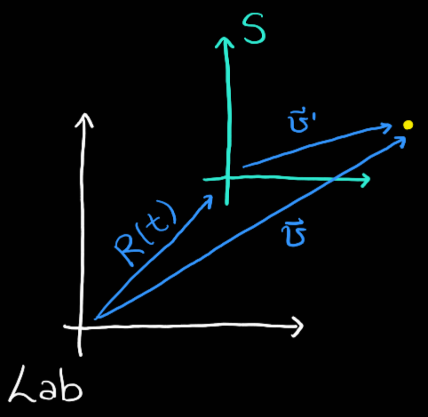
Sesión 14 (8 marzo 23): Repaso de ángulos de Euler.

10 marzo 23

(i) Ecuaciones de la Mec. en un sistema no inercial



$$\vec{v} = \vec{v}' + \vec{v}(t)$$

$$\mathcal{L} = \frac{1}{2}mv^2 - v(\vec{r})$$

$$\mathcal{L}' = \frac{1}{2}mv'^2 + m\vec{v}'\cdot\vec{v} + \cancel{\frac{1}{2}mv^2}^{\,0} - U(\vec{r})$$

pero $\frac{d\vec{r}'}{dt}\cdot\vec{v}(t) = \frac{d}{dt}\cancel{\{\vec{r}'\cdot\vec{v}(t)\}}^{\,0} - \vec{r}'\cdot\frac{d\vec{v}}{dt}$, entonces

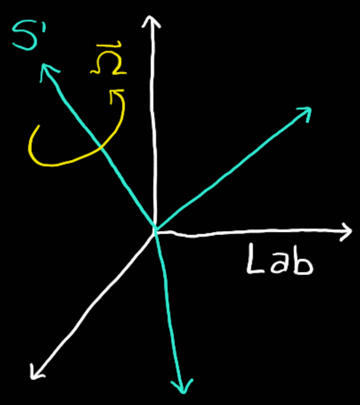
$$\mathcal{L}' = \frac{1}{2}mv'^2 - m\vec{r}'\cdot\frac{d\vec{v}}{dt} - U(\vec{r})$$

Por Euler-Lagrange se tiene que $\frac{d}{dt}\left(\frac{\partial L}{\partial \vec{v}}\right) - \frac{\partial L}{\partial \vec{r}'} = 0$, entonces

$$m\dot{\vec{v}} = -m\frac{d\vec{v}}{dt} + \frac{\partial U}{\partial \vec{r}} = \vec{F}^* + \vec{F}$$

fuerzas inerciales

$$m\frac{d}{dt}\left(\vec{v}' + v(t)\right) = -\frac{\partial U}{\partial r}$$



$$\vec{v} = \vec{v}' + \vec{\Omega}\times\vec{r}$$

$$\mathcal{L} = \frac{1}{2}mv^2 - U(\vec{r})$$

$$\mathcal{L}' = \frac{1}{2}mv'^2 + m\vec{v}'\cdot\vec{\Omega}\times\vec{r}' + \frac{1}{2}m(\Omega\times\vec{r})^2 - m\vec{r}\cdot\frac{d\vec{v}}{dt} - U(r)$$

$$d(\vec{\Omega}\times\vec{r}')^2 = 2(\vec{\Omega}\times\vec{r}')\cdot(\vec{\Omega}\times d\vec{r})$$

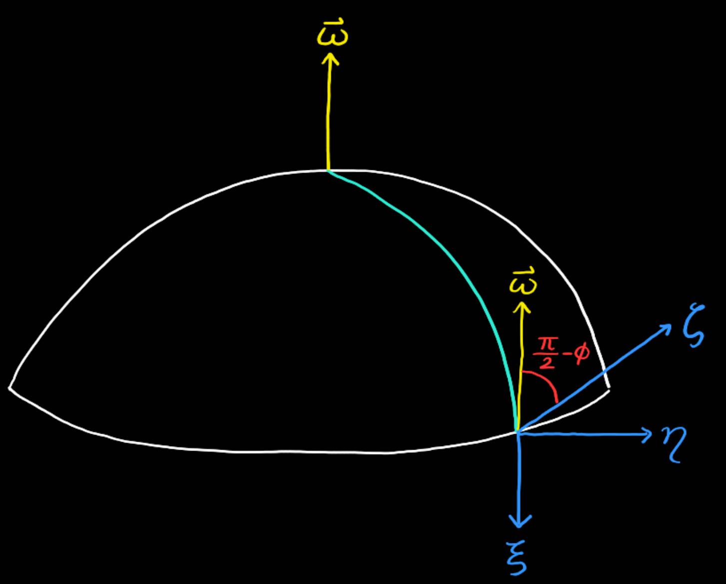
$$d(\varepsilon_{ijk}\Omega_i r_i)^2 = 2(\varepsilon_{ijk}\Omega_i r_j)(\varepsilon_{ijk}\Omega_i dr_j)$$

$$= 2(\vec{\Omega}\times\vec{r}')\cdot(\vec{\Omega}\times d\vec{r})$$

$$\Rightarrow d\mathcal{L}' = m\vec{v}'\cdot d\vec{v}' + m(\vec{\Omega}\times\vec{r}')\cdot d\vec{v}' + md\vec{r}'\cdot(\vec{v}'\times\vec{\Omega}) + m(\vec{\Omega}\times\vec{r}')\cdot(\vec{\Omega}\times d\vec{r}')$$
$$- m\frac{d\vec{v}'}{dt}\cdot d\vec{r}' - \left(\frac{\partial U}{\partial \vec{r}'}\right)\cdot d\vec{r}'$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \vec{v}'} = m\dot{\vec{v}}' + m\vec{\Omega}\times\vec{v}'; \qquad \frac{\partial \mathcal{L}}{\partial \vec{r}'} = m\vec{v}'\times\vec{\Omega} + m(\vec{\Omega}\times\vec{r}')\times\vec{\Omega} - m\frac{d\vec{v}'}{dt} - \frac{\partial U}{\partial \vec{r}'}$$

$$m\dot{\vec{v}}' = -\frac{\partial U}{\partial \vec{r}} - m\frac{d\vec{v}'(t)}{dt} \underbrace{- 2m\vec{v}'\times\vec{\Omega}}_{\text{Coriollis}} + \underbrace{m\vec{\Omega}\times(\vec{r}\times\vec{\Omega})}_{\text{Centrípeta}} + m\vec{r}\times\dot{\vec{\Omega}}$$



$$\vec{F} + \vec{C} = -m\vec{g}$$

Coriollis

$$m\frac{d\vec{v}}{dt} = -m\vec{g} + 2\vec{v}\times\vec{\omega}$$

$$\vec{\omega} = -\omega\cos\phi\,\hat{\xi} + \omega\sin\phi\,\hat{\eta}$$

$$\vec{v} = \frac{d\xi}{dt}\hat{\xi} + \frac{d\zeta}{dt}\hat{\zeta} + \frac{d\eta}{dt}\hat{\eta}$$

$$\frac{d^2\xi}{dt^2} = 2\omega\sin\phi\frac{d\eta}{dt}$$

$$\frac{d^2\eta}{dt^2} = -2\omega\sin\phi\frac{d\zeta}{dt} - 2\omega\cos\phi\frac{d\xi}{dt}$$

$$\Rightarrow \frac{d\xi}{dt} + gt = 2\omega\cos\phi - \eta; \quad \zeta(0) = k, \quad \eta(0) = 0, \quad \xi(0) = 0$$

$$\Rightarrow \frac{d^2\xi}{dt^2} + 4\omega^2\eta = ct, \quad c = 2\omega g\cos\phi$$

De donde $\eta = \frac{c}{4\omega^2}\left(t - \frac{\sin 2\omega t}{2\omega}\right) = \frac{g\cos\phi}{2\omega}\left(t - \frac{\sin 2\omega t}{2\omega}\right)$

Por lo tanto, $\eta \simeq \frac{gt^2}{3}\cos\phi\,\omega t$; $\xi \simeq \frac{gt^2}{6}\sin\phi(\omega t)^2$; $\zeta \simeq k - \frac{1}{2}gt^2\left(1 - \frac{\cos^2\phi}{3}(\omega t)^2\right)$