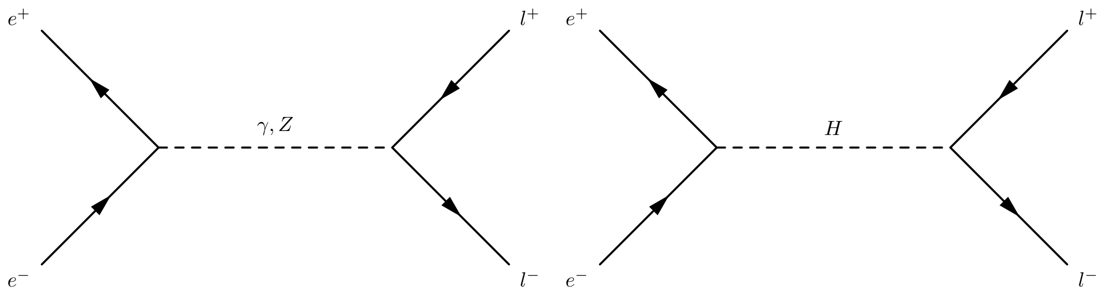
1 Di-lepton production in e^+e^- in the SM

1.1 Feynman graphs



e^+e^- annihiliation into γ is the more favourable choice compared to Z and H due to the mass difference in these.

1.2 Final states

The e and μ are stable particles and will therefore be detected as particle paths in the calorimeters. τ has a short life time and will therefore most likely decay before hitting the detector. Some decay modes from τ are

$\pi^\pm + \pi^0 + \nu_\tau$: 25.52 %

$e^\pm + \nu_e + \nu\tau$: 17.83 %

$\mu^\pm + \nu_\mu + \nu\tau$: 17.41 %

$\pi^\pm + \nu_\tau$: 10.83 %

$\pi^\pm + 2\pi^0 + \nu_\tau$: 9.30 %

$K^\pm + \nu_\tau$: 7.00 %

1.3 QED

Using the Feynman rules for scalar QED we get an amplitude

$$i\mathcal{M} = \bar{v}^{s'}(p')(-ie\gamma^\mu)u^s(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^r(k)(-ie\gamma^\nu)v^{r'}(k') \tag{1}$$

Where p and p' are the momentum for the incoming e^+e^-,and k and k' are the momentums of the outgoing $\tau^+\tau^-$. s and r are the spin indicies, q is the momentum of the force mediator and e is the elementary charge. On a more compact form leaving the spin superscripts implicit

$$i\mathcal{M} = \frac{ie^2}{q^2}\left(\bar{v}(p')\gamma^\mu u(p)\right)\left(\bar{u}(k)\gamma_\mu v(k')\right) \tag{2}$$

Then using $(\bar{v}\gamma^\mu u)^* = \bar{u}\gamma^\mu v$ to get the $|\mathcal{M}|^2$ leads to

$$|\mathcal{M}|^2 = \frac{e^4}{q^4}\Big(\bar{v}(p')\gamma^\mu u(p)\bar{u}(k)\gamma^\nu v(k')\Big)\Big(\bar{u}(k)\gamma_\mu v(k')\bar{v}(p')\gamma_\nu u(p)\Big) \tag{3}$$

Now taking the spins into account we have an expression on the form

$$\frac{1}{4}\sum_s\sum_{s'}\sum_r\sum_{r'}|\mathcal{M}(s,s'\to r,r')|^2. \tag{4}$$

With the 2 completeness relations

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m\,; \qquad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m \tag{5}$$

used in the the first parenthesis in equation (3) written out in spinor indicies, making it possible to move v and $\bar{v}$ next to each other we get

$$\sum_{s,s'} \bar{v}_a^{s'}(p')\gamma^\mu_{ab} u_b^s(p)\bar{u}_c^s(p)\gamma^\nu_{cd} v_d^{s'}(p') = (\not{p}' - m)_{da}\gamma^\mu_{ab}(\not{p} + m)_{bc}\gamma^\nu_{cd}$$

$$= \mathrm{tr}[(\not{p}' - m)\gamma^\mu(\not{p} + m)\gamma^\nu]$$

thus our squared matrix element becomes the product of 2 traces

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4q^4}\mathrm{tr}\left[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right]\mathrm{tr}\left[(\not{k} + m_\tau)\gamma_\mu(\not{k}' - m_\tau)\gamma_\nu\right] \tag{6}$$

Insert trace technology here Using the trace relations we get for the e part

$$\mathrm{tr}\left[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right] = 4\left[p^{'\mu}p^\nu + p^{'\nu}p^\mu - g^{\mu\nu}(p \cdot p' + m_e^2)\right] \tag{7}$$

and for the τ

$$\mathrm{tr}\left[(\not{k} + m_\tau)\gamma_\mu(\not{k}' + m_\tau)\gamma_\nu\right] = 4\left[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(k \cdot k' + m_\tau^2)\right] \tag{8}$$

There is a large difference in $m_e << m_\tau$ which makes it reasonable to set $m_e = 0$, multiplying the traces gives us a square matrix element of

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4}{q^4}\left[(p \cdot k)(p' \cdot k') + (p \cdot k')(p' \cdot k) + m_\tau^2(p \cdot p')\right] \tag{9}$$

Furterhmore we choose the center of mass reference frame and translates our momentums into kinematic variables instead, energies and angles.

$$\begin{array}{ccc} q^2 = (p + p')^2 = 4E^2 & ; & p \cdot p' = 2E^2 \\ p \cdot k = E^2 - E|\mathbf{k}|\cos\theta & ; & p \cdot k' = E^2 + E|\mathbf{k}|\cos\theta \end{array} \tag{10}$$

Rewriting eq. (9) in terms of E and θ we get

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4}{16E^4}\left[E^2(E - |\mathbf{k}|\cos\theta)^2 + E^2(E + |\mathbf{k}|\cos\theta)^2 + 2m_\tau^2 E^2]\right] \tag{11}$$

$$= e^4\left[\left(1 + \frac{m_\tau^2}{E^2}\right) + \left(1 - \frac{m_\tau^2}{E^2}\right)\cos^2\theta\right] \tag{12}$$

Now that we have $|\mathcal{M}|^2$ we can put it into a formula for $d\sigma/d\cos\theta$ derived in Peskin[1]

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{2E_A 2E_B|v_p - v_{p'}|}\frac{|\mathbf{k}|}{(2\pi)^2 4E_{CM}}|\mathcal{M}|^2 \tag{13}$$

In the center of mass frame the relative speed $|v_p - v_{p'}|$ becomes 2, and $E_p = E_{p'} = E_{CM}/2 = \sqrt{s}/2$. With a symmetry about the longitudinal direction we can make the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{|\mathbf{k}|}{32\pi^2 s^{3/2}}|\mathcal{M}|^2 = \frac{1}{32\pi^2 s}\sqrt{1 - 4\frac{m_\tau^2}{s}}|\mathcal{M}|^2 \tag{14}$$

Integrating with respect to $\cos\theta$ we aquire the total cross section. Keeping the prefactors out of the calculation since they are not dependent on $cos\theta$ and integrate our expression for the squared matrix element.

$$\int_{-1}^{1} d\cos\theta e^4|\mathcal{M}|^2 = e^4\left[\left(1 + 4\frac{m_\tau^2}{s}\right)\cos\theta + \frac{1}{3}\left(1 - 4\frac{m_\tau^2}{s}\right)\cos^3\theta\right]_{-1}^{1}$$

$$= \frac{8}{3}\left(1 + 2\frac{m_\tau^2}{s}\right)$$

[1]Page 107, Equation 4.84

Combining combining this with our differential cross section we get an expression for the total cross section

$$\sigma = \frac{4}{3}\frac{e^4}{32\pi^2 s}\sqrt{1-4\frac{m_\tau^2}{s}}\left(1+2\frac{m_\tau^2}{s}\right)$$
$$= \frac{2}{3}\frac{\alpha^2}{s}\sqrt{1-4\frac{m_\tau^2}{s}}\left(1+2\frac{m_\tau^2}{s}\right)$$

with $\alpha = e^2/(4\pi)$.

1.4 Rate

The distribution of number of events goes as $N = L \times \sigma$, where L is the luminosity and σ is the cross section.

1.5 $e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$

The $e^+e^- \to e^+e^-$ case we have to extra diagrams where a photon or a Z are exchanged between e^+e^- without anything changing the outcome, and would so give this process a higher probability of happening then the others. The τ has a much larger mass and would therefore require larger incoming energies to have enough momenta in the photon to decay to τ.

1.6 Electroweak

This unifcation requires both the $\mathcal{M}_{QED}$ and $\mathcal{M}_{WI}$ matrix elements. Using the feynman-rules from the weak model our $\mathcal{M}_{WI}$ will take the form

$$\mathcal{M}_{WI} = -\frac{g_Z^2}{s-m_Z^2+im_z\Gamma_Z}g_{\mu\nu}\left[\bar{v}(p')\gamma^\mu\frac{1}{2}(c_V^e - c_A^e\gamma^5)u(p)\right]\left[\bar{u}(k)\gamma^\mu\frac{1}{2}(c_V^\tau - c_A^\tau\gamma^5)v(k')\right] \tag{15}$$

where $1/(s-m_Z^2+im_Z\Gamma_Z) = P_Z(s)$ is the Z propogator, $c_{V/A}^{e/\tau}$ are the vector and axial-vector couplings of the Z to our leptons. It will be handy to rewrite those couplings to left- and right-handed chiral states as $c_V = c_L + c_R$ and $c_A = c_L - c_R$

$$-P_Z g_Z^2 g_{\mu\nu}\left[c_L^e\bar{v}(p')\gamma^\mu P_L u(p) + c_R^e\bar{v}(p')\gamma^\mu P_R u(p)\right]\left[c_L^\tau\bar{u}(k)\gamma^\nu P_L v(k') + c_R^\tau\bar{u}(k)\gamma^\nu P_R v(k')\right] \tag{16}$$

where P_L and P_R are the chiral projection operators $\frac{1}{2}(1\mp\gamma^5)$. Using these chiral projection operators on a particle state give the result

$$P_L u = u_\downarrow,\ P_R u = u_\uparrow,\ P_L v = v_\uparrow,\ P_R v = v_\downarrow$$

And with helicity combinations like $\bar{u}_\uparrow\gamma^\mu v_\uparrow$ giving zero matrix elements we are left with four helicity combinations

$$\mathcal{M}_{RR} = -P_Z g_Z^2 c_R^e c_R^\tau g^{\mu\nu}\left[\bar{v}_\downarrow(p')\gamma^\mu u_\uparrow(p)\right]\left[\bar{u}_\uparrow(k)\gamma^\nu v_\downarrow(k')\right] \tag{17}$$
$$\mathcal{M}_{RL} = -P_Z g_Z^2 c_R^e c_L^\tau g^{\mu\nu}\left[\bar{v}_\downarrow(p')\gamma^\mu u_\uparrow(p)\right]\left[\bar{u}_\downarrow(k)\gamma^\nu v_\uparrow(k')\right] \tag{18}$$
$$\mathcal{M}_{LR} = -P_Z g_Z^2 c_L^e c_R^\tau g^{\mu\nu}\left[\bar{v}_\uparrow(p')\gamma^\mu u_\downarrow(p)\right]\left[\bar{u}_\uparrow(k)\gamma^\nu v_\downarrow(k')\right] \tag{19}$$
$$\mathcal{M}_{LL} = -P_Z g_Z^2 c_L^e c_L^\tau g^{\mu\nu}\left[\bar{v}_\uparrow(p')\gamma^\mu u_\downarrow(p)\right]\left[\bar{u}_\uparrow(k)\gamma^\nu v_\uparrow(k')\right] \tag{20}$$

The combinations of these four-vector currents can be shown to come to a simpler form as with this example which is the same as in QED

$$g_{\mu\nu}[\bar{v}_\downarrow(p')\gamma^\mu u_\uparrow(p)][\bar{u}_\uparrow(k)\gamma^\nu v_\downarrow(k')] = s(1+\cos\theta)$$

using this on each helicity state we get

$$|\mathcal{M}_{RR}|^2 = |P_Z|^2 g_Z^4 (c_R^e)^2 (c_R^\tau)^2 (1+\cos\theta)^2 \tag{21}$$
$$|\mathcal{M}_{RL}|^2 = |P_Z|^2 g_Z^4 (c_R^e)^2 (c_L^\tau)^2 (1-\cos\theta)^2 \tag{22}$$
$$|\mathcal{M}_{LR}|^2 = |P_Z|^2 g_Z^4 (c_L^e)^2 (c_R^\tau)^2 (1-\cos\theta)^2 \tag{23}$$
$$|\mathcal{M}_{LL}|^2 = |P_Z|^2 g_Z^4 (c_L^e)^2 (c_L^\tau)^2 (1+\cos\theta)^2 \tag{24}$$

The spin average of the full matrix elemnt $|\mathcal{M}|^2$ is then the sum of each helicity state squared

$$<|\mathcal{M}|^2>=\frac{1}{4}\left(|\mathcal{M}_{RR}|^2+|\mathcal{M}_{LL}|^2+|\mathcal{M}_{RL}|^2+|\mathcal{M}_{LR}|^2\right)$$

writing the terms out gives us

$$\begin{aligned}<|\mathcal{M}|^2>=\frac{1}{4}|P_Z|^2 g_Z^4 s^2\Big(&\left[(c_R^e)^2(c_R^\tau)^2+(c_L^e)^2(c_L^\tau)^2\right](1+\cos\theta)^2\\&+\left[(c_R^e)^2(c_L^\tau)^2+(c_L^e)^2(c_R^\tau)^2\right](1-\cos\theta)^2\Big)\end{aligned}$$

This is the $|\mathcal{M}_{WI}|^2$. Now we have to find the cross terms $\mathcal{M}_{QED}^*\mathcal{M}_{WI}$ and $\mathcal{M}_{WI}^*\mathcal{M}_{QED}$. For the first combination we have to combine the QED part with each of the helicity combinations of the WI

$$\begin{aligned}\mathcal{M}_{QED}^*\mathcal{M}_{RR}&=|P_Z|^2 g_Z^4 e^2 c_R^e c_R^\tau[\bar{v}_\downarrow(k')\gamma^\mu u_\uparrow(k)][\bar{u}_\uparrow(p)\gamma_\mu v_\downarrow(p')][\bar{v}_\downarrow(p')\gamma^\nu u_\uparrow(p)][\bar{u}_\uparrow(k)\gamma_\nu v_\downarrow(k')]\\\mathcal{M}_{QED}^*\mathcal{M}_{RL}&=|P_Z|^2 g_Z^4 e^2 c_R^e c_L^\tau[\bar{v}_\uparrow(k')\gamma^\mu u_\downarrow(k)][\bar{u}_\uparrow(p)\gamma_\mu v_\downarrow(p')][\bar{v}_\downarrow(p')\gamma^\nu u_\uparrow(p)][\bar{u}_\downarrow(k)\gamma_\nu v_\uparrow(k')]\\\mathcal{M}_{QED}^*\mathcal{M}_{LR}&=|P_Z|^2 g_Z^4 e^2 c_L^e c_R^\tau[\bar{v}_\downarrow(k')\gamma^\mu u_\uparrow(k)][\bar{u}_\downarrow(p)\gamma_\mu v_\uparrow(p')][\bar{v}_\uparrow(p')\gamma^\nu u_\downarrow(p)][\bar{u}_\uparrow(k)\gamma_\nu v_\downarrow(k')]\\\mathcal{M}_{QED}^*\mathcal{M}_{LL}&=|P_Z|^2 g_Z^4 e^2 c_L^e c_L^\tau[\bar{v}_\uparrow(k')\gamma^\mu u_\downarrow(k)][\bar{u}_\downarrow(p)\gamma_\mu v_\uparrow(p')][\bar{v}_\uparrow(p')\gamma^\nu u_\downarrow(p)][\bar{u}_\downarrow(k)\gamma_\nu v_\uparrow(k')]\end{aligned}$$

Each of these prodcuts can be calculated as such

$$\begin{aligned}\bar{u}_\uparrow(p)\gamma^\mu v_\downarrow(p')&=2E(0,-1,i,0)\\\bar{u}_\downarrow(p)\gamma^\mu v_\uparrow(p')&=2E(0,-1,-i,0)\\\bar{v}_\uparrow(p')\gamma^\mu u_\downarrow(p)&=2E(0,-1,i,0)\\\bar{v}_\downarrow(p')\gamma^\mu u_\uparrow(p)&=2E(0,-1,-i,0)\\\bar{u}_\uparrow(k)\gamma^\mu v_\downarrow(k')&=2E(0,-\cos\theta,i,\sin\theta)\\\bar{u}_\downarrow(k)\gamma^\mu v_\uparrow(k')&=2E(0,-\cos\theta,-i,\sin\theta)\\\bar{v}_\uparrow(k')\gamma^\mu u_\downarrow(k)&=2E(0,-\cos\theta,i,\sin\theta)\\\bar{v}_\downarrow(k')\gamma^\mu u_\uparrow(k)&=2E(0,-\cos\theta,-i,\sin\theta)\end{aligned}$$

And when combining these results together

$$\begin{aligned}\mathcal{M}_{QED}^*\mathcal{M}_{RR}&=|P_Z|^2 g_Z^4 e^2 c_R^e c_R^\tau 16s(\cos\theta+1)^2\\\mathcal{M}_{QED}^*\mathcal{M}_{RL}&=|P_Z|^2 g_Z^4 e^2 c_R^e c_L^\tau 16s(\cos\theta-1)^2\\\mathcal{M}_{QED}^*\mathcal{M}_{LR}&=|P_Z|^2 g_Z^4 e^2 c_L^e c_R^\tau 16s(\cos\theta-1)^2\\\mathcal{M}_{QED}^*\mathcal{M}_{LL}&=|P_Z|^2 g_Z^4 e^2 c_L^e c_L^\tau 16s(\cos\theta+1)^2\end{aligned}$$

So the contribution to the total matrix element will be the sum of these

$$\begin{aligned}<\mathcal{M}_{QED}^*\mathcal{M}_{WI}>&=4s|P_Z|^2 g_Z^2 e^2\left[(c_R^e c_R^\tau+c_L^e c_L^\tau)(\cos\theta+1)^2+(c_R^e c_L^\tau+c_L^e c_R^\tau)(\cos\theta-1)^2\right]\\&=4s|P_Z|^2 g_Z^2 e^2\left(c_V^e c_V^\tau(\cos^2\theta+1)+2c_A^e c_A^\tau\cos\theta\right)\end{aligned}$$

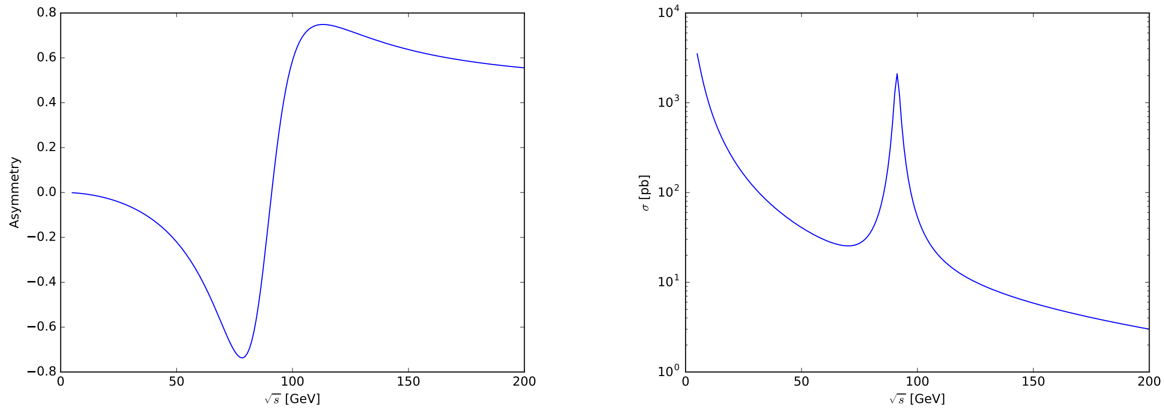
The same contribution will come from $\mathcal{M}_{WI}^*\mathcal{M}_{QED}$ thus giving us a total matrix element of

$$\begin{aligned}|\mathcal{M}|^2&=|\mathcal{M}_{QED}|^2+2\mathcal{M}_{QED}^*\mathcal{M}_{WI}+|\mathcal{M}_{WI}|^2\\&=e^4\left[\left(1+\frac{m_\tau^2}{E^2}\right)+\left(1-\frac{m_\tau^2}{E^2}\right)\cos^2\theta\right]\\&+8s|P_Z|^2 g_Z^2 e^2\left(c_V^e c_V^\tau(\cos^2\theta+1)+2c_A^e c_A^\tau\cos\theta\right)\\&+\frac{1}{4}|P_Z|^2 g_Z^4 s^2\Big(\left[(c_R^e)^2(c_R^\tau)^2+(c_L^e)^2(c_L^\tau)^2\right](1+\cos\theta)^2+\left[(c_R^e)^2(c_L^\tau)^2+(c_L^e)^2(c_R^\tau)^2\right](1-\cos\theta)^2\Big)\end{aligned}$$

which has grown to a rather unruly equation.

1.7 Comparison

Figure 2: $e^+e^- \to \tau^+\tau^-$



(a) Forward backward asymmetry

(b) Total cross section

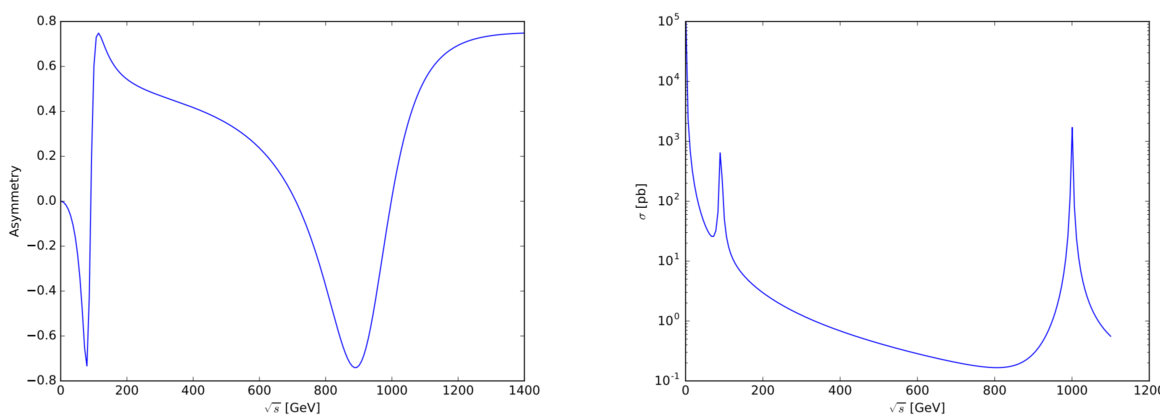
2 Dilepton production beyond the SM

2.1 Z'

Almost any grand unified theory perdicts the existence of a Z'. Z' would be involved in the same lepton production as Z only it would need much more energy due to its mass. Finding the Z' could give clues to how the forces relate to one another, or give information about the characteristics of extra dimensions.

2.2 Cross section and A_{FB}

Figure 3: $e^+e^- \to \gamma, Z, Z' \to \mu^+\mu^-$



(a) Forward backward asymmetry with Z' included

(b) Total cross section with

In the cross section graph we see two large spikes around the Z and Z' masses showing the favourability around these processes under these masses. We can see the same behaviour in the A_{FB} plot.

2.3 Z' and W' production

Z' would be produced in hadron colliders due to the need for high energies. The Z' would be produced from an $q\bar{q}$ annihilation from where it would decay into dilepton- or diquark pair. As for the W' it would alse come from a $q\bar{q}$

annihilation and decay into lepton and neutrino, or top and bottom quark. From an article from 2011[2] the ATLAS Experiment had a cross section using the cross section from $pp \to W'X$ at about $1\,fb^{-1}$, times the branching ratio of $W' \to e\nu$ which gives a cross section of about $100\,fb$ for a W' mass of $500\,GeV$. In the search for W' we are looking for a high missing transverse mass, in where the transverse mass is

$$m_T^2 = m^2 + p_x^2 + p_y^2$$

so it is the sum of the invariant mass and p_T.

2.4 Slepton pair production

There would be an energy difference in the signals

[2] http://pdg.lbl.gov/2011/reviews/rpp2011-rev-wprime-searches.pdf