Ondas

Recordemos que la ecuación de onda está dada por:

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0}$$

Cuya solución es:

$$\psi(x,t) = \psi_0 e^{i(kx-\omega t)} = |\psi_0| e^{i(\overbrace{kx-\omega t+\phi_0}^{\text{fase}})}$$

si y solamente si

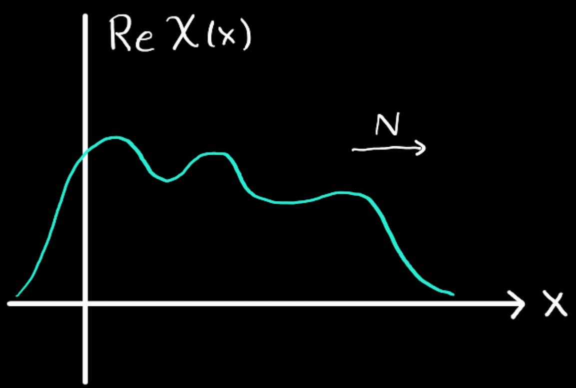
$$\boxed{\omega = kv}, \quad k \in \mathbb{R}$$

No dispersiva

Además, $\lambda = \frac{2\pi}{k}$ y $\tau = \frac{2\pi}{\omega}$.

$$\chi(x,t) = \sum_m \psi_{0m} e^{i(k_m x - \omega_m t)} \Leftrightarrow \omega_m = k_m v.$$

Si $t=0$,



La clave de la no dispersión recae en que la frecuencia es kv (así la velocidad es constante).

$$e^{ik_m(x-vt)}$$

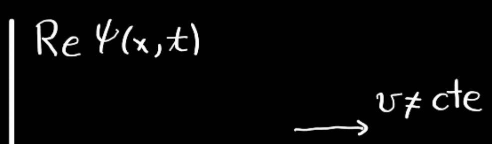
Suponga $\frac{\partial^2 \psi}{\partial x^2} + \frac{i}{\alpha}\frac{\partial \psi}{\partial t} = 0$; $\alpha = cte$, $[\alpha] = L^2/T$

$$\psi(x,t) = \psi_0 e^{i(kx-\omega t)}, \quad k \in \mathbb{R}$$

$$-k^2\psi(x,t) + \frac{i}{\alpha}(-i\omega)\psi(x,t) = 0$$

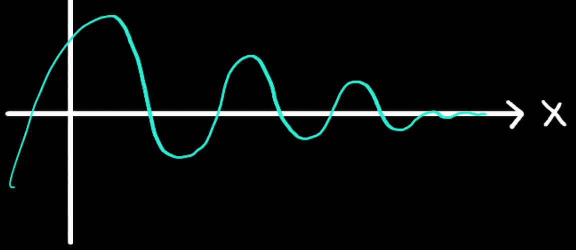
$$\psi_0 \neq 0 \text{ solución} \Leftrightarrow -k^2 + \frac{\omega}{\alpha} = 0 \Leftrightarrow \boxed{\omega = \alpha k^2}$$

Dispersiva



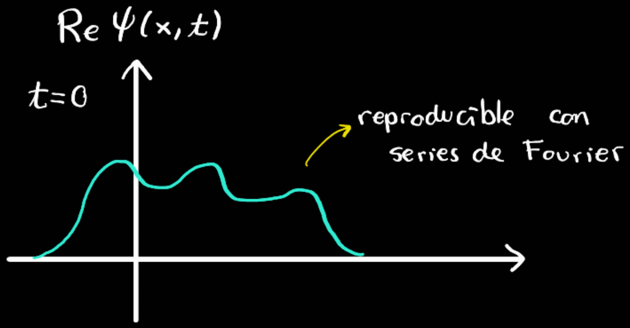
$$\psi_0 e^{ik(x-\frac{\omega}{k}t)} = \psi_0 e^{ih(x-\underbrace{\alpha k}_{v(k)=\alpha k} t)}$$

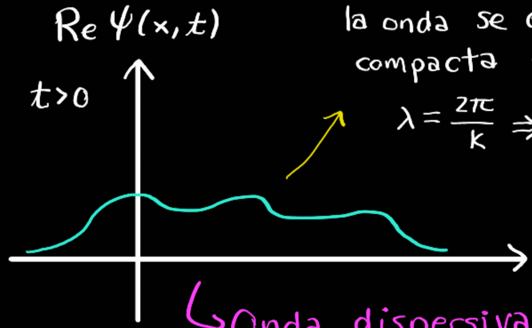
velocidad de la fase



$$\psi(x,t) = \sum_m \psi_{om}\, e^{i(k_m x - \omega_m t)}$$

$$\Leftrightarrow \omega_m = \alpha k_m^2$$





la onda se ensancha o compacta pues $\lambda = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{\lambda}$

Onda dispersiva

Interferencia:

- Interferencia constructiva

- Interferencia destructiva

ONDAS 3D (esféricas)

$\psi(x,y,z,t)$

$$\frac{\partial^2 \psi}{\partial x^2} \longrightarrow \nabla^2 \psi, \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Por lo cual, $$\nabla^2 \psi + \begin{cases} \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \psi \\ \frac{i}{\alpha} \frac{\partial}{\partial t} \psi \end{cases} = 0$$

Además $\psi(\vec{r},t) = \psi_0(\vec{r})\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ donde $\vec{k}$ es el vector de onda y $|\vec{k}| = \frac{2\pi}{\lambda}$ (i.e. la onda viaja en dirección $\vec{k}$).

ONDA PLANA

Dado t, $\forall \vec{r}$ $\vec{k}\cdot\vec{r} = cte$ $\therefore$ $\psi(\vec{r},t) = cte$. Pero,

$$\vec{k}\cdot\vec{r} = k_{ox}\, x + k_{oy}\, y + k_{oz}\, z = cte$$

Note que esto es un plano $\perp \vec{k}$

Tomando $\vec{k} = k_0 \hat{z}$ se obtiene

$$\psi(x,y,z,t) = \psi_0\, e^{i(\vec{h}\cdot\vec{r} - \omega t)} = \psi_0\, e^{i(h_0 z - \omega t)}$$