CS 511, Fall 2020, Handout 01

Syntax of Propositional Logic

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Syntax of the wff's of Propositional Logic

- Reading: [LCS, Section 1.3]

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- The wff's of propositional logic are obtained by applying the construction rules below, and only these, **finitely many times**.

  One **basis step**:

  0. every propositional atom (*i.e.*, propositional variable) p is a WFF

  Four **induction steps**:

  1. if φ is a wff, then so is $(\neg\varphi)$
  2. if φ and ψ are wff's, then so is $(\varphi \wedge \psi)$
  3. if φ and ψ are wff's, then so is $(\varphi \vee \psi)$
  4. if φ and ψ are wff's, then so is $(\varphi \rightarrow \psi)$

Remember this inductive definition: We use it later in syntax-directed proofs.

Syntax of the wff's of Propositional Logic

- More succintly, in **BNF (Backus Naur Form)**:

$$\varphi \ ::= \ p \mid (\neg\varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$$

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- Or, more abstractly by omitting parentheses, in **Extended BNF**:

$$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \rightarrow \psi$$

  Parentheses are used only to set an order of precedence among logical connectives $\{\neg, \wedge, \vee, \rightarrow\}$.

Parse Trees of wff's

- A fully-parenthesized wff:

$$\Big(\big(\neg((\neg P) \vee (Q \wedge (\neg P)))\big) \\ \rightarrow \big(\neg((\neg P) \rightarrow (Q \vee (\neg R)))\big)\Big)$$

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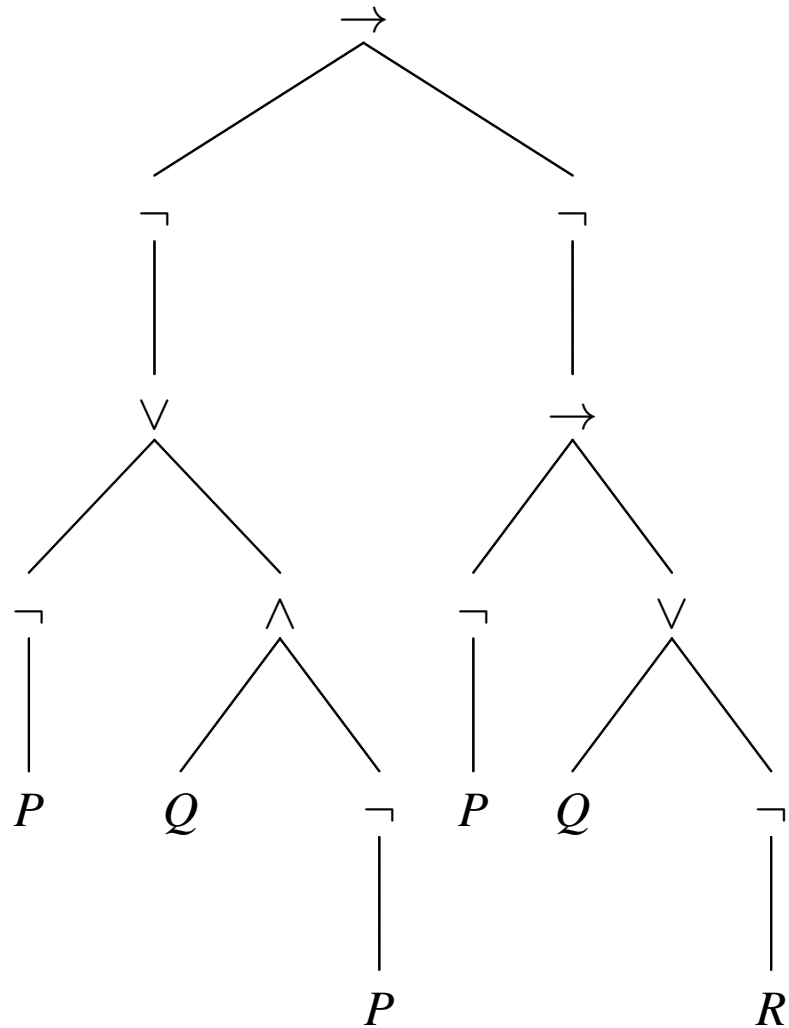
No parentheses in the parse tree



Parse trees are very nice,

but more difficult to store.

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