Formal Methods for High-Assurance Software Engineering

HomeWork Assignment 04

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September 2020

Problem 1. From Lecture Slides 11, entitled Quantified Boolean Formulas (QBF's): **part 1** Do the exercise on page 31.

Solution. We prove these equivalencies semantically, rather than providing a theoretical proof. Starting with the case that all wff's are quantifier-free.

$\Phi = (\phi \vee \psi_1) \wedge (\phi \vee \psi_2) \wedge (\phi \vee \psi_3)$
$\Psi = \exists y.(y \leftrightarrow \phi) \wedge (y \vee \psi_1) \wedge (y \vee \psi_2) \wedge (y \vee \psi_3)$

In order to show $\Phi \equiv \Psi$ it is sufficient to show that $\Phi \Leftrightarrow \Psi$, semantically speaking, we want to show that this wff is a tautology and the last column of it's truth table is all True

truth-table for WFF $(\Phi \Rightarrow \Psi) \wedge (\Psi \Rightarrow \Phi)$

ϕ	ψ_1	ψ_2	ψ_3	$[y=0]$	$[y=1]$	$\phi \vee \psi_1$	$\phi \vee \psi_2$	$\phi \vee \psi_3$	Φ	$[y=0] \leftrightarrow \phi$	$[y=1] \leftrightarrow \phi$	$[y=0] \vee \psi_1$	$[y=0] \vee \psi_2$	$[y=0] \vee \psi_3$	$[y=1] \vee \psi_1$	$[y=1] \vee \psi_2$	$[y=1] \vee$
F	T	F	F	F	F	T	F	F	F	F	F	F	F	F	F	F	T
F	T	F	F	F	F	T	F	F	F	F	F	F	F	F	F	F	T
F	T	F	F	T	F	T	F	F	F	F	F	T	T	F	F	F	T
F	T	F	F	T	F	T	F	F	F	F	F	T	T	F	F	F	T
F	T	F	T	F	F	T	F	F	F	T	T	F	F	F	F	F	T
F	T	F	T	F	F	T	F	F	F	T	T	F	F	F	F	F	T
F	T	F	T	T	F	T	F	F	F	T	T	T	T	F	F	F	T
F	T	F	T	T	F	T	F	F	F	T	T	T	T	F	F	F	T
F	T	T	F	F	F	T	F	T	T	F	F	F	F	F	F	F	T
F	T	T	F	F	F	T	F	T	T	F	F	F	F	F	F	F	T
F	T	T	F	T	F	T	F	T	T	F	F	T	T	F	F	F	T
F	T	T	F	T	F	T	F	T	T	F	F	T	T	F	F	F	T
F	T	T	T	F	F	T	F	T	T	T	T	F	F	F	F	F	T
F	T	T	T	F	F	T	F	T	T	T	T	F	F	F	F	F	T
F	T	T	T	T	T	T	F	T	T	T	T	T	T	T	F	T	T
F	T	T	T	T	T	T	F	T	T	T	T	T	T	T	F	T	T

part 2 Do the exercise on page 32.

Problem 3. LCS 2.3.3 on page 160
Write down a sentence of predicate logic which intuitively holds in a model iff the model has (respectively):
part a exactly three distinct elements

Problem 3. part b

Problem 3. part c

Problem 3.

Problem 4. LCS 2.3.9 on page 161
Prove the validity of the following sequents in predicate logic, where F , G, P , and Q have arity 1, and S has arity 0 (a 'propositional atom'):

part a

Solution. hws
part b

Solution. hws 5
part c

Solution. khodam
part d

Solution. sm p41

Problem 5.

Solution. `https://github.com/ro0zkhosh/CS511/blob/master/HW4/shahin_streamroller.py`

Problem 6.

Solution. `https://github.com/ro0zkhosh/CS511/blob/master/HW4/shahin_whokilledaunty.in`