NSSS
(Neutron Star Spin Sequences)

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1 What can it do?

The *NSSS* is based on the ISCO code by Stergioulas, & Friedman [2]. The grid size used by default is MDIVxSDIV=151x301, and it can be changed in the file *makefile*. This code is able to compute

- One neutron star with a given ratio of r_p/r_e,
- Sequences of neutron stars with constant rest mass (mass of the particles that make up the star, denoted by M_0) by changing the rotational frequency from the zero spin to maximum spin frequency (Kepler limit),
- Similar to the previous case, it will compute a sequence of neutron stars, but in this case the maximum spin is set by the user.

The parameters that the code accepts to compute either of the previous tasks are:
-f name of the file with the tabulated equation of state (EOS)
-e minimum central energy density (in g/cm^3)
-n number of sequences produced
-m central energy density for the maximum mass neutron star for a given equation of state
-s spin frequency (in Hz)
-r ratio of r_p/r_e (between 0 and 1)
-t maximum spin frequency

2 Computing one neutron star

To compute a single neutron star we need a tabulated equation of state (EOS), a value of the central energy density (ε_c), and the value of the quotient that tells us how oblate the neutron star is (Figure

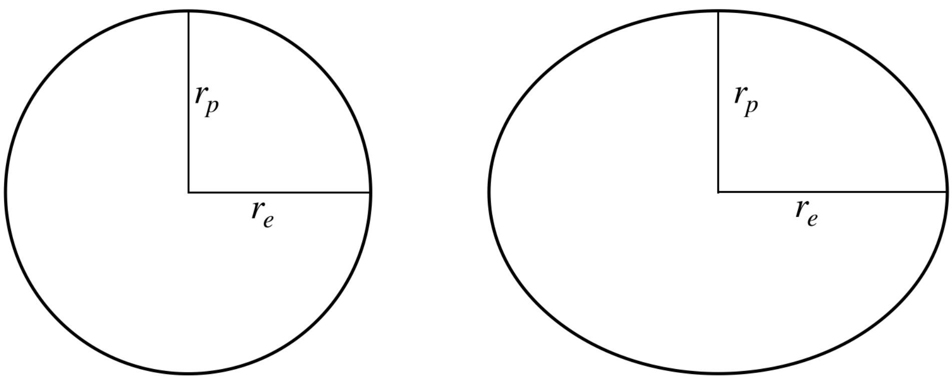

Figure 1: Left: Model of a non rotating neutron star with the two radii considered in *NSSS.c*, the equatorial radius, r_e, and the polar radius, r_p. In this case, they are equal to each other. Right: Rapidly rotating neutron star, where now $r_e > r_p$, because of the rotation.

1). This quotient is indicative of how fast this object is spinning. The faster it rotates, the more oblate it becomes. When we have a ratio of 1 the neutron star is spherical. The EOS and the central energy density will be used to create a zero-spin star with a certain mass and radius. Then, the quotient r_p/r_e, will be used to make the star oblate, and therefore, spinning. So, an example of the command line would be

./sequences -f *filename* -e 1.6e15 -r 0.9

3 Computing a sequence from zero spin to the Kepler limit

The sequences that are computed in this part are made of neutron stars with a constant value of rest mass (also called baryonic mass), in other words, no matter is being added to it.

To compute a sequence of neutron stars we need to have a value of central energy density, ε_c, a tabulated equation of state, and the value of ε_c at which we get the maximum-mass neutron star.

First, to create a sequence of neutron stars a non-rotating star is created using the previous description and this value of M_0 is stored. Now, the star need to rotate gradually faster until it reaches the break up limit (Kepler limit). To do this, the value of r_p/r_e is decreased and to find a neutron star with the same rest mass as the one stored the code makes use of the 3-point interpolation, and the following star takes the value of ε_c to continue and create another star This process of computing one neutron star continues as r_p/r_e is decreased (making the neutron star rotate faster) until the star gets to the Kepler limit, which is the maximum rotational frequency that the star can rotate at. Now the code has produce one sequence of neutron stars with the same value of M_0.

To compute another sequence the value of ε_c is increased and the code computes another sequence of stars in the same way.

This process can stop after a given number of sequences that we need (by input) or if it reaches the maximum mass for the equation of state being used.

An example of an input line to compute 10 sequences would be

./sequences -f *filename* -e 1.6e15 -n 10 -m 2.3327

The word *filename* can be replaced with the corresponding file of the equation of stat. The ten different EOS used can be found in Table 1, which also shows the value of ε_c needed to compute a neutron star with a mass of $M = 1M_\odot$, and the value of ε_c (used as input) needed to compute the highest mass neutron star for each EOS.

Table 1: Data corresponding to the value of R and ε_c when $M \sim 1M_\odot$ for each equation of state considered here. It also shows the value that ε_c has to have when the neutron star with the maximum value in mass is computed, and the corresponding values of radius and baryonic mass for the most massive neutron star.

	R and ε_c when $M \sim 1M_\odot$			ε_c when M_{max} is obtained			
EOS	$M \sim 1M_\odot$ ($M_\odot$)	R (km)	ε_c (g/cm^3)$\times 10^{15}$	M_{max} ($M_\odot$)	$M_{0,M}$ ($M_\odot$)	R_M (km)	ε_c (g/cm^3)$\times 10^{15}$
ABPR1	1.13982	11.5003	0.8	1.93468	2.26467	10.8059	2.3317
ABPR2	1.00871	10.4593	1.0	1.49662	1.69859	9.28537	3.3454
ABPR3	1.01155	11.4536	1.0	1.47598	1.65273	9.67557	3.1508
APR	1.05274	11.3806	0.8	2.23685	2.71921	9.89745	2.8437
BBB1	1.08652	11.1366	0.9	1.78951	2.08233	9.6527	3.07827
BBB2	1.10285	11.245	0.9	1.91853	2.26798	9.48855	3.7733
HLPS1	1.18626	9.80258	1.1	2.0423	2.51353	9.23051	3.1781
HLPS2	0.990614	11.9816	0.6	2.49576	3.05768	11.5378	1.9929
HLPS3	1.44232	13.6516	0.5	2.998219	3.7009	13.3676	1.455
L	1.22615	14.6968	0.4	2.71085	3.22955	13.7477	1.44216

4 Computing sequences up to a given value of spin frequency

This task is very similar to the previous one, to compute a series of sequences it takes a tabulated EOS, a value of ε_c and the value of ε_c at which the maximum-mass neutron star is obtained. The difference is that, in this case, the maximum rotational frequency is not the Kepler limit, it can be set by the user, no matter which EOS is being used. An example of the line of initialization for this task is

./sequences -f *filename* -e 1.6e15 -n 10 -m 2.3317 -t 800

The parameter **-t 800** tells the code to only compute neutron stars with the limiting value in frequency of 800 Hz.

5 Properties obtained from the code

The physical properties of the neutron star that this code outputs in the form of columns are seen in Table 2. Note that these are some of the parameters used in [1]

Table 2: Parameters obtained from the computation of a sequence of neutron stars

Parameter	Description
ε_c	Central energy density
M	Total mass (in $M_\odot$)
M_0	Baryonic mass, also know as rest mass (in $M_\odot$)
M_*	Mass when the neutron star is not rotating in a sequence (in $M_\odot$)
M_{max}	Maximum mass of a non-rotating NS for a given EOS (in $M_\odot$)
R	Radius of the NS (in km)
r_p/r_e	Ratio of the polar radius and the equatorial radius
R_*	Radius when the neutron star is not rotating in a sequence (in km)
ν	Rotational frequency (in Hz)
ν_K	Kepler limit for rotation (in Hz)
J	Angular momentum (in cm^{2}g/s)
T	Rotational kinetic energy (in g)
U	Gravitational binding energy (in g)
R_{max}	Maximum radius of a non-rotating neutron star for a given EOS (in km)
$M_{max/R_{max}}$	Quotient of the maximum mass and radius of the non-rotating NS for a given sequence

6 Output

The following output example is obtained when we input the following line of code

./sequences -f eos-master/eosL -e 0.36e15 -n 5 -m 1.44216 -t 800

This line will compute 5 sequences of neutron stars with the same M_0, which have a spin frequency that goes up to $\nu = 800$ Hz. And a line before the headings tells which is the limiting spin frequency. Notice that the output file also contains the data corresponding to the parameters in Table 2.

We can get the same output if we omit the command *-t 800*, but now the first output line will state *Computing sequences with spin frequency from 0 Hz to the Kepler limit.*

```
Computing star with spin frequency from 0 to 800 Hz
e_c       Mass     Mass_0   StatM    Radius   R-ratio  StatR     Spin    K freq
e15       Msun     Msun     Msun     km       --       km        Hz      Hz
Energy center = 0.36
0.36      0.99737  1.04661  0.99737  14.51368 1.000  14.51368  0.000  1047.09802
0.35978   0.99738  1.04661  0.99737  14.52606 0.998  14.51368  52.552  1044.55302
0.359603  0.99743  1.04662  0.99737  14.54023 0.996  14.51368  76.061  1042.68951
```

The following line of code is to compute one neutron star with the ratio $r_p/r_e = 0.9$

./sequences -f eos-master/eosL -e 0.36e15 -r 0.9

The output is the following set of data that belongs to this one star, denoted too by the sentence before the headings.

```
Computing one neutron star
e_c     Mass     Mass_0   StatM    Radius   R-ratio  StatR    Spin     K freq
e15     Msun     Msun     Msun     km       --       km       Hz       Hz
0.36 1.05561 1.10903 0.99737 15.34503 0.900 14.51368 384.982 995.61897
```

References

[1] Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 424, 823

[2] Stergioulas, N., & Friedman, J. L. 1995, ApJ, 444, 306