9.1: Sequences HW #'s 1, 7-27EOO

1) a) $\{a_n\} = \frac{1}{n^2}$ b) $\{a_n\} = \frac{(-1)^{n+1}}{n^2}$ c) $\{a_n\} = \frac{2n-1}{2(n)}$ d) $\{a_n\} = \frac{n^2}{\pi^{1/n+1}}$

7) $\left\{\frac{n}{n+2}\right\}_{n=1}^{+\infty} = \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}$; converges

Let $f(x) = \frac{x}{x+2}$

$\lim\limits_{x\to\infty} \frac{x}{x+2}$ (type $\frac{\infty}{\infty}$) $\overset{H}{=} \frac{1}{1} = 1$ $\therefore \lim\limits_{n\to\infty} \frac{N}{N+2} = 1$

11) $\{a_n\} = \left\{\frac{\ln n}{N}\right\}_{n=1}^{+\infty}$ $\ln(1), \frac{\ln(2)}{2}, \frac{\ln(3)}{3}, \frac{\ln(4)}{4}, \frac{\ln(5)}{5},$ converges

$\lim\limits_{n\to\infty} \frac{\ln(N)}{N}$ Let $f(x) = \frac{\ln(x)}{x}$

$\lim\limits_{x\to\infty} \frac{\ln x}{x}$ (type $\frac{\infty}{\infty}$) $\overset{H}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{\cancel{x}}^{0}}{1} = 0$ $\therefore \lim\limits_{n\to\infty} \frac{\ln(n)}{n} = 0$

15) $\left\{(-1)^n \cdot \frac{2n^3}{n^3+1}\right\}_{n=1}^{+\infty}$ $-2, \frac{16}{9}, -\frac{54}{28}, \frac{128}{65}, -\frac{250}{126}$; diverges

19) $\{n^2e^{-n}\}_{n=1}^{+\infty}$ $\frac{1}{e}, \frac{4}{e^2}, \frac{9}{e^3}, \frac{16}{e^4}, \frac{25}{e^5}$ converges

$\lim\limits_{n\to\infty} \{n^2e^{-n}\}_{n=1}^{\infty}$ Let $f(x) = x^2e^{-x}$

$\lim\limits_{x\to\infty} x^2e^{-x} = \lim\limits_{x\to\infty} \frac{x^2}{e^x} \left(\text{type } \frac{\infty}{\infty}\right) \overset{H}{=} \lim\limits_{x\to\infty} \frac{2x}{e^x} \overset{H}{=} \lim\limits_{x\to\infty} \frac{2}{\cancel{e^x}\,\infty} = 0 \quad \therefore \lim\limits_{n\to\infty} n^2e^{-n} = 0$

23) $\left\{\frac{2n-1}{2n}\right\}_{n=1}^{+\infty}$ converges

$\lim\limits_{n\to\infty} \frac{2n-1}{2n}$ Let $f(x) = \frac{2x-1}{2x}$

$\lim\limits_{x\to\infty} \frac{2x-1}{2x} \left(\text{type } \frac{\infty}{\infty}\right) \overset{H}{=} \lim\limits_{x\to\infty} \frac{2}{2} = \boxed{1} \quad \therefore \lim\limits_{n\to\infty} \frac{2n-1}{2n} = \underline{1}$

27) $\left\{(-1)^{n+1} \cdot \left(\frac{1}{n} - \frac{1}{n+1}\right)\right\}_{n=1}^{+\infty}$

$\lim\limits_{n\to\infty} (-1)^{\cancel{n+1}\,\infty} \cdot \left(\frac{1}{\cancel{n}\,\infty} - \frac{1}{\cancel{n}\,\infty+1}\right)$

$-1^{\infty} \cdot 0 = \boxed{0}$