Properties of the Hodgkin-Huxley equations

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1 Threshold

1.1 Question 1

Bisect the amplitude interval for sub-threshold and supra-threshold stimulating currents:

```
% Load Hodgkin-Huxley parameters
hhconst;
% Set up parameters for action potential simulation
amp1 = 6;
width1 = 1;
disp(amp1);
hhmplot(0, 50, 0);
for k=1:10
    amp1=(amp1+7)/2;
    amp1_rounded = round(amp1, 2); % Round amplitude to 2 decimal places
    disp(amp1_rounded);
    hhmplot(0, 50, k);
end
```

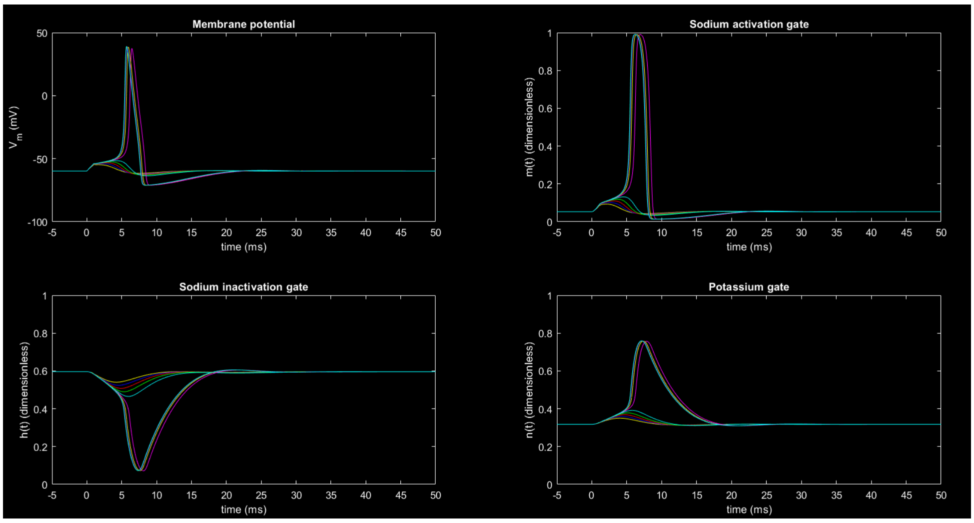


Figure 1: Simulated plots

```
amp0 = 6
amp1 = 6.50
amp2 = 6.75
amp3 = 6.88
amp4 = 6.94
amp5 = 6.97
amp6 = 6.98
amp7 = 6.99
amp8 = 7.00
```

```
amp9 = 7.00
amp10= 7.00
```

Action potentials become visible from an amplitude of 6.97 onwards. For amplitudes below this value, action potentials are not observed. To obtain a precise value, let's examine the plots near 6.97.

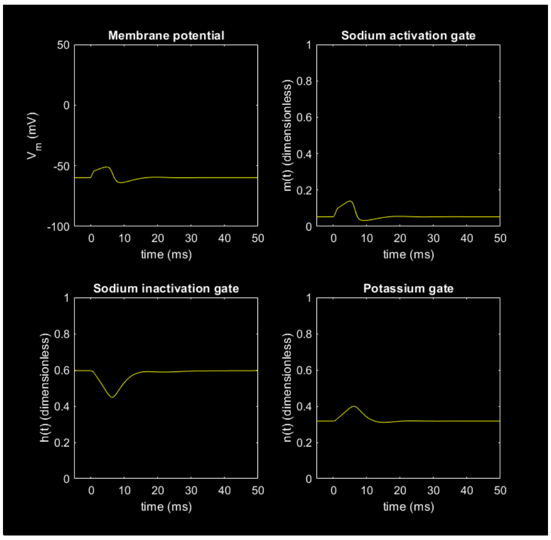


Figure 2: When amp = 6.95

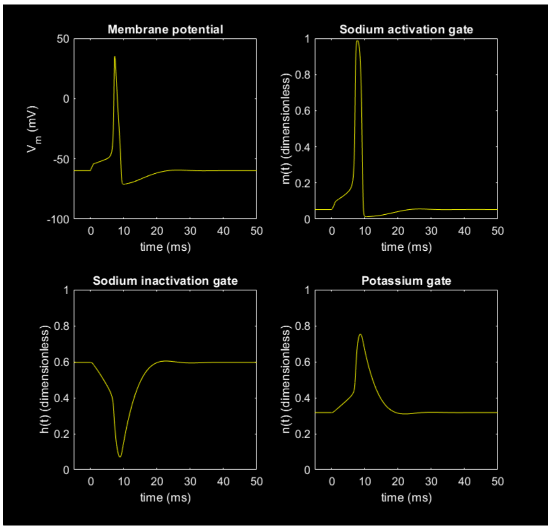


Figure 3: When amp = 6.96

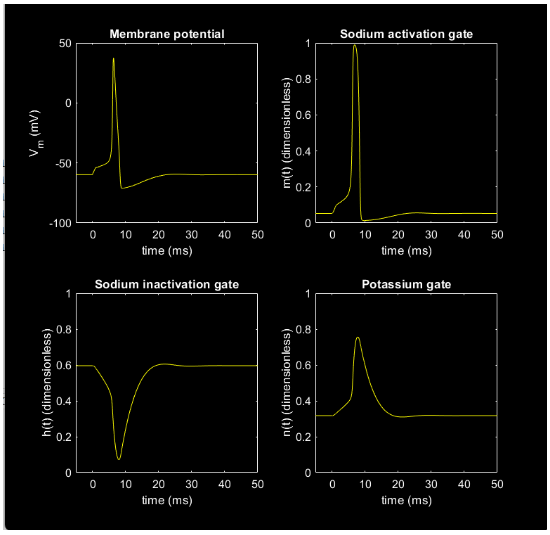


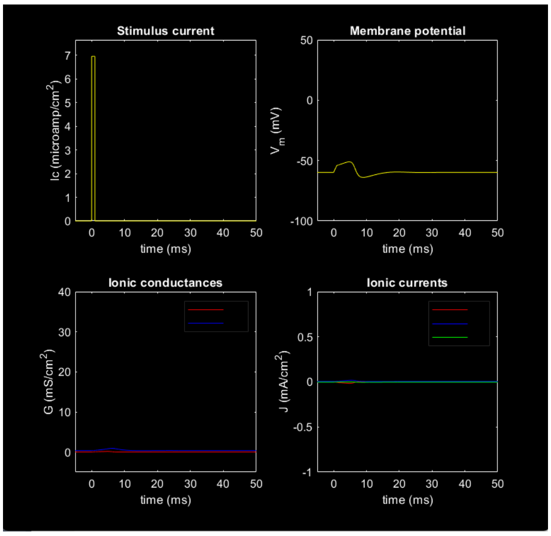
Figure 4: When amp = 6.97

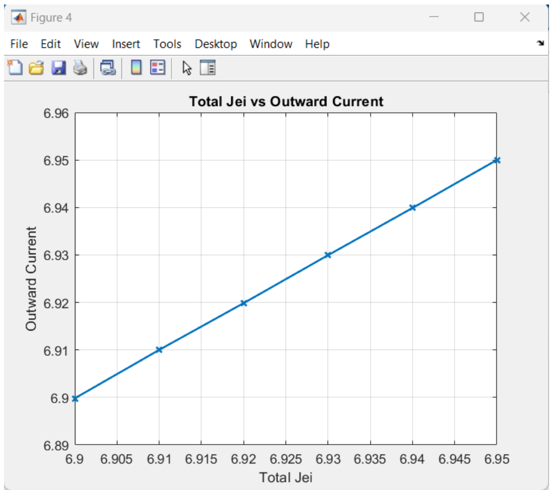
Estimated threshold stimulating current amplitude = 6.96

1.2 Question 2

```
% Load Hodgkin-Huxley parameters
hhconst;

% Set up parameters for action potential simulation
amp1 = 6.90;
width1 = 1;
disp(amp1);

totalJeiArray = [];
outwardCurrentArray = [];
```

```
for k = 1:6
    [qna, qk, ql] = hhsplot(0, 50);
    totalJei = width1 * amp1;
    outwardCurrent = qna + qk + ql;
    % Store the values in the arrays
    totalJeiArray = [totalJeiArray, totalJei];
    outwardCurrentArray = [outwardCurrentArray, outwardCurrent];

    amp1 = amp1 + 0.01;
    disp(amp1);
end

% Plot totalJei and outwardCurrent arrays with crosses and lines
figure;
plot(totalJeiArray, outwardCurrentArray, 'x-', 'LineWidth', 1.5);
hold on;

% Add labels and title
xlabel('Total Jei');
ylabel('Outward Current');
title('Total Jei vs Outward Current');
grid on;
```





Observations: These $\int_{t_o}^{t_f} \sum_k J_k \, dt$ and $\int_{t_o}^{t_f} J_{ei} \, dt$ values are almost equal.

2 Refractoriness

2.1 Question 3

Delay: 25 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 25;
amp2 = 13.4;
width2 = 0.5;
hhsplot(0,40);
```

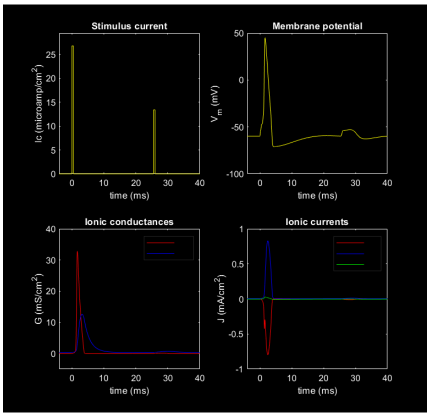


Figure 5: Delay=25, amp2=13.4

This indicates that at a delay of 25 ms, an amplitude of 13.4 is insufficient to evoke an action potential. However, by simulating for higher amplitudes, we observe the generation of an action potential starts at amplitude of 13.7.

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 25;
amp2 = 13.7;
width2 = 0.5;
hhsplot(0,40);
```

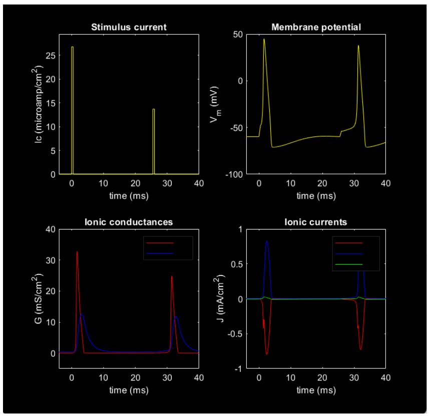


Figure 6: Delay=25, amp2=13.7

Delay: 20 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 20;
amp2 = 11.6;
width2 = 0.5;
hhsplot(0,40);
```

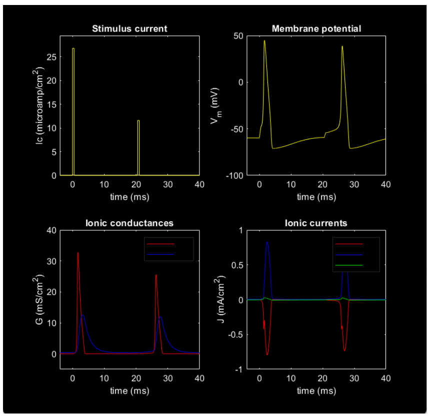


Figure 7: Delay=20, amp2=11.6

Delay: 18 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 18;
amp2 = 11.3;
width2 = 0.5;
hhsplot(0,40);
```

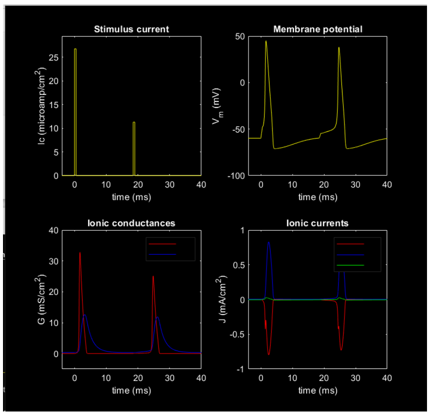


Figure 8: Delay=18, amp2=11.3

Delay: 16 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 16;
amp2 = 12.7;
width2 = 0.5;
hhsplot(0,40);
```

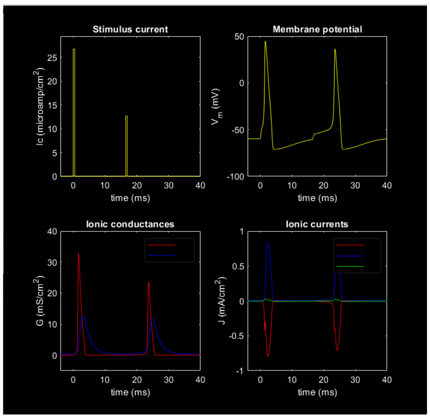


Figure 9: Delay=16, amp2=12.7

Delay: 14 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 14;
amp2 = 17;
width2 = 0.5;
hhsplot(0,40);
```

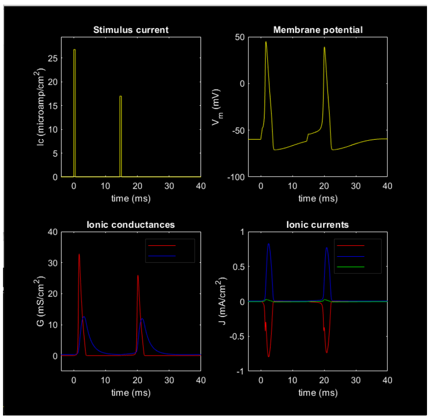


Figure 10: Delay=14, amp2=17

Delay: 12 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 12;
amp2 = 25.5;
width2 = 0.5;
hhsplot(0,40);
```

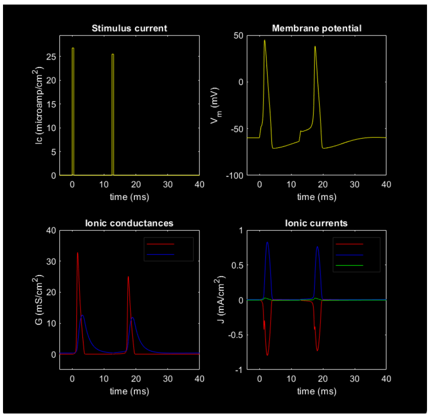


Figure 11: Delay=12, amp2=25.5

Delay: 10 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 10;
amp2 = 40.8;
width2 = 0.5;
hhsplot(0,40);
```

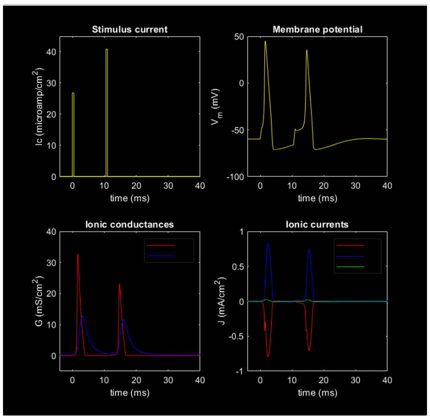


Figure 12: Delay=10, amp2=40.8

Delay: 8 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 8;
amp2 = 70.1;
width2 = 0.5;
hhsplot(0,40);
```

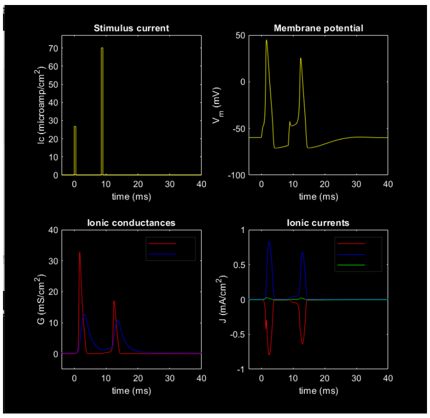


Figure 13: Delay=8, amp2=70.1

Delay: 6 ms

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 6;
amp2 = 145.2;
width2 = 0.5;
hhsplot(0,40);
```

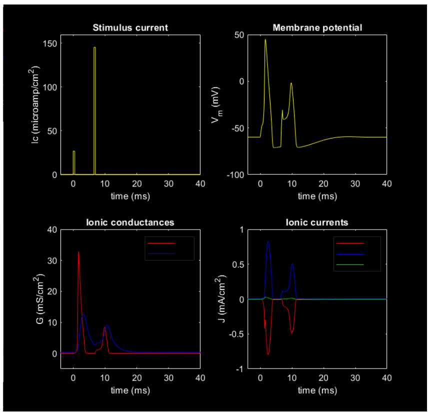


Figure 14: Delay=6, amp2=145.2

Summary :

Delay	I2
25	13.7
20	11.6
18	11.3
16	12.7
14	17
12	25.5
10	40.8
8	70.1
6	145.2

```
delay = [6; 8; 10; 12; 14; 16; 18; 20; 25];
I2 = [145.2; 70.1; 40.8; 25.5; 17; 12.7; 11.3; 11.6; 13.7];
I1 = 26.8;
% Calculate the ratio I2/I1
ratio = I2 ./ I1;
% Plot
figure;
plot(delay, ratio, 'b+-', 'LineWidth', 2); % 'b' for blue color, '+' marker
xlabel('Inter-pulse Interval (ms)');
ylabel('I2_{th}/I1_{th}');
title('I2/I1 Ratio');
grid on;
```

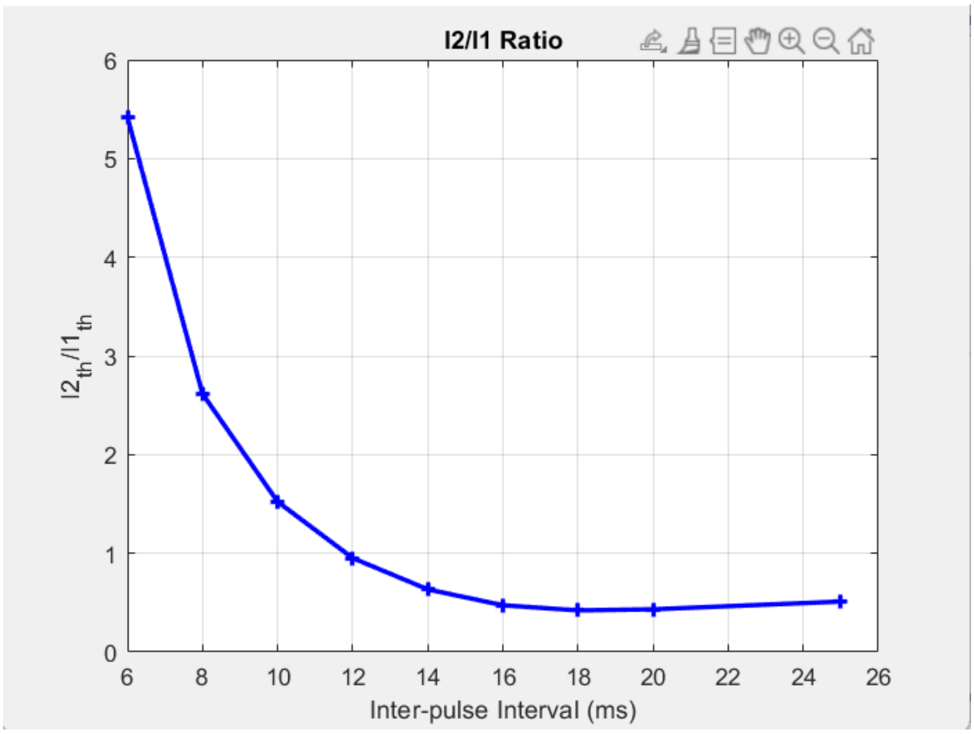


Figure 15: plot

2.2 Question 4

The graph clearly indicates that the necessary current exceeds five times the initial threshold at a 6 ms delay, indicating an absolute refractory period of 0–6 ms. Moreover, the required pulse falls below the original threshold value after 12 ms, suggesting a relative refractory period between 6 and 12 ms.

3 Repetitive activity

3.1 Question 5

a) Amplitude = 5 μAcm^{-2}

```
amp1 = 5;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

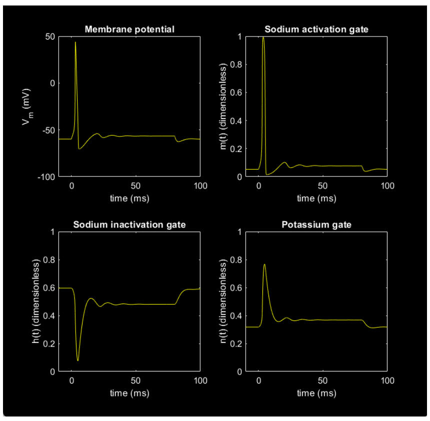


Figure 16: amp1=5

Number of action potentials : 1

b) Amplitude = 10 µAcm^{-2}

```
amp1 = 10;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

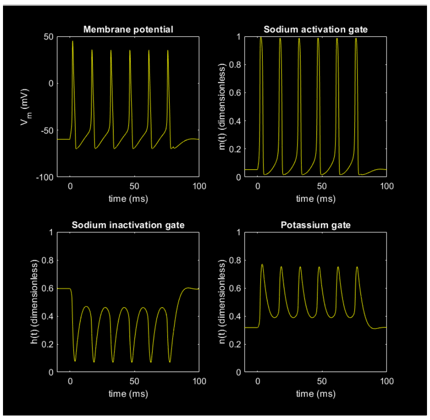


Figure 17: amp1=10

Number of action potentials : 6

c) Amplitude = 20 µAcm^{-2}

```
amp1 = 20;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

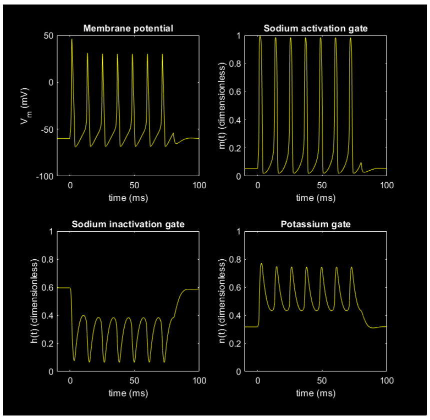


Figure 18: amp1=20

Number of action potentials : 7

d) Amplitude = 30 µAcm^{-2}

```
amp1 = 30;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

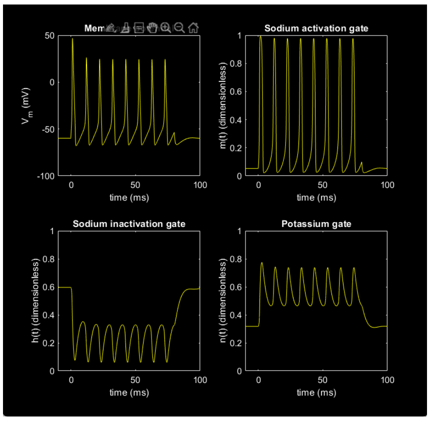


Figure 19: amp1=30

Number of action potentials : 8

e) Amplitude = 50 $\mu A cm^{-2}$

```
amp1 = 50;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

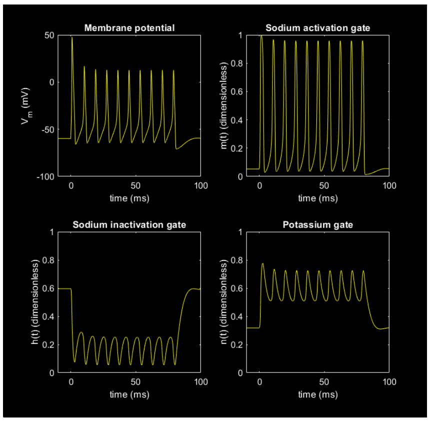


Figure 20: amp1=50

Number of action potentials : 10

f) Amplitude = 70 $\mu A cm^{-2}$

```
amp1 = 70;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

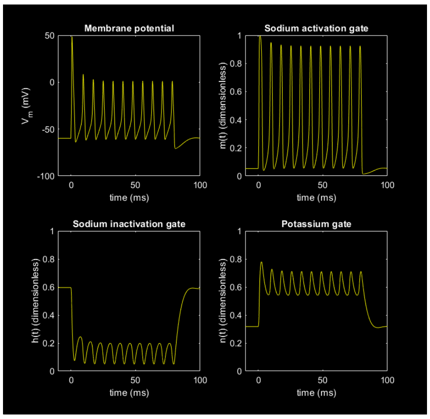


Figure 21: amp1=70

Number of action potentials : 11

g) Amplitude = 100 $\mu A cm^{-2}$

```
amp1 = 100;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

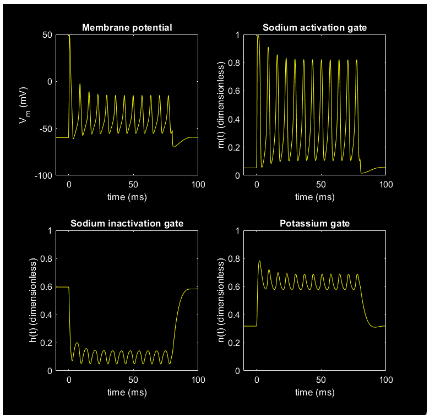


Figure 22: amp1=100

Number of action potentials : 12

amp1	No. of AP
5	1
10	6
20	7
30	8
50	10
70	11
100	12

Summary : Plotting Amplitude vs no. of action potentials

```
Amplitudes = [5, 10, 20, 30, 50, 70, 100];
No_of_Aps = [1, 6, 7, 8, 10, 11, 12];
% Create the plot
figure;
plot(Amplitudes, No_of_Aps, 'rx-', 'LineWidth', 2, 'MarkerSize', 10);

xlabel('Amplitude ( Acm ^{-2})', 'FontSize', 12);
ylabel('Number of Action Potentials', 'FontSize', 12);
title('Number of Action Potentials vs. Amplitude', 'FontSize', 14);

grid on;
```

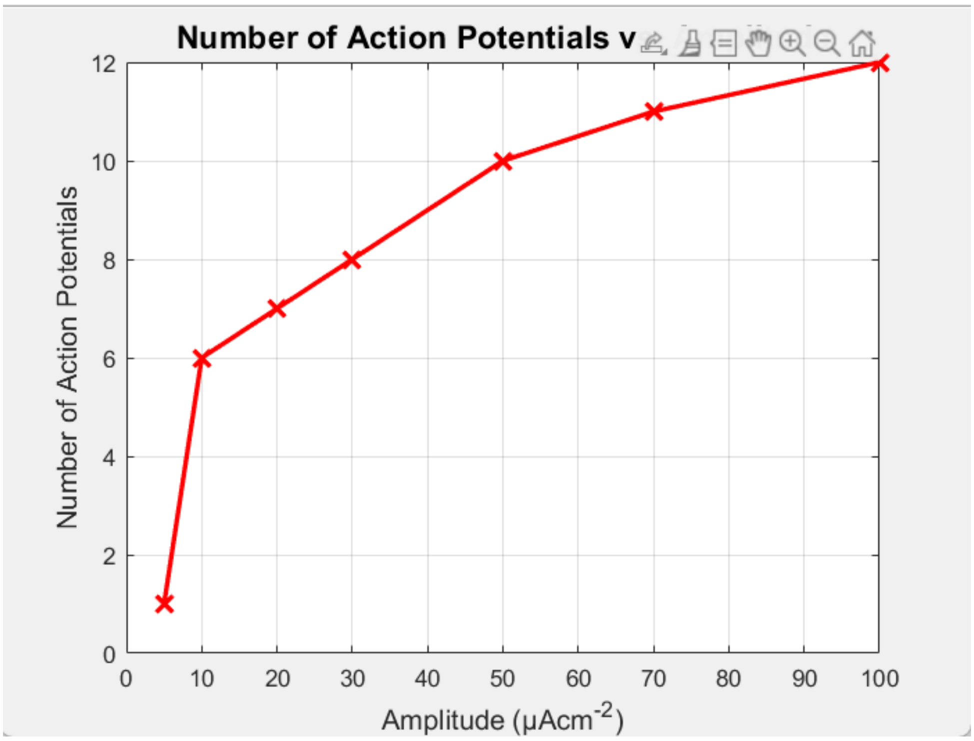


Figure 23: amp vs no. of APs plot

Based on the results, it can be observed that the frequency of action potentials increases with the rise in stimulus intensity amplitude. Additionally, the plots indicate that the amplitude of action potentials decreases as the stimulus intensity amplitude increases.

3.2 Question 6

```
amp1 = 200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```

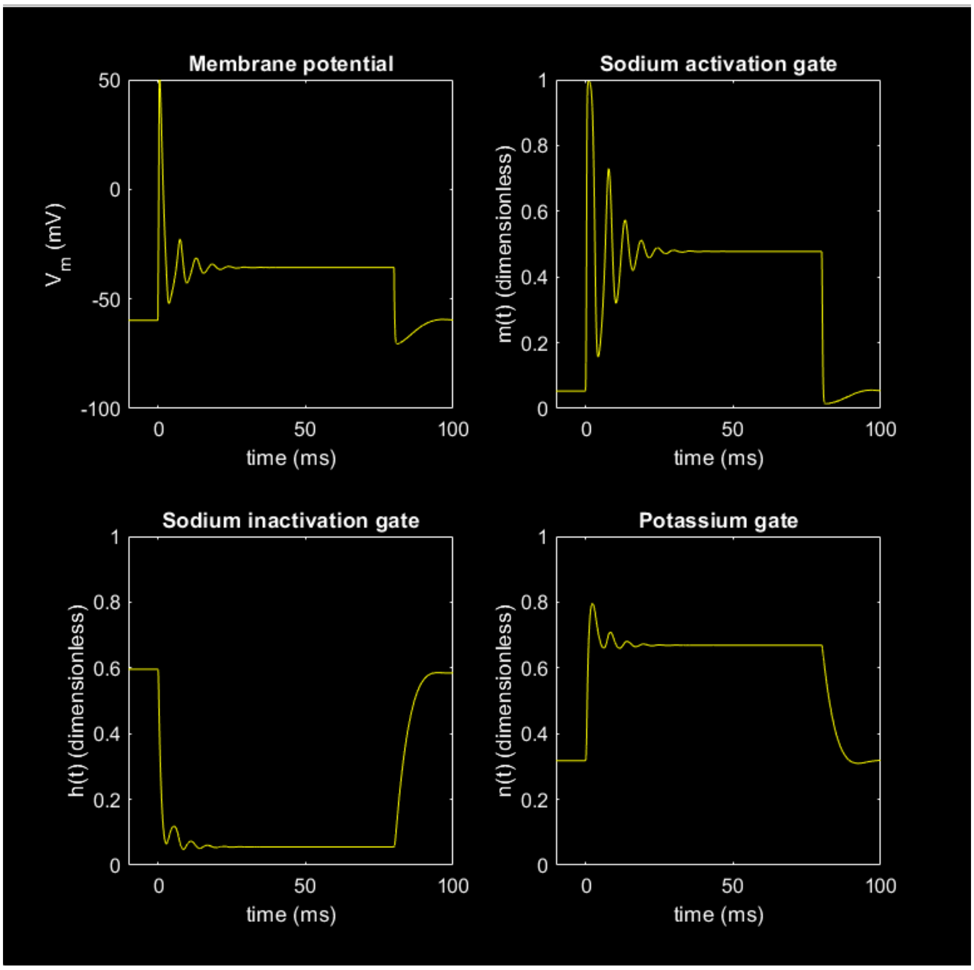


Figure 24: amp vs no. of APs plot

Sustained Depolarization:
At large stimulating amplitudes, the membrane potential stays excessively depolarized for a prolonged period. This prolonged depolarization can lead to the inactivation of voltage-gated sodium channels, preventing the generation of action potentials.

Dependence of Gating Variables on Voltage:
The h and n factors in the Hodgkin-Huxley model are critical in determining the membrane's excitability. At very depolarized potentials, the h (sodium inactivation) and n (potassium activation) factors can quickly move to their respective states that prevent the initiation of action potentials.

Sodium Channels (m and h):

- With increasing stimulus intensity, the membrane potential depolarizes more significantly.

- This results in a higher probability of sodium channels opening, leading to a greater inward sodium current during the action potential's upstroke.
- However, sustained depolarization causes h to decrease, leading to inactivation of sodium channels. Once inactivated, these channels cannot reopen until the membrane potential repolarizes sufficiently.

Potassium Channels (n):

- Conversely, n increases with depolarization, leading to more potassium channels opening.
- The increased potassium conductance helps counterbalance the depolarizing sodium influx.
- However, if the potassium conductance cannot sufficiently counter the prolonged sodium influx, the neuron remains in a depolarized state, contributing to the depolarization block.

4 Temperature dependence

4.1 Question 3

```
% Define temperature range
temperatures = [0, 5, 10, 15, 20, 24, 25, 26, 30];
% Set parameters
vclamp = 0;
amp1 = 20;
width1 = 0.5;
% Plot all action potentials in the same figure
figure;
hold on;
i=0;
% Loop through each temperature
for temp = temperatures
    % Set temperature
    tempc = temp;
    hhmplot(0, 30, i);
    i=i+1
end
```

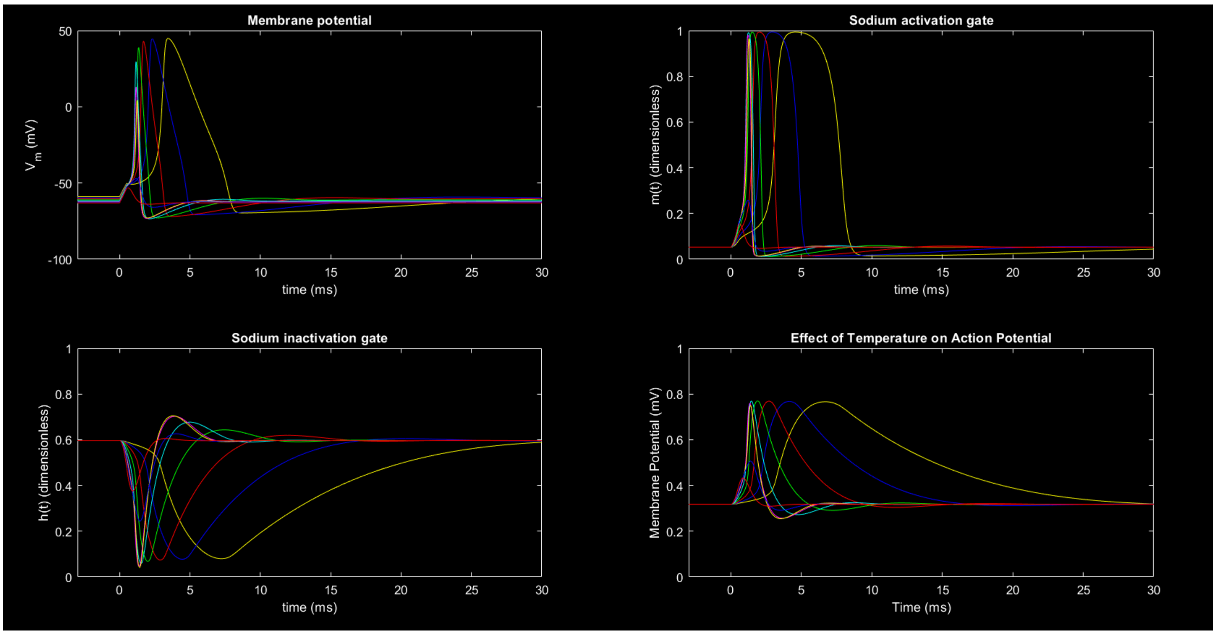


Figure 25: APs in different temperatures

a) Synaptic Transmission: When it's warmer, the release of chemicals that help signal between neurons at synapses becomes more efficient. This is because the little packages of chemicals in neurons fuse together faster. This can make signals between neurons stronger and work better.

b) Threshold and Resting Membrane Potential: When it gets warmer, the resting membrane potential usually goes down a bit. This happens because the sodium-potassium ATPase pump, which helps keep the balance of ions in the membrane, works harder. Also, the threshold for starting an action potential might go down, making neurons more likely to get excited.

c) Conduction Velocity: When it's warmer, action potentials travel faster along nerves. This happens because the time it takes for the membrane to recover after an action potential is shorter, and ions move faster across the nerve membrane. So, nerve signals travel faster.

d) Ionic Current Speed: At higher temperatures, the channels that let ions in and out of cells work faster. So, during an action potential, sodium ions rush in quicker, and potassium ions rush out faster during repolarization. This makes the action potential's ups and downs sharper, with a bigger peak and a shorter duration.

e) Refractory Periods: Warmer temperatures usually mean shorter breaks between action potentials. This happens because sodium channels, which help start action potentials, work faster. So, neurons can fire action potentials one after the other more quickly.