

15.

$$\int \frac{x^2 - 8x + 13}{(x-1)(x^2 - 4x + 5)} dx =$$

$$\frac{x^2 - 8x + 13}{(x-1)(x^2 - 4x + 5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2 - 4x + 5} = \frac{A(x^2 - 4x + 5) + (Bx+C)(x-1)}{(x-1)(x^2 - 4x + 5)} =$$

$$= \frac{Ax^2 - 4Ax + 5A + Bx^2 - Bx + Cx - C}{(x-1)(x^2 - 4x + 5)} = \frac{(A+B)x^2 + (-4A-B+C)x + 5A-C}{(x-1)(x^2 - 4x + 5)}$$

$$\begin{cases} A+B=1 \\ -4A-B+C=-8 \\ 5A-C=13 \end{cases} \quad \begin{matrix} -3A+C=-4 \\ 2A=6 \end{matrix}$$

$$\begin{cases} A=3 \\ B=-2 \\ C=2 \end{cases}$$

$$\begin{aligned} &= \int \frac{3}{x-1} dx + \int \frac{2-2x}{x^2-4x+5} dx = 3 \ln|x-1| - \int \frac{2x-4}{(x-2)^2+1} dx - \int \frac{2}{(x-2)^2+1} dx = \\ &= 3 \ln|x-1| - \ln((x-2)^2+1) - 2 \arctg(x-2) + C = \\ &= \ln \left| \frac{(x-1)^3}{x^2-4x+5} \right| - 2 \arctg(x-2) + C \end{aligned}$$

Итак, бегим проверку.

$$\begin{aligned} \left(\ln \left| \frac{(x-1)^3}{x^2-4x+5} \right| - 2 \arctg(x-2) \right)' &= (3 \ln|x-1| - \ln(x^2-4x+5) - 2 \arctg(x-2))' = \\ &= \frac{3}{x-1} - \frac{2x-4}{x^2-4x+5} - \frac{2}{x^2-4x+5} = \frac{x^2-8x+13}{(x-1)(x^2-4x+5)} \end{aligned}$$

Ответ: $\ln \left| \frac{(x-1)^3}{x^2-4x+5} \right| - 2 \arctg(x-2) + C.$