CS 511
Formal Methods for High-Assurance Software Engineering
Homework Assignment 04

Out: 25 September 2020
Due: Friday, 2 October 2020, by 11:59 pm

- There are six problems in this assignment. The first 4 problems have to be solved by hand. The last 2 problems are implementation/coding problems.
- You should submit a single "`.pdf`" file to Gradescope, where you include links to your scripts for Problem 5 and Problem 6. Your scripts should be stored in the repository of your Github account.
- For full credit in the homework, you need to complete 4 out of 6 problems in this assignment. Each is worth 4 points. Of course, you may want to try all 6 problems. You will get credit for all extra exercises you do (correctly!).

Problem 1 There are two parts in this problem, both from Lecture Slides 11, entitled ***Quantified Boolean Formulas (QBF's)***:

1. Do the exercise on page 31.

   Hint: Start with the simple case when the wff's φ, ψ_1, ψ_2, and ψ_3, are quantifier-free, *i.e.*, wff's of propositional logic.

2. Do the exercise on page 32.

   Hint: Start with the simpler case when all all the wff's φ_i and ψ_j are quantifier-free, and assume that θ is an arbitrary QBF wff where only propositional variables x and y appear free.

Problem 2 Go to the set of Lecture Notes with file name `2020-09-07.fCtC.pdf`, posted on Piazza under the "Resources" webpage. Do part 1 and part 2 in Exercise 27 on pages 20-21 .

Hint: You will find it useful to read Example 25 and Exercise 26 on pages 18-19-20.

Problem 3 [LCS, page 160], Exercise 2.3.3. Cross out the words "Try to" at the beginning of the exercise. First, do parts (a) and (b). Second, do part (c) adjusted as follows: (c) *Write down an infinite set of FO sentences which hold in a model iff the model has infinitely many distinct elements.*

Problem 4 [LCS, page 161], Exercise 2.3.9. Do parts (a), (b), (c), and (d) only.

Problem 5 Write a Z3Py script that solves the Schubert's Steamroller Problem. A script in the conventions of Prover9+Mace4 is already available and posted on Piazza under the "Resources" webpage, which you can take as a reference.

Problem 6 For this problem, you will find it useful to consult the Lecture slides and the scripts (in the conventions of Prover9+Mace4) in which Schubert's Steamroller is presented.

In two consecutive stages, translate the following sentences into wff's of first-order logic: (1) type-set the wff's using Latex, and (2) translate them into a script that you can run with Prover9+Mace4:

1. Someone who lives in Dreadbury Mansion killed Aunt Agatha.
2. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein.
3. A killer always hates his victim, and is never richer than his victim.
4. Charles hates no one that Aunt Agatha hates.
5. Agatha hates everyone except the butler.
6. The butler hates everyone not richer than Aunt Agatha.
7. The butler hates everyone Aunt Agatha hates.
8. No one hates everyone.
9. Agatha is not the butler.

For a uniform modeling of the problem from all the submissions we will receive, we ask you to only use the following constants and relations (all highlighted):

- **Constants:**
  A means "Agatha", B means "the butler", C means "Charles", and
  D means "Danbury Mansion".
- **Binary relations:**
  $\text{LivesIn}(a, b)$ means "a lives in b", $\text{Killed}(a, b)$ means "a killed b",
  $\text{Hates}(a, b)$ means "a hates b", and $\text{RicherThan}(a, b)$ means "a is richer than b".

Your first-order wff's should therefore be written over the signature:

$$\{\ A,\ B,\ C,\ D,\ \text{LivesIn}(\ ,\ ),\ \text{Killed}(\ ,\ ),\ \text{Hates}(\ ,\ ),\ \text{RicherThan}(\ ,\ )\ \}$$

Use your script to answer the question: **Who killed Aunt Agatha?** by using Prover9+Mace4.
We will test your script in the same way.