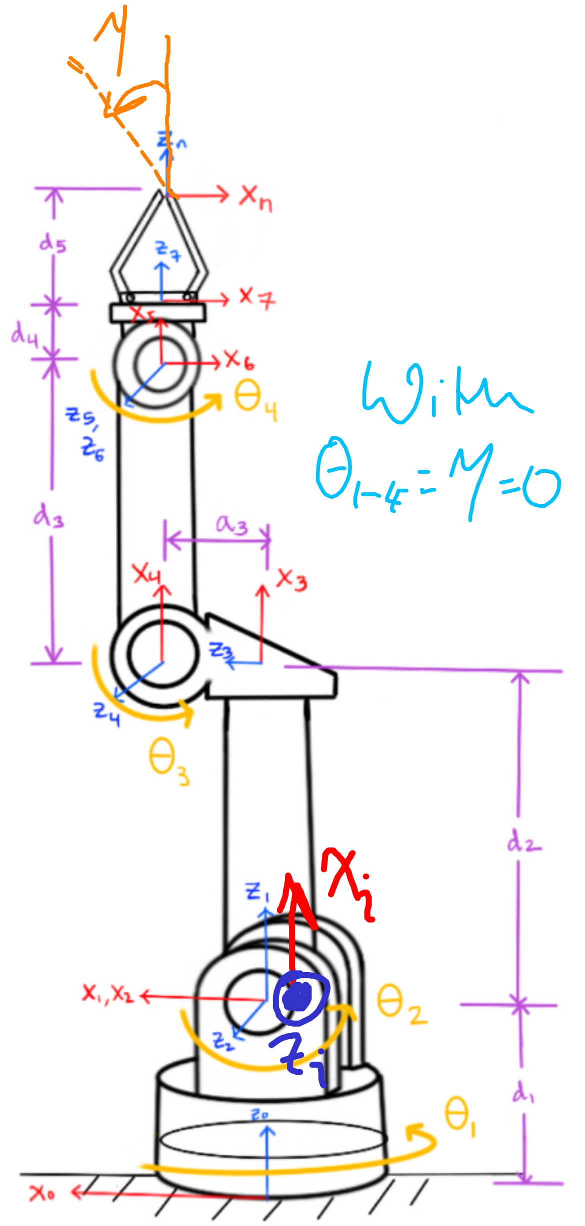

With $\theta_{1-4} = \gamma = 0$

L	θ	d	α	a
1	θ_1	d_1	0	0
2	0	0	$-\pi/2$	0
3	$\theta_2 - \pi/2$	0	$-\pi/2$	d_2
4	0	a_2	$+\pi/2$	0
5	θ_3	0	0	d_3
6	$\theta_4 - \pi/2$	0	$-\pi/2$	0
7	0	d_4	0	0
n	0	d_5	0	0

$\theta_1 = [-\pi, +\pi]$

$\theta_2 = [0, +\frac{2\pi}{3}]$

$\theta_3 = [0, +\pi]$

$\theta_4 = [-\frac{2\pi}{3}, +\frac{2\pi}{3}]$

Linkage length projected onto xy_0 plane uses elements of $c_1, c_2, s_2, s_3,$ and s_4:

^0x_n: Contribution from θ_2: $d_2 s_2 + a_2 c_2$

Contribution from θ_3: $d_3 s_3$

Contribution from θ_4: $(d_4 + d_5) s_4$

$$\Rightarrow c_1\left[d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_{234}\right]$$

$$\Rightarrow {}^0y_n = s_1\left[d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_{234}\right]$$

Linkage length in z_0 axis uses elements of c_2, s_2, c_3, and c_4:

^0z_n: Contribution from θ_2: $d_2 c_2 - a_2 s_2$

Contribution from θ_3: $d_3 c_3$

Contribution from θ_4: $(d_4 + d_5) c_4$

$$\Rightarrow d_1 + d_2 c_2 - a_2 s_2 + d_3 c_{23} + (d_4 + d_5) c_{234}$$

Finally, $\gamma = \theta_2 + \theta_3 + \theta_4$

Now solve for inverse:

$$\frac{^0y_n}{^0x_n} = \tan(\theta_1) \cdot \cancel{\frac{[d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_{234}]}{[d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_{234}]}}$$

$$\Rightarrow \theta_1 = \operatorname{atan}\left(\frac{^0y_n}{^0x_n}\right) = \operatorname{atan2}(^0y_n, {}^0x_n)$$

Past θ_1, the rest of the robot becomes a planar manipulator projected onto the xz_1 plane.

$${}^i x_n = d_2 c_2 - a_2 s_2 + d_3 c_{23} + (d_4 + d_5) c_{234}$$
$${}^i y_n = d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_{234}$$
$${}^i z_n = 0$$

Substitute $\gamma = \theta_2 + \theta_3 + \theta_4$:

$${}^i x_n = d_2 c_2 - a_2 s_2 + d_3 c_{23} + (d_4 + d_5) c_\gamma$$
$${}^i y_n = d_2 s_2 + a_2 c_2 + d_3 s_{23} + (d_4 + d_5) s_\gamma$$

$$\Rightarrow {}^i x_n - (d_4 + d_5) c_\gamma = d_2 c_2 - a_2 s_2 + d_3 c_{23}$$
$${}^i y_n - (d_4 + d_5) s_\gamma = d_2 s_2 + a_2 c_2 + d_3 s_{23}$$

$\Rightarrow$ Let $x' = {}^i x_n - (d_4 + d_5) c_\gamma$, $y' = {}^i y_n - (d_4 + d_5) s_\gamma$

$$\Rightarrow \left.\begin{aligned} x' - d_2 c_2 + a_2 s_2 &= d_3 c_{23} \\ y' - d_2 s_2 - a_2 c_2 &= d_3 s_{23} \end{aligned}\right\}$$

Square all 4 sides and add...

$$\Rightarrow \cancel{x'^2} - \cancel{x' d_2 c_2} + \cancel{x' a_2 s_2} - \cancel{x' d_2 c_2} + \cancel{d_2^2 c_2^2}$$
$$- \cancel{d_2 a_2 c_2 s_2} + \cancel{x' a_2 s_2} - \cancel{d_2 a_2 c_2 s_2} + \cancel{a_2^2 s_2^2}$$
$$+ y'^2 - \cancel{y' d_2 s_2} - \cancel{y' a_2 c_2} - \cancel{y' d_2 s_2} + \cancel{d_2^2 s_2^2}$$
$$+ \cancel{d_2 a_2 c_2 s_2} - \cancel{y' a_2 c_2} + \cancel{d_2 a_2 c_2 s_2} + \cancel{a_2^2 c_2^2}$$
$$= d_3^2 c_{23}^2 + d_3^2 s_{23}^2$$

$$\Rightarrow \cancel{x'^2} - \cancel{2x'd_2c_2} + \cancel{2x'a_2s_2} + \cancel{d_2^2c_2^2} + \cancel{a_2^2s_2^2}$$
$$+ \cancel{y'^2} - \cancel{2y'd_2s_2} - \cancel{2y'a_2c_2} + \cancel{d_2^2s_2^2} + \cancel{a_2^2c_2^2}$$
$$= d_3^2c_{23}^2 + d_3^2s_{23}^2 = d_3^2(c_{23}^2 + s_{23}^2)$$

$$\Rightarrow (-2x'd_2 - 2y'a_2)c_2 + (2x'a_2 - 2y'd_2)s_2$$
$$+ (x'^2 + y'^2 + d_2^2 + a_2^2 + d_3^2) = 0$$

This follows the form $Pc_\beta + Qs_\beta + R = 0$. To solve for it, define δ such that:

$$c_\delta = P/\sqrt{P^2+Q^2}, \quad s_\delta = Q/\sqrt{P^2+Q^2}$$
$$\Rightarrow \delta = \operatorname{atan2}\left(Q/\sqrt{P^2+Q^2},\ P/\sqrt{P^2+Q^2}\right)$$
$$\Rightarrow c_\delta c_\beta + s_\delta s_\beta + R/\sqrt{P^2+Q^2} = 0$$
$$\Rightarrow \beta = \delta \pm \cos^{-1}\left(-R/\sqrt{P^2+Q^2}\right)$$

Using this: $\theta_2 = \delta \pm \cos^{-1}\left(-R/\sqrt{P^2+Q^2}\right)$

Where: $\delta = \operatorname{atan2}\left(Q/\sqrt{P^2+Q^2},\ P/\sqrt{P^2+Q^2}\right)$

$P = -2x'd_2 - 2y'a_2$, $Q = 2x'a_2 - 2y'd_2$,

$R = x'^2 + y'^2 + d_2^2 + a_2^2 + d_3^2$

From before: $x' - d_2 c_2 + a_2 s_2 = d_3 c_{23}$

$y' - d_2 s_2 - a_2 c_2 = d_3 s_{23}$

$$\Rightarrow s_{23} = \frac{y' - d_2 s_2 - a_2 c_2}{d_3}, \quad c_{23} = \frac{x' - d_2 c_2 + a_2 s_2}{d_3}$$

$$\Rightarrow \tan(\theta_2 + \theta_3) = \frac{\frac{y' - d_2 s_2 - a_2 c_2}{\cancel{d_3}}}{\frac{x' - d_2 c_2 + a_2 s_2}{\cancel{d_3}}}$$

$$\Rightarrow \theta_2 + \theta_3 = \operatorname{atan2}(y' - d_2 s_2 - a_2 c_2, x' - d_2 c_2 + a_2 s_2)$$

$$\Rightarrow \theta_3 = \operatorname{atan2}(y' - d_2 s_2 - a_2 c_2, x' - d_2 c_2 + a_2 s_2) - \theta_2$$

Finally: $\gamma = \theta_2 + \theta_3 + \theta_4 \Rightarrow \theta_4 = \gamma - \theta_2 - \theta_3$

Furthermore, to resolve the (x, y) coordinates of the tool frame in the 1 frame:

$[{}^1x_n \ {}^1y_n \ {}^1z_n]^T = {}^1p_n$, $[{}^0x_n \ {}^0y_n \ {}^0z_n]^T = {}^0p_n$

$$\Rightarrow {}^1p_n = {}^1_0T \cdot {}^0p_n = ({}^0_1T \cdot {}^1_2T \cdot {}^2_3T)^{-1} \cdot {}^0p_n$$