CS 511 Formal Methods for High-Assurance Software Engineering

Homework Assignment 03 - Selected Solution

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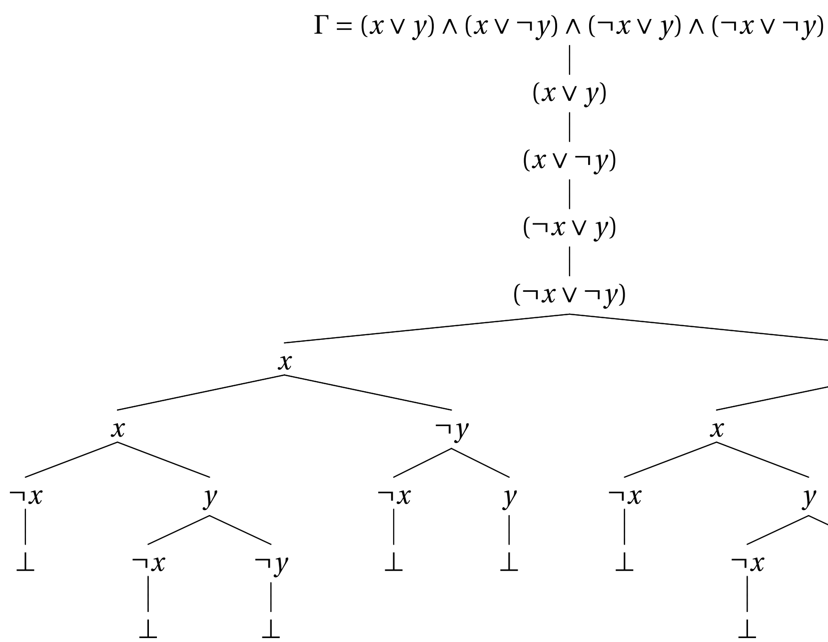
Problem 1. Consider the wff Φ, which is in CNF, in the last exercise in Lecture Slides 08 as following where $k = 2$ ($k = 1$ is the smallest member but yields to the same length for both methods):

$$\phi_1 = (x \lor y) \land (x \lor \neg y) \land (\neg x \lor y) \land (\neg x \lor \neg y)$$

In resolution method, we have following derivation:

1	$(x \lor y)$	
2	$(x \lor \neg y)$	
3	$(\neg x \lor y)$	
4	$(\neg x \lor \neg y)$	
5	x	resolve $1, 2$
6	$\neg x$	resolve $3, 4$
7	$\bot$	$\neg$e $4, 5$

In tableaux method, we have following derivation:



It is obvious that the tableaux method takes more step than the resolution method. Since tableaux method is required to search on every branches in order to derive the desired result. On the other hand, the resolution method only need to resolve the certain branch leading the desired result.

Problem 2.

Part 1. Fact: satisfiability of ϕ can be determined by first checking if ROBDD(ϕ) is equal to the ROBDD with a single terminal label "0", in which case ϕ is unsatisfiable, otherwise ϕ is satisfiable.

Part 2. Using a DFS(deep first search) algorithm to search for the path from root to the leaf labeled with 1. If there exists a path, then return satisfiable and this path, otherwise return unsatisfiable.

Algorithm 1 Find the Satisfiability `finSat`$(\psi, ROBDD(\psi), r, p)$

Require: a wff ψ with variables $\{x_1, x_2, \cdots, x_3\}$. ROBDD($\psi$) constructed in the order of $x_1 < x_2 < \cdots < x_n$, the current node r and the current path from root to current node p,

    if child(r) = None and label(r) = 1 **do**
return p;
    else if child(r) = None and label(r) = 0 **do**
return None;
    else do
        for r_i in children(r) **do**
            `let` $l =$ `finSat`$(\psi, ROBDD(\psi), r_i, r_i :: p)$
            if $l \neq None$ **do**
return l;
return $None$

Part 3. Using a DFS(deep first search) algorithm to search for the unique path from root to the leaf labeled with 1. If the # of path is greater than 0, then .

Algorithm 2 Find the Satisfiability `finSat`$(\psi, ROBDD(\psi), r)$

Require: a wff ψ with variables $\{x_1, x_2, \cdots, x_3\}$. ROBDD($\psi$) constructed in the order of $x_1 < x_2 < \cdots < x_n$, the current node r

    `let` $c = 0$
    if child(r) = None and label(r) = 1 **do**
return 1;
    else if child(r) = None and label(r) = 0 **do**
return 0;
    else do
        for r_i in children(r) **do**
            $c+=$ `finSat`$(\psi, ROBDD(\psi), r_i)$
return c

(BFS(breath first search) is also a good method.)

Problem 3.

$$
\begin{aligned}
\theta_k \stackrel{\text{def}}{=} \bigwedge_{1\leqslant i,j\leqslant k-1}^{i+j=k} \Big\{ q_{i,j} \;\rightarrow\; & \bigwedge\{\neg q_{i',j'} \mid i'+j'=k+1, (i,j)\neq(i'j')\} \wedge && \text{(if a queen is in } (i,j) \text{ on anti. } k\text{)} \\
& \bigwedge\{\neg q_{i',j} \mid 1\leqslant i' < i\} \wedge && \text{(no queen north of } (i,j)\text{)} \\
& \bigwedge\{\neg q_{i,j'} \mid 1\leqslant j' < j\} \wedge && \text{(no queen west of } (i,j)\text{)} \\
& \bigwedge\{\neg q_{i',j'} \mid 1\leqslant i' < i, 1\leqslant j' < j, i-j=i'-j'\} && \text{(no queen north-west of } (i,j)\text{)} \\
\Big\} &
\end{aligned}
$$

Problem 4. According to the Compactness theorem, we know Γ is satisfiable iff every finite subset of Γ is satisfiable. By the Problem 3, we know the infinite set of formula of θ_i is finitely satisfiable. Therefore the *Infinite Queens Problem* is fully satisfiable by compactness.

Problem 5. `https://piazza.com/class_profile/get_resource/ke1gp4ep1z513t/kfldq45b2i467m`

Problem 6. `https://piazza.com/class_profile/get_resource/ke1gp4ep1z513t/kfldqwpsa291io`