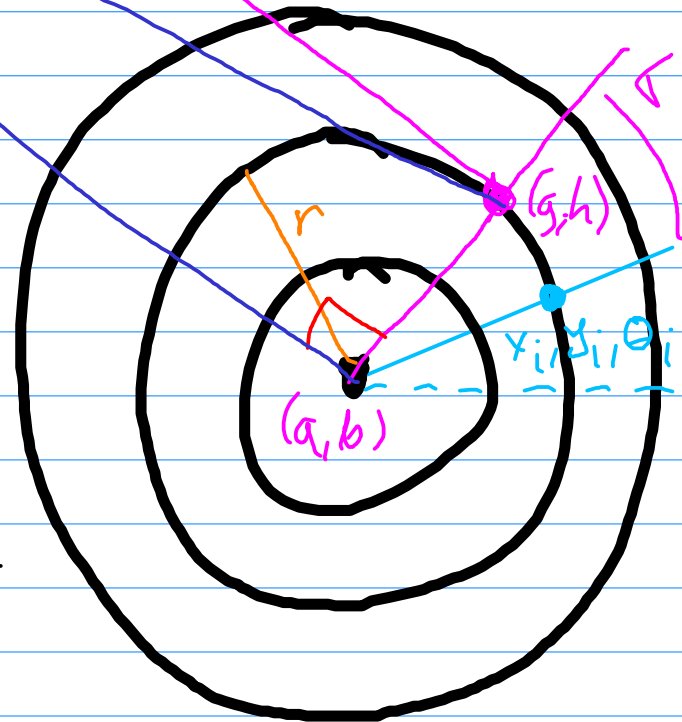
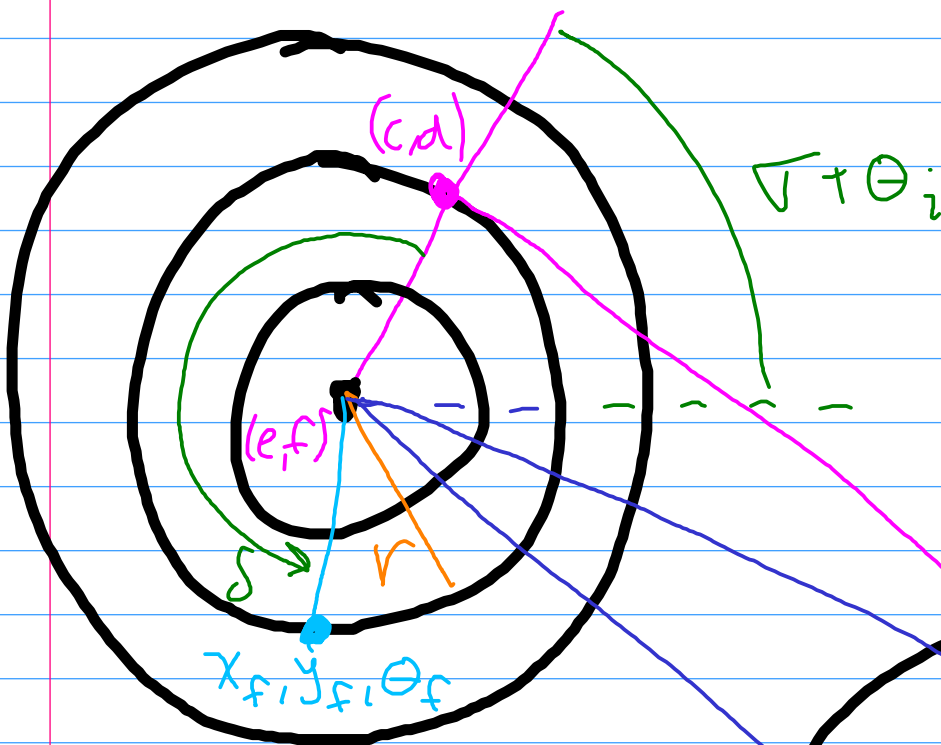


known: $x_i, y_i, \theta_i, x_f, y_f, \theta_f, r$
 find: σ, δ



$$\theta_f = \theta_i + \sigma + \delta \Rightarrow \delta = \theta_f - \theta_i - \sigma$$

$$e = x_f - r \cos \theta_f, \quad f = y_f - r \sin \theta_f$$

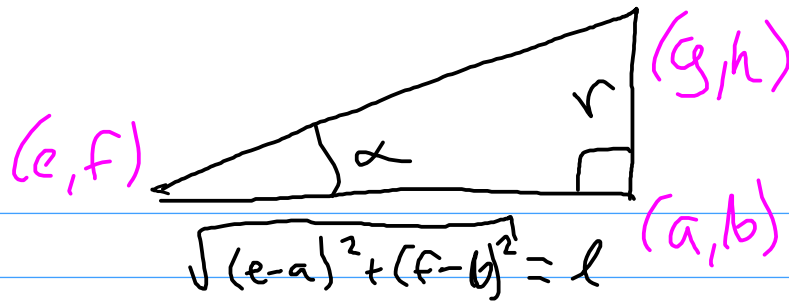
$$a = x_i - r \cos(\theta_i), \quad b = y_i - r \sin(\theta_i)$$

$$c = e + r \cos(\sigma + \theta_i) = x_f - r \cos(\theta_f) + r \cos(\sigma + \theta_i)$$

$$d = f + r \sin(\sigma + \theta_i) = y_f - r \sin(\theta_f) + r \sin(\sigma + \theta_i)$$

$$g = a + r \cos(\sigma + \theta_i) = x_i - r \cos(\theta_i) + r \cos(\sigma + \theta_i)$$

$$h = b + r \sin(\sigma + \theta_i) = y_i - r \sin(\theta_i) + r \sin(\sigma + \theta_i)$$



$$\alpha = \arctan(r/l)$$

$$g = e + r \cos(\alpha) = e + r \cos(\arctan(r/l)) = e + \frac{r}{\sqrt{\frac{r^2}{l^2} + 1}}$$

$$g = x_i - r \cos(\theta_i) + r \cos(\sigma + \theta_i)$$

$$\Rightarrow r \cos(\sigma + \theta_i) = g - x_i + r \cos(\theta_i)$$

$$\Rightarrow \cos(\sigma + \theta_i) = \frac{g - x_i}{r} + \cos \theta_i$$

$$\Rightarrow \frac{g - x_i}{r} = \frac{e + \frac{r}{\sqrt{\frac{r^2}{l^2} + 1}}}{r} = \frac{x_f - r \cos(\theta_f) + \frac{r}{\sqrt{\frac{r^2}{l^2} + 1}}}{r}$$

$$= \frac{x_f}{r} - \cos(\theta_f) + \frac{1}{\sqrt{\frac{r^2}{l^2} + 1}}$$

$$\cos(\sigma + \theta_i) = \frac{x_f}{r} - \cos(\theta_f) + \frac{1}{\sqrt{\frac{r^2}{l^2} + 1}} + \cos(\theta_i)$$

$$\sigma + \theta_i = \cos^{-1}\left(\frac{x_f}{r} - \cos(\theta_f) + \frac{1}{\sqrt{\frac{r^2}{l^2} + 1}} + \cos(\theta_i)\right)$$

$$\sigma = \cos^{-1}\left(\frac{x_f}{r} - \cos(\theta_f) + \frac{1}{\sqrt{\frac{r^2}{l^2} + 1}} + \cos(\theta_i)\right) - \theta_i$$

$$\delta = \theta_f - \theta_i - \sigma$$