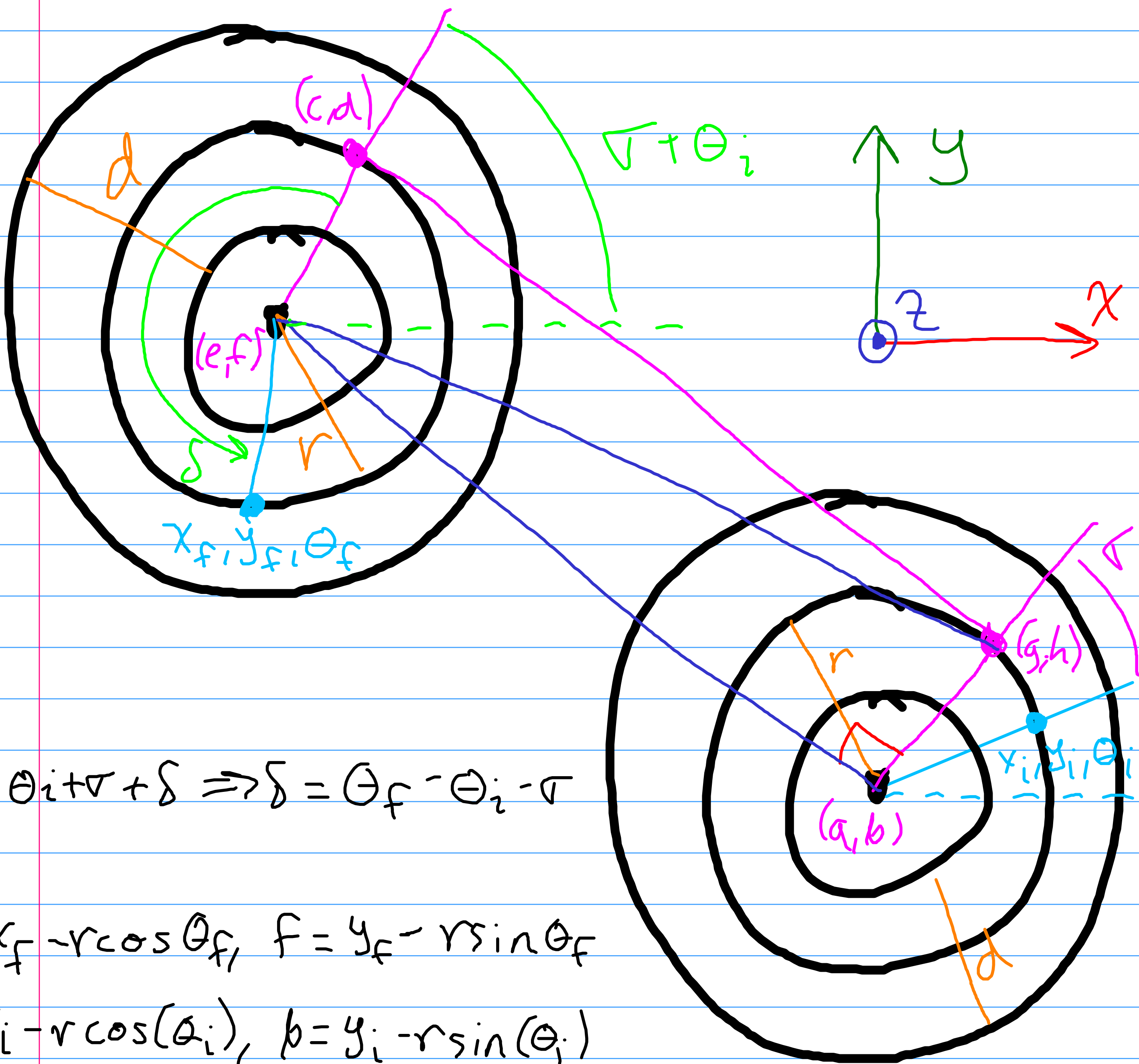
known: $x_i, y_i, \theta_i, x_f, y_i, \theta_f, r$

find: σ, δ



$$\theta_f = \theta_i + \sigma + \delta \Rightarrow \delta = \theta_f - \theta_i - \sigma$$

$$e = x_f - r\cos\theta_f, \quad f = y_f - r\sin\theta_f$$

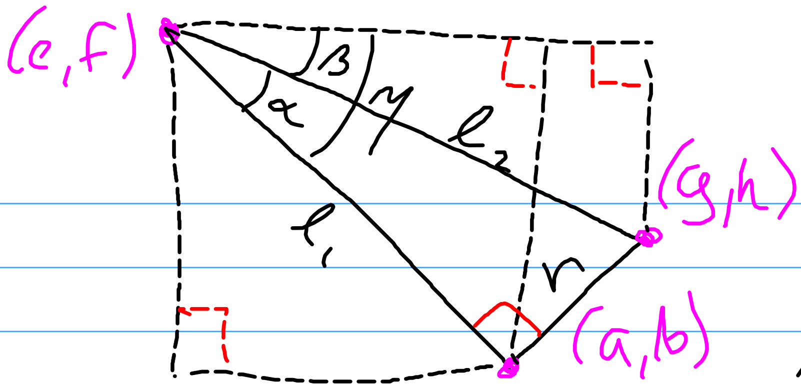
$$a = x_i - r\cos(\theta_i), \quad b = y_i - r\sin(\theta_i)$$

$$c = e + r\cos(\sigma+\theta_i) = x_f - r\cos(\theta_f) + r\cos(\sigma+\theta_i)$$

$$d = f + r\sin(\sigma+\theta_i) = y_f - r\sin(\theta_f) + r\sin(\sigma+\theta_i)$$

$$g = a + r\cos(\sigma+\theta_i) = x_i - r\cos(\theta_i) + r\cos(\sigma+\theta_i)$$

$$h = b + r\sin(\sigma+\theta_i) = y_i - r\sin(\theta_i) + r\sin(\sigma+\theta_i)$$



$$l_1 = \sqrt{(a-e)^2+(f-b)^2}$$

$$l_2 = \sqrt{l_1^2 + r^2}$$

$$= \sqrt{(a-e)^2+(f-b)^2+r^2}$$

$$\gamma = \alpha + \beta \Rightarrow \beta = \gamma - \alpha$$

$$\alpha = -\operatorname{atan}(r/l_1), \quad \gamma = \operatorname{atan2}(f-b, a-e)$$

$$\Rightarrow \beta = \operatorname{atan2}(f-b, a-e) + \operatorname{atan}(r/l_1)$$

$$\cos\beta = \frac{g-e}{l_2} \Rightarrow g = e + l_2 \cos\beta$$

$$\Rightarrow g = x_f - r\cos(\theta_f) + l_2\cos(\beta)$$

From before: $g = x_i - r\cos\theta_i + r\cos(\sigma+\theta_i)$

$$\Rightarrow x_i - r\cos(\theta_i) + r\cos(\sigma+\theta_i) = x_f - r\cos(\theta_f) + l_2\cos(\beta)$$

$$\Rightarrow \cos(\sigma+\theta_i) = \frac{1}{r}\left[x_f - x_i + r\cos(\theta_i) - r\cos(\theta_f) + l_2\cos(\beta)\right]$$

$$= \frac{x_f - x_i + l_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)$$

$$\Rightarrow \sigma + \theta_i = \cos^{-1}\left(\frac{x_f - x_i + l_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)\right)$$

$$\Rightarrow \sigma = \cos^{-1}\left(\frac{x_f - x_i + l_2\cos(\beta)}{r} + \cos(\theta_i) - \cos(\theta_f)\right) - \theta_i$$

$$\Rightarrow \delta = \theta_f - \theta_i - \sigma$$

Known: $r, d, R_w, \dot{\omega}_0, \sigma, \delta, \ell_1$

Assumption: Outside wheels (the ones that will be rotating the fastest) have $\dot{\omega}_0 = \dot{\omega}_{max}$

Find: * Δt for both turns and linear move

* Angular velocity of inside wheels for both turns

Let S be the arc length traveled during a turn.

$$S_{oft} = (r+d)\sigma, \quad S_{oft} = R_w \Delta W_{oft}$$

$$\Delta W_{oft} = \dot{\omega}_{oft} \Delta t_{ft} \text{ with } \dot{\omega}_{oft} = \dot{\omega}_0$$

$$\Rightarrow \Delta t_{ft} = \Delta W_{oft} / \dot{\omega}_0, \quad \Delta W_{oft} = S_{oft} / R_w$$

$$\Rightarrow \boxed{\Delta t_{ft} = S_{oft} / \dot{\omega}_0 \cdot R_w}$$

$$S_{ift} = r\sigma, \quad S_{ift} = R_w \Delta W_{ift}$$

$$\Delta W_{ift} = \dot{\omega}_{ift} \Delta t_{ft} \Rightarrow \dot{\omega}_{ift} = \Delta W_{ift} / \Delta t_{ft}$$

$$\Rightarrow \Delta W_{ift} = S_{ift} / R_w \Rightarrow \boxed{\dot{\omega}_{ift} = S_{ift} / R_w \cdot \Delta t_{ft}}$$

For the second turn, the calculations are the same, just that S_{ost} and S_{ist} are solved with an angular displacement of δ instead of σ.

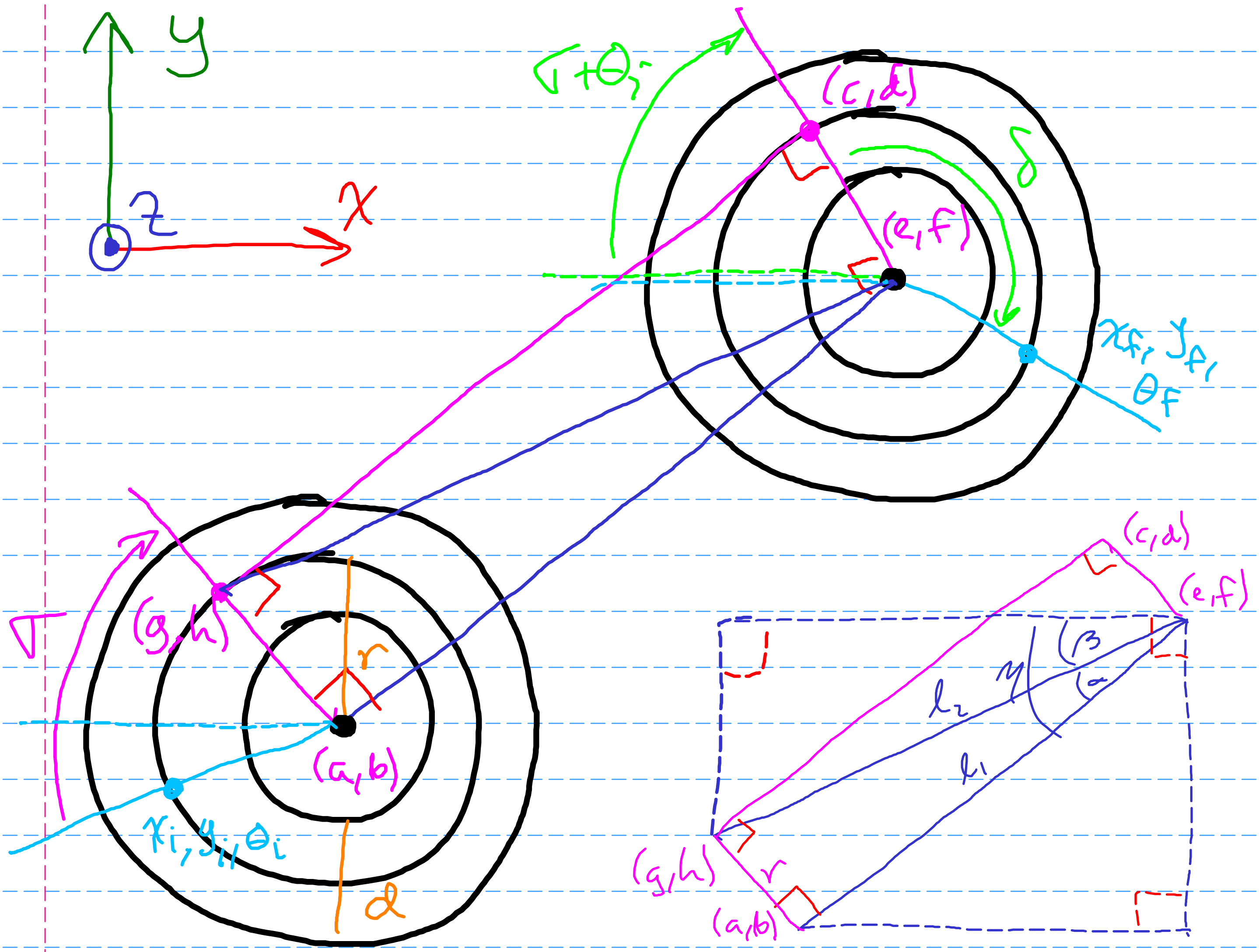
For the linear movement from (g,h) to (c,d), all wheels can travel at the "outside" wheel speed.

$$l_1 = R_w \Delta\omega_m \Rightarrow \Delta\omega_m = l_1 / R_w$$

$$\Delta\omega_m = \dot{\omega}_m \Delta t_m \Rightarrow \Delta t_m = \Delta\omega_m / \dot{\omega}_m$$

$$\Rightarrow \boxed{\Delta t_m \quad l_1 / R_w \cdot \dot{\omega}_m}$$

See what happens/changes for other cases:



$$l_1 = \sqrt{(e-a)^2+(f-b)^2},\quad l_2 = \sqrt{l_1^2+r^2} = \sqrt{(e-a)^2+(f-b)^2+r^2}$$

$$\mu = \alpha+\beta \Rightarrow \beta = \mu-\alpha$$

$$\alpha = -\text{atan}(r/l_1),\quad \mu = \text{atan2}(f-b,\, e-a)$$

$$\Rightarrow \beta = \text{atan2}(f-b,\, e-a) + \text{atan}(r/l_1)$$

$$\cos\beta = \frac{e-g}{l_2} \Rightarrow g = e - l_2\cos\beta$$

$$\Rightarrow g = x_f - r\cos(\theta_f) - l_2\cos(\beta)$$