Normal Distribution, Skewness & Kurtosis: A Beginner's Guide

Gaussian Normal Distribution

What is a Normal Distribution?
Simple Definition: A normal distribution is a bell-shaped curve where data is symmetrically distributed around the average (mean).
Think of it like: A perfect hill where most people are in the middle, and fewer people are at the very top or bottom.

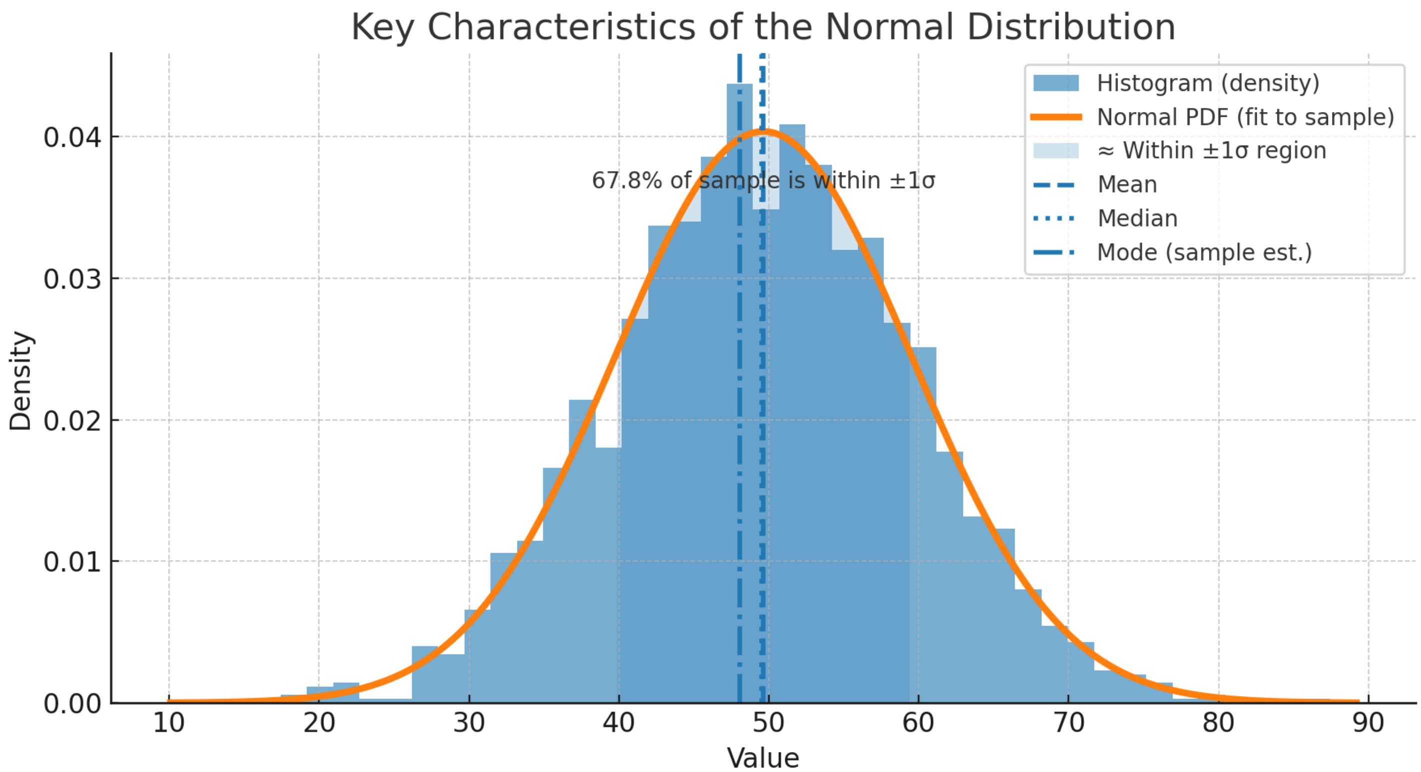
Key Characteristics of Normal Distribution

1. Bell-Shaped Curve
- Looks like a bell when you draw it
- Smooth, continuous curve
- Single peak in the middle

2. Symmetrical
- Left side is a mirror image of the right side
- If you fold it in half at the center, both sides match perfectly

3. Mean = Median = Mode
- All three measures of central tendency are the same
- They all occur at the center peak of the curve

4. Most Data Near the Center
- Majority of values cluster around the average
- Fewer values at the extremes (very high or very low)



What the chart shows—in simple words

- **Bell-shaped curve:** the smooth line forms a bell. That's the normal (Gaussian) curve fitted to the sample.
- **Symmetry:** the curve and the bars look the same on both sides of the center.
- **Mean = Median = Mode:** the three vertical lines (dashed, dotted, dash-dot) sit on top of each other at the center peak—showing they're (approximately) equal for normal data.
- **Most data near the center:** the light-shaded band marks the region from **mean − 1σ to mean + 1σ**. You can see most bars lie under this band. In our sample it's labeled on the plot (about **≈68%**), which is what the normal rule predicts.

Real-World Examples of Normal Distribution

Example 1: Human Heights
Scenario: Heights of adult men in a country
Distribution:

- **Average height**: 5'9" (most common)
- **Many people**: 5'7" to 5'11" (near average)
- **Some people**: 5'5" to 6'1" (moderately different)
- **Few people**: Under 5'3" or over 6'3" (extreme heights)

Why it's normal: Most men are average height, with fewer very tall or very short men.

Example 2: IQ Scores
Scenario: Intelligence test scores in the population
Distribution:

- **Average IQ**: 100 (center of curve)
- **Most people**: 85-115 (near average intelligence)
- **Some people**: 70-130 (below or above average)
- **Few people**: Below 70 or above 130 (very low or very high)

Why it's normal: Intelligence naturally clusters around average, with fewer extremely high or low scores.
Example 3: Product Manufacturing

Scenario: Weight of chocolate bars from a factory
Distribution:
- **Target weight**: 100g (what we aim for)
- **Most bars**: 98-102g (close to target)
- **Some bars**: 96-104g (slight variations)
- **Few bars**: Under 95g or over 105g (defects)

Why it's normal: Good manufacturing produces consistent results with small, random variations.

The Famous 68-95-99.7 Rule
This rule tells you exactly how data is spread in a normal distribution:
68% Rule (1 Standard Deviation)
- **68% of all data** falls within 1 standard deviation of the mean
- **Example**: If average height is 5'9" with std dev 3", then 68% of men are between 5'6" and 6'0"

95% Rule (2 Standard Deviations)
- **95% of all data** falls within 2 standard deviations of the mean
- **Example**: 95% of men are between 5'3" and 6'3"

99.7% Rule (3 Standard Deviations)
- **99.7% of all data** falls within 3 standard deviations of the mean
- **Example**: 99.7% of men are between 5'0" and 6'6"

Practical Application: Test Scores
Scenario: SAT scores are normally distributed
- **Mean**: 500 points
- **Standard deviation**: 100 points

What this means:
- **68% of students** score between 400-600
- **95% of students** score between 300-700
- **99.7% of students** score between 200-800
- **Only 2.5%** score above 700 (high achievers)
- **Only 2.5%** score below 300 (need extra help)

Why Normal Distribution is Important
1. Prediction
Example: If you know test scores are normally distributed, you can predict what percentage of students will need tutoring (those below 400).
2. Quality Control
Example: If chocolate bar weights are normally distributed around 100g, you know 95% should be between 96-104g. Anything outside this range indicates a problem.
3. Decision Making
Example: Insurance companies use normal distribution to set premiums based on typical claims.
4. Statistical Analysis
Example: Many statistical tests assume normal distribution, making it the foundation for data analysis.

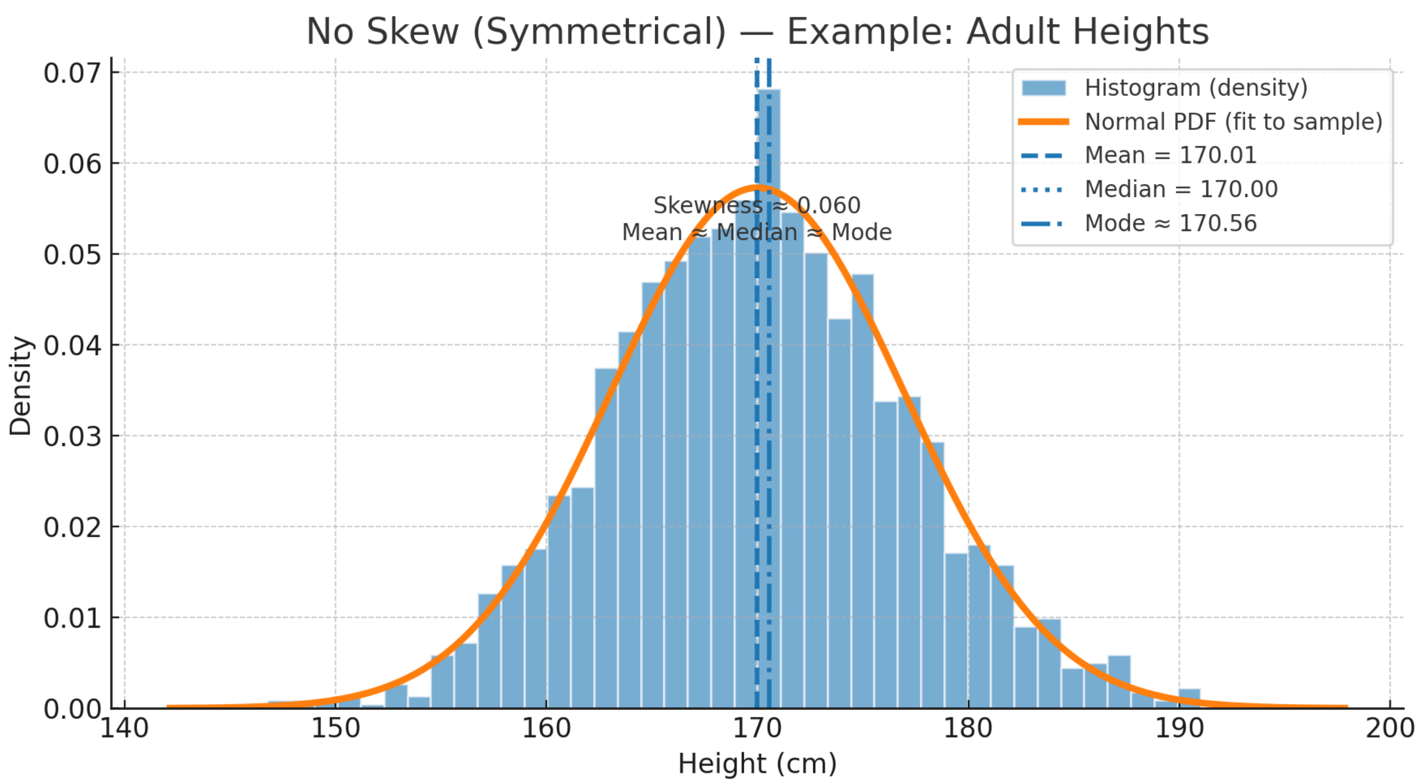
Skewness

What is Skewness?

Simple Definition: Skewness measures how lopsided or asymmetrical your data distribution is.
Think of it like: Imagine a seesaw - skewness tells you if one side is heavier than the other.

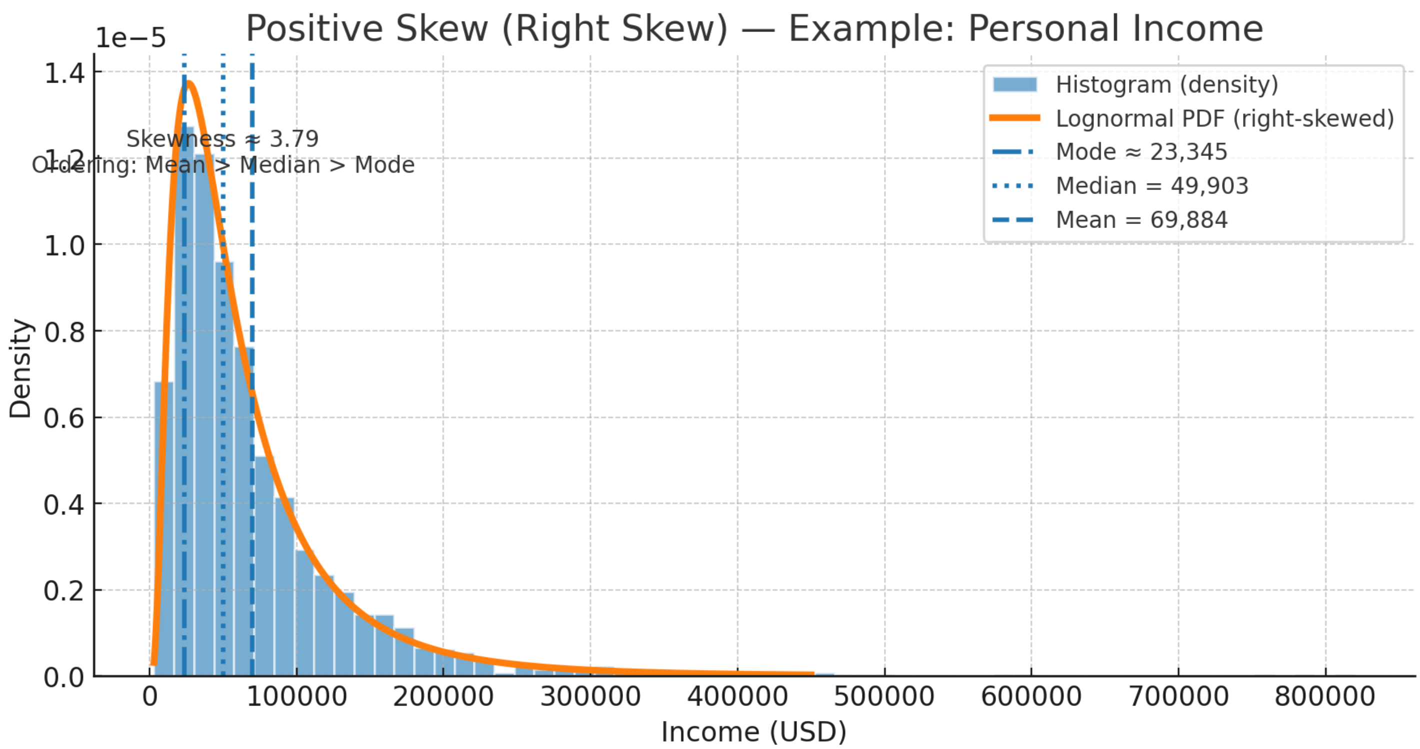
Types of Skewness
1. No Skew (Symmetrical)
Description: Data is evenly distributed on both sides of the mean **Shape**: Perfect bell curve
Characteristic: Mean = Median = Mode
Example: Heights of adult population

- Equal numbers of people above and below average height
- Smooth, symmetrical distribution



- The histogram and the smooth curve form a **bell** that is **symmetric**.
- The three vertical lines (mean, median, mode) land in the **same place** (or almost), which is typical when there's **no skew**.
- The printed **skewness** value is close to **0**, confirming symmetry.
- Interpretation: about as many people are **above** the average height as **below**; deviations on each side are similar.

2. Positive Skew (Right Skew)
Description: Most data is on the left, with a long tail stretching to the right **Shape**: Looks like a hill with a long gentle slope to the right **Characteristic**: Mean > Median > Mode
Visual Analogy: Like a ski jump - steep on the left, long gentle slope on the right



- Most observations sit **on the left**, and there's a **long right tail** (a few very large values).
- The vertical lines show **Mode (left) < Median (middle) < Mean (right)**.
  - The **mean** gets pulled **rightward** by the big incomes.
- The **skewness** value is **positive** (greater than 0), which quantifies the right-tail asymmetry.
- Interpretation: many people earn around the lower/middle range, and a smaller number earn **much more**, stretching the distribution to the right.

Real-World Examples of Positive Skew:
Example 1: Household Income
- **Most families**: Earn $30,000-$80,000 (clustered on left)
- **Some families**: Earn $100,000-$200,000 (middle)
- **Few families**: Earn $500,000+ (long tail on right - millionaires)

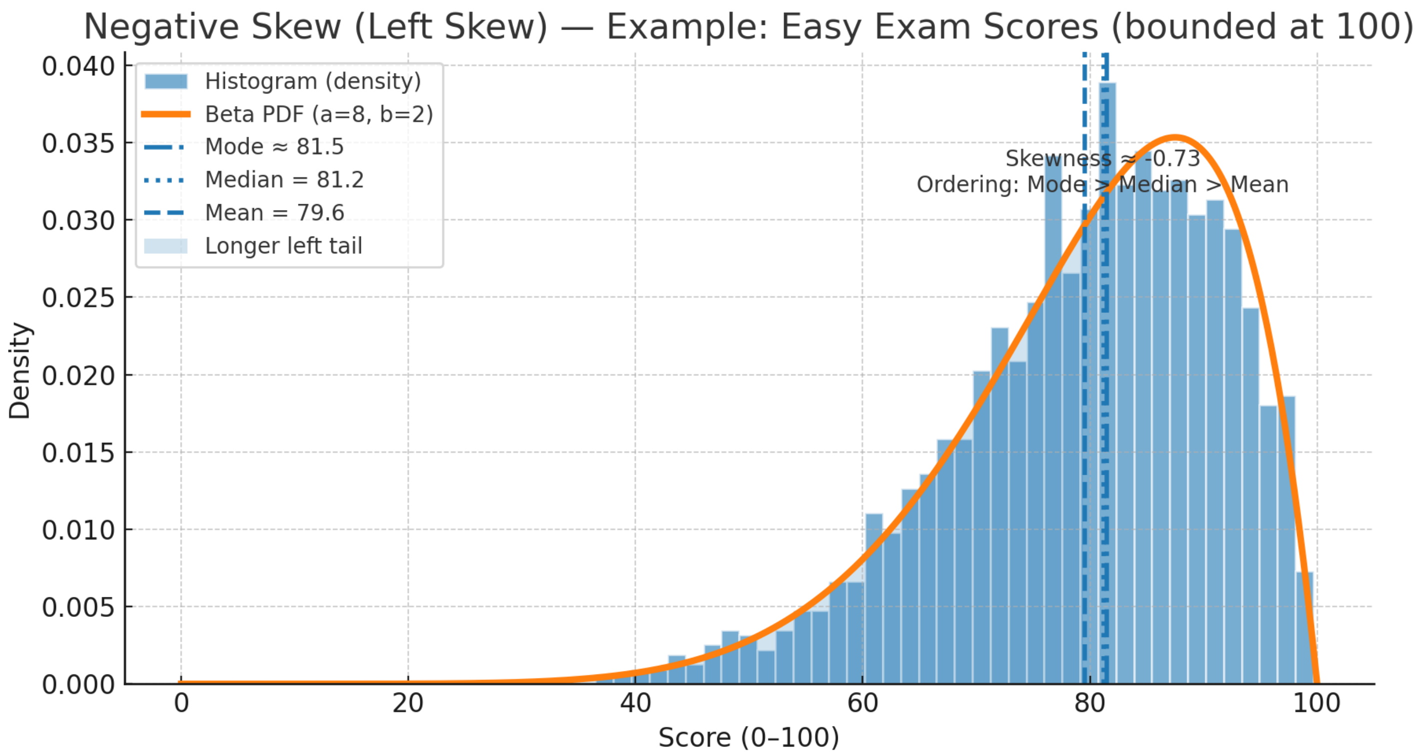
Why: There are many middle-class families but few extremely wealthy ones.

Example 2: House Prices in a City
- **Most houses**: $150,000-$400,000 (affordable range)
- **Some houses**: $500,000-$800,000 (luxury homes)
- **Few houses**: $2,000,000+ (mansions create the long tail)

Example 3: Website Visit Duration
- **Most visitors**: Stay 1-5 minutes (quick browse)
- **Some visitors**: Stay 10-20 minutes (engaged users)
- **Few visitors**: Stay 60+ minutes (very engaged, create right tail)

3. Negative Skew (Left Skew)

Description: Most data is on the right, with a long tail stretching to the left **Shape**: Looks like a hill with a long gentle slope to the left **Characteristic**: Mode > Median > Mean
Visual Analogy: Like a cliff - gentle slope on the left, steep drop on the right



- **Shape:** Most scores are high (right side), and a **long tail extends left** to lower scores. That's negative (left) skew.
- **Center measures:** The vertical lines show
  Mode (peak) > Median > Mean.
  Low outliers pull the **mean** left the most; the **median** moves a little; the **mode** stays near the high cluster.
- **Skewness value:** The printed number is **negative**, confirming left skew.
- **Shaded area:** Emphasizes the **longer left tail** (more probability on the low side compared with a symmetric bell).

Real-World Examples of Negative Skew:
Example 1: Age at Retirement
- **Most people**: Retire at 62-67 (clustered on right)
- **Some people**: Retire at 55-60 (early retirement)
- **Few people**: Retire at 40-50 (very early retirement creates left tail)

Why: Most people retire around normal age, but some retire much earlier.

Example 2: Test Scores (Well-Prepared Class)

- **Most students**: Score 80-95% (high performers)
- **Some students**: Score 65-75% (average)
- **Few students**: Score 40-60% (struggling students create left tail)

Example 3: Product Ratings

- **Most products**: Rated 4-5 stars (satisfied customers)
- **Some products**: Rated 3 stars (neutral)
- **Few products**: Rated 1-2 stars (very dissatisfied, create left tail)

How to Identify Skewness

Look at Mean vs Median:

Positive Skew: Mean > Median

- The few high values pull the average up
- Example: Income data where mean salary > median salary

Negative Skew: Mean < Median

- The few low values pull the average down
- Example: Test scores where mean score < median score

No Skew: Mean ≈ Median

- Values are evenly distributed
- Example: Heights where mean height ≈ median height

Practical Implications of Skewness

For Positive Skew (Income Example):

- **Median income** better represents typical family ($65,000)
- **Mean income** misleading due to millionaires ($85,000)
- **Business decision**: Target products for median income level

For Negative Skew (Test Scores Example):

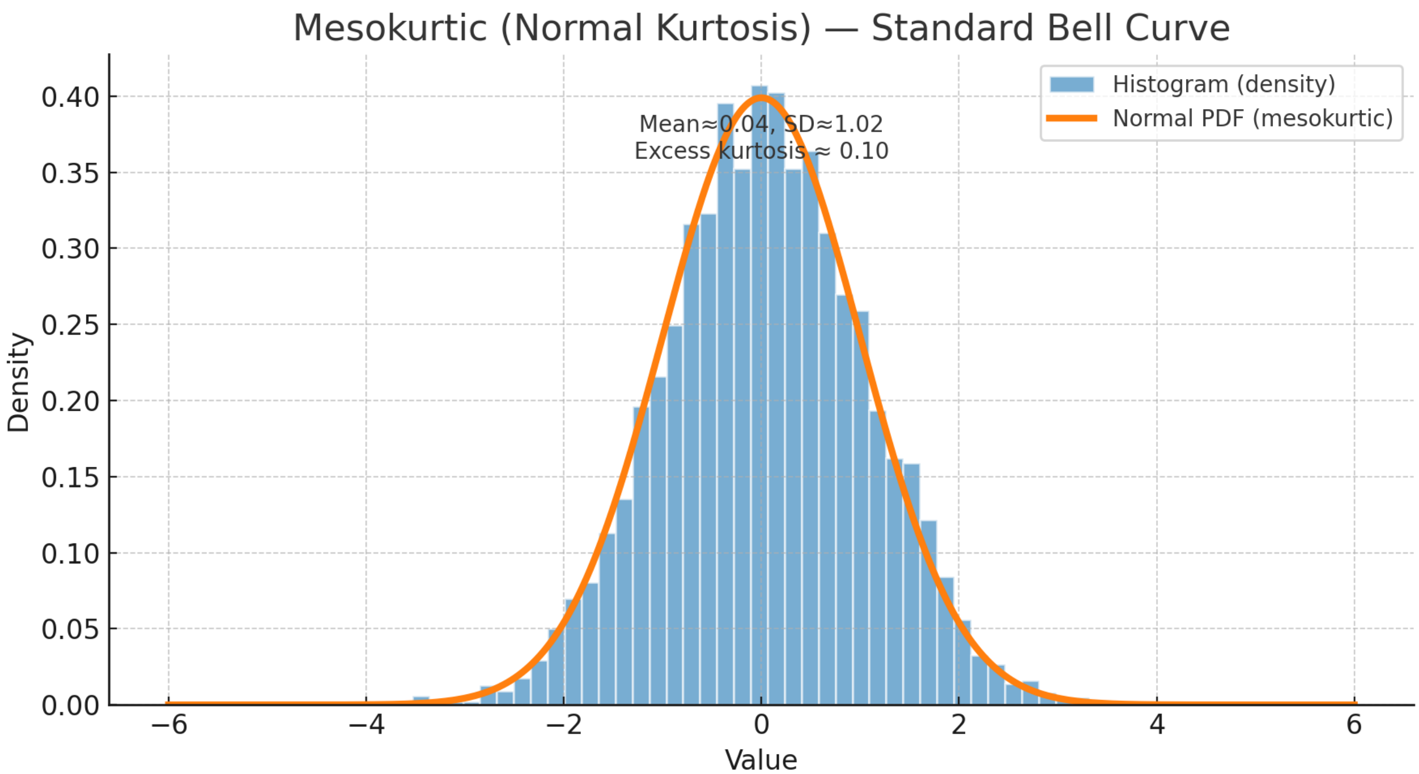
- **Few struggling students** need extra help
- **Most students** performing well
- **Teaching decision**: Provide remedial support for the tail

Kurtosis

What is Kurtosis?

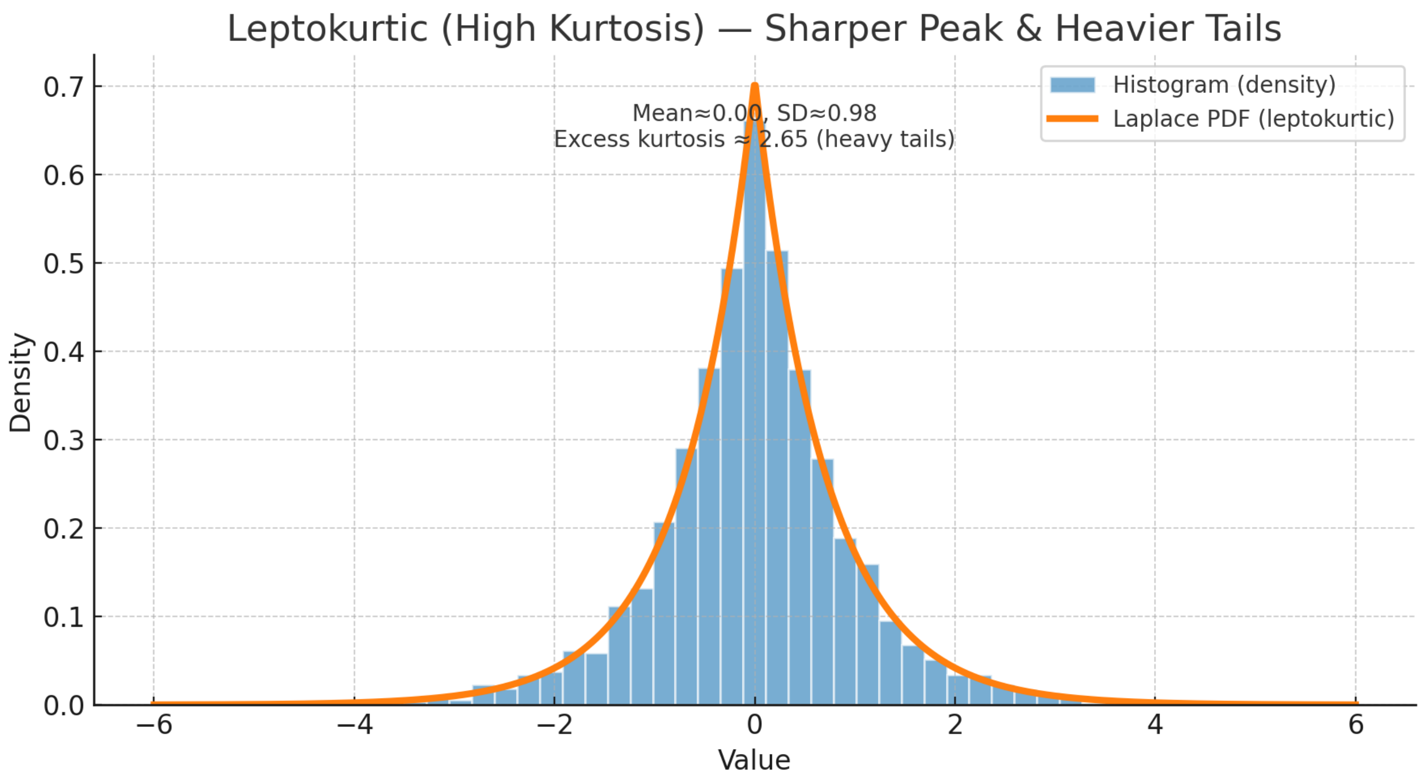
Simple Definition: Kurtosis measures how "peaked" or "flat" your distribution is compared to a normal distribution, and how heavy the tails are.
Think of it like: Comparing different mountain shapes - some are sharp peaks, others are flat hills.

Types of Kurtosis
1. Mesokurtic (Normal Kurtosis)
Description: Exactly like a normal distribution **Shape**: Standard bell curve **Tails**: Normal thickness **Peak**: Normal height
Example: Human heights - perfect bell curve with normal peak and tails



- The histogram aligns well with the smooth **Normal** curve.
- **Excess kurtosis ≈ 0** (printed on the chart): neither heavy nor thin tails.
- **What it means:** A balanced peak and tails. Outliers occur as often as the normal model predicts.

2. Leptokurtic (High Kurtosis)
Description: Sharper peak and heavier (thicker) tails than normal **Shape**: Tall, narrow peak with fat tails **Characteristic**: More extreme values than expected
Visual Analogy: Like a sharp mountain peak with wide base

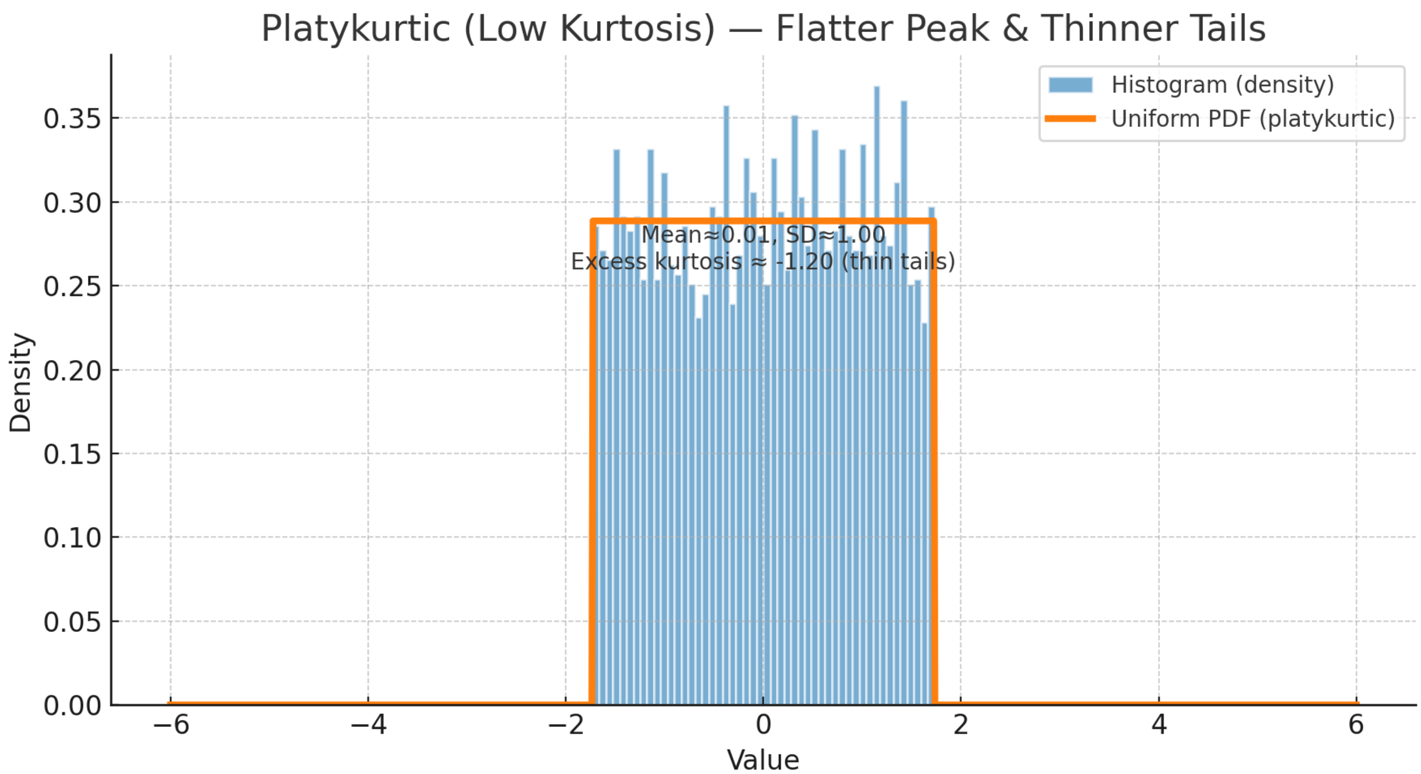


- The **Laplace** curve rises **sharper** at the center and its tails fall off more slowly.
- **Excess kurtosis > 0** (you'll see a large positive number, often around ~3 for Laplace), confirming **heavier tails**.
- **What it means:** More probability in the **center and tails**—so you see **more extreme values** (outliers) than a normal curve would predict. Think daily returns with occasional jumps, large insurance claims, etc.

Real-World Examples of Leptokurtic:
Example 1: Stock Market Returns
- **Most days**: Small gains/losses around 0% (sharp peak)
- **Some days**: Moderate changes ±2-5%
- **Few days**: Extreme crashes or booms ±10-20% (fat tails)
- **Interpretation**: More extreme market events than normal distribution would predict

Example 2: Website Server Response Times
- **Most requests**: Processed in 100-200ms (sharp peak)
- **Some requests**: Take 300-500ms
- **Few requests**: Take 5,000+ms due to server issues (heavy tail)
- **Interpretation**: More extreme delays than normal

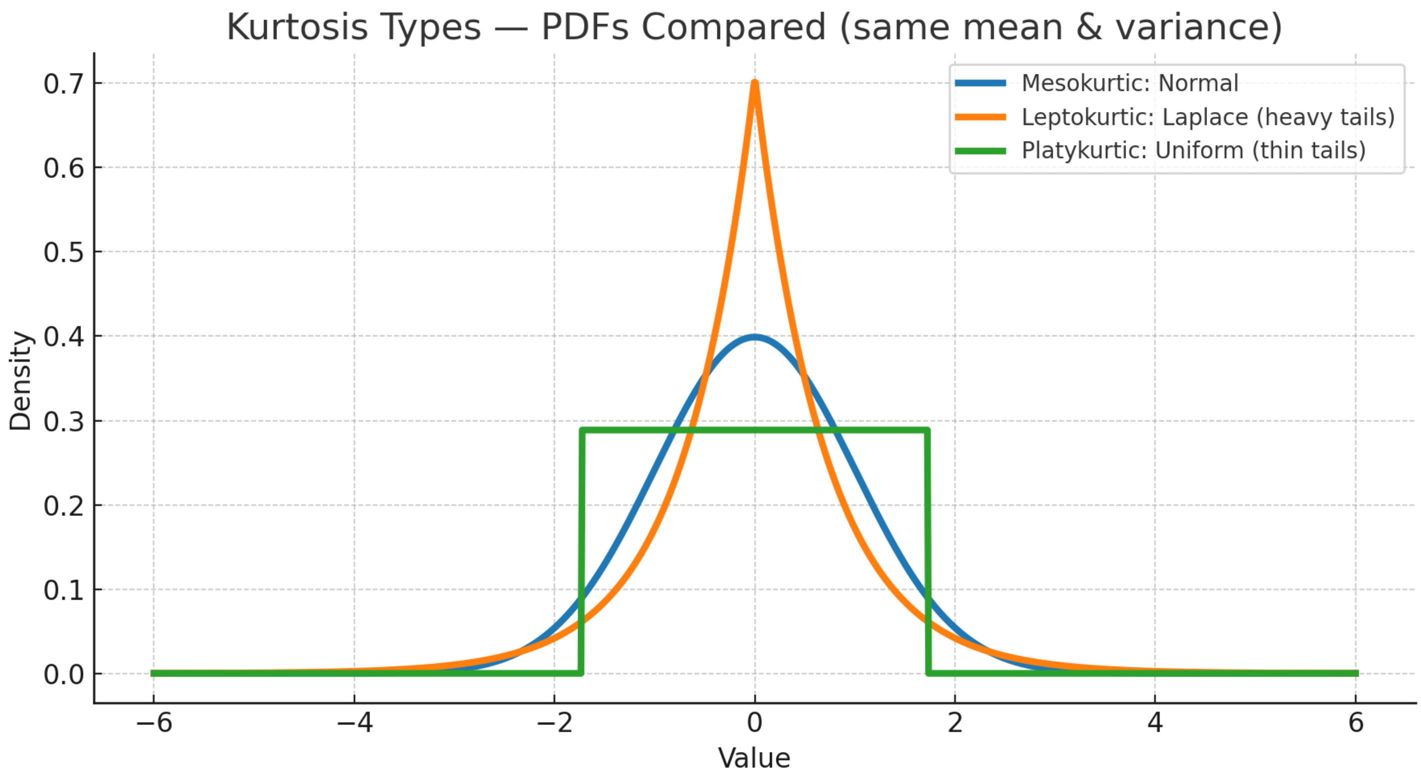
3. Platykurtic (Low Kurtosis)
Description: Flatter peak and thinner tails than normal **Shape**: Wide, flat distribution **Characteristic**: Fewer extreme values than expected
Visual Analogy: Like a flat hill or plateau



- The **Uniform** curve is flat with **finite** ends (no long tails).
- **Excess kurtosis < 0** (about **−1.2** for uniform), indicating **thin tails** and a flatter top than normal.
- **What it means:** Fewer outliers than normal. Many processes with strong caps/limits look more platykurtic.

Real-World Examples of Platykurtic:
Example 1: Uniform Random Numbers
- **Distribution**: Every number equally likely
- **Shape**: Flat line (rectangular)
- **Interpretation**: No peak, very thin tails

Example 2: Student Grades (Easy Test)
- **Most students**: Score between 80-95%
- **Few students**: Score below 75% or above 98%
- **Shape**: Flat distribution across high scores
- **Interpretation**: Test was too easy, little discrimination



Comparative overlay (PDFs only)

- All three curves are plotted together (same center and variance):
    - **Normal** sits in the middle.
    - **Laplace** shows a **taller peak** and **fatter tails** (leptokurtic).
    - **Uniform** shows a **flatter middle** and **abrupt ends** (platykurtic).

Understanding Kurtosis in Practice

High Kurtosis (Leptokurtic) Implications:

Finance Example: Stock with high kurtosis

- **Risk**: More likely to have extreme price movements
- **Strategy**: Need larger safety margins
- **Decision**: Consider more conservative investment

Quality Control Example: Product measurements with high kurtosis

- **Issue**: More defects than normal distribution predicts
- **Action**: Investigate process for sudden changes
- **Solution**: Improve process control

Low Kurtosis (Platykurtic) Implications:

Marketing Example: Customer satisfaction scores with low kurtosis

- **Observation**: Responses spread evenly across scale
- **Interpretation**: Customers have mixed, moderate opinions
- **Action**: Improve product to create more satisfied customers

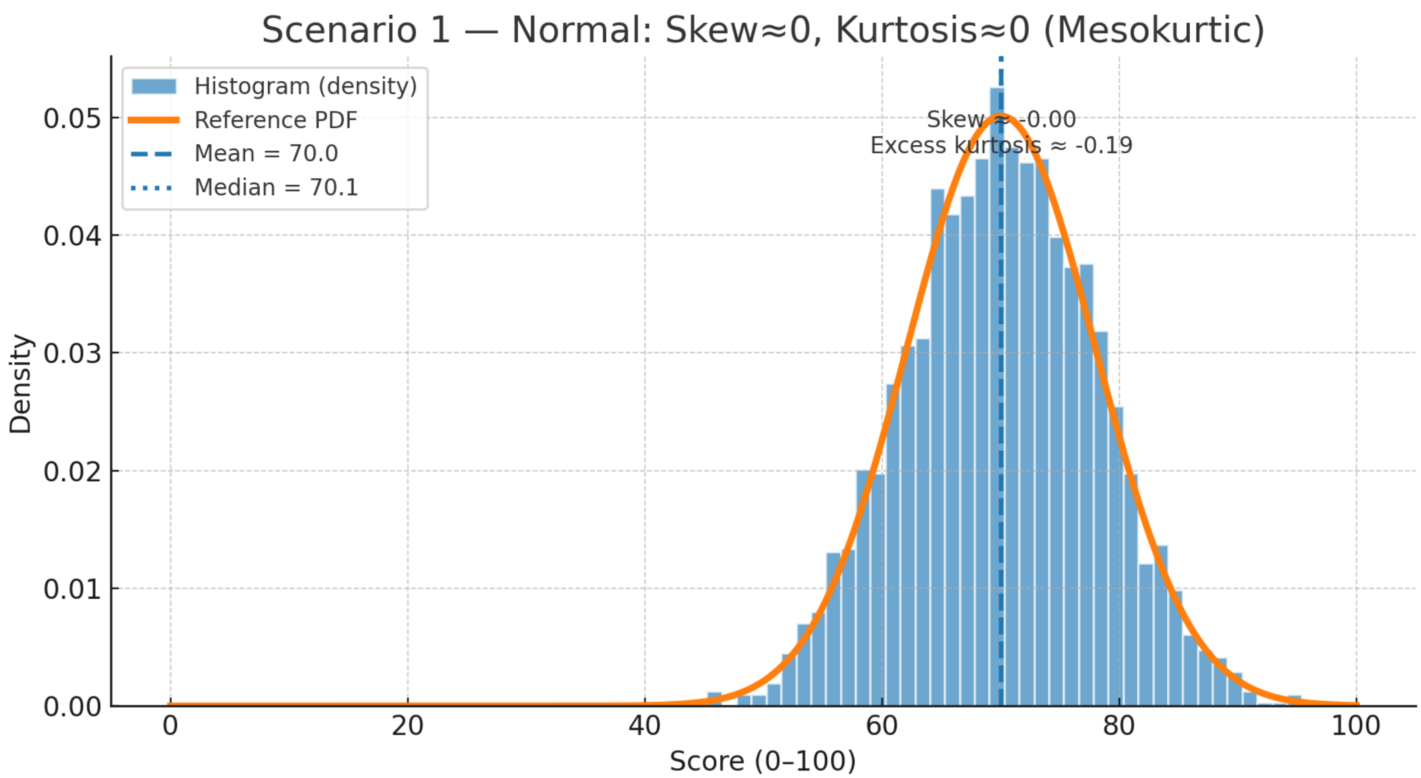
Education Example: Test scores with low kurtosis

- **Observation**: Scores spread evenly, no clear peak
- **Interpretation**: Test doesn't discriminate well between students
- **Action**: Redesign test for better assessment

Combining Skewness and Kurtosis
Real-World Example: Employee Productivity Scores

	mean	median	std (ddof=1)	skewness	excess kurtosis	min	max
Scenario 1: Normal	70.046	70.08	7.958	-0.002	-0.192	45.225	95.394
Scenario 2: +Skew, High Kurt	35.707	33.267	10.881	1.234	2.096	20.046	100
Scenario 3: -Skew, Low Kurt	76.906	77.931	9.784	-0.579	0.243	30.728	97.491

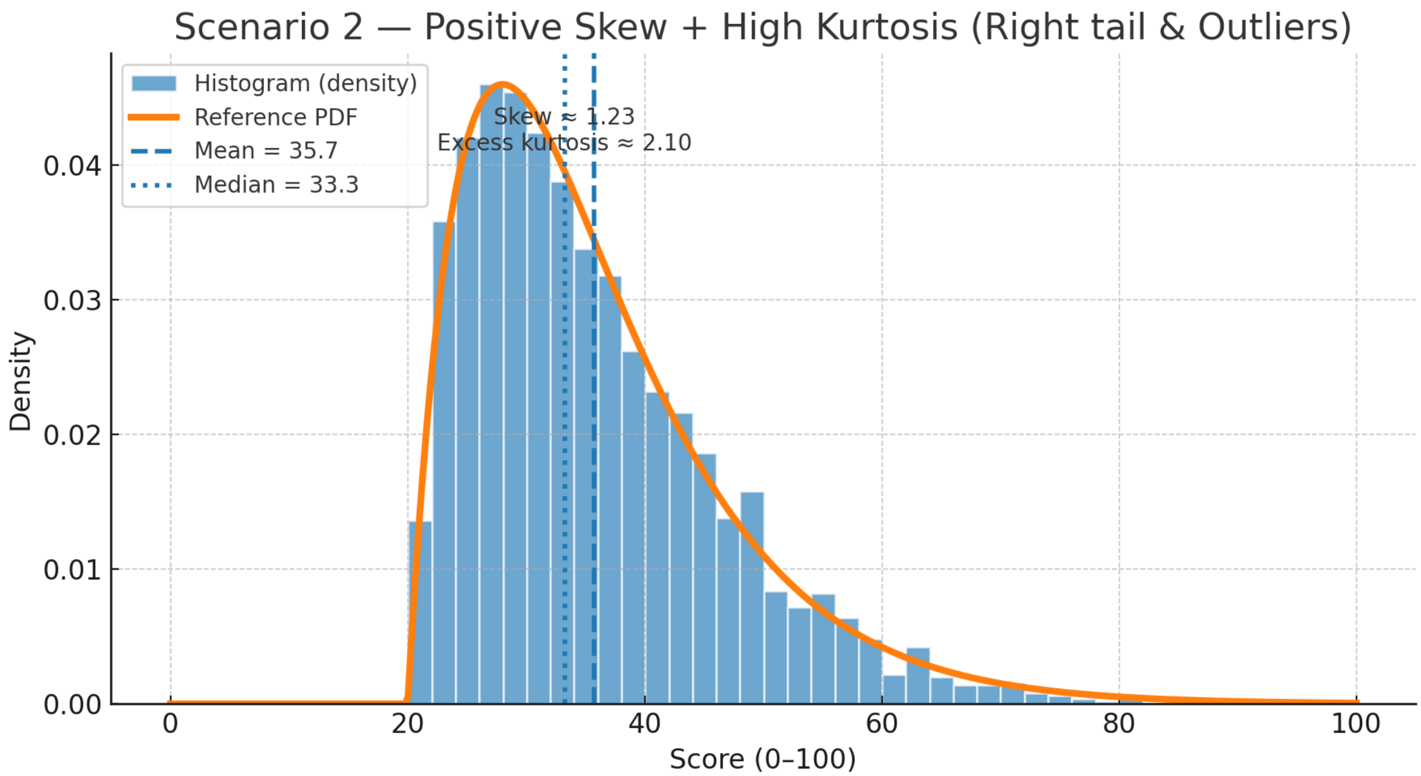
Scenario 1: Normal Distribution
- **Skewness**: 0 (symmetrical)
- **Kurtosis**: Normal (mesokurtic)
- **Interpretation**: Balanced workforce with predictable performance
- **Management**: Standard performance management works



- **What you see:** A symmetric bell of scores around ~70. The mean and median are almost identical; the tails look "normal thickness."
- **What it means:** Performance is balanced and predictable—few extreme highs or lows.
- **Management action:** Standard performance management (clear goals, routine feedback) is appropriate.

Scenario 2: Positive Skew + High Kurtosis

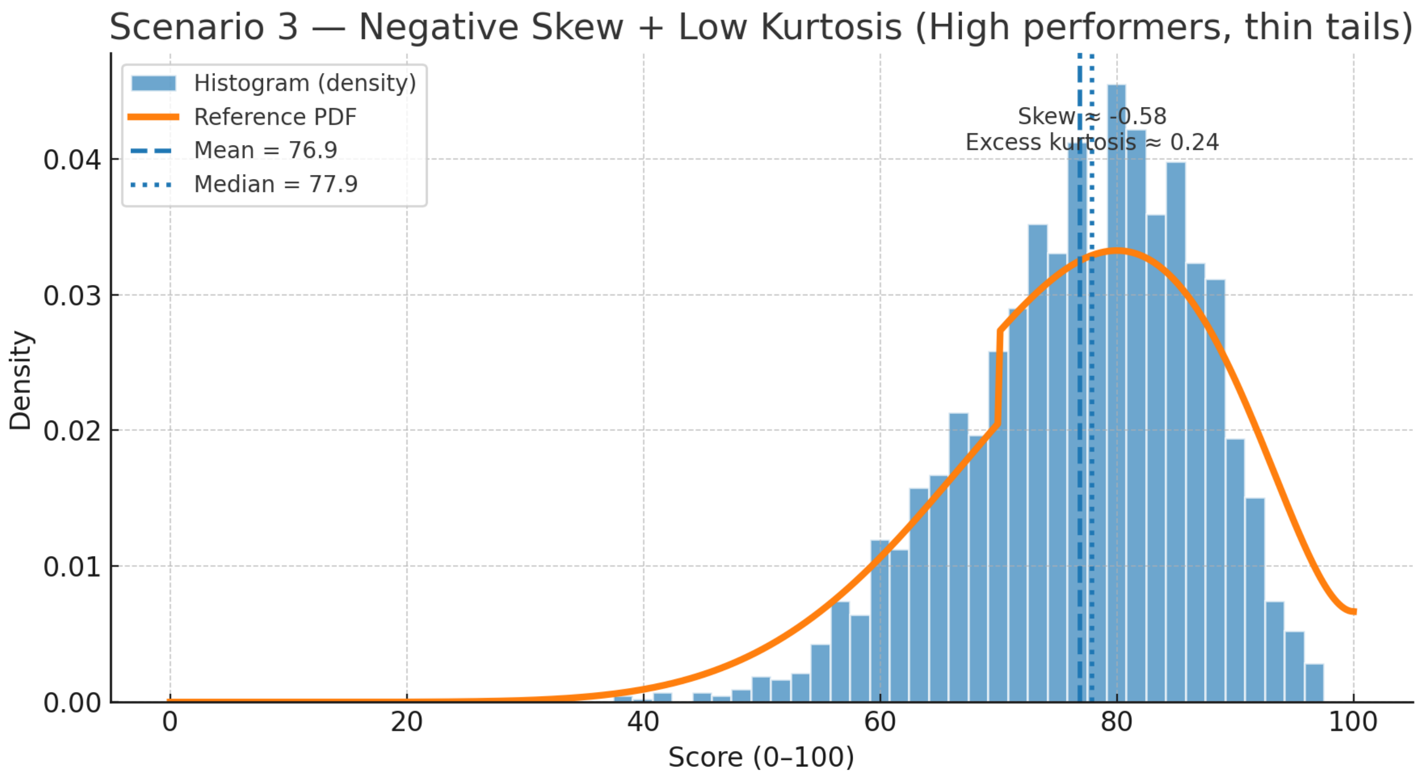
- **Skewness**: Positive (most employees average, few superstars)
- **Kurtosis**: High (some extremely high performers)
- **Interpretation**: Most employees average, but some exceptional performers
- **Management**: Identify and retain top performers, develop others



- **What you see:** Most employees cluster around a moderate score, with a **long right tail** and **more outliers** than normal. The **mean > median** (pulled up by the top performers). The chart text shows **positive skew** and **high excess kurtosis**.
- **What it means:** A handful of **exceptional performers** lift the average; the group has **fat tails** (occasional very high scores).
- **Management action:** Identify, reward, and **retain top performers**; create stretch opportunities and **develop** the rest.

Scenario 3: Negative Skew + Low Kurtosis

- **Skewness**: Negative (most employees high performers, few struggling)
- **Kurtosis**: Low (performance spread evenly)
- **Interpretation**: Generally high-performing team with consistent results
- **Management**: Focus on helping the few struggling employees



- **What you see:** Most employees score **high**, with a **left tail** of a few struggling individuals. Mean < median (pulled left by the few low scores). Kurtosis is **lower** (thinner tails / more even spread near the top).
- **What it means:** A generally **high-performing** team with few underperformers and **few extremes** overall.
- **Management action: Support the small struggling group** (coaching, resources) and keep sustaining excellence for the majority.

Practical Applications Summary

For Business Decision Making:

Normal Distribution: Use standard statistical methods and planning **Positive Skew**: Focus on median, plan for typical customer/employee **Negative Skew**: Address the struggling minority, celebrate high performers **High Kurtosis**: Prepare for extreme events, increase safety margins **Low Kurtosis**: Expect consistent, predictable outcomes

For Risk Management:

High Kurtosis: Higher risk of extreme events (market crashes, system failures) **Low Kurtosis**: Lower risk, more predictable outcomes **Skewness**: Identifies which direction extreme events are likely

Key Takeaways

Normal Distribution:

- **What**: Bell-shaped, symmetrical curve
- **When**: Heights, IQ scores, measurement errors
- **Use**: Prediction, quality control, statistical testing

Skewness:

- **What**: Measures asymmetry (lopsidedness)
- **Positive**: Income, house prices (few very high values)
- **Negative**: Test scores, product ratings (few very low values)
- **Use**: Choose mean vs median, understand data bias

Kurtosis:

- **What**: Measures peak sharpness and tail thickness
- **High**: Stock returns, server response times (more extremes)
- **Low**: Uniform data, easy tests (fewer extremes)
- **Use**: Risk assessment, outlier prediction

Understanding these concepts helps you:

1. **Choose the right statistics** (mean vs median)
2. **Set appropriate expectations** (normal vs skewed outcomes)
3. **Manage risk** (prepare for extreme events)
4. **Make better decisions** (based on actual data patterns)