CS 511, Fall 2020, Addendum 06 (C)
Scripts for Prover9 and Mace4

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transitivity of subset relation

▶ If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

▶ More formally, $(A \subseteq B) \wedge (B \subseteq C) \rightarrow (A \subseteq C)$.

▶ Formal definition of "$\subseteq$" using quantifiers and "$\in$":

  $(A \subseteq B)$ iff $\forall x(x \in A \rightarrow x \in B)$

▶ The following script verifies the *transitivity of the subset relation*:

```
formulas(assumptions).
  all x all y (subset(x,y) <-> (all z (member(z,x) -> member(z,y)))).
end_of_list.

formulas(goals).
  all x all y all z (subset(x,y) & subset(y,z) -> subset(x,z)).
end_of_list.
```

Schubert's *Steamroller Problem*

This was a *challenge problem* for the early ATP's in the 1980's and 1990's. It was a *challenge* because the search space exceeded the capacity of most computers at the time. It is no longer a *challenge* for current ATP's.

1+2. **Wolves, foxes, birds, caterpillars, and snails are animals, and there are some of each of them.**

3. *Also there are some grains, and grains are plants.*

4. **Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants.**

5. *Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which are in turn much smaller than wolves.*

6. **Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails.**

7. *Caterpillars and snails like to eat some plants.*

8. Prove there is an animal that likes to eat a grain-eating animal (where a grain-eating animal is one that eats all grains).

Schubert's *Steamroller Problem*

▶ **Wolves, foxes, birds, caterpillars, and snails are animals , and there are some of each of them .**

1. $\forall x \ \big(\text{Wolf}(x) \vee \text{Fox}(x) \vee \text{Bird}(x) \vee \text{Caterpillar}(x) \vee \text{Snail}(x) \\ \rightarrow \text{Animal}(x)\big)$

2. $\exists x \ \text{Wolf}(x) \ \wedge \ \exists x \ \text{Fox}(x) \ \wedge \\ \exists x \ \text{Bird}(x) \ \wedge \ \exists x \ \text{Caterpillar}(x) \ \wedge \ \exists x \ \text{Snail}(x)$

▶ *Also there are some grains, and grains are plants.*

3. $\exists x \ \text{Grain}(x) \ \wedge \ \forall x \ \big(\text{Grain}(x) \rightarrow \text{Plant}(x)\big)$

(Reminder: quantifiers bind more tightly than propositional connectives.)

Schubert's *Steamroller Problem*

▶ **Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants.**

$$
\begin{aligned}
4. \quad \forall x \ \Big(&\text{Animal}(x) \to \\
&\Big(\ \forall y \ \big(\text{Plant}(y) \to \text{Eats}(x, y)\big) \ \vee \\
&\quad \forall z \ \big(\text{Animal}(z) \ \wedge \ \text{Smaller}(z, x) \ \wedge \\
&\qquad \exists u \ \big(\text{Plant}(u) \wedge \text{Eats}(z, u)\big) \to \ \text{Eats}(x, z)\big)\Big)\Big)
\end{aligned}
$$

(Reminder: quantifiers bind more tightly than propositional connectives.)

Schubert's *Steamroller Problem*

▶ *Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which are in turn much smaller than wolves.*

5. $\forall x \, \forall y \, \big(\mathsf{Caterpillar}(x) \, \wedge \, \mathsf{Bird}(y) \, \rightarrow \, \mathsf{Smaller}(x, y)\big) \, \wedge$
$\forall x \, \forall y \, \big(\mathsf{Snail}(x) \, \wedge \, \mathsf{Bird}(y) \, \rightarrow \, \mathsf{Smaller}(x, y)\big) \, \wedge$
$\forall x \, \forall y \, \big(\mathsf{Bird}(x) \, \wedge \, \mathsf{Fox}(y) \, \rightarrow \, \mathsf{Smaller}(x, y)\big) \, \wedge$
$\forall x \, \forall y \, \big(\mathsf{Fox}(x) \, \wedge \, \mathsf{Wolf}(y) \, \rightarrow \, \mathsf{Smaller}(x, y)\big)$

(Reminder: quantifiers bind more tightly than propositional connectives, and "$\wedge$" binds more tightly than "$\rightarrow$".)

Schubert's *Steamroller Problem*

▶ **Wolves do not like to eat foxes or grains, while birds like to eat caterpillars but not snails.**

6. $\forall x \, \forall y \, \big(\text{Wolf}(x) \, \wedge \, \big(\text{Fox}(y) \vee \text{Grain}(y) \big) \, \rightarrow \, \neg \text{Eats}(x, y) \big) \, \wedge$
$\forall x \, \forall y \, \big(\text{Bird}(x) \, \wedge \, \text{Caterpillar}(y) \, \rightarrow \, \text{Eats}(x, y) \big) \, \wedge$
$\forall x \, \forall y \, \big(\text{Bird}(x) \, \wedge \, \text{Snail}(y) \, \rightarrow \, \neg \text{Eats}(x, y) \big)$

▶ *Caterpillars and snails like to eat some plants.*

7. $\forall x \, \big(\text{Caterpillar}(x) \vee \text{Snail}(x) \, \rightarrow \, \exists y \, \big(\text{Plant}(y) \wedge \text{Eats}(x, y) \big) \big)$

(Reminder: quantifiers bind more tightly than propositional connectives, and "$\wedge$" and "$\vee$" bind more tightly than "$\rightarrow$".)

Schubert's *Steamroller Problem*

▶ Prove there is an animal that likes to eat a grain-eating animal (where a grain-eating animal is one that eats all grains).

$$8. \quad \exists x \, \exists y \, \Big(\, \mathsf{Animal}(x) \, \wedge \, \mathsf{Animal}(y) \, \wedge \, \mathsf{Eats}(x, y) \, \wedge$$
$$\forall z \, \big(\mathsf{Grain}(z) \, \rightarrow \, \mathsf{Eats}(y, z) \big) \Big)$$

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