Creation of Multiple Random Piecewise Polytropic Equation of States

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1 Introduction

Here you can find a way to create a set of random Equations of State (EOS). The purpose of this document is not to go over the theory and the methods that have been used by the authors, but it has as a goal to give you an idea for these methods and show you how you can use our codes. If you want to learn more about the EOS models you can take a look at the references that we provide. In these notes we will be using the notation as listed below:

Symbol	Quantity	Units
P	Pressure	dyne cm^{-2}
ϵ	Energy density/c^2	g cm^{-3}
ρ	Mass density	g cm^{-3}
h	Specific enthalpy	$(cm/s)^2$
N	Relativistic enthalpy	$(cm/s)^2$

2 Piecewise Polytropes (PP)

This method is based on the assumption that it is separated into 3 polytropic regions (The detailed process can be found in [1]). For each region we have to define the constant K and the adiabatic index Γ. Also, we have to define the "borders" for each region ρ_1 and ρ_2, which are the baryon mass densities that separate each area. For the crust we can use one of the pre-existed EOSs ([2] (eosNV), the [3](eosFPS), or the [4] (eosBPS)). From the crust EOS file we can choose the "border" density for the crust (ρ_{crust}). Therefore, the EOS profile of the PP is going to look as follows:

$$P(\rho) = \begin{cases} P_{crust} & \rho \leq \rho_{crust} \\ K_1\rho^{\Gamma_1} & \rho_{crust} \leq \rho \leq \rho_1 \\ K_2\rho^{\Gamma_2} & \rho_1 \leq \rho \leq \rho_2 \\ K_3\rho^{\Gamma_3} & \rho_2 \leq \rho \end{cases} \tag{2.1}$$

Lastly we have to define properly the Ks, Γs, ρ_1 and ρ_2. In [5] the authors give a range of possible values for them based on the fact that the speed of sound has to be less than the speed of light, and on the fact that the EOS has to support the heaviest observed neutron star. Based on that, we take random values within this given range and we create a set of random EOSs.

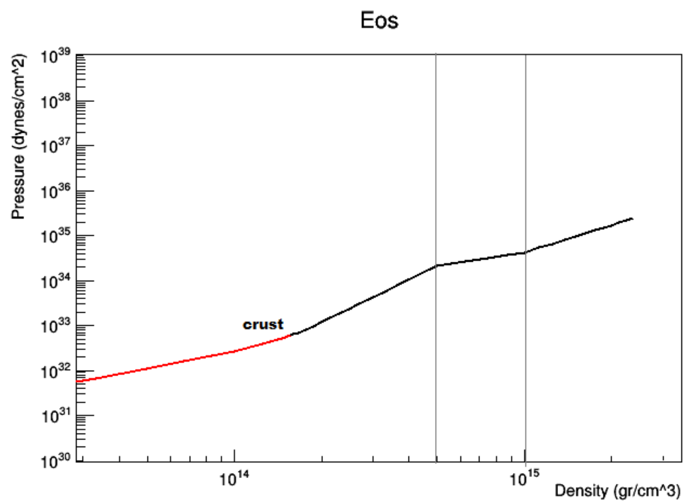

Figure 2.1: Piecewise Polytrope

3 The code

3.1 Compiling the code

The code is written in C++, but it uses some tools that come from ROOT which is a Data Analysis Framework that has been developed by CERN [6]. To run the code, you have to download Root and then you have to type the word "root" followed by the name of the file in the terminal window (i.e "root Polytropes.cpp"). Once it runs, the code asks from the user the number of the random EOSs that he or she wants to create.
"Give the number of EOS"
If you give the number and press "Enter" the code will run by using the predefined parameters.

3.2 The outputs

Once the code finishes with the compiling process, it is going to create a number of EOS files (which depends on user's choice).

The name of the file is "eosPol+number". In the first line of the table we give the number of inputs. Below that, you are going to find 4 columns. The first column is the energy density in g/cm^3, the second is the Pressure in $dyne/cm^-2$, the third is the Relativistic enthalpy in cm^2/s^2 and the last one is the baryon density in $1/cm^3$ (is the mass density divided by 1.66 10^{-24} g). The file is going to look as follows:

80			
7.8	1.01e+08	1	4.6988e+24
7.86	1.01e+09	1.15795e+08	4.73494e+24
7.9	1.01e+10	1.26945e+09	4.75904e+24
7.9	1.01e+10	1.26945e+09	4.75904e+24
.	.	.	.
.	.	.	.
.	.	.	.

Also, a file with the name "EosTable" is created, and includes 8 columns. In each column we have the name of the EOS, the pressure at the border with the crust, Γ_1, Γ_2, Γ_3, ρ_1 (g/cm^3), ρ_2 (g/cm^3), and the name of the crust EOS.

Furthermore, the code solves the TOV equations by using the Runge-Kutta4 method. In file "eosPol+number+table" we provide the Mass and Radius of a neutron star for different values of the central energy density. This file is composed by 4 columns. The first one is the mass in $M_\odot$, the second one is the Radius in km, the third one is the speed of sound divided by the speed of light at the center and the last one is the central energy at the center in g/cm^3.

Finally, the code creates a plot for the Mass-Radius curves with the name "Mass_Radius.png". You can plot the EOSs by using the "plot.cpp" code. Once you have the EOS files you type "root plot.cpp" in the terminal and the code is going to ask you for the number of the EOS files. The code will give you back a plot with the name "plotEOS.png".

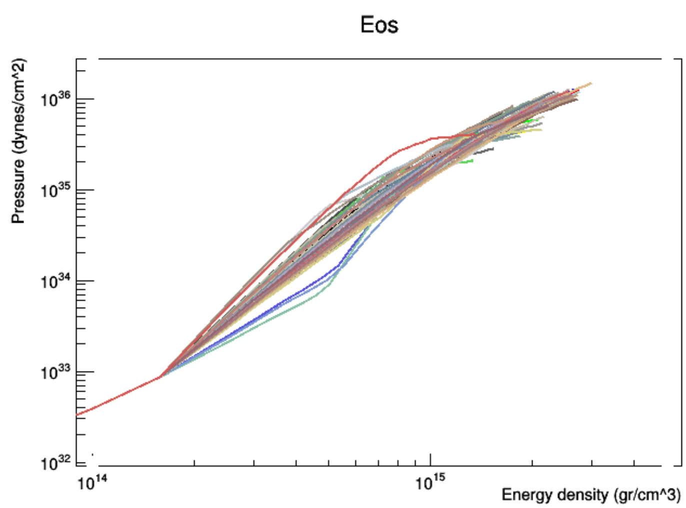

Figure 3.1: plotEOS.png

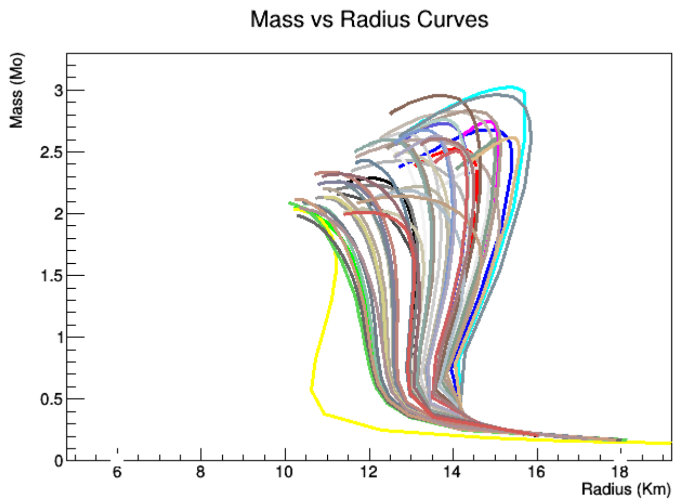

Figure 3.2: "Mass_Radius.png"

3.3 How to change the pre-defined parameters

The main program starts at line 435 "void Polytropes()". Before that point someone can find the functions that we used in our code. At line 463 you can choose the crust EOS by changing the value of the "sEOSnum" parameter. 0 is the NV EOS, 1 is the FPS EOS and 2 is the BPS EOS. Next, you can choose the value of the crust density border by changing "d_crust".
"Maxd0" represents the maximum value that the central mass density can take. By definition is 8.3 (*2.28 10^14 g/cm^3) where this value came from the Hebeler et. al. paper. You can change the mass of the heaviest observed neutron star at "Max_mass_limit". All the EOSs can support a star with such mass.
Finally, at line 551 you can change the values of the range of possible values of Γ_1, Γ_2, Γ_3, ρ_1 and ρ_2.

4 Acknowledgements

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References

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