CS 511 Formal Methods for High-Assurance Software Engineering
Homework Assignment 02

Selected Solution

(by Jiawen Liu)

Problem 1.

(a) $\phi_1 = (\neg p \vee \neg q) \wedge (p \vee \neg q) \wedge (\neg q \vee q)$
$\phi_1 = \neg p \wedge \neg q$

(b) $\phi_2 = (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$
$\phi_2 = r \wedge (\neg p \vee q)$

(c) $\phi_3 = (\neg r \vee \neg s \vee \neg q) \wedge (\neg r \vee s \vee \neg q) \wedge (r \vee \neg s \vee \neg q) \wedge (r \vee \neg s \vee q) \wedge (r \vee s \vee \neg q)$
$\phi_3 = \neg q \wedge (r \vee \neg s)$

Problem 2.

Part 1. Write a natural-deduction proof of the following WFF:

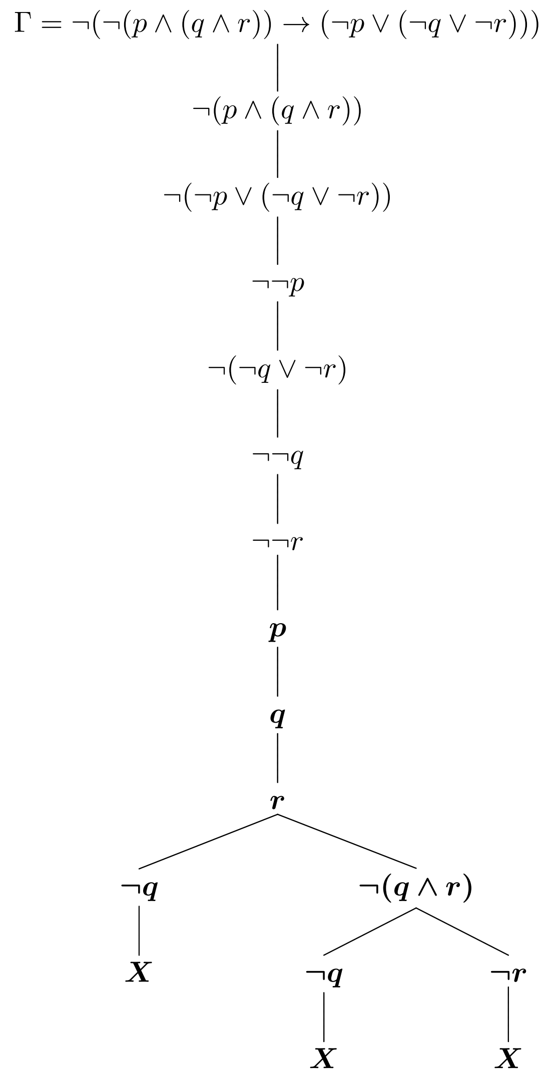
$$\phi_1 = \neg(p \wedge (q \wedge r)) \rightarrow (\neg p \vee (\neg q \vee \neg r))$$

1	$\neg(p \wedge (q \wedge r))$	assume
2	$\neg(\neg p \vee (\neg q \vee \neg r))$	assume
3	$\neg p$	assume
4	$\neg p \vee (\neg q \vee \neg r)$	$\vee\text{i}_1 3$
5	$\perp$	$\neg\text{e } 2, 4$
6	$\neg\neg p$	$\neg\text{i } 2-5$
7	p	$\neg\neg\text{e } 6$
8	$\neg q$	assume
9	$\neg q \vee \neg r$	$\vee\text{i}_1\ 8$
10	$\neg p \vee (\neg q \vee \neg r)$	$\vee\text{i}_2\ 9$
11	$\perp$	$\neg\text{e } 2, 10$
12	$\neg\neg q$	$\neg\text{i } 2-11$
13	q	$\neg\neg\text{e } 12$
14	$\neg r$	assume
15	$\neg q \vee \neg r$	$\vee\text{i}_2\ 14$
16	$\neg p \vee (\neg q \vee \neg r)$	$\vee\text{i}_2\ 15$
17	$\perp$	$\neg\text{e } 2, 16$
18	$\neg\neg r$	$\neg\text{i } 2-17$
19	r	$\neg\neg\text{e } 18$
20	$q \wedge r$	$\wedge\text{i } 7, 13$
21	$p \wedge (q \wedge r)$	$\wedge\text{i } 20, 19$
22	$\perp$	$\neg\text{e } 1, 21$
23	$(\neg p \vee (\neg q \vee \neg r))$	$\neg\text{e}2-22$
24	$\neg(p \wedge (q \wedge r)) \rightarrow (\neg p \vee (\neg q \vee \neg r))$	$\rightarrow\text{i } 1-23$

Part 2. Use the tableaux method to show the validity of following de Morgan's Law:

$$\phi_1 = \neg(\neg(p \wedge (q \wedge r)) \rightarrow (\neg p \vee \neg q \vee \neg r))$$

Proving by showing its negation is a contradiction.



Problem 3.

(a) $\phi_n^{row} = \bigwedge_{i=1}^{n} \bigvee_{k=1}^{n} \Big\{ q_{i,k} \wedge \bigwedge\{\neg q_{i,j} | j = 1, \cdots, n \wedge j \neq k\} \Big\}$

(b) $\phi_n^{col} = \bigwedge_{j=1}^{n} \bigvee_{k=1}^{n} \Big\{ q_{k,j} \wedge \bigwedge\{\neg q_{i,j} | i = 1, \cdots, n \wedge i \neq k\} \Big\}$

(c) $\phi_n^{diag1} = \bigwedge \Big\{ \neg q_{i_1,j_1} \vee \neg q_{i_2,j_2} \mid i_1, j_1, i_2, j_2 \in \{1, \cdots, n\} \ s.t., i_1 \neq i_2 \wedge j_1 \neq j_2 \wedge i_1 - j_1 = i_2 - j_2 \Big\}$

(d) $\phi_n^{diag1} = \bigwedge \Big\{ \neg q_{i_1,j_1} \vee \neg q_{i_2,j_2} \mid i_1, j_1, i_2, j_2 \in \{1, \cdots, n\} \ s.t., i_1 \neq i_2 \wedge j_1 \neq j_2 \wedge i_1 + j_1 = i_2 + j_2 \Big\}$

Problem 4. Γ is not even finitely satisfiable. Consider the case of $n = 4$, i.e., ϕ_4. It has exactly 2 possible solutions. Each of the 2 solution can be extended in a unique way to simultaneously satisfy ϕ_4 and ϕ_5. However, no matter how a solution for $\{\phi_4, \phi_5\}$ is extended, it will never satisfy the ϕ_6. We can conclude from this that, using the same set Q of propositional variables, with the same interpretation of indices $\{i, j\}$, the set $\{\phi_4, \phi_5, \phi_6\}$ is not satisfiable. Even though we can still write each wff in $\{\phi_4, \phi_5, \phi_6\}$ and every other wff for $n \geq 7$ over a distinct private set of variables, the resulting infinite model will not correspond

to a solution for *Infinite Queens Problem*.
In other words, the solution for $n-1$ doesn't guarantee a solution for n. We cannot induct from ϕ_{n-1} that ϕ_n has a solution.

Problem 5. `https://piazza.com/class_profile/get_resource/ke1gp4ep1z513t/kfafm2ksek7a6`

Problem 6. `https://piazza.com/class_profile/get_resource/ke1gp4ep1z513t/kfafmcsgp5mor`