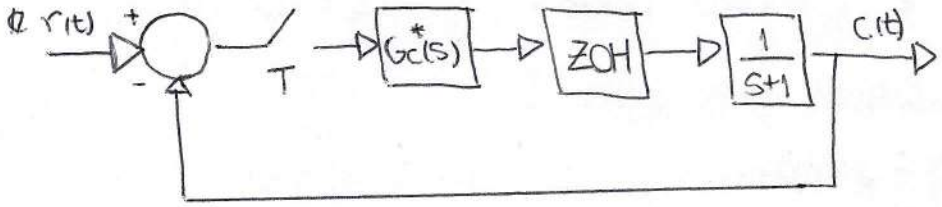
Exercício 3

Nome: Rodrigo Nunes Reis

Matrícula: 16/0144094

1.



$$G(z) = \frac{K}{1-z^{-1}} \cdot (1-z^{-1}) \cdot Z\left\{\frac{1}{s(s+1)}\right\} = K \cdot Z\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$\Rightarrow G(z) = K \cdot \left(\frac{z}{z-1} - \frac{ze^T}{ze^T-1}\right) = K \cdot \frac{(e^T-1)z}{e^T z^2 - (1+e^T)z + 1}$$

a) Obtida a função de malha aberta $G(z)$, obteve-se o LGR por meio do comando rlocus (ver figuras abaixo).

b) Da expressão $1+G(z)=0$ tiramos o polinômio característico $P(z)$:

$$P(z) = e^T z^2 - (1+e^T)z + 1 + K(e^T-1)z = 0$$

Para o critério de Routh modificado utilizamos $z = \frac{s+1}{s-1}$. Logo

$$P(s) = e^T\left(\frac{s+1}{s-1}\right)^2 + (-1-e^T+Ke^T-K)\left(\frac{s+1}{s-1}\right) + 1 = 0$$

$$\Rightarrow (Ke^T-K)s^2 + (2e^T-2)s - Ke^T + 2e^T + K + 2 = 0$$

s^2	Ke^T-K	$-Ke^T+2e^T+K+2$
s	$2e^T-2$	
	$-Ke^T+2e^T+K+2$	

Condições p/ estabilidade:

(I) $Ke^T-K > 0 \Rightarrow K > 0$, $(e^T-1) > 0$

(II) $-Ke^T+2e^T+K+2 > 0 \Rightarrow K < \frac{2+2e^T}{e^T-1}$

Desse modo, obtemos K_{cr}:

$$K_{cr} = \frac{2+2e^T}{e^T-1}$$

Assim sendo:

- $T = 0,5$: $K_{cr} = 8,166$
- $T = 1$: $K_{cr} = 4,328$
- $T = 2$: $K_{cr} = 2,626$

c) Dado $P(z) = e^T z^2 + (K(e^T-1) - e^T - 1) z + 1 = 0$

Substituímos $K = 2$ e resolvemos para cada T.

- $T = 0,5$: $z = 0,5794 \pm j\,0,4332$
- $T = 1$: $z = 0,3823 \pm j\,0,5787$
- $T = 2$: $z = 0,1059 \pm j\,0,7922$

d) Por meio do Simulink obtemos:

- $T = 0,5$: $M_p = 48,3\%$ $t_s = 8\,s$
- $T = 1$: $M_p = 39,5\%$ $t_s = 9\,s$
- $T = 2$: $M_p = 72,9\%$ $t_s = 8\,s$

e) Por meio do Simulink obtemos:

- $T = 0,5$: $e_{ss} = 0,2608$
- $T = 1$: $e_{ss} = 0,5001$
- $T = 2$: $e_{ss} = 1,007$

2.

$$G(z) = Z\{G_{ho}(s) \cdot G(s)\} = (1-z^{-1}) Z\left\{\frac{G(s)}{s}\right\} = (1-z^{-1}) Z\left\{\frac{1}{s^2(s+2)}\right\} = 0,01758 \cdot \frac{z+0,8753}{(z-1)(z-0,6703)}$$

a) Requisitos: $\zeta = 0,5$ e $t_s = 2\,s$

$$t_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = 4 \qquad \omega_d = \omega_n \sqrt{1-\zeta^2} = 3,464$$

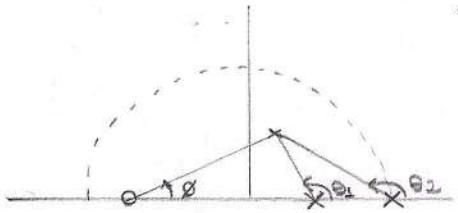
$z = \exp(sT)$ com $s = -\zeta\omega_n + \omega_d$

$|z| = e^{0,4}$ $\angle z = 39,70°$

$z = 0,5158 + j0,4281$

$$\angle G(z) = \phi - \theta_1 - \theta_2 = -231,26°$$

$$\Delta = -180° - \angle G(z)$$
$$= 51,27°$$



Deveremos projetar $G_D(z) = \frac{z - z_c}{z - p_c}$ com $\phi_c - \theta_c = 51,27°$

Podemos cancelar o pólo em 0,6703 : $z_c = 0,6703$. Assim:

$$\theta_c = \phi_c - 51,27° \Rightarrow \theta_c = \tan^{-1}\left(\frac{0,5158 - 0,6703}{0,4281}\right) - 51,27° = 58,58°$$

Assim $\theta_c = \tan^{-1}\left(\frac{0,5158 - p_c}{0,4281}\right) \Rightarrow p_c = \frac{0,5158 \tan\theta_c - 0,4281}{\tan\theta_c} = 0,2543$

Para a condição de módulo : $|G_D(z)G(z)|_{z=z_0} = 1$

$$\Rightarrow K \cdot 0,01758 \left[\frac{z + 0,8753}{(z-1)(z-0,2543)}\right]_{z=z_0} = 1 \Rightarrow K = 12,67$$

Logo : $G_D(z) = 12,67 \cdot \frac{z - 0,6703}{z - 0,2543}$

b) A partir de $t = 2s$ $\max(e(t)) = 0,021 \approx 2\%$. Pode-se dizer que o tempo de acomodação foi cumprido.

c) $e_{ss} = 0,3594$

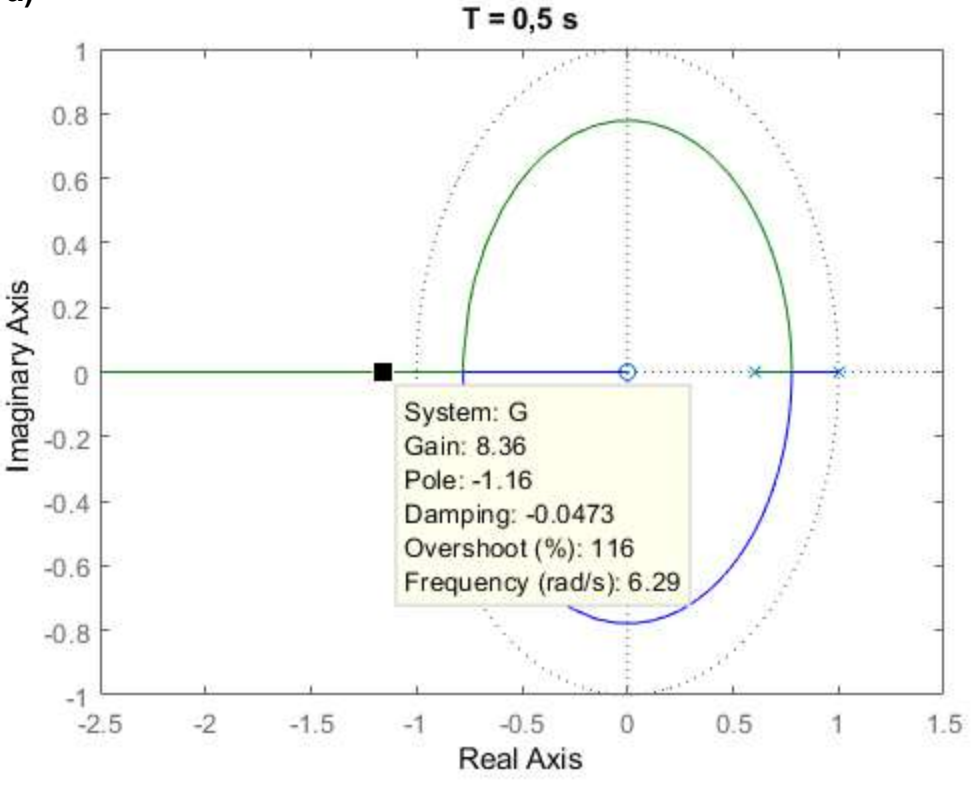
d) Podemos adicionar um compensador que altere pouco a posição do pólo mas que aumente o K_v em três.

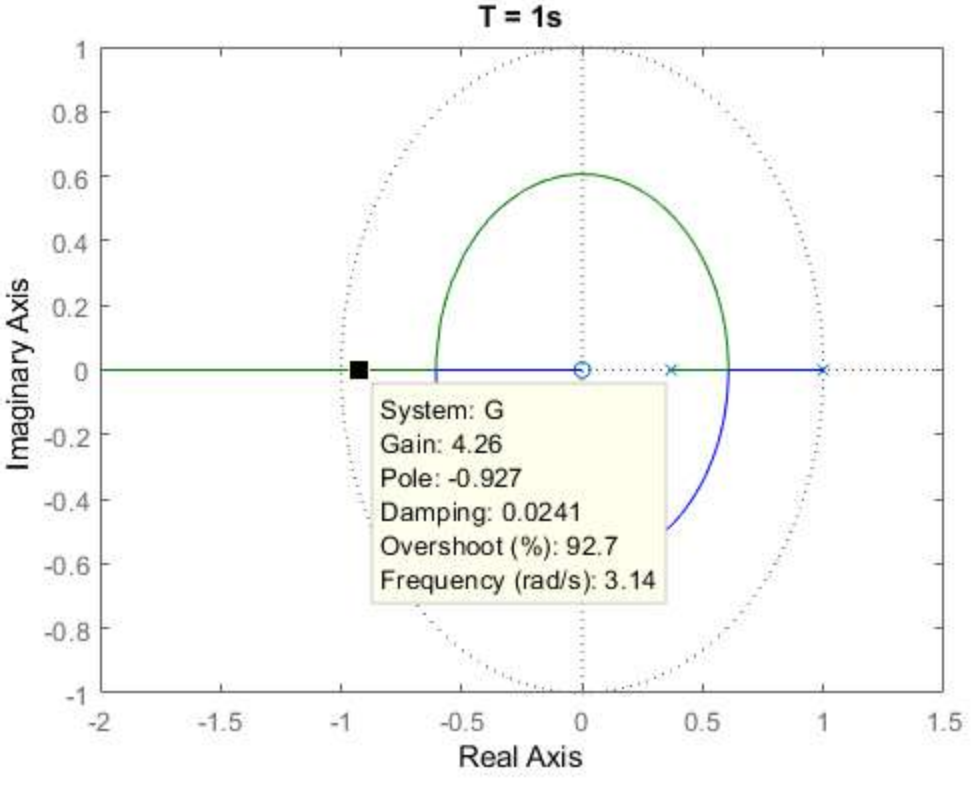
$G_c(z) = \frac{z - 0,97}{z - 0,99}$ Note que $\lim_{z \to 1}[G_c(z)] = 3$

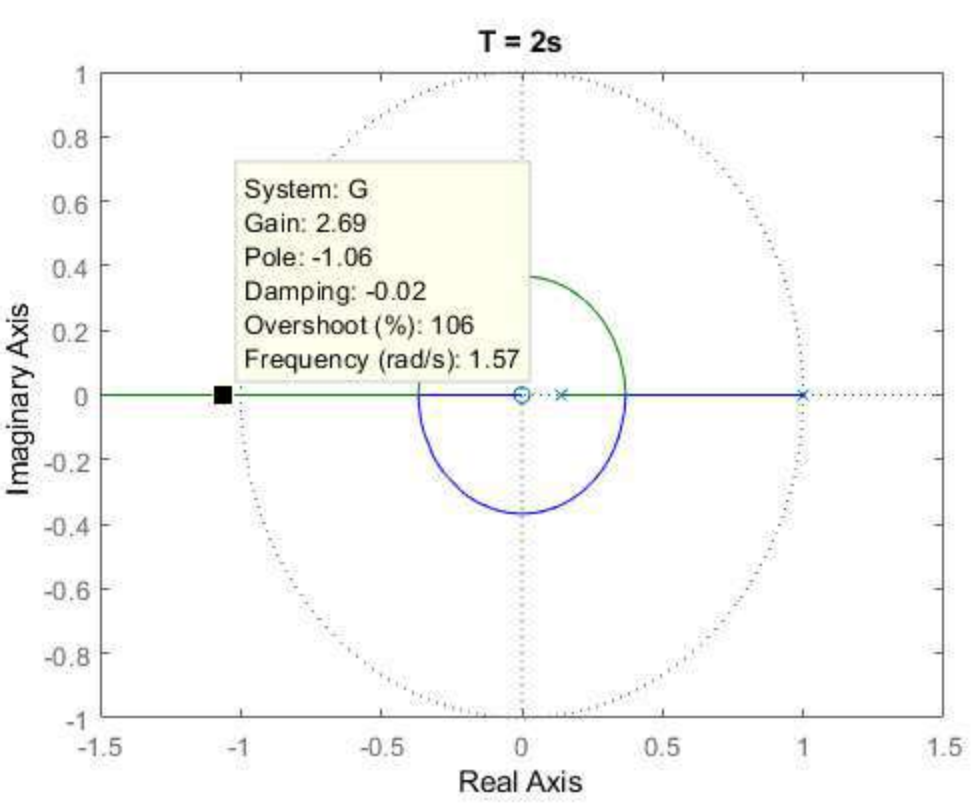
Logo $\lim_{z \to 1} G_c(z) \cdot \overbrace{\left(\frac{1 - z^{-1}}{T}\right) G_D(z)G(z)}^{K_v} = 3 K_v$ como queríamos.

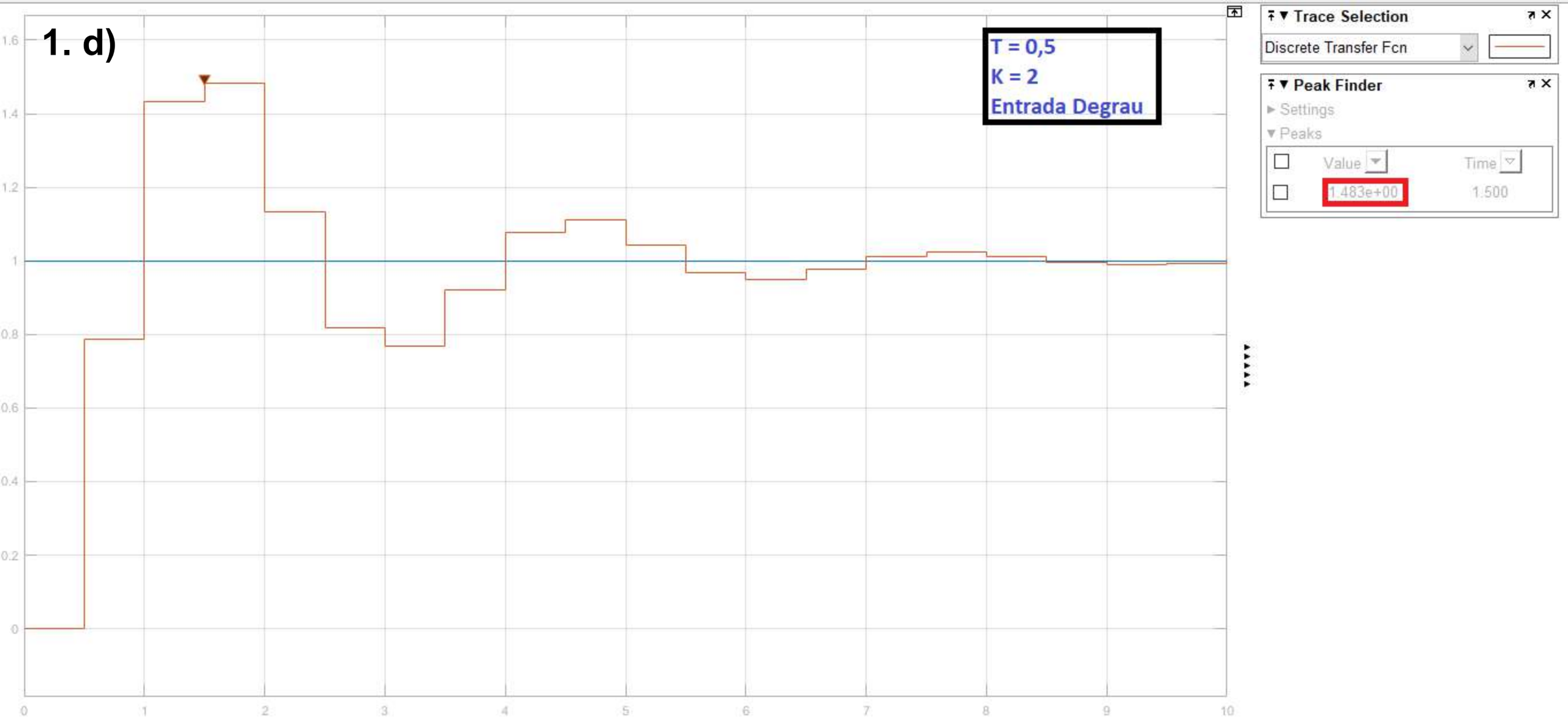
Com efeito, o erro estacionário diminui para 0,1963. Por outro lado, a resposta transitória se deteriorou : o tempo de acomodação dobrou e a convergência para o valor final foi extremamente lenta.

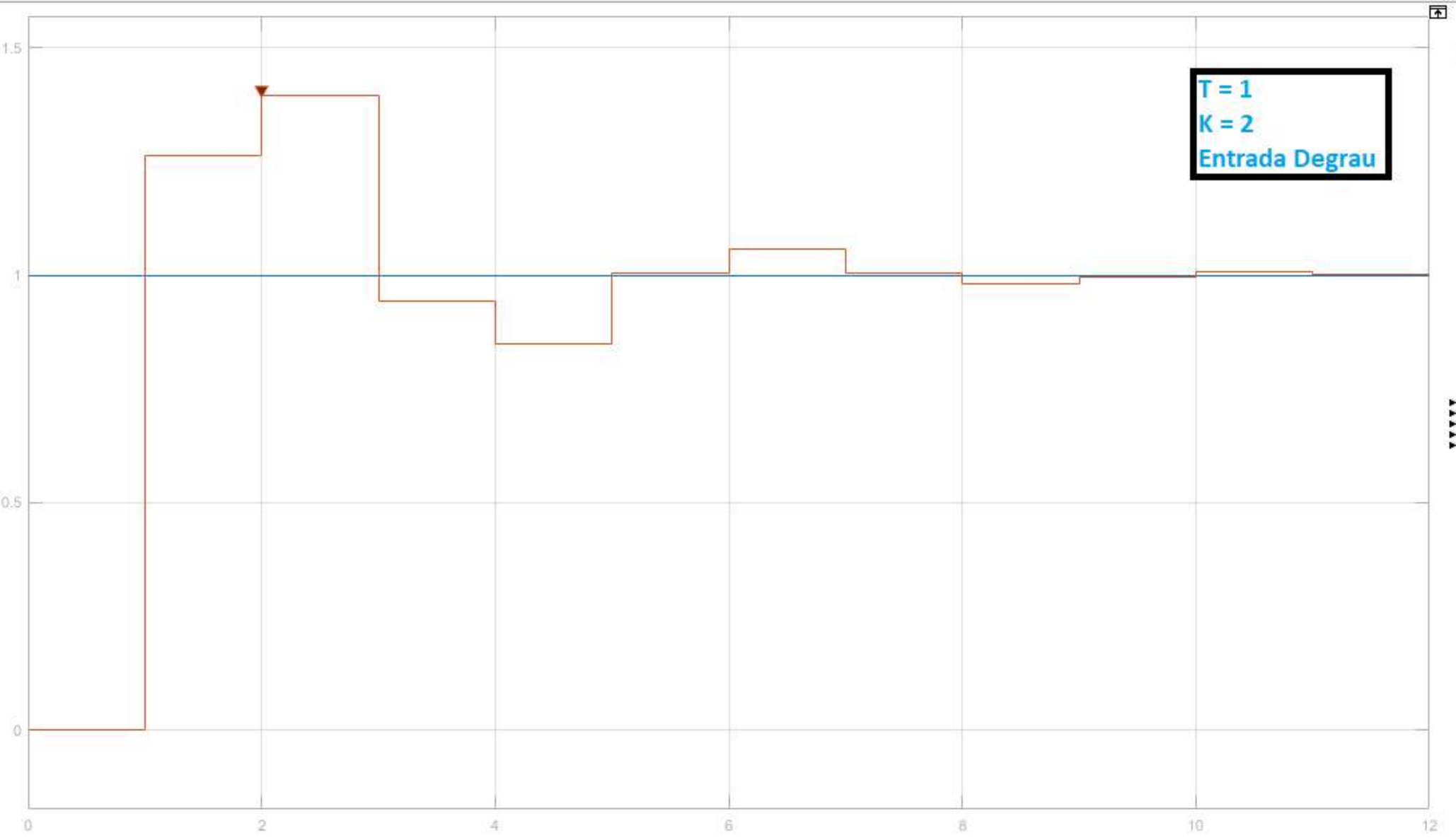
1. a)


T = 0,5 s
Imaginary Axis
Real Axis
System: G
Gain: 8.36
Pole: -1.16
Damping: -0.0473
Overshoot (%): 116
Frequency (rad/s): 6.29
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5


T = 1s
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
System: G
Gain: 4.26
Pole: -0.927
Damping: 0.0241
Overshoot (%): 92.7
Frequency (rad/s): 3.14


T = 2s
System: G
Gain: 2.69
Pole: -1.06
Damping: -0.02
Overshoot (%): 106
Frequency (rad/s): 1.57
Imaginary Axis
Real Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.5
-1
-0.5
0
0.5
1
1.5


1. d)
T = 0,5
K = 2
Entrada Degrau
Trace Selection
Discrete Transfer Fcn
Peak Finder
Settings
Peaks
Value
Time
1.483e+00
1.500
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10



Trace Selection

Discrete Transfer Fcn

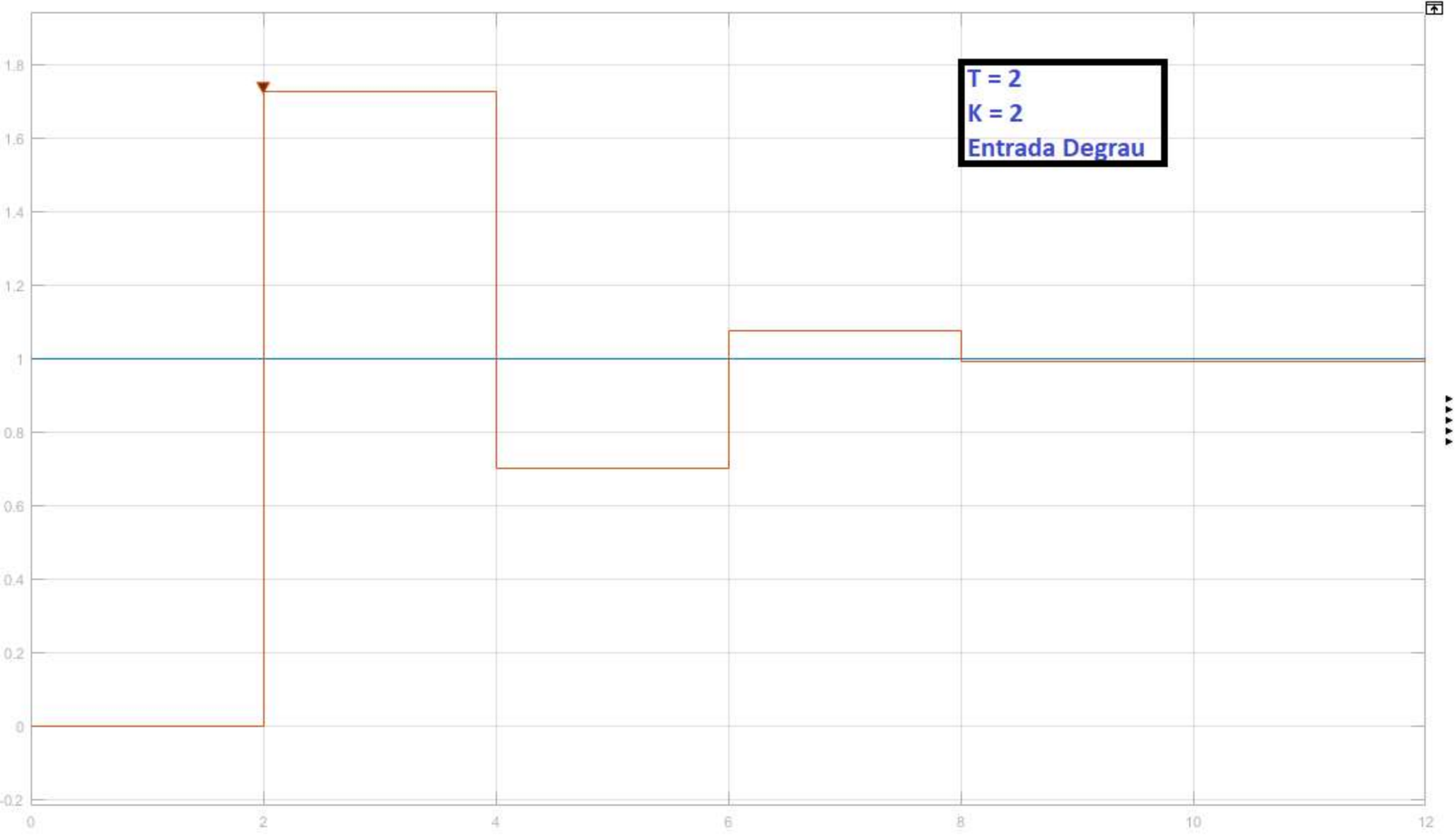
Peak Finder

Settings

Peak Threshold:	-Inf
Max Num of Peaks:	1
Min Peak Distance:	1
Peak Excursion:	0
Label Format:	X + Y

Peaks

Value	Time
1.395e+00	2.000



Trace Selection

Discrete Transfer Fcn

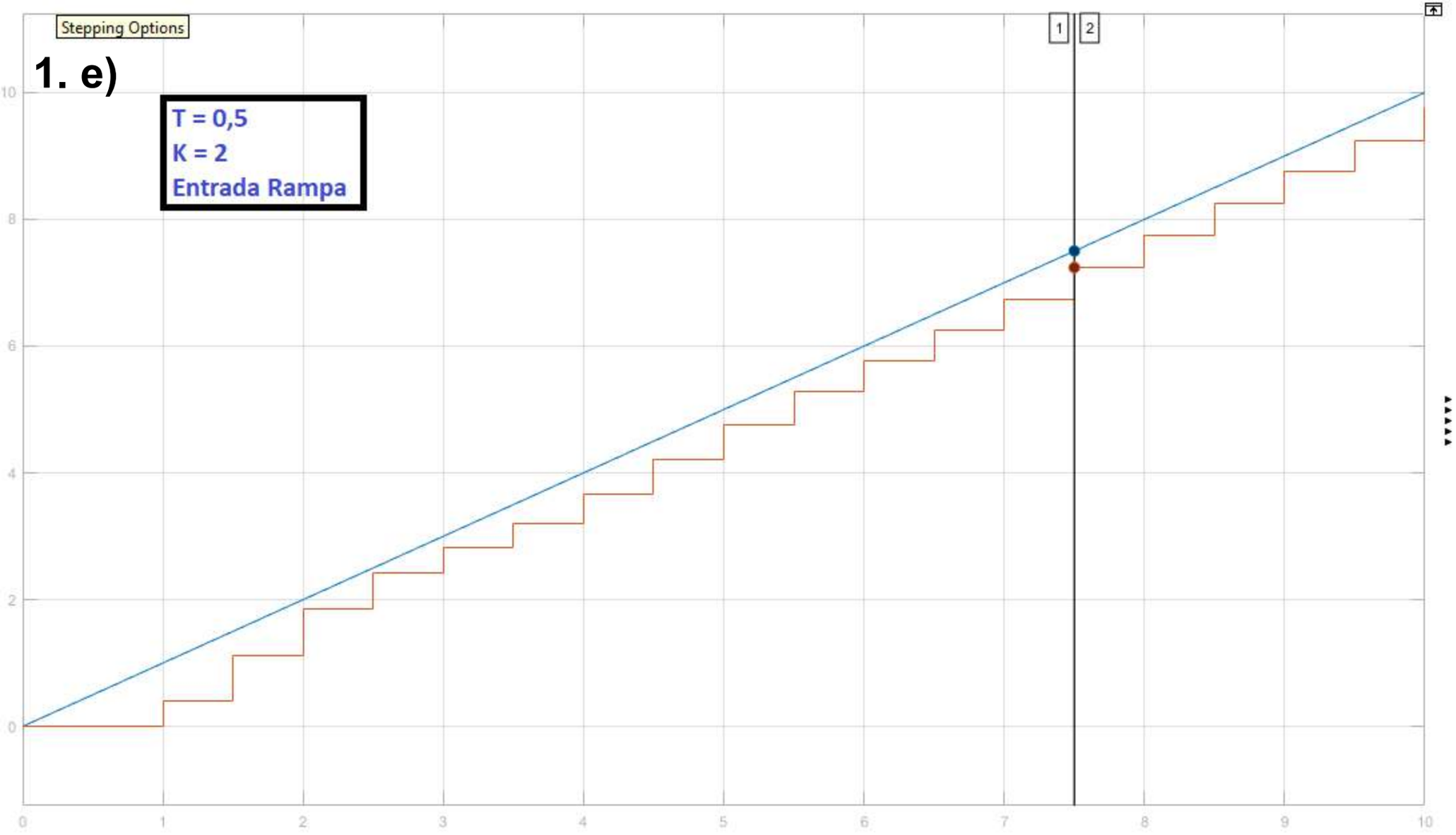
Peak Finder

Settings

Peak Threshold:	-Inf
Max Num of Peaks:	1
Min Peak Distance:	1
Peak Excursion:	0
Label Format:	X + Y

Peaks

Value	Time
1.729e+00	2.000



Trace Selection

Discrete Transfer Fcn

Peak Finder

Settings

Peaks

Value Time

-- --

Cursor Measurements

Settings

Screen cursors

Horizontal Vertical

Waveform cursors

1 | Ramp

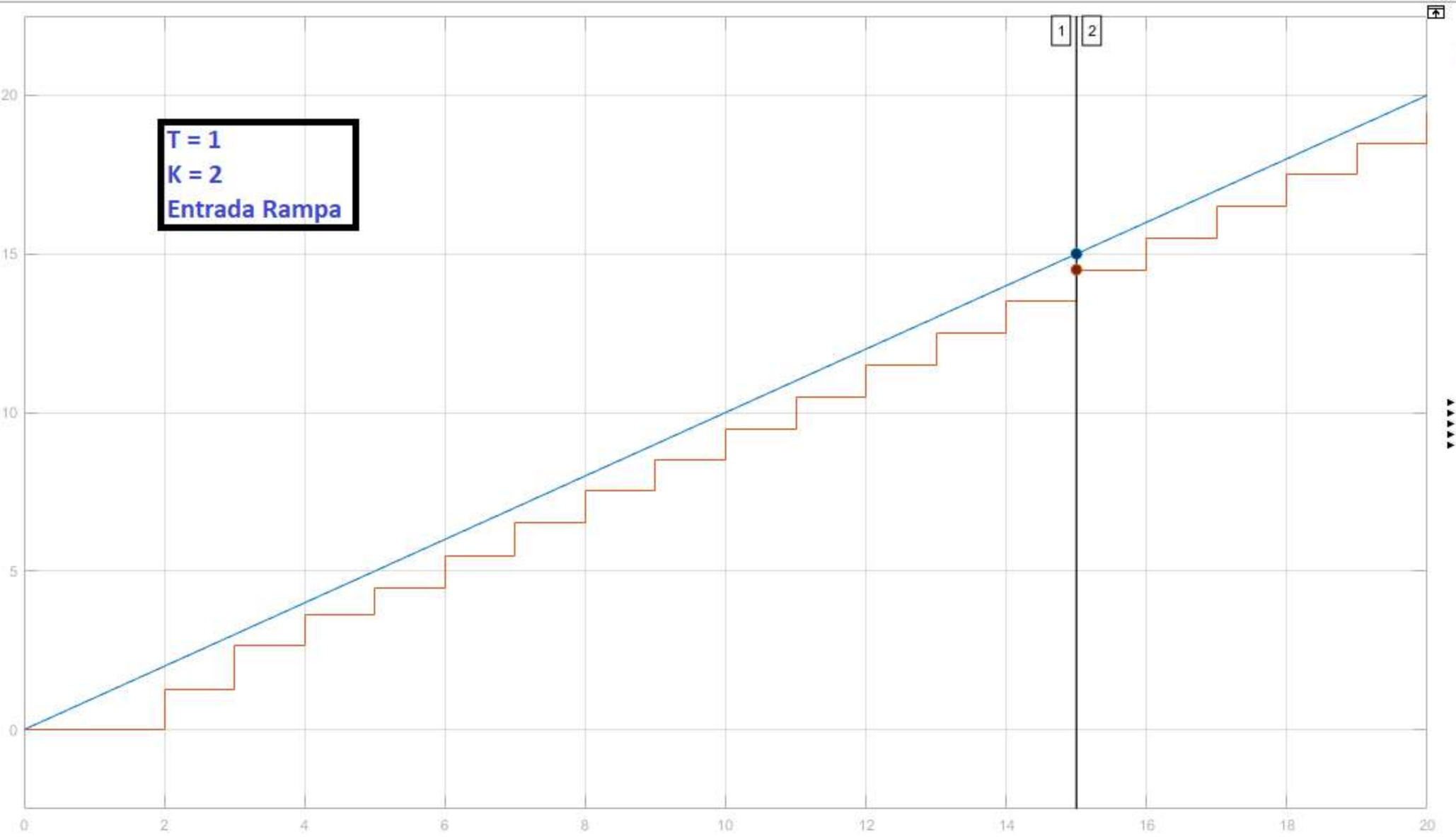
2 | Discrete Transfer Fcn

Lock cursor spacing

Snap to data

Measurements

	Time	Value
1	7.500	7.500e+00
2	7.500	7.239e+00
ΔT	34.639 fs	ΔY	2.608e-01
1 / ΔT	28.869 THz	
ΔY / ΔT	7.529 (/ps)	



Trace Selection

Discrete Transfer Fcn

Cursor Measurements

Settings

Screen cursors

Horizontal Vertical

Waveform cursors

1 | Ramp

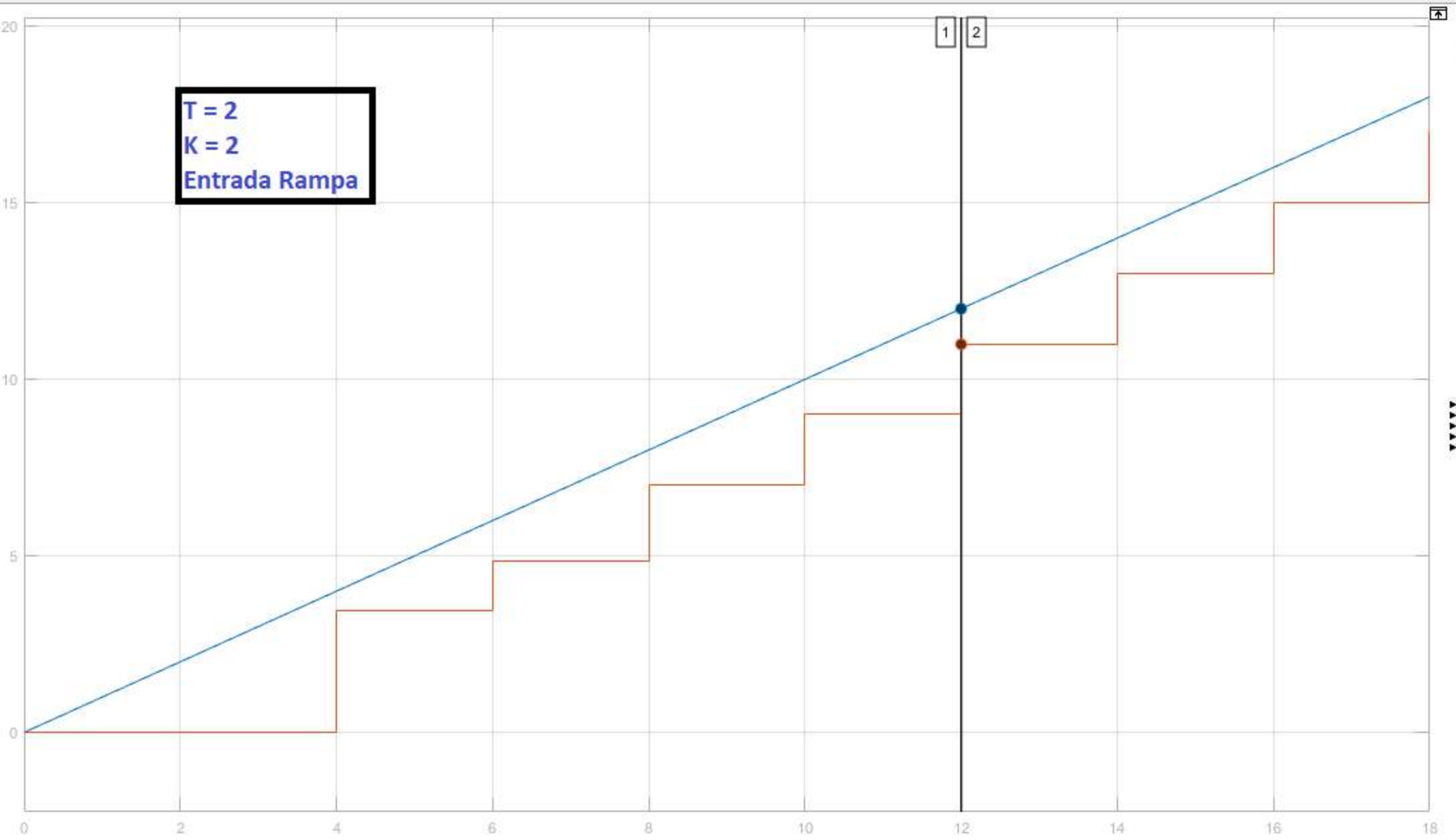
2 | Discrete Transfer Fcn

Lock cursor spacing

Snap to data

Measurements

	Time	Value
1	15.000	1.500e+01
2	15.000	1.450e+01
ΔT	69.278 fs	ΔY	5.001e-01
1 / ΔT	14.435 THz	
ΔY / ΔT	7.219 (/ps)	



Trace Selection

Ramp

Cursor Measurements

Settings

Screen cursors

Horizontal Vertical

Waveform cursors

1 | Ramp

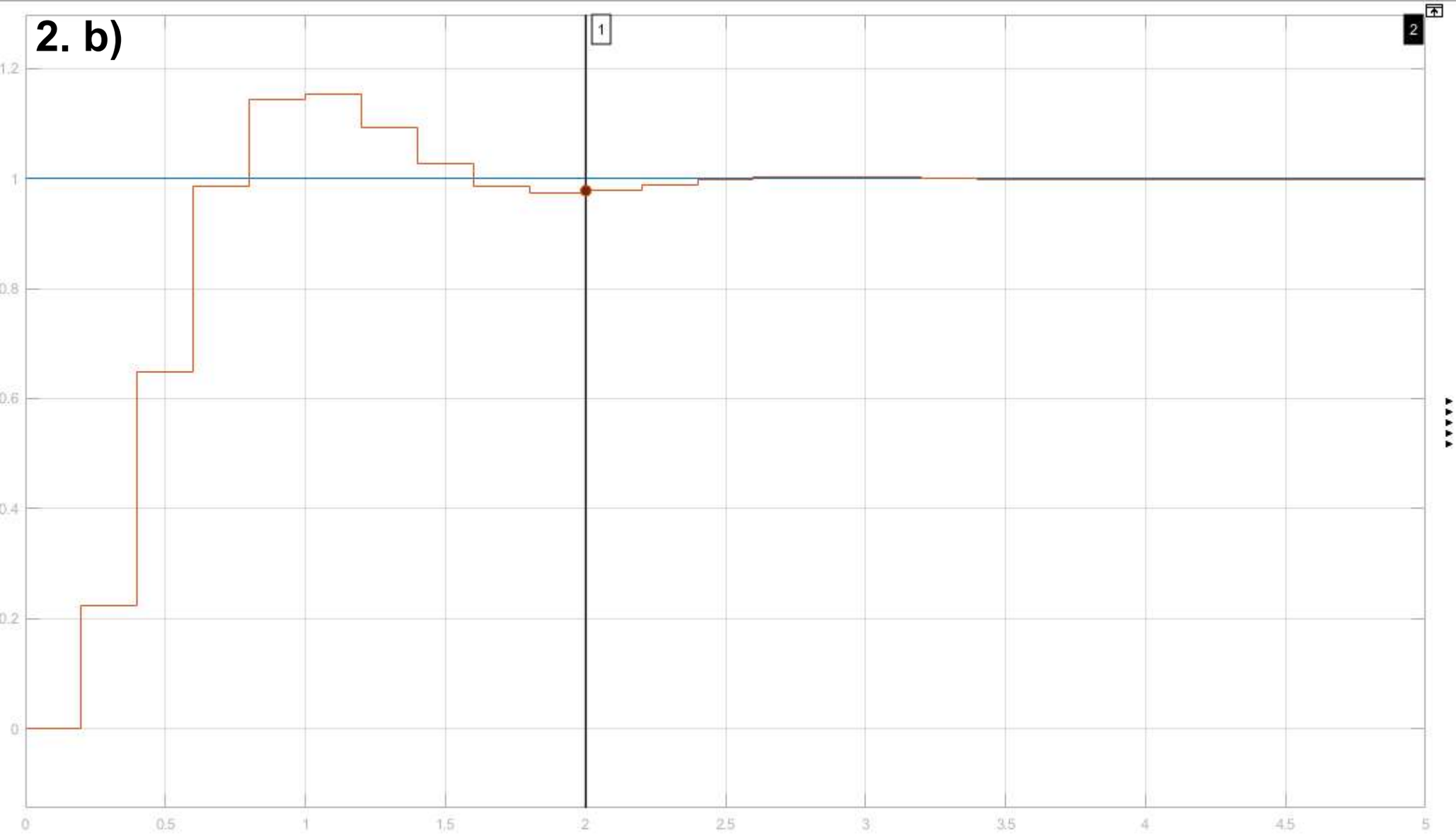
2 | Discrete Transfer Fcn

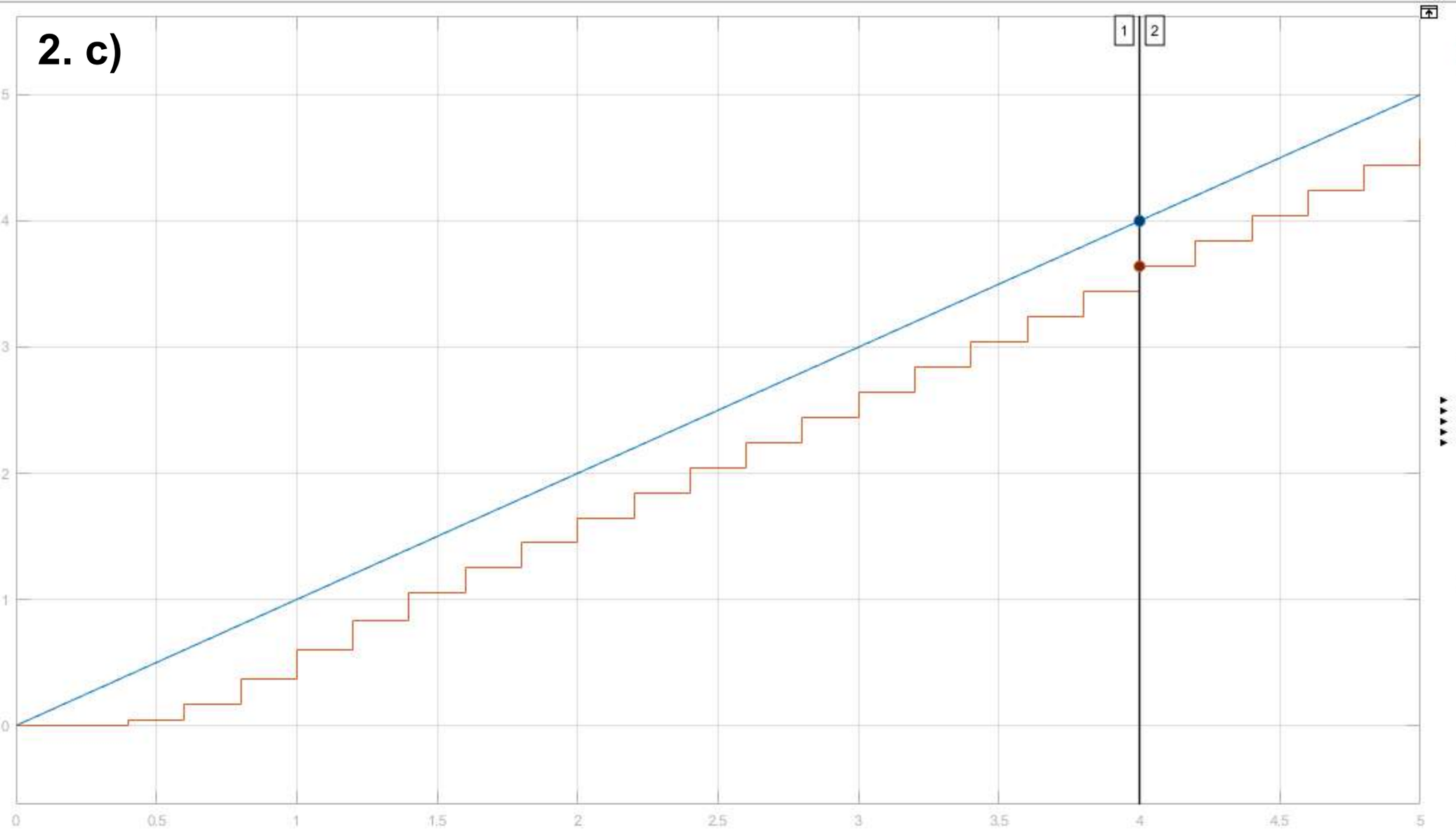
Lock cursor spacing

Snap to data

Measurements

	Time	Value
1 |	12.000	1.200e+01
2 |	12.000	1.099e+01
ΔT	0.000 s	ΔY 1.007e+00
1 / ΔT		Inf Hz
ΔY / ΔT		Inf (/s)


2. b)
1
2
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Trace Selection
Step
Cursor Measurements
Settings
Screen cursors
Horizontal
Vertical
Waveform cursors
1 | Discrete Transfer Fcn
2 | Step
Lock cursor spacing
Snap to data
Measurements
Time
Value
1 | 2.001
9.779e-01
2 | 6.000
–
ΔT 3.999 s
ΔY –
1 / ΔT
250.082 mHz
ΔY / ΔT
–



Trace Selection

Ramp

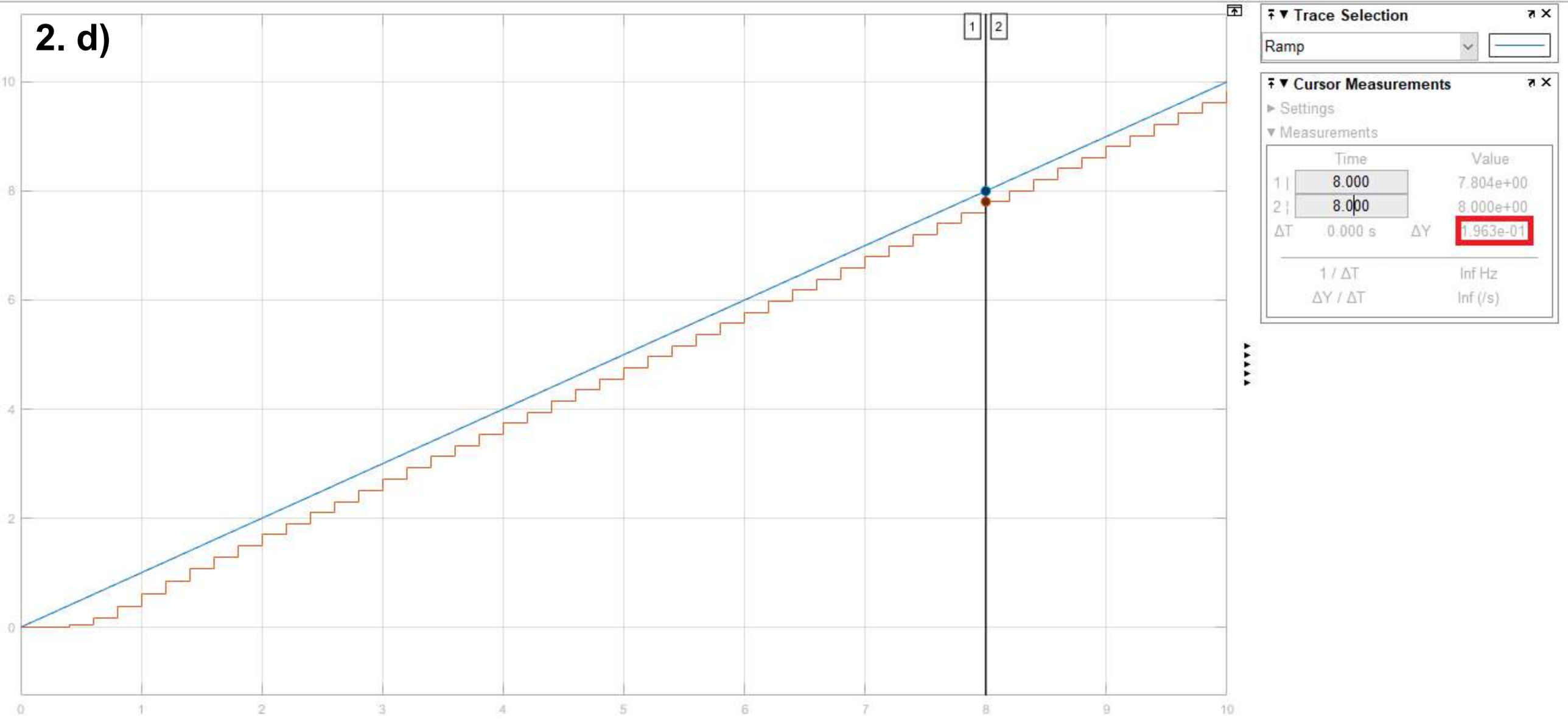
Cursor Measurements

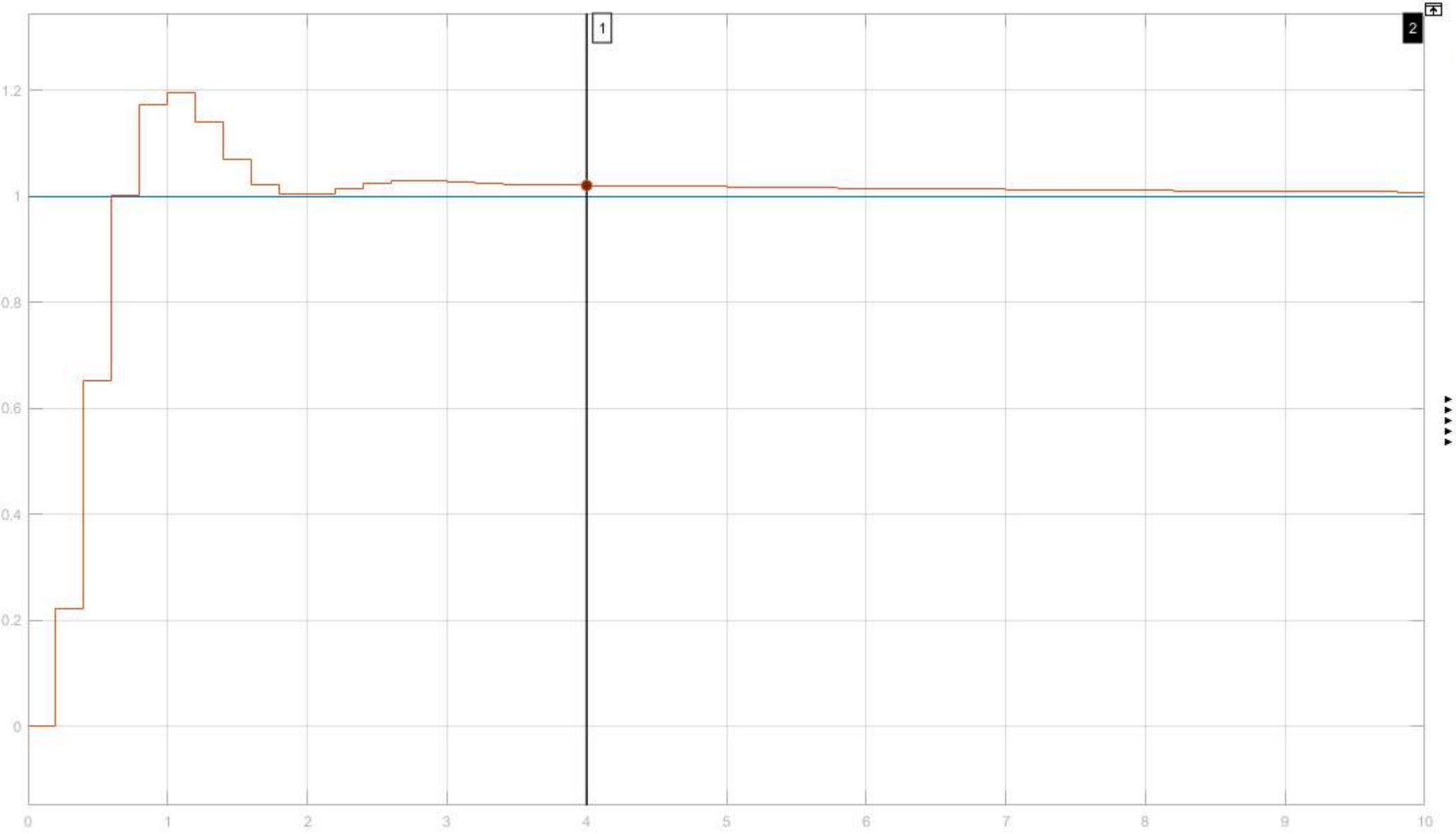
Settings

- Screen cursors
  - Horizontal
  - Vertical
- Waveform cursors
  - 1 | Ramp
  - 2 | Discrete Transfer Fcn
- Lock cursor spacing
- Snap to data

Measurements

	Time	Value
1 |	4.000	4.000e+00
2 |	4.000	3.641e+00
ΔT	0.000 s	ΔY	3.594e-01
1 / ΔT		Inf Hz
ΔY / ΔT		Inf (/s)


2. d)
1
2
Trace Selection
Ramp
Cursor Measurements
Settings
Measurements
Time
Value
1
8.000
7.804e+00
2
8.000
8.000e+00
ΔT
0.000 s
ΔY
1.963e-01
1 / ΔT
Inf Hz
ΔY / ΔT
Inf (/s)
0
2
4
6
8
10
0
1
2
3
4
5
6
7
8
9
10



Trace Selection

Step

Cursor Measurements

Settings

Measurements

	Time		Value
1	4.000		1.020e+00
2	12.500		–
ΔT	8.500 s	ΔY	–
	1 / ΔT		117.647 mHz
	ΔY / ΔT		–