

~24. Для начала, найдём искомую функцию

$$f(x) = -\sin^2 x.$$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + o((x-x_0)^n).$$

$$f(x_0) = 0$$

$$f'(x_0) = 2\sin x_0 \cos x_0 = 0$$

$$f''(x_0) = 2(\cos^2 x_0 - \sin^2 x_0) = 2$$

$$f'''(x_0) = -8\sin x_0 \cos x_0 = 0$$

$$f^{(iv)}(x_0) = 8\sin^2 x_0 - 8\cos^2 x_0 = -8$$

$$f(x) = x^2 + \frac{1}{3}x^4 + o(x^4)$$

$$\lim_{x \rightarrow 0} \frac{x^2 - \sin^2 x}{x^4} = \lim_{x \rightarrow 0} \frac{x^2 + x^2 + \frac{1}{3}x^4 + o(x^4)}{x^4} = \lim_{x \rightarrow 0} \frac{x^4 \left(\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{3} \right)}{x^4} = \frac{1}{3}$$

Ответ: $\frac{1}{3}$.