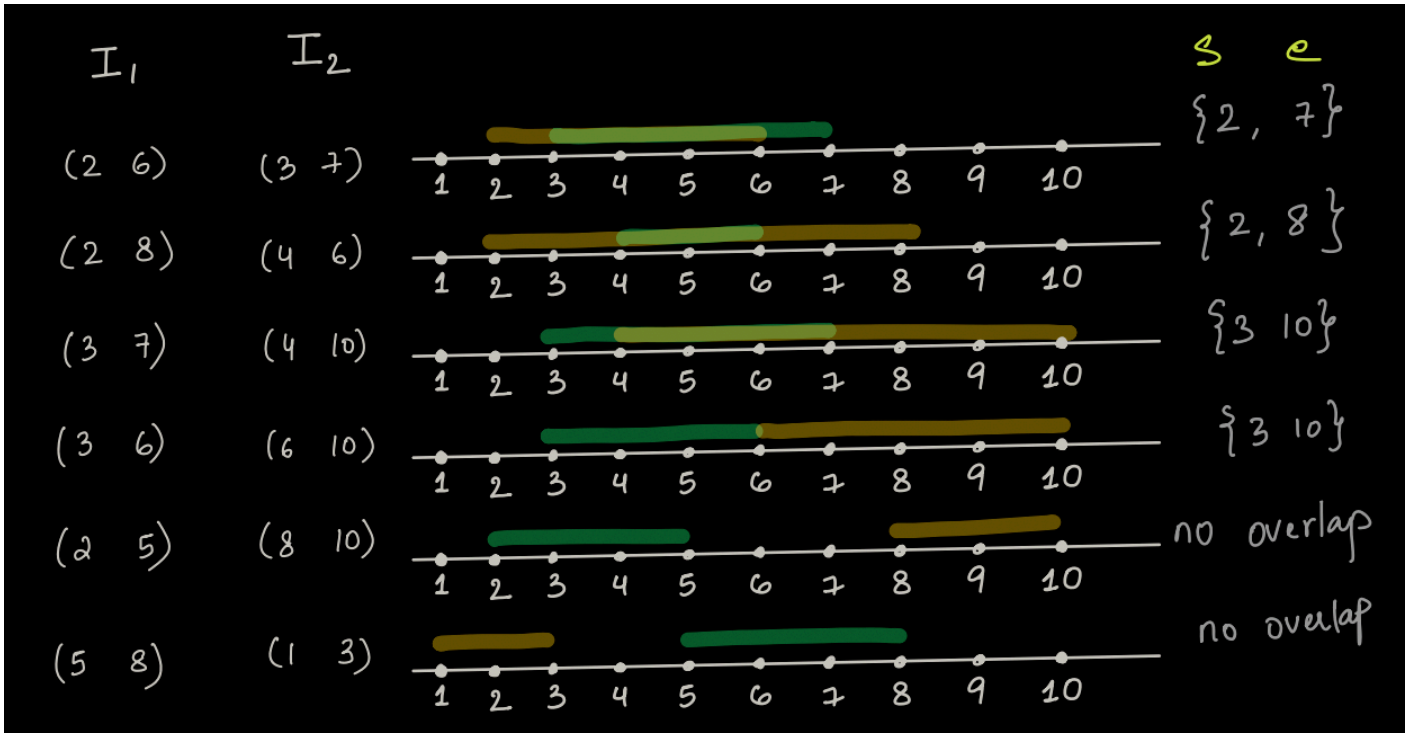
Lecture | Advanced DSA: Arrays 3 - Interview Problems

Merge Intervals

An Interval is defined by start and end time, where start <= end.

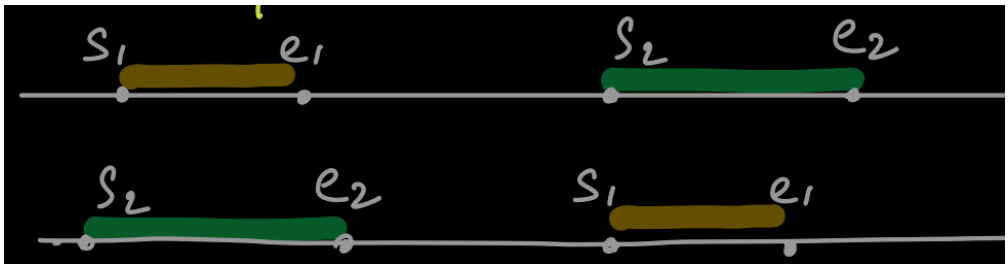
Say we are given a list of Intervals, we will have to merge them if they overlap.

Let's look at them below -



Non-Overlapping Condition

Say there are two Intervals, I1 {s1 e1} & I2 {s2 e2}
Then the condition for them to not overlap is -




```
if(s2 > e1 || s1 > e2)
```

So, if above condition is not followed, it says that Intervals are overlapping!

How to merge overlapping Intervals ?

[I1.start , I1.end] & [I2.start , I2.end]
After merging -
[min(I1.start, I2.start) , max(I1.end, I2.end)]

Question

If the intervals [3, 8] and [5, 12] are given, do they overlap?

Choices

- [x] Yes
- [ ] No

Explanation:

Answer: Yes

The intervals [3, 8] and [5, 12] overlap because 8 is greater than 5. The overlapping area is [5, 8]

Question

What is the correct way to represent the merged result of intervals [6, 10] and [8, 15]?

Choices

- [x] [6, 15]

- [ ] [6, 8, 10, 15]
- [ ] [6, 10] and [8, 15]
- [ ] [8, 10]

Explanation:

[6, 15]

This is because the merging of intervals involves combining overlapping intervals into a single, continuous interval

Problem 1 : Merge sorted Overlapping Intervals

Problem Statement
Given a sorted list of overlapping intervals, sorted based on start time, merge all overlapping intervals and return sorted list.

Input:
Interval[] = {(0,2), (1,4), (5,6), (6,8), (7,10), (8,9), (12,14)}

Output:

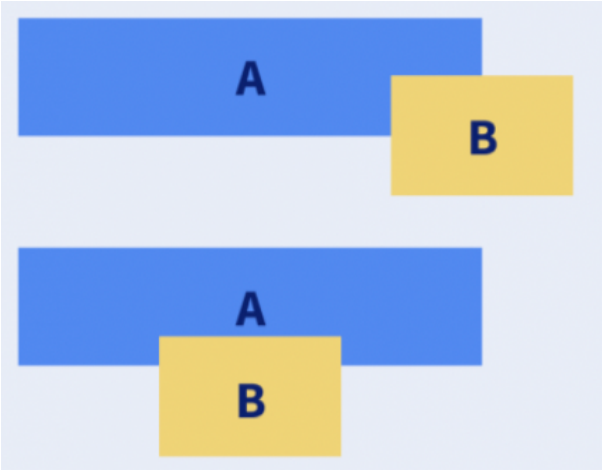
{(0,4), (5,10), (12,14)}

Explanation:

Interval 1	Interval 2		Answer Interval List
0-2	1-4	Overlapping	0-4
0-4	5-6	Not Overlapping	0-4, 5-6
5-6	6-8	Overlapping	0-4, 5-8
5-8	7-10	Overlapping	0-4, 5-10
5-10	8-9	Overlapping	0-4, 5-10
5-10	12-14	Not Overlapping	0-4, 5-10, 12-14

The Array Is Sorted Based on Start Time. What Is the Overlapping Condition?

Say start time of A < start time of B



Overlapping Condition -

If start of B <= end of A

Question

Given a sorted list of overlapping intervals, sorted based on start time, merge all overlapping intervals and return sorted list.

Input:

Interval[] = { (1,10), (2, 3), (4, 5), (9, 12)}

Choices

- [x] (1, 12)
- [ ] (1, 10), (9, 12)
- [ ] (1, 9), (9, 12)
- [ ] No Change

Problem 1 Approach

- Create an array to store the merged intervals.
- If the current and ith intervals overlaps, merge them. In this case update the current interval with the merged interval.
- Else, insert the current interval to answer array since it doesn't overlap with any other interval and update the current Interval to ith Interval.

Dry Run

Input:

Interval[] = {(0,2), (1,4), (5,6), (6,8), (7,10), (8,9), (12,14)}

Explanation:

current	ith		After merging	answer list
0-2	1-4	Overlapping	0-4	
0-4	5-6	Not Overlapping	Not needed	0-4
5-6	6-8	Overlapping	5-8	0-4
5-8	7-10	Overlapping	5-10	0-4
5-10	8-9	Overlapping	5-10	0-4
5-10	12-14	Not Overlapping	Not needed	0-4, 5-10
12-14	end			

At the end, we are left with the last interval, so add it to the list.

Pseudocode

```
//Already a class/structure will be present for Interval
//We will only need to create an answer array of type Interval

list < Interval > ans;

// Current Segment
int cur_start = A[0].start, cur_end = A[0].end;

for (int i = 1; i < A.size(); i++) {

  // if i'th interval overlaps with current interval
  if (A[i].start <= cur_end) {
    // merging them
    cur_end = max(cur_end, A[i].end);
  } else {
    //adding current interval to answer.
    //create a new Interval
    Interval temp(cur_start, cur_end); //if struct is declared, otherwise if class is d
    ans.push_back(temp);

    // update cur Interval to ith
    cur_start = A[i].start;
    cur_end = A[i].end;
  }
}
Interval temp(cur_start, cur_end);
ans.push_back(temp);
return ans;
```

Complexity

Time Complexity: O(N)
Space Complexity: O(1)

Problem 2 Sorted Set of Non Overlapping Intervals

Given a sorted list of overlapping intervals based on start time, insert a new interval such that the final list of intervals is also sorted and non-overlapping.

Print the Intervals.

Example 1:

N = 9

(1,3)
(4,7)
(10,14)
(16,19)
(21,24)
(27,30)
(32,35)
(38,41)
(43,50)

New Interval

(12, 22)

Explanation:

ith	new Interval	Overlaps?	Print
(1,3)	(12,22)	No	(1,3)
(4,7)	(12,22)	No	(4,7)
(10,14)	(12,22)	Yes; merged: (10,22)	
(16,19)	(10,22)	Yes; merged: (10,22)	
(21,24)	(10,22)	Yes; merged: (10,24)	
(27,30)	(10,22)	No; small new Interval gets printed first	(10,22)
(32,35)			(32,35)
(38,41)			(38,41)
(43,50)			(43,50)

Please Note: Once the new Interval gets printed, all the Intervals following it also gets printed.

More Examples

Example 2:

N = 5

(1,5)

(8,10)

(11,14)

(15,20)

(21,24)

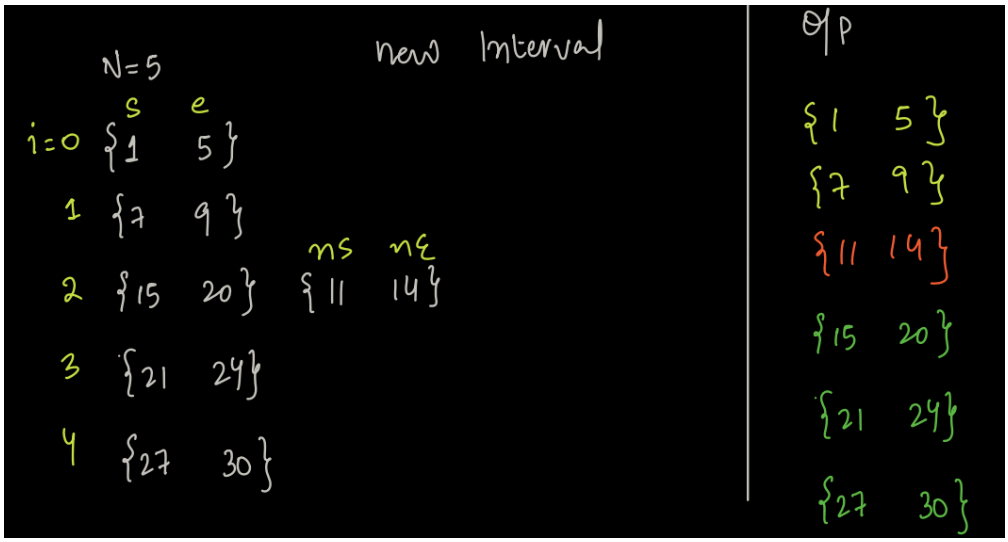
New Interval

(12, 24)

ith	new Interval	Overlaps?	Print
(1,5)	(12, 24)	No	(1,5)
(8,10)	(12, 24)	No	(8,10)
(11,14)	(12, 24)	Yes; merged:(11, 24)	
(15,20)	(11, 24)	Yes; merged:(11, 24)	
(21,24)	(11, 24)	Yes; merged:(11, 24)	

We are done with all the intervals but left the new Interval at the end; in this case we have to print the new Interval.

Example 3:



Question

If the sorted set of non-overlapping intervals is [1, 5], [6, 10], and [12, 15], what happens if you add the interval `[4, 7]` such that the final list of intervals is also sorted and non-overlapping.?

Choices

- [x] [1, 10] and [12, 15]
- [ ] [1, 5], [4, 7], [6, 10], [12, 15]
- [ ] [1, 5] and [6, 10] only
- [ ] No change

Explanation:

(1,5)
(6,10)
(12,15)

New Interval
(4, 7)

ith	new Interval	Overlaps?	Print
(1,5)	(4, 7)	Yes; merged:(1, 7)	
(6,10)	(1, 7)	Yes; merged:(1, 10)	
(12,15)	(1, 10)	No; small new Interval gets printed first	(1, 10)
(12,15)			(12, 15)

Thus after merging, the intervals are [1, 10] and [12, 15]

Problem 2 Pseudocode

```
void merge(int Interval[], int nS, int nE) {
    for (int i = 0; i < N; i++) {
        int L = Interval[i].start, R = Interval[i].end;

        //Not Overlapping
        if (nS > R) {
            print({
                L,
                R
            });
        }
        // new Interval is not overlapping and is smaller
        // print new Interval and then all the remaining Intervals
        else if (L > nE) {
            print({
                nS,
                nE
            });

            for (int j = i; j < N; j++) {
                print({
                    Interval[j].start,
                    Interval[j].end
                })
            }
            return;
        } else {
            nS = min(L, nS);
            nE = max(R, nE);
        }
    }
    print({
        nS,
        nE
    });
}
```

Complexity

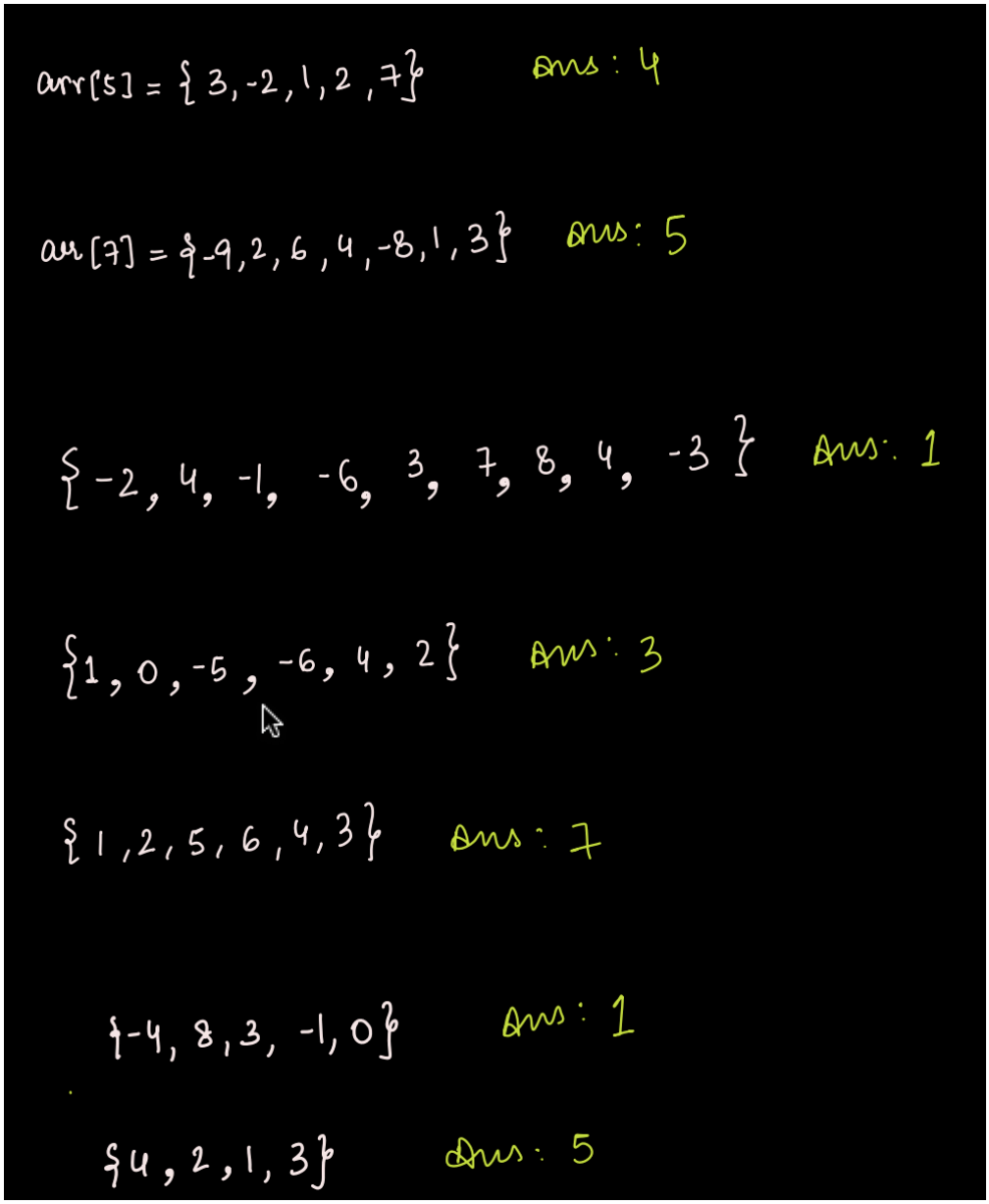
Time Complexity: O(N)

Space Complexity: O(1)

Problem 3 Find First Missing Natural Number

Given an unsorted array of integers, Find first missing Natural Number.

Examples



Question

In the array [5, 3, 1, -1, -2, -4, 7, 2], what is the first missing natural number?

Choices

- [x] 4
- [ ] 6
- [ ] -3
- [ ] 8

:::warning
Please take some time to think about the solution approach on your own before reading further.....
:::

Find First Missing Natural Number Solution Approach

Approach 1: Brute Force

Check for all the numbers from 1 till we get the answer
T.C - O(N * ans)

Here, in the worst case answer can go uptil N+1, in case if all numbers from 1 to N are present in the array.

Example - {4, 1, 3, 2}
Here we will have to iterate till 5, ie. N+1.

Idea

Can we utilise the fact that answer can be out of 1 to N+1?
If any number other than 1 to N is present, then missing is out of 1 to N only.
If all elements from 1 to N are present, then it will be N+1.

Say we start checking if 1 is present or not, we somehow want to mark the presence of 1.
Now, since we can't use extra space, so we can use indices to mark the presence of a number.

Index 0 can be used to mark presence of 1.
Index 1 can be used to mark presence of 2.

Index 2 can be used to mark presence of 3.
so on....

Now how do we mark the presence ?

One Solution:

```
We can set element at that index as negative.
```

But what if negative number is part of the input?

Let's assume only positive numbers are present
We can use indices to mark the presence of a number.
We can set element at that index as negative.

A[ ] = {8, 1, 4, 2, 6, 3}

```
N = 6
Answer can be from [1 7]
So, if any number is beyond this range, we don't care!
```

index	element	presence marked at index	state of the array
0	8	don't care, since out of answer range	
1	1	0	{-8, 1, 4, 2, 6, 3}
2	4	3	{-8, 1, 4, -2, 6, 3}
3	2	1	{-8, -1, 4, -2, 6, 3}
4	6	5	{-8, -1, 4, -2, 6, -3}
5	3	2	{-8, -1, -4, -2, 6, -3}

Now, we can just iterate from left to right and whichever element is not ve-, we can return i+1 as the answer.
Example: {-8, -1, -4, -2, 6, -3}
Here, index: 4 is +ve, hence 5 is the answer.

NOTE: Since we are marking elements as negative, so when checking presence of a certain number, we'll have to consider the absolute value of it.

Pseudocode

```
for (int i = 0; i < N; i++) {
    int ele = abs(A[i]);

    if (ele >= 1 && ele <= N) {
        int idx = ele - 1;
        A[idx] = -1 * abs(A[i]);
    }
}
```

Find First Missing Natural Number For Negative Numbers

How to resolve for negative numbers ?

Will negatives ever be our answer?

NO!

So, we don't have to care about them!

Should we change them to ve+ ?

NO! They may fall in our answer range.

Should we mark them 0?

NO! Then we will not be able to mark the presence of a number!

We can just change negative number to a number that is out of our answer range. It can be N+2.

```
for (int i = 0; i < N; i++) {
    if (A[i] <= 0) {
        A[i] = N + 2;
    }
}

for (int i = 0; i < N; i++) {
    int ele = abs(A[i]);

    if (ele >= 1 && ele <= N) {
        int idx = ele - 1;
        A[idx] = -1 * abs(A[i]);
    }
}

for (int i = 0; i < N; i++) {
    if (A[i] > 0) return i + 1;
}
return N + 1;
```

> Please show a dry run on - {4, 0, 1, -5, -10, 8, 2, 6}

Complexity

Time Complexity: O(N)

Space Complexity: O(1)