Appendix for
Conformalized Reachable Sets for Obstacle Avoidance With Spheres

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I. Proof of Theorem 5

Proof. The convex hull of two conformalized spheres encloses the convex hull of the corresponding ground truth spheres with probability:

$$\mathbb{P}(\mathrm{TC}_{i,j} \subseteq \hat{\mathrm{TC}}_{i,j}) \geq \prod_{j'=j}^{j+1} \mathbb{P}(\mathcal{B}_{j',i} \subseteq \hat{\mathcal{B}}_{j',i}(\Delta_{j'})) \geq (1-\epsilon)^2 \tag{1}$$

where $\mathrm{TC}_{i,j} = co(\bigcup_{j'=j}^{j+1}(\mathcal{B}_{j',i}))$ and $\hat{\mathrm{TC}}_{i,j} = co(\bigcup_{j'=j}^{j+1}(\hat{\mathcal{B}}_{j',i}(\Delta_{j'}))$. This probability extends across all the joints as follows:

$$\mathbb{P}(\bigcup_{j=1}^{n_q} \mathrm{TC}_{i,j} \subseteq \bigcup_{j=1}^{n_q} \hat{\mathrm{TC}}_{i,j}) \geq (1-\epsilon)^{n_q+1} \tag{2}$$

Because $\mathrm{FO}_i \subseteq \bigcup_{j=1}^{n_q} \mathrm{TC}_{i,j}$ and $\bigcup_{j=1}^{n_q} \hat{\mathrm{TC}}_{i,j} \subseteq \mathcal{S}\hat{\mathcal{F}}\mathcal{O}_i$, the following guarantee holds:

$$\mathbb{P}(\mathrm{FO}_i \subseteq \mathcal{S}\hat{\mathcal{F}}\mathcal{O}_i) \geq (1-\epsilon)^{n_q+1} \tag{3}$$

Therefore, the probability that the signed distance between the ground truth forward occupancy FO_i and obstacle set $\mathscr{O}$ remains positive is bounded by:

$$\mathbb{P}(\mathbf{s_d}(\mathrm{FO}_i, \mathscr{O}) > 0) \tag{4}$$

$$= \mathbb{P}(\mathrm{FO}_i \subseteq \mathcal{S}\hat{\mathcal{F}}\mathcal{O}_i) \cdot \mathbb{P}(\mathbf{s_d}(\mathcal{S}\hat{\mathcal{F}}\mathcal{O}_j, \mathscr{O}) > 0) \tag{5}$$

$$\geq (1-\epsilon)^{n_q+1} \cdot \mathbb{P}(s(\mathcal{S}\hat{\mathcal{F}}\mathcal{O}_j, \mathscr{O}) > 0) \tag{6}$$

If we enforce $\mathbb{P}(\mathbf{s_d}(\mathcal{S}\hat{\mathcal{F}}\mathcal{O}_j, \mathscr{O}) > 0) = 1$, then the following probability holds:

$$\mathbb{P}(\mathbf{s_d}(\mathrm{FO}_i, \mathscr{O}) > 0) \geq (1-\epsilon)^{n_q+1}. \tag{7}$$

□

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