Homework I: Computational Methods and Optimization

Year: 2023/2024 **Deadline**: January 26, 2024

Problem 1 (2)
Solve the unidimensional optimization problem using the Golden Section Method.

$$6e^{-2\lambda} + 2\lambda^2$$

Problem 2 (4)

$$f(\mathbf{x}) = 8x_1^2 + 8x_2^2 - 80\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} + 80\sqrt{x_1^2 + x_2^2 + 20x_2 + 100} - 5x_1 - 5x_2$$

a) Write the necessary conditions for the minimum. Solve these equations to obtain the solution.

Starting at the point $(x_1, x_2) = (4, 6)$:

b) Solve the minimization problem using Hooke-Jeeves method.
c) Solve the minimization problem using DFP (Davidon-Fletcher-Powell).
d) Solve the minimization problem using CG Fletcher-Reeves (CG-FR) method.
e) Analyze the results and compare the methods' performance.

Problem 3 (6)
Consider the following problem,

$$\min_{x_1, x_2} \left(x_1^4 - 2x_1^2 x_2 + x_1^2 + x_1 x_2^2 - 2x_1 + 4\right)$$

sujeito a,

$$0.25x_1^2 + 0.75x_2^2 - 1 \le 0$$

$$2x_1^2 + x_2^2 - 2 = 0$$

$$x_1 \in [0,5],\ x_2 \in [0,5]$$

a) Write the KKT necessary conditions.
b) Write the Augmented Lagrangian formulation for this problem.
c) Solve the problem using the Augmented Lagrangian method with DFP.
d) Write the (exterior) Penalty formulation for this problem.
e) Solve the penalized problem using CG (FR).
f) Analyze the results and compare the methods.

Problem 4 (8)
Consider the following problem,

$$\min_{x_1, x_2} \left(2x_1^2 - 3x_1 x_2 + x_2^2\right)$$

$$x_1^2 - x_2 + 3 \le 0$$

$$3x_1 + 2x_2 - 6 \le 0$$

$$x_1 \ge 0, x_2 \ge 0$$

a) Write the KKT necessary conditions.
b) Write the Augmented Lagrangian formulation for this problem.
c) Solve using the Augmented Lagrangian method with Rosenbrock.
d) Solve using the Augmented Lagrangian method with CG (FR).
e) Write the (exterior) Penalty formulation for this problem.
f) Solve using the penalty method with Rosenbrock.
g) Solve using the penalty method with CG (FR).
h) Analyze the results and compare the methods.

Note: Use Mathematica, MATLAB or Python to program the algorithms and solve the problems.
Send by email the final report (in pdf format) and the codes with the algorithms (in a zipped folder), using the email subject: HWI_MCO_2023, *student number*.