Principio D'Alambert

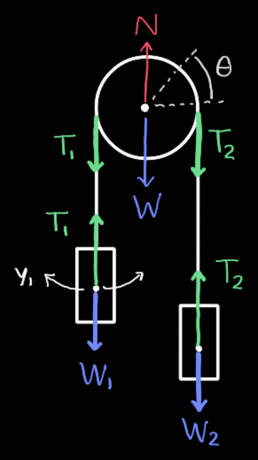
$$\boxed{\sum_{k=1}^{3}\left\{X_k - m_k\ddot{x}_k + \sum_i \frac{\partial F_i}{\partial x_k}\lambda_i\right\}\delta x_k = 0} \qquad (1)$$

El número de grados de libertad es $f = 3n - r$.

Si hay reestricciones geométricas se tendrá $\boxed{m_k\ddot{x}_k = X_k + \sum_i \frac{\partial F_i}{\partial x_k}\lambda_i}$ (2)

Por otro lado, si lo hacemos para torcas dado $\vec{\tau} = I\vec{\alpha}$, entonces

$$\boxed{I_k\alpha_k = I_k\ddot{\theta}_k = \tau_k + \sum \frac{\partial F_i}{\partial \theta_k}\lambda_i} \qquad (3)$$



Restricciones geométricas $r=2$

(i) $-\dot{y}_1 = a\dot{\theta} \Rightarrow dy_1 + a\,d\theta = 0 = dF_1$

(ii) $\dot{y}_2 = a\dot{\theta} \Rightarrow dy_2 - a\,d\theta = 0 = dF_2$

$\lambda_2 = -T_2$

Grados de libertad $3n = 3$

$$\left.\begin{aligned} m_1\ddot{y}_1 &= -W_1 + \lambda_1 \\ m_2\ddot{y}_2 &= -W_2 + \lambda_2 \end{aligned}\right\} \text{ vea } (2)\ldots \left(\text{aquí } \sum \frac{\partial F_i}{\partial x_k}\lambda_i = \lambda_i\right)$$

$$I\ddot{\theta} = a\lambda_1 + \lambda_2 a \} \text{ vea } (3)\cdots \left(\text{aquí } \sum \frac{\partial F_i}{\partial \theta_k}\lambda_i = a(\lambda_1 + \lambda_2)\right)$$

La Energía

Multiplicamos (2) por $\dot{x}_k dt \Rightarrow dt\sum_k m_k\dot{x}_k\ddot{x}_k = dT$

$$\Rightarrow \boxed{dt\sum_k d\left(\frac{1}{2}m_k\dot{x}_k^2\right) = dT}$$

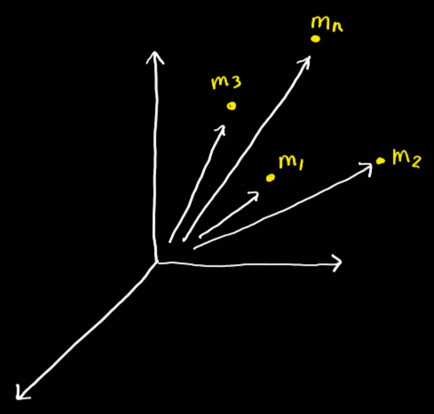
$$\Rightarrow \underbrace{dt\,\dot{x}_k}_{dx_k} X_k + \sum_i dt\,\dot{x}_k\frac{\partial F_i}{\partial x_k}\lambda_i = X_k\,dx_k + \sum \frac{\partial F_i}{\partial x_k}dx_k\,\lambda_i$$

Si depende de t la restricción geométrica tenemos

$$dF_i = \sum_{k=1}^{3n}\frac{\partial F_i}{\partial x_k}dx_k + \frac{\partial F_i}{\partial t}dt$$

$$dT = dW - \frac{\partial F_i}{\partial t}dt$$

Ejercicio: Considere un sistema de partículas



$$\sum_k\left(-\dot{\vec{P}}_k + \vec{F}_k^{ext} + \sum^{3}\vec{F}_{ik}\right)\cdot\delta S_k = 0$$

$$\sum_k\dot{\vec{P}}_k = \sum_k\vec{F}_k^{ext} \Rightarrow \dot{\vec{P}} = \vec{F}^{ext}$$

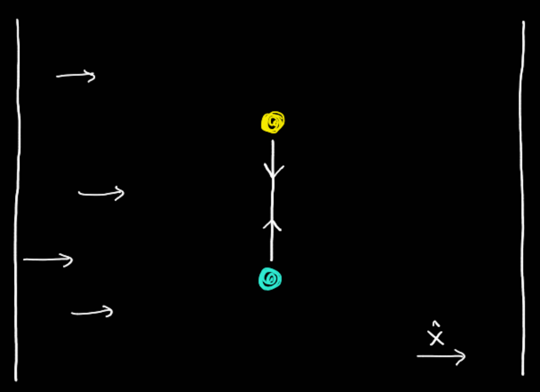
Donde $\sum\vec{P}_k = \sum m_k\vec{v}_k = M\vec{v}$ velocidad del centro de masa

$$\Rightarrow \vec{v} = \frac{1}{M}\sum m_k\vec{v}_k$$

$$\Rightarrow \vec{R}_{CM} = \frac{1}{M}\sum m_k\vec{r}_k$$

centro de masa

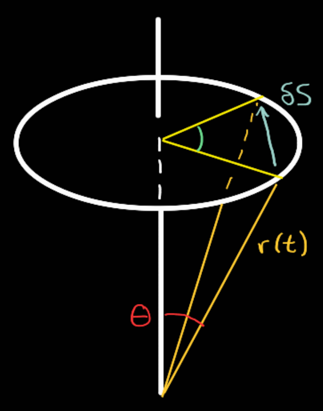
Problema. Tenemos una carga neutra y una con carga tal que están conectadas entre sí. Una corriente pasa de modo que la que está cargada se desplaza.



¿Cómo se mueve el centro de masa?

R: Se mueve en dirección $\hat{x}$ $\longrightarrow$

Problema. Suponga se tiene una partícula rotando de forma circular



$$|\delta\vec{S}| = r\sin\theta\,\delta\phi$$

$$\delta\vec{s} = \delta\vec{\phi}\times\vec{r}$$

$$\sum_k\left(F_k^{ext} + \sum_i \vec{F}_{ik} - \dot{\vec{p}}_k\right)\cdot\left(\delta\vec{\phi}_k\times\vec{r}\right) = 0 \qquad (4)$$

Considere que $\vec{A}\cdot\vec{C}\times\vec{B} = \vec{B}\cdot\vec{A}\times\vec{C} = \vec{C}\cdot\vec{A}\times\vec{B}$

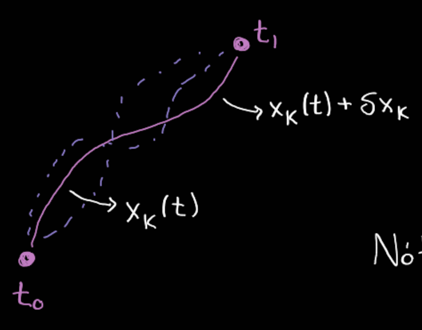
$$\dot{L}_k \rightsquigarrow \frac{dL}{dt} = \tau_{ext}$$

$$(4) \Rightarrow \delta\vec{\phi}\cdot\sum_k\left[\overbrace{(\vec{r}_k\times\vec{F}_k^{ext}}^{\tau_k} + \sum_i(\vec{r}_k\times\vec{F}_{ik}) - \overbrace{(\vec{r}_k\times\dot{\vec{p}}_k)}^{\dot{L}_k}\right] = 0$$

$$\Rightarrow \frac{1}{2}\sum_i\sum_k(\vec{r}_k\times\vec{F}_{ik}) - (\vec{r}_i\times\vec{F}_{ik})$$

$$\Rightarrow \frac{1}{2}\sum((\vec{r}_k-\vec{r}_i)\times\vec{F}_{ik}$$

Principio de Hamilton



$$\sum_{k=1}^{n}\Big\{(m_k\ddot{x}_k - X_k)\,\delta x_k + (m_k\ddot{y}_k - Y_k)\,\delta y_k + (m_k\ddot{z}_k - Z_k)\,\delta z_k\Big\} = 0$$

Nótese $\ddot{x}_k\,\delta x_k = \frac{d}{dt}(\dot{x}_k\,\delta x_k) - \dot{x}_k\,\delta\dot{x}_k$ donde $\frac{d}{dt}(\delta x_k) = \delta\dot{x}_k$

$$= \frac{d}{dt}(\dot{x}_k\,\delta x_k) - \delta\left(\tfrac{1}{2}\dot{x}_k^2\right)$$

Por lo cual,

$$\frac{d}{dt}\sum_k m_k\left(\dot{x}_k\,\delta x_k + \dot{y}_k\,\delta y_k + \dot{z}\,\delta z_k\right) = \sum_k \overbrace{\tfrac{1}{2}m_k\,\delta\left(\dot{x}_k^2 + \dot{y}_k^2 + \dot{z}_k^2\right)}^{\delta T_k} + \sum_k\left(X_k\,\delta x_k + Y_k\,\delta y_k + Z_k\,\delta z_k\right)$$

$$= \delta T + \delta W$$

$$\sum_k m_k\left(\dot{x}_k\,\delta x_k + \dot{y}_k\,\delta y_k + \dot{z}_k\,\delta z_k\right)\Big|_i^f = \int \delta T + \delta W\,dt \Rightarrow \int(\delta T + \delta W)\,dt = 0$$