# 1 Method

The Hamiltonian for the we are using has the following form

$$H = \sum_{i}^{N} \left( \frac{-\hbar^2}{2m} \nabla_i^2 + V_{ext}(\boldsymbol{r}_i) \right) + \sum_{i<j}^{N} V_{int}(\boldsymbol{r}_i, \boldsymbol{r}_j) \tag{1}$$

where the external potential given by the boson trap

$$V_{ext}(\boldsymbol{r}) = \begin{cases} \frac{1}{2} m \omega_{ho}^2 r^2 & \text{Spherical} \\ \frac{1}{2} m [\omega_{ho}^2 (x^2 + y^2) + \omega_z z^2] & \text{Elliptical} \end{cases} \tag{2}$$

and a repulsive potential due to bosons interaction given by

$$V_{int}(|\boldsymbol{r}_i - \boldsymbol{r}_j|) = \begin{cases} \inf & |\boldsymbol{r}_i - \boldsymbol{r}_j| \leq a \\ 0 & |\boldsymbol{r}_i - \boldsymbol{r}_j| > a \end{cases} \tag{3}$$

As for the trial wavefunction for the ground state with $N$ atoms

$$\Psi_T(\mathbf{R}) = \Psi_T(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N, \alpha, \beta) = \prod_i g(\alpha, \beta, \mathbf{r}_i) \prod_{i<j} f(a, |\mathbf{r}_i - \mathbf{r}_j|), \tag{4}$$

with $\alpha, \beta$ as variational parameters. The correlation function $f(a, |\mathbf{r}_i - \mathbf{r}_j|)$ is given by

$$f(a, |\mathbf{r}_i - \mathbf{r}_j|) = \begin{cases} 0 & |\mathbf{r}_i - \mathbf{r}_j| \leq a \\ (1 - \frac{a}{|\mathbf{r}_i - \mathbf{r}_j|}) & |\mathbf{r}_i - \mathbf{r}_j| > a. \end{cases} \tag{5}$$

Benchmarking for $10^6$ cycles for analytical and numerical solutions

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 5.000000e-01 | 0.000000e+00 | 0.996284 | 723 |
| 10 | 5.000000e+00 | 0.000000e+00 | 0.996449 | 920 |
| 100 | 5.000000e+01 | 0.000000e+00 | 0.996320 | 2715 |
| 500 | 2.500000e+02 | 0.000000e+00 | 0.996494 | 10450 |

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.000000e+00 | 0.000000e+00 | 0.996363 | 746 |
| 10 | 1.000000e+01 | 0.000000e+00 | 0.996489 | 1090 |
| 100 | 1.000000e+02 | 0.000000e+00 | 0.996347 | 4204 |
| 500 | 5.000000e+02 | 0.000000e+00 | 0.996431 | 18594 |

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.500000e+00 | 0.000000e+00 | 0.996373 | 780 |
| 10 | 1.500000e+01 | 0.000000e+00 | 0.996321 | 1198 |
| 100 | 1.500000e+02 | 0.000000e+00 | 0.996450 | 5396 |
| 500 | 7.500000e+02 | 0.000000e+00 | 0.996357 | 24110 |

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 5.000000e-01 | 0.000000e+00 | 0.968346 | 473 |
| 10 | 5.000000e+00 | 0.000000e+00 | 0.968118 | 593 |
| 100 | 5.000000e+01 | 0.000000e+00 | 0.968481 | 1837 |
| 500 | 2.500000e+02 | 0.000000e+00 | 0.968320 | 7329 |

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.000000e+00 | 0.000000e+00 | 0.968367 | 470 |
| 10 | 1.000000e+01 | 0.000000e+00 | 0.967963 | 711 |
| 100 | 1.000000e+02 | 0.000000e+00 | 0.968089 | 2808 |
| 500 | 5.000000e+02 | 0.000000e+00 | 0.968799 | 12433 |

| Analytical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.500000e+00 | 0.000000e+00 | 0.968941 | 484 |
| 10 | 1.500000e+01 | 0.000000e+00 | 0.967473 | 792 |
| 100 | 1.500000e+02 | 0.000000e+00 | 0.968197 | 3623 |
| 500 | 7.500000e+02 | 0.000000e+00 | 0.969128 | 16629 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 4.999999e-01 | 1.915135e-15 | 0.996353 | 839 |
| 10 | 4.999999e+00 | -2.351896e-12 | 0.996342 | 2724 |
| 100 | 4.999999e+01 | 3.310561e-10 | 0.996470 | 68701 |
| 500 | 2.500000e+02 | 5.456968e-10 | 0.996473 | 1410677 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 9.999999e-01 | 2.009504e-14 | 0.996349 | 1026 |
| 10 | 9.999999e+00 | 4.661160e-12 | 0.996371 | 6074 |
| 100 | 9.999999e+01 | -7.275958e-12 | 0.996320 | 228597 |
| 500 | 4.999999e+02 | -1.082662e-08 | 0.996328 | 5160175 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.500000e+00 | 1.243450e-13 | 0.996377 | 1198 |
| 10 | 1.500000e+01 | 6.025402e-12 | 0.996434 | 9108 |
| 100 | 1.500000e+02 | 1.615263e-09 | 0.996397 | 429843 |
| 500 | 7.499999e+02 | -9.709038e-08 | 0.996334 | 9671595 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 5.000000e-01 | 3.122502e-14 | 0.968660 | 600 |
| 10 | 5.000000e+00 | -3.304024e-13 | 0.968201 | 2433 |
| 100 | 5.000000e+01 | 6.593837e-11 | 0.967691 | 67993 |
| 500 | 2.500000e+02 | 3.885361e-09 | 0.968383 | 1390252 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 9.999999e-01 | 6.494805e-14 | 0.968163 | 751 |
| 10 | 9.999999e+00 | 1.747935e-12 | 0.968789 | 5478 |
| 100 | 9.999999e+01 | 1.618901e-10 | 0.968061 | 235928 |
| 500 | 5.000000e+02 | 2.732850e-08 | 0.969589 | 5144598 |

| Numerical | | | | |
|---|---|---|---|---|
| N particles | $< E >$ | Variance | Accepted | Time [ms] |
| 1 | 1.500000e+00 | 1.509903e-14 | 0.968088 | 917 |
| 100 | 1.500000e+02 | 2.619345e-10 | 0.967844 | 415179 |
| 100 | 1.500000e+02 | -7.275958e-11 | 0.968416 | 413230 |
| 500 | 7.499999e+02 | -3.352761e-08 | 0.969971 | 9641421 |