Self-assessment questions: learning units 4-6 Section 1

Question A
Draw Venn diagrams to show that $(A' + B) = (B \cup C) \cap A$ is not an identity for all subsets A, B and C of U. Draw the diagrams in stages.

Question B
Provide a counterexample, and then use it to show that $(A' + B) \neq (B \cup C) \cap A$.

Question C
Using the sets X = {1, 2} and Y = {2, 3}, show that

$$(X \cup Y) \times (X \cap Y) = (X \times (X \cap Y)) \cup (Y \times (X \cap Y)).$$

Does this example show that this is an identity? Justify your answer.

Question D
Prove that $(X \cup Y) - W' = (X \cap W) \cup (Y \cap W)$ is an identity for all subsets X, Y and W of a universal set U.

HINTS:

➢ Compile Venn diagrams step by step. Partial credit will be given for those Venn diagrams that are correct, even if some final diagram is incorrect.

➢ Remember to provide headings for Venn diagrams and to name the sets inside your diagrams. (Otherwise your diagrams will not have meaning.) Shade only the area relevant to the heading. Draw the universal set for each diagram.

➢ When two sets are not equal, a counterexample should be provided. Choose small sets that have (a) member(s) in the region(s) where the two final Venn diagrams differ. Sets are indicated by curly brackets { , , }. Note: **First** determine the LHS, then the RHS, and **then** compare the final sets in your conclusion. Note: Do **not** attempt to provide some formal proof.

➢ Make sure that you understand how to apply the definitions of union, intersection, difference, complement and symmetric difference in a Venn diagram. It will help if you think of A + B as being the set $(A \cup B) - (A \cap B)$, which has as members those elements that live in A or in B, but not in both.

➢ If the two final Venn diagrams of a given expression have the same areas shaded, a proof is required to show that the given expression is an identity. Notation in a proof is important. Note that a **symbols** (eg "$\cap$") should be used as **connectives** for **sets** (eg $Y \cap W$), and **words** (eg "and") should be used as **connectives** in **sentences** (eg $x \in Y$ **and** $x \in W$).

➢ How do we prove two sets equal? A formal proof is required. In a proof we can use the connective "iff" in a bi-directional proof, or "if…then…" when the proof is given in two halves.

➢ Connectives give form to an argument, so if you leave out the connectives "iff" or "if..., then…", your proof is not convincing. The definitions of union, intersection, difference, complement, symmetric difference and Cartesian product should be applied in proofs.

➢ Make sure that brackets are included when necessary. Remember, "$x \in Y$ and $x \in W$ or $x \in X$" could have different meanings depending on where brackets are placed.

➢ Cartesian products have ordered pairs as members. When an expression includes a Cartesian product and a formal proof is required to show that it is an identity, start the proof with

$(u, v) \in$……. (The members of a Cartesian product are ordered pairs.)

Question E

Let R be the relation on $\mathbb{Z}$ (the set of integers) defined by

$(x, y) \in R$ iff $|y| = |x|$.

a) Give (i) an ordered pair in R, showing why it belongs to R, and
(ii) an ordered pair not in R, showing why it does not belong to R.
(Use at least one negative integer in one of the pairs.)

b) By doing the appropriate tests, show that R is an equivalence relation.

HINTS:

➢ Make sure that you understand the definitions of the properties of relations. You should be able to apply these definitions in proofs. For example, for transitivity, start the proof with: Suppose $(x, y) \in R$ and $(y, z) \in R$, then *use* this information (refer to the definition of the given relation) to prove that $(x, z) \in R$.
Note: An *example* will *not prove* that a relation has a certain property.

➢ Use the definition of a given relation (in this case $(x, y) \in R$ iff $|y| = |x|$) when attempting to prove that the relation has certain properties. General statements such as "for each (x, y) there is a (y, x)" will not convince anybody that a relation has some property.

➢ The definition for reflexivity of a relation does *not* say "If $(x, y) \in R$ then $x = y$" (two variables *x* and *y* are involved here). When it is required to prove that a relation R is reflexive on *A*, you should prove that "for every $\mathbf{x} \in A$, we have $(\mathbf{x}, \mathbf{x}) \in R$". In your proof only *one* variable should play a role.

➢ Do not use the contents of the proofs for theorems 6.1 & 6.2 (pp 93 – 96 in the study guide) to try and prove that some relation has properties such as reflexivity, symmetry or transitivity. You should apply the definitions of these concepts to prove that a relation has these properties.

Question F

Let P and R be relations on $A = \{1, 2, 3, \{1\}, \{2\}, \{3\}\}$ given by
$P = \{(1, \{1\}), (\{1\}, 1), (1, 2), (2, 1)\}$ and $R = \{(1, \{1\}), (3, \{3\}), (2, \{2\}), (\{2\}, \{2\}), (\{3\}, 3)\}$.

a) Test whether P has the following properties: irreflexive; reflexive; symmetric; antisymmetric; transitive.
b) Does R satisfy trichotomy?
c) Determine the relations $R \circ R$ and $R \circ P$ (ie P; R).
d) Give the subset T of R where $(A, B) \in T$ iff $A \subseteq B$.
e) Give a partition B of the set $A = \{ 1, 2, 3, \{1\}, \{2\}, \{3\} \}$.