9.6 1-25 odd

1)
$$\sum_{k=1}^{\infty} (-1)^{k+1} \cdot \frac{1}{2k+1}$$

1) positive because $x > 1$ & fraction is never negative

2) $\frac{1}{2(k+1)+1} \cdot \frac{2k+1}{1} = \frac{2k+1}{2k+3}$ always < 1

3) $\lim_{k \to \infty} \frac{1}{2k+1} = 0$ $\therefore$ AST converges

3)
$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$$

1) Positive for $k > 1$

2) $\frac{k+3}{3(k+1)+1} \cdot \frac{3k+1}{k+1} = \frac{(3k+1)(k+3)}{(3k+4)(k+1)} = \frac{3k^2+9k+k+3}{3k^2+3k+4k+4}$

$= \frac{3k^2+10k+3}{3k^2+7k+4} > 1$ Since a_n is Increasing $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$ is divergent

5) $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$

1) positive for all $k > 1$

2) $\dfrac{1}{e^{k+1}} \cdot \dfrac{e^k}{1} = \dfrac{e^k}{e^k \cdot e} = \dfrac{1}{e} < 1$

3) $\displaystyle\lim_{k\to\infty} \frac{1}{e^k} = 0$ $\quad\therefore$ Series converges by AST

7) $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k = \sum_{k=1}^{\infty} -\frac{3^k}{5^k}$

$$c = \lim_{k\to\infty} \left| -\frac{3^{(k+1)}}{5^{(k+1)}} \cdot -\frac{5^k}{3^k} \right|$$

$$= \lim_{k\to\infty} \frac{\cancel{3^k}\cdot 3}{\cancel{5^k}\cdot 5} \cdot \frac{\cancel{5^k}}{\cancel{3^k}} = \frac{3}{5} < 1$$

$\therefore$ Series absolutely converges by abs. ratio test

9) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^k}{k^2} = a_n$

$|a_n| = \sum_{k=1}^{\infty} \frac{3^k}{k^2}$

$c = \lim_{k\to\infty} \frac{3^{(k+1)}}{(k+1)^2} \cdot \frac{k^2}{3^k}$

$= \lim_{k\to\infty} \frac{\cancel{3^k}\cdot 3}{k^2+k+k+1} \cdot \frac{k^2}{\cancel{3^k}}$

$= \lim_{k\to\infty} \frac{3k^2}{k^2+2k+1} = 3 > 1$ $\therefore$ series diverges by absolute ratio test

11) $\sum_{k=1}^{\infty} (-1)^k \frac{k^3}{e^k} = a_n$

$|a_n| = \sum_{k=1}^{\infty} \frac{k^3}{e^k}$

$c = \lim_{k\to\infty} \frac{(k+1)^3}{e^{(k+1)}} \cdot \frac{e^k}{k^3}$

$= \lim_{k\to\infty} \frac{(k+1)^3}{\cancel{e^k}\cdot e} \cdot \frac{\cancel{e^k}}{k^3} = \lim_{k\to\infty} \frac{(k+1)^3}{ek^3}$

$$= \frac{1}{e} \lim_{k \to \infty} \left(\frac{k+1}{k}\right)^3 = \frac{1}{e} \cdot \frac{1}{1} = \frac{1}{e} < 1$$

$\therefore$ Series converges by ratio test

13) $$\sum_{k=1}^{\infty} (-1)^{k+1} \cdot \frac{1}{3k} = a_n$$

$$|a_n| = \sum_{k=1}^{\infty} \frac{1}{3k}$$

$$c = \frac{1}{\cancel{3}(k+1)} \cdot \frac{\cancel{3}k}{1} = \frac{k}{k+1} = 1$$

AST:

1) Positive $\forall$ $k > 1$

2) $$\frac{1}{\cancel{3}(k+1)} \cdot \frac{\cancel{3}k}{1} = \frac{k}{k+1} < 1$$

3) $$\lim_{k \to \infty} \frac{1}{3k} = 0$$

$\therefore$ Conditionally convergent by AST

15) $$\sum_{k=1}^{\infty} \frac{(-4)^k}{k^2} = a_n$$

$$|a_n| = \sum_{k=1}^{\infty} \frac{4^k}{k^2}$$

$$c = \lim_{k\to\infty} \frac{4^{k+1}}{(k+1)^2} \cdot \frac{k^2}{4^k}$$

$$c = \lim_{k\to\infty} \frac{\cancel{4^k}\cdot 4}{(k+1)^2} \cdot \frac{k^2}{\cancel{4^k}} = \lim_{k\to\infty} \frac{4k^2}{(k+1)^2}$$

$$= \lim_{k\to\infty} \frac{4k^2}{k^2+k+k+1} = \lim_{k\to\infty} \frac{4k^2}{k^2+2k+1} =$$

$$= \lim_{k\to\infty} \frac{4}{1+\cancel{\frac{2}{k}}^{0}+\cancel{\frac{1}{k^2}}^{0}} = 4$$ $\therefore$ Series diverges by ratio test

17) $$\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k} = \sum_{k=1}^{\infty} \frac{(-1)^k}{k}$$

AST:

1) a_n is always positive $k>1$
2) $f'(x) = \frac{1}{x} = \frac{1}{-x^2}$ always decreasing

3) $\lim_{k \to \infty} \frac{1}{k} = 0 \quad \therefore \sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k}$

is conditionally convergent

19) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+2}{k(k+3)} = a_n$

$|a_n| = \frac{k+2}{k(k+3)}$

$c = \lim_{k \to \infty} \frac{k+3}{(k+1)(k+4)} \cdot \frac{k(k+3)}{k+2}$

$= \lim_{k \to \infty} \frac{(k+3) \cdot (k^2+3k)}{(k^2+4k+k+4)(k+2)}$

$= \lim_{k \to \infty} \frac{k^3+3k^2+3k^2+9k}{k^3+4k^2+k^2+4+2k^2+8k+2k+8}$

$$= \lim_{k\to\infty} \frac{k^3+6k^2+9k}{k^3+7k^2+10k+12} \cdot \frac{1/k^3}{1/k^3} = 1$$

$\therefore$ Inconclusive

$\therefore$

$$|a_n| = \lim_{k\to\infty} \frac{k+2}{k(k+3)} \qquad \sum_{k=1}^{\infty} \frac{k}{k^2} = \sum_{k=1}^{\infty} \frac{1}{k}$$

$$c = \lim_{k\to\infty} \frac{k+2}{\cancel{k}(k+3)} \cdot \frac{\cancel{k}}{1} = \lim_{k\to\infty} \frac{k+2}{k+3} \cdot \frac{1/k}{1/k} = 1$$

$\therefore$ divergent by abs ratio test

AST:

$$\sum_{k=1}^{\infty} \frac{k+2}{k(k+3)} = \sum_{k=1}^{\infty} \frac{k+2}{k^2+3k}$$

1) Positive $k > 1$

2)

$$\frac{k+3}{(k+1)^2+3(k+1)} \cdot \frac{k^2+3k}{k+2}$$

$$\frac{k+3}{k^2+k+k+1+3k+3} \cdot \frac{k^2+3k}{k+2}$$

$$\frac{(k+3)(k^2+3k)}{(k^2+5k+4)(k+2)}$$

$$= \frac{k^3+3k^2+3k^2+9k}{k^3+5k^2+4k+2k^2+10k+8}$$

$$= \frac{k^3+6k^2+9k}{k^3+7k^2+14k+8} < 1$$

3)

$$\lim_{k\to\infty} \frac{k+2}{k(k+3)} = \lim_{k\to\infty} \frac{k+2 \cdot 1/k^2}{k^2+3k \cdot 1/k^2}$$

$$= \lim_{k\to\infty} \frac{\cancel{\frac{1}{k}}^{0} + \cancel{\frac{2}{k^2}}^{0}}{1+\cancel{\frac{3}{k}}^{0}} = 0$$

$\therefore$ Series is conditionally convergent

21) $$\sum_{k=1}^{\infty} \sin\frac{k\pi}{2}$$

1, 0, -1, 0

$\lim_{k\to\infty} \sin\left(\frac{k\pi}{2}\right)$ DNE, ocillates between 1 & -1

$\therefore \sum_{k=1}^{\infty} \sin\frac{k\pi}{2}$ diverges

23) $\sum_{k=2}^{\infty} (-1)^k \cdot \frac{1}{k\ln k} = a_k$

AST:

1) Positive $k > 1$

2) Decreasing

3) $\lim_{k \to \infty} \frac{1}{k\ln k} = 0$

$\therefore$ conditionally convergent

25) $\sum_{k=2}^{\infty} \left(-\frac{1}{\ln k}\right)^k$

$c = \lim_{k \to \infty} \sqrt[k]{\left|\left(-\frac{1}{\ln k}\right)^k\right|}$

$= \lim_{k \to \infty} \frac{1}{\ln k} = 0$

$\therefore$ Series absolutely convergent by root test