Hemos visto hasta el momento que

**Libros recomendados**
Marion, Landau

(i)

(ii) $\vec{F} = m\vec{a} \longrightarrow S = \int L(q,\dot{q})\,dt \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$

(iii)

**Idea útil**

Teorema de Euler para funciones homogéneas

$$T = \frac{1}{2}\sum_k q_k \frac{\partial T}{\partial q_k}$$

Ejemplo: $f(x) = x^2$

$$\Rightarrow f(x) = \frac{1}{2}(x)\frac{\partial f}{\partial x} = \frac{1}{2}(2x^2) = x^2$$

Dada $S = \int L(q,\dot{q})dt$, buscamos $\delta S = 0$ (minimizar). Además, recordemos que la ecuación de Euler-Lagrange es

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0$$

Note entonces,

$$\frac{dL}{dt} = \sum_k \frac{\partial L}{\partial q_k}\dot{q}_k + \sum_k \frac{\partial L}{\partial \dot{q}_k}\ddot{q}_k \Rightarrow 2\frac{dT}{dt} = \sum_k \ddot{q}_k \frac{\partial T}{\partial \dot{q}_k} + \sum_k \dot{q}_k \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)$$

$$\Rightarrow L = T - V$$

Lado derecho: $2\frac{dT}{dt} - \frac{dL}{dt} = \frac{dT}{dt} + \frac{dV}{dt} = \frac{dE}{dt} \Rightarrow E = T + V$

$$\frac{dE}{dt} = \sum_k \ddot{q}_k \frac{\partial V}{\partial \dot{q}_k} + \sum_k \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_k}\right)\dot{q}_k + \frac{\partial L}{\partial t}$$

(a) $E = cte$ si $T$ no depende de $T$

(b) Si $V = V(t) \Rightarrow \frac{d}{dt}(T+V) = \frac{\partial V}{\partial t}$

(c) Si $V = V(q,\dot{q}) - \sum \dot{q}_k \frac{\partial V}{\partial \dot{q}_k} = cte$

Observese que $T + V = \sum \dot{q}_k \underbrace{\frac{\partial L}{\partial \dot{q}_k}}_{p_k:\text{ momentos}} - L$. Como la energía libre de Helmholtz es $F = U - \frac{\partial U}{\partial S}T$, con $U = U(S,V,N)$ tal que $F = F(T,V,N)$. De forma tal que,

$$T + V = H(q_k, p_k)$$

# Principio Variacional

(i) Buscamos $y(x)$ tal que

$$J = \int_{x_1}^{x_2} f\{y(x), y'(x); x\}\,dx \qquad f \text{ es un funcional}$$

J es valor extremo.

**Definición.** Función vecina $y = y(\alpha, x)$ tal que $y(0,x) = y(x)$

(*) $y(\alpha, x) = y(x) + \alpha\, y(x)$

Tenemos entonces que $J(\alpha) = \int_{x_1}^{x_2} f(y, y'; x)\,dx$, $\left.\frac{\partial J}{\partial \alpha}\right|_{\alpha=0} = 0$.

$$\frac{\partial J}{\partial \alpha} = \int\left\{\frac{\partial f}{\partial y}\frac{\partial y}{\partial \alpha} + \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial \alpha}\right\}dx = \int\left\{\frac{\partial f}{\partial y}\eta(x) + \frac{\partial f}{\partial y'}\eta'(x)\right\}dx$$

$$\Rightarrow \int dx \frac{\partial f}{\partial y'}\eta'(x) = \left(\frac{\partial f}{\partial y'}\eta(x)\right)_{x_1}^{x_2} - \int \eta(x)\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)dx$$

$$\Rightarrow \left.\frac{\partial J}{\partial \alpha}\right|_{\alpha=0} = \int\left(\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right)\underline{\eta(x)}\,dx = 0$$