An-Najah National University
Department of Mathematics
Calculus I (Midterm A)
Tuesday 21/03/2023

Student Name: Key
Student Number:
Instructor:
Meeting's Time:

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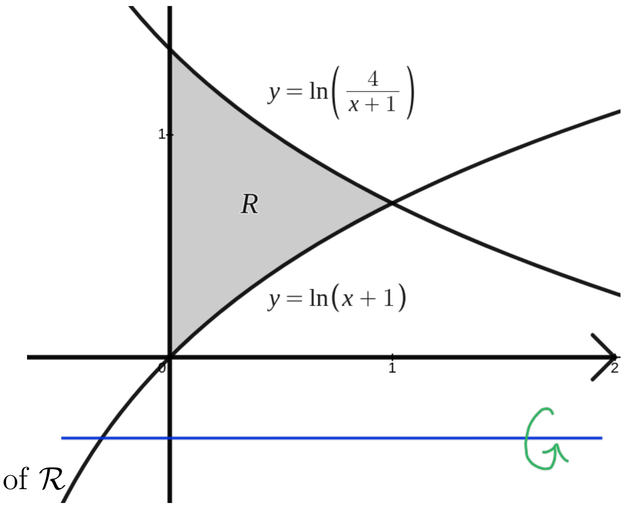
1. Let $\mathcal{R}$ be the region that enclosed by the curves:
$\boldsymbol{y = \ln(x+1)}$, $\boldsymbol{y = \ln\left(\frac{4}{x+1}\right)}$, and the y–axis.



(a) (2 points) **Setup** an integral that represents the area of $\mathcal{R}$

$$A = \int_0^1 \left[\ln\left(\frac{4}{x+1}\right) - \ln(x+1)\right] dx$$

$$A = \int_0^1 \left[\ln 4 - 2\ln(x+1)\right] dx.$$

(b) ($2\frac{1}{2}$ points) **Setup** (<u>do not evaluate</u>) an integral that represents the volume of the solid obtained by revolving (rotating) the region $\mathcal{R}$ about $\boldsymbol{y = -1}$

$$V = \pi \int_0^1 \left[\left(\ln\left(\frac{4}{x+1}\right) + 1\right)^2 - \left(\ln(x+1) + 1\right)^2\right] dx$$

(c) ($2\frac{1}{2}$ points) **Setup** (<u>do not evaluate</u>) an integral that represents the volume of the solid obtained by revolving (rotating) the region $\mathcal{R}$ about the y–axis

$$V = 2\pi \int_0^1 x\left(\ln\left(\frac{4}{x+1}\right) - \ln(x+1)\right) dx$$

$$= 2\pi \int_0^1 x\left[\ln 2 - 2\ln(x+1)\right] dx$$

2. Evaluate the following integrals

(a) (3 points) $\int \frac{e^x}{\sqrt{e^x+1}}\,dx$

let $y = \sqrt{e^x+1} \Rightarrow e^x+1 = y^2$

$e^x dx = 2y\,dy$.

$\therefore I = \int \frac{2y\,dy}{y} = \int 2dy = 2y + c$.

$\therefore \boxed{I = 2\sqrt{e^x+1} + c}$

(b) (3 points) $\int r^3 \ln r\, dr$

$u = \ln r$ $\quad dv = r^3$

$du = \frac{1}{r}dr$ $\quad v = \frac{r^4}{4}$

$\therefore I = \frac{r^4}{4}\ln r - \frac{1}{4}\int r^3 dr$

$\boxed{I = \frac{1}{4}r^4 \ln r - \frac{1}{16}r^4 + c}$

(c) (3 points) $\int \frac{1}{1-\cos x}\,dx$

$= \int \frac{1+\cos x}{1-\cos^2 x}dx = \int \frac{1+\cos x}{\sin^2 x}dx$.

$= \int [\csc^2 x + \cot x \csc x]\, dx$

$\boxed{I = -\cot x - \csc x + c}$

3. (6 points) Determine whether each of the following statements is **TRUE** or **FALSE**.

T F $\displaystyle\int_0^2 (x-x^3)\,dx$ represents the area under the curve $y = x - x^3$ from 0 to 2.

T F $\displaystyle\int_{-1}^{1} \frac{e^{x^2}}{1+e^{-x}}\,dx = \int_{-1}^{1} \frac{e^{x^2}}{1+e^{x}}\,dx$

T F If f is continuous on $[a,b]$ and $g(x) = \displaystyle\int_a^x f(t)\,dt$, then g is differentiable on (a,b).

T F $\displaystyle\int_0^1 f(2x)\,dx = 2\int_0^1 f(x)\,dx =$

T F $\displaystyle\int_0^1 \sin^6 x\,dx \geq 0$

T F $\displaystyle\int 3e^2\,dx = e^3 + C.$

4. (15 points) Choose the correct answer

(1) An appropriate trigonometric substitution for the integral $\boxed{\displaystyle\int \frac{x^3}{(9-4x^2)^{3/2}}\,dx}$ is

a. $\boxed{x = \frac{3}{2}\cos\theta}$ b. $\boxed{x = \frac{2}{3}\cos\theta}$

c. $\boxed{x = \frac{3}{2}\sec\theta}$ d. $\boxed{x = \frac{3}{2}\tan\theta}$

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(2) $\displaystyle\int \frac{1}{1+x^2}\,dx$ is

a. $\sec^{-1} x + c$ b. $-\csc^{-1} x + c$

c. $-\cot^{-1} x + c$ d. $\sin^{-1} x + c$

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(3) $\displaystyle\int_0^{\pi/4} \sec x \tan x\,dx =$

a. $1-\sqrt{2}$ b. $\sqrt{2}-1$

c. $\sqrt{2}+1$ d. else

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(4) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n} \frac{6}{n}\left(5+\frac{i}{n}\right)^5 =$

a. 1. b. $6^6 - 5^6$.

c. $\dfrac{6^6-5^6}{6}$. d. else.

(5) If $\boxed{f(x) = \begin{cases} 2 & \text{if } 0 \le x \le 1 \\ 2x & \text{if } 1 < x \le 3 \end{cases}}$. Then f_{avg} on $[0, 3]$ is

a. 10/3
b. 6
c. 3
d. Else

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(6) If $\boxed{f(x) = \begin{cases} \ln(x^2 - x + 1) & \text{if } 0 \le x \le 1 \\ (1 - x)\sin x & \text{if } 1 < x \le 3 \end{cases}}$ and $\boxed{g(x) = \int_0^x f(t)\, dt}$. Then $\boxed{g'\left(\frac{\pi}{2}\right) =}$

a. $\pi/2$
b. 0
c. $1 - \pi/2$
d. else

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(7) $\int_0^2 |1 - x|\ dx =$

a. -1
b. 0
c. 1
d. else

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(8) $\boxed{\lim_{x \to 3} \frac{x}{x - 3} \int_3^x \frac{\cos(t - 3)}{t}\ dt =}$

a. -1
b. 4
c. 1
d. else

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(9) $\int \ln x\ dx =$

a. $x \ln x$
b. $\ln x + C$
c. $(x - 1)\ln x + C$
d. else

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(10) If $\boxed{\int_0^1 e^{x^2}\ dx = K}$. Then $\boxed{\int_{-1}^1 \frac{e^{x^2}}{1 + e^x}\ dx =}$

$\boxed{\textbf{Hint: } \frac{1}{1+t} + \frac{1}{1+t^{-1}} = 1}$

a. K
b. $2K$
c. $1 - K$
d. e^K

If $\int_0^1 e^{x^2}dx = k$ then $\int_{-1}^1 \frac{e^{x^2}}{1+e^x}dx =$

$$\frac{1}{1+t} + \frac{1}{1+t^{-1}} = 1 \quad \text{let } t=e^x \Rightarrow$$

$$\frac{1}{1+e^x} + \frac{1}{1+e^{-x}} = 1 \quad / * e^{x^2}$$

$$\Rightarrow \frac{e^{x^2}}{1+e^x} + \frac{e^{x^2}}{1+e^{-x}} = e^{x^2} \quad / \int_{-1}^1$$

$$\int_{-1}^1 \frac{e^{x^2}}{1+e^x}dx + \int_{-1}^1 \frac{e^{x^2}}{1+e^{-x}}dx = \int_{-1}^1 \underbrace{e^{x^2}}_{\text{even}}dx = 2\int_0^1 e^{x^2}dx$$

but $\int_{-1}^1 \frac{e^{x^2}}{1+e^x}dx = \int_{-1}^1 \frac{e^{x^2}}{1+e^{-x}}dx$ from (T or F)

$$\Rightarrow 2\int_{-1}^1 \frac{e^{x^2}}{1+e^x}dx = 2\int_0^1 e^{x^2}dx = 2K$$

$$\therefore \boxed{\int_{-1}^1 \frac{e^{x^2}}{1+e^x}dx = K}$$