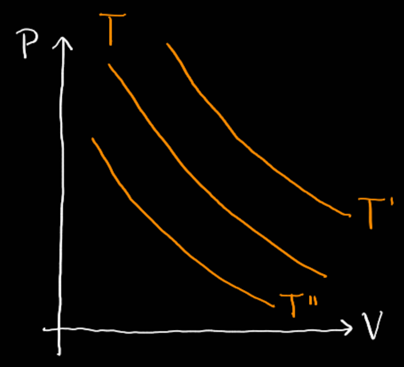
22 feb 23

Procesos (expansión o compresión)

Isotérmico $T = cte$
Adiabático $đQ = 0$

$\Big\}$ quasiestático

Ejemplo gas ideal



$T' > T > T''$

$$p = \frac{N}{V} kT$$

$$p = \frac{cte}{V} \Rightarrow pV = cte$$

$$\left(\frac{\partial p}{\partial V}\right)_{N,T} < 0, \quad \text{gral}$$

Adiabático $N = cte$

$$V^{\gamma-1} T = cte$$

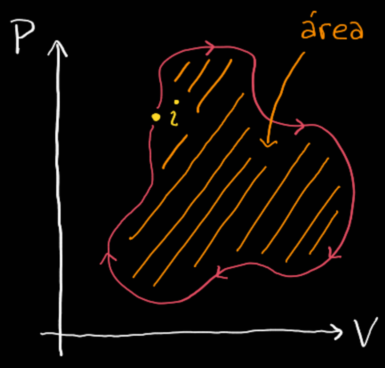
$$\gamma = \frac{C_p}{C_V} > 1$$

Si V disminuye (compresión), T aumenta
Si V aumenta (expansión), T disminuye

$$\Rightarrow V^{\gamma-1}\left(\frac{pV}{NK}\right) = cte \Rightarrow V^{\gamma-1} pV = cte \Rightarrow \boxed{pV^{\gamma} = cte}$$

Ciclo de Carnot
Sadi

Ciclo
($N = cte$)
"pistón"



Variable de estado

$$i = \underbrace{p_i, V_i, E_i, T_i, \ldots}$$

$$\int_1^2 dA = A_2 - A_1$$

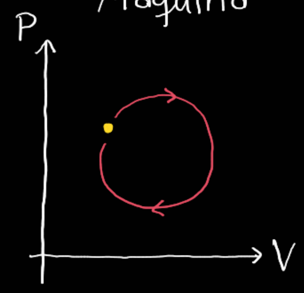
Dado el ciclo en cuestión tenemos que

$\Delta A = 0, \quad \forall A$ variable de estado

$\Delta E = 0, \quad$ como $\Delta E = W + Q \Rightarrow Q = -W$

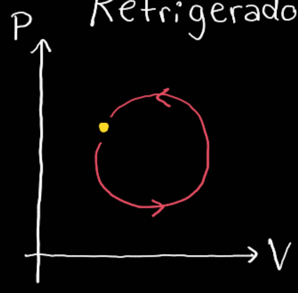
Sabemos que $W = -\oint_C p\,dV = -$(Área dentro del ciclo).

Máquina



$\oint p\,dV > 0$
$W < 0$
$Q > 0$

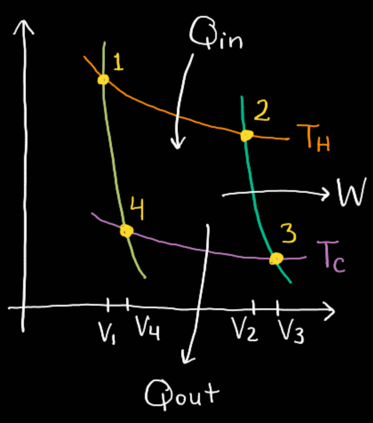
Refrigerador



$\oint p\,dV < 0$
$W > 0$
$Q < 0$

Potencia del ciclo $= \dfrac{|W|}{t} \to 0$ si $t \to \infty$ (¡esto no ocurre en la vida real!)

Ciclo de un gas (cualquiera) $N = cte$



$1 \to 2$: expansión isotérmica a T_H
$2 \to 3$: expansión adiabática $T_H \to T_C$, $T_H > T_C$
$3 \to 4$: compresión isotérmica a T_C
$4 \to 1$: compresión adiabática $T_C \to T_H$

$$\Delta E_T = \Delta E_{12} + \Delta E_{23} + \Delta E_{34} + \Delta E_{41}, \quad \Delta E_{ij} = E_j - E_i$$
$$= \cdots = 0$$

Tenemos que

1→2: $W_{12}<0 \quad \boxed{Q_{in}>0}$ *

2→3: $W_{23}<0 \quad Q_{23}=0$

3→4: $W_{34}>0 \quad \boxed{Q_{out}<0}$ *

4→1: $W_{41}>0 \quad Q_{41}=0$

*Hipótesis
↳ después veremos por qué es así

$$\Delta E_T = W_T + Q_T = 0, \quad Q_T = -W_T$$

$$W_T = W_{12}+W_{23}+W_{34}+W_{41} < 0$$

$$Q_T = Q_{in} + Q_{out} > 0$$

$$= Q_{in} - |Q_{out}| > 0 \quad \therefore Q_{in} > |Q_{out}|$$

$$Q_T = -W_T \Rightarrow Q_{in} - |Q_{out}| = -W_T$$

$$\underbrace{Q_{in}}_{\text{entrada energía}} = \underbrace{|Q_{out}|}_{\text{calor "PERDIDO"}} + \underbrace{|W_T|}_{\text{trabajo ÚTIL}}$$

Eficiencia (del ciclo)

$$\eta = \frac{|W_T|}{Q_{in}} \leq 1 \quad \text{1ra Ley} \quad (\eta < 1 \text{ por 1° y 2° Ley})$$

$$\eta = \frac{Q_{in} - |Q_{out}|}{Q_{in}}$$

Gas ideal

$$\boxed{E = C_V T, \quad p = \frac{N}{V}kT, \quad C_V > 0}$$

1→2: $T_H = cte, \quad \Delta E_{12} = 0, \quad Q_{in} = -W_{12}$

$$W_{12} = -\int_{V_1}^{V_2} p\,dV = -NkT_H \int_{V_1}^{V_2} p\,dV = -NkT_H \ln\left(\frac{V_2}{V_1}\right) < 0$$

$$Q_{in} = -W_{12} = NkT_H \ln\left(\frac{V_2}{V_1}\right) > 0$$

2→3: $T_H \to T_C, \quad \Delta E_{23} = E_3 - E_2 = C_V(T_C - T_H)$

$$W_{23} = \Delta E_{23} < 0, \quad Q_{23} = 0$$

Como es adiabático $V_2^{\gamma-1} T_H = V_3^{\gamma-1} T_C$

3→4: $T_C = cte, \quad \Delta E_{34} = 0,$

$$W_{34} = -NkT_C \ln\left(\frac{V_4}{V_3}\right) = NkT_C \ln\left(\frac{V_3}{V_4}\right) > 0$$

$$Q_{out} = -W_{34} = -NkT_C \ln\left(\frac{V_3}{V_4}\right) < 0$$

4→1: $T_C \to T_H, \quad Q_{41} = 0$

$$\Delta E_{41} = C_V(T_H - T_C), \quad W_{41} = C_V(T_H - T_C)$$

$$V_4^{\gamma-1} T_C = V_1^{\gamma-1} T_H$$

Por otra parte note que

(*) $\Delta E_T = \Delta E_{12} + \Delta E_{23} + \Delta E_{34} + \Delta E_{41} = C_V(T_C - T_H) + C_V(T_H - T_C) = 0$

(*) $W_T = W_{12} + W_{23} + W_{34} + W_{41}$

$$= -NkT_H \ln\left(\frac{V_2}{V_1}\right) + \cancel{C_V(T_C - T_H)} + NkT_C \ln\left(\frac{V_3}{V_4}\right) + \cancel{C_V(T_H - T_C)}$$

Pero, $V_2^{\gamma-1} T_H = V_3^{\gamma-1} T_C$ y $V_1^{\gamma-1} T_H = V_4^{\gamma-1} T_C$ y por tanto

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

entonces $\boxed{W_T = -Nk(T_H - T_C)\ln\left(\frac{V_2}{V_1}\right) < 0}$. Consecuentemente,

$$Q_{in} = +NkT_H \ln\left(\frac{V_2}{V_1}\right) > 0$$

$$Q_{out} = -NkT_H \ln\left(\frac{V_3}{V_4}\right) = -NkT_H \ln\left(\frac{V_2}{V_1}\right) < 0$$

$$\Rightarrow Q_{in} + Q_{out} = Nk(T_H - T_C)\ln\left(\frac{V_2}{V_1}\right) = -W_T, \quad W_T < 0$$

Por lo cual, $\eta = \frac{|W_T|}{Q_{in}} = 1 - \frac{|Q_{out}|}{|Q_{in}|} = 1 - \frac{\cancel{Nk}\, T_c \cancel{\ln\left(\frac{V_2}{V_1}\right)}}{\cancel{Nk}\, T_H \cancel{\ln\left(\frac{V_2}{V_1}\right)}} \Rightarrow \boxed{\eta = 1 - \frac{T_C}{T_H}}$ ... gas ideal

Note que $\eta = 1$ si $T_C = 0$ ¿? ... pero de acuerdo con la 3ª Ley $T = 0$ es inalcanzable