9.3 1-15 odd 37

1) a) $2 + \frac{2}{5} + \frac{2}{5^2} + \dots + \frac{2}{5^{k-1}} + \dots$

$S_1 = \boxed{2}$

$S_2 = \frac{10}{5} + \frac{2}{5} = \boxed{\frac{12}{5}}$

$S_3 = \frac{50}{25} + \frac{10}{25} + \frac{2}{25} = \boxed{\frac{62}{25}}$

$S_4 = \frac{2}{1} + \frac{2}{5} + \frac{2}{5^2} + \frac{2}{5^3} = \boxed{\frac{312}{125}}$

Geometric $|r| < 1$, $\boxed{\text{converge}}$ $|r| \geq 1$ diverge

$a = 2 \qquad r = \boxed{\frac{1}{5}}$

$$\frac{2}{\frac{5}{5} - \frac{1}{5}} = \frac{2}{\frac{4}{5}} = \frac{10}{4} = \frac{5}{2} \qquad \lim_{n \to \infty} S_n = \frac{5}{2}$$

Since convergent, State the sum

$$\boxed{\frac{5}{2}\left(1 - \left(\frac{1}{5}\right)^n\right)}$$

$n = 1$

$$\frac{5}{2}\left(\frac{5}{5} - \frac{1}{5}\right) = \frac{5}{2}\left(\frac{4}{5}\right) = \frac{20}{10} = 2$$

$n = 2$

$$\frac{5}{2}\left(\frac{25}{25} - \frac{1}{25}\right) = \frac{5}{2}\left(\frac{24}{25}\right) = \frac{120}{50} = \frac{12}{5}$$

b) $\frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \dots + \frac{2^{k-1}}{4}$

$S_1 = \boxed{\frac{1}{4}}$

$S_2 = \boxed{\frac{3}{4}}$

$S_3 = \boxed{\frac{7}{4}}$

$S_4 = \boxed{\frac{15}{4}}$

Geometric $\boxed{r < 1,}$ covergent $r \geq 1,$ $\boxed{\text{Divergent}}$

$a = \frac{1}{4}$ $\boxed{r = 2}$ Divergent

$\lim_{k \to \infty} S_k = \frac{\cancel{2^{k-1}}^{\infty/4}}{4} = +\infty$

Sum:

$$-\frac{1}{4}(1 - 2^n)$$

$n = 1$

$$-\frac{1}{4}(1-2) = -\frac{1}{4}(-1) = \frac{1}{4}$$

c) $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{(k+1)(k+2)} + \dots$

$S_1 = \boxed{\frac{1}{6}}$

$S_2 = \frac{1}{6} + \frac{1}{12} = \boxed{\frac{1}{4}}$

$S_3 = \frac{1}{4} + \frac{1}{20} = \frac{6}{20} = \boxed{\frac{3}{10}}$

$S_4 = \frac{3}{10} + \frac{1}{5 \cdot 6} = \frac{9}{30} + \frac{1}{30} = \boxed{\frac{1}{3}}$

$$\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$$

$$\frac{1}{(k+1)(k+2)} = \frac{A}{k+1} + \frac{B}{k+2}$$

$$1 = A(k+2) + B(k+1)$$

Let $k = -1$; let $k = -1$

$1 = A(1)$ | $1 = B(-1)$

$A = 1$ | $B = -1$

$$\frac{1}{(k+1)(k+2)} = \frac{1}{k+1} - \frac{1}{k+2}$$

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{1}{n+1} - \frac{1}{n+2}$$

$$= \lim_{n \to \infty} \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{3}}\right) \cancel{+} \cdots \cancel{+} \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2}\right)$$

$$= \lim_{n \to \infty} \frac{1}{2} - \cancel{\frac{1}{n+1}}^{\,0} = \boxed{\frac{1}{2}}$$

Sum: $\boxed{\frac{1}{2} - \frac{1}{n+2}}$

3) $\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1}$

$1, -\frac{3}{4}, \frac{9}{16}, -\frac{27}{64}$

Geometric

$a = 1 \quad r = -\frac{3}{4}$, converges

$$\frac{1}{\frac{4}{4} + \frac{3}{4}} = \boxed{\frac{4}{7}}$$

5) $\sum_{k=1}^{\infty} (-1)^{k-1} \cdot \frac{7}{6^{k-1}}$

$7, -\frac{7}{6}, \frac{7}{36}$

Geometric

$a = 7 \quad r = -\frac{1}{6} \qquad \left|-\frac{1}{6}\right| < 1$, Convergent

$$\frac{7}{\frac{6}{6} + \frac{1}{6}} = \frac{42}{7} = \boxed{6}$$

7) $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3}$

$$\frac{1}{(k+2)(k+3)} = \frac{A}{(k+2)} + \frac{B}{k+3}$$

$1 = A(k+3) + B(k+2)$

Let $k = -2$ | Let $k = -3$

$1 = A(1)$ | $1 = B(-1)$

$A = 1$ | $B = -1$

$$\sum_{k=1}^{\infty} \frac{1}{(k+2)} - \frac{1}{(k+3)}$$

$$\frac{1}{3} + \frac{1}{k+3}$$

$$\lim_{n\to\infty} \frac{1}{3} + \frac{1}{\cancel{n+3}}{}^{0}_{\infty} = \boxed{\frac{1}{3}}$$

9)
$$\sum_{k=1}^{\infty} \frac{1}{9k^2+3k-2} = \sum_{k=1}^{\infty} \frac{1}{(3k-1)(3k+2)}$$

$$\frac{1}{(3k-1)(3k+2)} = \frac{A}{3k-1} + \frac{B}{3k+2}$$

$$1 = A(3k+2) + B(3k-1)$$

Let $k = -\frac{2}{3}$

$$1 = B\left(-\frac{6}{3} - 1\right) = B\left(-\frac{9}{3}\right) = -3B$$

$$\boxed{B = -\frac{1}{3}}$$

Let $k = \frac{1}{3}$

$$1 = A(3)$$

$$\boxed{A = \frac{1}{3}}$$

$$\sum_{k=1}^{\infty} \frac{1}{(3k-1)(3k+2)} = \sum_{k=1}^{\infty} \frac{\frac{1}{3}}{3k-1} - \frac{\frac{1}{3}}{3k+2} = \sum_{k=1}^{\infty} \frac{1}{9k-3} - \frac{1}{9k+6}$$

$$\frac{1}{6} - \frac{1}{15} + \frac{1}{15} - \frac{1}{24}$$

$\frac{1}{6} - \frac{1}{9n+6}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{6} - \frac{1}{\cancel{9n+6}}^{\,0}_{\,\infty} = \boxed{\frac{1}{6}}$ Convergent

11) $\sum_{k=3}^{\infty} \frac{1}{k-1}$

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Harmonic Series $\Rightarrow$ Diverges

13) $\sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}} = \frac{4^3}{1} + \frac{4^4}{7} + \frac{4^5}{49}$

Geometric Series

$a = 64 \quad r = \frac{4}{7}$

$\frac{64}{\frac{7}{7} - \frac{4}{7}} = \frac{64}{\frac{3}{7}} = \boxed{\frac{448}{3}}$

15)
a) 5 b) 3

d) 9 c) 7

37) $\sum_{k=1}^{\infty} \frac{1}{k} - \frac{1}{k+2} = \left(1 - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \dots + \left(\frac{1}{k+1} - \frac{1}{k+2}\right)$

$$\lim_{n \to \infty} \frac{3}{2} - \cancel{\frac{1}{n+2}}^{0} = \boxed{\frac{3}{2}}$$