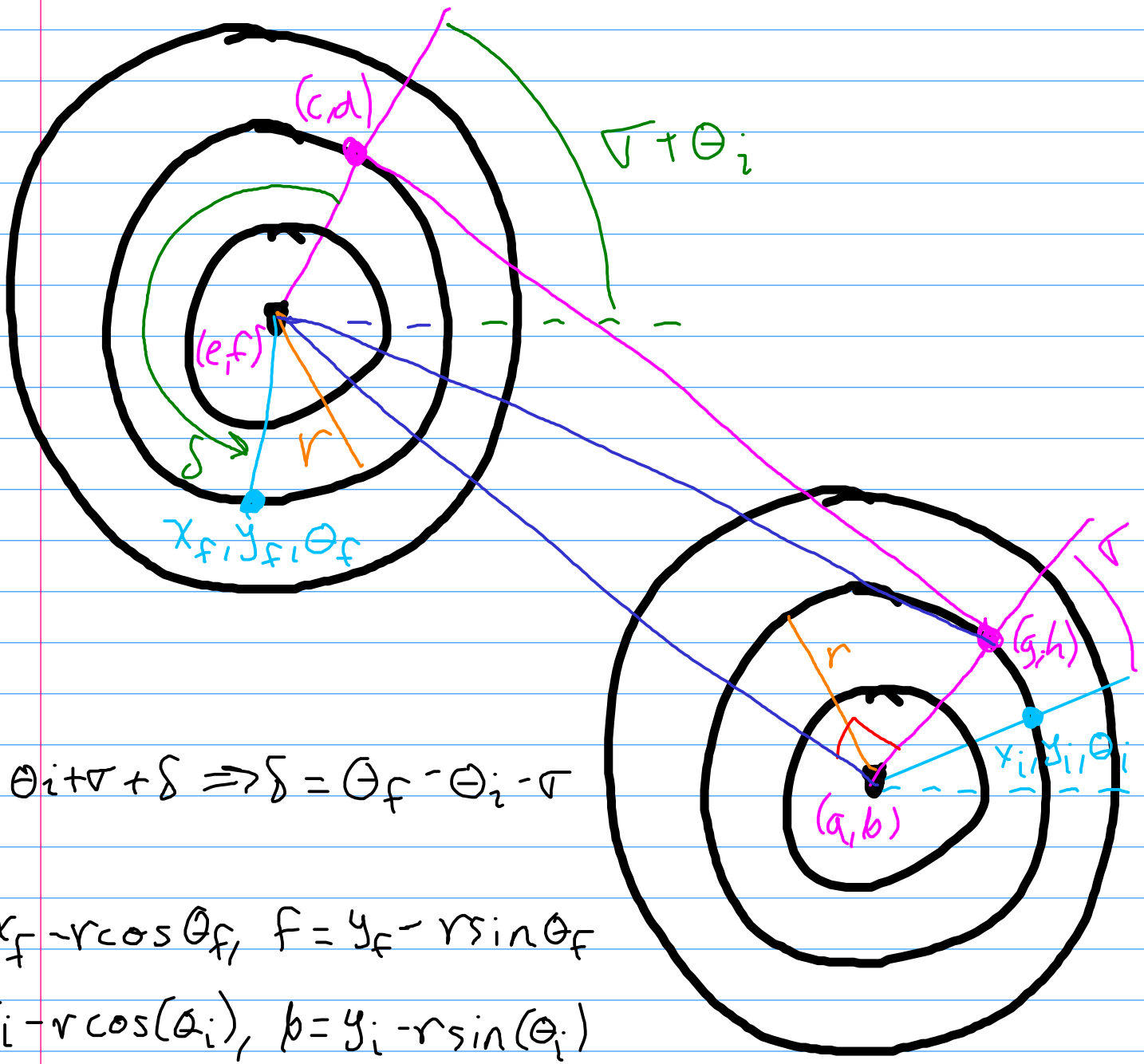
known: $x_i, y_i, \theta_i, x_f, y_f, \theta_f, r$

find: σ, δ



$\theta_f = \theta_i + \sigma + \delta \Rightarrow \delta = \theta_f - \theta_i - \sigma$

$e = x_f - r\cos\theta_f, \quad f = y_f - r\sin\theta_f$

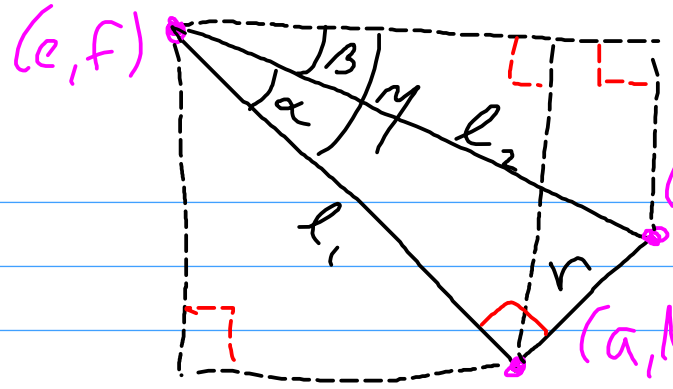
$a = x_i - r\cos(\theta_i), \quad b = y_i - r\sin(\theta_i)$

$c = e + r\cos(\sigma+\theta_i) = x_f - r\cos(\theta_f) + r\cos(\sigma+\theta_i)$

$d = f + r\sin(\sigma+\theta_i) = y_f - r\sin(\theta_f) + r\sin(\sigma+\theta_i)$

$g = a + r\cos(\sigma+\theta_i) = x_i - r\cos(\theta_i) + r\cos(\sigma+\theta_i)$

$h = b + r\sin(\sigma+\theta_i) = y_i - r\sin(\theta_i) + r\sin(\sigma+\theta_i)$



$$\ell_1 = \sqrt{(e-a)^2+(f-b)^2}$$

$$\ell_2 = \sqrt{\ell_1^2 + r^2}$$

$$= \sqrt{(e-a)^2+(f-b)^2+r^2}$$

$$\gamma = \alpha+\beta \Rightarrow \beta = \gamma - \alpha$$

$$\alpha = \text{atan}(r/\ell_1), \quad \gamma = \text{atan}\left(\frac{f-b}{e-a}\right)$$

$$\Rightarrow \beta = \text{atan}\left(\frac{f-b}{e-a}\right) - \text{atan}(r/\ell_1)$$

$$\cos\beta = \frac{e-g}{\ell_2} \Rightarrow g = e - \ell_2\cos\beta$$

$$\Rightarrow g = x_f - r\cos(\theta_f) - \ell_2\cos(\beta)$$

From before: $g = x_i - r\cos\theta_i + r\cos(\sigma+\theta_i)$

$$\Rightarrow x_i - r\cos(\theta_i) + r\cos(\sigma+\theta_i) = x_f - r\cos(\theta_f) - \ell_2\cos(\beta)$$

$$\Rightarrow \cos(\sigma+\theta_i) = \frac{1}{r}\left[x_f - x_i + r\cos(\theta_i) - r\cos(\theta_f) - \ell_2\cos(\beta)\right]$$

$$= \frac{x_f - x_i - \ell_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)$$

$$\Rightarrow \sigma + \theta_i = \cos^{-1}\left(\frac{x_f - x_i - \ell_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)\right)$$

$$\Rightarrow \sigma = \cos^{-1}\left(\frac{x_f - x_i - \ell_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)\right) - \theta_i$$

$$\Rightarrow \delta = \theta_f - \theta_i - \sigma$$