№9. $\int \frac{dx}{5-3\cos x} = \left[\begin{array}{l} u = tg\frac{x}{2};\ \cancel{dx}\ dx = \frac{2du}{u^2+1} \\ \cos x = \frac{1-u^2}{u^2+1} \end{array} \right] =$

$= \int \frac{2du}{(u^2+1)\left(5 - \frac{3(1-u^2)}{u^2+1}\right)} = \int \frac{2du}{(u^2+1)\left(5+3\cdot\frac{u^2+1-2}{u^2+1}\right)} =$

$= \int \frac{2du}{(u^2+1)\left(5+3-\frac{6}{u^2+1}\right)} = \int \frac{2du}{8(u^2+1)-6} = \int \frac{2du}{8u^2+2} = \int \frac{du}{4u^2+1} =$

$= \frac{1}{4}\int \frac{du}{u^2+\frac{1}{4}} = \frac{1}{2} arctg\, 2u + C = \frac{1}{2} arctg\left(2 tg\frac{x}{2}\right) + C.$

Проверка:

$\left(\frac{1}{2} arctg\left(2 tg \frac{x}{2}\right)\right)' = \frac{1}{2} \cdot \frac{2}{1+4tg^2\frac{x}{2}} \cdot \frac{1}{2\cos^2\frac{x}{2}} = \frac{1}{2\cos^2\frac{x}{2} + 8\sin^2\frac{x}{2}} =$

$= \frac{1}{2+6\sin\frac{x}{2}} = \frac{1}{2+3(1-\cos x)} = \frac{1}{5-3\cos x}$

Ответ: $\frac{1}{2} arctg\left(2 tg\frac{x}{2}\right) + C.$