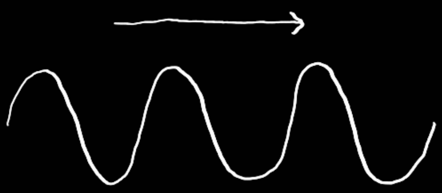
Ondas (Lineales)



(i) sonido

(ii) cuerda

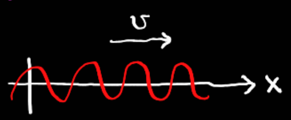
(iii) olas

(iv) EM

(v) Gravitacionales

ψ amplitud de onda

Ondas no dispersivas

1D $\quad \psi(x,t)$ 

Todo onda de este tipo obedece:

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0}$$

ecuación de onda

Para resolver la ec. de onda proponemos: $\psi(x,t) = f(x - vt) = f(z)$

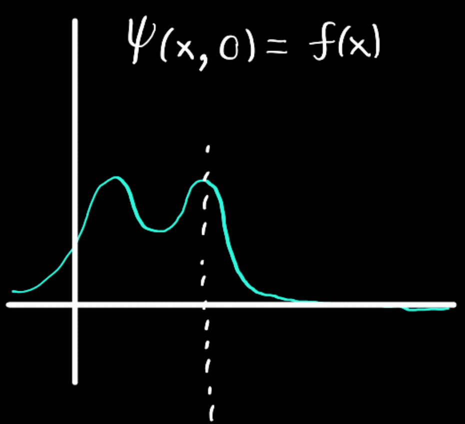
$$\Rightarrow \frac{\partial f}{\partial x} = \frac{df}{dz}\frac{\partial z}{\partial x} = \frac{df}{dz} \qquad \frac{\partial \psi}{\partial t} = \frac{df}{\partial z}\frac{\partial z}{\partial t} = -v\frac{df}{dz}$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x}\frac{df}{dz} = \frac{d^2 f}{dz}\frac{\partial z}{\partial x} = \frac{d^2 f}{dz^2}$$

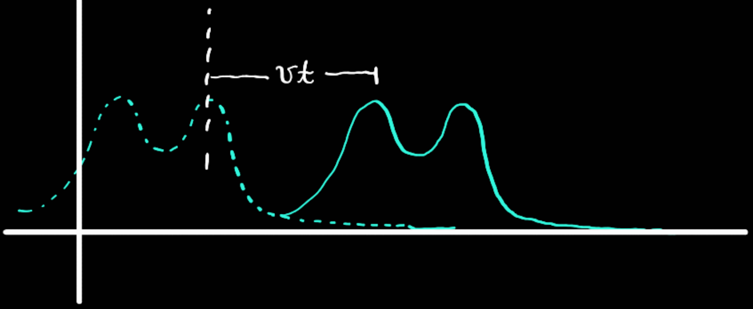
$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial t}\left(-v\frac{df}{dz}\right) = -v\frac{\partial}{\partial t}\frac{df}{dz} = -v\frac{d^2 f}{dz}(-v) = v^2\frac{d^2 t}{dz^2}$$

$$\Rightarrow \begin{array}{l} \dfrac{\partial^2 \psi}{\partial x^2} = \dfrac{d^2}{dz^2} f \\[2ex] \dfrac{\partial^2 \psi}{\partial x^2} = v^2 \dfrac{d^2}{dz^2} f \end{array} \Rightarrow \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

$\therefore \psi(x,t) = f(x - vt)$ es solución



$\psi(x,0) = f(x - vt)$



Proponemos otra solución: $\psi(x,t) = \psi_0 \, e^{i(kx-\omega t)}$

$$\frac{\partial \psi}{\partial x} = \psi_0 \, e^{i(kx-\omega t)} ik$$

$$\frac{\partial^2 \psi}{\partial x^2} = \psi_0 \, e^{i(kx-\omega t)} (ik)(ik) = -k^2 \psi_0 e^{i(kx-\omega t)} = -k^2 \psi(x,t)$$

$$\frac{\partial \psi}{\partial t} = \psi_0 \, e^{i(kx-\omega t)} (-i\omega)$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0 \, e^{i(kx-\omega t)} (-i\omega)^2 = -\omega^2 \psi(x,t)$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 t}{\partial t^2} = 0 \Rightarrow -k^2 \psi(x,t) + \frac{\omega^2}{v^2}\psi(x,t) = 0$$

$$\Leftrightarrow k^2 = \frac{\omega^2}{v^2} \qquad (\text{si } \psi_0 \neq 0)$$

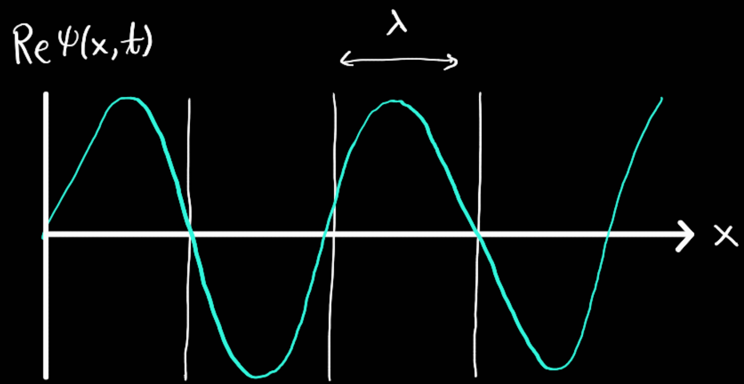
$$\Leftrightarrow \omega = kv$$

$$\psi(x,t) = \psi_0 \, e^{i(kx-\omega t)}$$

$$= \psi_0 \, e^{ik(x - \frac{\omega}{k} t)} \qquad \psi_0 \in \mathbb{C}$$

$$= \psi_0 \, e^{ik(x - vt)} \qquad 0 \leq \phi_0 < 2\pi$$

$$\psi(x,t) = |\psi_0| \, e^{i(kx - \omega t + \phi_0)}$$



$\therefore e^{i\lambda x} = 1$

k: # de onda o vector de onda

τ: periodo; tiempo en que se repite

$\lambda = \frac{2\pi}{k}$ long. de onda

$\tau = \frac{2\pi}{\omega}$ periodo onda

$\psi(x,t) = \psi_0 \, e^{i(hx - \omega t)}$; $h \in \mathbb{R}$,

$\omega = kv$

$\nu = \frac{1}{\tau}$

$\omega = 2\pi\nu$

$\Rightarrow v = \frac{\lambda}{\tau}$

Sea $\Psi_1(x,t) = \Psi_{01}\, e^{i(k_1 x - \omega_1 t)}$ es onda $\Leftrightarrow$ $\omega_1 = k_1 v$

$k_1 \neq k_2$

$\Psi_2(x,t) = \Psi_{02}\, e^{i(k_2 x - \omega_2 t)}$ es onda $\Leftrightarrow$ $\omega_2 = k_2 v$

$\chi(x,t) = a\,\Psi_1(x,t) + b\,\Psi_2(x,t)$; $a, b \in \mathbb{C}$ es solución

$$\chi(x,t) = \sum_m \Psi_{0m}\, e^{i(k_m x - \omega_m t)}$$

las ondas no se dispersan pues la velocidad es la misma para todas las ondas, es decir $\omega = k v$