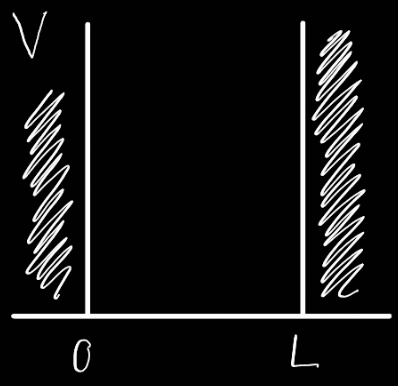
Partícula en una caja



$$H = \frac{\hat{p}^2}{2m} + V(x)$$

$$\hat{H}|\phi_n\rangle = E_n|\phi_n\rangle$$

$$E_n = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n^2 , \quad n = 2, \dots, \infty$$

$$\phi_n(x) = \langle x|\phi_n\rangle = \begin{cases} \frac{2}{L}\sin\left(\frac{n\pi x}{L}\right) & 0 \le x \le L \\ 0 & x \le 0, \quad x \ge L \end{cases}$$

$|\psi\rangle$ arbitraria $\Rightarrow \psi(x) = \langle x|\psi\rangle, \quad \langle\psi|\psi\rangle = 1$

$$\psi(x) = \sum_{n=1}^{\infty}\langle x|\phi_n\rangle\langle\phi_n|\psi\rangle = \sum_{n=1}^{\infty}\phi_n(x)a_n = \sum_{n=1}^{\infty}a_n\phi_n(x)$$

$a_n = \langle\phi_n|\psi\rangle$ ¿?

$$a_n = \int_{-\infty}^{\infty}\langle\phi_n|x\rangle\langle x|\psi\rangle dx$$

$$= \int_{-\infty}^{\infty}\phi_n^*(x)\psi(x)\,dx = \int_0^L \phi_n^*(x)\psi(x)dx$$

$|a_n|^2$: prob. de hallar el eigenvalor del sistema en $|\psi\rangle$ en de la enegía $\psi(x)$

$$\langle\psi|\psi\rangle = \int_{-\infty}^{\infty}\langle\psi|x\rangle\langle x|\psi\rangle dx = \int_{-\infty}^{\infty}|\psi(x)|^2 dx = 1$$

$$= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\int_{-\infty}^{\infty} a_m^*\phi_m^*(x)a_n\phi_n(x)\,dx$$

$$= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}a_m^* a_n \int_{-\infty}^{\infty}\underbrace{\phi_m^*(x)\phi_n(x)dx}_{\delta_{mn}}$$

$$= \sum_{n=1}^{\infty}a_n^* a_n = 1$$

$|a_n|^2 \longrightarrow$ probabilidad

$t=0 \quad |\psi\rangle \longrightarrow |\psi(t)\rangle = $ ¿?

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

$$|\psi\rangle = \sum_{n=1}^{\infty}a_n|\phi_n\rangle, \qquad a_n = \langle\phi_n|\psi\rangle$$

$$|\psi(t)\rangle = \sum_{n=1}^{\infty}a_n e^{-iE_nt/\hbar}|\phi_n\rangle$$

$$\langle\psi(t)|\hat{A}(\hat{x},\hat{p})|\psi(t)\rangle = \checkmark$$

¡cambia en el tiempo!

$$\langle\psi(t)|\hat{p}|\psi(t)\rangle = \int_{-\infty}^{\infty}\psi^*(x,t)\frac{\hbar}{i}\frac{\partial}{\partial x}\psi(x,t)\,dx$$

$$\psi(x,t) = \sum_{n=1}^{\infty}a_n e^{-iE_nt/\hbar}\phi_n(x)$$

Sea $t=0$ y $|\psi\rangle = |\phi_\ell\rangle$ donde ℓ dado $(\ell =$

$$\Rightarrow \begin{matrix} a_\ell = 1 \\ a_n = 0 \quad \forall n \neq \ell \end{matrix} \quad \Rightarrow |\psi(t)\rangle = e^{-iE_\ell t/\hbar}|\phi_\ell\rangle$$

$$\langle \psi(t)|\, \hat{A}\,|\psi(t)\,|\,\psi(t)\rangle = e^{iE_\ell t/\hbar}\langle \phi_\ell|\hat{A}|\phi_\ell\rangle\, e^{-iE_\ell t/\hbar} = \langle \phi_\ell|\hat{A}|\phi_\ell\rangle$$

donde $|\phi_\ell\rangle$ son los ESTADOS ESTACIONARIOS.

$$\hat{H}_{real} = \hat{H} + \cancel{\hat{H}_{rad}} + \cancel{\hat{H}_{at\text{-}rad}}$$

Dado un edo. arbitrario $|\psi(t)\rangle = \sum_{n=1}^{\infty} \overbrace{(a_n e^{-iE_n t/\hbar})}^{|a_n e^{-iE_n t/\hbar}|^2 = |a_n|^2} |\phi_n\rangle$

$\hat{A}|x_n\rangle = \alpha_n |x_n\rangle$, entonces

$$|\langle x_k|\psi(t)\rangle|^2 = \left|\sum_{n=1}^{\infty} a_m e^{-iE_n t/\hbar}\langle x_k|\phi_n\rangle\right|^2$$

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_m^* e^{iE_m t/\hbar} a_n e^{-iE_n t/\hbar}$$

$\langle \phi_m|x_k\rangle\langle x_k|\phi_n\rangle$ cambia en el tiempo

Ejemplo: $f(x) = A\cos(\alpha\pi x)$, $0 \le x \le L$

$$f(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right), \quad a_n = \int_0^L \phi_n^*(x) f(x)\,dx$$

...¡Métalo a Mathematica!

$$|\phi_n\rangle = \int_{-\infty}^{\infty} |p\rangle\langle p|\phi_n\rangle\,dp$$

$\langle p|x\rangle$

$$\int_0^L \frac{1}{\sqrt{2\pi\hbar}} e^{-ipx/\hbar}\phi_n(x)\,dx$$

$$= \frac{1}{2}\sqrt{\frac{L}{\pi\hbar}}\left[(-1)^n e^{-ipL/\hbar} - 1\right]\left[\frac{1}{p\frac{L}{\hbar} - n\hbar} - \frac{1}{p\frac{L}{\hbar} + n\hbar}\right]$$

$$\begin{matrix}\langle p|\phi_n\rangle = \tilde{\phi}_n(p) \\ \langle x|\phi_n\rangle = \phi_n(x)\end{matrix} \quad \Rightarrow \langle x|\phi_n\rangle = \int_{-\infty}^{\infty}\langle x|p\rangle\langle \phi|\phi_n\rangle\,dp$$

$$\phi_n(x) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{ipx/\hbar}\,\tilde{\phi}_n(p)\,dp$$