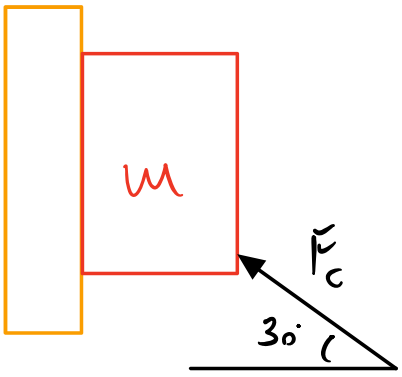
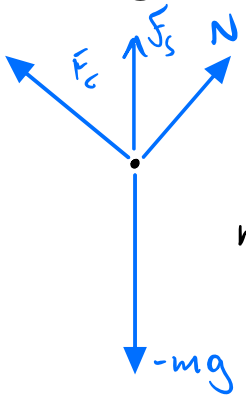
PS7:

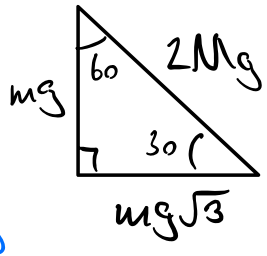
Problem 1)



when $\theta \to 30°$ $\quad \mu_s = \frac{\sqrt{3}}{5}$

$F_c = 2Mg$

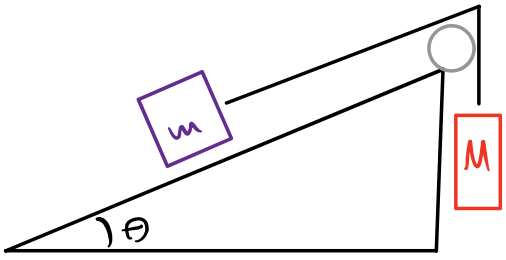


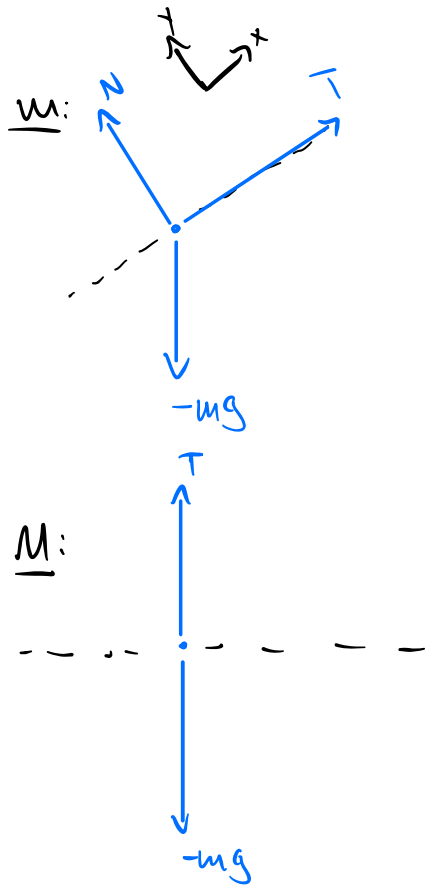


a) $\sum F_y = m\vec{a} = \boxed{-mg - \frac{\sqrt{3}}{5}}$

b) $\boxed{-mg + \frac{\sqrt{3}}{5}}$

2)

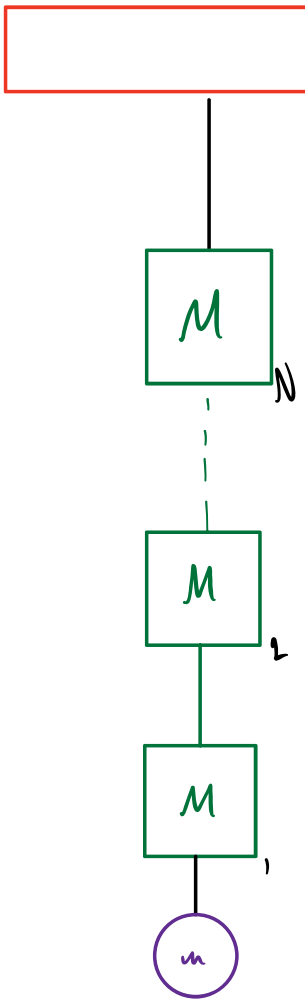




$$\sum F_{yM} = m\cancel{a}^{0} = -mg + T + \frac{1}{\sqrt{3}}$$

$$m = \frac{T + \frac{1}{\sqrt{3}}}{g} = \frac{T+1}{g\sqrt{3}}$$

3)



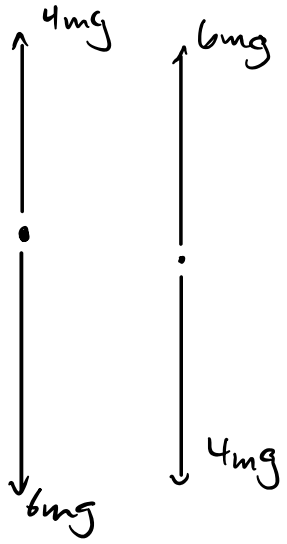
m:

$$\sum F_y = m\cancel{a_y}^{0} = T_{max} - mg$$

$T_{max} = mg$

$$\boxed{m = \frac{T_{max}}{g}}$$

4)



a)

$$\sum F_{6Y} = 6m\vec{a}_{6Y} = (5mg - t) - 6mg$$

$$\sum F_{4Y} < 4ma_{4Y} = -4mg + 5mg - t$$

$$a_{4Y} = \frac{g}{4} - \frac{t}{4m}$$

$$a_{6Y} = -\frac{g}{4} + \frac{t}{4m}$$

B)

$$\sum F_{3Y} : 3ma_{3Y} = -3mg + T$$

$$-7ma_{7Y} = 7mg - T$$

$$3ma_{3Y} = -3mg + T$$

$$-7ma_{7Y} + 3ma_{3Y} = 4mg$$

$$10ma_{3X} = 4mg$$

$$10a_{3X} = 4g$$

$$\boxed{a_{3X} = \frac{2}{5}g \qquad a_{7Y} = -\frac{2}{5}g}$$

$$6m\left(-\frac{g}{4} + \frac{t}{4m}\right) = -mg - t$$

$$-\frac{6mg}{4} + \frac{3t}{4} = -mg + t$$

$$-\frac{6mg}{4} + mg = t - \frac{3t}{4}$$

$$m\left(-\frac{1}{2}g\right) = \frac{t}{4}$$

$$m = \frac{2t}{-4g} \qquad \boxed{m = -\frac{t}{2g}}$$

5) a)

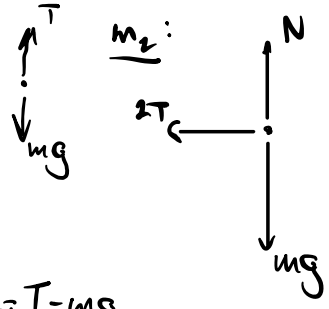
$$0 = -\frac{dy_1}{dt} + 2\frac{dx_2}{dt}$$

$$V_{y_1} = 2V_{x_2}$$

$$\frac{dV_{y_1}}{dt} = 2\frac{dV_{x_2}}{dt}$$

$$\boxed{a_{y_1} = 2a_{x_2}}$$

5b) m_1: m_2:



$$\sum F_{m_1 y} = m\vec{a}_{m_1 x} = T - mg$$

$$\sum \vec{F}_{m_1 x} = m\vec{a}_{m_1 x} = 0$$

$$\sum \vec{F}_{m_2 x} = m\vec{a}_{m_2 y} = N - mg$$

$$\sum \vec{F}_{m_2 x} = m\vec{a}_{m_2 x} - 2T$$

c) $T = m\vec{a}_{m_2 x}$

$$a_{m_1 y} = \frac{m_2 a_{m_2 x}}{m_1} - \frac{m_1 g}{m_1}$$

$$\Rightarrow \frac{m_2 \cdot a_{m_2 x}}{2m_1} - g$$

$$\Rightarrow \frac{m_2 \cdot \frac{a_{m_2 y}}{2}}{2m_1} - g$$

$$\Rightarrow \frac{m_2 \cdot a_y}{4m_1} - g = a_y$$

$$a_y\left(1 - \frac{m_2}{4m_1}\right) = -g \qquad a_y\left(\frac{4m_1 - m_2}{4m_1}\right)$$

$$\boxed{a_y = \frac{-4gm_1}{4m_1 - m_2}}$$

$$a_{x_1} = \frac{-4m_1 g}{4m_1 - m_2}$$

$$a_{x_2} = \frac{-2m_1 g}{4m_1 - m_2}$$

b) a)

$$\sum F_x : ma_x = T\cos(\theta) - f_s$$

$$\sum F_y : ma_y = T\sin(\theta) + N - mg$$

$$N = mg - T\sin(\theta)$$

$$ma_x = T\cos(\theta) - (\mu mg - \mu T\sin(\theta))$$

$$ma_x = T\cos(\theta) - \mu mg + \mu T\sin(\theta)$$

$$a_x = \frac{T\cos(\theta) - \mu mg + \mu T\sin(\theta)}{m}$$

$$a_x = \frac{T\cos\theta}{m} - mg + \frac{\mu T\sin\theta}{m}$$

$$a_x = -\mu g + \left(\frac{T\cos(\theta) + \mu T\sin\theta}{m}\right)$$

$$a_x = -\mu g + \frac{T}{m}\left(\cos\theta + \mu\sin\theta\right)$$

b) $$\frac{da_x}{dt} = 0 - \frac{T}{m}\sin(\theta) + \frac{T\mu}{m}\cos\theta$$

$$\frac{da_x}{dt} = \frac{T}{m}\left(\mu\cos(\theta)\cdot\sin\theta\right) = 0$$

$$M\cos\theta - \sin\theta = 0$$

$$\boxed{M = \tan\theta}$$

$$a^2 + b^2 = c^2$$

$$h^2 + \frac{h^2}{M^2} = T^2$$

$$T = \sqrt{\frac{M^2h^2 + h^2}{M^2}} \Rightarrow T = h\frac{\sqrt{M^2+1}}{M}$$

$$\sin\theta\Big|_{a_{max}} = \frac{h}{h\frac{\sqrt{M^2+1}}{M}} = \boxed{\frac{M}{\sqrt{M^2+1}}}$$

$$\cos\theta\Big|_{a_{max}} = \frac{\frac{h}{M}}{h\frac{\sqrt{M^2+1}}{M}} = \boxed{\frac{1}{\sqrt{M^2+1}}}$$