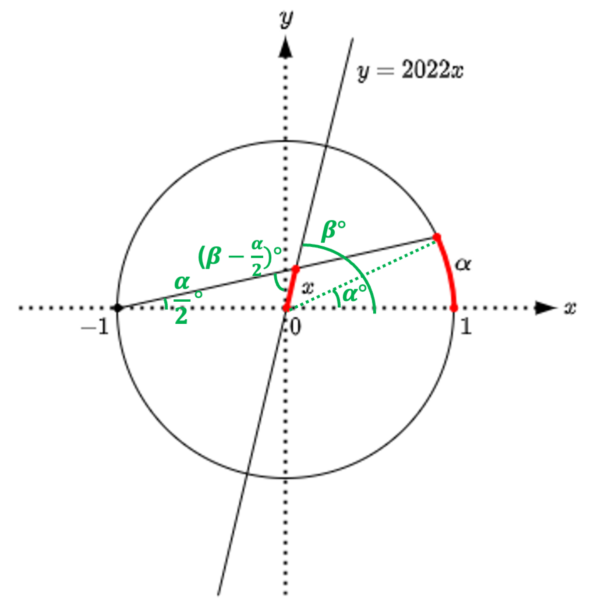
A detailed version for the formula part of TetCTF-2022 Intended Solutions.



According to the law of sines:

$$\frac{a}{sin(A)} = \frac{b}{sin(B)} \tag{1}$$

We have:

$$\frac{x}{sin(\frac{\alpha}{2})} = \frac{1}{sin(\beta - \frac{\alpha}{2})} \tag{2}$$

That is:

$$x = \frac{sin(\frac{\alpha}{2})}{sin(\beta - \frac{\alpha}{2})} \tag{3}$$

According to the Euler's formula:

$$sin(x) = \frac{e^{i \cdot x} - e^{-i \cdot x}}{2 \cdot i} \tag{4}$$

We have:

$$\begin{aligned} x &= \frac{e^{i \cdot \frac{\alpha}{2}} - e^{-i \cdot \frac{\alpha}{2}}}{e^{i \cdot (\beta - \frac{\alpha}{2})} - e^{-i \cdot (\beta - \frac{\alpha}{2})}} \\ &= \frac{e^{i \cdot \alpha} - 1}{e^{i \cdot \beta} - e^{-i \cdot (\beta + \alpha)}} \end{aligned} \tag{5}$$

That is:

$$e^{i \cdot \alpha} = \frac{1 + x \cdot e^{i \cdot \beta}}{1 + x \cdot e^{-i \cdot \beta}} \tag{6}$$

According to the Euler's formula:

$$tan(x) = \frac{e^{i \cdot x} - e^{-i \cdot x}}{i \cdot (e^{i \cdot x} + e^{-i \cdot x})} \tag{7}$$

We have:

$$\begin{aligned} e^{i \cdot \beta} &= \sqrt{\frac{1 + i \cdot tan(\beta)}{1 - i \cdot tan(\beta)}} \\ &= \sqrt{\frac{1 + i \cdot 2022}{1 - i \cdot 2022}} \end{aligned} \tag{8}$$

Finally we have:

$$e^{i \cdot \alpha} = \frac{1 + x \cdot \sqrt{\frac{1+i \cdot 2022}{1-i \cdot 2022}}}{1 + x \cdot \frac{1}{\sqrt{\frac{1+i \cdot 2022}{1-i \cdot 2022}}}} \tag{9}$$