Sequences of RNS with constant central energy density

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This document is a copy of sections 3.1 and 3.2 from my Master's thesis

1 The code

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For our research we used the RNS code [1] (written in C) which numerically solves these equations for a given EOS, its central energy density and a value of the $R_{pole}/R_{equator}$. The authors use $R_{pole}/R_{equator}$ instead of the frequency of a NS because in the Newtonian limit we can have two solutions, in the case where we study the Maclaurin spheroids. The RNS code uses the techniques that have been developed in [2] in order to solve the equations that come from [3].

A brief explanation of the techniques that have been used in [2] has as follows. At first, the authors make an initial guess for the values of the metric potentials, the energy density and the angular velocity. Then they integrated the equations that give the values for each potential (and can be found in their paper). By using the new potentials they found new values for the energy density and the angular velocity. They repeated that process until the values of the parameters do not have any significant change. For the integration they used the Simpson's method.
Next, by using Newton-Raphson method they solved some equations that are given in their paper, in order to find the frequency at the equator, the pole and the center, the point with the maximum density, the equatorial radius and the parameter C which comes into one of the equations that relates the enthalpy. These parameters are essential for later, where by integration in the whole space they calculate the macroscopic parameters that we have introduced before.

[4] for his thesis modified the RNS code to create sequences of rotating NSs by keeping the baryon mass constant. The modified code has the name NSSS (Neutron Stars Spin Sequences). In this thesis we modified this code and we created sequences of rotating NSs by keeping constant the central energy density.

The equations that [3] provide are with respect to s and μ parameters, therefore in section 2 we give a brief description of the s - μ space. Next, 3, we are

going to show how we modified this code in order to serve our purpose. The table below gives the notations we use in the next chapters:"

ϵ_c	Central energy density
M	Total mass (in M $\odot$)
M_0	Rest mass, also known as baryonic mass (in M $\odot$)
M_*	Mass of the first nonrotating NS in a sequence (in M $\odot$)
R_e	Equatorial radius of the NS (in km)
r_p /r_e	Ratio of the polar radius and the equatorial radius
R_*	Equatorial radius of the first nonrotating NS in a sequence (in km)
Ω	$2\pi\times$Rotational frequency (in Hz)
Ω_K	Kepler limit for rotation (in Hz)
J	Angular momentum (in cm 2 g/s)
T	Rotational kinetic energy (in $M_\odot$)
W	Gravitational binding energy (in $M_\odot$)

Table 1.1: Stergioulas' notation

2 The μ - s space

"We said before that instead of θ ($0 \leq \theta \leq \pi/2$) and r ($0 \leq r \leq \infty$), we use the new coordinate system μ ($0 \leq \mu \leq 1$) and s ($0 \leq s \leq 1$). The main problem of this transformation is that it is impossible to transfer from s to r at point s=1 ($r \to \infty$) inside the code. So, instead of going to infinity, someone can define a number SMAX which denotes a distance far away from the star. This is also the maximum possible value for s. Now, the code is ready to divide the μ - s space in small pixels.

The number of the pixels is given by the user, as he or she has to define the values SDIV and MDIV, which correspond to the number of intervals that we split the s-axis and μ-axis respectively. The authors give a suggestion for the values of these parameters in their code (i.e. for SMAX we can use 0.9999 and by default MDIV$\times$SDIV=201$\times$401), but of course someone can choose something else. This can happen by changing these values in the "Makefile", but remember that every time that something is changed in this file, the object files must be erased (the files with ".o" at the end). After you type the command "make" in a terminal window, the code is prepared to run with the new values that have been chosen.

We can find the value of s and μ for the point (i_μ,i_s) from the following equations:

$$\mu(i_\mu) = \frac{i_\mu - 1}{MDIV - 1} \tag{2.1}$$

$$s(i_s) = SMAX \frac{i_s - 1}{SDIV - 1} \tag{2.2}$$

where i_s=1,2,3....SDIV and i_μ = 1,2,3...MDIV. Keep in mind that $i_s = 1$ corresponds to the center of the star, i_s = SDIV corresponds to infinity, $i_\mu = 1$ corresponds to the equatorial plane and i_μ = MDIV corresponds to the polar direction. Also, the equatorial radius is represented by the point s = 0.5, m = 0. The figure 2.1 represents this μ - s space."

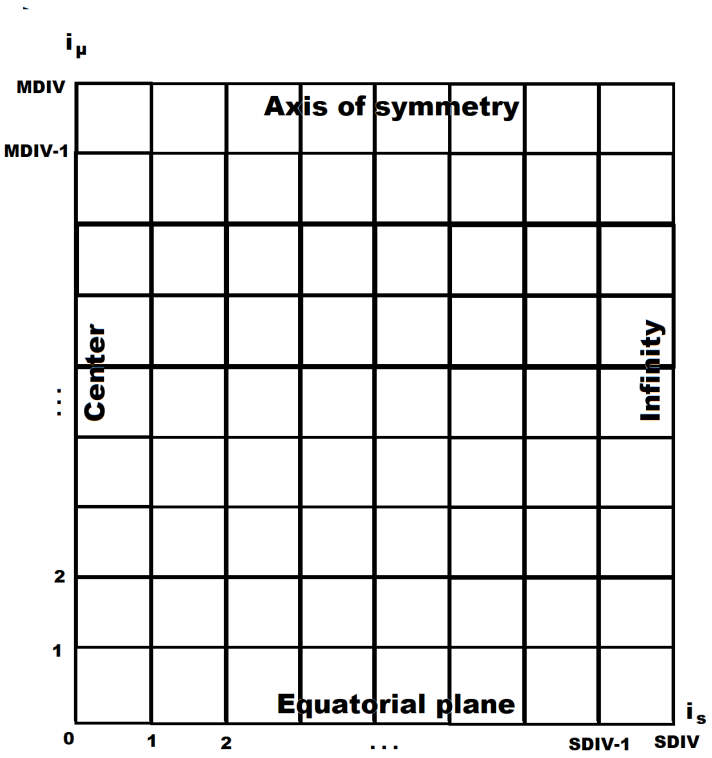

Figure 2.1: μ - s space

3 Sequences of rotating neutron stars with constant central energy density

"For purposes that are going to be explained in the next chapter, we want to investigate the case where we have a sequence of neutron stars with constant central energy density. To do that we modified a little bit the NSSS code.

The **first star in each sequence** must be non-rotating, so we start from a star where its $R_{ratio} = 1$. R_{ratio} is the polar radius divided by the equatorial radius, and for a non-rotating star the polar radius is equal to the polar radius.

As the spin value increases in a sequence, the polar radius decreases and the equatorial radius increases, so the shape of the NS changes from sphere to an ellipsoid (The figure below shows us how the shape of a NS changes in a sequence). This means that the R_{ratio} value has to be less or equal to the unity, and of course larger than 0. So, changing the R_{ratio} value, is equivalent to changing the frequency.

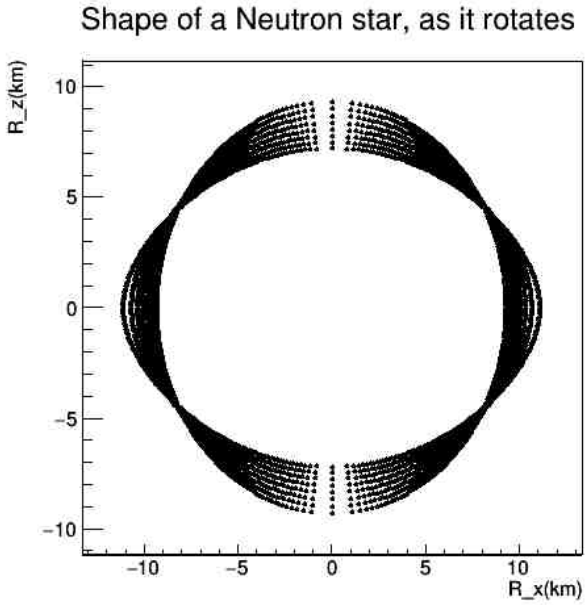

Figure 3.1: Example for the change of the shape of a NS in a sequence

The next stars are created, and their R_{ratio} changes by 0.005. This value has been chosen just for the purpose of this thesis. The user can choose a different value by changing the ratio_ch parameter in the "nsss.c" file.

The **last star in each sequence** rotates with a frequency a little bit smaller than the Kepler frequency. The sequence stops after the creation of this star.

The first sequence that has been created has a central energy density equals to the value that the user chose (we talk about that in the next paragraphs). When the first sequence reaches the Kepler frequency the whole process is repeated with a new value for ϵ_c that is e_ch=0.05 10^{15} g/cm^3 smaller than before. Sequences are created until their central energy density becomes smaller than energy_min=0.22 10^{15} g/cm^3 in the case where we use the Polytropic EOSs and energy_min=0.32 10^{15} in the case where we use the speed of sound model (Again energy_min and e_ch can be modified by the user). These values represent the point just before the crust EOS or the chiral effective field theory band take place.

After we choose the values of ratio_ch, energy_min and e_ch we must run the

makefile and the code is ready. We can run the code by writing the command "./nsss" plus other information that is necessary for the code to solve the problem. Such information is the EOS that we want to use and the central energy density that is going to remain constant in the sequence.

To choose the EOS that is going to be used by the program, "-f eos_file_name" has to be included in the command line that runs the code. Also, a "segmentation fault" message is given in the case where the EOS file is not included in the same directory as the NSSS files.

By including -e in the command line we can choose the value of the energy density of the NS at the center. Now, we can run our modified code by using the command "./nsss -f eos_file -e ϵ_c".
On the screen the following parameters are printed out: e_c, M, M_0, R_e, R_{ratio}, Spin, Kepler frequency for each step. The screen looks as follows:
NS_data_eos_file_name.txt
NS_data_eos_file_name.txteos_file_name_table.txt
eosSA2, MDIVxSDIV=201x401
Computing one neutron star

e_c	Mass	Mass_0	Radius	R-ratio	Spin	K freq
e15	Msun	Msun	km	–	Hz	Hz
3.5	1.27762	1.47349	7.54216	1.000	0.000	3162.89213
3.5	1.28285	1.47963	7.56923	0.990	385.440	3120.20129
3.5	1.28795	1.48537	7.59843	0.980	549.147	3098.65182
3.5	1.29316	1.49122	7.62834	0.970	674.264	3080.72261
3.5	1.29846	1.49719	7.65883	0.960	779.150	3064.56960
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More than that a file with the name "NS_data_eos_file_name.txt" has been created in the same directory. Each column in this file represents the following values:
ϵ_c, M, M_0, Mstat, R_e, R_{ratio}, Rstat, ν, ν_{kepler}, J, T, W, R_{ratio}_S
in the units that have been stated before. R_{ratio}_S is the same as the R_{ratio} but by using the Schwarzschild-like metric. This parameter is useful for reasons that are going to be clarified later on. "

3.1 EOS file structure

Finally, "let's say a few things about the proper structure that an EOS file must have in order to be accepted by the RNS code. On the **first line** an EOS file must have the number of inputs that are included in the file. In the **next lines** we have four columns. Each column represents a specific quantity. The first column is the energy density in g/cm^3, the second one represents the pressure in $dynes/cm^2$, the third one is the relativistic enthalpy N in $(cm/s)^2$ and the fourth one is the baryon number density in cm^{-3}."

4 Acknowledgements

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References

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