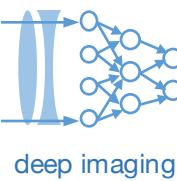


Lecture 22: Reinforcement Learning

Machine Learning and Imaging

BME 590L
Roarke Horstmeyer



Resources for this lecture

Stanford CS231n, Lecture 17

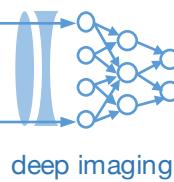
Berkeley CS 294: Deep Reinforcement Learning

http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_3_rl_intro.pdf

V. Mnih et al., “Human-level control through deep reinforcement learning,” Nature (2016)

Technical note: Q-Learning

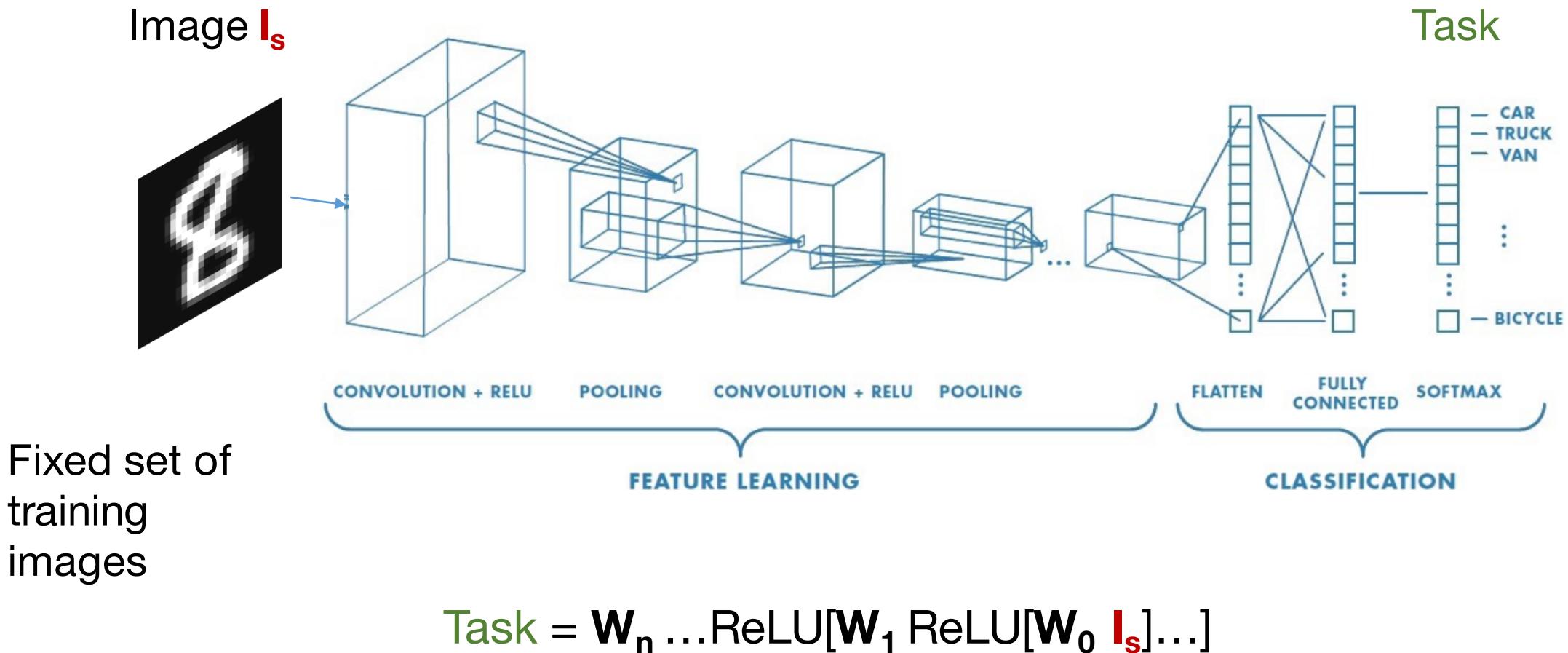
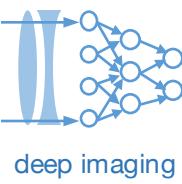
<http://www.gatsby.ucl.ac.uk/~Dayan/papers/cjch.pdf>

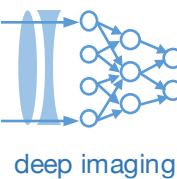


Reinforcement learning - in a nutshell

- So far, we've looked at:
 - 1) Decisions from fixed images (classification, detection, segmentation)

CNN's





Reinforcement learning - in a nutshell

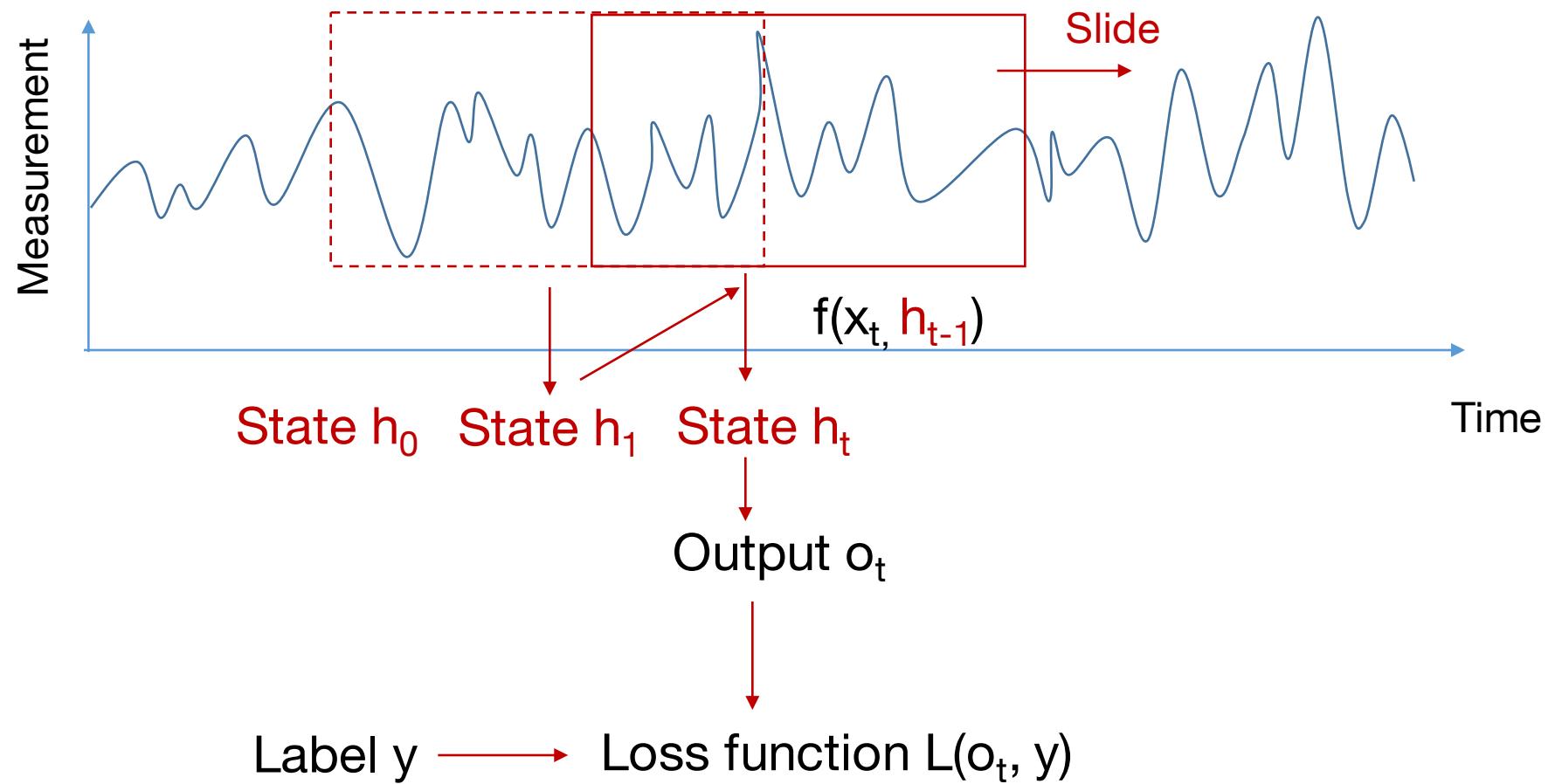
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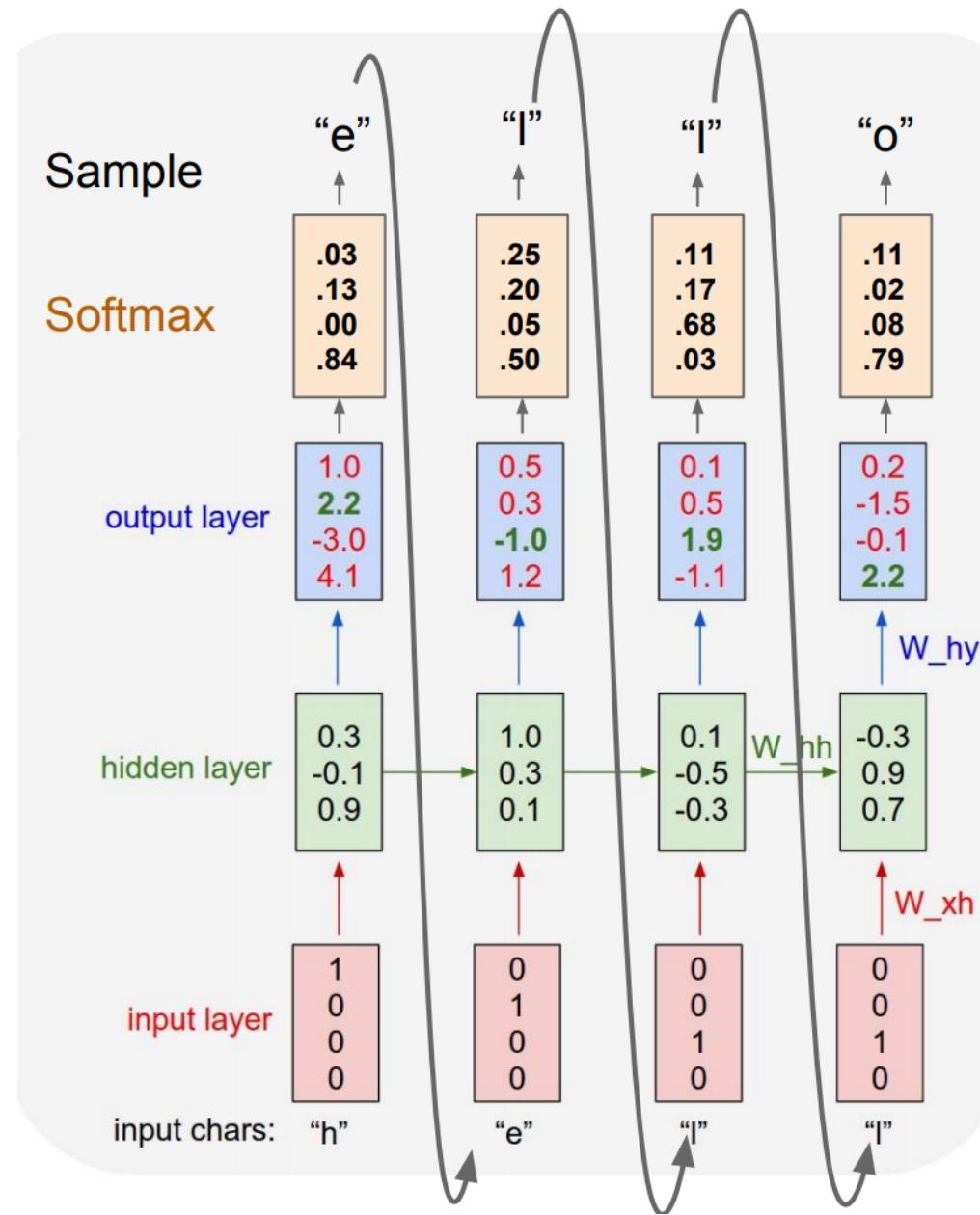
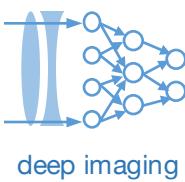
CNN's

- 2) Decisions from time-sequence data (captioning as classification, etc.)

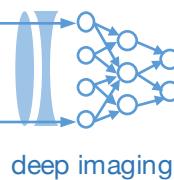
RNN's

Fixed set of
temporal sequences





From Stanford CS231n Lecture 10 slides



Reinforcement learning - in a nutshell

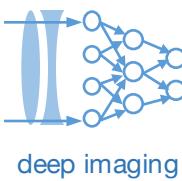
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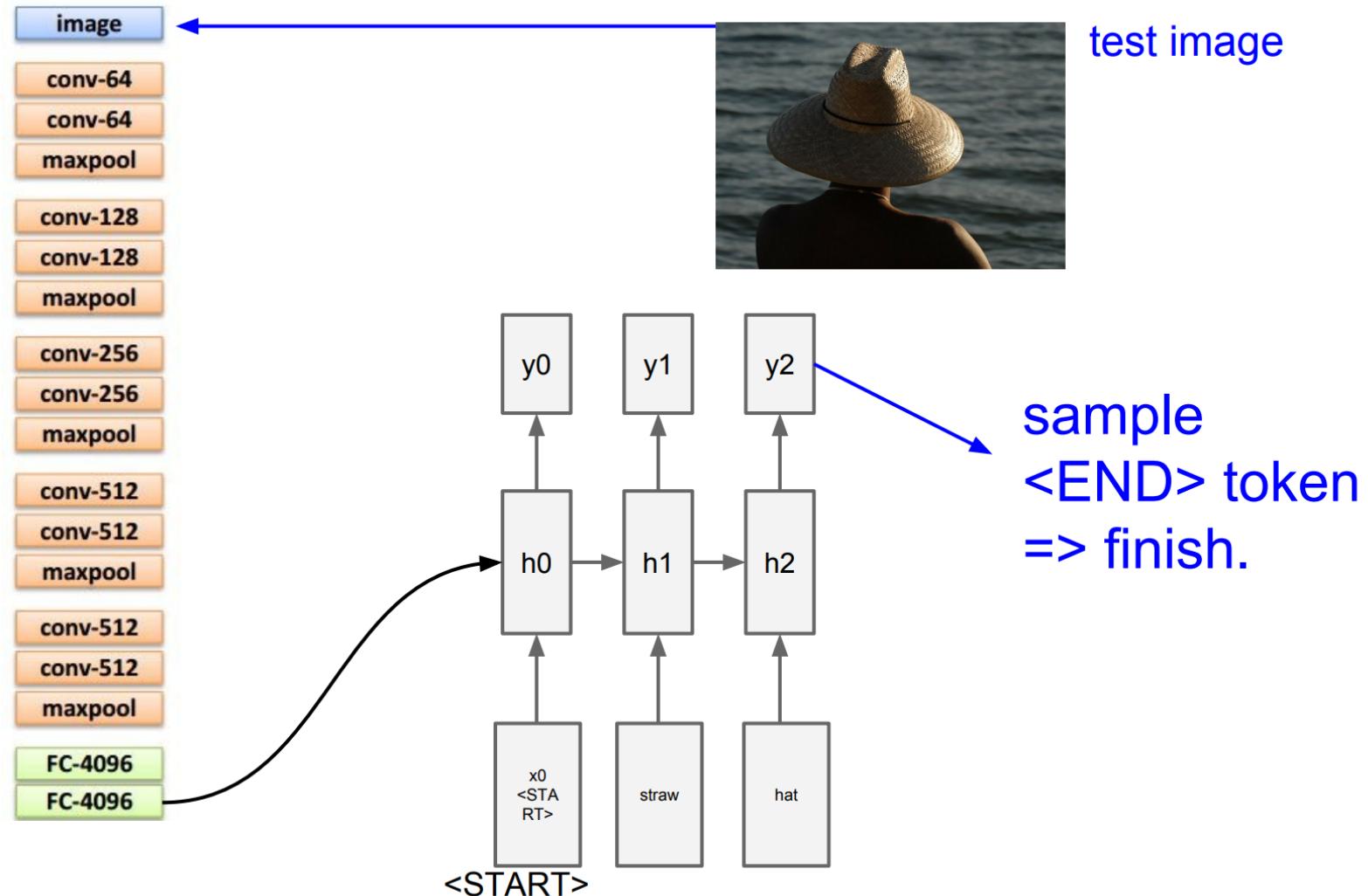
- 2) Decisions from time-sequence data (captioning as classification, etc.)

Decisions from images and time-sequence data (video classification, etc.)

RNN's



Example: Image captioning

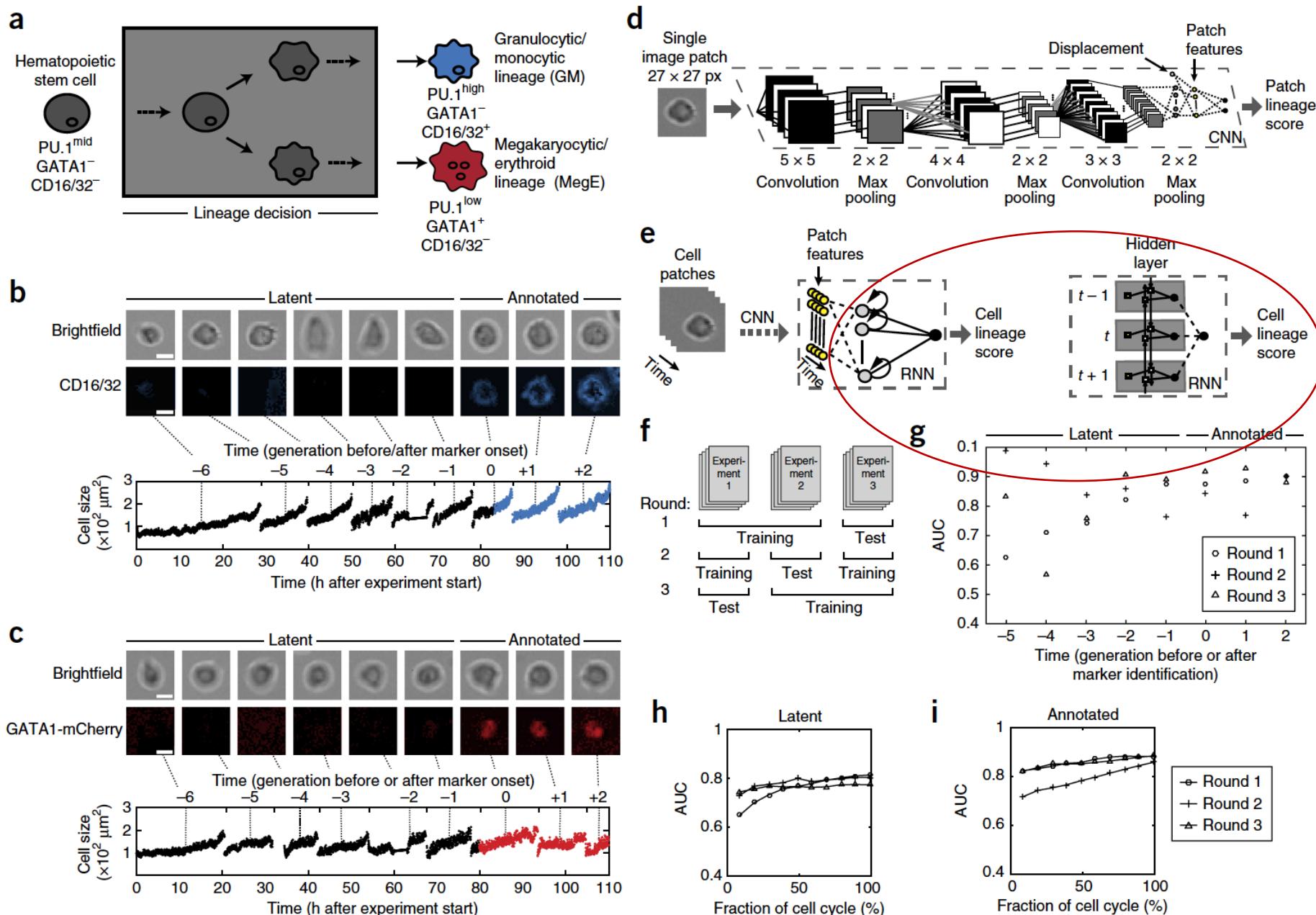


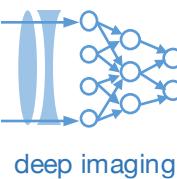
From Stanford CS231n Lecture 10 slides



Prospective identification of hematopoietic lineage choice by deep learning

Felix Buggenthin^{1,6}, Florian Buettner^{1,2,6}, Philipp S Hoppe^{3,4}, Max Endele³, Manuel Kroiss^{1,5}, Michael Strasser¹, Michael Schwarzfischer¹, Dirk Loeffler^{3,4}, Konstantinos D Kokkaliaris^{3,4}, Oliver Hilsenbeck^{3,4}, Timm Schroeder^{3,4}, Fabian J Theis^{1,5} & Carsten Marr¹





Reinforcement learning - in a nutshell

- So far, we've looked at:
 - 1) Decisions from fixed images (classification, detection, segmentation)

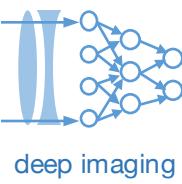
CNN's

- 2) Decisions from time-sequence data (captioning as classification, etc.)
Decisions from images and time-sequence data (video classification, etc.)

RNN's

- Now, we're going to consider decisions for *dynamic data*
 - Most successful application: dynamic image data
e.g.: video games, images of a Go game, car turning through obstacles

Reinforcement Learning

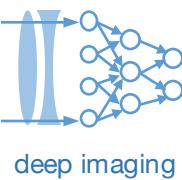


The step from fixed video to dynamic video



Outcome:
Cell type B

Goal: examine *all* data
to make final decision

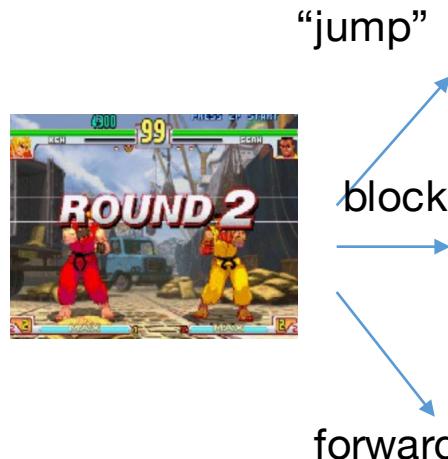


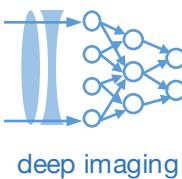
The step from fixed video to dynamic video



Outcome:
Cell type B

Goal: examine *all* data
to make final decision





The step from fixed video to dynamic video



Outcome:
Cell type B

Goal: examine *all* data
to make final decision

Goal: decide on path
through data to get to
final result



“jump”
block
forward

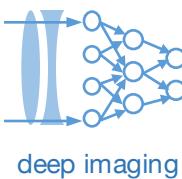


“jump”
block
forward

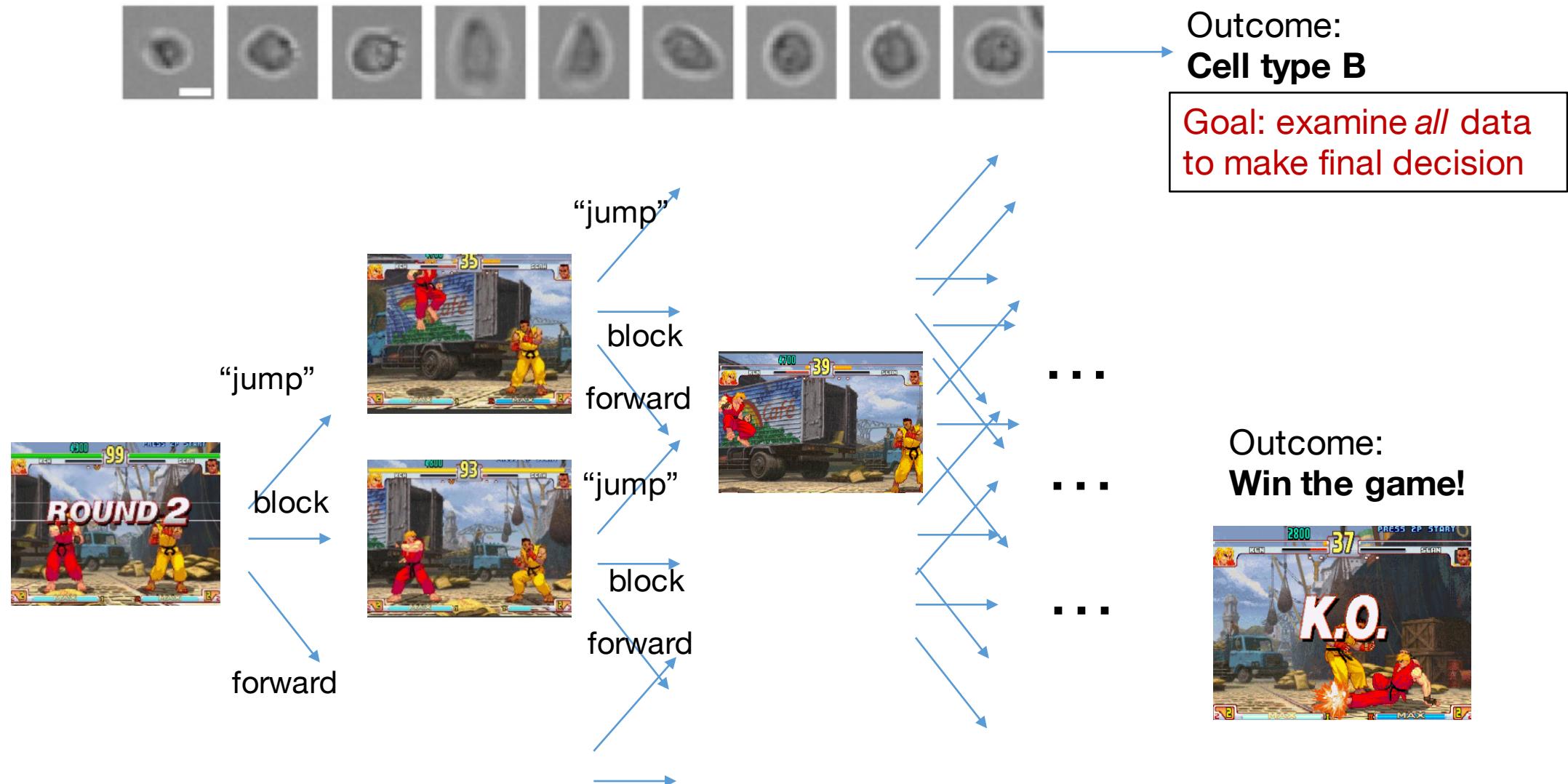


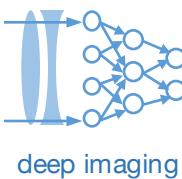
“jump”
block
forward

→

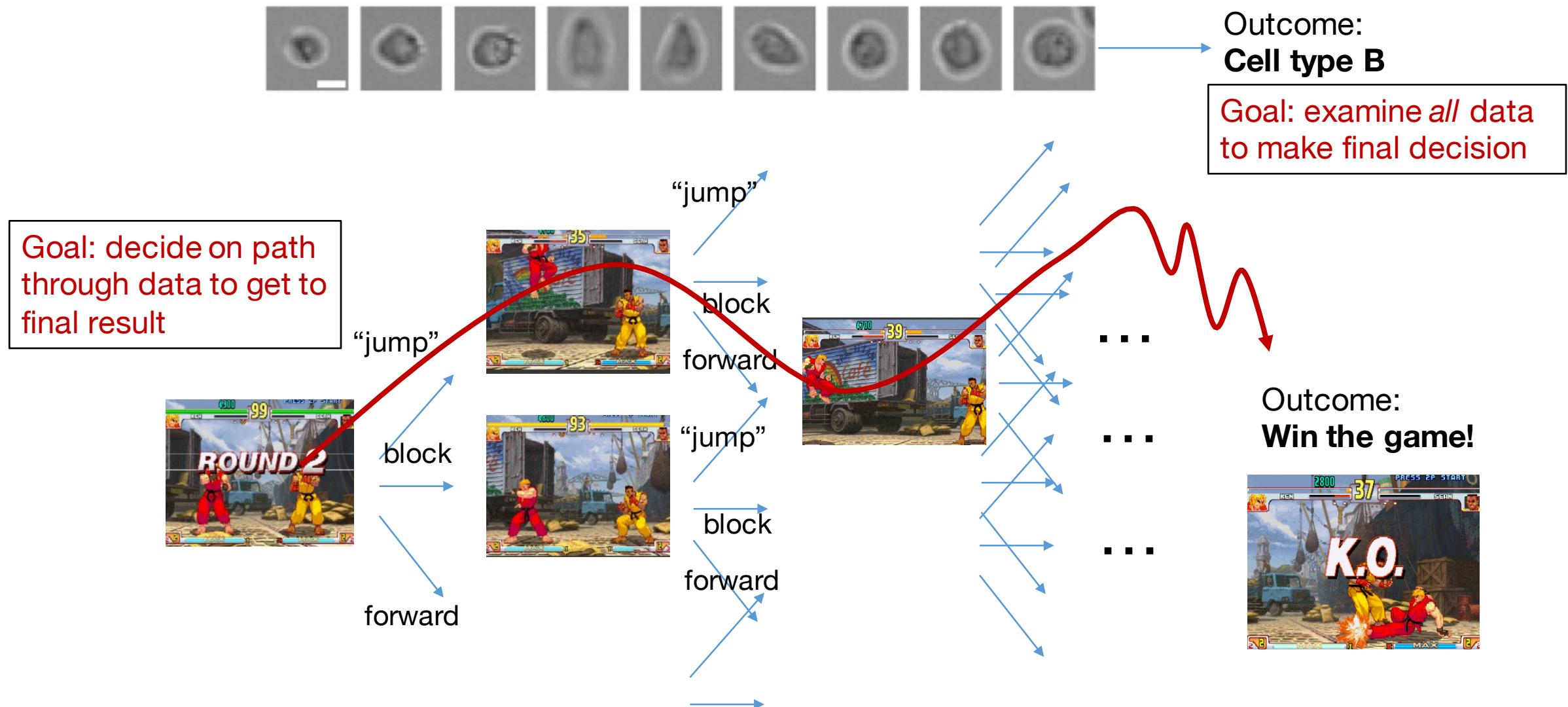


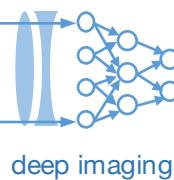
The step from fixed video to dynamic video





The step from fixed video to dynamic video



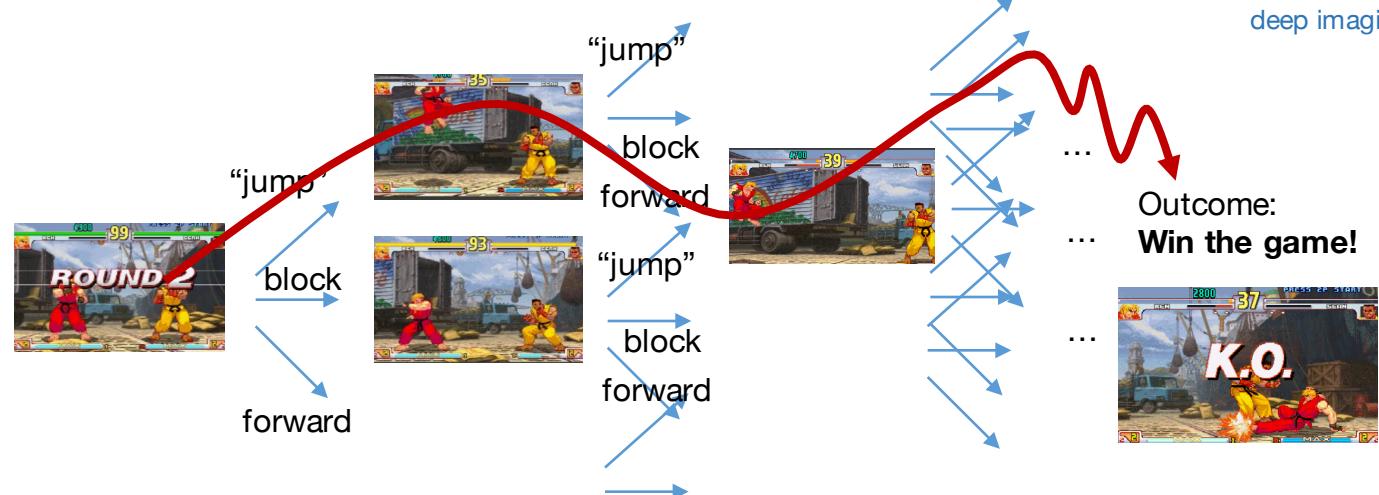


Supervised ML



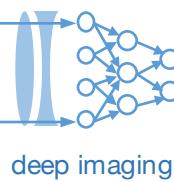
Outcome:
Cell type B

Reinforcement learning



- Fixed image sequence
- Goal: match to known label
(large labeled dataset needed)
- Output: label
- Examines all data

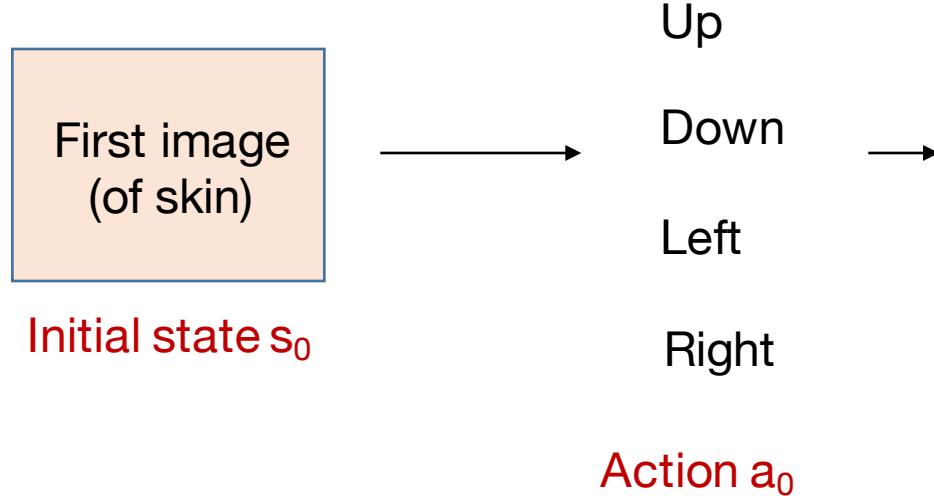
- Dynamic image sequence
- Goal: get to known desired outcome
(no labels needed, really...)
- Output: sequence of actions
- Not possible to examine *all* data

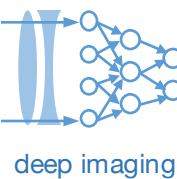


Terms and notation

Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible

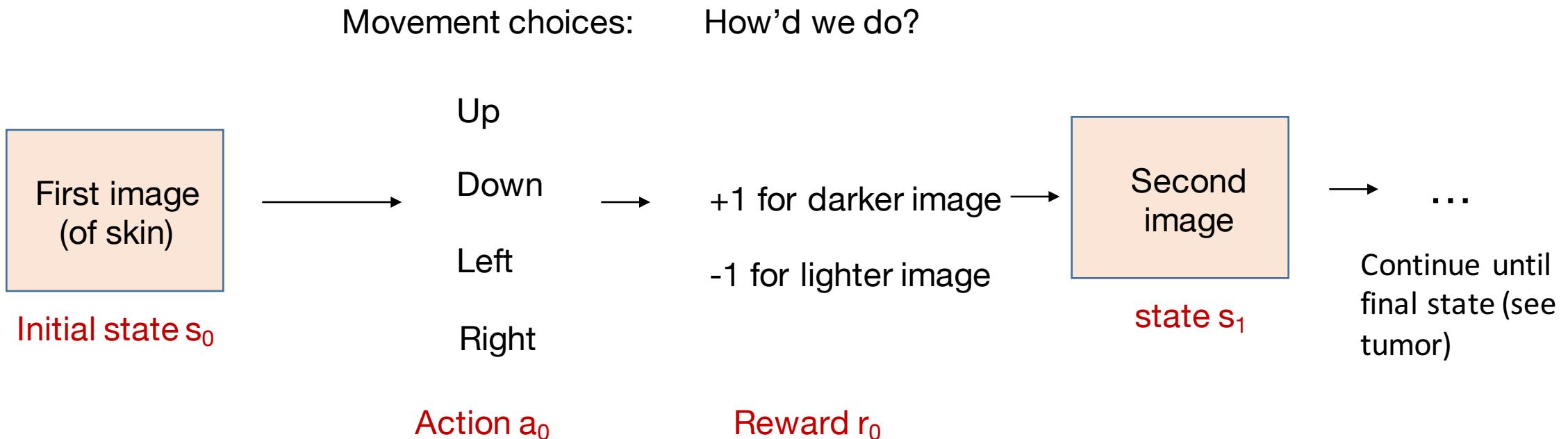
Movement choices:

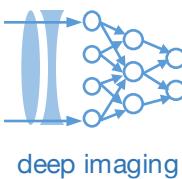




Terms and notation

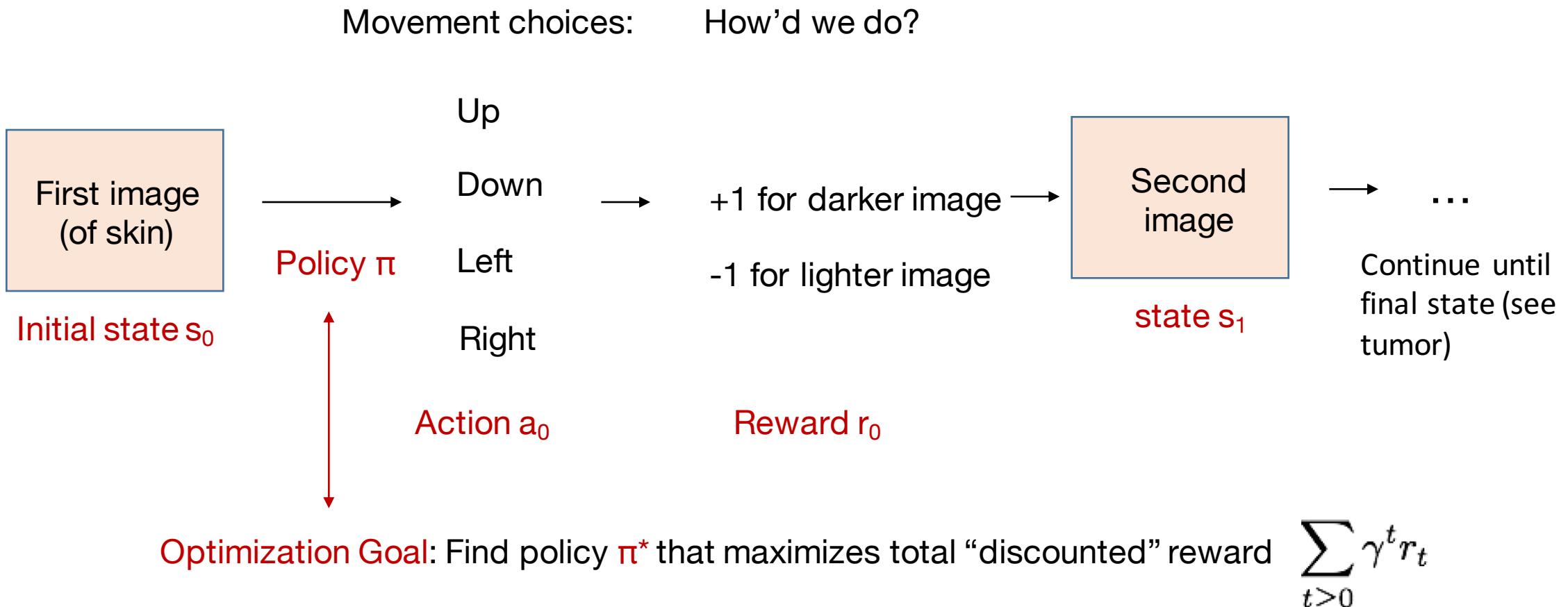
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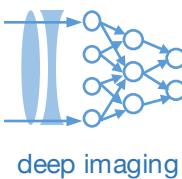




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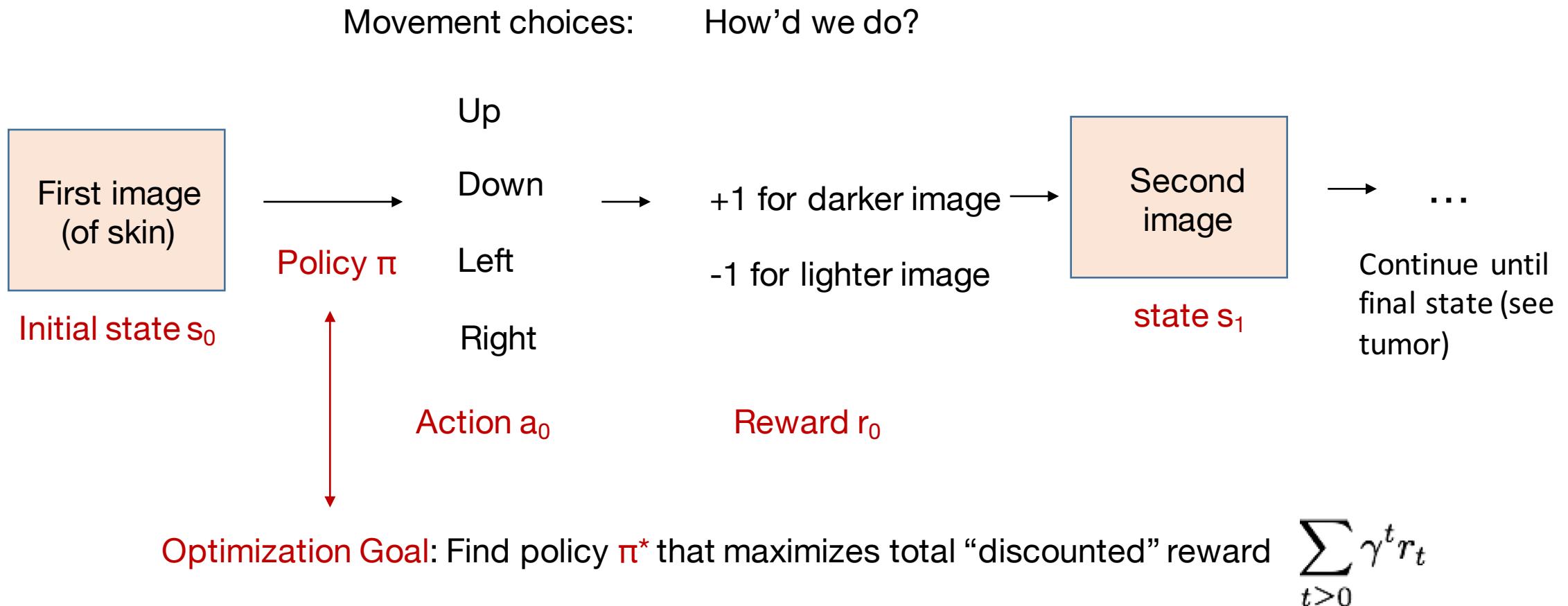
Example situation: Preparing for surgery using a robotically controlled instrument with an endoscope camera. You want endoscope to guide itself to tumor as quickly as possible





TL;DR

-> Use a CNN to map images to actions, optimize CNN with respect to loss function that depends on reward *in a recursive manner*



A simple MDP: Grid World

actions = {

1. right →

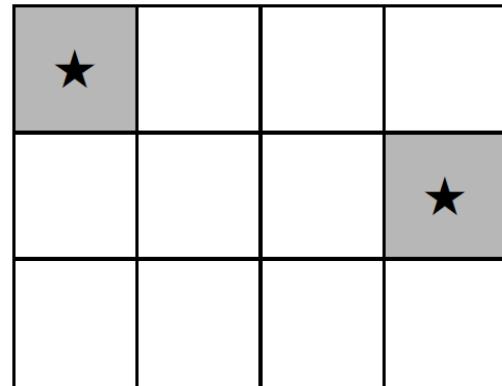
2. left ←

3. up ↑

4. down ↓

}

states

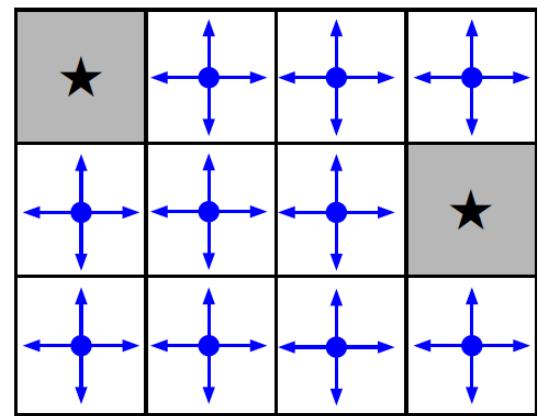


Set a negative “reward”
for each transition
(e.g. $r = -1$)

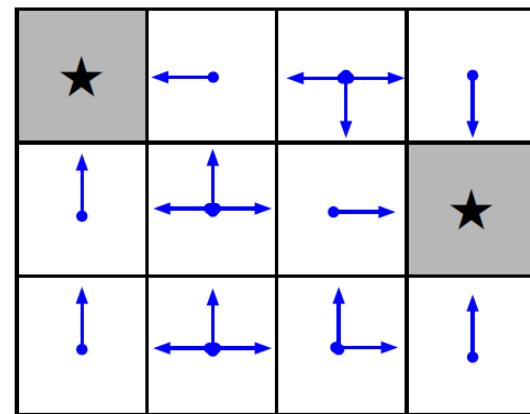
Objective: reach one of terminal states (greyed out) in
least number of actions

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A simple MDP: Grid World

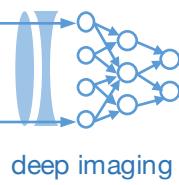


Random Policy



Optimal Policy

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Let's jump into the math....

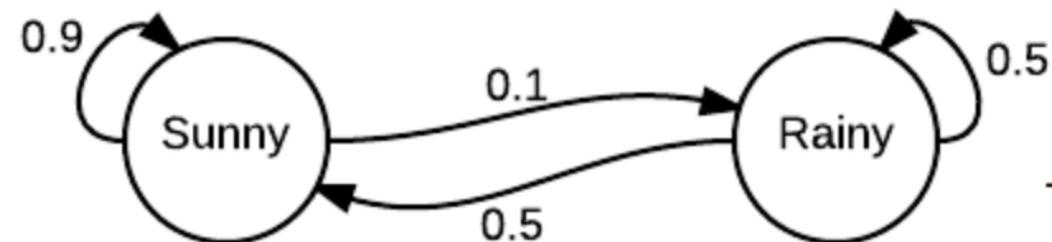
Definition of a Markov process:

$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n)$$

Let's jump into the math....

Definition of a Markov process:

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2 states: Sunny and Rainy

$$\mathbf{x}^{(0)} = [1 \quad 0]$$

The weather on day 2 can be predicted by:

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} P = [1 \quad 0] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = [0.9 \quad 0.1]$$

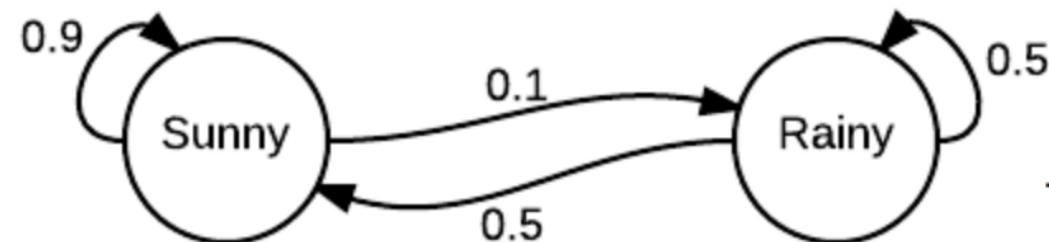
Thus, there is a 90% chance that day 2 will also be sunny.

https://en.wikipedia.org/wiki/Markov_chain

Let's jump into the math....

Definition of a Markov process:

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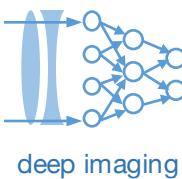
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Thus, there is a 90% chance that day 2 will also be sunny.

Transition matrix – try to learn this from state to state

https://en.wikipedia.org/wiki/Markov_chain



Assume transition between states follows Markov process

$$P(s_{t+1} | s_t, s_{t-1} \dots s_0) = P(s_{t+1} | s_t)$$

Markov chain

$$\mathcal{M} = \{\mathcal{S}, \mathcal{T}\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{T} – transition operator

$$p(s_{t+1} | s_t)$$



Andrey Markov

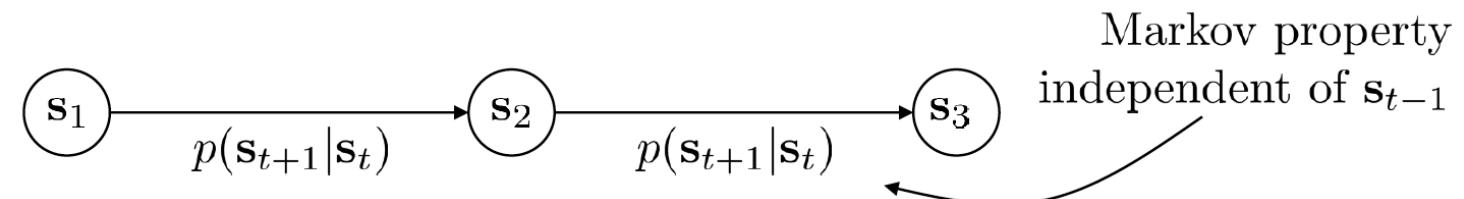
why “operator”?

$$\text{let } \mu_{t,i} = p(s_t = i)$$

$\vec{\mu}_t$ is a vector of probabilities

$$\text{let } \mathcal{T}_{i,j} = p(s_{t+1} = i | s_t = j)$$

$$\text{then } \vec{\mu}_{t+1} = \mathcal{T} \vec{\mu}_t$$



Add in dependence on action: Markov *decision* process

$$P(s_{t+1}|s_t) \Rightarrow P(s_{t+1} | s_t, a_t) = P(s_{t+1} | s_t, a_t, \dots s_0, a_0)$$

Markov decision process

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

\mathcal{S} – state space

states $s \in \mathcal{S}$ (discrete or continuous)

\mathcal{A} – action space

actions $a \in \mathcal{A}$ (discrete or continuous)

\mathcal{T} – transition operator (now a tensor!)

$$\text{let } \mu_{t,j} = p(s_t = j)$$

$$\text{let } \xi_{t,k} = p(a_t = k)$$

$$\text{let } \mathcal{T}_{i,j,k} = p(s_{t+1} = i | s_t = j, a_t = k)$$

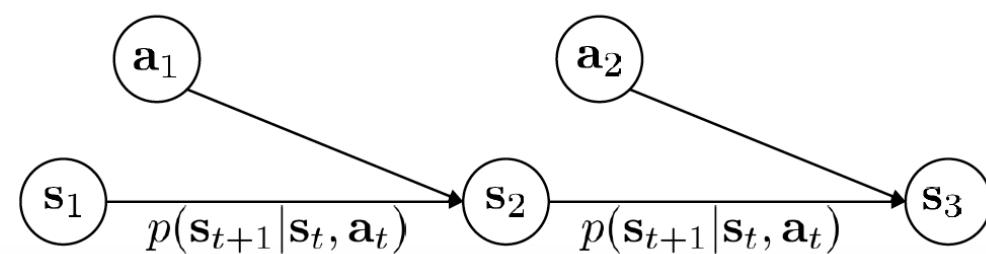
$$\mu_{t,i} = \sum_{j,k} \mathcal{T}_{i,j,k} \mu_{t,j} \xi_{t,k}$$

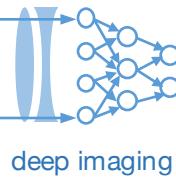


Andrey Markov



Richard Bellman





Add in dependence on action: Markov *decision* process

$P(s_{t+1}|s_t, a_t)$ can include reward $r(s_t, a_t)$

Markov decision process

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

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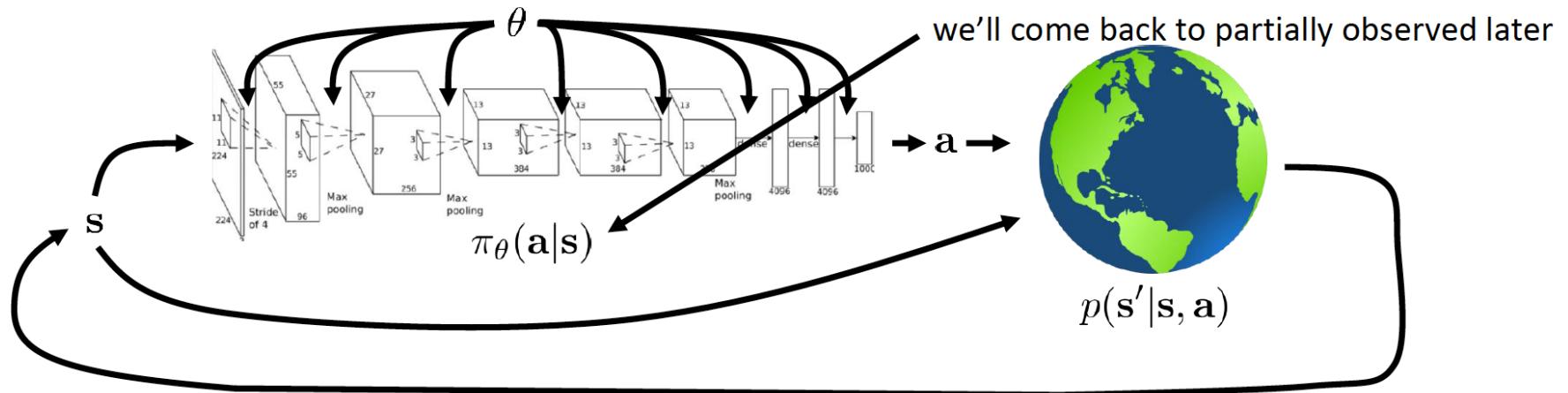
\mathcal{T} – transition operator (now a tensor!)

r – reward function

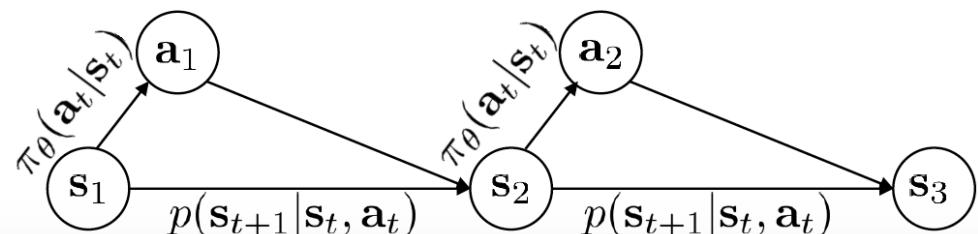
$$r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

$r(s_t, a_t)$ – reward

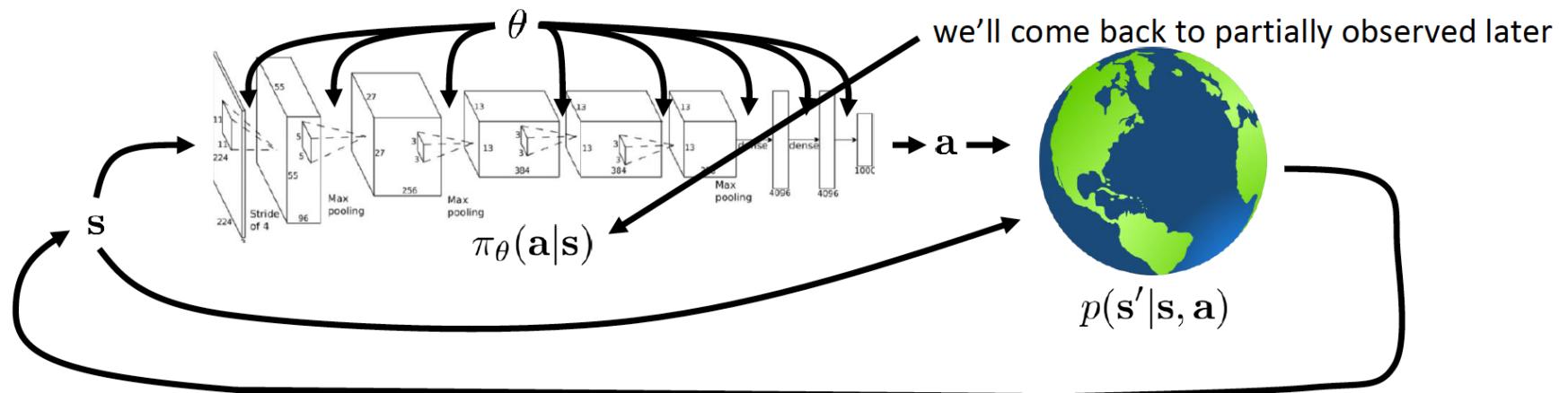
The goal of reinforcement learning



$$p_{\theta}(s_1, a_1, \dots, s_T, a_T) = \underbrace{p(s_1)}_{\pi_{\theta}(\tau)} \prod_{t=1}^T \underbrace{\pi_{\theta}(a_t | s_t) p(s_{t+1} | s_t, a_t)}_{\text{Markov chain on } (s, a)}$$

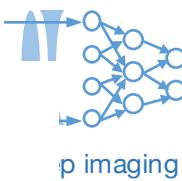


The goal of reinforcement learning



$$p_\theta(s_1, a_1, \dots, s_T, a_T) = \underbrace{p(s_1)}_{\pi_\theta(\tau)} \prod_{t=1}^T \underbrace{\pi_\theta(a_t | s_t) p(s_{t+1} | s_t, a_t)}_{\text{Markov chain on } (s, a)}$$

$$\theta^\star = \arg \max_\theta \sum_{t=1}^T E_{(s_t, a_t) \sim p_\theta(s_t, a_t)} [r(s_t, a_t)]$$



The optimal policy π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)?
Maximize the expected sum of rewards!

Formally: $\pi^* = \arg \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | \pi \right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot | s_t), s_{t+1} \sim p(\cdot | s_t, a_t)$

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The optimal policy π^*

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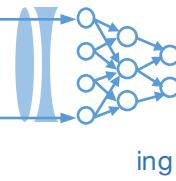
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The **Q-value function** at state s and action a , is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi \right]$$

**Don't have access
to all policies, so
use Q in practice**



Bellman equation

The optimal Q-value function Q^* is the maximum expected cumulative reward achievable from a given (state, action) pair:

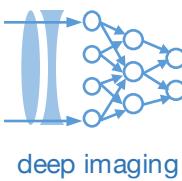
$$Q^*(s, a) = \max_{\pi} \mathbb{E} \left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi \right]$$

Q^* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s', a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

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Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning!**

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Solving for the optimal policy: Q-learning

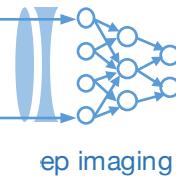
Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$



Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot)} [(y_i - Q(s, a; \theta_i))^2]$

where $y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$

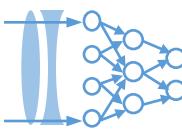
Iteratively try to make the Q-value close to the target value (y_i) it should have, if Q-function corresponds to optimal Q^* (and optimal policy π^*)

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i) \right] \nabla_{\theta_i} Q(s, a; \theta_i)$$

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[Mnih et al. NIPS Workshop 2013; Nature 2015]  sep imaging

Case Study: Playing Atari Games



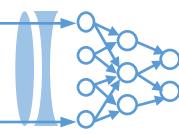
Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

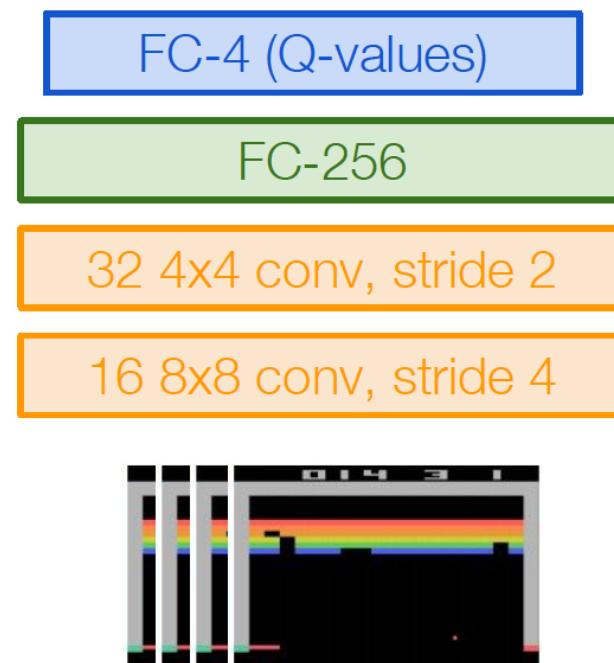
Reward: Score increase/decrease at each time step

From Stanford CS231n Lecture 17



Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1), Q(s_t, a_2), Q(s_t, a_3), Q(s_t, a_4)$

Input: state s_t

Current state s_t : 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

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Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

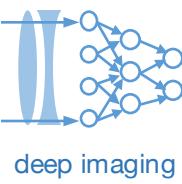
- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand side) => can lead to bad feedback loops

Address these problems using **experience replay**

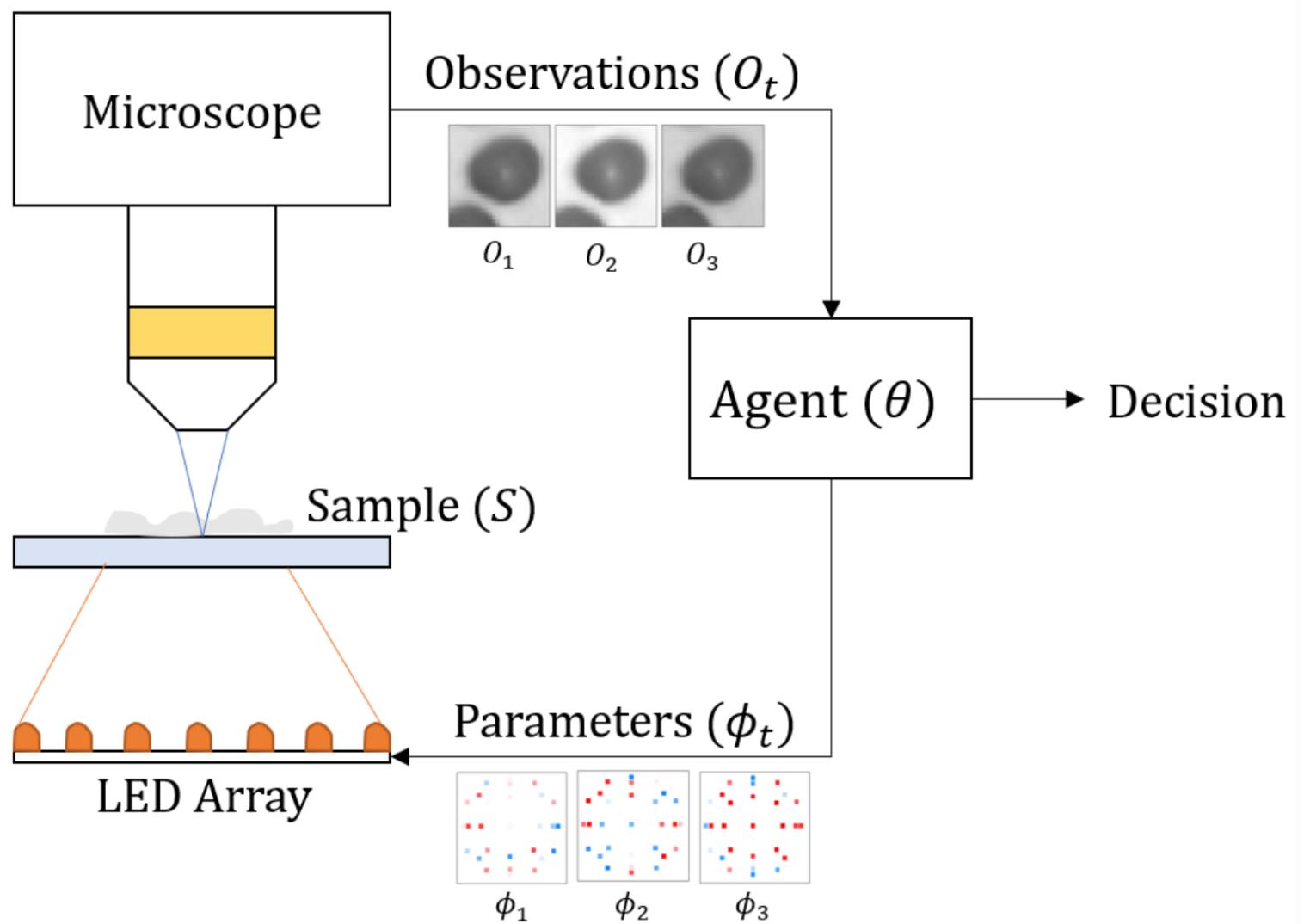
- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

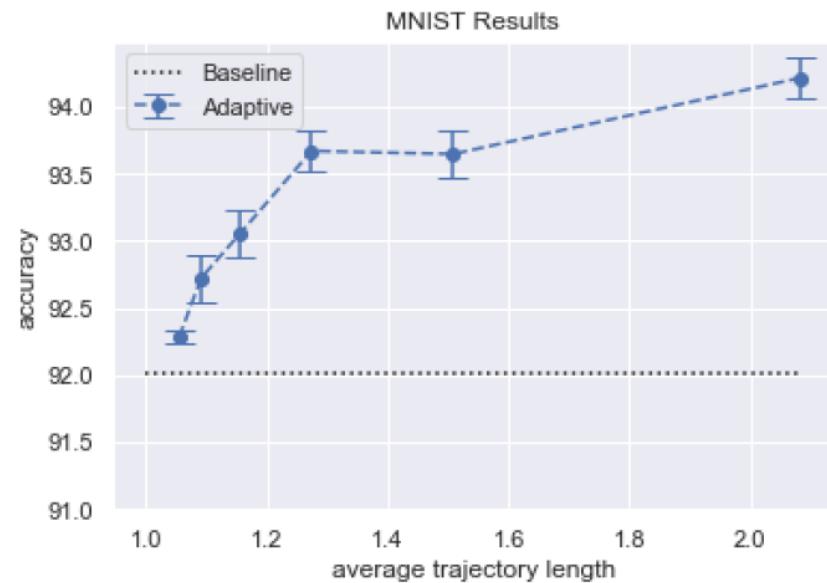
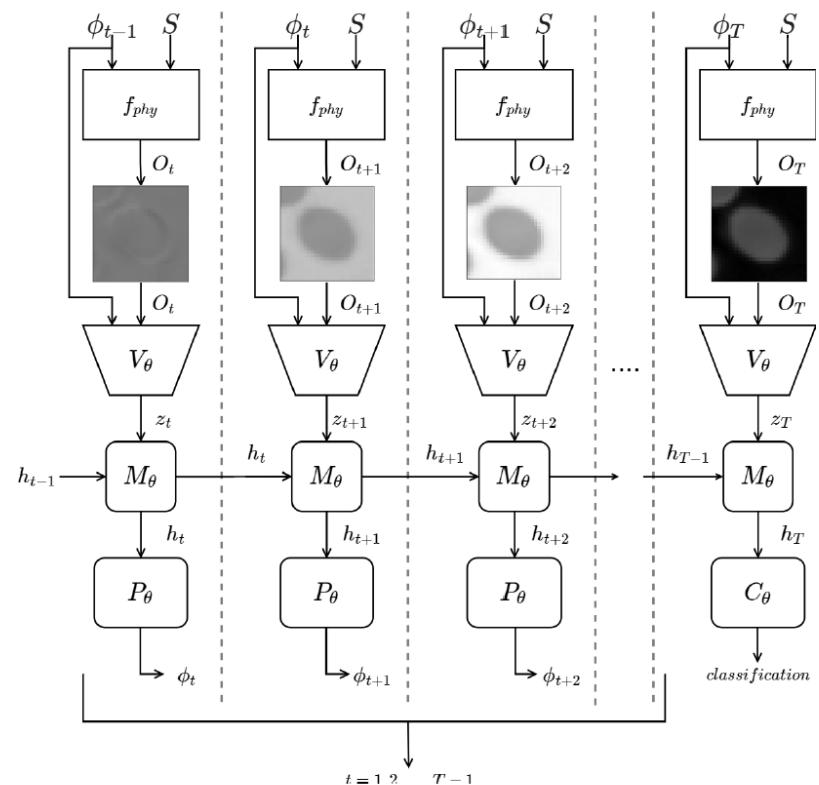
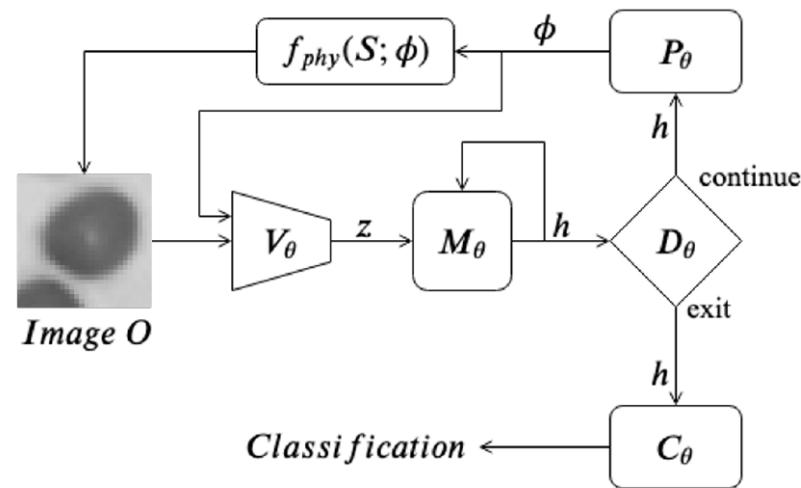
Each transition can also contribute
to multiple weight updates
=> greater data efficiency

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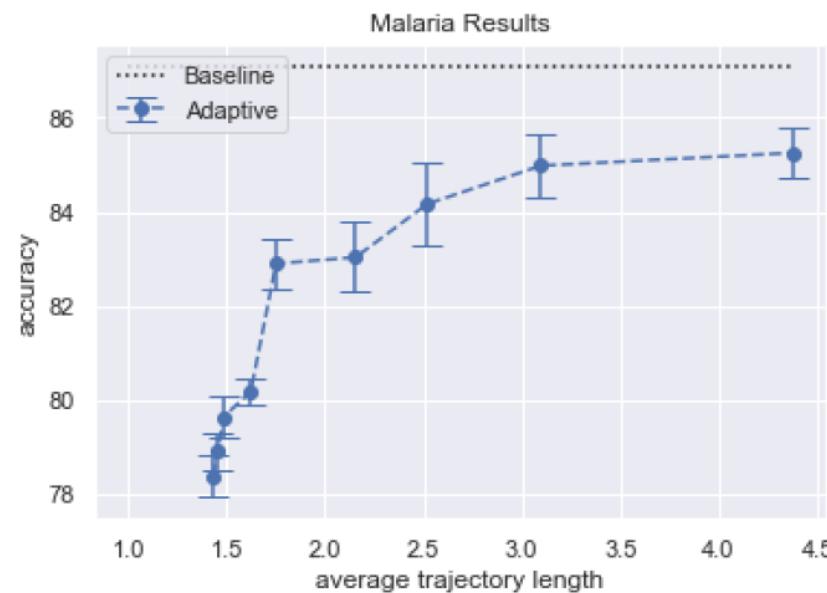


How can this be applied to optimized imaging?

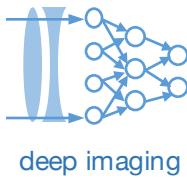


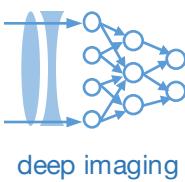


(a) MNIST results



(b) Malaria results





How can this be applied to optimized imaging?

