№10. $\int\limits_1^{\infty} \frac{\operatorname{arctg} x}{1+x^2}\,dx = \lim\limits_{t\to\infty}\int\limits_1^{t} \frac{\operatorname{arctg} x}{1+x^2}\,dx = \lim\limits_{t\to\infty}\int\limits_1^{t} \operatorname{arctg} x \, d\operatorname{arctg} x =$

$= \lim\limits_{t\to\infty} \frac{\operatorname{arctg}^2 x}{2}\Big|_1^t = \frac{1}{2}\left(\frac{\pi^2}{4} - \frac{\pi^2}{16}\right) = \frac{1}{2}\left(\frac{3\pi^2}{16}\right) = \frac{3\pi^2}{32}$

Ответ: $\frac{3\pi^2}{32}$.