3. A solution for the *Infinite Queens Problem* can be inferred from a combinatorial game on the south-east quadrant of the Cartesian plane. Starting with $0$, the positions are assigned natural numbers along successive upward antidiagonals, starting from the north-west corner, as shown in Figure 3. We call a traversal of the positions in increasing order of their assigned numbers a *good traversal*. In the game, a queen is initially placed anywhere on the board, and the players take turns moving it along a good traversal *in reverse*, *i.e.*, to a lower-numbered position, which is moreover a queen's move away. The first player unable to move loses. We now define a process of placing infinitely many queens, one at a time:

   > Consider a good traversal of the south-east quadrant. When a position $(i, j)$ is visited, if $(i, j)$ is a queen's move away from all the previously placed queens, then place a queen in $(i, j)$, else leave $(i, j)$ empty and proceed to the next position.

   From the preceding game and the process just defined of placing infinitely many queens, it is possible to prove the following result:

   > Every row and every column in the south-east quadrant is eventually occupied by exactly one queen. (By our definition of the process, every diagonal and every antidiagonal is necessarily occupied by at most one queen.)[8]

   Let the antidiagonals of the south-east quadrant be uniquely identified by the positive integers $1,\ 2,\ 3, \ldots$ starting from the top. (Antidiagonal $1$ is empty, antidiagonal $2$ is "*0*", antidiagonal $3$ is "*1 2*", antidiagonal $4$ is "*3 4 5*", etc., moving upwards along every antidiagonal.) Your task is to define, for every $k \geqslant 2$, a propositinal wff $\theta_k \in \mathsf{WFF}_{\mathsf{PL}}(\mathcal{Q})$ which is satisfied iff two conditions hold:

   (a) at most one queen occupies antidiagonal $k$, and

   (b) if a queen occupies antidiagonal $k$, then it cannot be attacked by any queen which is placed in a position to the north and/or west of antidiagonal $k$.

   We start at $k = 2$ because antidiagonal $1$ is empty. Observe that antidiagonal $k$ includes exactly $k - 1$ positions, namely $\{(k-1, 1),\ (k-2, 2),\ (k-3, 3),\ \ldots\ , (1, k-1)\}$.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | ⋯ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $\boxed{\mathit{0}}$ | *2* | *5* | *9* | *14* | *20* | *27* | *35* | *44* | *54* | ⋯ | |
| 2 | *1* | *4* | *8* | $\boxed{\mathit{13}}$ | *19* | *26* | *34* | *43* | *53* | ⋯ | | |
| 3 | *3* | $\boxed{\mathit{7}}$ | *12* | *18* | *25* | *33* | *42* | *52* | ⋯ | | | |
| 4 | *6* | *11* | *17* | *24* | $\boxed{\mathit{32}}$ | *41* | *51* | ⋯ | | | | |
| 5 | *10* | *16* | $\boxed{\mathit{23}}$ | *31* | *40* | *50* | ⋯ | | | | | |
| 6 | *15* | *22* | *30* | *39* | *49* | ⋯ | | | | | | |
| 7 | *21* | *29* | *38* | *48* | ⋯ | | | | | | | |
| 8 | *28* | *37* | *47* | ⋯ | | | | | | | | |
| 9 | *36* | *46* | ⋯ | | | | | | | | | |
| 10 | *45* | ⋯ | | | | | | | | | | |
| 11 | ⋯ | | | | | | | | | | | |
| ⋮ | | | | | | | | | | | | |

Exercise 13 in 2020-09-07.fCtC.pdf

**Figure 3:** Positions in the south-east quadrant of the Cartesian plane are identified by the natural numbers (in italics), starting with *0* at the north-west corner and continuing along successive upwards antidiagonals. The figure shows only antidiagonals 2, 3,. . ., 11 (antidiagonal 1 is empty). The boxed positions are the first 5 queens that are placed according to the traversal defined in part 3 of Exercise 17. In the shown fragment of the quadrant, antidiagonals 2, 5, 6, 8, and 9, are each occupied by a queen while antidiagonals 3, 4, 7, 10, and 11, are not.