9.5 1-49 EOO

1) Guess: convergent

a) $\sum_{k=1}^{\infty} \frac{1}{5k^2-k}$

Divergence Test

let $f(x) = \frac{1}{5x^2-x}$

$$\lim_{x\to\infty} \frac{1}{5x^2-x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\cancel{\frac{1}{x^2}}^{0}}{5-\cancel{\frac{1}{x}}^{0}} = 0$$

Inconclusive

$$\sum_{k=1}^{\infty} \frac{1}{5k^2-k} > \frac{1}{5}\sum \frac{1}{k^2}$$ convergent p-series

limit comparison test

$$c = \lim_{k\to\infty} \frac{1}{5k^2-k} \cdot \frac{5k^2}{1}$$

$$= \lim_{k\to\infty} \frac{5k}{5k-1} = \lim_{k\to\infty} \frac{5}{5-\cancel{\frac{1}{k}}^{0}} = 1$$

$\therefore \sum_{k=1}^{\infty} \frac{1}{5k^2-k}$ & $\sum_{k=1}^{\infty} \frac{1}{5k^2}$ will converge together

by limit comparison test

b) $\sum_{k=1}^{\infty} \frac{3}{k-\frac{1}{4}} > \sum_{k=1}^{\infty} \frac{3}{k}$ $p=1$, Divergent p-series

limit comparison test

$c = \lim_{k\to\infty} \frac{\cancel{3}}{k-\frac{1}{4}} \cdot \frac{k}{\cancel{3}} = \lim_{k\to\infty} \frac{k}{k-\frac{1}{4}} = \lim_{k\to\infty} \frac{1}{1-0}$

$= \boxed{1}$

$\therefore$ By LCT $\sum_{k=1}^{\infty} \frac{3}{k-\frac{1}{4}}$ & $\sum_{k=1}^{\infty} \frac{3}{k}$ will diverge together

5) $\sum_{k=1}^{\infty} \underbrace{\frac{4k^2-2k+6}{8k^7+k-8}}_{a_n}$ $\qquad \sum_{k=1}^{\infty} \underbrace{\frac{4k^2}{8k^7}}_{b_n} = \sum^{\infty} \frac{1}{2k^5}$

$c = \lim_{k\to\infty} \frac{4k^2-2k+6}{8k^7+k-8} \cdot \frac{2k^5}{1}$

$= \lim_{k\to\infty} \frac{8k^7-4k^6+6k^5}{8k^7+k-8} \cdot \frac{\frac{1}{k^7}}{\frac{1}{k^7}} = \lim_{k\to\infty} \frac{\frac{8\cancel{k^7}}{\cancel{k^7}} - \cancel{\frac{4k^6}{k^7}}^{0} + \cancel{\frac{6k^5}{k^7}}^{0}}{\frac{8\cancel{k^7}}{\cancel{k^7}} + \cancel{\frac{k}{k^7}}^{0} - \cancel{\frac{8}{k^7}}^{0}}$

$$= \frac{8}{8} = 1 \qquad \therefore \sum_{k=1}^{\infty} \frac{4k^2 - 2k + 6}{8k^7 + k - 8} \;\&\; \sum_{k=1}^{\infty} \frac{4k^2}{8k^7}$$

Will converge together since $\sum_{k=1}^{\infty} \frac{1}{2k^5}$ is a convergent p-series

9)

$$a_n = \sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{8k^2 - 3k}} \qquad b_n = \frac{1}{(8k^2)^{1/3}}$$

Divergent p-series

$$c = \lim_{k \to \infty} \frac{1}{(8k^2)^{1/3} - (3k)^{1/3}} \cdot \frac{(8k^2)^{1/3}}{1}$$

$$= \lim_{k \to \infty} \frac{8^{1/3} \cdot k^{2/3}}{(8)^{1/3} \cdot (k^{2/3}) - (3^{1/3})(k)^{1/3}}$$

$$= \lim_{k \to \infty} \frac{8^{1/3} \cdot k^{\cancel{2}/3}}{\cancel{k}^{1/3}\left[(8k)^{1/3} - (3^{1/3})\right]}$$

1/3

$$= \lim_{k \to \infty} \frac{(8k)^{1/3}}{(8k)^{1/3} - (3^{1/3})} \cdot \frac{\frac{1}{k^{1/3}}}{\frac{1}{k^{1/3}}}$$

$$= \lim_{k\to\infty} \frac{8^{1/3}}{8^{1/3} - \sqrt[3]{\cancel{\frac{3}{k}}^{0}}} = \boxed{1}$$

$\therefore$ a_n & b_n will diverge by comparison test since b_n is a divergent p-series ($p = 2/3$)

13) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k}$ ✓ Divergent p-series $p=1$

$\therefore \displaystyle\sum_{k=1}^{\infty} \frac{1}{5k}$ is divergent

17) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$

$$c = \lim_{k\to\infty} \cancel{\sqrt[k]{\left(\frac{3k+2}{2k-1}\right)^{k}}} = \lim_{k\to\infty} \frac{3k+2 \cdot \frac{1}{k}}{2k-1 \cdot \frac{1}{k}}$$

$$= \lim_{k\to\infty} \frac{3 + \cancel{\frac{2}{k}}^{0}}{2 - \cancel{\frac{1}{k}}^{0}} = \frac{3}{2}$$

$\therefore \displaystyle\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ diverges by root test

21) False; limit comparison test takes two sequences and takes a limit of their quotient

25) $\sum_{k=0}^{\infty} \frac{7^k}{k!}$

Ratio test

$$c = \lim_{k \to \infty} \frac{7^{k+1}}{(k+1)!} \cdot \frac{k!}{7^k} = \lim_{k \to \infty} \frac{\cancel{7^k} \cdot 7}{(k+2)\cancel{k!}} \cdot \frac{\cancel{k!}}{\cancel{7^k}}$$

$$= \lim_{k \to \infty} \frac{7}{k+1} = 0 \quad \therefore \sum_{k=0}^{\infty} \frac{7^k}{k!} \text{ converges}$$

by ratio test

29) $\sum_{k=1}^{\infty} \frac{k^{50}}{e^k}$

ratio test

$$c = \lim_{k \to \infty} \frac{(k+1)^{50}}{e^{k+1}} \cdot \frac{e^k}{k^{50}}$$

$$= \lim_{k\to\infty} \frac{(k+1)^{50}}{\cancel{e^k} \cdot e^1} \cdot \frac{\cancel{e^k}}{k^{50}} = \lim_{k\to\infty} \frac{(k+1)^{50}}{e k^{50}}$$

$$= \frac{1}{e} \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^{50} = \frac{1}{e} \lim_{k\to\infty} \left(\frac{1+\frac{1}{k}}{1}\right)^{50}$$

$$= \frac{1}{e} \cdot 1^{50} = \boxed{\frac{1}{e}} < 1 \quad \therefore \sum_{k=1}^{\infty} k^{50} \cdot e^{-k} \text{ converges}$$

by ratio test

33) $$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}} = \sum_{k=1}^{\infty} \frac{1}{(k^2+k)^{1/2}}$$

$$= \sum_{k=1}^{\infty} \frac{1}{k + k^{1/2}} < \sum_{k=1}^{\infty} \frac{1}{k}$$ Divergent p-series

$\therefore \sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$ is divergent by comparison test

37)

$$\sum_{1=1}^{\infty} \frac{1}{1+(k)^{1/2}}$$

divergence Test

$$\lim_{k\to\infty} \frac{1}{1+(k)^{1/2}} = 0 \quad \therefore \text{ inconclusive}$$

$$\sum_{k=1}^{\infty} \frac{1}{1+k^{1/2}} < \sum_{k=1}^{\infty} \frac{1}{k^{1/2}} \swarrow \text{Divergent p-series}$$

$\therefore \sum_{k=1}^{\infty} \frac{1}{1+\sqrt{k}}$ diverges by comparison test

41) $$\sum_{k=0}^{\infty} \frac{(k+4)!}{4!\,k!\,4^k} = \sum_{k=0}^{\infty} \frac{(k+4)\cancel{k!}}{4!\,\cancel{k!}\,4^k} = \sum_{k=0}^{\infty} \frac{k+4}{4!\,4^k}$$

ratio test

$$c = \lim_{k\to\infty} \frac{k+5}{4!\,4^{(k+1)}} \cdot \frac{4!\,4^k}{k+4}$$

$$= \lim_{k\to\infty} \frac{k+5}{\cancel{4!}\,\cancel{4^k}\cdot 4} \cdot \frac{\cancel{4!}\,\cancel{4^k}}{k+4} = \lim_{k\to\infty} \frac{k+5}{4(k+4)} = \lim_{k\to\infty} \frac{1+\cancel{\frac{5}{k}}^{0}}{4+\cancel{\frac{16}{k}}^{0}}$$

$$= \frac{1}{4} \qquad \therefore \sum_{k=0}^{\infty} \frac{(k+4)!}{4!\,k!\,4^k} \text{ is convergent by ratio test}$$

45)

$$\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k^2}$$

$$\lim_{k\to\infty} \frac{\tan^{-1} k}{k^2} = 0, \text{ Inconclusive}$$

$$C = \lim_{k\to\infty} \frac{\tan^{-1}(k+1)}{(k+1)^2} \cdot \frac{k^2}{\tan^{-1}(k)}$$

$$= \lim_{k\to\infty} \frac{\tan^{-1}(k+1)}{k^2+k+k+1} \cdot \frac{k^2}{\tan^{-1}(k)}$$

$$= \lim_{k\to\infty} \frac{\overset{\frac{\pi}{2}}{\cancel{\tan^{-1}(k+1)}} \cdot k^2}{(k^2+2k+1) \cdot \underset{\pi/2}{\cancel{\tan^{-1}(k)}}}$$

$$= \lim_{k\to\infty} \frac{k^2}{k^2+2k+1} = \lim_{k\to\infty} \frac{1}{1+\cancel{\frac{2}{k}}^{0}+\cancel{\frac{1}{k^2}}^{0}} = 1$$

$\therefore$ the ratio test is inconclusive

$$\sum_{k=1}^{\infty} \frac{\tan^{-1}(k)}{k^2} \leq \frac{\pi}{2} \sum_{k=1}^{\infty} \frac{1}{k^2}$$ convergent p-series

So, by comparison test, any smaller series will also be convergent

$$\therefore \sum_{k=1}^{\infty} \frac{\tan^{-1}(k)}{k^2}$$ is convergent

49) $$\sum_{k=1}^{\infty} \frac{\ln k}{3^k}$$

$$c = \lim_{k \to \infty} \frac{\ln(k+1)}{3^{(k+1)}} \cdot \frac{3^k}{\ln(k)}$$

$$= \lim_{k \to \infty} \frac{\ln(k+1)}{\cancel{3k}+3} \cdot \frac{\cancel{3k}}{\ln k} = \lim_{k \to \infty} \frac{\ln(k+1)}{3 \ln k}$$

$$= \frac{1}{3} \lim_{k \to \infty} \frac{\ln(k+1)}{\ln k} \overset{H}{=} \frac{\frac{1}{k+1}}{\frac{1}{k}} = \frac{k}{k+1} = \boxed{1}$$

$\therefore$ Ratio test is inconclusive

$$c = \lim_{k \to \infty} \sqrt[k]{\frac{\ln k}{3^k}} = \lim_{k \to \infty} \frac{\sqrt[k]{\ln k}}{3}$$

$$= \lim_{k \to \infty} \frac{(\ln k)^{1/k}}{3} = \boxed{\frac{1}{3}} < 1$$

$\therefore$ convergent by root test