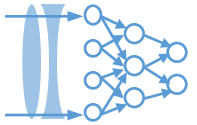


Lecture 15: Examples of Physical Layers for CNNs

Machine Learning and Imaging

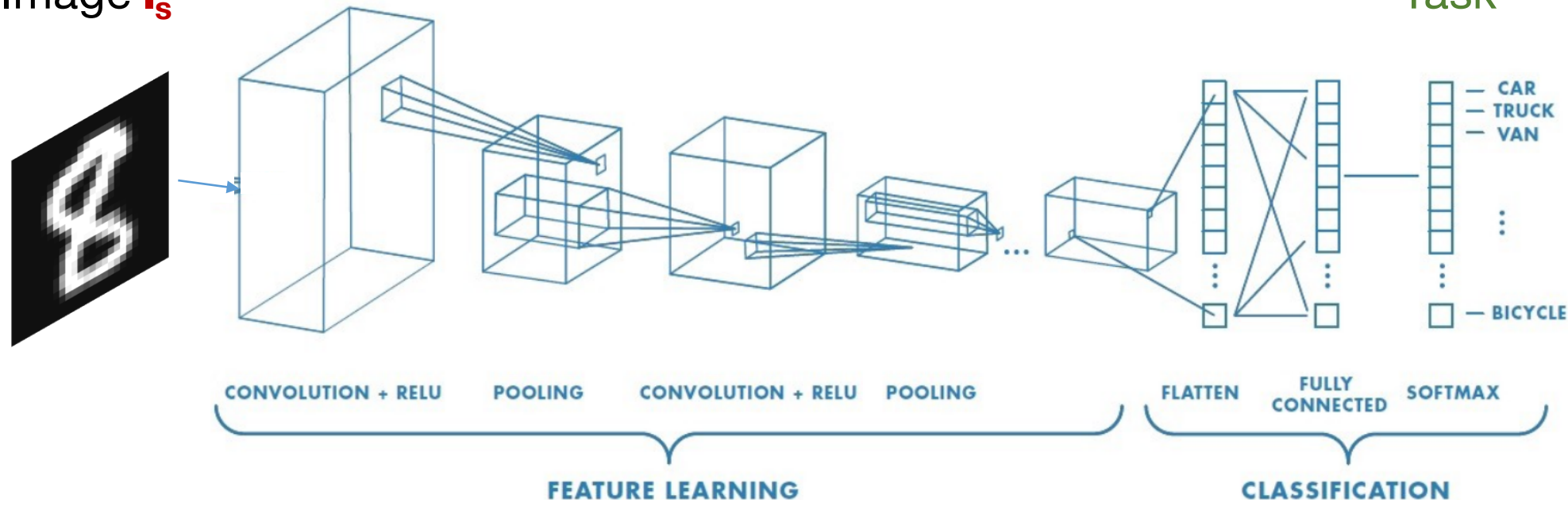
BME 590L
Roarke Horstmeyer

Bringing together physical and digital image representations

deep imaging

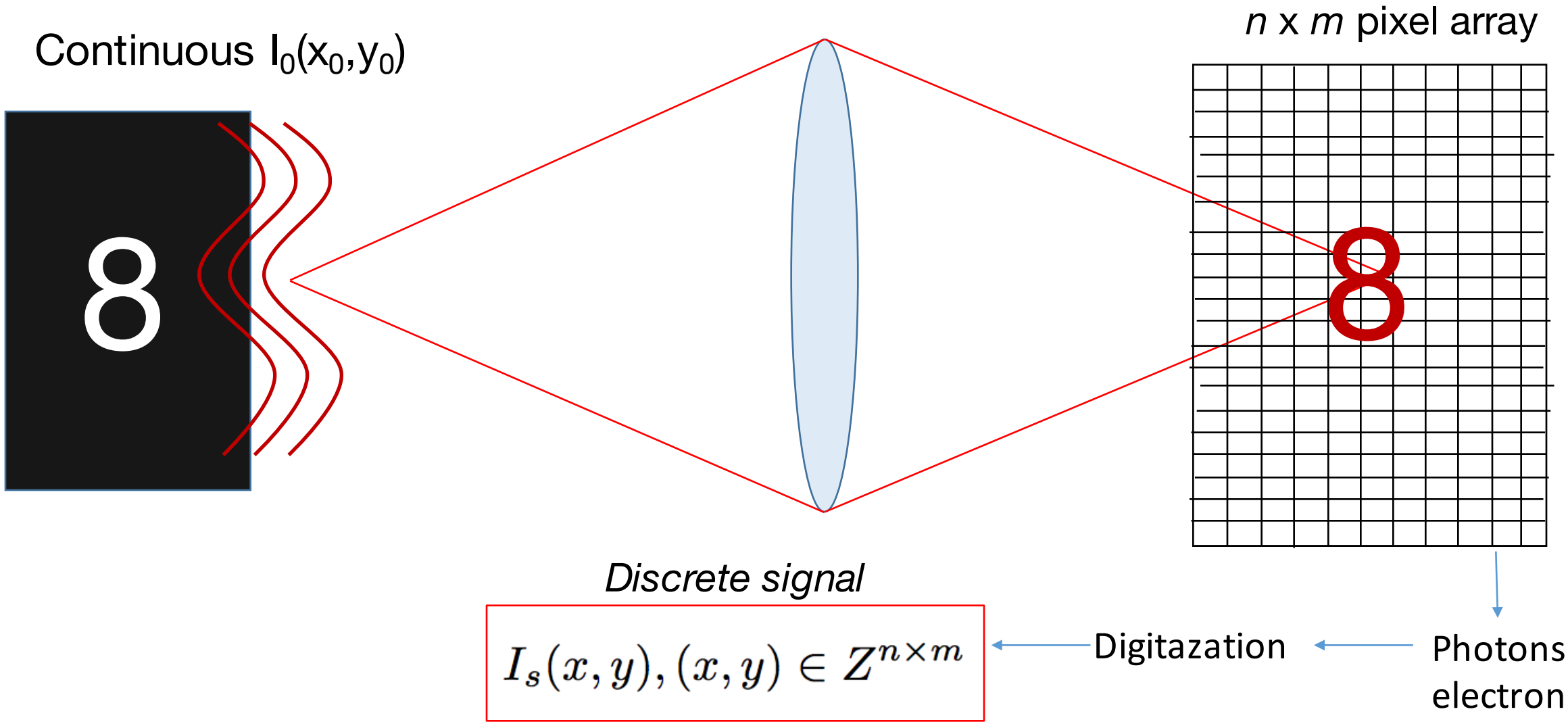
Image I_s

Task

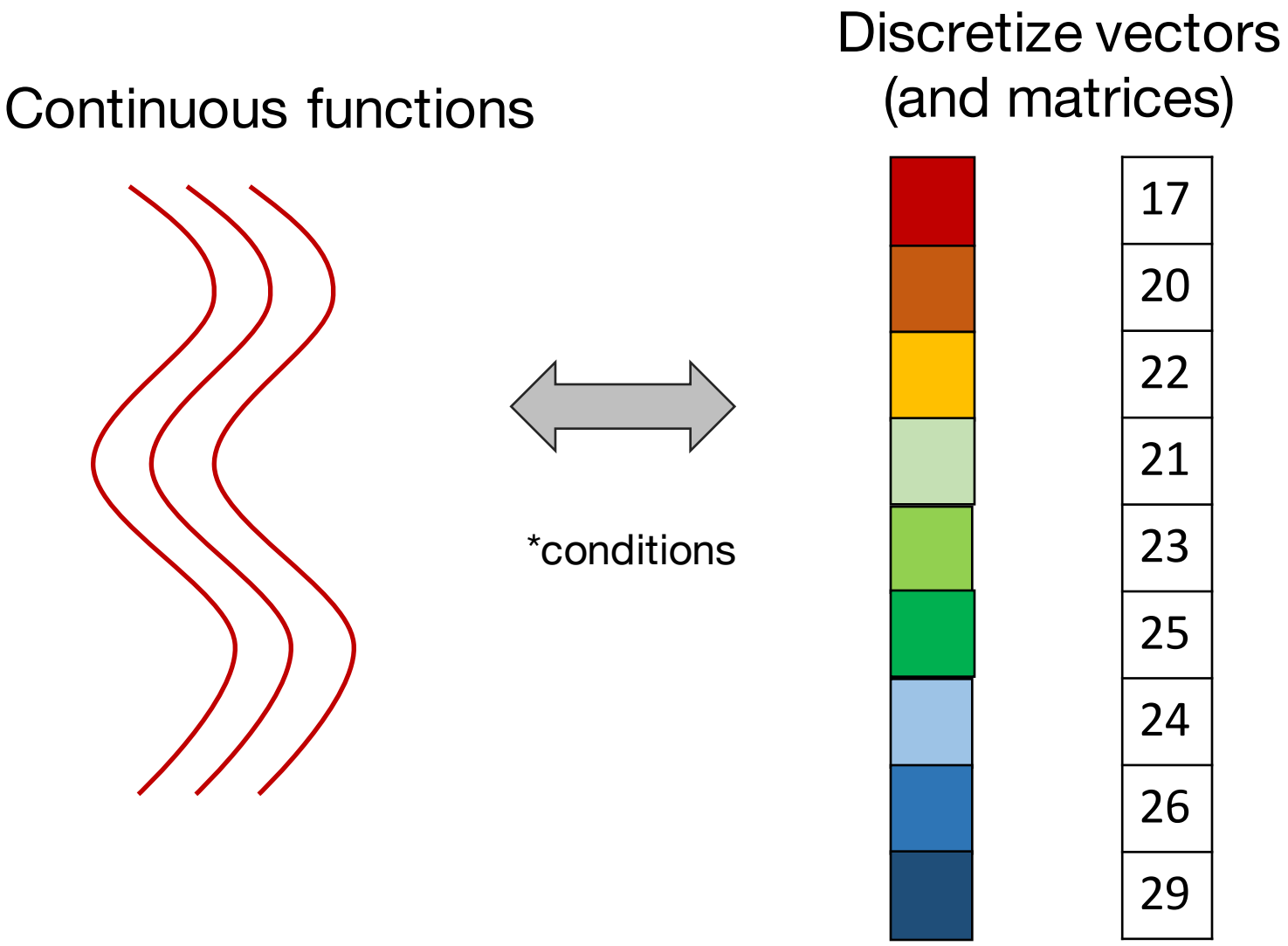


$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, I_s] \ldots]$$

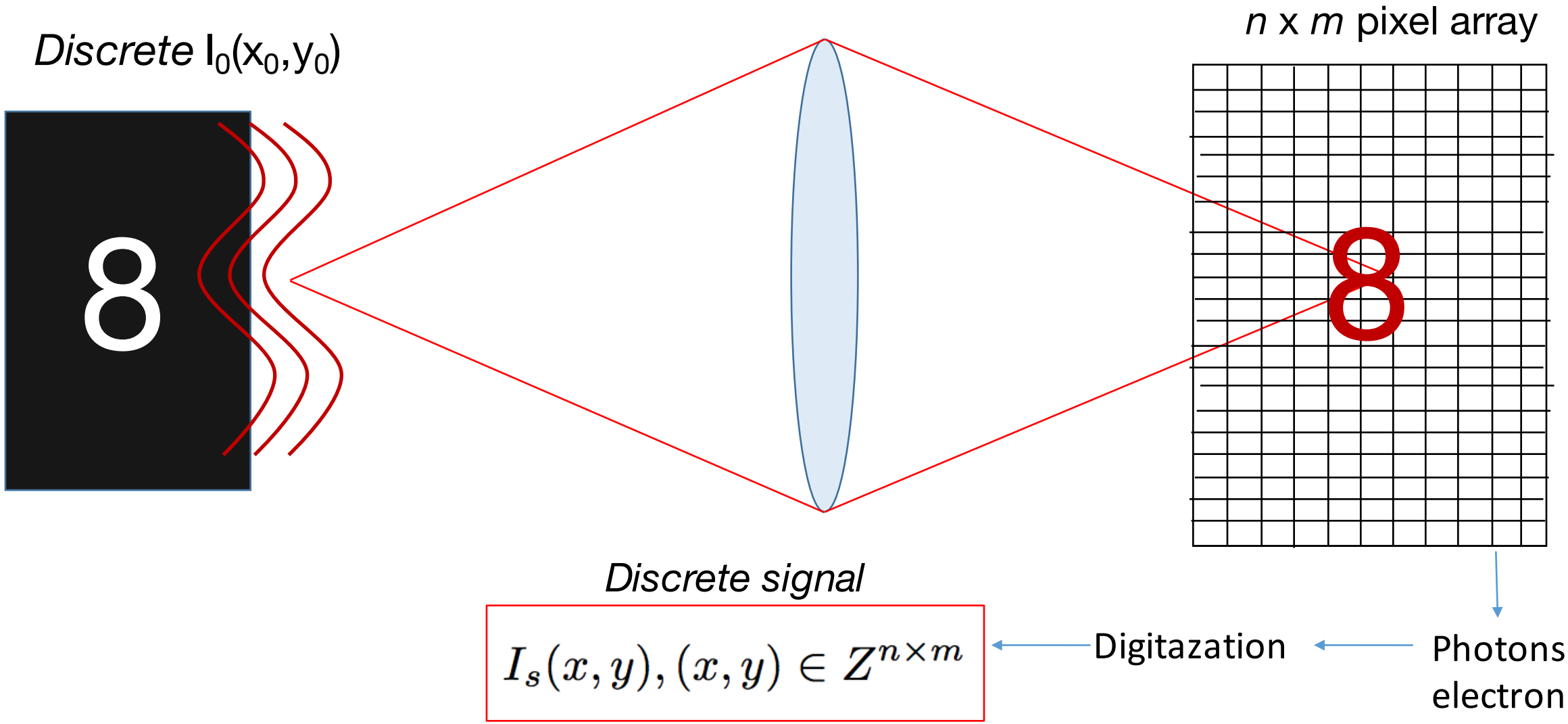
Simple model of image formation



What does the Sampling Theorem mean for us?



Simple model of image formation



Bringing together physical and digital image representations

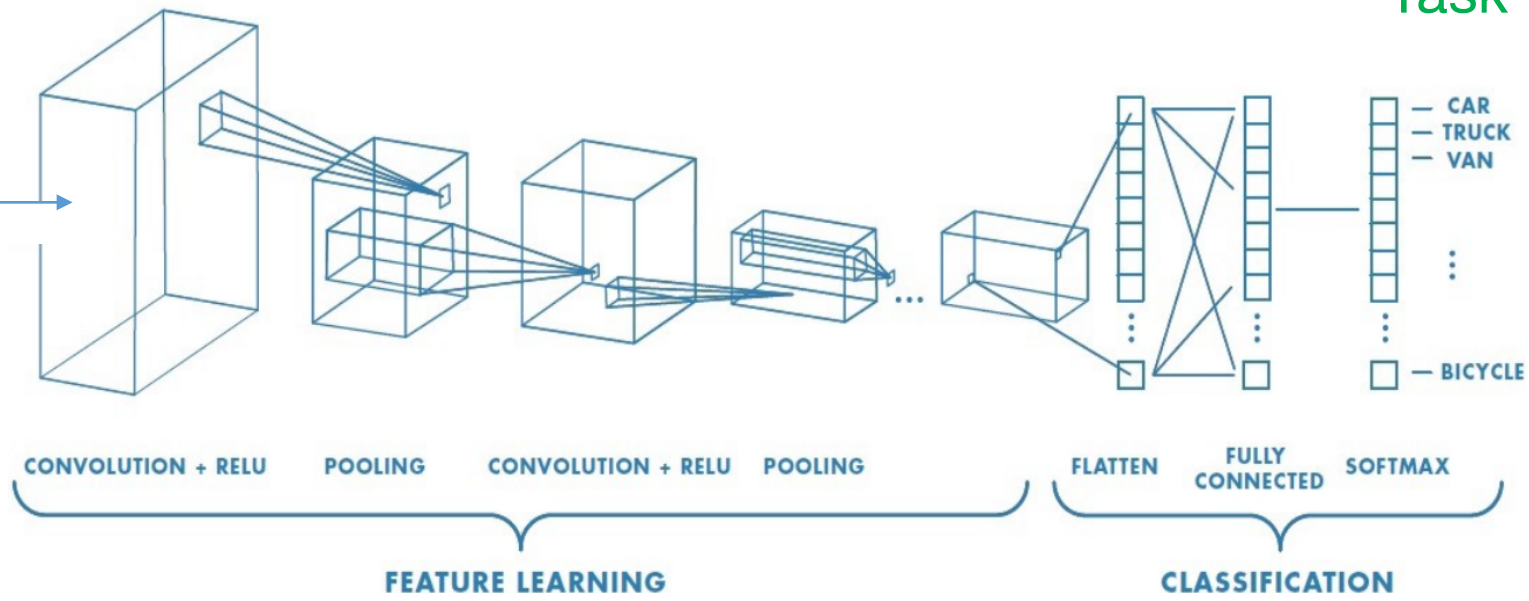
deep imaging

Physical Layers

Digital Layers

Task

Physical Object I_0 → f[ ] → Digitized image I_s →



$$I_s = f[I_0]$$

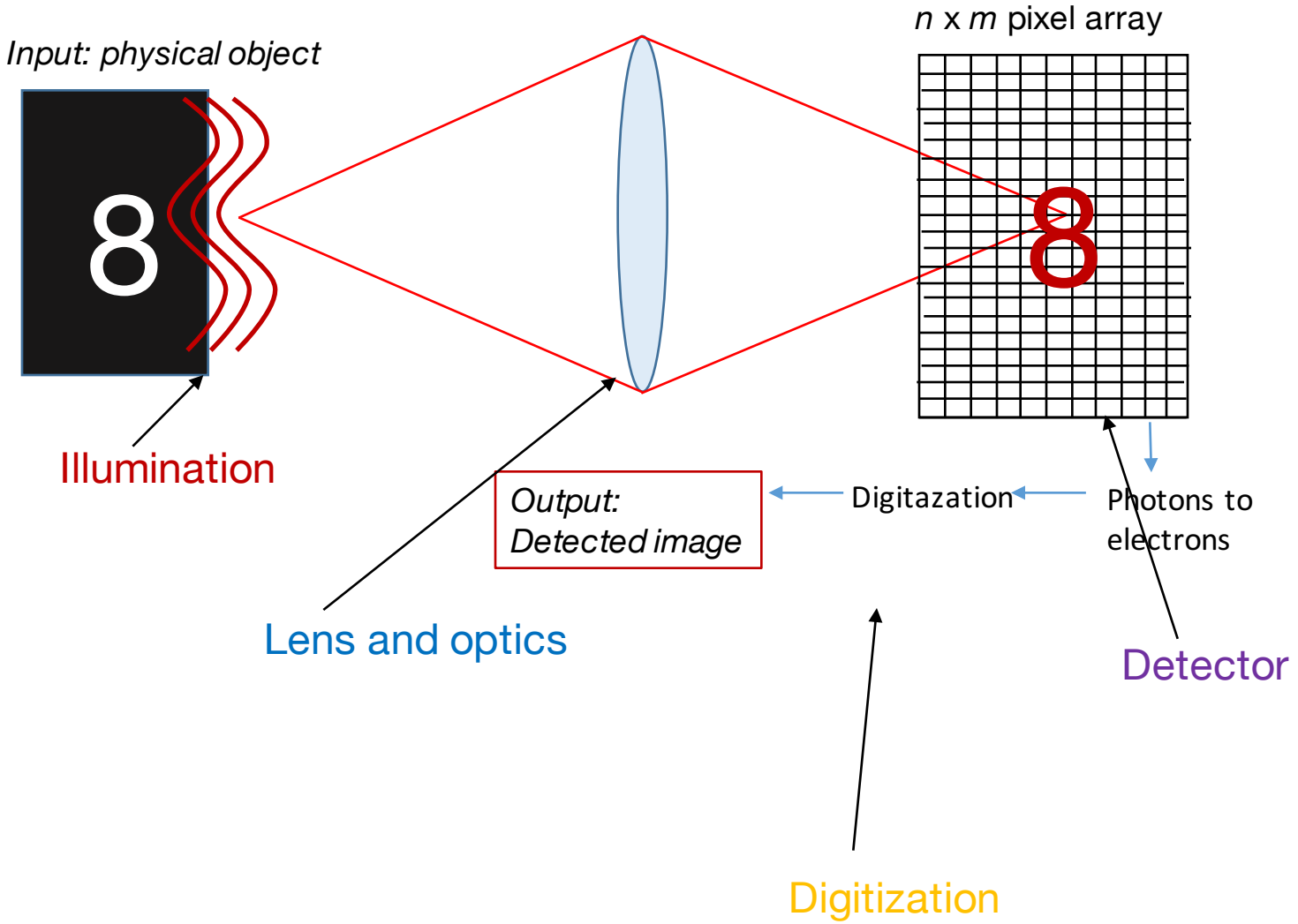
Digital layers

Physical layers

$$\text{Task} = W_n \ldots \text{ReLU}[W_1 \, \text{ReLU}[W_0 \, f[I_0]] \ldots]$$

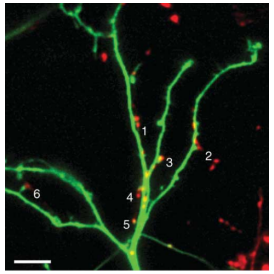
What physical parameters effect image formation f[ ]?

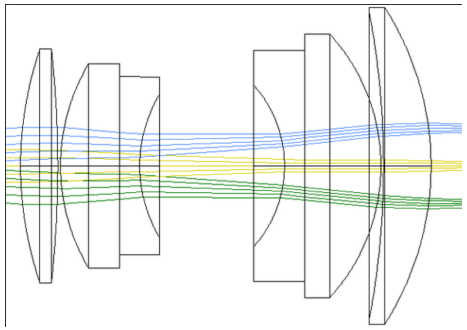
- Illumination
  - Spatial pattern
  - Angle of incidence
  - Color, polarization
- Lens and optics
  - Position/orientation
  - Shape
  - Focus
  - Transparency
- Detector
  - Pixel size
  - Pixel shape & fill factor
  - Color filters
  - Other filters
- Digitization
  - E to P curves
  - Digitization schemes/thresholds
  - Data transmission, multiplexing
- Physical object



First model of light we'll consider – Incoherent light

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

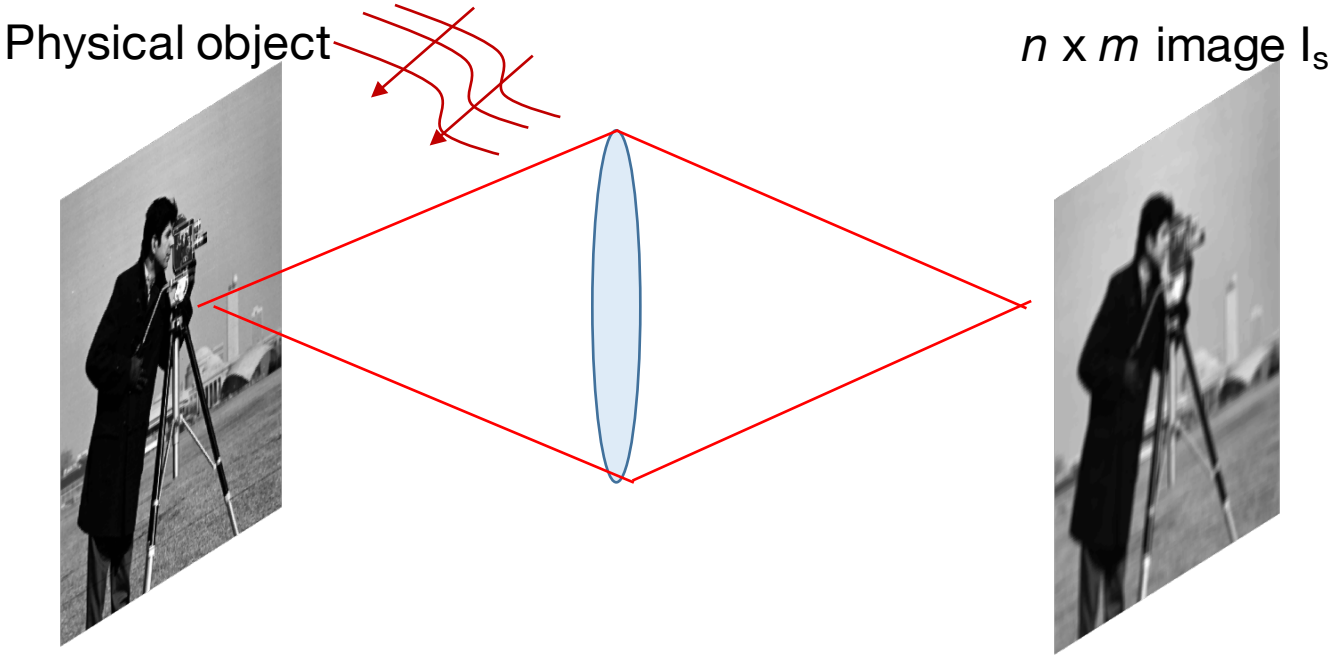




- Real, non-negative
- Models absorption and brightness

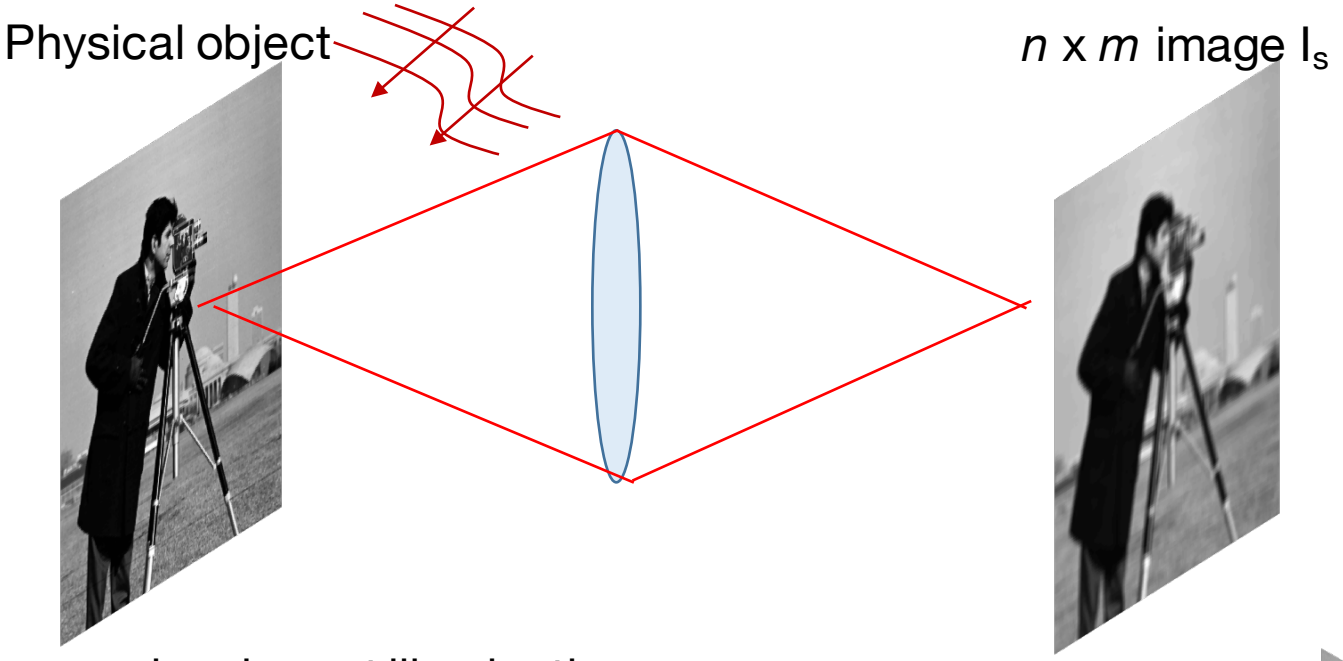
$$I_{tot} = I_1 + I_2$$

Example #1: Optimized illumination pattern (one color)



- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Example #1: Optimized illumination pattern (one color)



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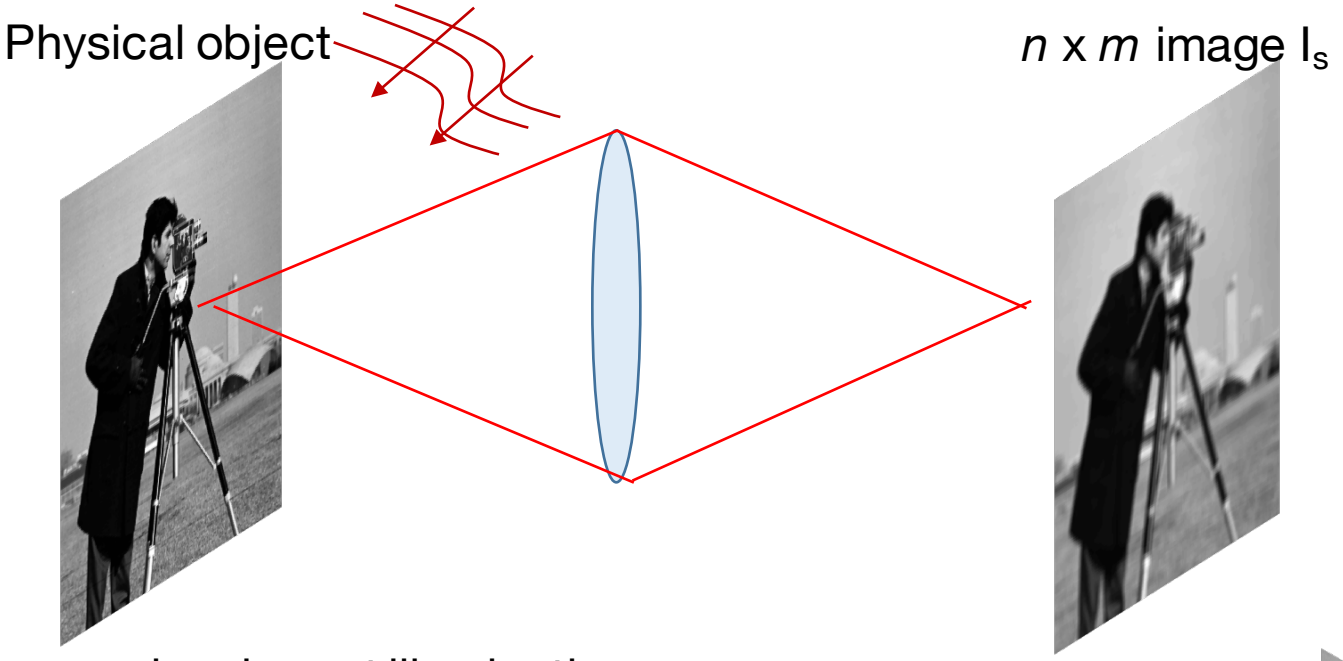
Example #1: Optimized illumination pattern (one color)



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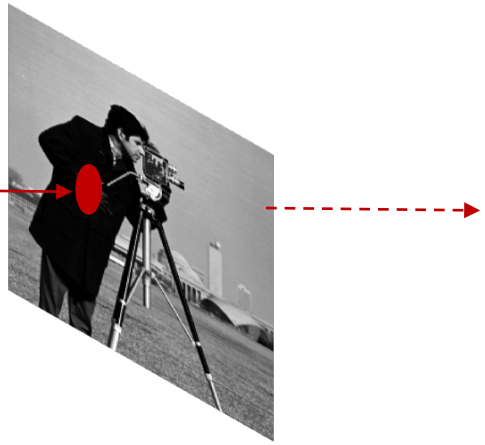
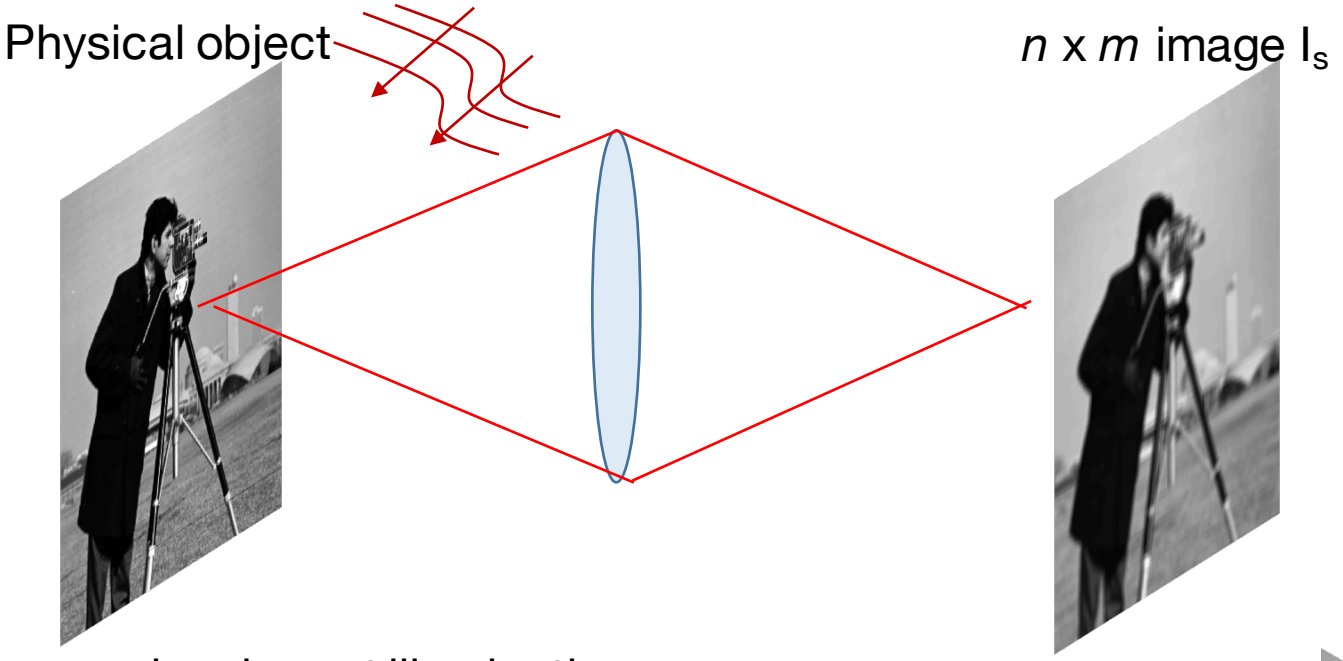


Example #1: Optimized illumination pattern (one color)



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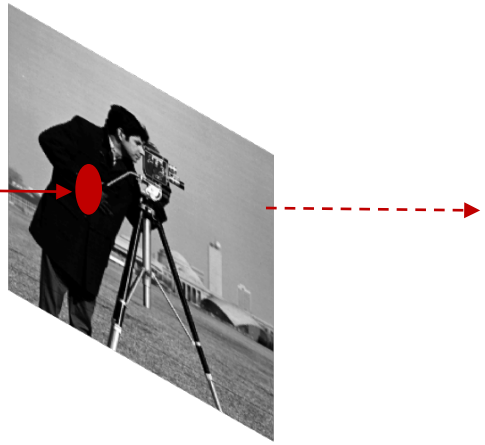
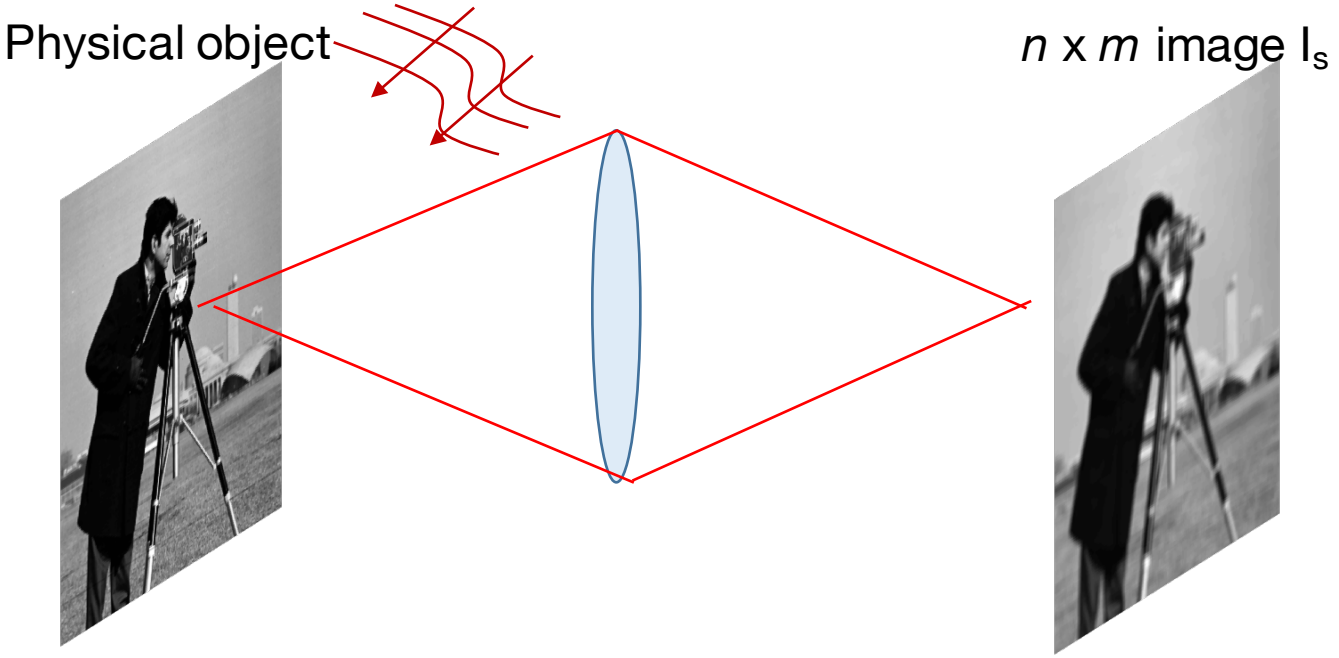


Object absorption: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Example #1: Optimized illumination pattern (one color)



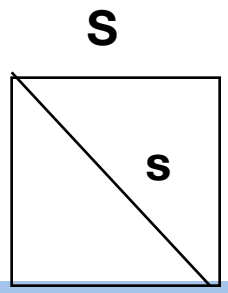
- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Modeling incoherent illumination

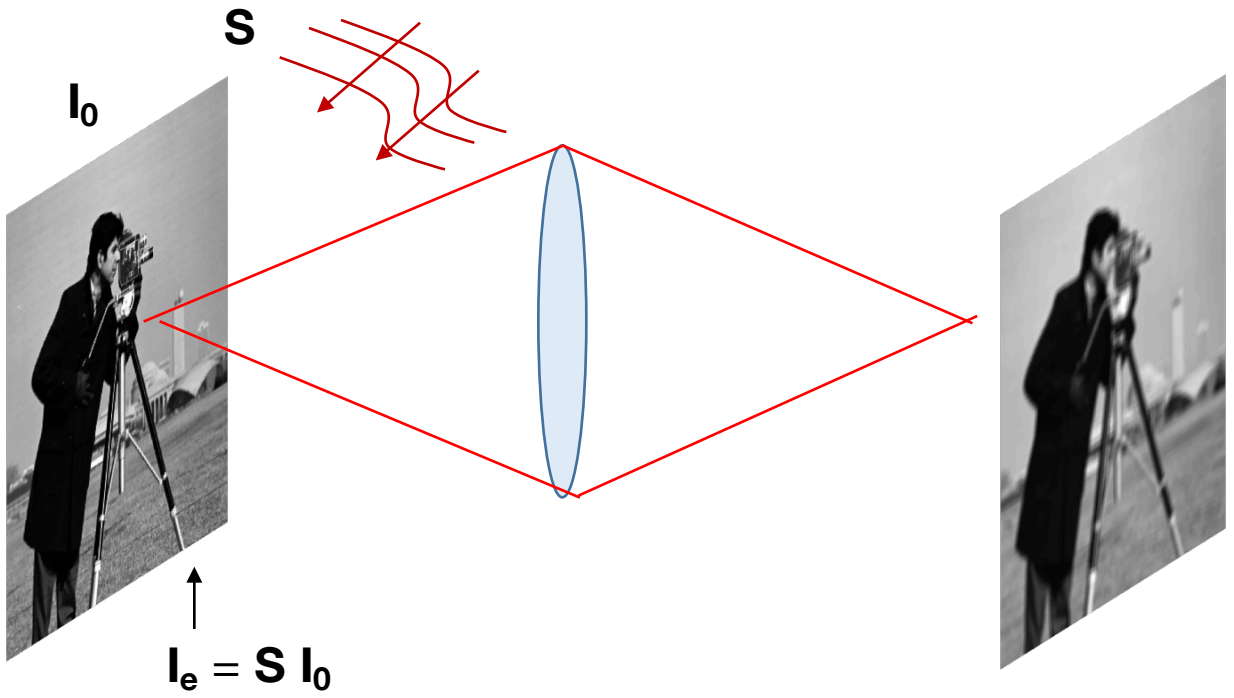
$$\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$$

$$\mathbf{I_e} = \mathbf{S}\,\mathbf{I_0}$$

$\mathrm{diag}(\mathbf{S}) = \mathbf{s}$



Example #1: Optimized illumination pattern (one color)



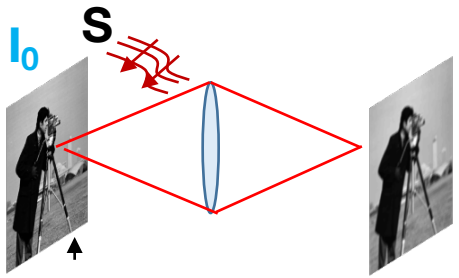
First, assume perfect camera:
intensity at image plane $I_p = I_e = S\ I_0$

Example #1: Optimized illumination pattern (one color)

deep imaging

Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

Example #1: Optimized illumination pattern (one color)



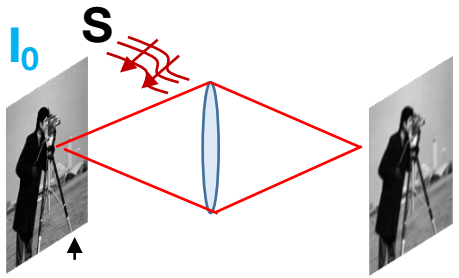
$[I_e(x, y), y]$

Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

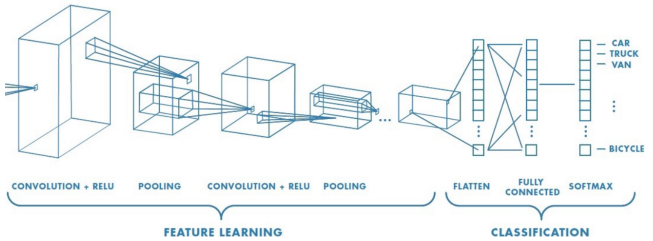
$I_e(x,y) = \mathbf{S}\ I_0(x,y)$

Physical Layer

Example #1: Optimized illumination pattern (one color)



$[I_e(x, y), y]$



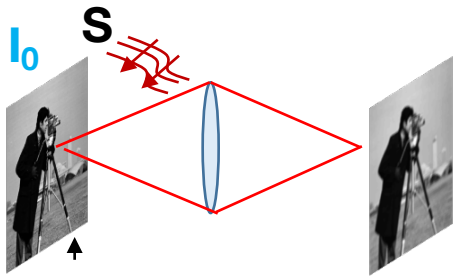
Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

$I_e(x,y) = \mathbf{S}\ I_0(x,y)$

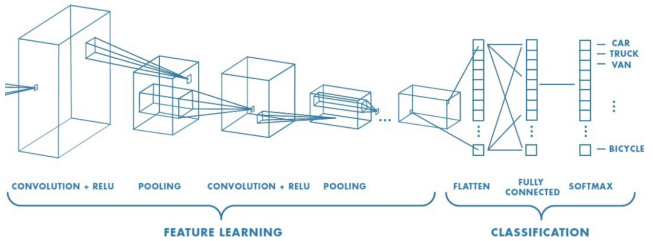
Physical Layer

Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\,\mathrm{ReLU}[\mathbf{W_0}\,I_e] \ldots]$

Example #1: Optimized illumination pattern (one color)



$[I_e(x, y), y]$



Training data:
$[I_0(x, y), y]$
I_0:100 x 100
Label y: 1x2

$I_e(x,y) = S\ I_0(x,y)$

Physical Layer

Task = $W_n \ldots \text{ReLU}[W_1 \text{ReLU}[W_0 I_e] \ldots]$

Tensorflow 1.0 code

```
training_images = tf.placeholder(tf.float32, [image_size, None])
training_labels = tf.placeholder(tf.float32, [None, 3])
illumination_pattern = tf.truncated_normal([image_size, 1], stddev = 0.1)
illumination_matrix = tf.linalg.diag(illumination_pattern)
illumianted_images = tf.matmul(illumination_matrix, training_images)

#Now, train CNN of your choice using [illumianted_images, training_labels]
#Output of train/test will be CNN weights, classification performance AND illumination_pattern!
```

See CoLab Notebook for implementation in Tensorflow 2.0

Option 1: tf.linalg.matmul

1.0 code

```
training_images = tf.placeholder(tf.float32, [image_size, None])
training_labels = tf.placeholder(tf.float32, [None, 3])
illumination_pattern = tf.truncated_normal([image_size, 1], stddev = 0.1)
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```
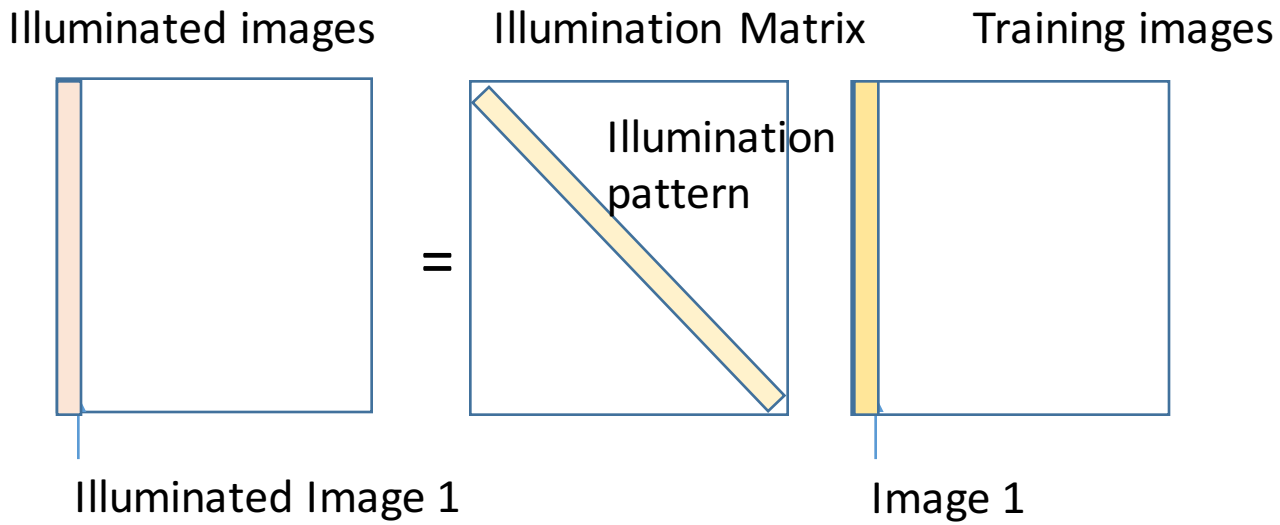



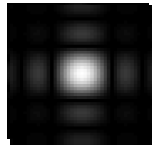
Option 2: tf.linalg.multiply (see CoLab Notebook)

We can also add in some lens blur

Lenses blur and rescale images:
(We'll learn how exactly next few weeks)

Input intensity

Convolution filter h



Output intensity

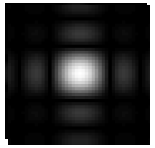
We can also add in some lens blur

Lenses blur and rescale images:
(We'll learn how exactly next few weeks)

Input intensity

Convolution filter h

$*$



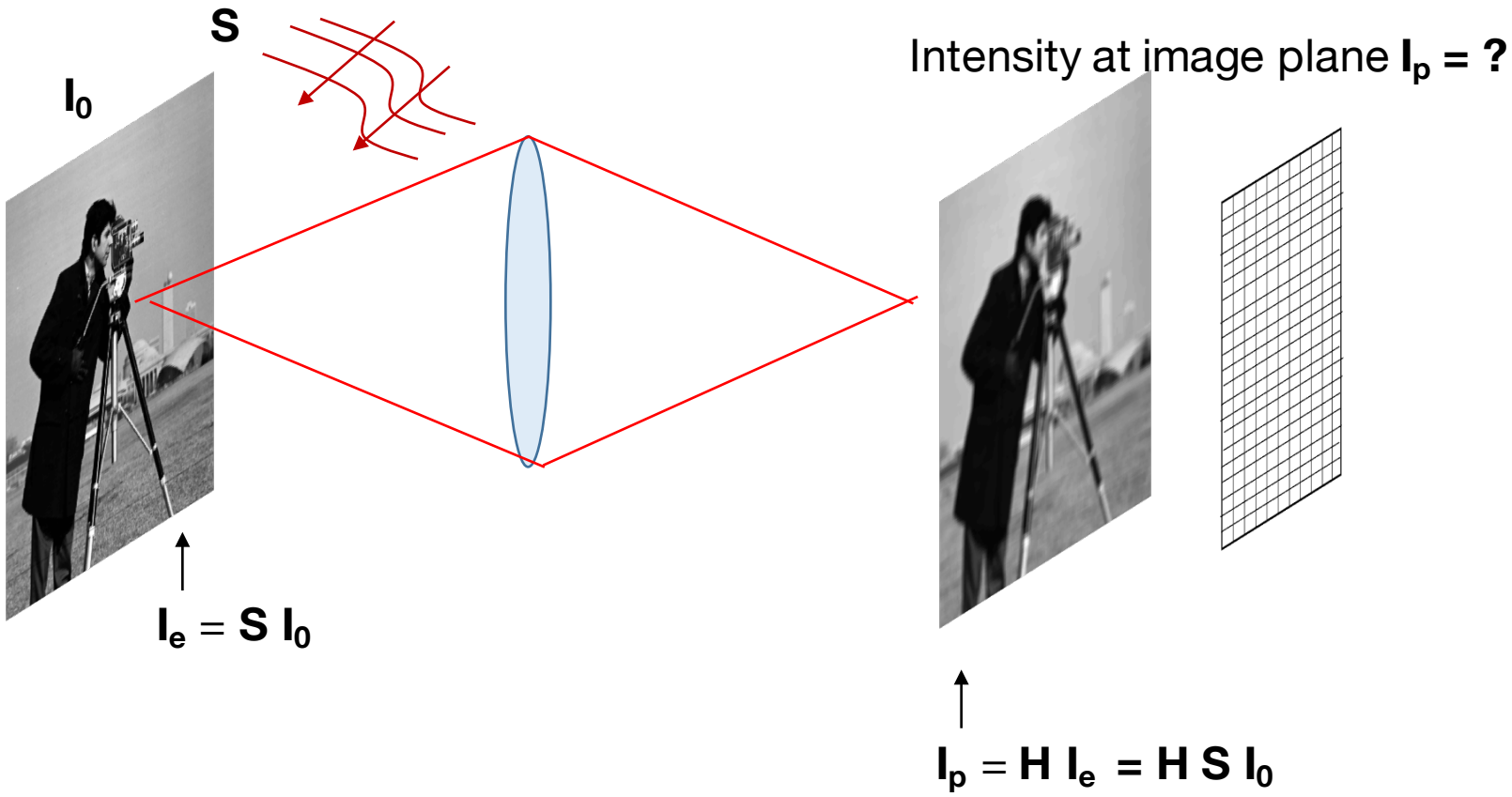
$=$

Output intensity

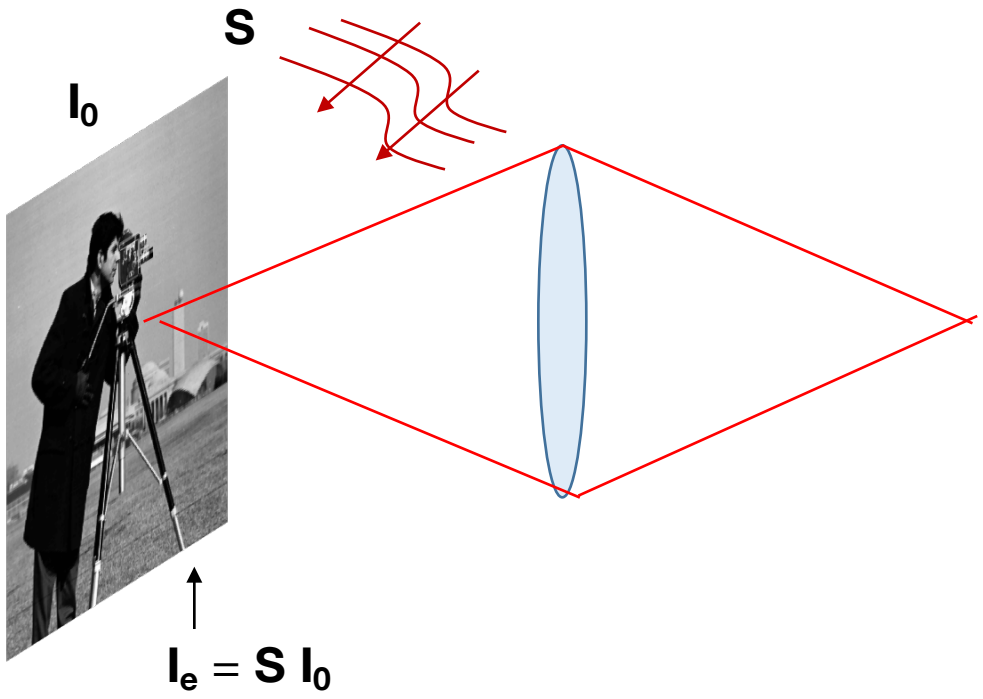
$\mathbf{I_e}(x/M, y/M) \quad * \quad \mathbf{h}(x,y) \quad = \quad \mathbf{I_p}(x,y)$

Assuming we've resized by M, $\mathbf{I_p = I_e * h = H I_e}$

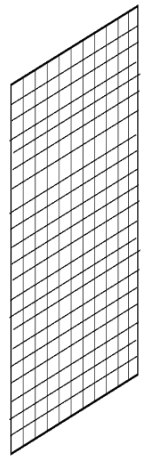
Simple mathematical model of image formation

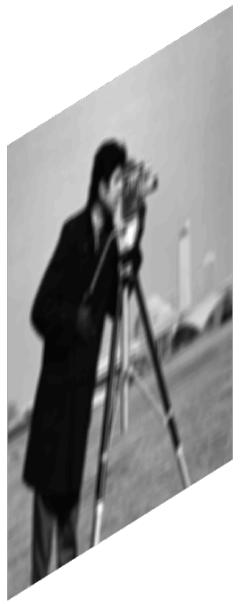


Simple mathematical model of image formation



Discretization by detector ?





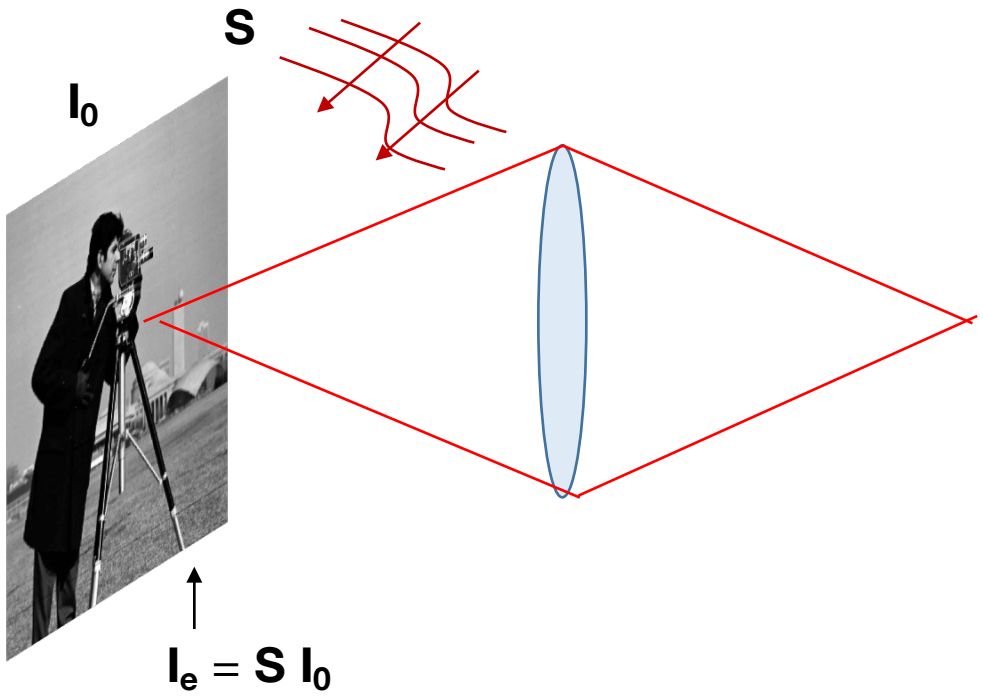
$I_p = H\ I_e = H\ S\ I_0$

Use downsampling matrix (sum-pooling)

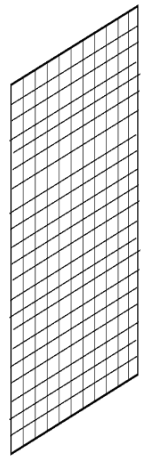
$D =$

$$\begin{bmatrix} 0.5\ 0.5\ 0\ 0\ 0\ \ldots. & & \\ & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. & \\ & & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. \\ \vdots & & \vdots \end{bmatrix}$$

Simple mathematical model of image formation



Discretization by detector?



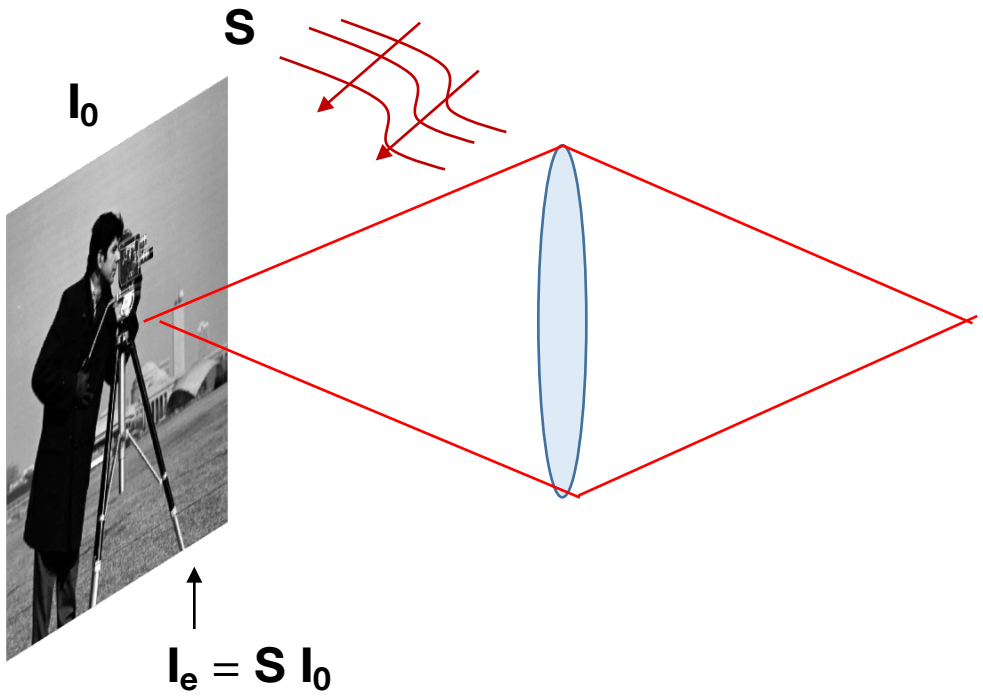
$$I_s = D\ I_p = D\ H\ S\ I_0$$

$D =$

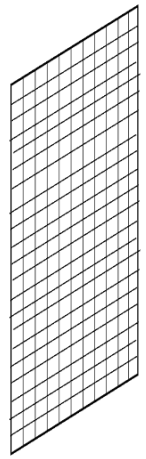
```
w1 w2 0 0 0 ....
      w3 w4  0 0 0 ....
            w5 e6  0 0 0 ....
```

*Can make these learnable weights

Simple mathematical model of image formation



Noise caused by detector?



$$I_s = D\ I_p + N$$

Can also add in detector-dependent noise N = k * np.random.randn(dx, dy)

(zero-mean Gaussian noise example)

deep imaging

Physical Layers

Digital Layers

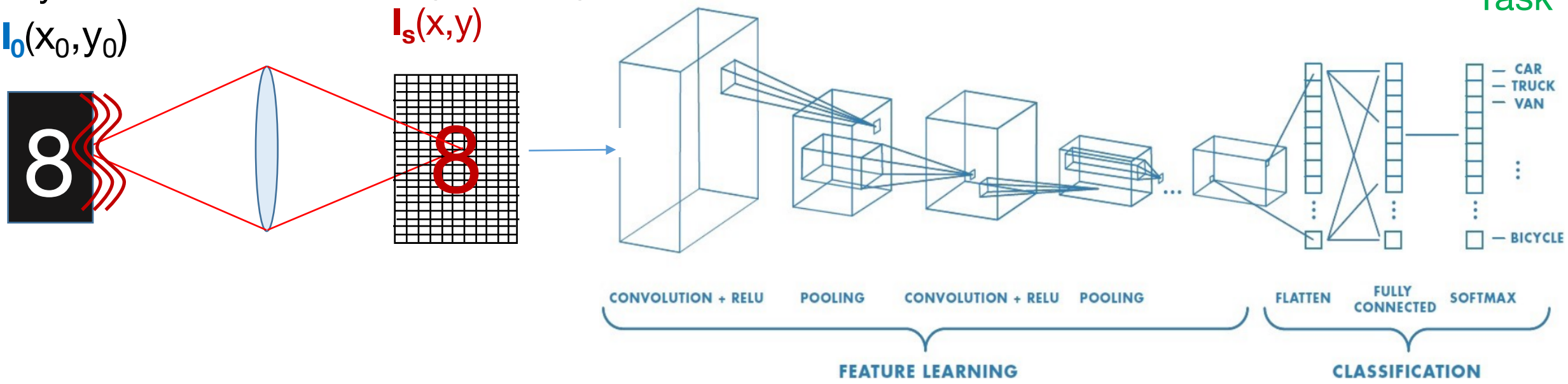
Physical world

$I_0(x_0,y_0)$

Digital Image

$I_s(x,y)$

Task



Digital layers

Physical layers

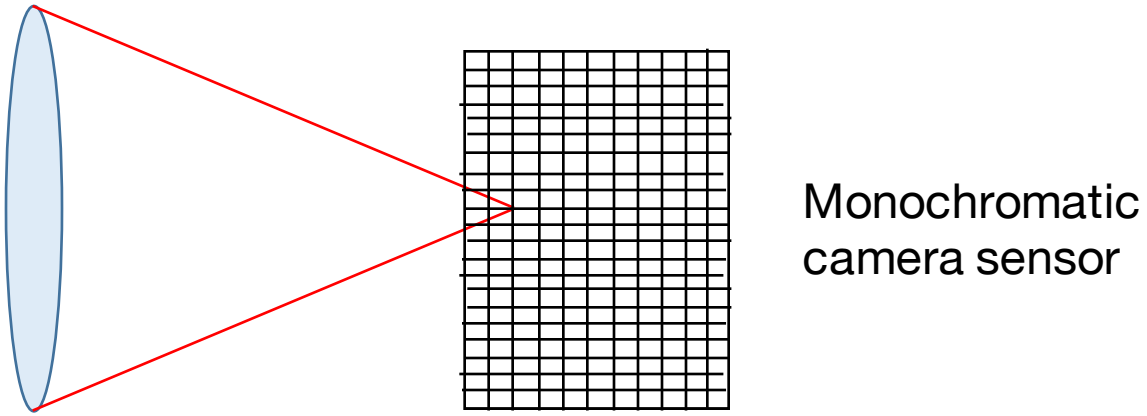
$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, f[I_0]]\ldots]$$

$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, \mathbf{D} \, \mathbf{H} \, \mathbf{S} \, I_0]\ldots]$$

Example #2: Optimized color filter for a grayscale camera

"Ground truth" object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

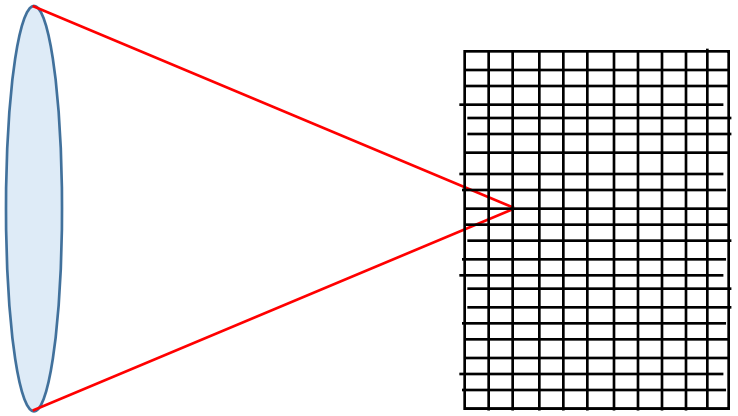
Example #2: Optimized color filter for a grayscale camera



Monochromatic
camera sensor

"Ground truth" object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

Example #2: Optimized color filter for a grayscale camera

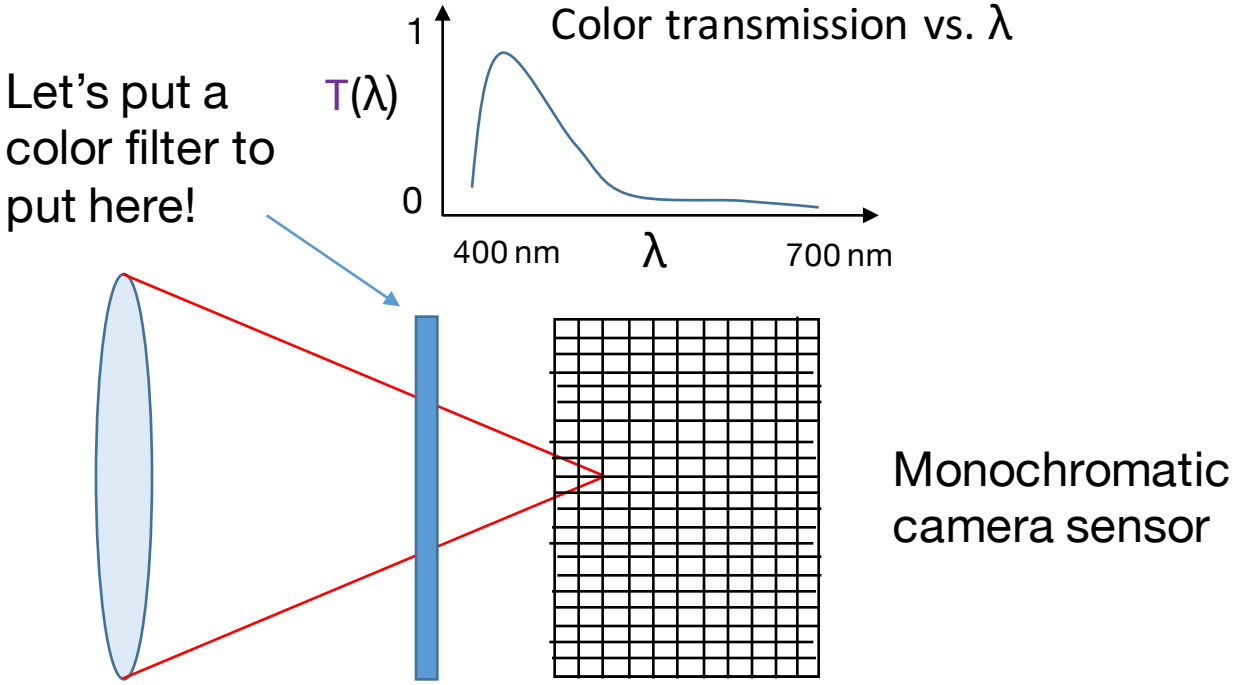


Monochromatic camera sensor

"Ground truth" object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

$$I_s(x, y) = \sum_{\lambda} I_0(x, y, \lambda)$$

Example #2: Optimized color filter for a grayscale camera

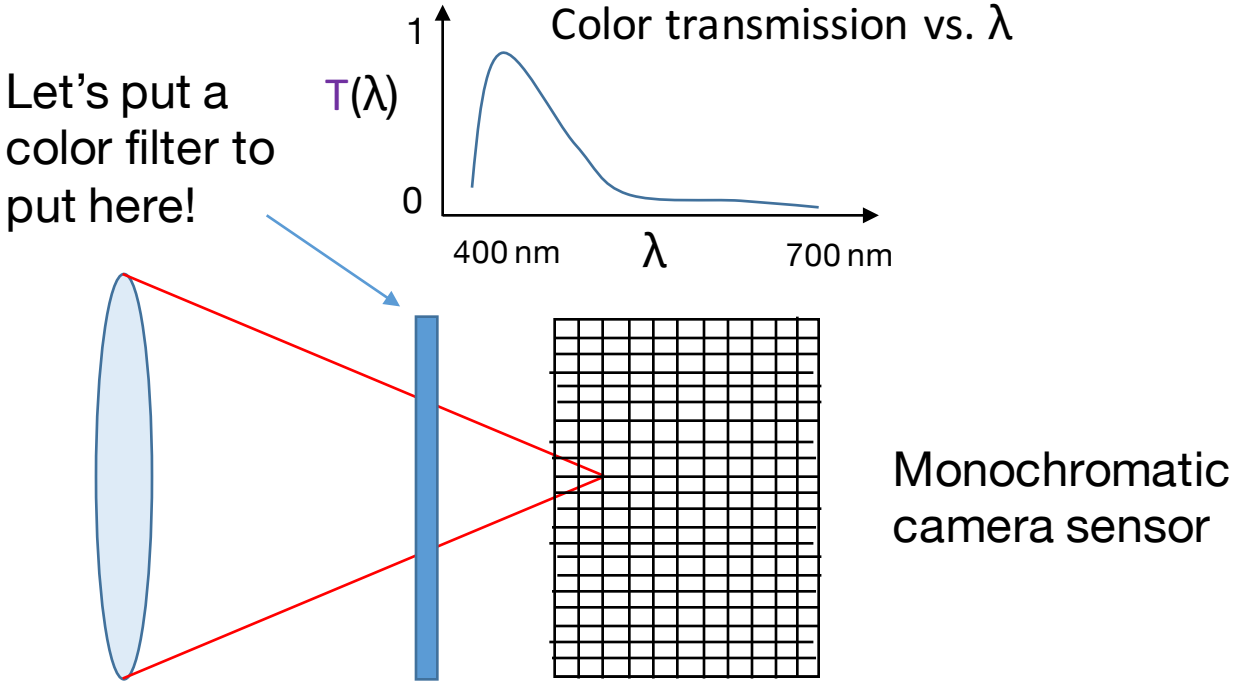


"Ground truth" object:

$I_0(x, y, \lambda)$

100 x 100 pix. x 30 spectral channels

Example #2: Optimized color filter for a grayscale camera

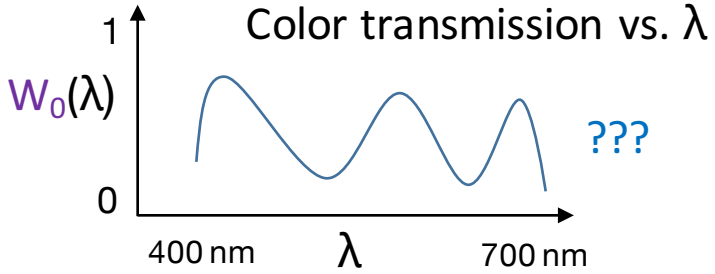


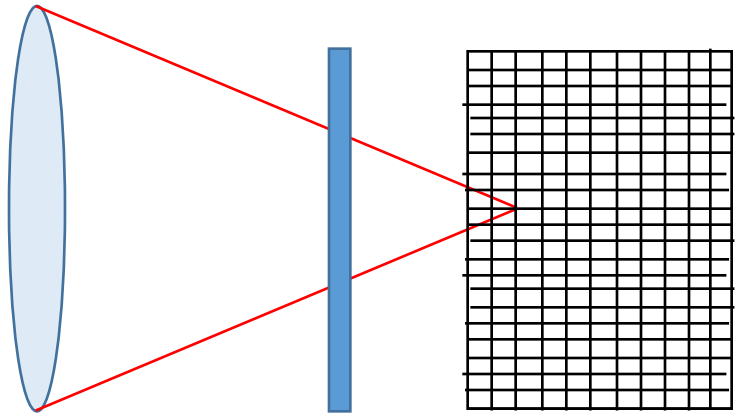
"Ground truth" object:
$I_0(x, y, \lambda)$
100 x 100 pix. x 30 spectral channels

$$I_s(x, y) = \sum_{\lambda} T(\lambda)\, I_0(x, y, \lambda)$$

Example #2: Optimized color filter for a grayscale camera

Design optimal color filter for classification:





Monochromatic camera sensor

Training data:
$[I_0(x, y, \lambda), y]$
I_0:100 x 100 pix. x 30
Label y: 1x3 - pepper, broccoli, green beans

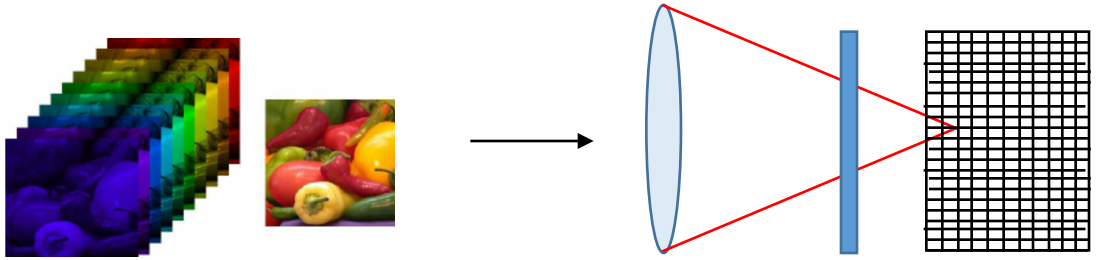
$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

Physical Layer

Example #2: Optimized color filter for a grayscale camera

Training data:
$[I_0(x, y, \lambda), y]$
I_0:100 x 100 x 30
Label y: 1x3

Example #2: Optimized color filter for a grayscale camera

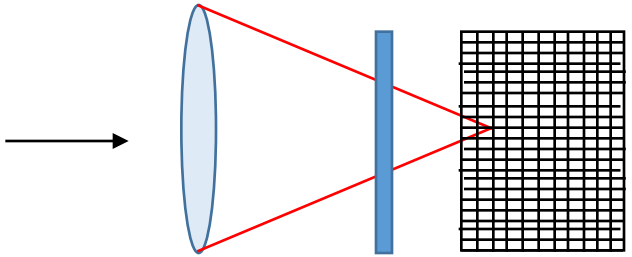


Training data:
$[I_0(x, y, \lambda), y]$
I_0:100 x 100 x 30
Label y: 1x3

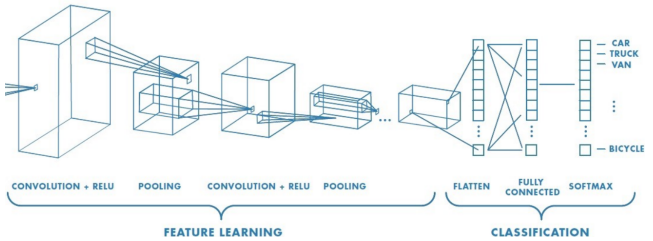
$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

Physical Layer

Example #2: Optimized color filter for a grayscale camera



$[I_s(x, y), y]$



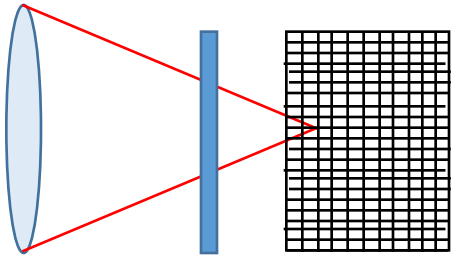
Training data:
$[I_0(x, y, \lambda), y]$
I_0:100 x 100 x 30
Label y: 1x3

$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

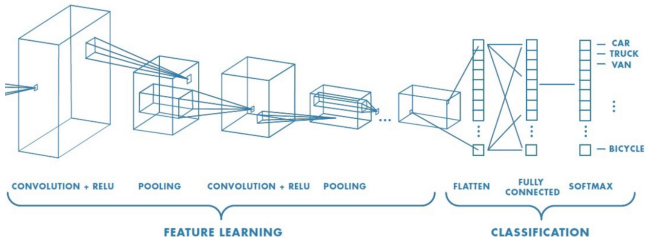
Physical Layer

Task = $\mathbf{W_n} \ldots \mathrm{ReLU}[\mathbf{W_1}\, \mathrm{ReLU}[\mathbf{W_0}\, I_s] \ldots]$

Example #2: Optimized color filter for a grayscale camera



$[I_s(x, y), y]$



Training data:
$[I_0(x, y, \lambda), y]$
I_0:100 x 100 x 30
Label y: 1x3

$$I_s(x, y) = \sum_{\lambda} W_0(\lambda)\, I_0(x, y, \lambda)$$

Physical Layer

Task = $\mathbf{W_n}$ …ReLU[$\mathbf{W_1}$ ReLU[$\mathbf{W_0}$ I_s] …]

```
multispectral_data = tf.placeholder(tf.float32, [None, num_colors, image_size])
veg_labels = tf.placeholder(tf.float32, [None, 3])
filter_weights = tf.truncated_normal([num_colors, 1], stddev = 0.1)
filtered_images = tf.einsum('aij,jk->aik', multispectral_data, filter_weights)

#Now, train CNN of your choice using [filtered_images, veg_labels]
#Output of training/testing will be CNN weights, classification performance AND filter_weights!
```

Example implementation with Tensorflow 1.0 code

Tensorflow: operations to sum along 3rd (or higher) dimension

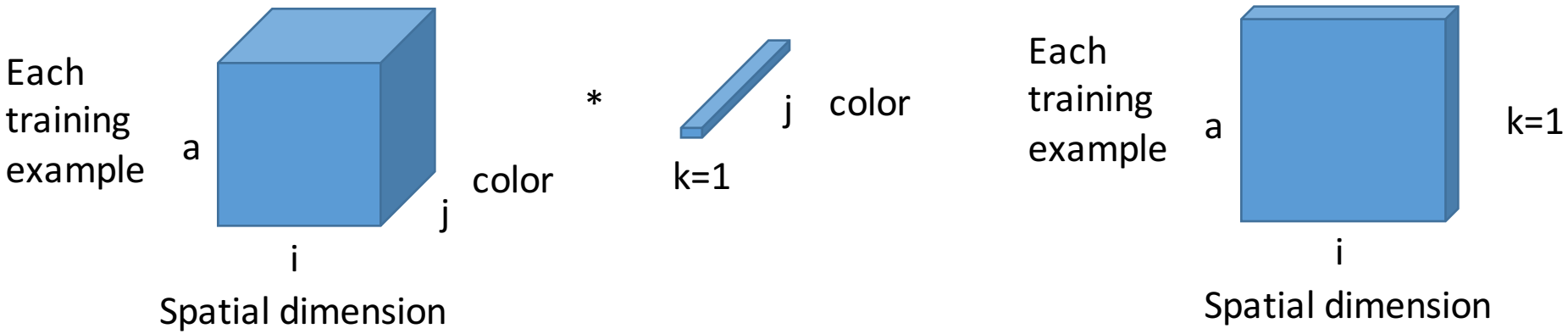
deep imaging

Option 1: Einsum (shown as Tensorflow 1.0 code, and also applicable in Tensorflow 2.0)

```
filtered_images = tf.einsum('aij,jk->aik', multispectral_data, filter_weights)
```

Option 2: tf.reduce_sum

```
filtered_images = tf.reduce_sum(multispectral_data * filter_weights, axis=2)
```



Option 3: Locally connected conv2D with a 1x1 filter size

https://github.com/keras-team/keras/blob/master/keras/layers/local.py#L183

Summary: simple physical layers for incoherent imaging

- Deal with sample/image intensities I, real and non-negative
- Effect of illumination is element-wise multiplication

$$\mathbf{I}_e(x,y) = \mathbf{S}\, \mathbf{I}_0(x,y)$$

- Imaging systems blur the object via point-spread function matrix **H**

$$\mathbf{I}_b(x,y) = \mathbf{H}\, \mathbf{I}_0(x,y)$$

- Discrete pixels down-sample the object via

$$\mathbf{I}_d(x,y) = \mathbf{D}\, \mathbf{I}_0(x,y)$$

- Add noise into measurement

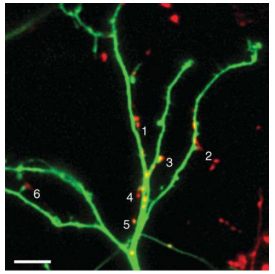
$$\mathbf{I}_N(x,y) = \mathbf{D}\, \mathbf{I}_0(x,y) + \mathbf{N}$$

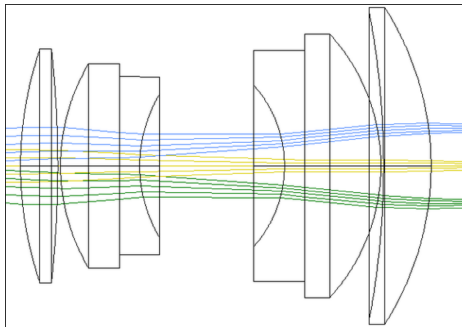
- Different colors add linearly

$$\mathbf{I}_s(x,\, y) = \sum_{\lambda} \mathbf{I}_0(x,\, y,\, \lambda)$$

What is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

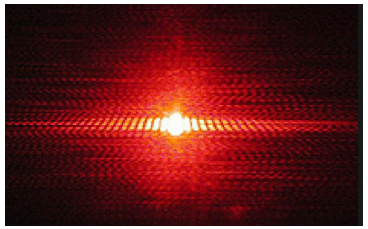


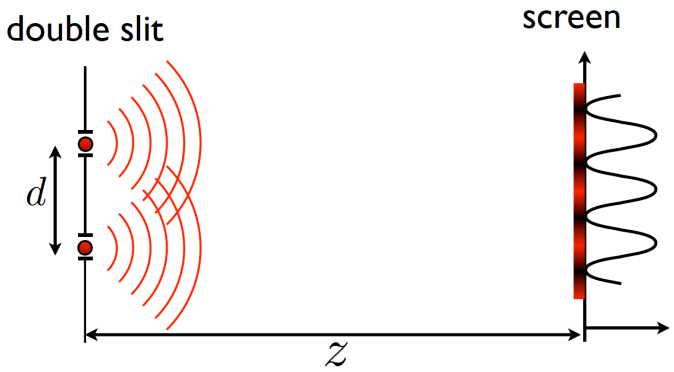


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves





- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

Next class: Modeling coherent radiation as a wave