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Osciladores lineales

Sea $\mathcal{L} = \frac{1}{2} m_{\alpha\beta}(q)\, \dot{q}_\alpha \dot{q}_\beta - V(\{q\})$ tal que $V(q)$ tiene mínimo $\left.\frac{\partial V}{\partial q}\right|_{q=q_0} = 0.$

Tomaremos

$$q_i = q_0 + x_i, \quad m_{\alpha\beta} = M_{\alpha\beta} + (\quad) x_i, \quad V(q) = V(q_0) + \left(\frac{1}{2}\frac{\partial^2 V}{\partial q^2}\right) x^2$$

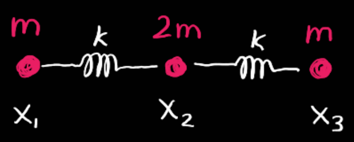
En general, para $\alpha, \beta = 1, 2$

$$\mathcal{L} = \sum_{\alpha=1}^{n} \left[\frac{1}{2} m_{ij}^{\alpha}(q^\alpha)\, \dot{q}_i^\alpha \dot{q}_j^\alpha\right] - V(q)$$

aquí $V(q) = \frac{1}{2} \left.\frac{\partial V}{\partial q_\alpha \partial q_\beta}\right|_{q=q_0} x_i x_j$. De modo tal que

$$\mathcal{L} = \sum_\alpha \left[\frac{1}{2} M_{ij}\, \dot{x}_i \dot{x}_j - \frac{1}{2} K_{ij}^\alpha x_i x_j\right]$$

dada $\ddot{x} = -\omega^2 x$, entonces $\ddot{x}_i = -\Lambda_{ij} \cdot x \Rightarrow \ddot{\underline{x}} = -\mathbb{\Lambda} \cdot x$

Ejemplo: 

$$\mathcal{L} = \frac{1}{2} m (\dot{x}_1 + 2\dot{x}_2 + \dot{x}_3) - \frac{1}{2} k (x_2 - x_1)^2 - \frac{1}{2} k (x_3 - x_2)^2$$

$$m\ddot{x}_1 = k(x_2 - x_1) \quad \Rightarrow \quad \ddot{x}_1 = \omega^2 (x_2 - x_1)$$

$$2m\ddot{x}_2 = -k(x_2 - x_1) + k(x_3 - x_2) \quad \Rightarrow \quad \ddot{x}_2 = -\frac{1}{2}\omega^2 (x_2 - x_1) + \frac{1}{2}\omega^2 (x_3 - x_2)$$

$$m\ddot{x}_3 = -k(x_3 - x_2) \quad \Rightarrow \quad \ddot{x}_3 = -\omega^2 (x_3 - x_2)$$

$$\Rightarrow \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = \omega^2 \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

diagonalizando se obtiene $\lambda_1 = -2$, $\lambda_2 = -1$ y $\lambda_3 = 0$.