CS 511, Fall 2020, Handout 03 (Part A)

Semantics of Propositional Logic

Assaf Kfoury

September 03, 2020

some familiar truth-tables:

logical "or" ($\vee$) and logical "and" ($\wedge$)

x	y	$x \vee y$
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

x	y	$x \wedge y$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

logical "implication" ($\rightarrow$)

x	y	$x \rightarrow y$
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

and similarly for "negation" ($\neg$) and many other logical connectives . . . .

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y
T	**T**
T	**F**
F	**T**
F	**F**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$
T	**T**	**F**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$
T	**T**	**F**	**F**
T	**F**	**F**	**T**
F	**T**	**T**	**F**
F	**F**	**T**	**T**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$	$x \to \neg y$
T	**T**	**F**	**F**	**F**
T	**F**	**F**	**T**	**T**
F	**T**	**T**	**F**	**T**
F	**F**	**T**	**T**	**T**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$	$x \to \neg y$	$y \vee \neg x$
T	**T**	**F**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**
F	**T**	**T**	**F**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$	$x \to \neg y$	$y \vee \neg x$	$(x \to \neg y) \to (y \vee \neg x)$
T	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \overset{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$	$x \to \neg y$	$y \vee \neg x$	$(x \to \neg y) \to (y \vee \neg x)$
T	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**

- propositional wff φ is **satisfiable** if **there is** an assignment of truth-values to the propositional atoms which makes φ true.
- propositional wff φ is a **tautology** if **every** assignment of truth-values to the propositional atoms makes φ true.

from *propositional formulas* to *truth-tables*

consider propositional wff (**w**ell-**f**ormed **f**ormula): $\varphi \stackrel{\text{def}}{=} (x \to \neg y) \to (y \vee \neg x)$:

- start with all the propositional atoms in the wff φ
- incrementally, consider each sub-wff of φ, from innermost to outermost

x	y	$\neg x$	$\neg y$	$x \to \neg y$	$y \vee \neg x$	$(x \to \neg y) \to (y \vee \neg x)$
T	**T**	**F**	**F**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**

- propositional wff φ is **satisfiable** if **there is** an assignment of truth-values to the propositional atoms which makes φ true.
- propositional wff φ is a **tautology** if **every** assignment of truth-values to the propositional atoms makes φ true.
- $\varphi \triangleq (x \to \neg y) \to (y \vee \neg x)$ is **satisfiable**, but is **not a tautology**.

Another More Complicated Truth-Table

not of a single wff, but of a sequent $\boxed{(P \wedge \neg Q) \to R,\ \neg R,\ P \vdash Q}$, which was shown **formally derivable** by the proof rules at the end of **Handout 02**.

Another More Complicated Truth-Table

not of a single wff, but of a sequent $(P \wedge \neg Q) \to R,\ \neg R,\ P \vdash Q$, which was shown **formally derivable** by the proof rules at the end of **Handout 02**.

P	Q	R	$\neg Q$	$\neg R$	$P \wedge \neg Q$	$(P \wedge \neg Q) \to R$
T	**T**	**T**	**F**	**F**	**F**	**T**
T	**T**	**F**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**T**	**T**	**F**	**T**

Another More Complicated Truth-Table

not of a single wff, but of a sequent $(P \wedge \neg Q) \to R,\ \neg R,\ P \vdash Q$, which was shown **formally derivable** by the proof rules at the end of **Handout 02**.

P	Q	R	$\neg Q$	$\neg R$	$P \wedge \neg Q$	$(P \wedge \neg Q) \to R$
T	**T**	**T**	**F**	**F**	**F**	**T**
T	**T**	**F**	**F**	**T**	**F**	**T**
T	**F**	**T**	**T**	**F**	**T**	**T**
T	**F**	**F**	**T**	**T**	**T**	**F**
F	**T**	**T**	**F**	**F**	**F**	**T**
F	**T**	**F**	**F**	**T**	**F**	**T**
F	**F**	**T**	**T**	**F**	**F**	**T**
F	**F**	**F**	**T**	**T**	**F**	**T**

- when all the premises (shaded in gray) evaluate to **T**, so does the conclusion (shaded in green) – this occurs in **row 2** of the truth table,
- in such a case we write $(P \wedge \neg Q) \to R,\ \neg R,\ P \models Q$.

(THIS PAGE INTENTIONALLY LEFT BLANK)