Ch1.1 **Laws of Addition|DeMorgan**
1.If A_1,A = disjoint, $P(A_1 \cup A_2) = P(A_1+A_2)$
2.$(A \cap B)^c = A^c \cup A^c$ 3.P(AUB)=P(A)+P(B)-P(AnB)

Ch1.2 **Permutations**
1. Multip. Principle- total # of outcomes= mn

2. Extended MP - P experiments $p_1=n_1$outcomes $p_2=n_2$ outcomes Total # outcomes = $n_1n_2n_3$.....

3. Permutations - ordered arrangement n=set of size n
a.With replacement: n^r r=sample of size r
b.without replacement: n(n-1)(n-2)..(n-r+1)

Ch1.3 **Combinations**
1. Binomial Coefficients - unordered samples
a. no replacement =$\binom{n}{r}$
2. Maximum Likelihood Estimate =
$$\frac{\binom{t}{r}\binom{n-t}{m-r}}{\binom{n}{m}}$$

3. Choose:
$\binom{n}{r} = \frac{n!}{r!(n-r)!}$ and $\binom{52}{13\ 13\ 13\ 13} = \frac{52!}{(13!)^4}$

Ch1.4 **Conditional Probability**
1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$
2. $P(A \cap B) = P(A|B)P(B)$
3. $P(A \cap B \cap C) = P(C|A \cap B)P(B|A)P(A)$

Law of Total Probability - **$P(B) = P(B|A_i)P(A_i)$**

Bayes Rule - **$P(A_j|B) = \frac{P(B|Aj)P(Aj)}{\sum i P(B|Ai)P(Ai)}$**

Ch1.5 Independence - P(AnB)=P(A)P(B)
Pairwise independent - A,B,C, any 2 are independent
Mutually Independent- $P(A_1 n A_2 n A_3) = P(A_1)P(A_2)P(A_3)$

Ch4.1 Expected Values μ - measure of center
Discrete Case - E(X) = $\sum$ xP(X=x) = $\sum_x xP_x(x)$

1.If Y=g(x),frequency func = p(x), then E(Y) → $\sum_x g(x)p(x)$

Continuous Case - E(X) = $\int_{-\infty}^{\infty} x f_x(x)dx$
1.If Y=g(x), density function = f(x), then E(Y) → $\int_{-\infty}^{\infty} g(x)p(x)dx$

Expected Value Properties:
1. E(X+c) = E(X) + c
2. E(aX)= aE(X)
3. E(a y(X) + b) = aE(y(X)) + b
4. E(XY)=E(X)E(Y)$_{\text{only if independent}}$
5. E(X+Y) = E(X) + E(Y)

4.2 Variance σ^2 - measure of spread
Var(X) = $E(X^2)-[E(X)^2]$
1. Var(X+ b) = Var(X)
2. Var(bX) = b^2Var(X)
3. Var(a y(X) + b) = a^2Var(y(X))
4. If X,Y are independently jointly distributed r.v.s, then
Var(X+Y) = Var(X) +Var(Y)

e.g→Var(5X-Y) =Cov(5X-Y,5X-Y)=25Var(X)-10Cov(X,Y)+Var(X)

4.3 Covariance Properties
Covariance Properties:
1. If X and Y are **independent**, Cov(X,Y) = 0
2. Cov(X,X) = Var(X) |Cov(Y,Y) = Var(Y)
3. Cov(c,X) = 0
4. Cov(Y,-Y) = -Var(Y)
5. Cov(X,Y) = E(XY)-E(X)E(Y)
6. Var(X-Y) = Cov(X-Y, X-Y)
= Var(X) +Var(Y) - 2Cov(X,Y)

4.4 Correlation
ρ = Corr(X,Y) = $\frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

Functions of a Random Variable PMF
If y=g(x) and strictly monotonic, find inverse. e.g, Y=$-X^3$+2 → g(x)=x=$_3\sqrt{}$ y-2

Ch 2.1 Discrete Random Var.
CDF → F(x)=P(X ≤x), -∞ <x<∞

Ch 2.2 **Discrete Distributions**
pmf |Frequency Function|lower case
a.$p(x_i)=P(X=x_i)$ and $\sum p(x_i)=1$
e.g. p(x=0)=⅛,p(x=1)=⅜ p(x=2)= ½

1.Bernoulli $p(k)=p(X=k)=p^k(1-p)^{1-k}I_{\{0,1\}}(k)$
(only 1/0): $p(1) = P(X = 1) = p$
$p(0) = P(X = 0) = 1 - p$

2.Binomial- ~ B(n,p)
a.X=# of successes in n trails at least/exactly
Exactly - $p(k)=p(X=k)=\binom{n}{r}p^k(1-p)^{n-k}I_{\{0,1,..n\}}(k)$

3.Geometric (∞ trials (k), up to first and including 1st success):
$p(1-p)^{k-1}I_{\{0,\infty\}}(k)$

3.Negative Binomial
∞ trials (k), up to and including rth success(r≥1):
- $\binom{k-1}{r-1}p^r(1-p)^{k-r}$
- $\binom{k-1}{r-1} = \binom{k-1}{k-r}$

4. Poisson
$\frac{(\lambda t)^k}{k!}e^{-\lambda t}I\{0,\infty\}(k)$

Ch2.3 Continuous Random Variables
PDF For a<b, P(a<x<b) = $\int_a^b f(t)dt$
(f(x)≥0, f piecewise continuous, $\int_{-\infty}^{\infty} f(x)dx = 1$)

1.Uniform pdf: f(x) = $I_{[0,1]}$(x) or
$\frac{1}{b-a}$ $I_{[a,b]}(x)$

2.Exponential → X~Exp(λ)
$f(x) = \lambda e^{-\lambda x} I_{[0,\infty)}(x)$ → memoryless

3.Gamma → X ~G(α , λ)
f(x) =
$\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$

4.Normal → μ(- ∞ < μ < ∞) and σ >0
f(x) =
$\frac{1}{\sigma(\sqrt{2\pi})}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
Z-Score → $\frac{x-\mu}{\sigma}$

2.4 Functions of a Random Variable PDF
If x is continuous with pdf $f_x(x)$ and g is differentiable, strictly monotonic
The pdf of Y=g(x)

→ $f_y(y)=f_x(g^{-1}(y))|\frac{dg^{-1}(y)}{dy}|$

4.5 Conditional E(X)

Discrete - E(Y|X=x) = $\sum_y$yP(Y=y|X=x) = $\sum_y yP_{y|x}(y|x)$
Discrete Function -E(h(y)|X=x) →
$\sum_y$h(y)P(Y=y|X=x) = $\sum_y h(y)P_{y|x}(y|x)$
Conditional PMF:same as pdf, but use $p_{Y|X}$

Continuous- E(Y|X=x)=$\int_{-\infty}^{\infty} y f_{Y|X}(Y|X)dy$
Continuous Function- E(h(y)|X=x) → $\int_{-\infty}^{\infty} h(y)f_{Y|X}(Y|X)dy$
Conditional PDF - $f_{Y|X}(Y|X) = f_{xy}(x,y)/f_x(x)$ e.g.x^2/2x I[0,1](y)

4.6 Law of iterated E(X)
E(Y)=E(E(Y|X))

4.7 Conditional Var(X)
X,Y are jointly distributed, x is a possible value of the rv, then
$Var(Y|X=x)=E[Y^2|X=x]-E(Y|X=x)]^2$

4.8 Chebyshev's Inequality
X= rv with E(X)=μ and Var(X)= σ ^{2}
P(|X-μ |)>t ≤ $\frac{\sigma^2}{t^2}$

5.2 Convergence in probability
If Z_1,Z_2,Z_3.....is a sequence of random variables such that for any t>0,
→ $\lim_{n\to\infty}$ P(|Z_n-Y|>t = 0,
Then $\{Z_n\}^\infty_{n=1}$ is said to converge to probability Y as n→ ∞ .
Z_n→Y as n→ ∞

Ch 3.1 Joint Distributions
CDF → F(x,y) = P(X≤ x,Y≤y)

Ch 3.2 Joint Discrete PMF
PMF → p(x,y)=P(X=x,Y=y)=P(X(w)=x,Y(w)=y)

1.*Marginal Frequency* - $p_x(x) = \sum_i p(x,y_i)$ e.g (⅔ or ½)

2.*Multinomial Distribution* →
$(N_1,N_2,N_3..N_r)$~Multinomial
n = # of independent trials
r =outcomes for each trail
p_k = Probability of outcome k k=1,...,r,
N_k=# of times outcome k occurs in n trials
$\binom{n}{n1\ n2\ n3..\ nr}p_1^{n1}\cdot p_2^{n2}\cdot\ldots p_r^{nr}$ $_{n1+n2+..nr=n}$

3.3 Joint Continuous PDF
PDF→ f(x,y)Є A = $\int\int_A f(x,y)dxdy$
Properties :
1. f(x,y) ≥ 0 for all (x,y)
2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$
3.f(x,y) is piecewise continuous

Marginal PDF → $f_x(x) = \int_{-\infty}^{\infty} f(x,y)dy$ →e.g.$x^2I_{[0,2]}(x)$
use interval given for bounds

3.3b Independence → F(X,Y)=$F_x(X)F_Y(Y)$

3.4 Distribution of Sums of Random Variables

Discrete → Let X,Y have joint PMF p(x,y), if Z = X+Y then PMF of Z =
$P_Z(z)=P(Z=z)=P(X+Y=Z)=\sum_x p(x,z-x)=\sum_y p(z-y,y)$
(Make a table for visualization)

Continuous → Let X,Y, have joint PDF f(x,y), if Z=X+Y then PDF of Z =
$f_Z(z)=\int_{-\infty}^{\infty} f(x,z-x)dx=\int_{-\infty}^{\infty} f(z-y,y)dy$
!!To find P(Xn≥function) draw out graph, integrate
!!To find P(1<x<2) pdf(2)-pdf(1)-compute directly
Independent → If X,Y are independent, use convolution formula

3.5 Finding Joint Density
Let X,Y be continuous random variables w/joint PDF $f_{xy}(x,y)$
$U=g_1(x,y)$ $x=h_1(u,v)$
$V=g_2(x,y)$ $y=h_2(u,v)$
J (x,y) = det | |

Joint PDF →
$f_{U,V}(u,v) = f_{X,Y}(h_1(u,v), h_2(u,v))|J^{-1}(h_1(u,v), h_2(u,v))|$

5.1 Law of Large Numbers - As sample size increases, the sample mean gets closer to underlying mean μ .
Let $X_1X_2,...X_i$ be a sequence of independent random variables with $E(X_i) = \mu$ $Var(X_i)=\sigma^2$.
Let $\bar{X}_n = \frac{1}{n}\sum^n_{i=i}X_i$ = takes on values 0 or 1, n = number of trials, then
$P(|\bar{X}_n-\mu| > \varepsilon) \to 0$ as $\lim_{n\to\infty}$

5.3 Central Limit Theorem → SAMPLE > 30
$X_1X_2X_3$ are iid with μ and σ^2
Let $S_n = \sum^n_{i=1}(X_i-\mu)$ Then $Z_n=\frac{Sn}{\sigma\sqrt{n}}$ → Z~N(0,1) as n → ∞

$Z_n = \frac{Xn-\mu}{\frac{\sigma}{(\sqrt{n})}}$ P(|X-μ|<c)=p(-c<X-μ<c)

$P(\frac{-c}{\sigma/\sqrt{n}}<\frac{X-\mu}{\sigma/\sqrt{n}}<\frac{c}{\sigma/\sqrt{n}}) = \varphi(\frac{c}{\sigma/\sqrt{n}})-\varphi(\frac{-c}{\sigma/\sqrt{n}})$

Where $X_n = \frac{1}{n}\sum^n x_i$

Finding Mean of cdf F(x)=$\int_1 f(y_1)+\int_2 f(y_2)$..
Finding Median of cdf F(x) → F(x)=½, or look at[a,b]/2

Multinomial Probability

1.A computer will independently generate 7 random numbers on the interval 0 to 1. What is the probability that 3 of the numbers will be from interval 0 t0 ½, 3 will be in interval ½ to ⅔ and 1 will be in the interval ⅔ to 1?

n=7 independent trials| $n_1=3, n_2=3, ,n_3=1$ | $p_1=½$ $p_2=⅓$ $p_3=⅓$

$=\binom{7}{3\ 3\ 1}(½)^3(⅙)^3(⅓)^1$

2.If 7 balanced dice are rolled, what is the probability that each of the six numbers will appear at least once?

$=6\binom{7}{2\ 1\ 1\ 1\ 1\ 1}(⅙)^7$

3.Suppose $Z_1Z_2Z_3Z_4$ are independent N(0,1) rvs w $Z_{(1)} \leq Z_{(2)} \leq Z_{(3)} \leq Z_{(4)}$. What is the $P(Z_1<0, Z_2\geq 0, ,Z_3>2)$? (draw)

z score = $x-\mu/\sigma$ since it's N(0,1), $\mu = 0, \sigma = 1$, $x-0/1 = x$

Look at chart, $z_1=p(x<0) = .50$, $z_2 \in [0,2]$, $z_3=1-p(2)$

$\rightarrow \binom{4}{1\ 1\ 2}\Phi(0)[\Phi(2)-\Phi(0)][1-\Phi(2)]^2$

$\rightarrow 4!/2!\ (.5)(.9772-.5)(1-.9772)^2$

Probability Density Function ORDERED STAT

1.Give PDF for RVs $Z_1Z_2Z_3Z$

The function for N(0,1) = $f(x) = 1/\sigma(\sqrt{2\pi})e^{-(x-\mu)2/2\sigma^2}$

$\rightarrow f(x)=f(z_1,z_2,z_3,z_4)=4!\ (z_1)(z_2)(z_3)(z_4)$ for $z_1<z_2<z_3<z_4$

$= 24(\frac{1}{\sqrt{2\pi}})^4e^{-½(z1^2+z2^2+z3^3+z4^2)}$

$=6/\pi^2\ e^{-½(z1^2+z2^2+z3^3+z4^2)}$ for $z_1<z_2<z_3<z_4$

Law of Large Numbers

1.Suppose the continuous rvs X_1,X_2,X_3.. are iid with common pdf function f(x) =

Find the expected value of $X_{40}=1/40 \sum X_i$

$E(X_i) = \int_0^2 (x^2/2)dx = 4/3$

$E(X_{40}) = X_n = \frac{1}{n}\sum^n x_i = \frac{1}{40}(\frac{4}{3})(40) = 4/3$

2.Find the variance of $X_{40}= X_{40}=1/40 \sum X_i$

$Var(X) = E(X^2)-(E(X)^2) \rightarrow E(X^2) = \int_0^2 \frac{x3}{2}=x^4/8|^2_0=2$

$E(X)^2=4/3=16/9 \rightarrow 2-16/9 = 2/9$

$Var(X) = 2/9, Var(X_{40})=Var(1/40(X_1)....1/40(X_{40})$

$Var(cX)=c^2Var(X) \rightarrow 1/40^2(2/9)(40) \rightarrow 1/40(2/9)$

Central Limit Theorem

Use the CLT to approximate $(X_{40}\leq 1.5)$. Express your answer using Φ.

The CLT = $Z_n=\frac{Xn-\mu}{\frac{\sigma}{(\sqrt{n})}}$, $\sigma = \sqrt{2/3}$, $\mu = 4/3$ n = 40

$\frac{X40-\mu}{\frac{\sigma}{(\sqrt{n})}} \leq \frac{1.5-4/3}{\frac{\sqrt{2}}{(3\sqrt{40})}} = \varphi\ (\sqrt{180}[1.5-(4/3)] = \varphi\ (\sqrt{5})$

CE|A Company ships packages with μ =15 and σ = 10. Assuming that packages come from a large number of different customers, find probability that 100 packages will have a total weight exceeding 1700lb.

100>30, so use $Z_n=\frac{Xn-\mu}{\frac{\sigma}{(\sqrt{n})}} \rightarrow P(\sum^{100}X_i > 1700)$

$\rightarrow P(\frac{1}{100}\sum^{100}X_i > 17) \quad \rightarrow \frac{17-15}{10/sqrt100} = P(Z_{100}>2)=1-P(Z_{100}\leq 2)$

$=1-.9772$

F2018: 3 independent observations are randomly drawn from a normal distribution μ and $\sigma^2=12$

1.Probability the sample **mean** of 3 observations lies within 1 unit of μ ?(Find $p(X_3-\mu\ |) \leq 1$)

Use $P(|X-\mu|<c)=p(-c<X-\mu<c)$

$P(\frac{-c}{\sigma/\sqrt{n}}<\frac{X-\mu}{\sigma/\sqrt{n}}<\frac{c}{\sigma/\sqrt{n}}) = \varphi\ (\frac{c}{\sigma/\sqrt{n}})-\varphi\ (\frac{-c}{\sigma/\sqrt{n}})$

$\sigma = \sqrt{12}, \sqrt{n} = \sqrt{3}\ \sqrt{12/3}= \sqrt{4} = 2$

$P(-1<x-\mu<1) = P(\frac{-1}{2}<\frac{X-\mu}{2}<\frac{c}{2})$

$= \varphi\ (.5)- \varphi\ (-.5) = .6915 - (1-.6915)$

What is the probability the sample **median** lies within 1 unit of μ ?

$P(|X_{(2)}-\mu\ | \leq 1) =$

Law of Total Probability

1.Suppose you plan to toss a fair coin until either a head appears or the coin is tossed three times, whichever happens first. This experiment is then performed a second time. What is the probability that the second experiment requires more tosses than the first?

X_1=# of tosses in experiment 1

X_2=# of tosses in experiment 2

$P(X_2>X_1) = P(X_2>X_1 | X_1 =1)P(X_1 =1) + P(X_2>X_1 | X_1 =2)P(X_1 =2) +P(X_2>X_1 | X_1 =3)P(X_1 =3)$

$P(X_2>X_1) = P(X_2>1)P(X_1 =1) + P(X_2>2)P(X_1 =2) +P(X_2>3)P(X_1 =3)$

$P(T_1)P(H_1) + P(T_1nT_2)P(T_1nT_2)+0 = 5/16$

Bayes Theorem

1 marble is drawn from an urn that contains 2 R, 3 W, 4 B. If a R or W is drawn, we return to urn. If B is drawn, that marble with another B are returned to urn. Then a second is drawn at random.

a.If it is given that two selected marbles are the same color, what is the probability they are both red?

$P(R_1nR_2|\text{Same Color} =$

$\rightarrow P(R_1nR_2)/P(R_1nR_2)+P(W_1nW_2)+P(B_1nB_2)$

$\rightarrow P(R_2|R_1)P(R_1)/P(R_2|R_1)P(R_1)+P(W_2|W_1)P(W_1)+P(B_2|B_1)P(B_1)$

$\rightarrow (2/9)^2/(2/9)^2+(3/9)^2+(4/9)(5/10)=4/31$

Conditional Expectation E(X|Y=y)E(Y|X=x)

8.5 Let X and Y have the joint density $f(x,y) = e^{-y}$, $0\leq x\leq y$

Find E(X|Y=y) and E(Y|X=x)

Use $E(Y|X=x)=\int_{-\infty}^{\infty}yf_{Y|X}(Y|X)dy$

Conditional PDF - $f_{Y|X}(Y|X) = f_{xy}(x,y)/f_x(x)$

Chebyshev's Inequality

Suppose X is a random variable with E(X) = μ and Var(X) = σ^2> 0. Consider $P(|X- \mu\ |> 4\ \sigma\ )$

A: t= 4σ Plugging into the formula, I get $P(|X- \mu|> 4\sigma) \leq \frac{\sigma^2}{16\sigma^2}$

= 1/16

PMF Change of Variables

6.10A discrete random variable X has the following probability mass function:

$$p_X(k) = \begin{cases} 1/6, & k=-2,-1,0,1 \\ 1/3, & k=2 \\ 0, & \text{otherwise.} \end{cases}$$

If the random variable $Y = -X^3+2$, give the probability mass function for Y.

5.6 PMF = $p(x,y) = \{cxy, \quad (x,y) = (1,1)(2,1)(2,2)(3,1)$
$\{0, \quad$ otherwise

a.Find PMF for RV $V=(X-2)^2$

→When V=0, x=2 → p(2,1)+p(2,2)=.2+.4=.6

→When V=1, x=1 → $1=(x-2)^2$→ $+-1=(x-2)^2$

→p(1,1)+p(3,1)=.4

Order Statistics

7.8 Suppose X1, X2,....X10 are iid Exponential (λ =3), and let $X_{(1)}\leq X_{,(2)}\leq ...\leq X_{(10)}$ represent the corresponding order statistics.

Find the probability density function for the third quartile $X_{(8)}$

$f(x)=3e^{-3x}I(0,+\infty)(x)$

$F(x)=1-e^{-3x}$

pdf for $_{\text{for } k=1,..n,}$ $X_{(k)} = \frac{n!}{(k-1)!(n-k)!}f(x)F(x)]^{k-1}[1-F(x)]^{n-k}$

n=10,k=8

What is $P(X_{(8)}<⅕)$? Use PDF of $X_{(8)}$ and set up $\int_0^{1/5}X8\ dx$

PDF Conversion Functions of Random Var.

1.If the rv X has an exponential distribution with parameter λ =3, find the pdf for the rv $Y=X^2$.

$f_x(x)=\lambda\ e^{-\lambda x}$, strictly monotonic, use $f_y(y)=f_x(g^{-1}(y))|\frac{dg^{-1}(y)}{dy}|$

$g(x)=Y=X^2$ $g^{-1}(x) = \sqrt{Y}$, $d/dx(g^{-1}(x))=1/y^{-½}$

$\rightarrow fx(x)=3e^{-\lambda/\sqrt{y}}|\frac{1}{2sqrt(y)}|I_{[0,\infty)}(y)$

2.If r.v. U is uniform on [-2,2] find density for U^2+1 and |U|

USE CDF Method:

$F_x(x)=P(X\leq x)=P(U^2-1\leq x)=P(U^2\leq x+1)=P(-\sqrt{x+1}\leq U\leq \sqrt{x+1})$

→ Find max - min → $F_u(\sqrt{x+1})-F_u(-\sqrt{x+1})$

→Find $f_x(x)=dF(x)/dx \rightarrow f_u(\sqrt{x+1})½(x+1)^{-½}-f_u(-\sqrt{x+1})-½(x+1)^{-1/2}-$

$\rightarrow(½)(¼)(x+1)^{-½}+(½)(¼)(x+1)^{-½}$

$\rightarrow 1/4\sqrt{x+1}\ I_{[-1,3]}(x)$

F2018 : X has Exponential Distribution with λ =1. Find PDF for r.v. Y=|X-1|. **(Sketch y=|x-1| first) USE CDF, ∫, then d/dy**

$f_x(x)=e^{-x}I_{[0,\infty)}(x)$ possible values for y → $[0,\infty)$

$@Y=[0,1]F_Y(y)=P(Y\leq y)=P(1-y\leq X\leq 1+y)= \int_{1-y}^{1+y}e^{-x}dx$

$\rightarrow -e^{-1-y}+e^{y-1}$, $f_y(y)=dF_y(y)/dy = -e^{-1-y}+e^{y-1}$

$@Y>1, P(0\leq x\leq y+1) =\int^{y+1}_0 e^{-x}dx$

$\rightarrow 1-e^{-y-1}$, $f_y(y)=dF_y(y)/dy=e^{-y-1}$

$f_yy=e^{-y-1}I_{[0,\infty)}(y) + e^{y-1}I_{[0,1]}(y)$

7.10 Suppose X,Y are independent Unif(0,1) random variables, find the pdf of Z=X times Y

Use Jacobian Joint density formula

PDF

4.8 .A random variable Y has cdf:

$$F(y) = \begin{cases} 0 & y<1 \\ \ln y & 1\leq y\leq e \\ 1 & e<y. \end{cases}$$

a.Find the corresponding pdf f(y)

$\frac{1}{y}I_{[1,e]}(y)$

b.Find P(2<Y<2.5) → ln(2.5)-ln(2)

Law of Iterated E(X)

Suppose X,Y are jointly distributed random variables s.t. E(X)=10, E(Y)=14, E(Y|X)=cX, find c.

A:E(Y)=E(E(Y|X))=E(cX)=cE(X) 14=10c, c =14/10

Mixed Probability Problem

1.We plan to independently toss a fair coin 6 times. X=#of heads observed after first tail. If no tails, x=0. Find P(X=3)

$\rightarrow \binom{5}{3}(\frac{1}{2})^6+\binom{5}{4}(\frac{1}{2})^6+(\frac{1}{2})^6 = (\frac{15}{2^6})$

Order Statistics Formulas -

$V= X_{(1)}=\min\{X_1,X_2,X_3...X_n\}$

$U= X_{(n)}=\max\{X_1,X_2,X_3...X_n\}$

Range of Values → R=U-V

cdf for $U = X_{(n)} = F_U(u)=[F(u)]^n$

pdf for $U = X_{(n)}= f_U(u)=nf(u)[F(u)]^{n-1}$

cdf for $V = X_{(1)}$: $F_V(v)= 1-[1-F(v)]^n$

pdf for $V = X_{(1)}$: $f_V(v)= nf(v)[1-F(v)]^{n-1}$

pdf for $_{\text{for } k=1,..n,}$ $X_{(k)} =\frac{n!}{(k-1)!(n-k)!}f(x)F(x)]^{k-1}[1-F(x)]^{n-k}$

Joint pdf for X,Y = $f_{X,Y}(xy)=n!f(x)f(y)$

Joint pdf for $V=X_1$, $U=X_n$ =

$f_{V,U}(v,u)=n(n-1)f(v)f(u)[F(u)-F(v)]^{n-2}$ for v<u