Pylos Strategy Guide

Pylos is an abstract strategy game for two players. Each player takes turns placing a black or white marble onto a 4x4 grid (the bottom level), or on top of four other marbles as part of another level. If a player has no marbles left, they lose.

Whoever can place a marble on the top of the pyramid wins. If it is possible to move a marble already on the board to a higher level, the player may do so. Also, if a player creates a 2x2 square of their colour, they may take up to 2 marbles off the board and put them back in their collection.

Pylos is a very simple game, with very fews rules. However, there are subtleties that exist, even in the opening 12 moves that aren't immediately obvious.

This guide is going to mainly focus on some things that can happen in those first few moves. The notation we will use is given here: http://boardgamegeek.com/thread/305365/towards-pylos-notation. The notation is Level, Row, Column, where Level 1 is the bottom level, and the rows start at the bottom, from A-D, and the columns start from the left, 1-4.

Opening Theory

Each player has 15 marbles, so first player (white) will need to 'break serve' at some point, and save a marble to avoid black claiming the top of the pyramid. At a beginner level, players generally avoid given their opponent a leg up by forming a (mulitcoloured) square for their opponent to move a marble onto.

However, it turns out, if boths players follow this strategy whilst blocking (one colour) squares from being formed, the board usually turns out something like this. You can see in Figure 1 that both players have played six marbles, with four holes remaining. It is now white's turn, and so the first player will have to be the one to 'fill in' the bottom level first, giving a leg up to black. This is one of the ways in which white's 'first mover advantage' is limited.

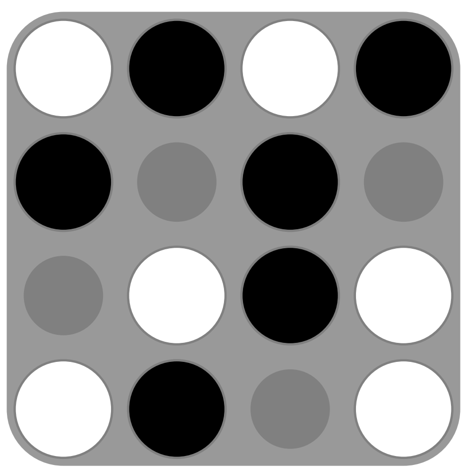

Figure 1: A typical opening.

So maybe white wants to sacrifice the next level earlier, in the face of this inevitable 'filling up' of the board. And yes, maybe this is how advanced players play. However, after playing a few dozen games we wondered if 4 was the maximum number of holes left such that all remaining moves make a square.

After many games against a computer, a position emerged that did indeed have this property! In

Figure 2, you can see that white has played six marbles, black only five. Second player has no choice but to fill in a gap and make a square.

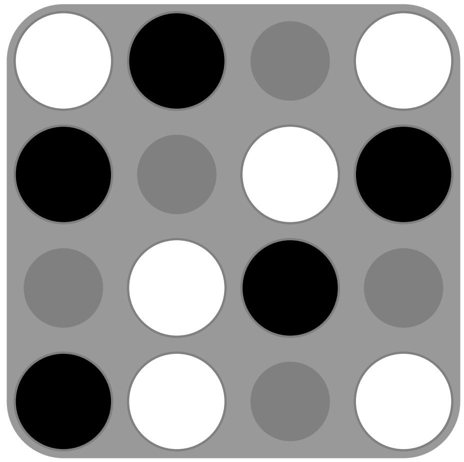

Figure 2: The wall.

I have been calling this position 'the wall' as there is a clear distinction between areas of the board. It is notable for its symmetry. The discovery of this formation led us to wonder if there were other such positions. Note the the marble at 1A1 can be shifted to 1B1 (or rather, the hole shifted from 1B1 to 1A1) without losing the property that every move creates a square. I have been referring to this position as simple the 'broken symmetry' formation, as we have disrupted the symmetry achieved by the wall position.

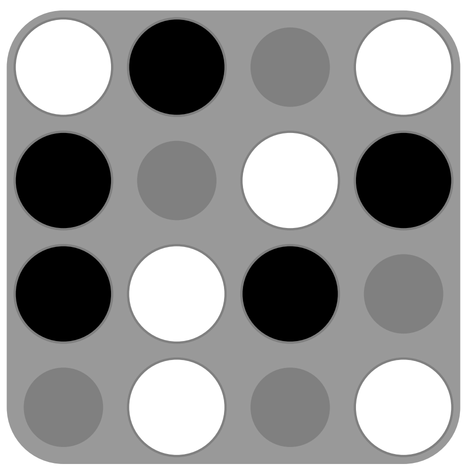

Figure 3: Broken symmetry.

Of course, we quickly realised that breaking the symmetry on one side meant that we could break the symmetry on the other side as well! So we reach another position that is symmetric, which I will call 'the square', due to the 3x3 square with a hole in the centre.

Now we have discovered (not counting reflectional and rotational symmetry or marble colouring) three different positions that, if first player can force, allow him to be the first to the second layer of the pyramid. Do any more of these positions exist? We would obviously like a proof that these are the only three positions possible.

After some thought, we were able to come up with the following.

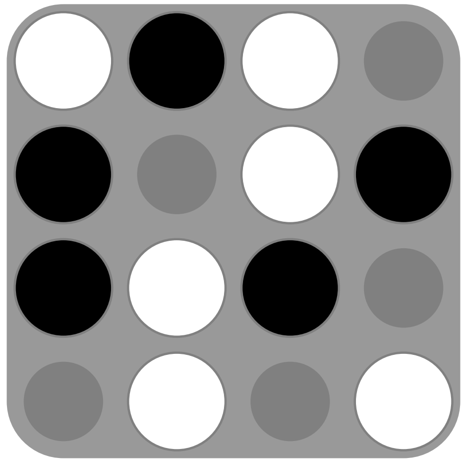

Figure 4: The square.

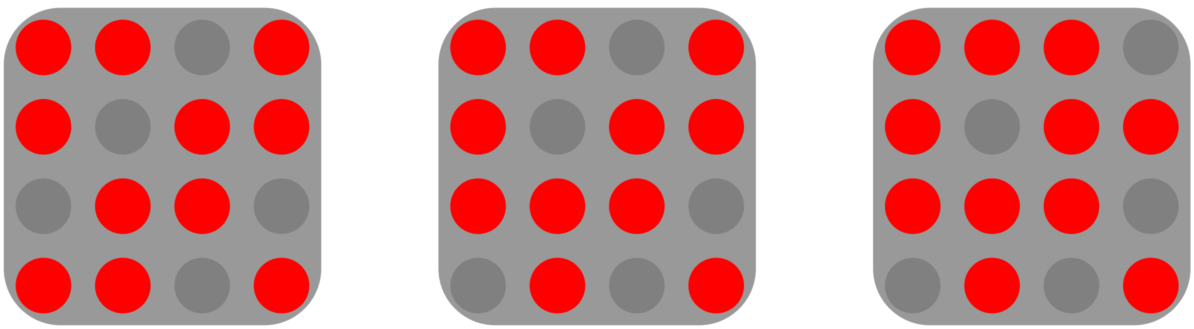

Figure 5: From left to right: the wall, broken symmetry and the square.

Theorem

There are only three possible Pylos positions with five holes, such that no squares are formed but filling any hole creates a square.

Proof

It is clear that the 2x2 square in the middle of the board must contain at least one hole. Let us begin then, with a hole at 1B2. This immediately rules out the rest of the points in that quarter of the board from being holes. Note that holes are shown in blue, marbles in red.

Now, that's one hole taken care of. We are looking for solutions with four more holes, and only three quarters remain. By the pidgeon-hole principle, one of the quarters must contain two holes. By exhaustion, one can show that neither the top left quadrant (nor bottom right, by symmetry) can afford to accommodate two holes. A little more thought shows that the holes must be situated at 1C4 and 1D3. This rules out the squares at 1C2, 1D2, 1B3 and 1B4 from being holes.

Thus, the solutions are as above with holes at (1A3,1C1), (1A3,1D1), (1A4,1C1) or (1A4,1D1). Since (1A3,1D1) is really just a reflection of (1A4,1C1), we have rediscovered the three solutions previously mentioned.

We can follow similar thought processes to show there are no surprises when it comes to three, six, or any other number of holes. There exist no solutions for any number except 4 and 5.

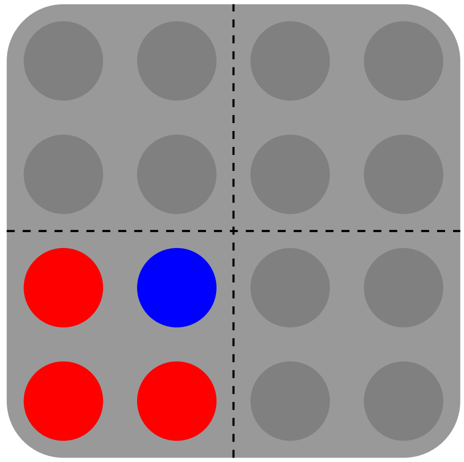

Figure 6: First step in the proof.

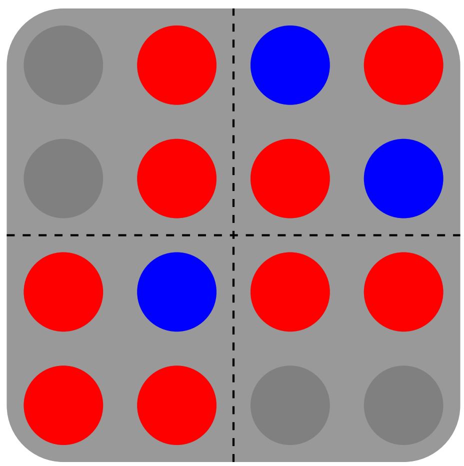

Figure 7: Second step in the proof.

If it's not looking like white can set it up, then a more regular game will probably commence, where I have little real advice yet, except for the following.

Middle Game

Claiming the centre of the second level can be useful. If only because you cannot make a square on the second level without it. However, the four pieces beneath this centre piece can never move. Often it is more useful to have free pieces around the edges of the board in order to move them up to higher levels. Our strategy is usually a balance between this and trying to limit our opponent forced moves.

In the following game, shown in Figure 8, white's attempt to gain the upper hand and counteract black's second mover advantage results in an atypical opening. White's attempts to create an all white square are blocked by black, resulting in the creation of a 2×2 square in the centre of the board. White leaps at the opportunity to claim the centre of the second level. However, this piece can never move, thus it traps forever the three white pieces below it. Black has more free agents and opportunities to save moves by moving marbles up from the bottom level to the second.

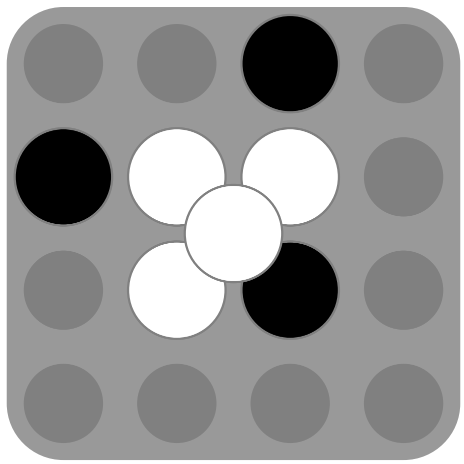

Figure 8: 1B1, 1C4, 1C2, 1C1, 1C3, 1B3, 2B2: White has claimed the centre of the second tier, but has trapped 3 of his own pieces to do it.