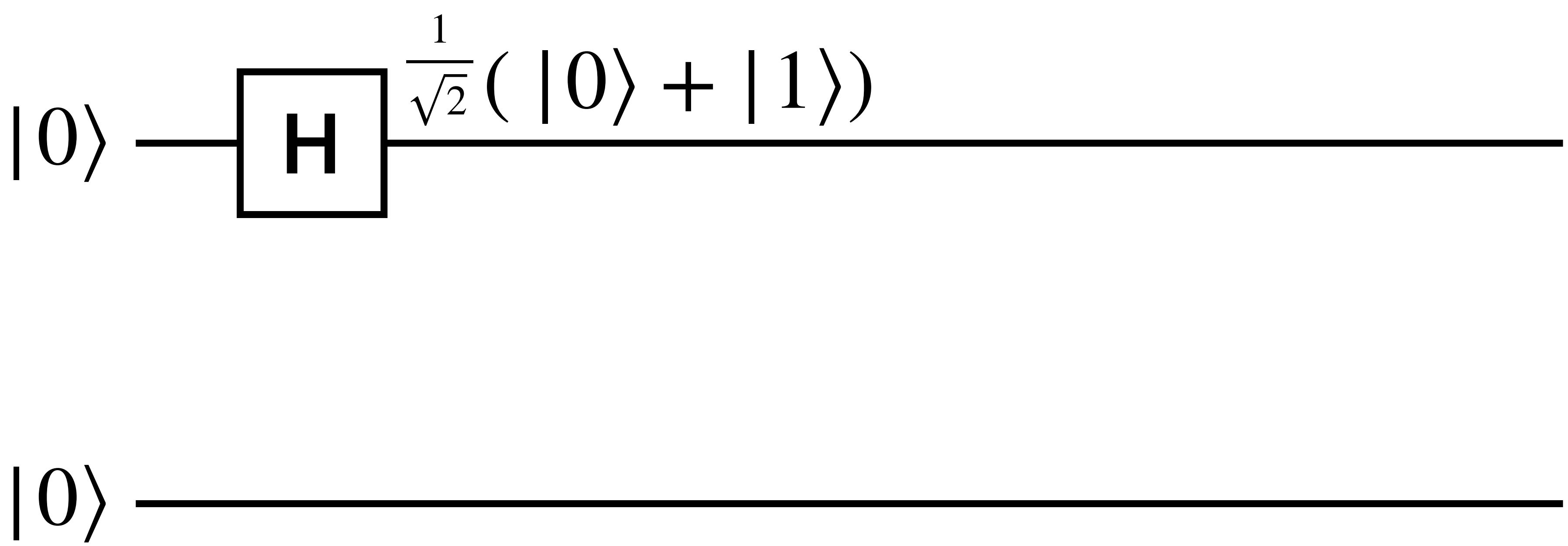
Bell Pair



Bell Pair



Bell Pair



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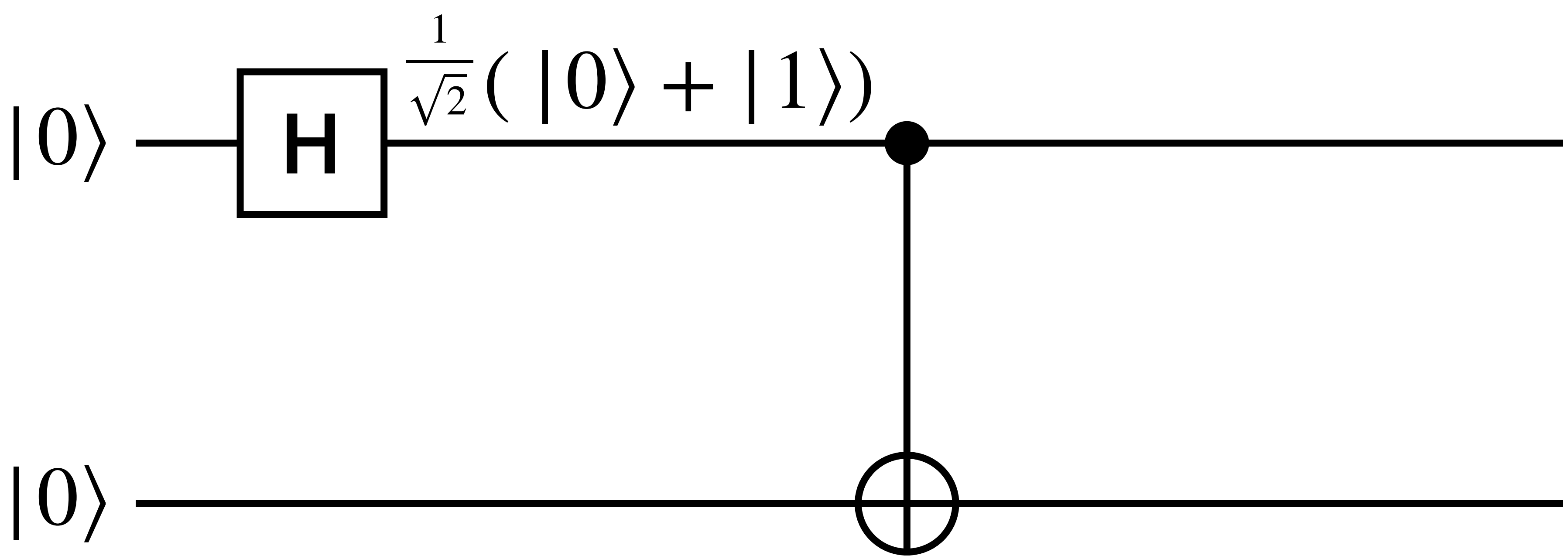
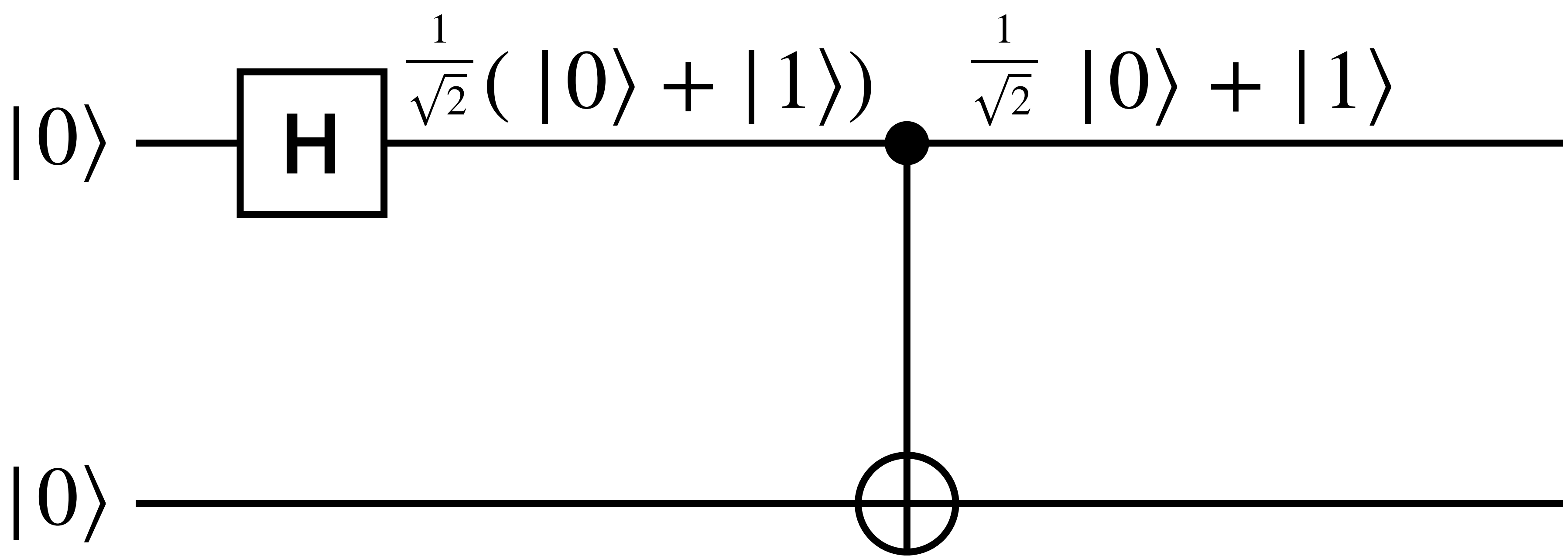
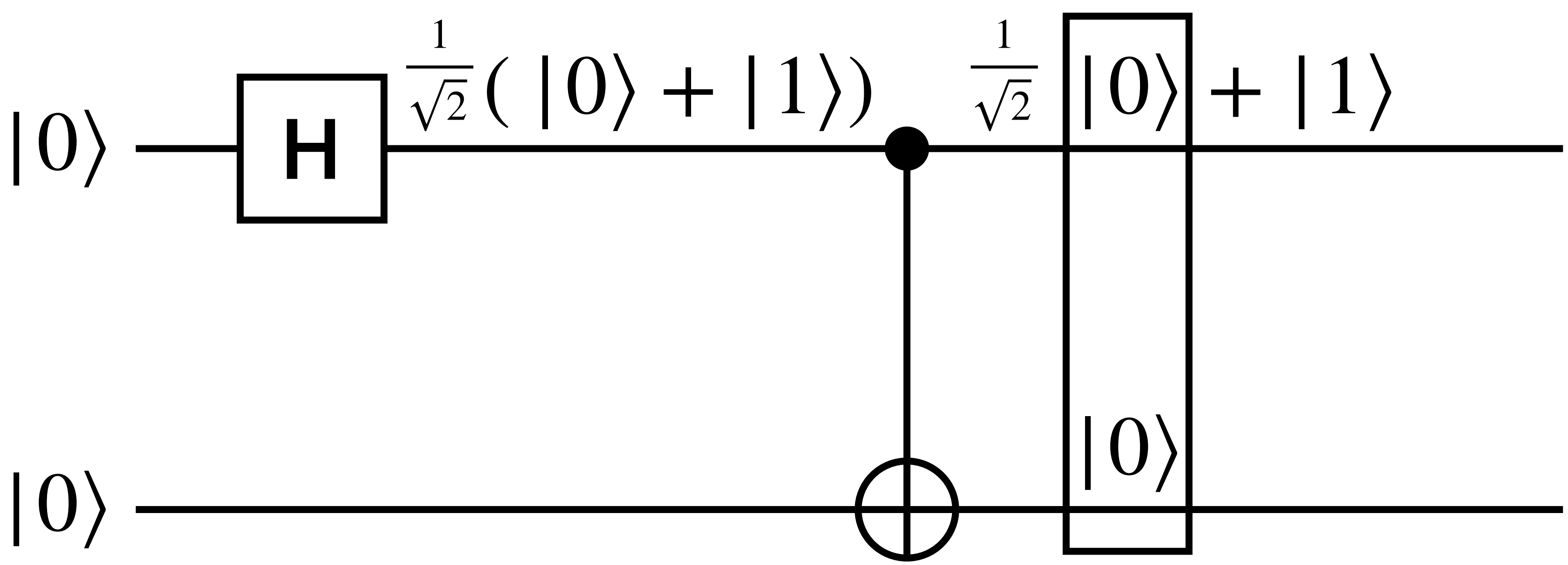


Bell Pair

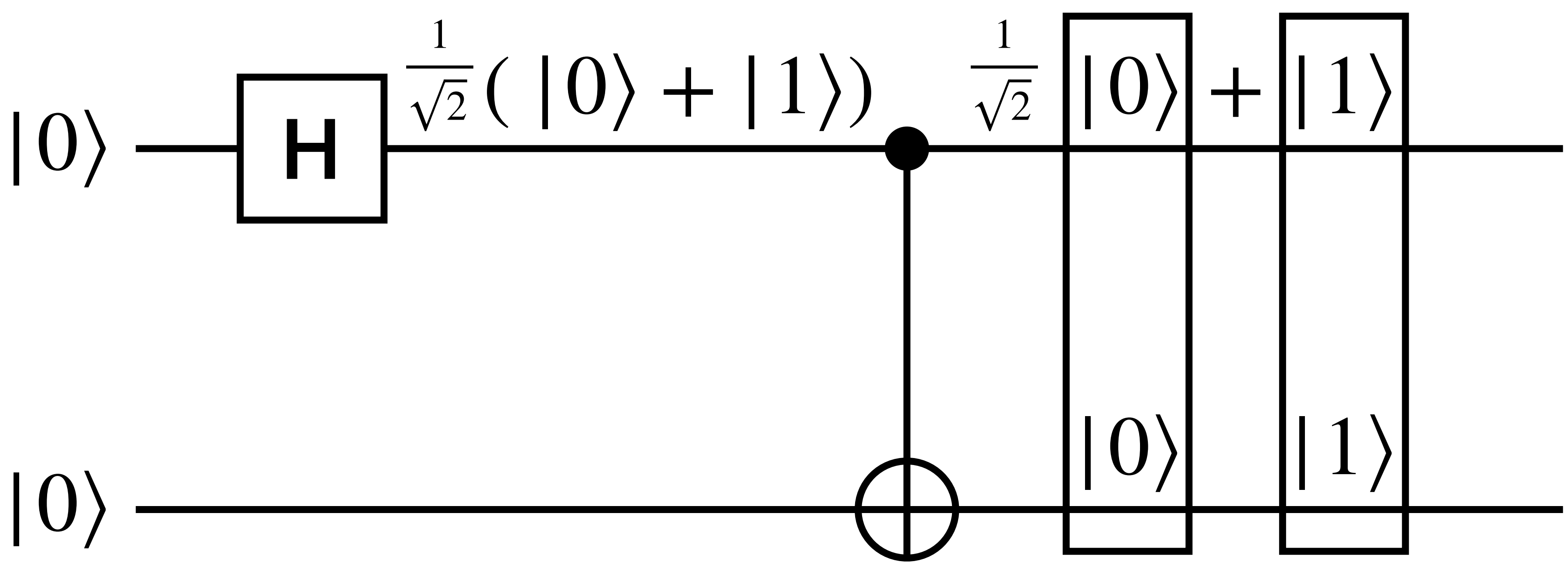

$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$\frac{1}{\sqrt{2}}\ |0\rangle + |1\rangle$
$|0\rangle$

Bell Pair



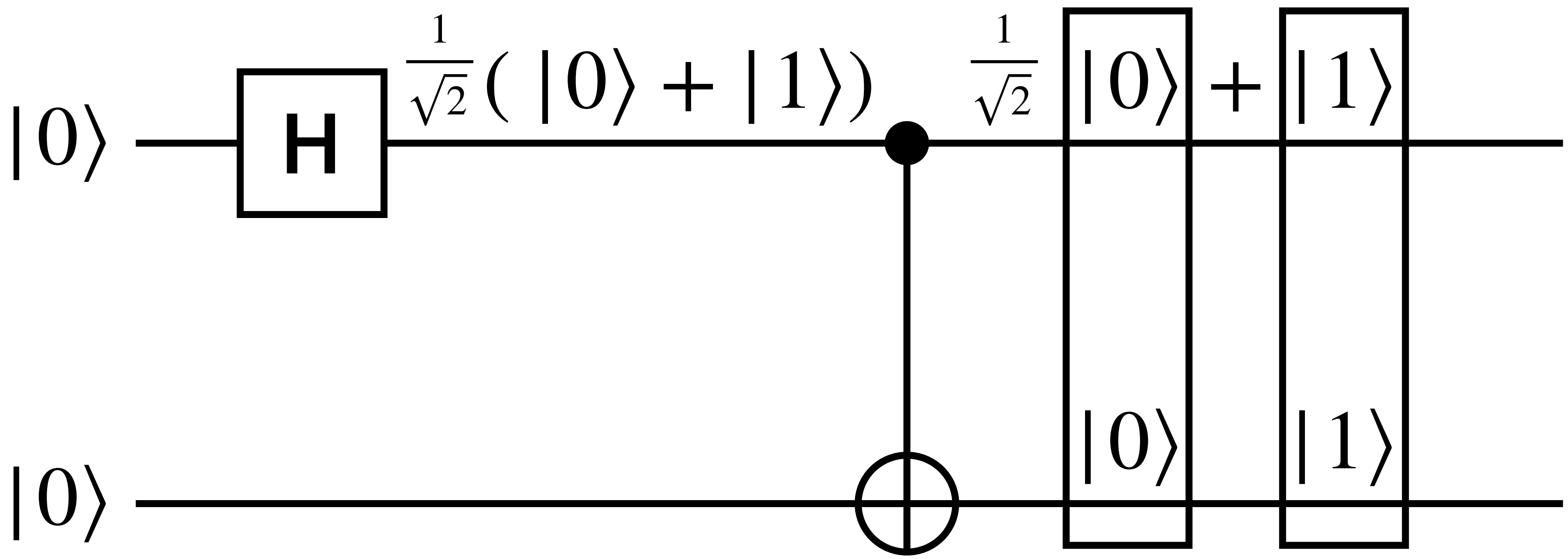
$|0\rangle$
$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$\frac{1}{\sqrt{2}}$
$|0\rangle$
$|0\rangle$
$+ |1\rangle$

Bell Pair



$|0\rangle$
H
$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
$\frac{1}{\sqrt{2}}$
$|0\rangle$
+
$|1\rangle$
$|0\rangle$
$|1\rangle$
$|0\rangle$

Bell Pair



$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Gottesman-Knill

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- We call H + S + CNOT the *Clifford Set.* (We can construct the Pauli gates from H + S.)

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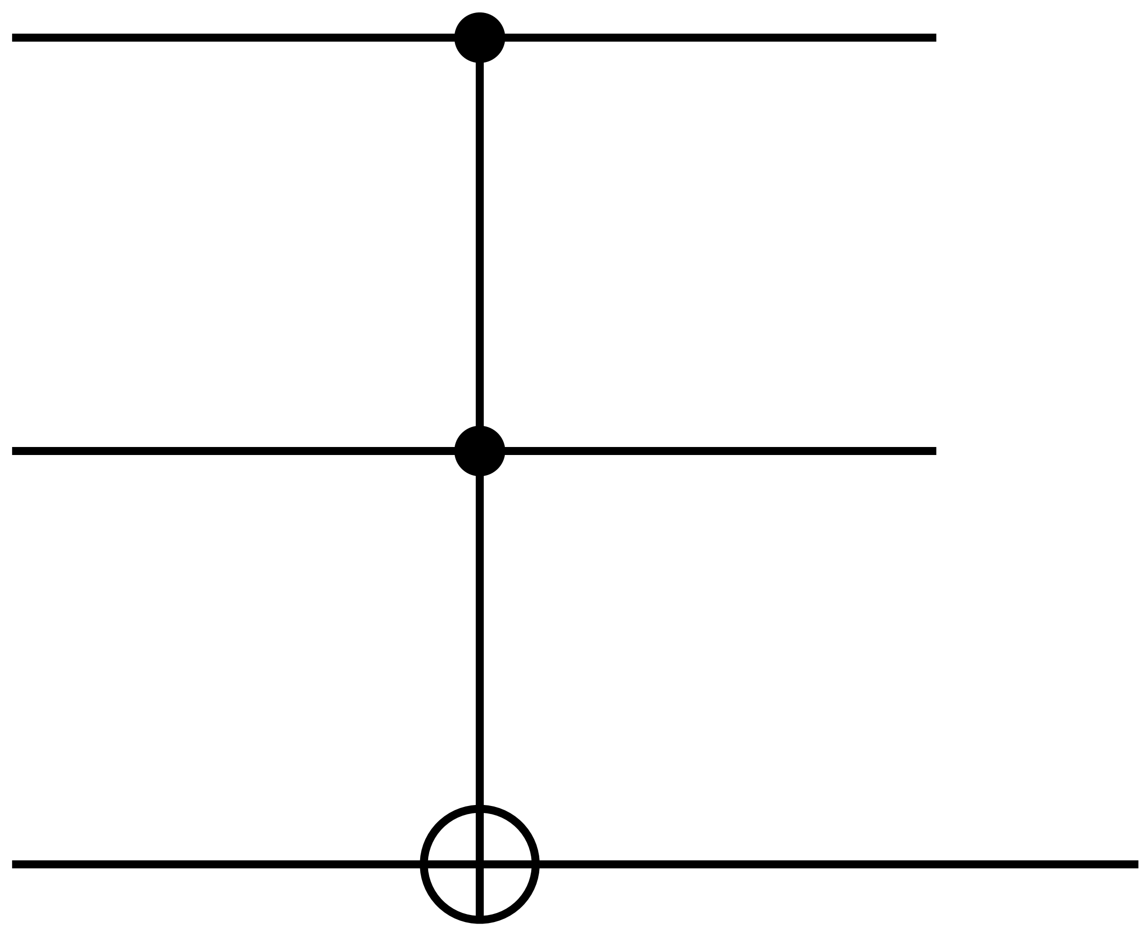
Gottesman-Knill

- We call H + S + CNOT the *Clifford Set.* (We can construct the Pauli gates from H + S.)
- *Gottesman-Knill Theorem*: Any Clifford circuit can be *efficiently simulated* on a classical computer.
- Hence, Clifford circuits must not be *universal* for quantum computation.

Two Roads to Universality

Tofolli (CCNOT)

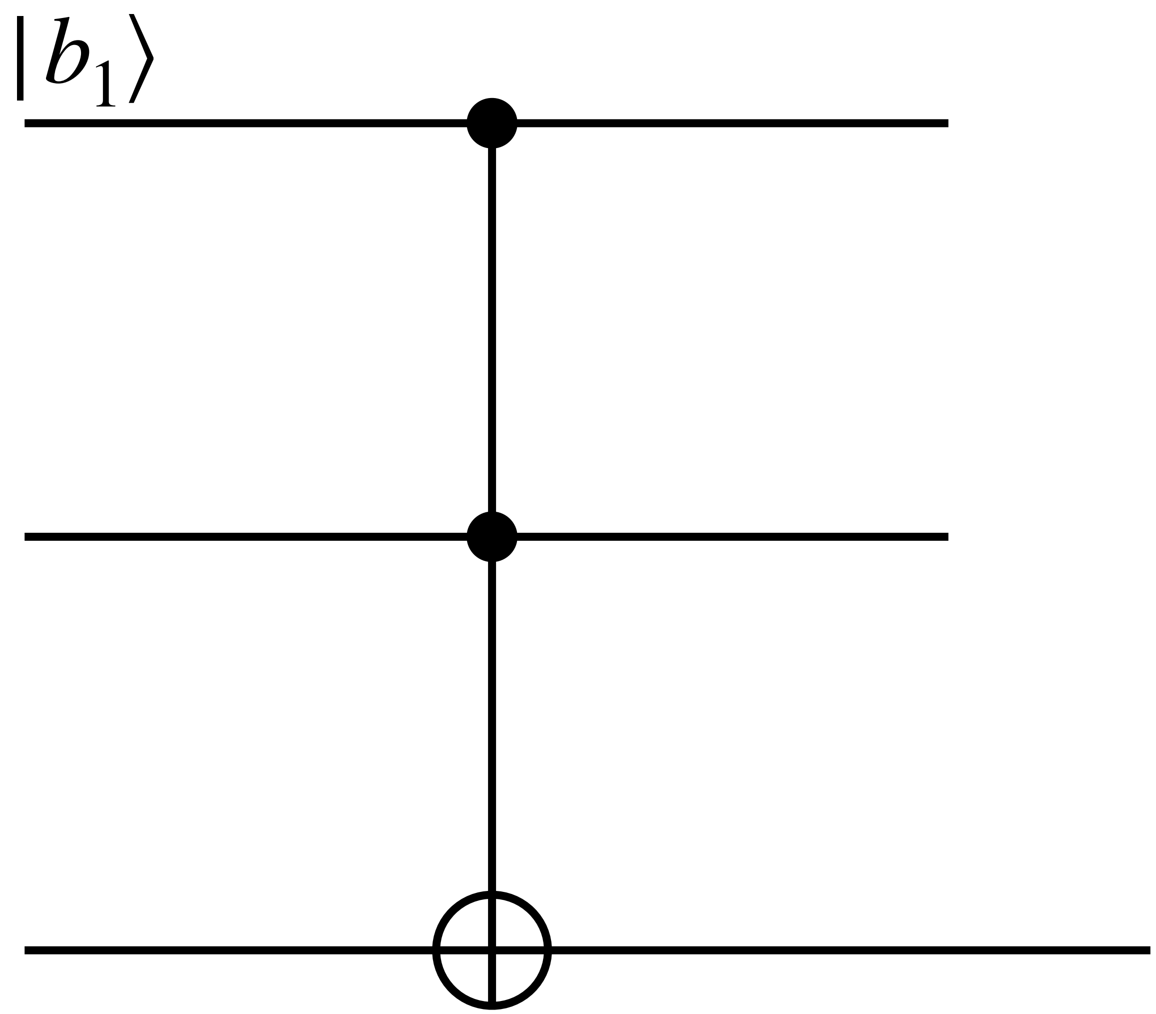
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Two Roads to Universality

Tofolli (CCNOT)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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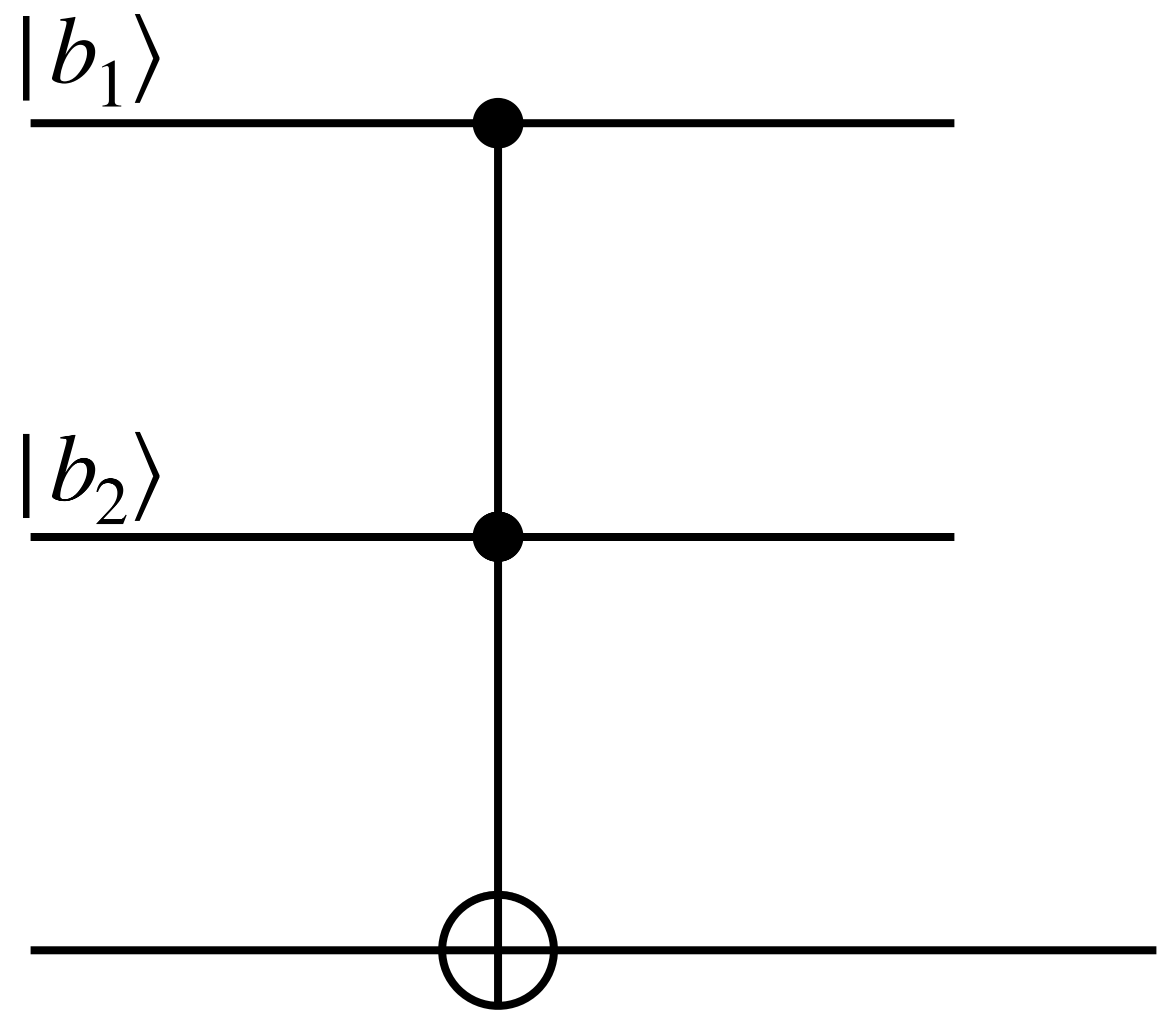
Tofolli (CCNOT)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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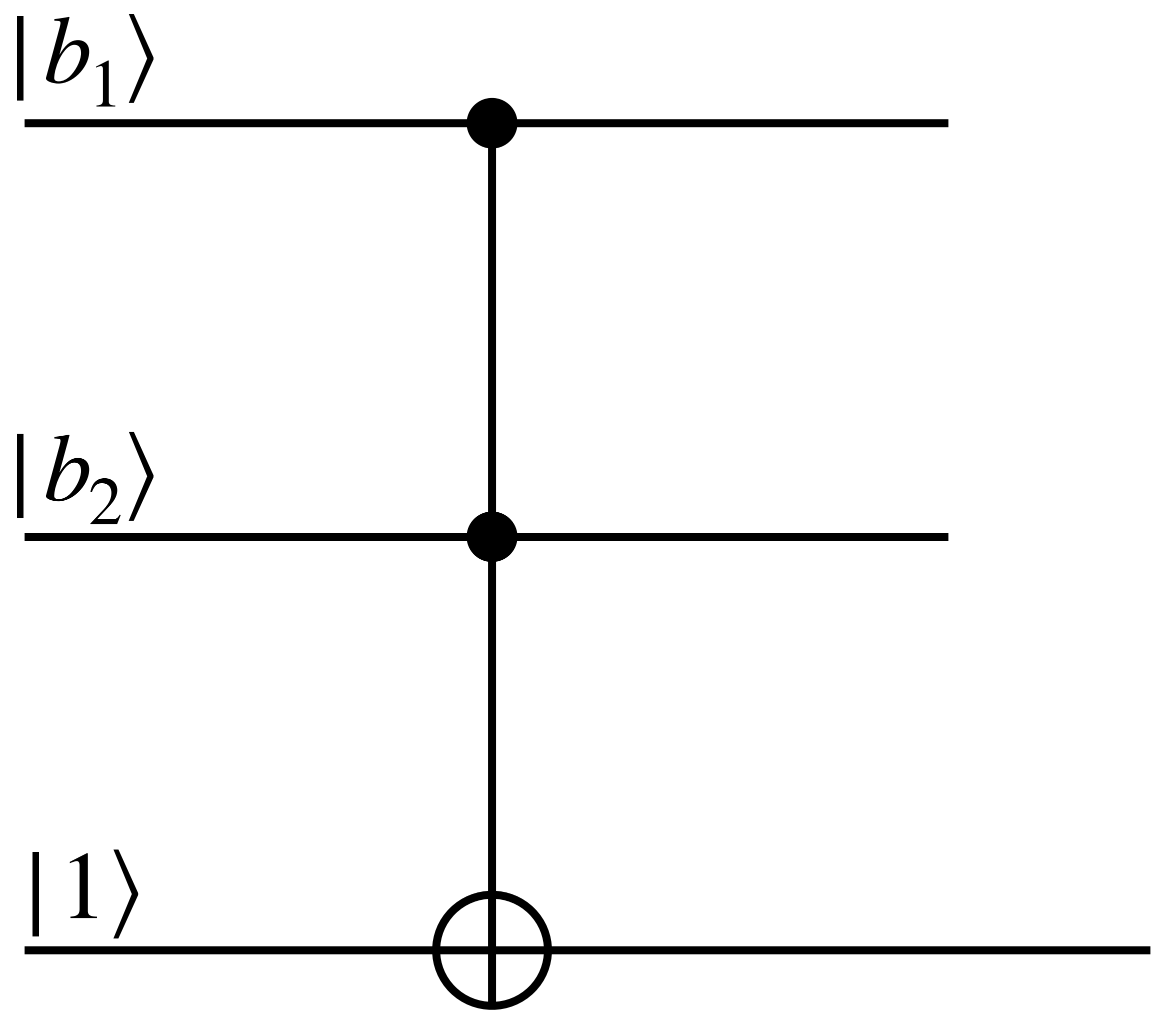
Tofolli (CCNOT)

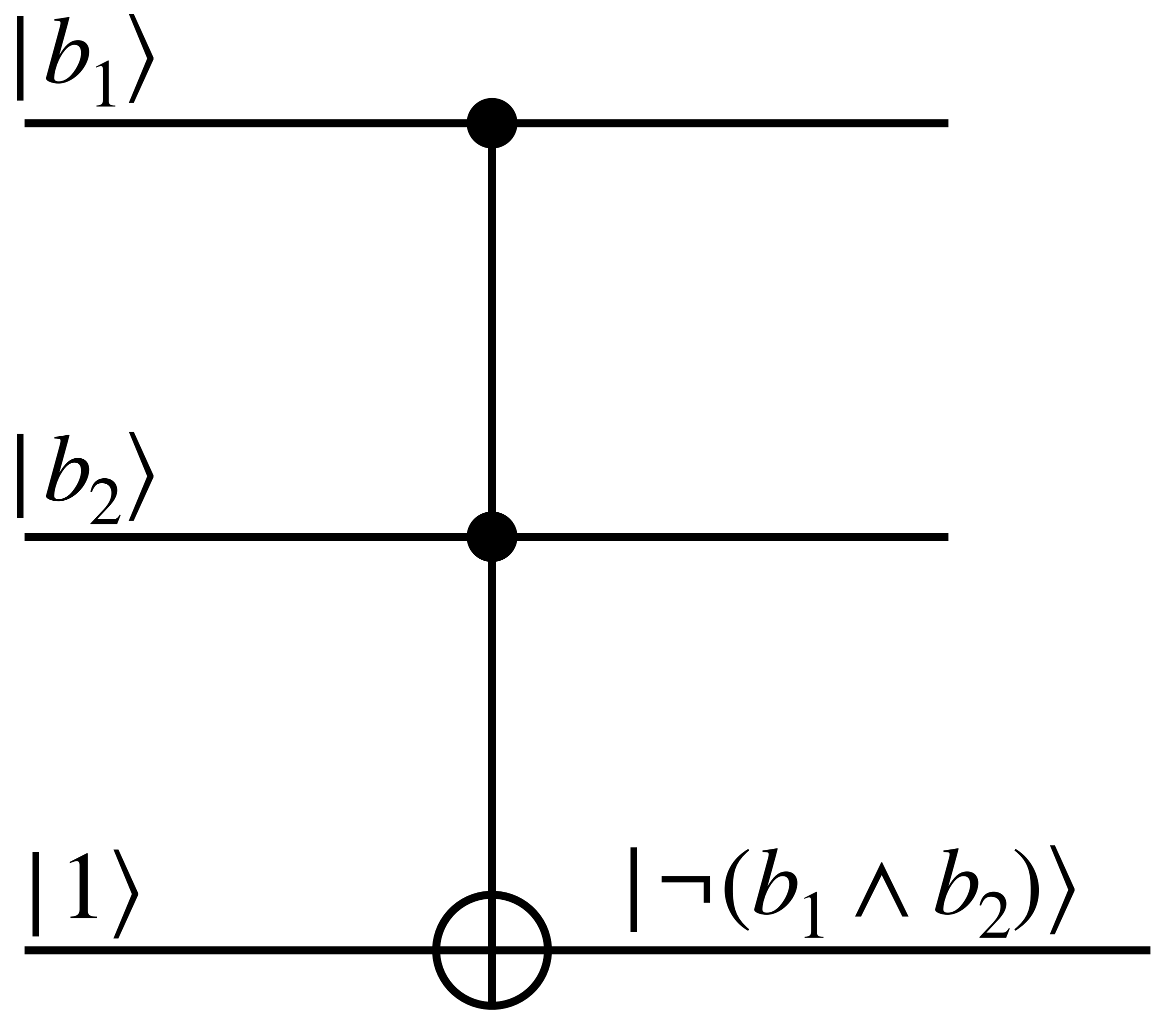
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Two Roads to Universality

Tofolli (CCNOT)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



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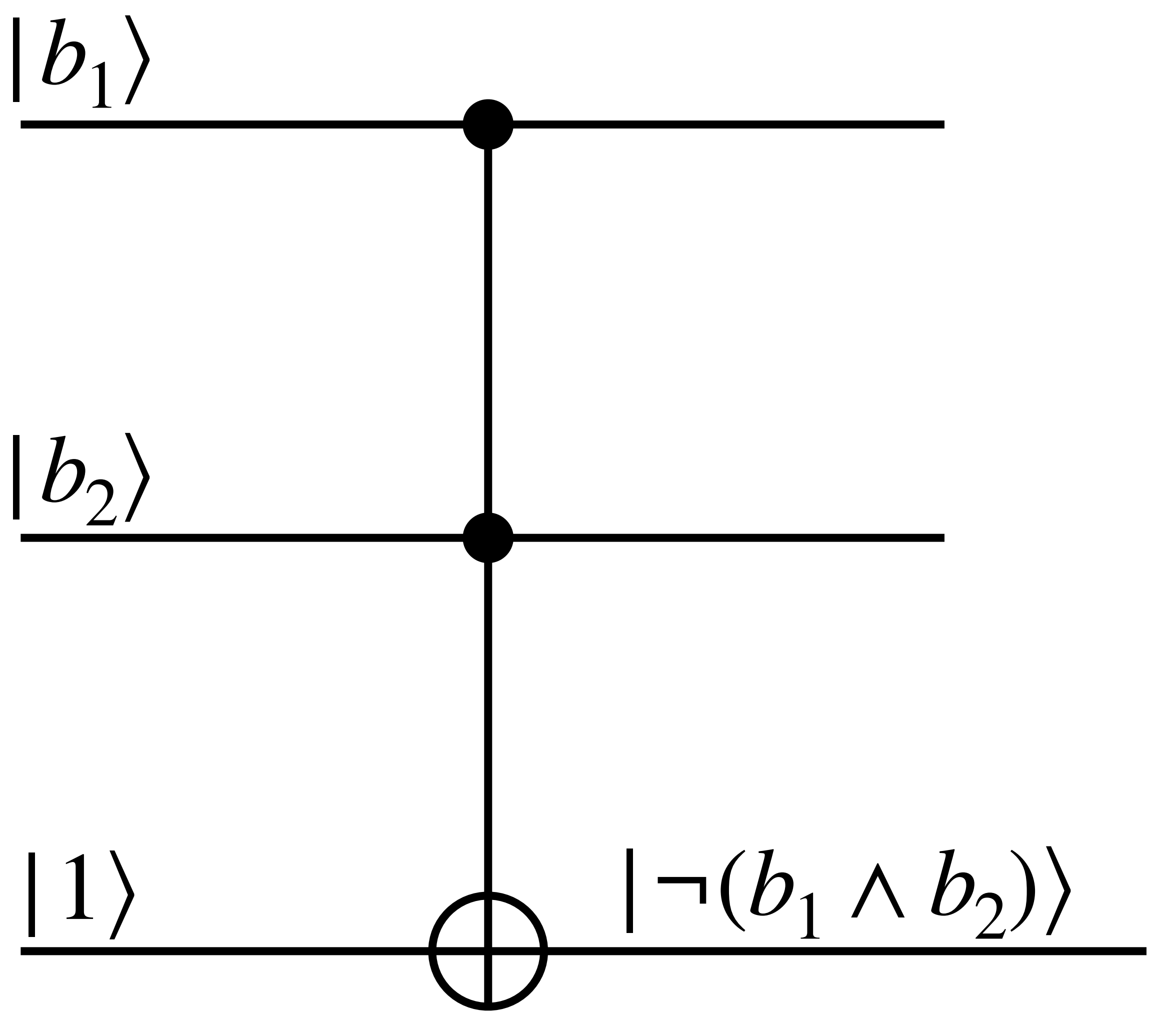

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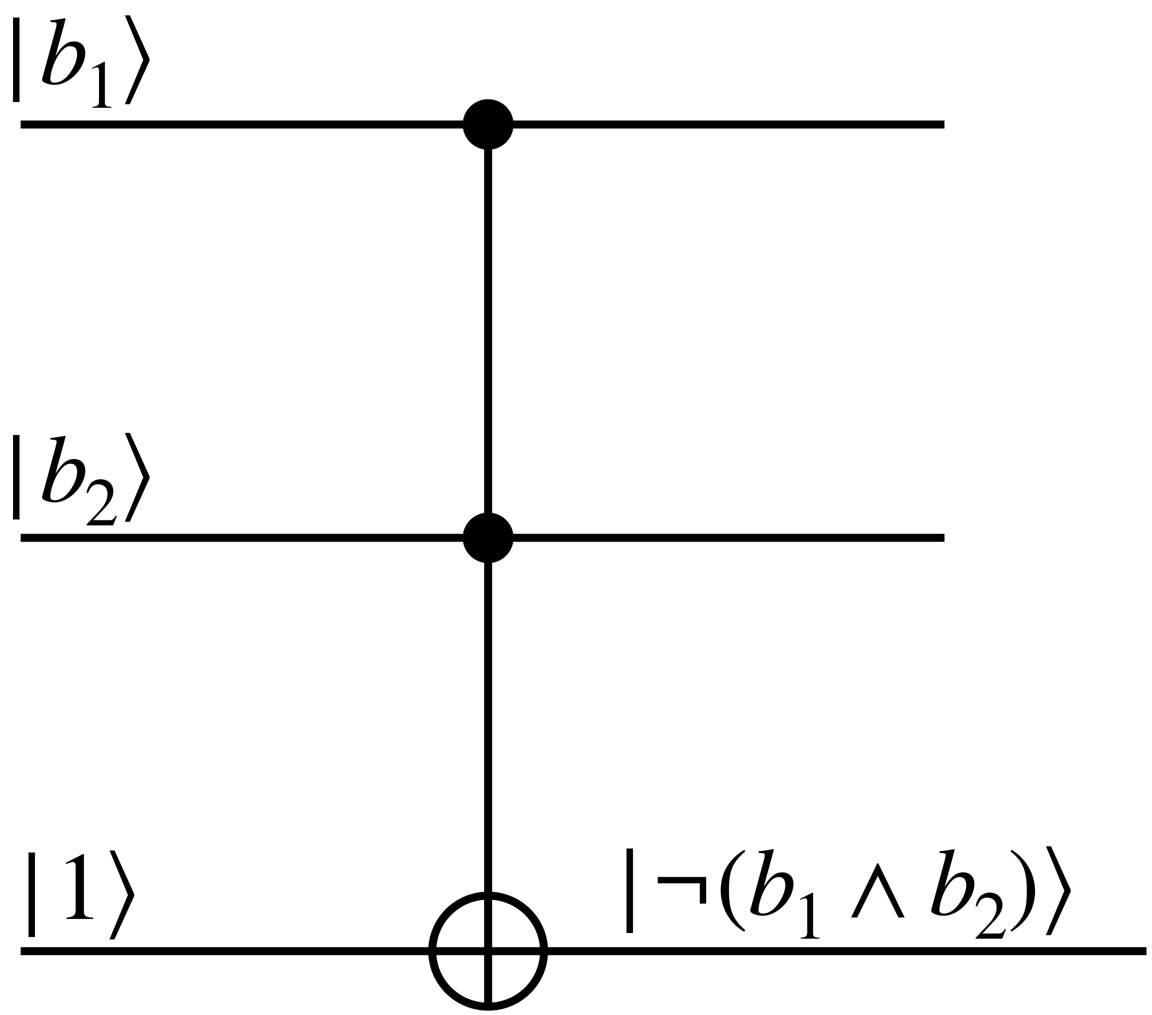
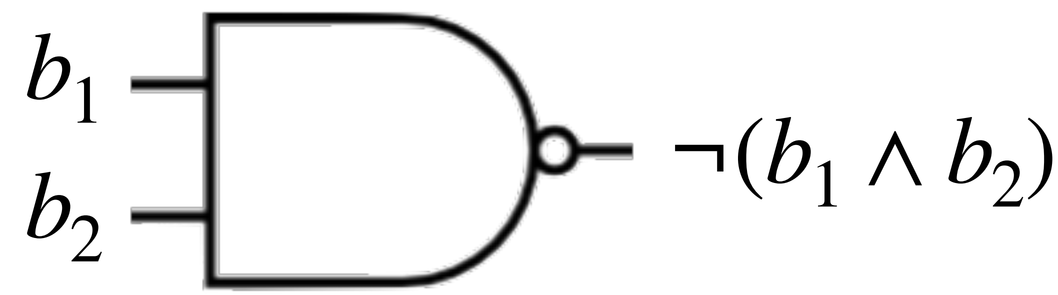
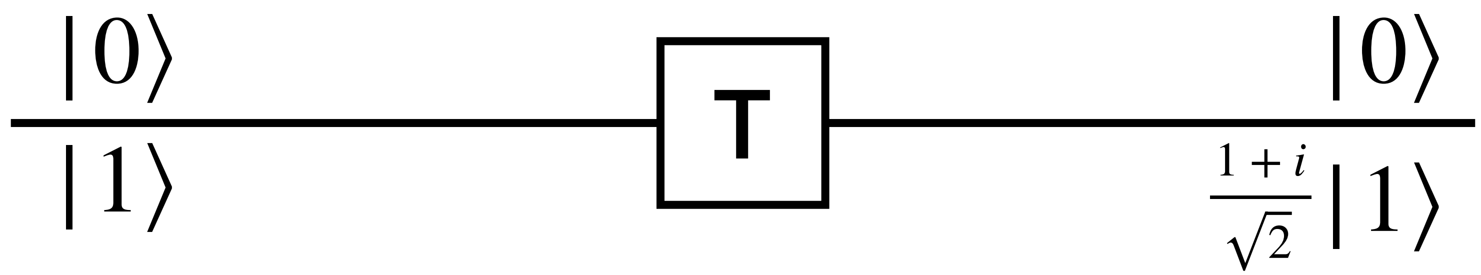


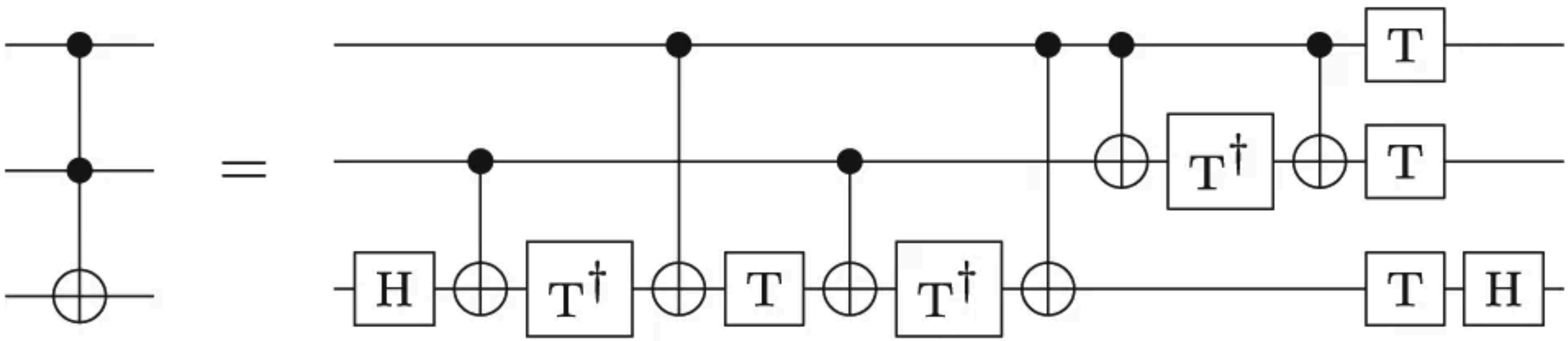


Two Roads to Universality

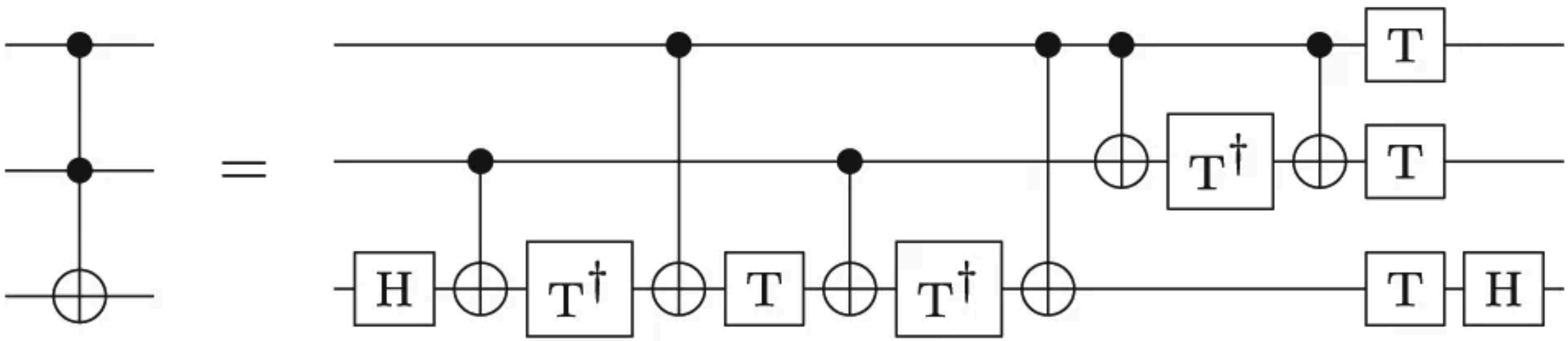
$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$



Two Roads to Universality

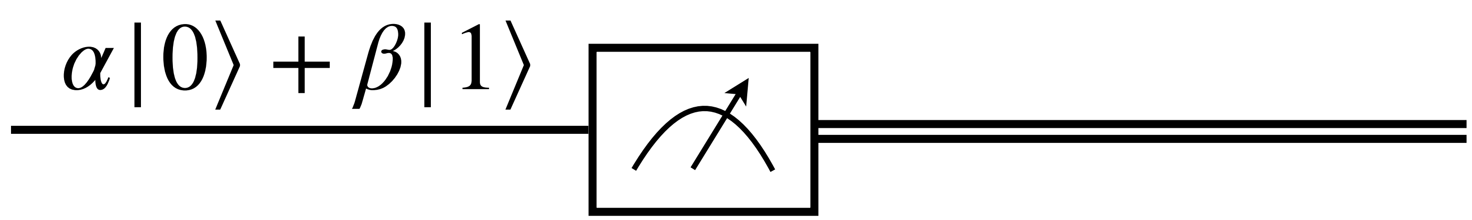


Two Roads to Universality

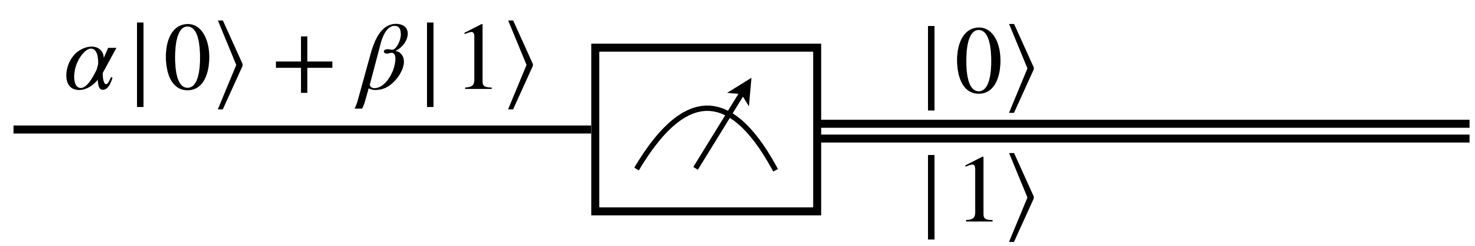


Lesson: T gates and Tofollis are *expensive*

Measurement



Measurement



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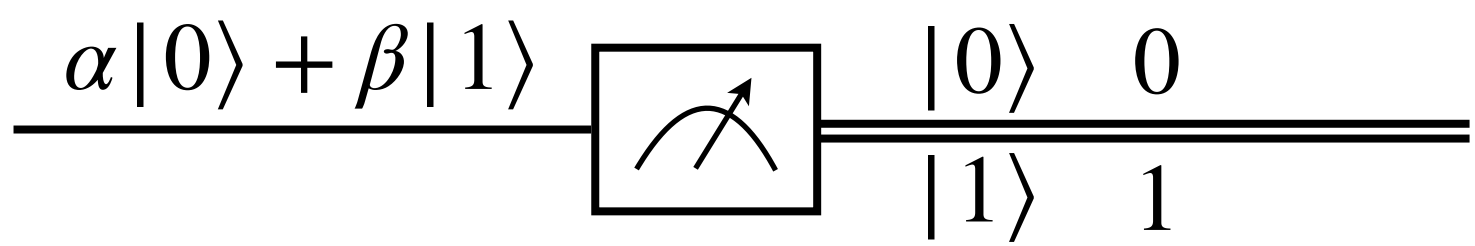
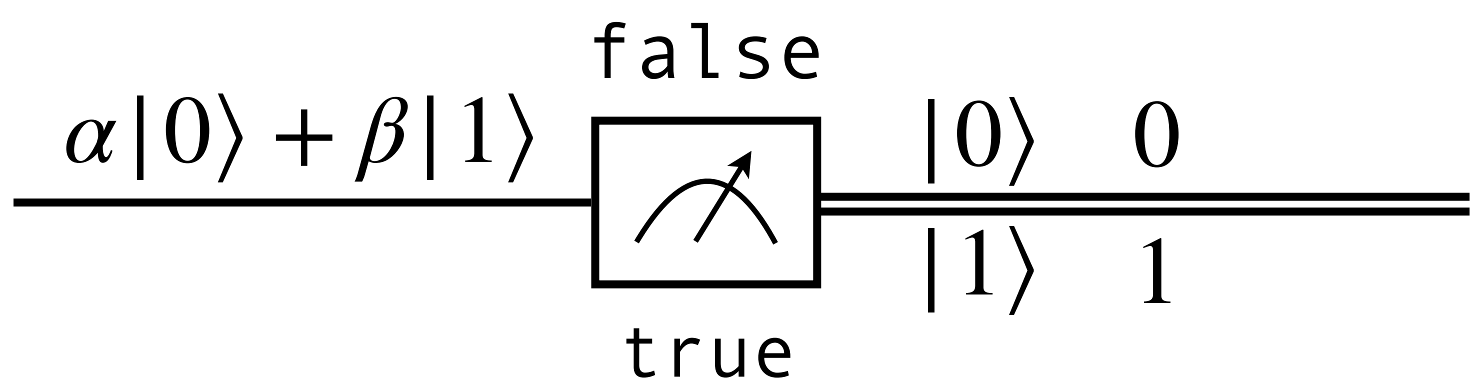


Measurement



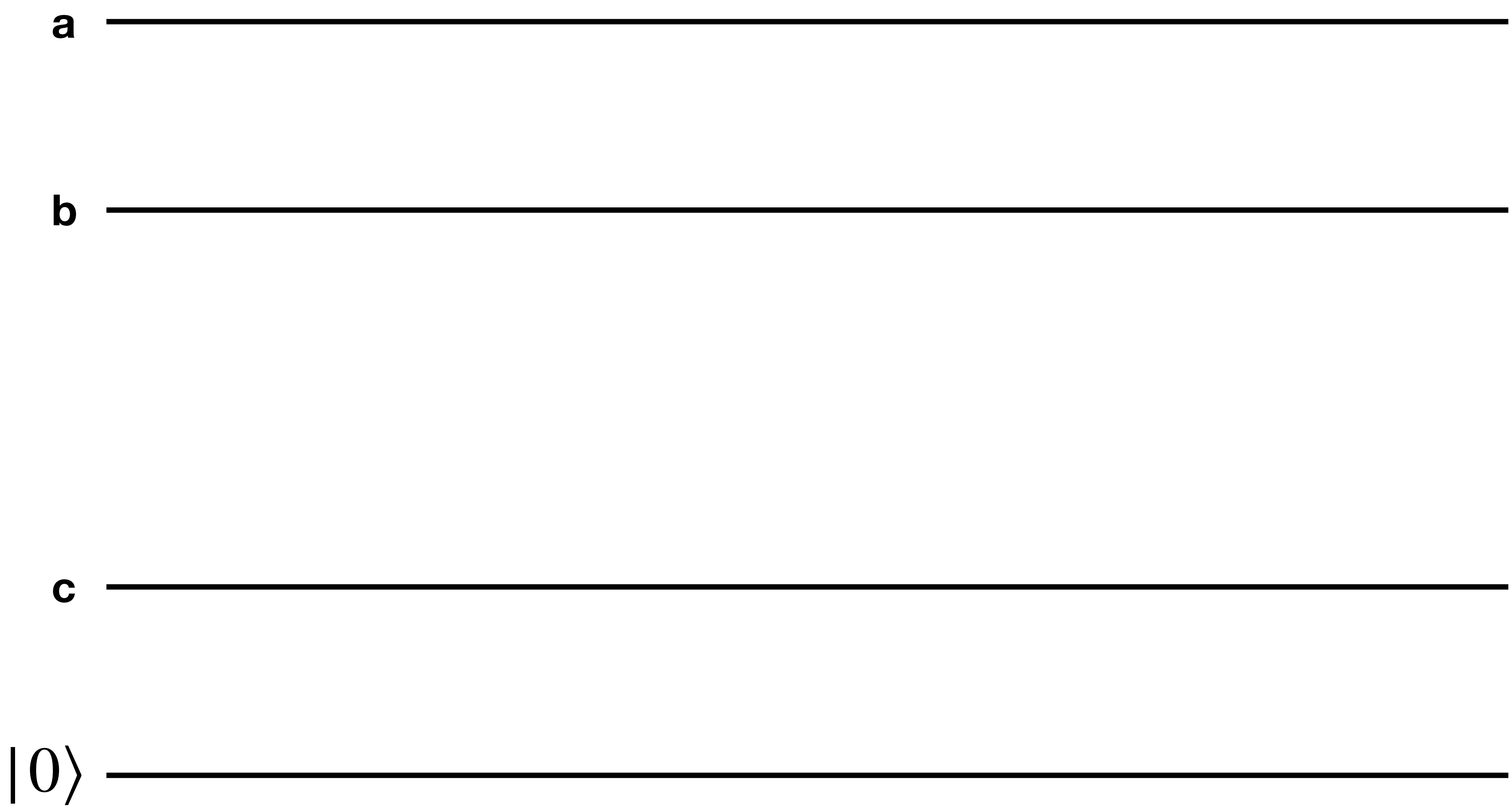
Ancillae

a

b

c

Ancillae



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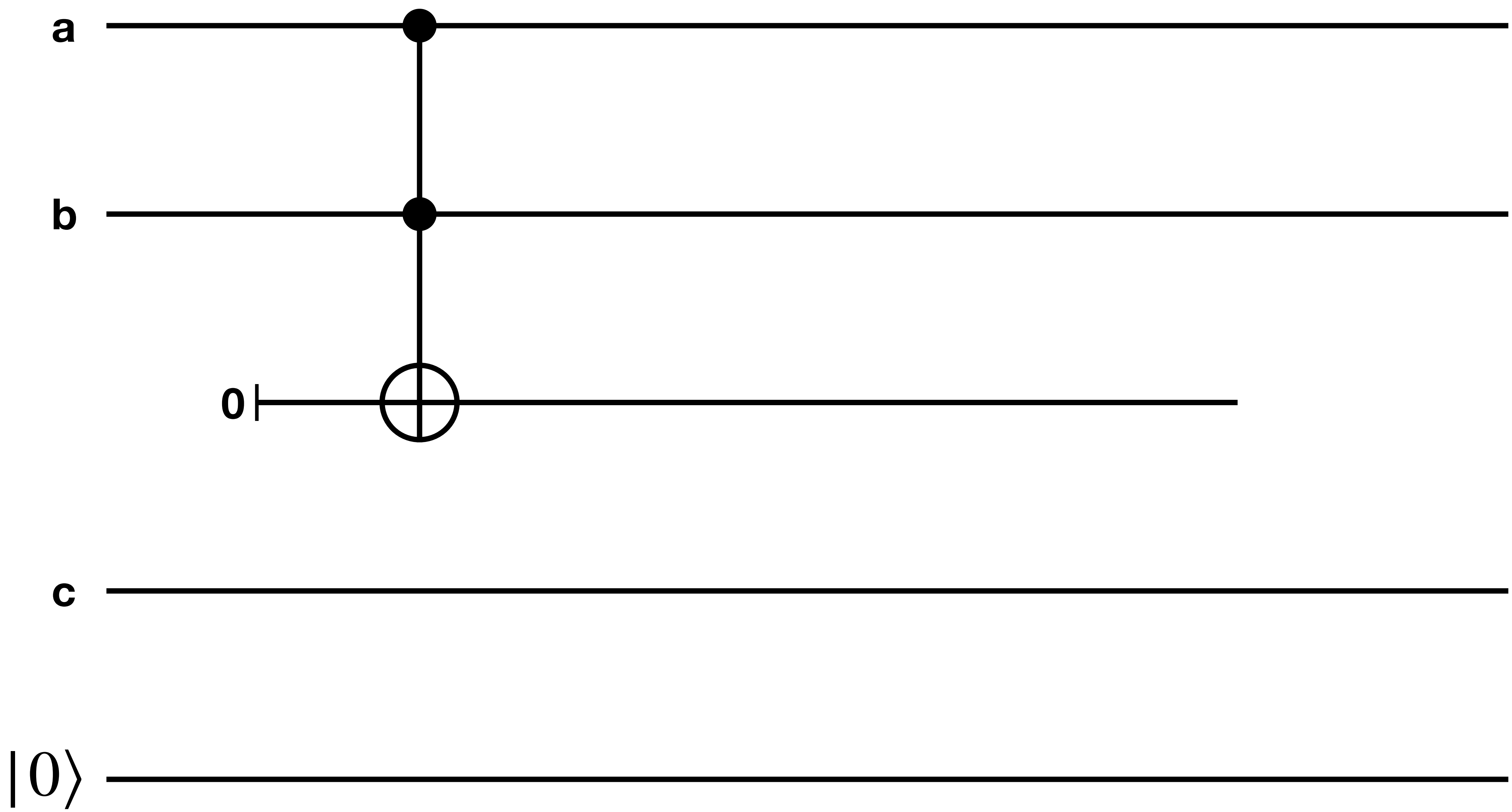

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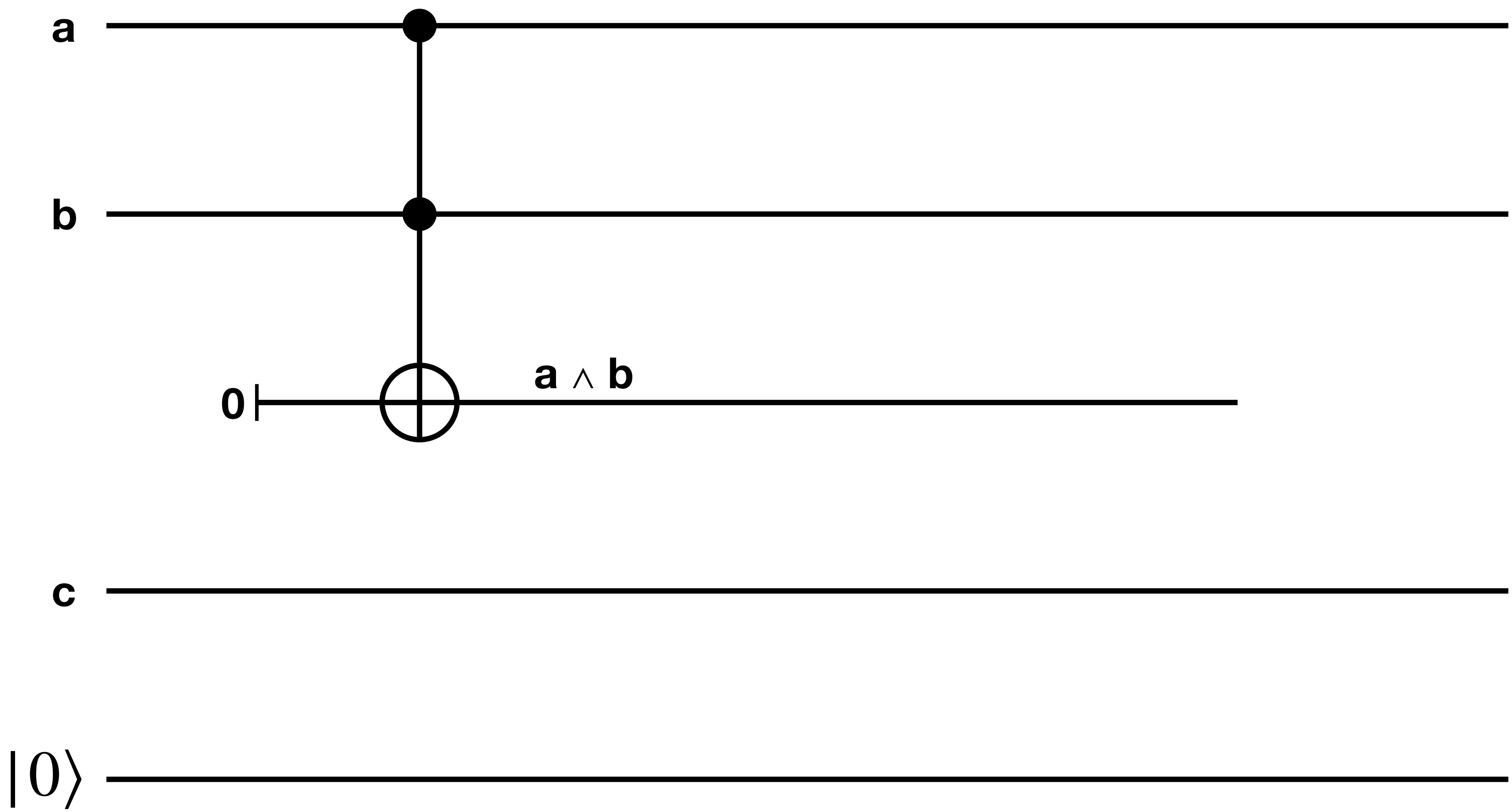

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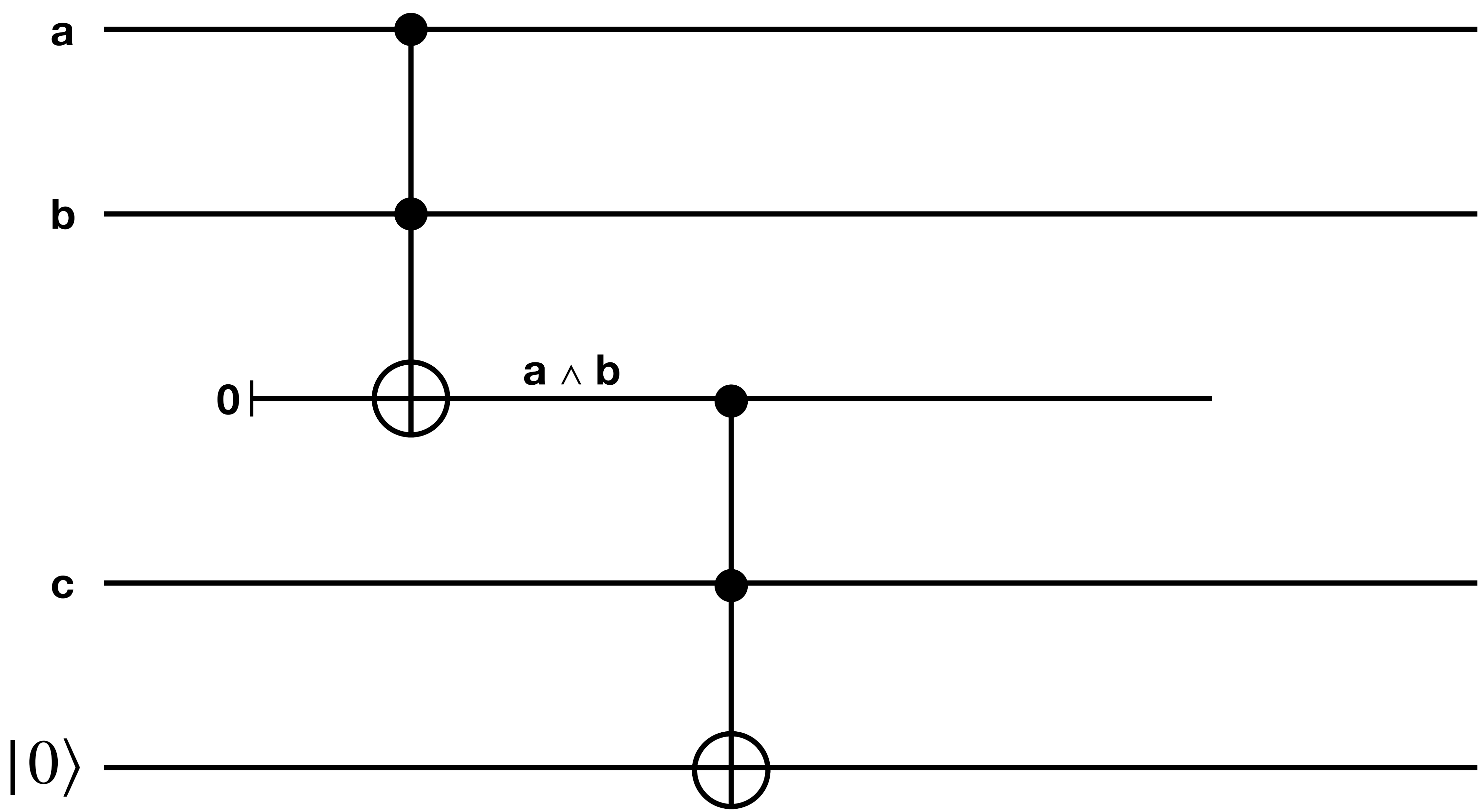

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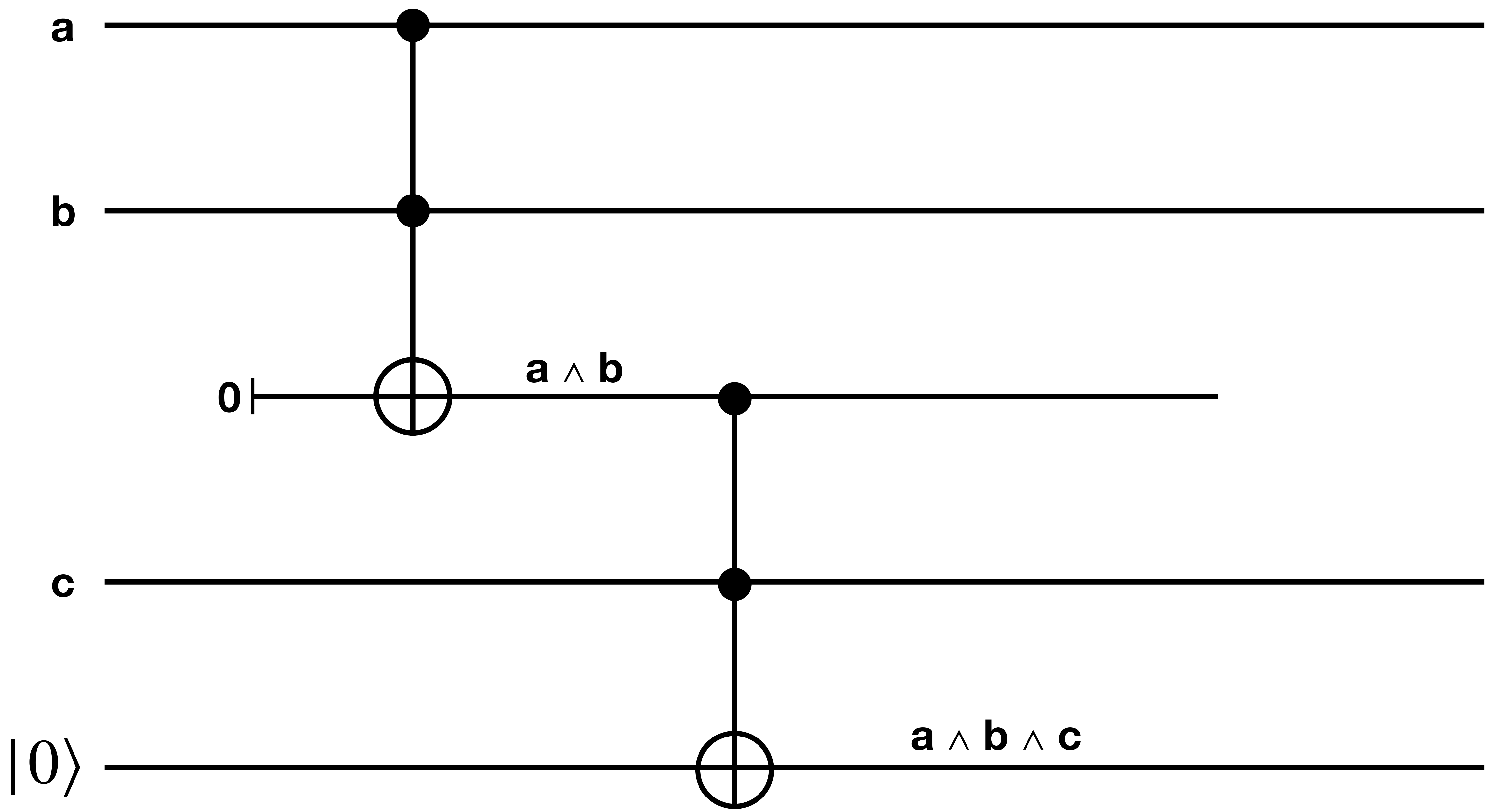

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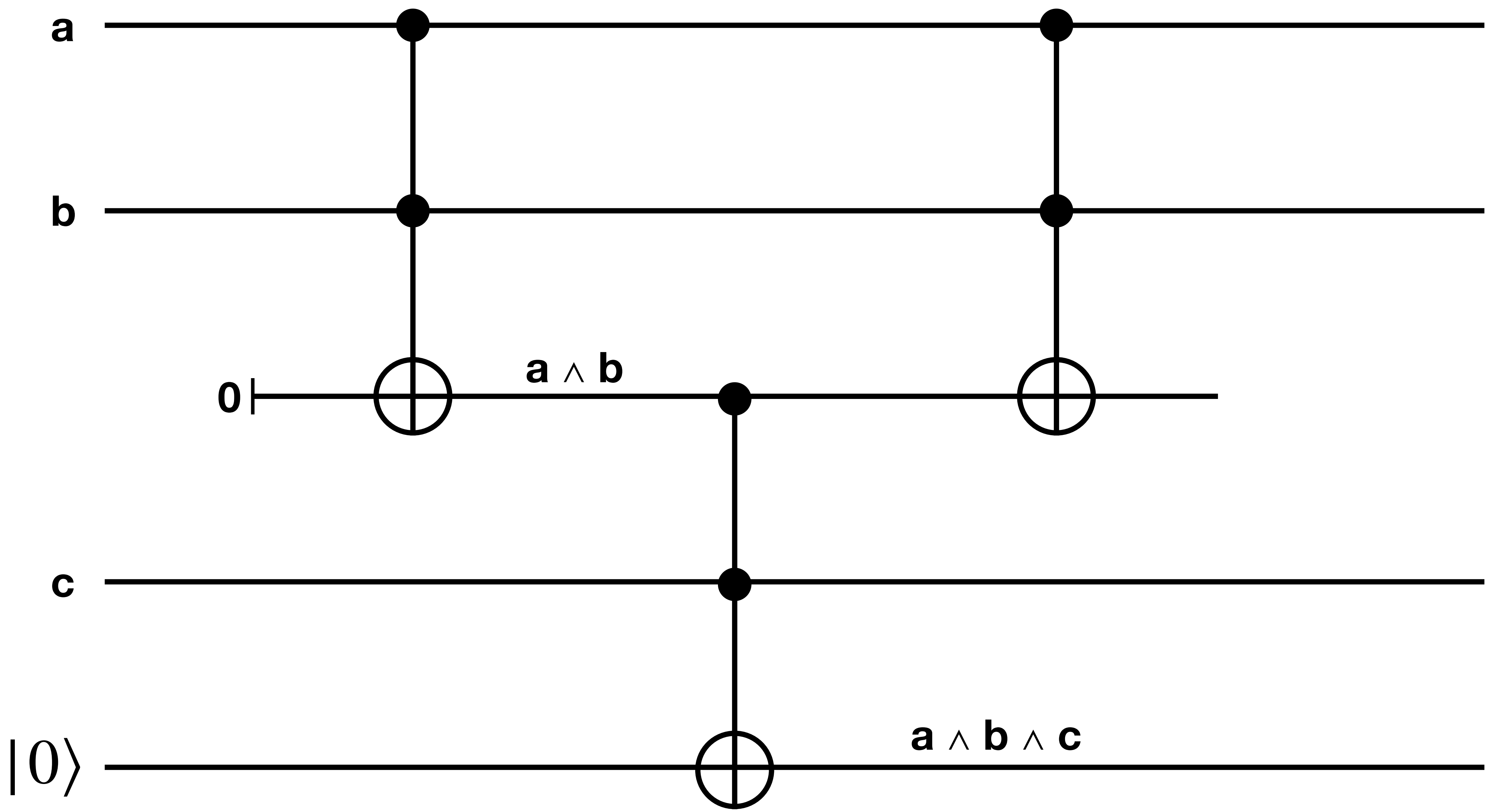
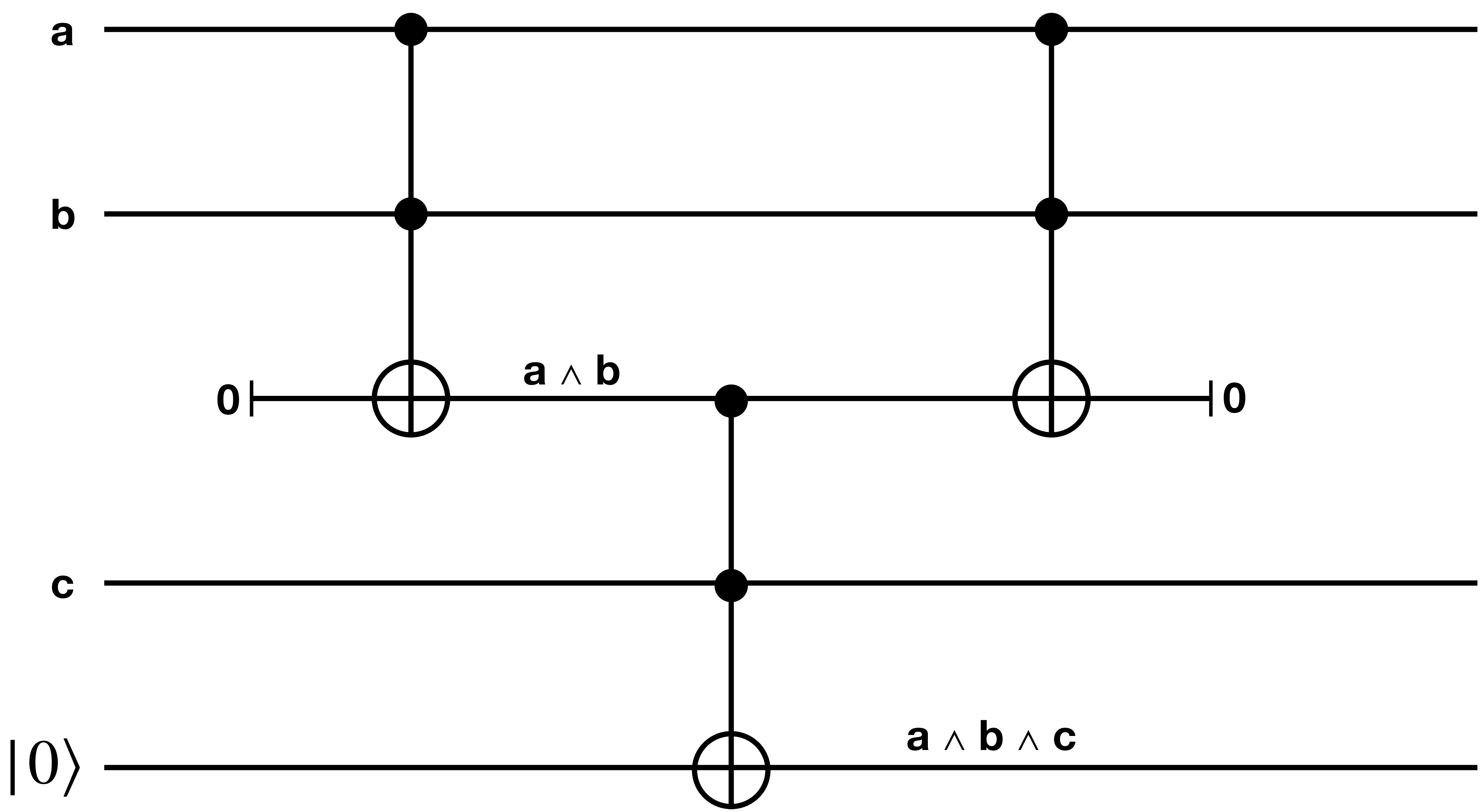
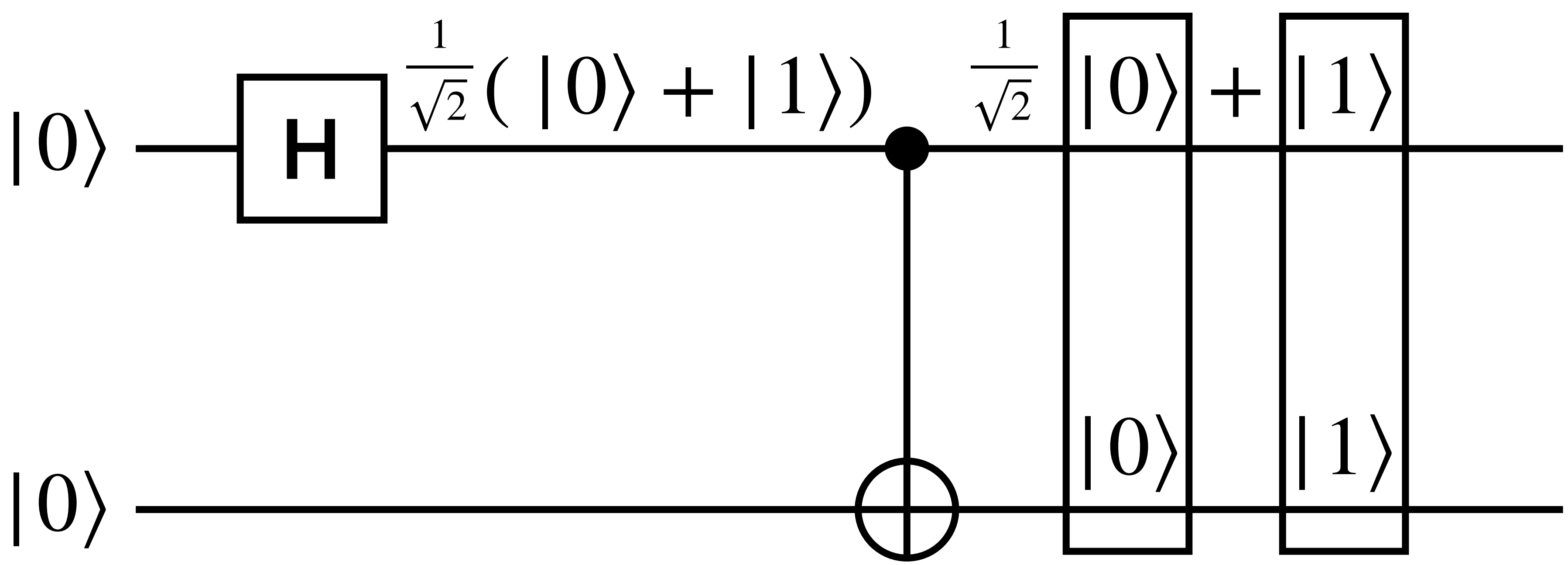


Ancillae

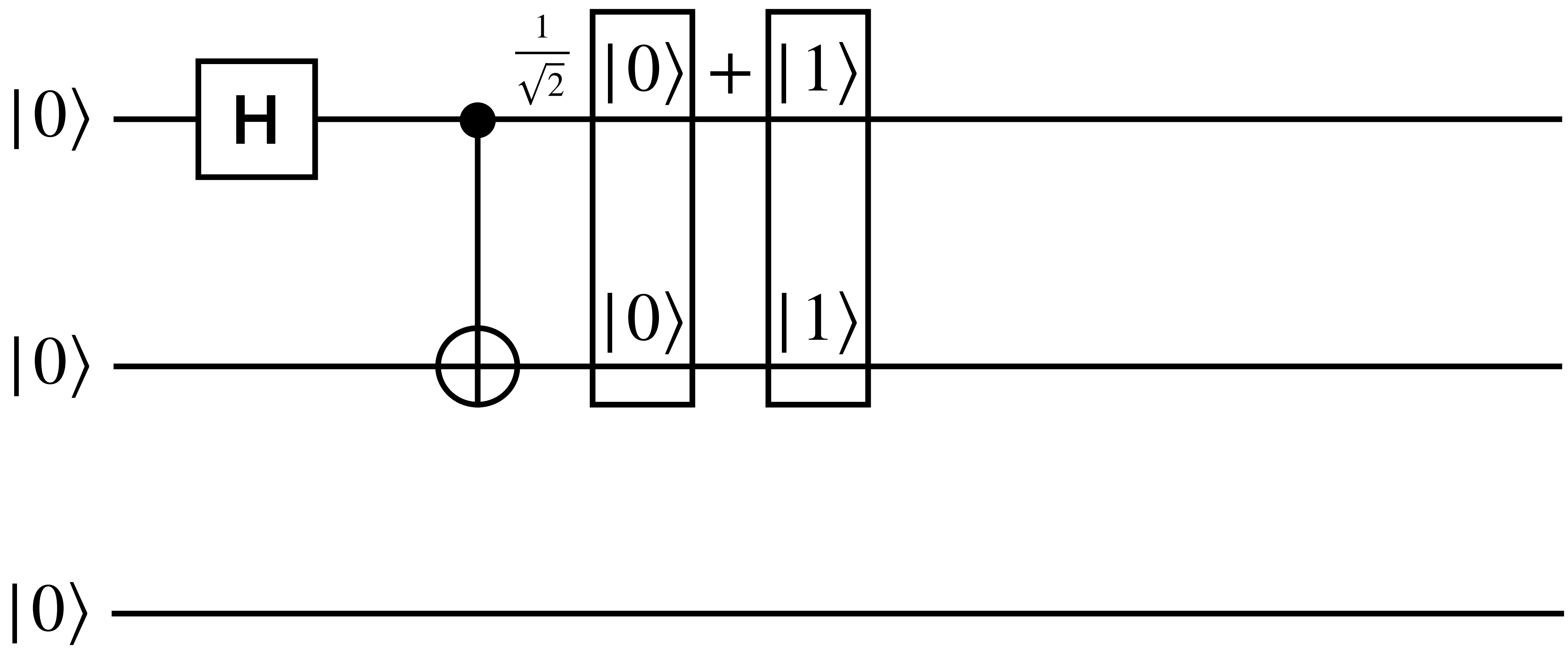


Bell Pair



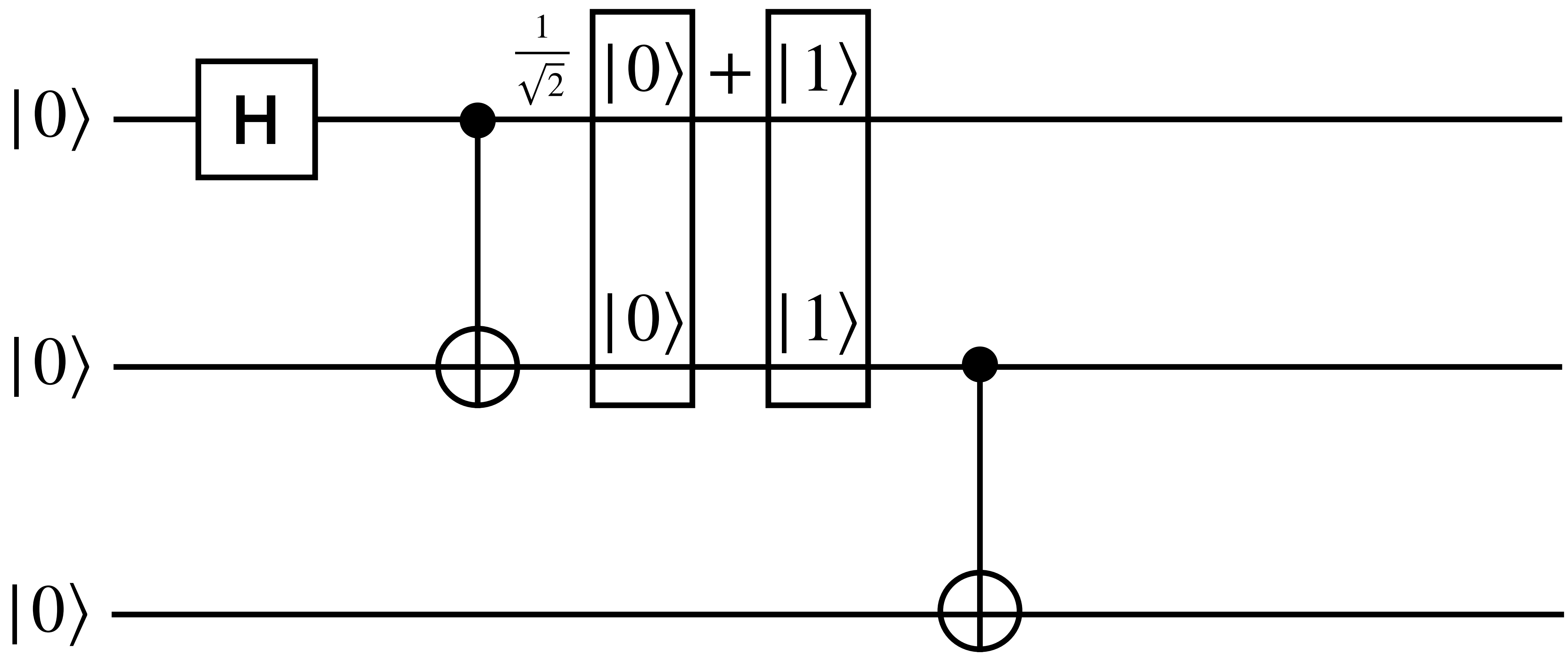
$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

GHZ State



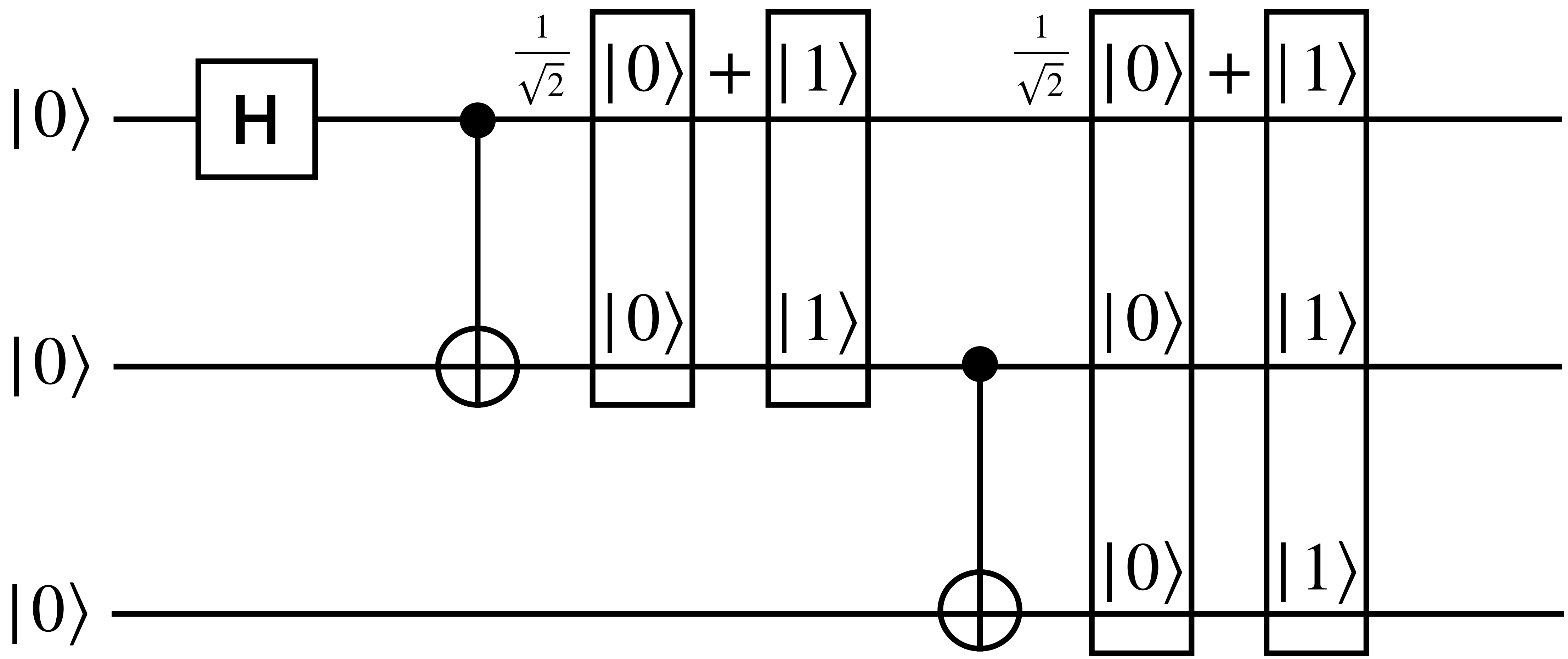
$|0\rangle$
H
$\frac{1}{\sqrt{2}}$
$|0\rangle$
+
$|1\rangle$
$|0\rangle$
$|0\rangle$
$|1\rangle$
$|0\rangle$

GHZ State

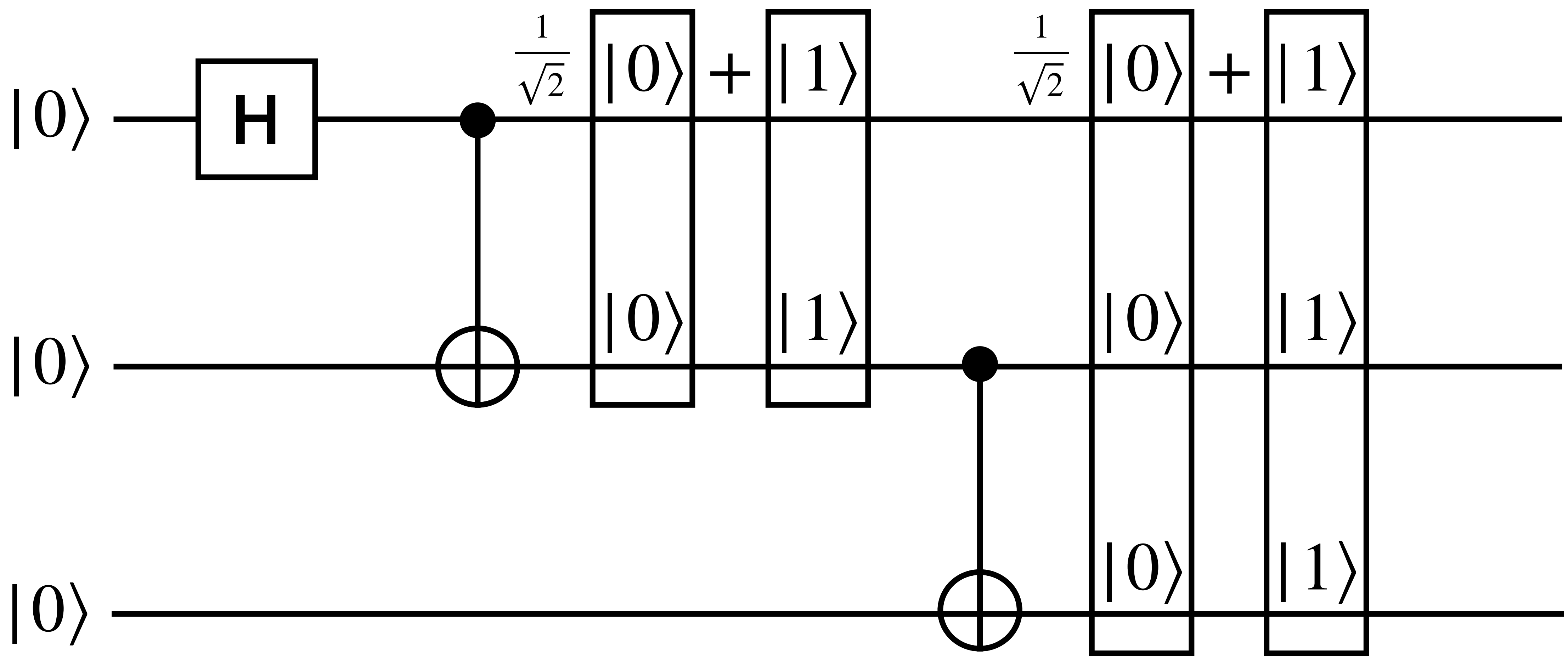


$|0\rangle$
$|0\rangle$
$|0\rangle$
H
$\frac{1}{\sqrt{2}}$
$|0\rangle$
$|0\rangle$
+
$|1\rangle$
$|1\rangle$

GHZ State



GHZ State



$$\frac{1}{\sqrt{2}}(\,|000\rangle + |111\rangle)$$

GHZ Program

```
Definition GHZ3:
  H 0;
  CNOT 0 1;
  CNOT 1 2.
```

GHZ Program

```
Definition GHZ (n : int):
  H 0;
  for i in range(0,n):
    CNOT i (i+1)
```

GHZ Functional Program

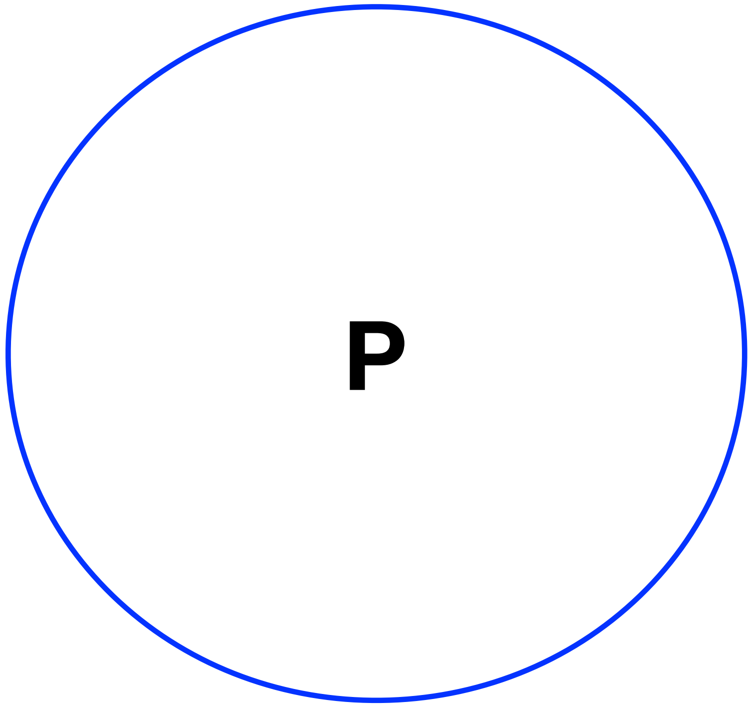
```
GHZ : nat -> Circuit
GHZ 0 = H 0
GHZ (n+1) = GHZ n; CNOT n (n+1)
```

Teleportation

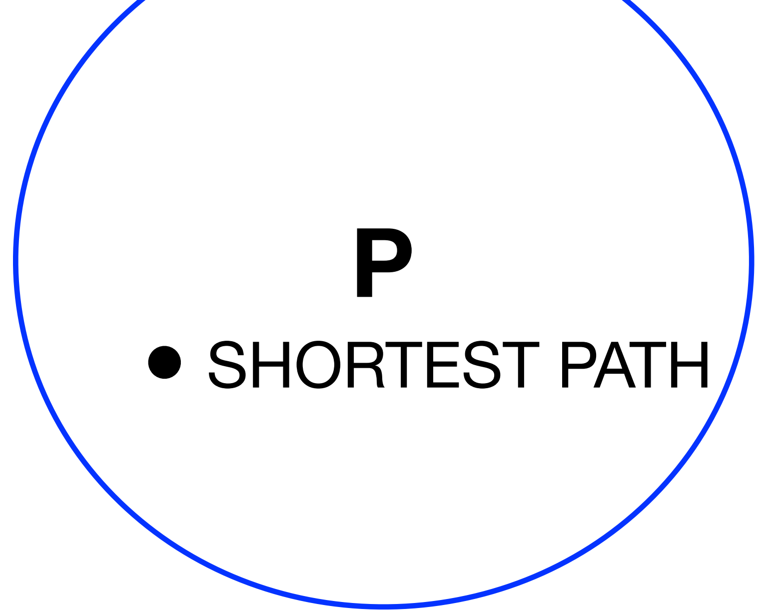
```
Definition teleport(q):
  q1,q2 = bell()
  b1,b2 = alice(q,q1)
  q’    = bob(b1,b2,q2)
  return q’
```

Quantum Complexity

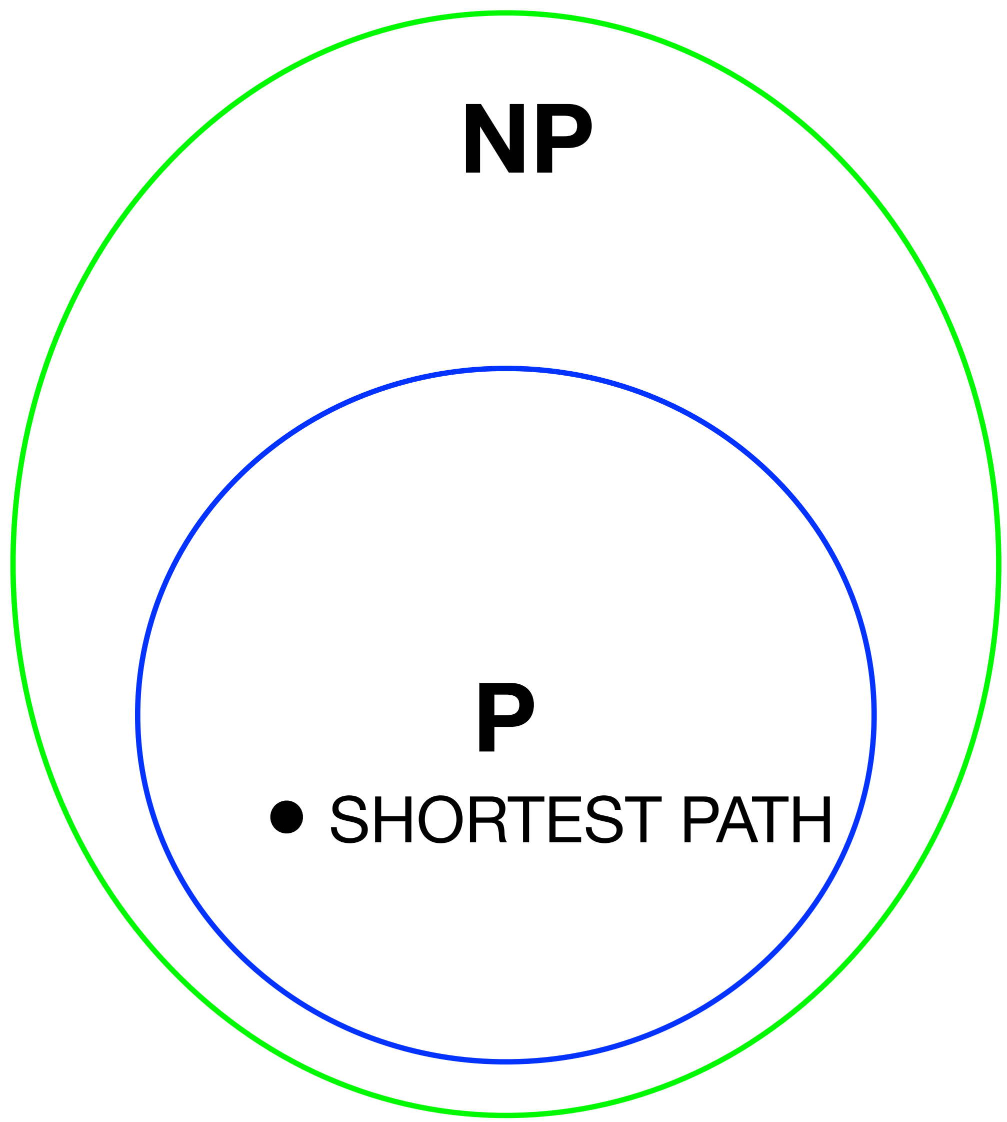
Quantum Complexity



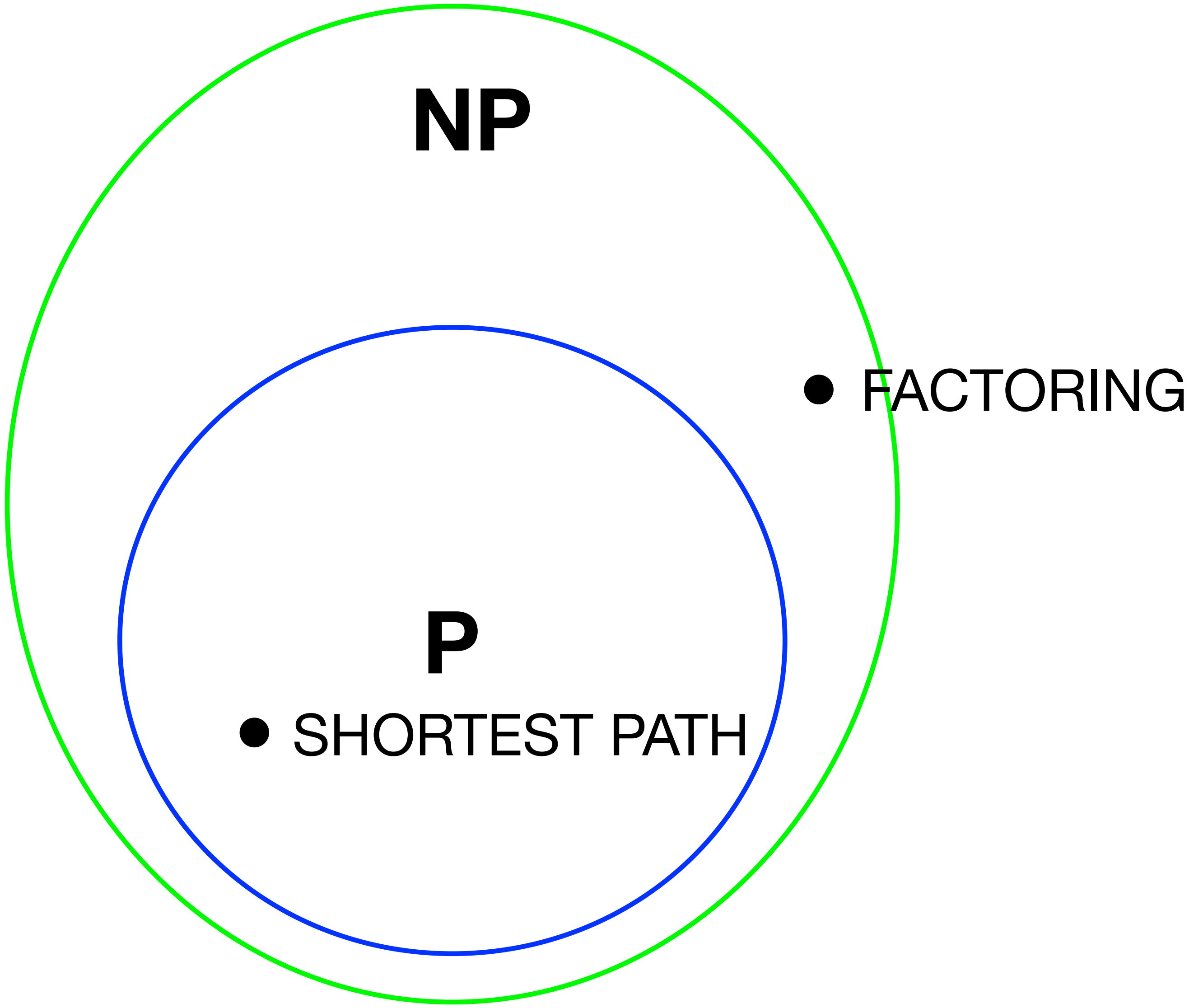
Quantum Complexity



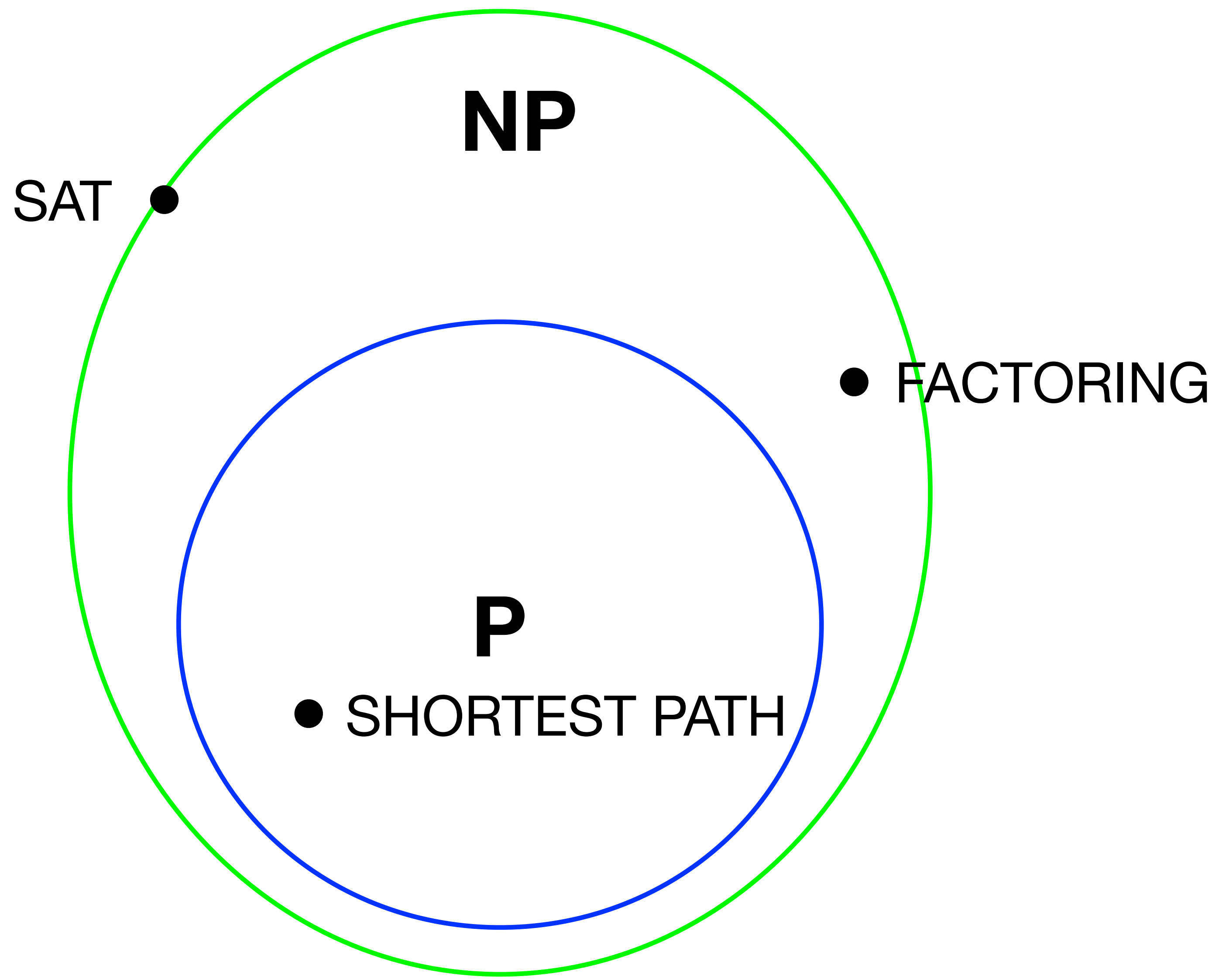
Quantum Complexity



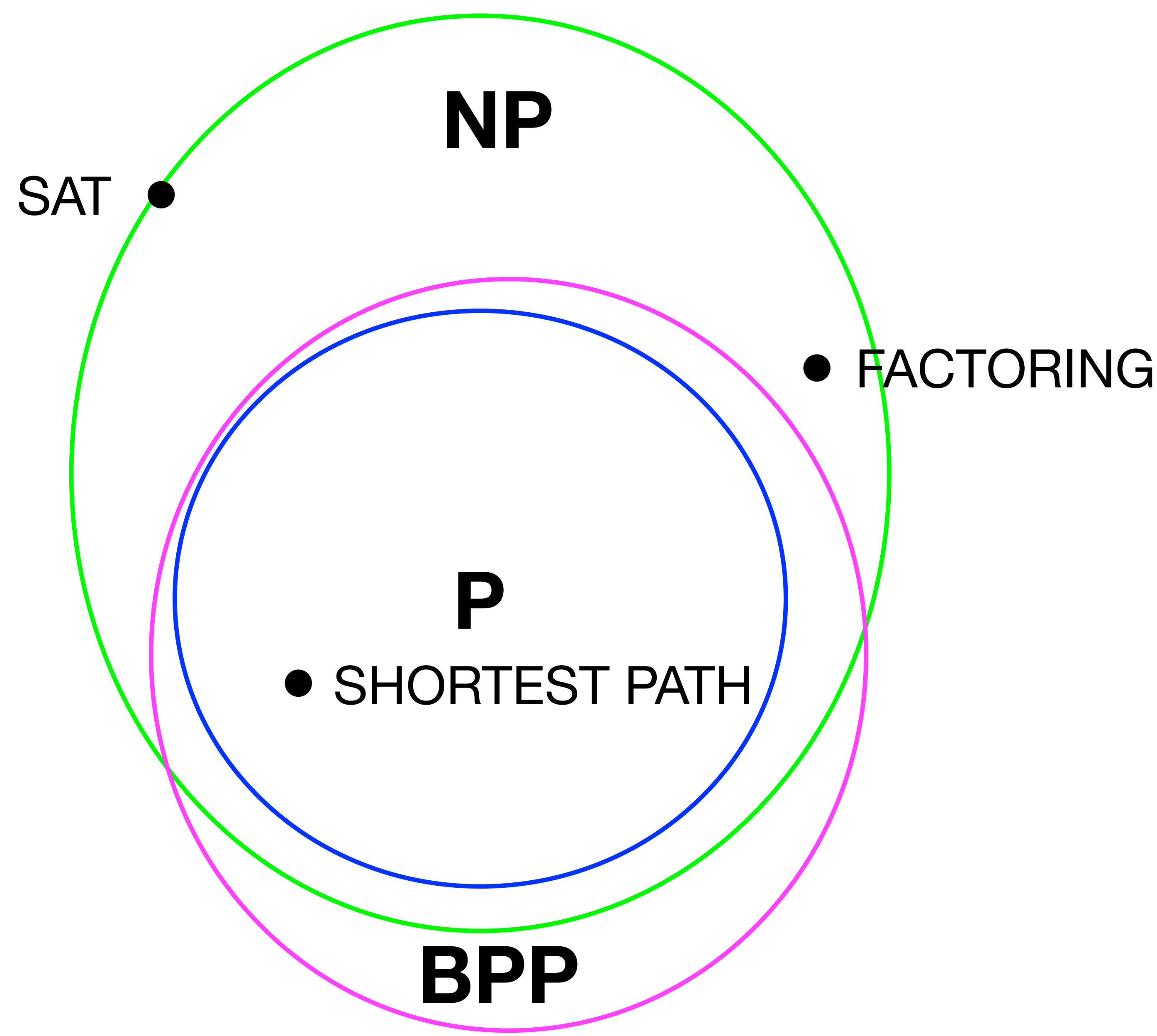
Quantum Complexity



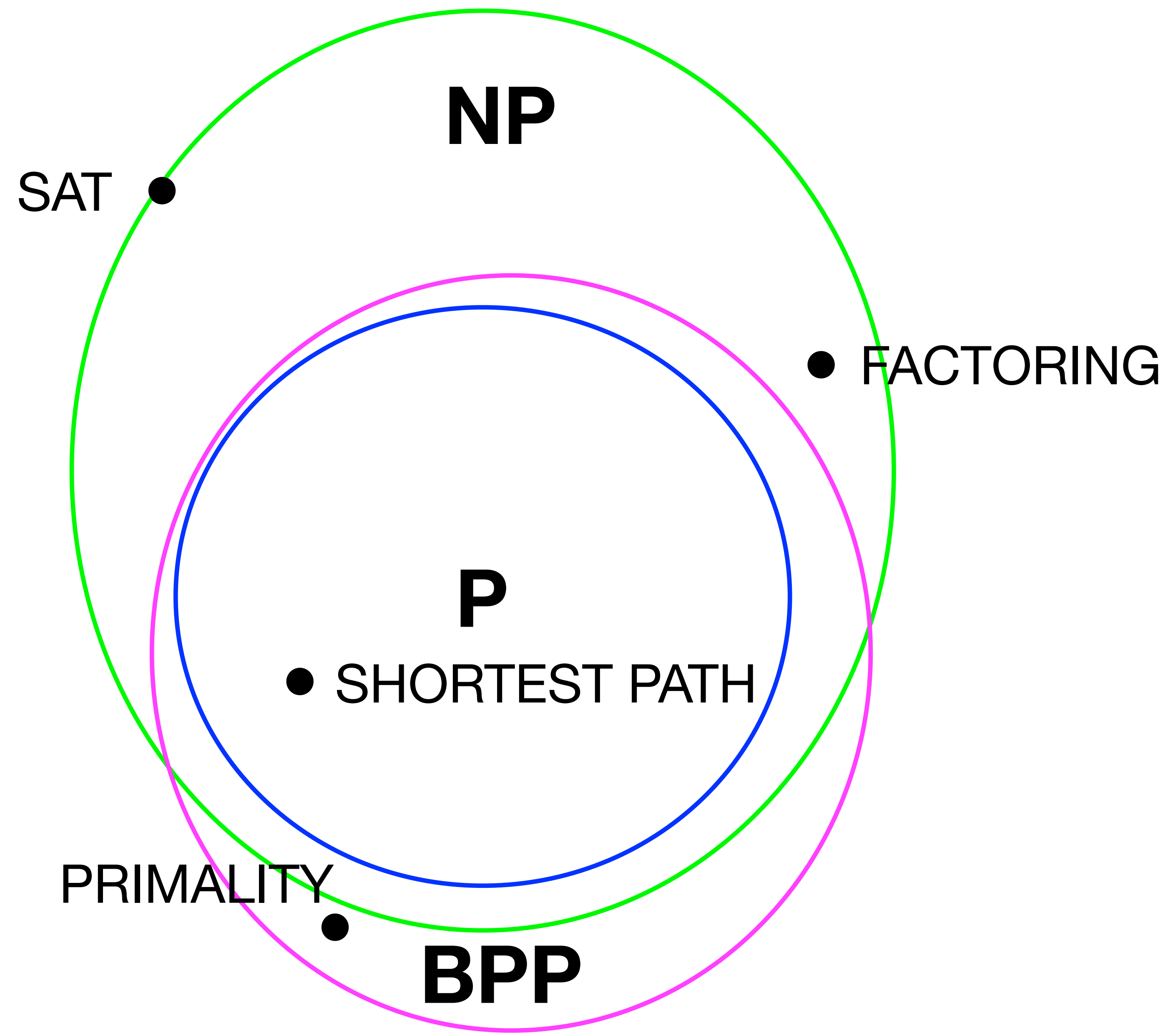
Quantum Complexity



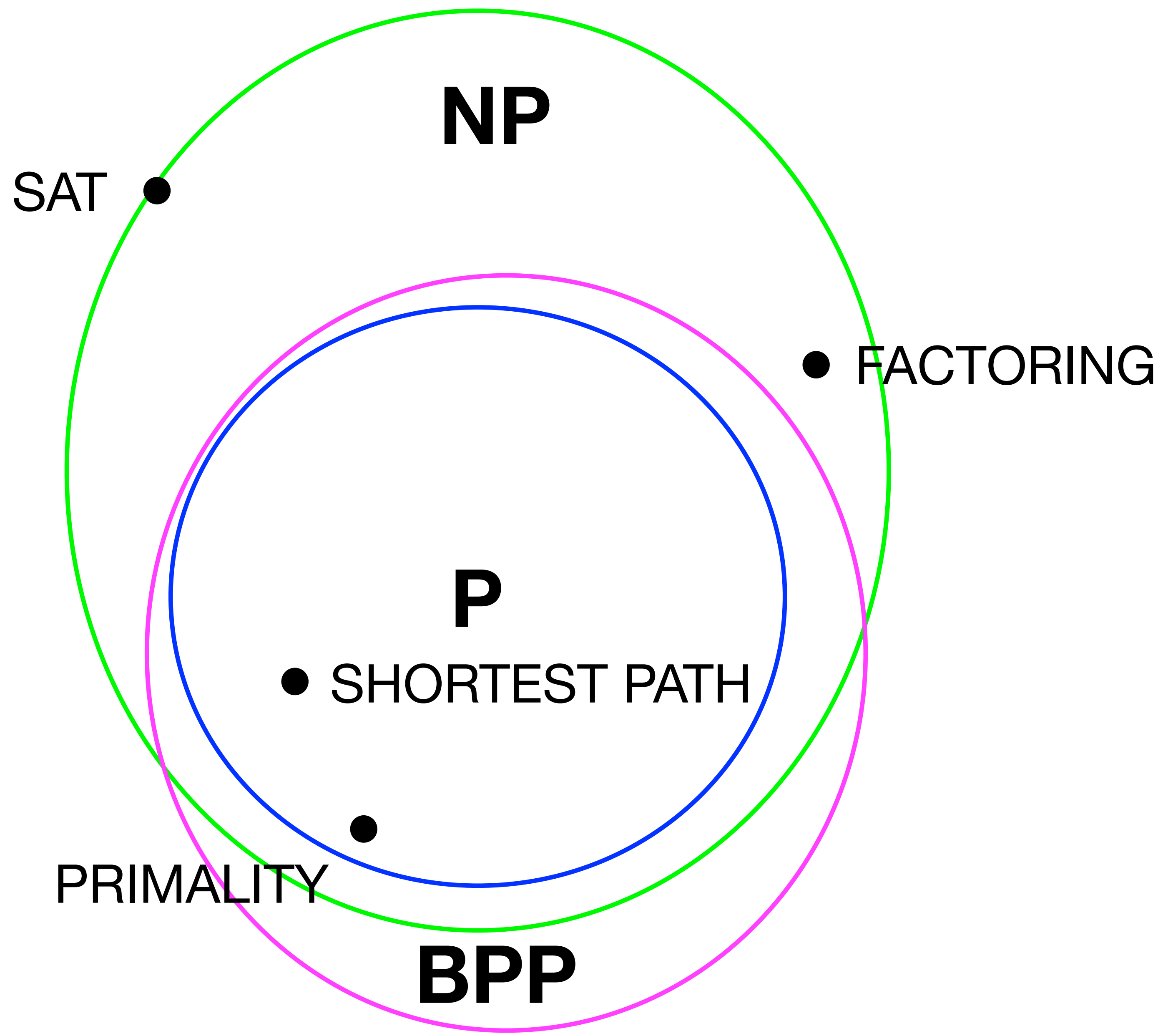
Quantum Complexity



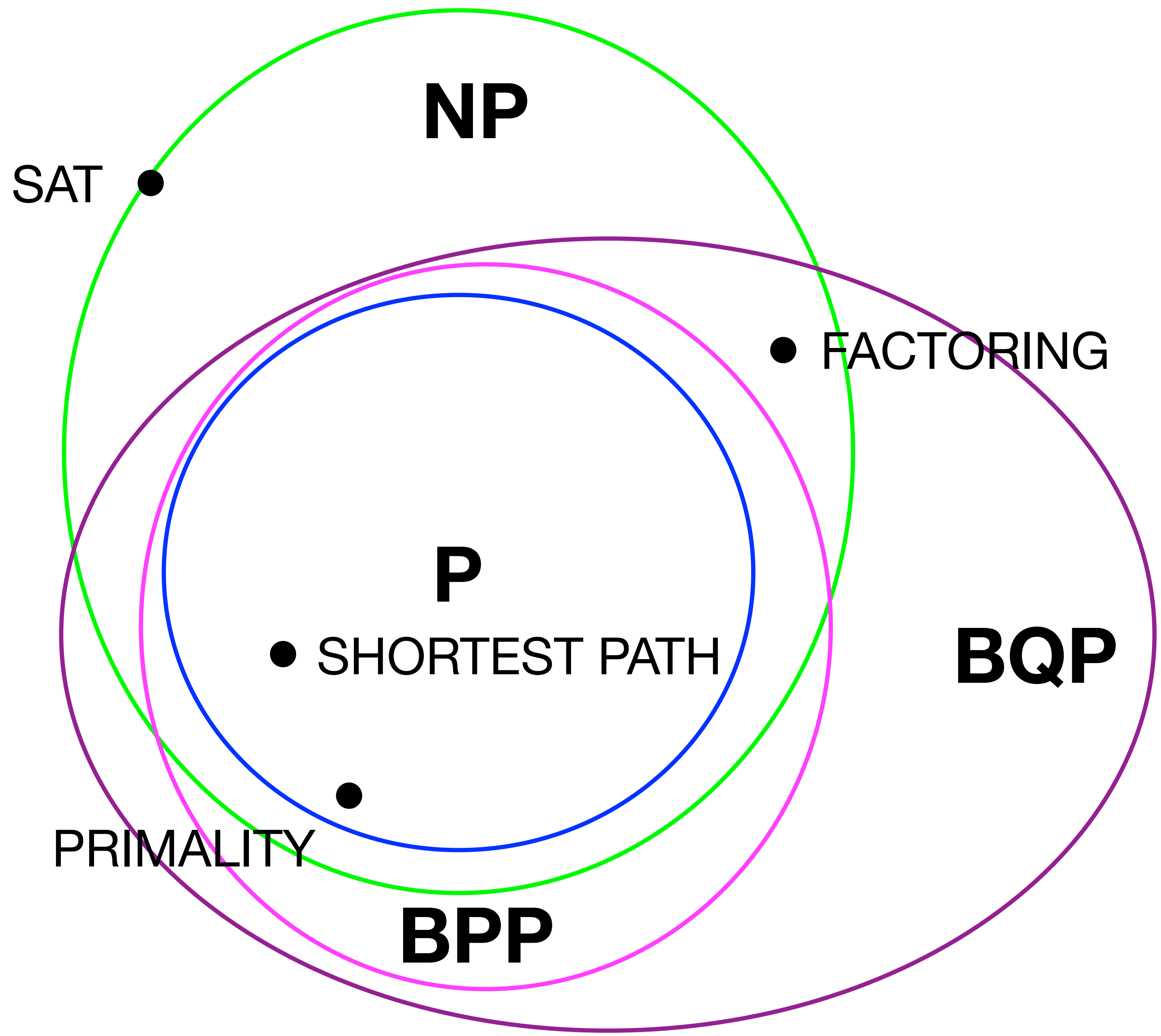
Quantum Complexity



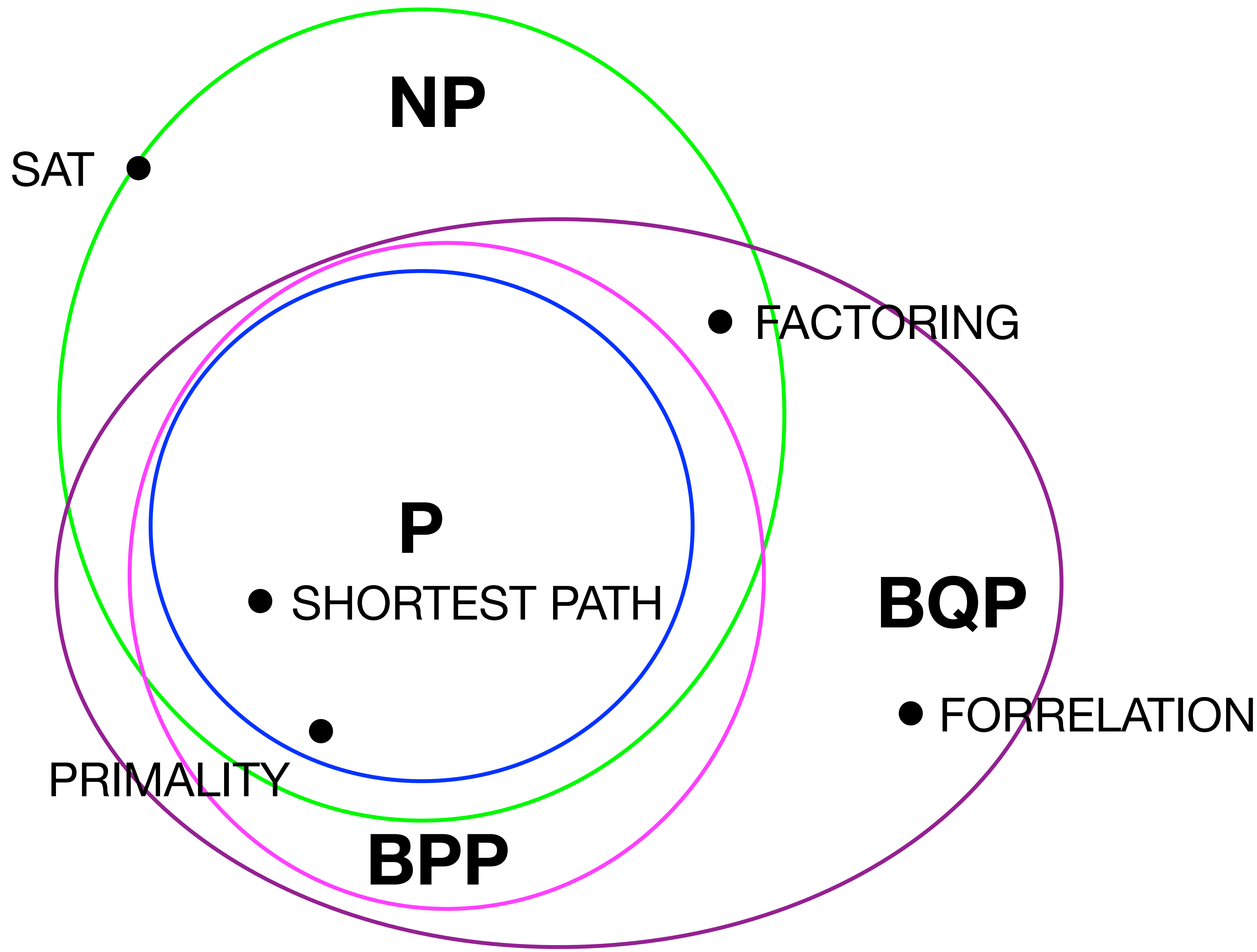
Quantum Complexity



Quantum Complexity



Quantum Complexity



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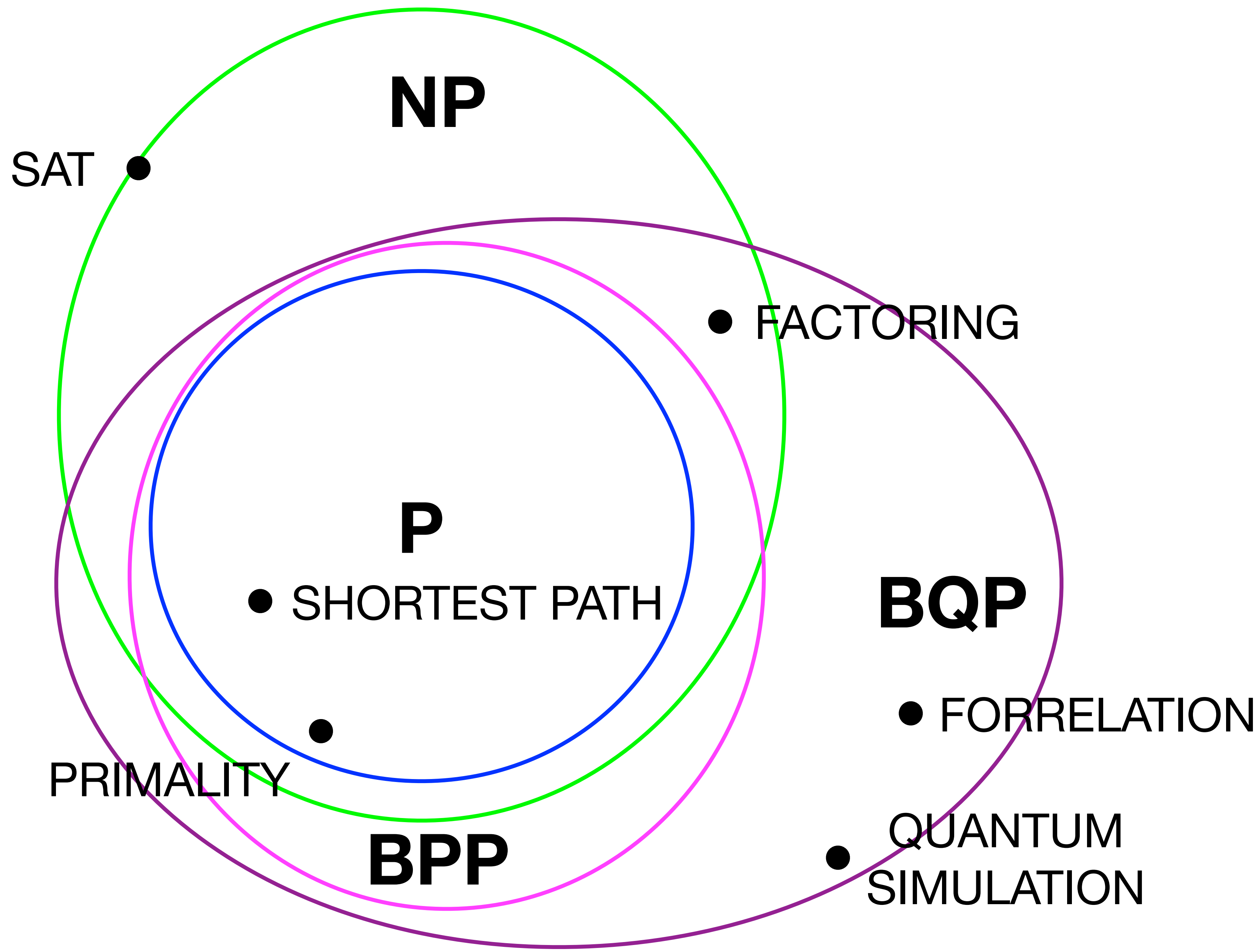
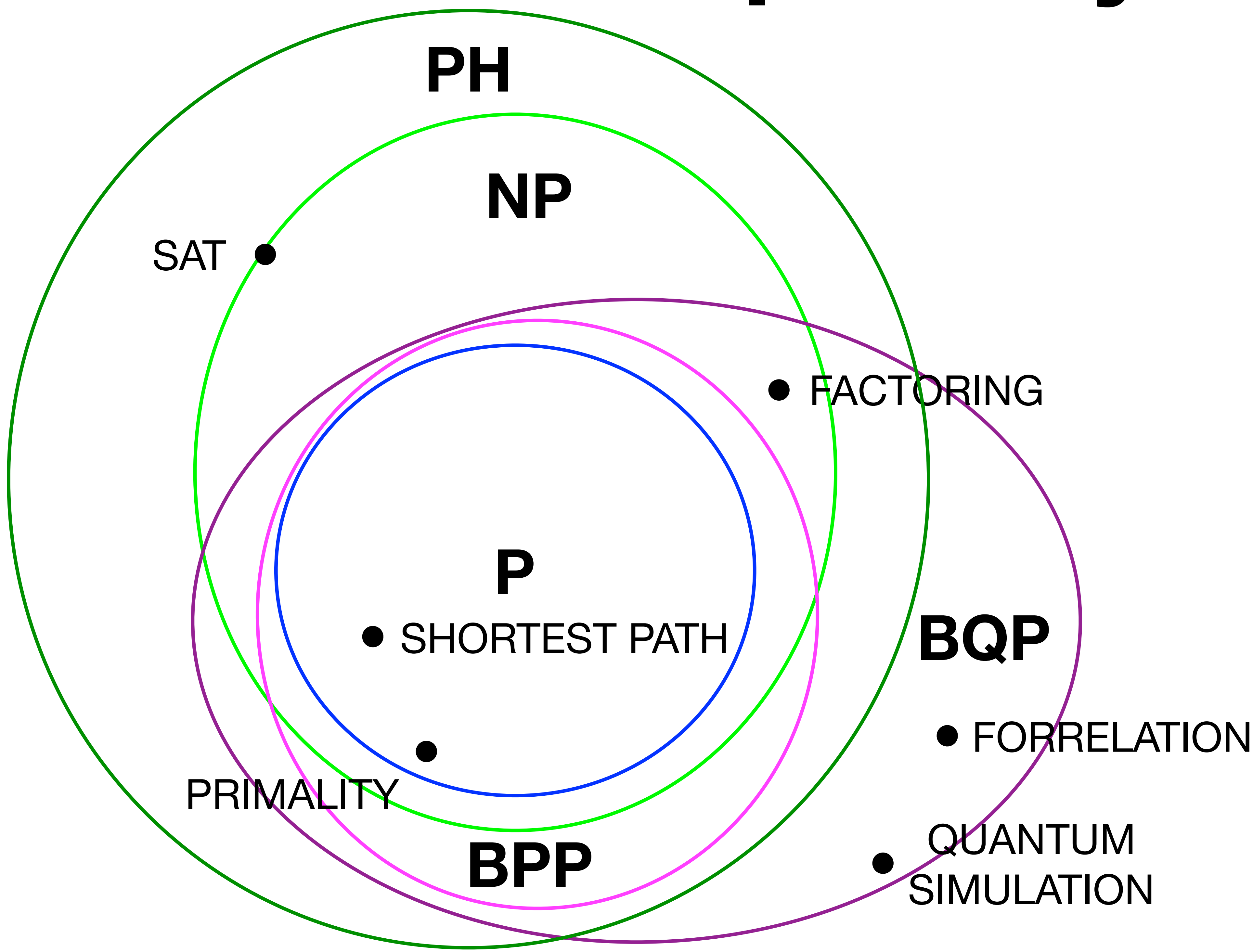
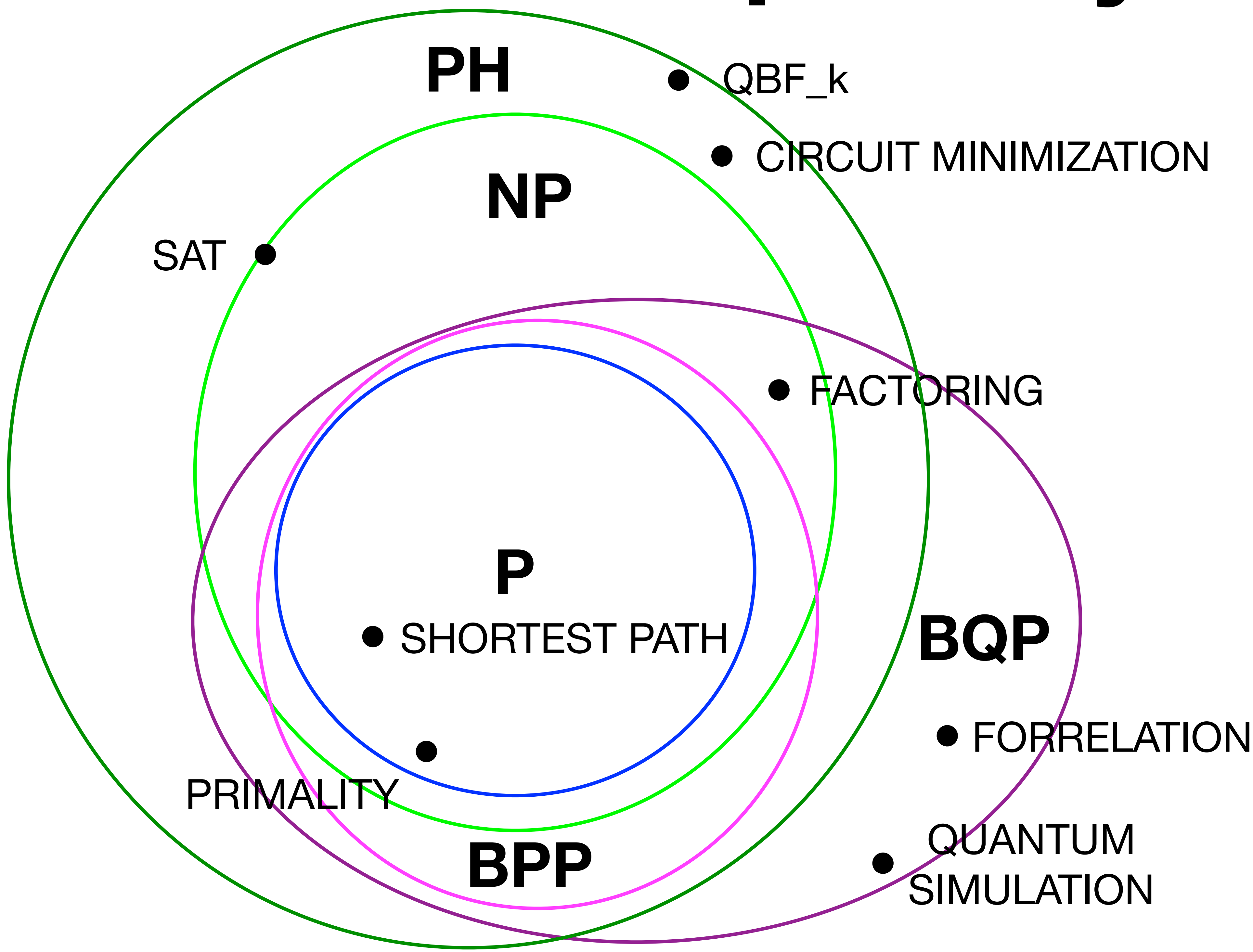


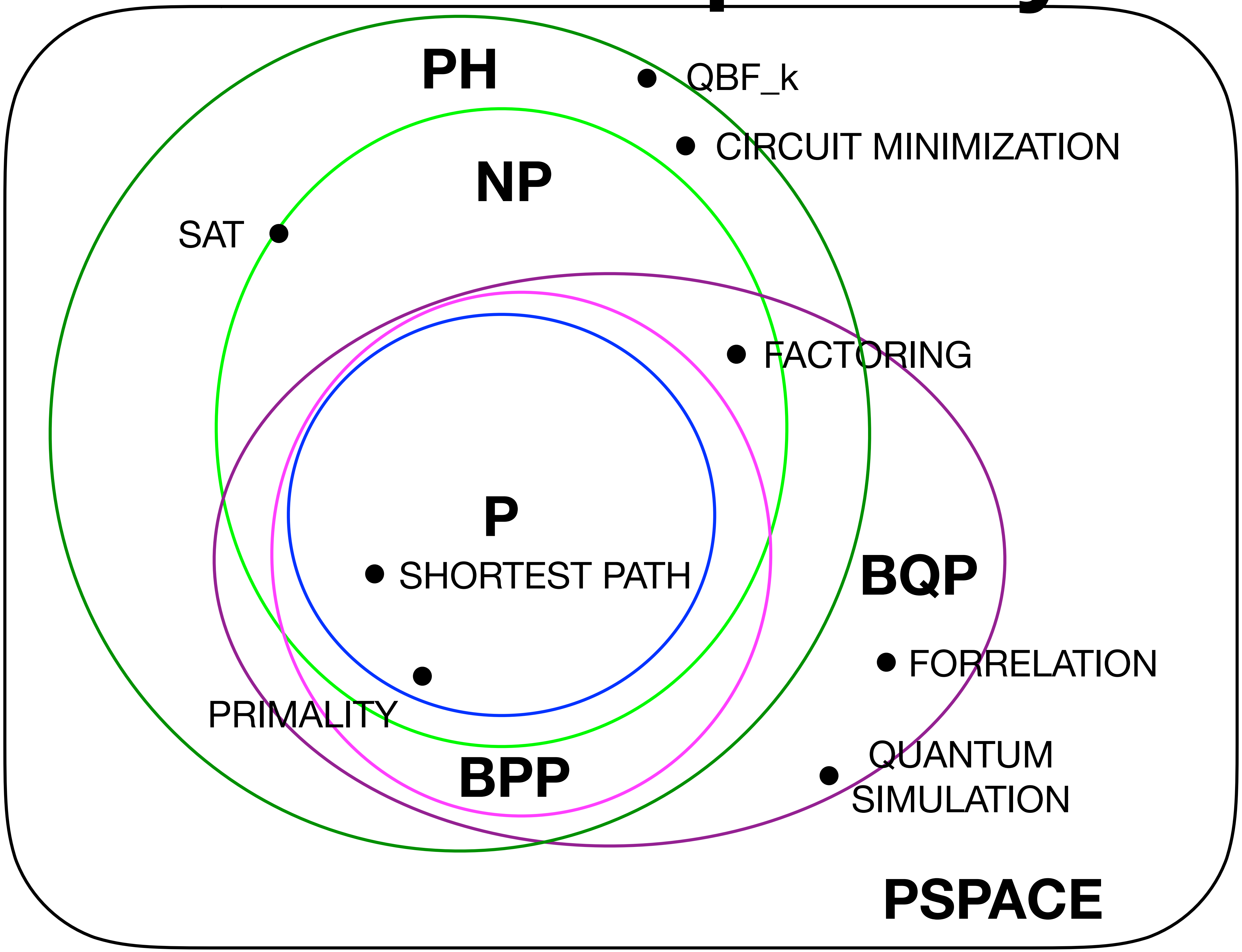
Quantum Complexity



Quantum Complexity

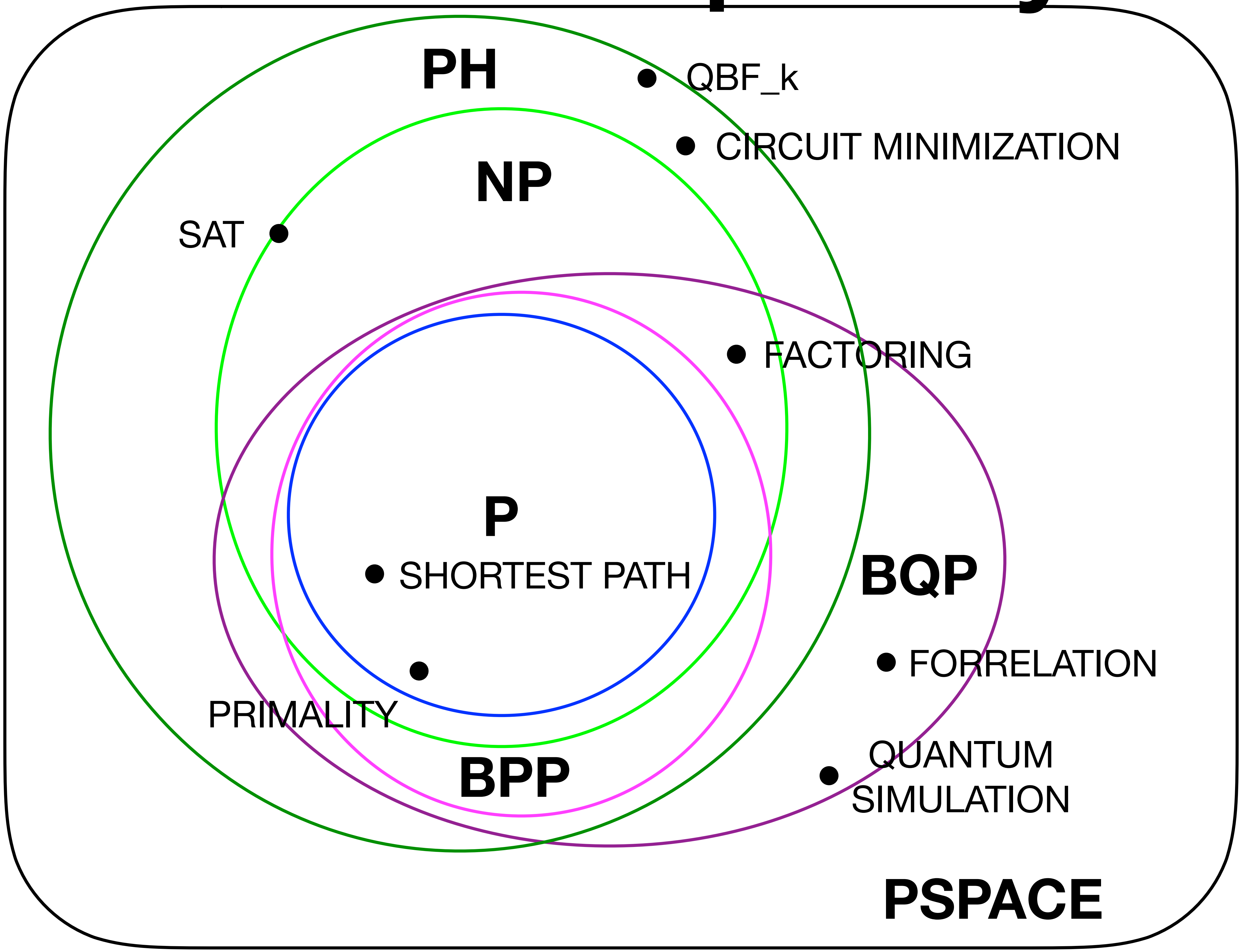


Quantum Complexity



Quantum Complexity

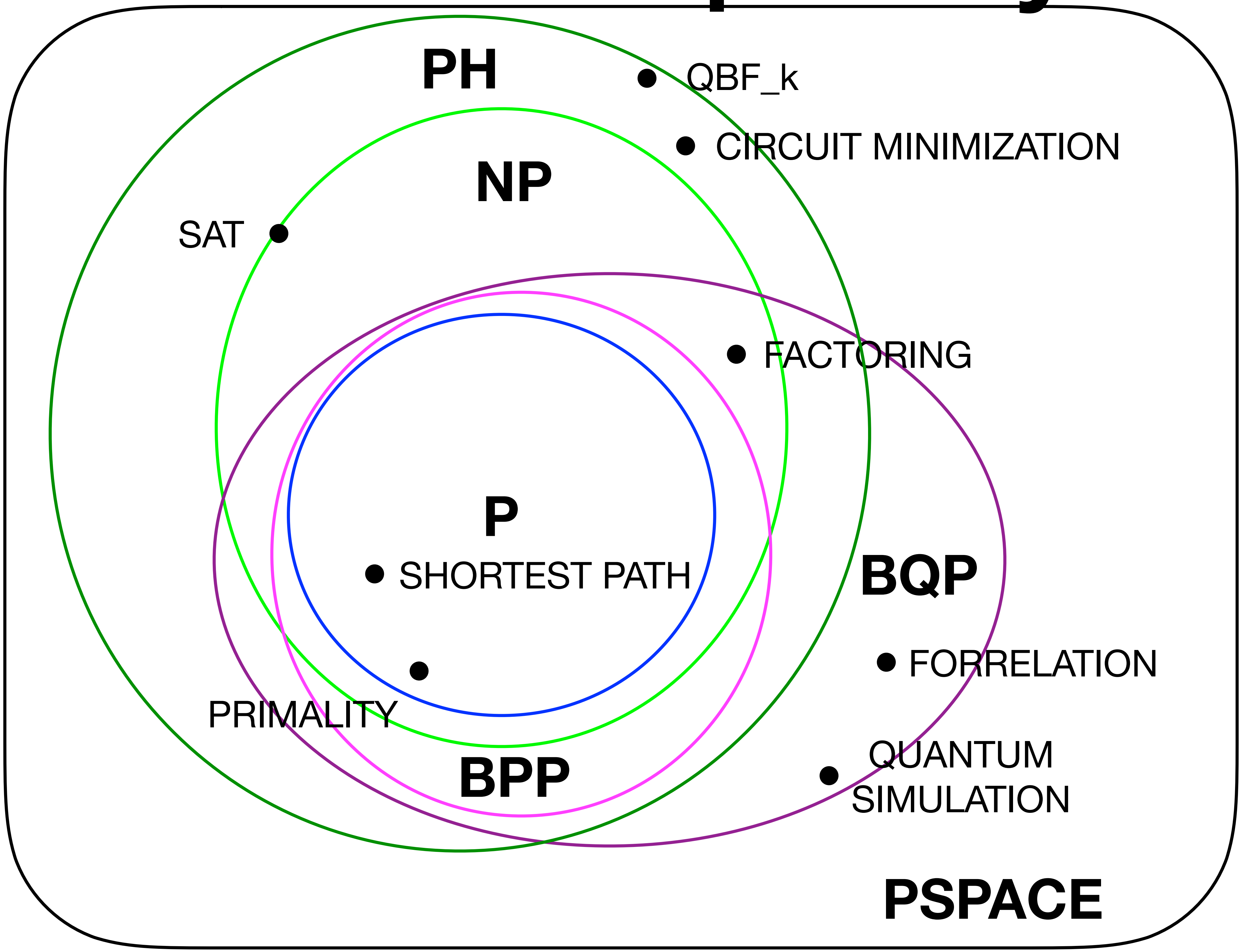
Common Beliefs



Quantum Complexity

Common Beliefs

$P \subset NP$

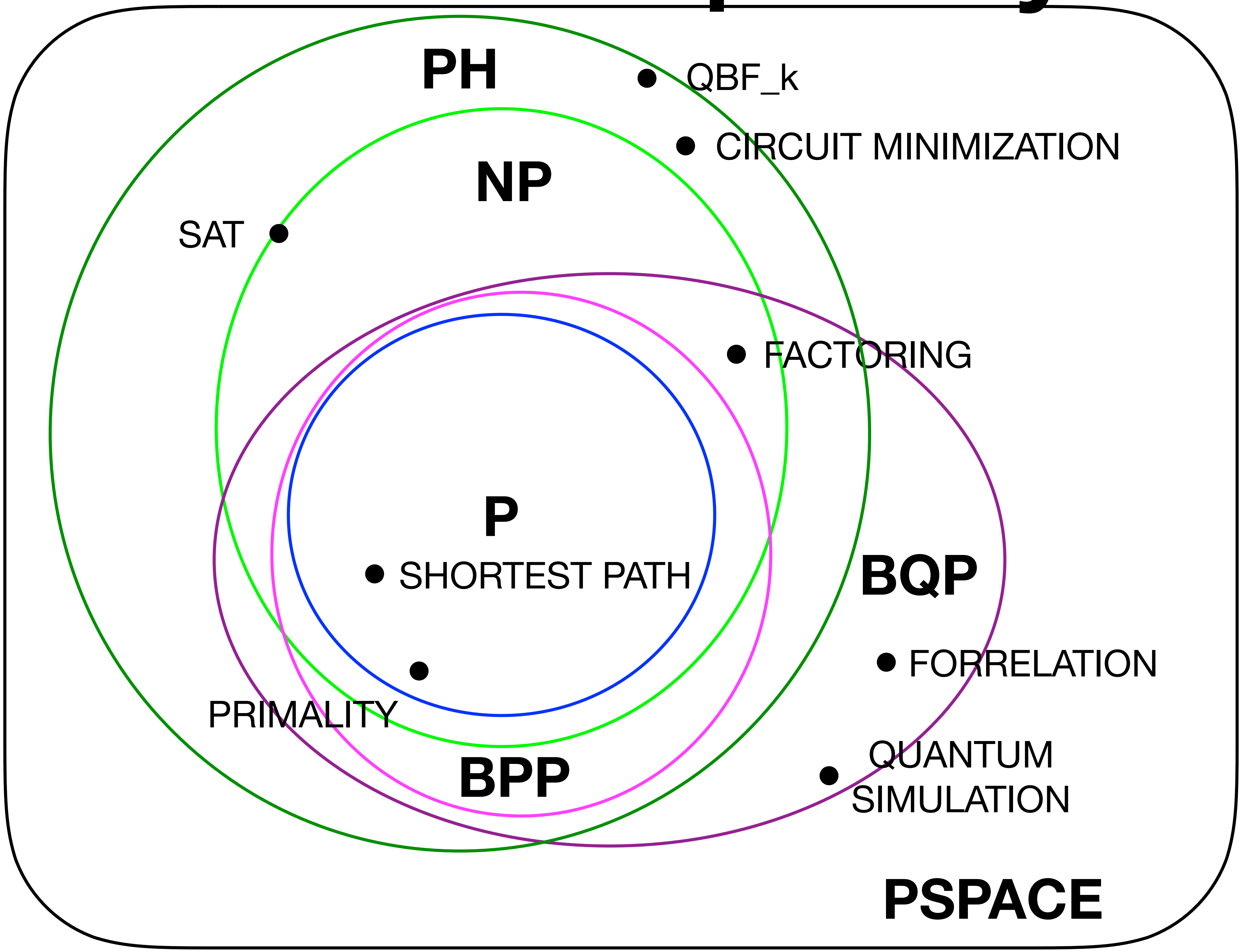


Quantum Complexity

<u>Common Beliefs</u>

$P \subset NP$

$P \subset BQP$



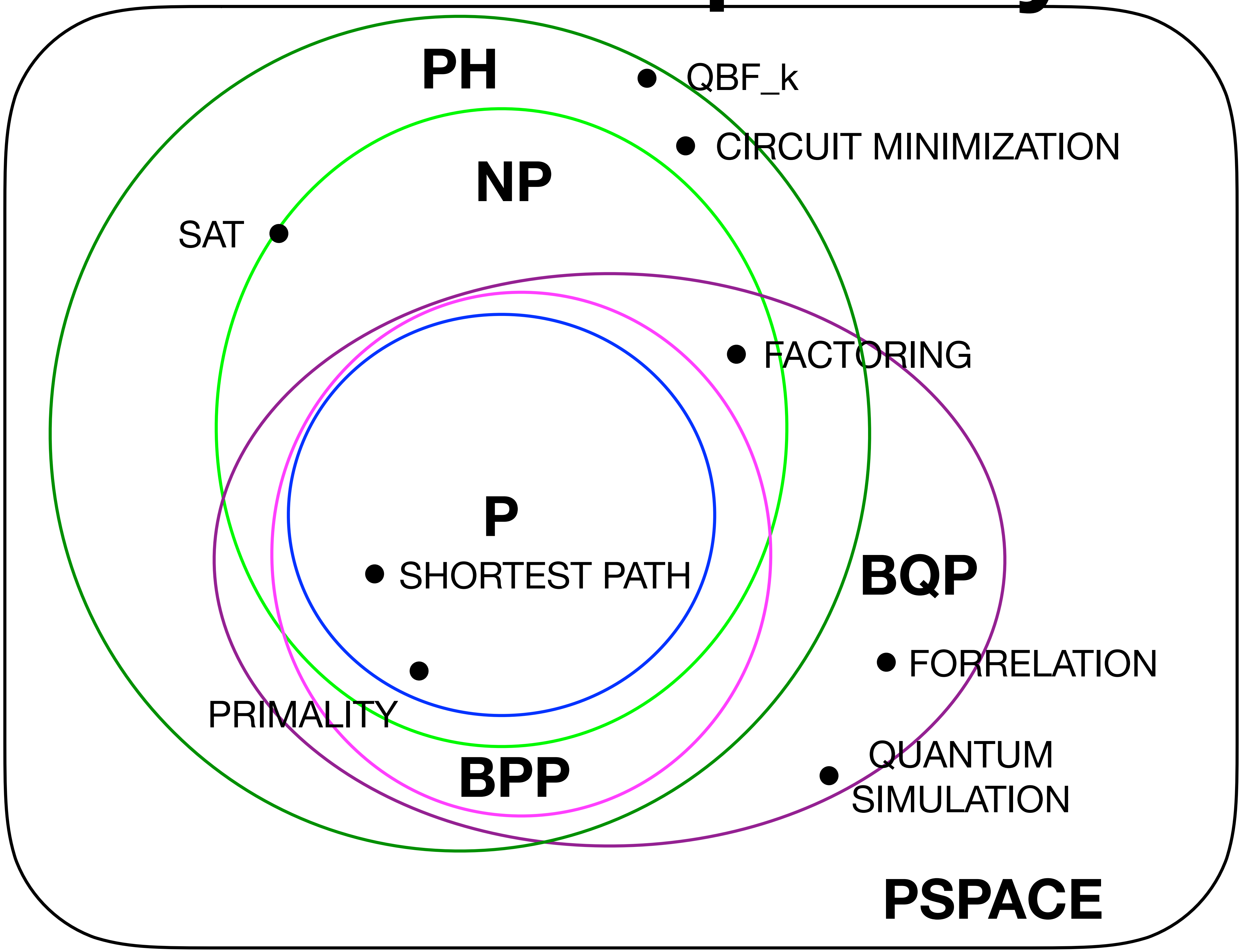
Quantum Complexity

Common Beliefs

$P \subset NP$

$P \subset BQP$

$P = BPP$



Quantum Complexity

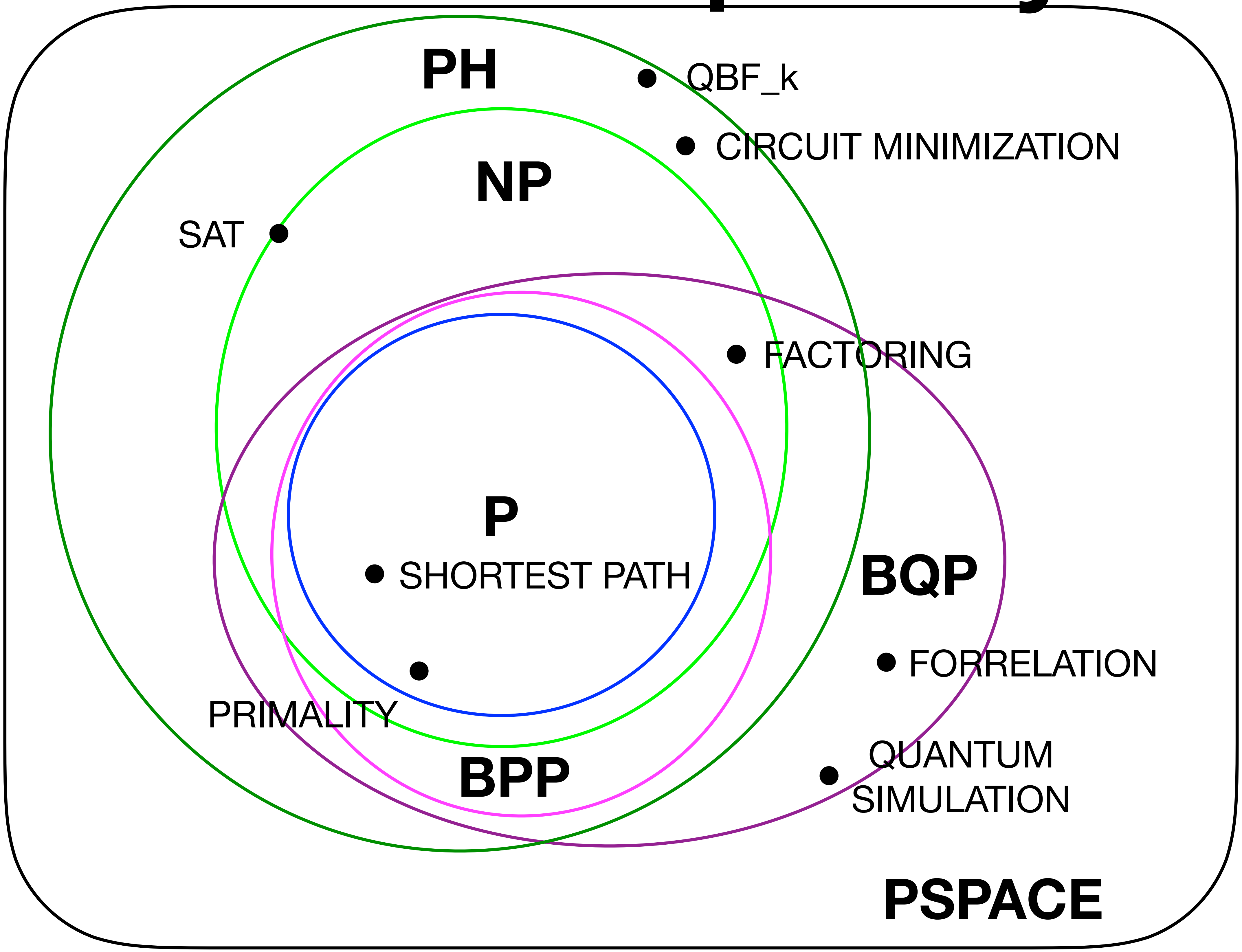
<u>Common Beliefs</u>

$P \subset NP$

$P \subset BQP$

$P = BPP$

$NP \not\subseteq BQP$



Quantum Complexity

Common Beliefs

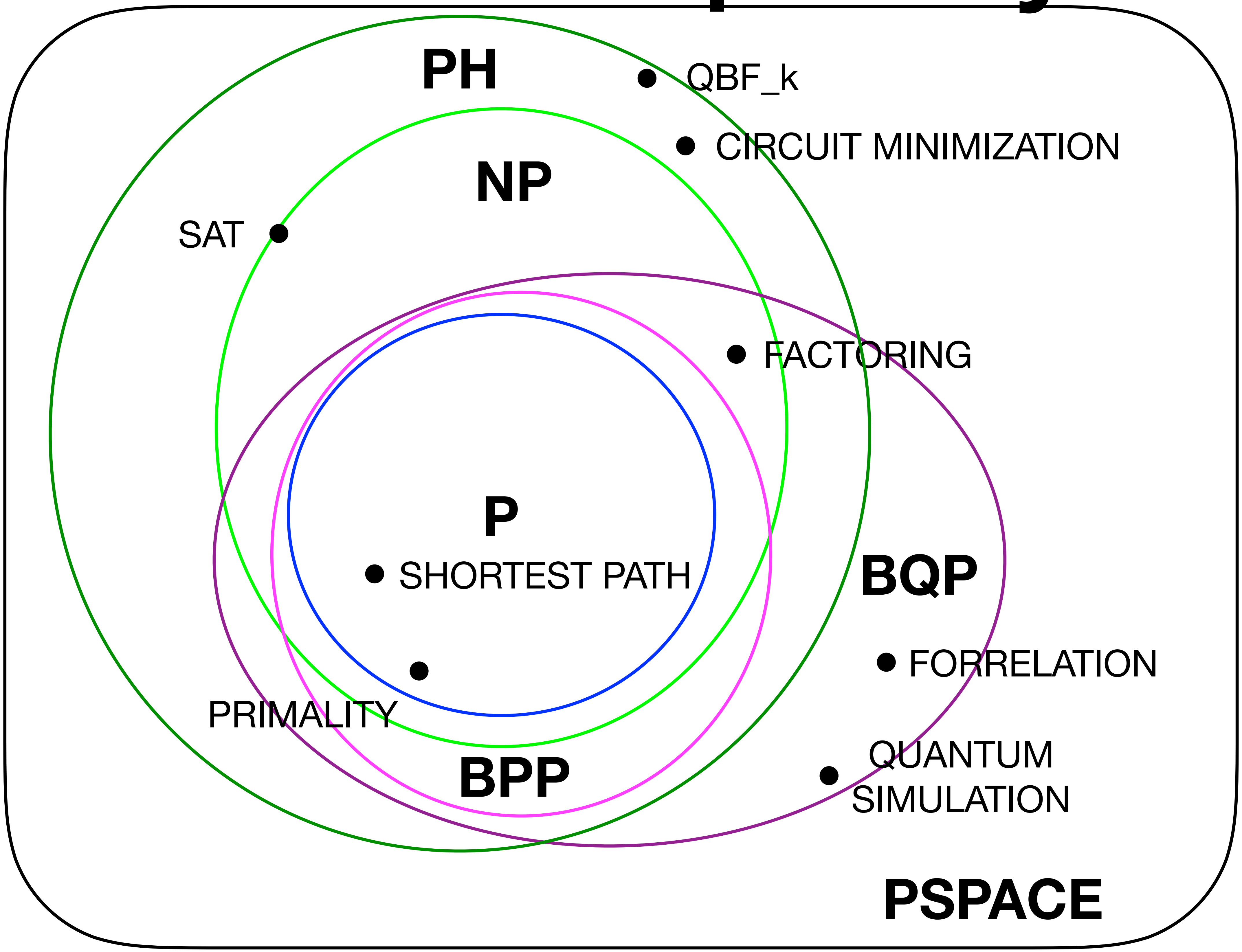
$P \subset NP$

$P \subset BQP$

$P = BPP$

$NP \not\subseteq BQP$

$BQP \not\subseteq NP$



Quantum Complexity

<u>Common Beliefs</u>

$P \subset NP$

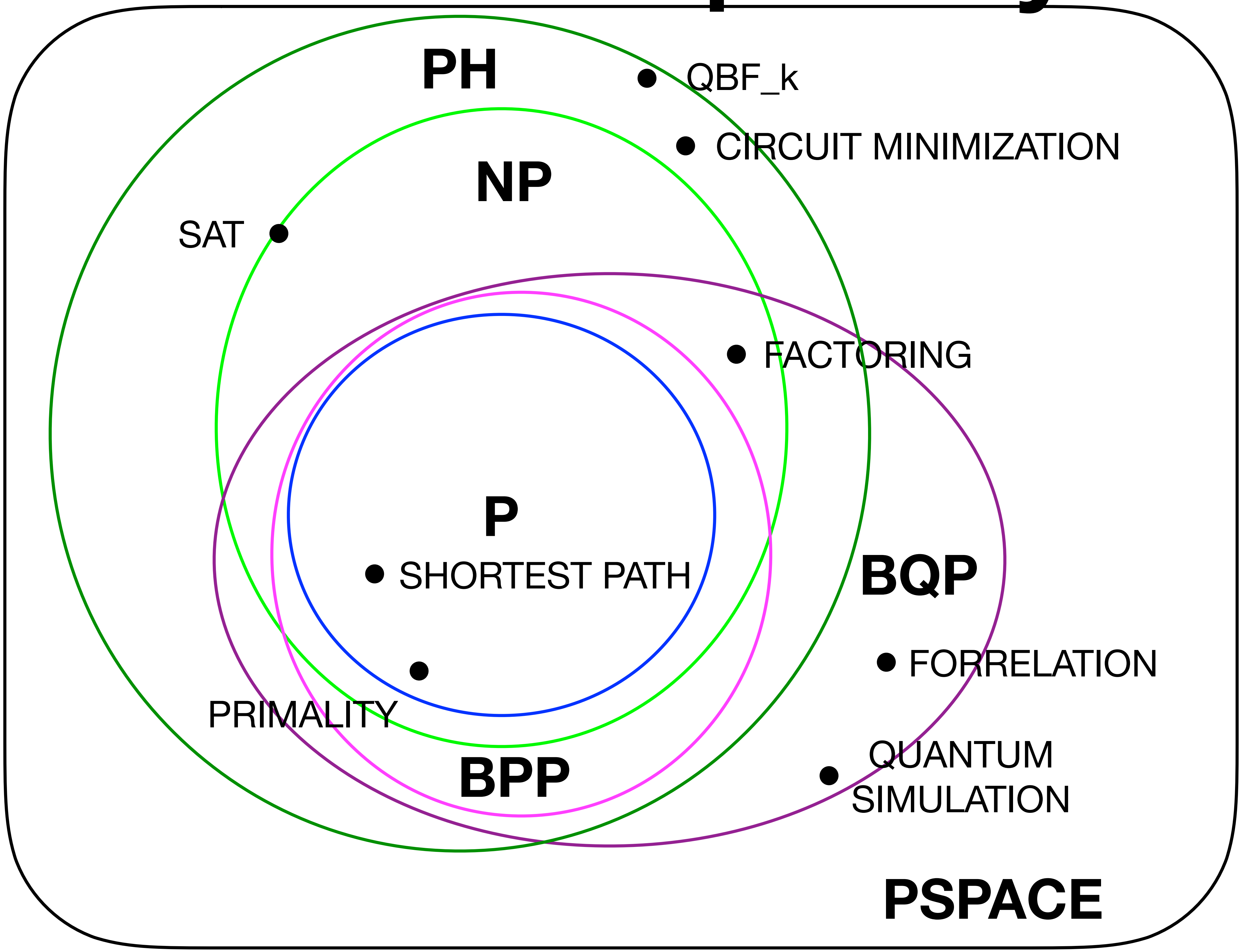
$P \subset BQP$

$P = BPP$

$NP \not\subseteq BQP$

$BQP \not\subseteq NP$

$BQP \not\subseteq PH$



Quantum Complexity

<u>Common Beliefs</u>

$P \subset NP$

$P \subset BQP$

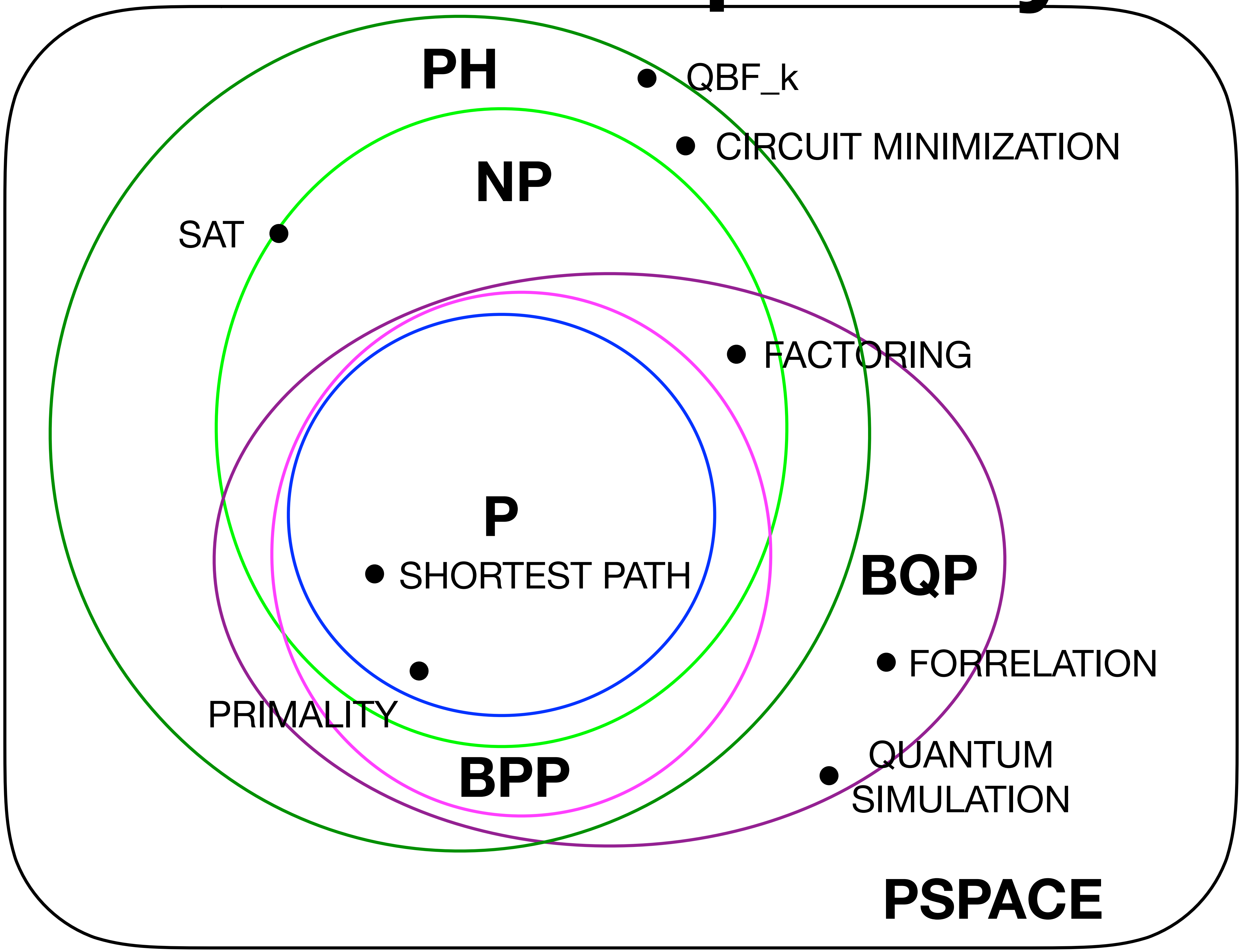
$P = BPP$

$NP \not\subseteq BQP$

$BQP \not\subseteq NP$

$BQP \not\subseteq PH$

(Raz & Tal, 2018)



Resources

- John Watrous' Lecture Notes on QC (2006)
- Quantum Country: https://quantum.country/qcvc
- Microsoft's *Quantum Katas* in Q#
- IIAS/HUJI Winter School (esp. Aharonov lectures)
- Complexity Zoo & Quantum Algorithm Zoo
- Scott Aaronson's blog