№6.

$$\int \frac{dx}{x(1+\sqrt[3]{x^2})} = \begin{bmatrix} x = t^3 \Rightarrow dx = 3t^2 dt \\ \sqrt{x} = t^2 \end{bmatrix} =$$

$$= \int \frac{3t^2 dt}{t^3(1+t^2)} = 3\int \frac{dt}{t(1+t^2)} = 3\int \frac{(1+t^2-t^2)dt}{t(1+t^2)} =$$

$$= 3\int \frac{dt}{t} - 3\int \frac{t\,dt}{1+t^2} = 3\ln|t| - \frac{3}{2}\ln(1+t^2) + C =$$

$$= 3\ln\left|\frac{t}{(1+t^2)^{\frac{1}{2}}}\right| + C = 3\ln\left|\frac{\sqrt[3]{x}}{\sqrt{1+\sqrt[3]{x^2}}}\right| + C$$

Произведем проверку:

$$\left(3\ln\left|\frac{\sqrt[3]{x}}{\sqrt{1+\sqrt[3]{x^2}}}\right|\right)' = 3\cdot\frac{\sqrt{1+\sqrt[3]{x^2}}}{\sqrt[3]{x}}\cdot$$

$$\cdot\frac{\frac{1}{3}x^{-\frac{2}{3}}\sqrt{1+\sqrt[3]{x^2}} - \sqrt[3]{x}\cdot\frac{1}{2\sqrt{1+\sqrt[3]{x^2}}}\cdot\frac{2}{3}x^{-\frac{1}{3}}}{1+\sqrt[3]{x^2}} =$$

$$= \frac{x^{-\frac{2}{3}}(1+\sqrt[3]{x^2})-1}{\sqrt[3]{x}(1+\sqrt[3]{x^2})} = \frac{x^{-\frac{2}{3}}}{\sqrt[3]{x}(1+\sqrt[3]{x^2})} = \frac{1}{x(1+\sqrt[3]{x^2})}$$

Ответ: $3\ln\left|\frac{\sqrt[3]{x}}{\sqrt{1+\sqrt[3]{x^2}}}\right| + C$.