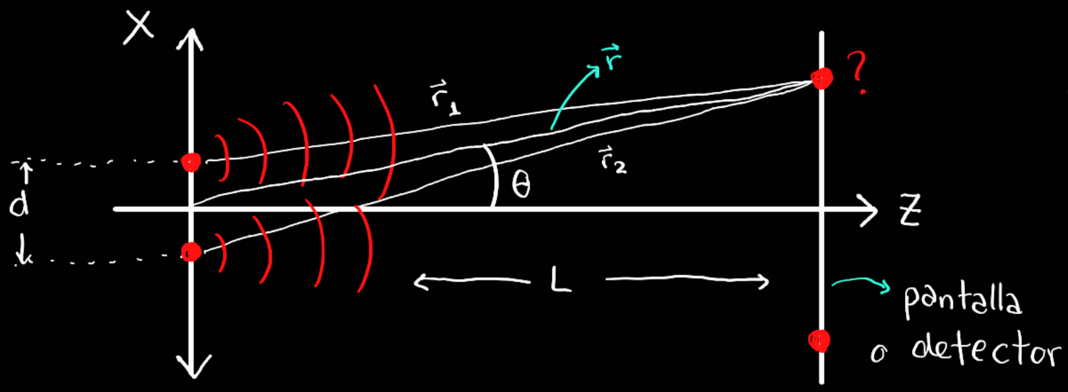
# INTERFERENCIA

(Problema de Young de las 2 rendijas)




$L \gg d \Rightarrow |\vec{r}| = r \gg d$

$$\Psi_1(\vec{r},t) = \frac{A}{r_1} e^{i(kr_1 - \omega t)}, \quad A \in \mathbb{R}$$

$$r_1 = \sqrt{r^2 + \frac{d^2}{4} - dr\sin\theta}$$

$$\Psi_2(\vec{r},t) = \frac{A}{r_2} e^{i(kr_2 - \omega t)}, \quad A \in \mathbb{R}$$

$$r_2 = \sqrt{r^2 + \frac{d^2}{4} + dr\sin\theta}$$

$$\Psi(\vec{r},t) = \Psi_1(\vec{r},t) + \Psi_2(\vec{r},t)$$

Como $r \gg d$, entonces $r_1 = \sqrt{r^2 + \frac{d^2}{4} - rd\sin\theta} = r\sqrt{1 + \cancel{\frac{d^2}{4r^2}} - \frac{d}{r}\sin\theta}$. Pues $d/r \ll 1$

Taylor 1er orden

Así pues, $r_1 \simeq r\left(1 - \frac{d}{r}\sin\theta\right)^{1/2} \Rightarrow r_1 \simeq r\left(1 - \frac{1}{2}\frac{d}{r}\sin\theta\right)$

$r_2 \simeq r\left(1 + \frac{d}{r}\sin\theta\right)^{1/2} \Rightarrow r_2 \simeq r\left(1 + \frac{1}{2}\frac{d}{r}\sin\theta\right)$

$$\therefore \quad \Psi(\vec{r},t) \simeq \frac{A}{r} e^{i(kr(1 - \frac{1}{2}\frac{d}{r}\sin\theta) - \omega t)} + \frac{A}{r} e^{i(kr(1 + \frac{1}{2}\frac{d}{r}\sin\theta) - \omega t)}$$

$$\Psi(r,\theta,t) \simeq \frac{A}{r} e^{-i\omega t}\left[e^{i(kr - \frac{kd}{2}\sin\theta)} + e^{i(kr + \frac{kd}{2}\sin\theta)}\right]$$

$$= \frac{A}{r} e^{i(kr - \omega t)}\left[e^{\frac{-ikd}{2}\sin\theta} + e^{\frac{ikd}{2}\sin\theta}\right]$$

$\cos\left(\frac{kd}{2}\sin\theta\right) = \cos\left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)$

$$\boxed{\Psi(r,\theta,t) \simeq \frac{2A}{r} e^{i(kr - \omega t)} \cos\left(\frac{kd}{2}\sin\theta\right) \quad r \gg d}$$

## Intensidad de una onda

$\Psi(\vec{r},t) \Rightarrow I(\vec{r},t) \equiv |\Psi(\vec{r},t)|^2$. De modo tal que,

$$\boxed{I(r,\theta,t) \cong 4\frac{A^2}{r^2}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}$$

Dado que $\cos^2\alpha = \frac{1}{2}(1 + \cos 2\alpha)$ se sigue

$$I(r,\theta,t) \simeq \frac{\overset{2}{\cancel{4}}A^2}{r^2}\frac{1}{\cancel{2}}\left[1-\cos\left(\frac{2\pi d}{\lambda}\sin\theta\right)\right] = \frac{A^2}{r^2}+\frac{A^2}{r^2}-\frac{2A^2}{r^2}\cos\left(\frac{2\pi d}{\lambda}\sin\theta\right)$$

$$\boxed{\begin{aligned}\psi_1(r,\theta,t) &\simeq \frac{A}{r}e^{i(kr-\omega t)}e^{-\frac{ikd}{2}\sin\theta}\\ \psi_2(r,\theta,t) &\simeq \frac{A}{r}e^{i(kr-\omega t)}e^{i(kr-\omega t)}\end{aligned}}$$

$$I(r,\theta,t) \simeq I_1(r,\theta) + I_2(r,\theta) \underbrace{-\,2\frac{A^2}{r^2}\cos\left(\frac{2\pi d}{\lambda}\sin\theta\right)}_{\text{Interferencia}}$$

$\frac{2\pi d}{\lambda}\sin\theta = 2n\pi \;;\quad n\in\mathbb{N}$ $\qquad \cos(2n\pi)=1 \;\}$ Interferencia constructiva

$$I(r,\theta) \simeq \frac{4A^2}{r^2} \simeq 4I_1 \simeq 4I_2$$

$\frac{\cancel{2\pi} d}{\lambda}\sin\theta = \cancel{2}n\cancel{\pi} \Rightarrow \boxed{d\sin\theta = n\lambda}$ $\quad$ si $n=0 \Rightarrow d\sin\theta = 0 \Rightarrow \theta = 0$

si $n=1 \Rightarrow d\sin\theta=\lambda \Rightarrow \sin\theta = \frac{\lambda}{d} \ll 1$

Si $\frac{\lambda}{d}\ll 1$ $\quad \exists n = 0, 1, 2, \dots, n_{máx} \Rightarrow d\sin\theta = n_{máx}\lambda$

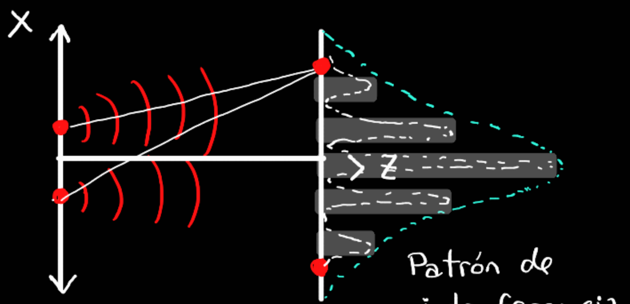
$n=0 \qquad \theta_0 = 0$

$n=1 \qquad \sin\theta_1 = \frac{\lambda}{d}$

$n=2 \qquad \sin\theta_2 = 2\frac{\lambda}{d}$

$\vdots$

$n=n_{máx} \qquad \sin\theta_{máx} = n_{máx}\frac{\lambda}{d}$




Patrón de interferencia

Ahora bien, si ocurre que $\frac{2\pi d}{\lambda}\sin\theta = (2n+1)\pi \qquad n = 0, 1, 2\dots$

$\Rightarrow \quad d\sin\theta = \frac{2n+1}{2}\lambda, \qquad \frac{d}{\lambda} \ll 1$

$\cos\left(\frac{2\pi d}{\lambda}\sin\theta\right) = -1 \Rightarrow I(r,\theta,t)=0$ ¡! $\;\}$ Interferencia destructiva

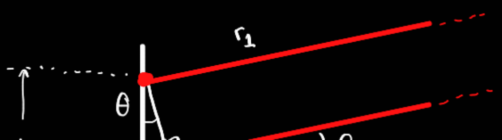
$\Delta = |r_1 - r_2|$

diferencia de camino óptico

En resumen:

$d\sin\theta \cong n\lambda$ $\quad$ constructiva $\quad$ si $|r_1-r_2| = \Delta = n\lambda$

$d\sin\theta \cong \frac{2n+1}{2}\lambda$ $\quad$ destructiva $\quad$ si $|r_1-r_2| = \Delta = \frac{2n+1}{2}\lambda$

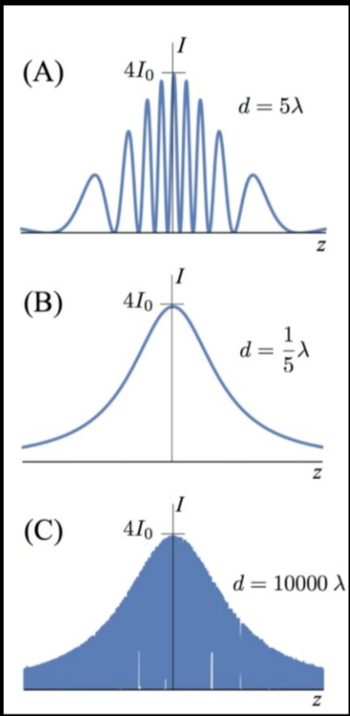



Separación entre dos máximos


$\Delta \simeq d \sin\theta$

$$\sin\theta_1 = \frac{\lambda}{d}$$

$$\sin\theta_0 = 0 \quad \Rightarrow \quad \theta_0 - \theta_1 = \arcsin\frac{\lambda}{d}$$




* $\frac{\lambda}{d} < 1$ garantiza $\sin\theta < 1$
* $\lambda \lesssim d$ $\quad \cos\left(\frac{2\pi d}{\lambda}\sin\theta\right) = \cos(2n\pi) = \cos((2n+1)\pi)$
* $\lambda \ll d$ $\quad \sin\theta_1 = \frac{\lambda}{d} \ll 1 \Rightarrow \theta_1 \approx \frac{\lambda}{d}$

Los detectores "promedian"

$$\frac{1}{2\pi}\int_0^{2\pi} \cos^2\alpha \, d\alpha = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}(1+\cos 2\alpha)\, d\alpha$$

$$= \frac{1}{4\pi}\int_0^{2\pi} (1 + \underbrace{\cancel{\cos 2\alpha}}_{0})\, d\alpha = \frac{1}{2}$$

Entonces, $\quad I = \frac{A^2}{r^2} + \frac{A^2}{r^2} + \frac{2A^2}{r^2}\cos\left(\frac{2\pi d}{\lambda}\sin\theta\right) = \frac{4A^2}{r^2}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$

$$\bar{I} = \frac{4A^2}{r^2}\underbrace{\cancel{\cos^2\left(\frac{2\pi d}{\lambda}\sin\theta\right)}}_{1/2}$$

La interferencia es $\lambda \lesssim d$

$$\bar{I} \approx \frac{2A^2}{r^2}, \qquad I \simeq I_1 + I_2$$