Lecture 18: Physical layers with coherent fields

Machine Learning and Imaging

BME 590L
Roarke Horstmeyer

deep imaging

Announcements

1. Project proposals due this Thursday before class.
    - Thanks for coming to meet with myself, Kevin and Amey!
    - I'll email you if revisions are needed, in which case these will be due Tues. Nov 12.

2. Will return Homework #2 today after class

3. Homework #4 was assigned on Friday November 1st, and will be due Friday November 15th at midnight (you can turn in the written component either Thursday in class, or place in a folder outside of my office by Friday at midnight)

4. The final homework #5 will be assigned Thursday Nov. 14th and will be due (likely) the weekend after Thanksgiving (Nov. 30)

Today: Review of wave optics, then work towards

Let's take a step back: how does light propagate?

Maxwell's equations
without any charge

$$\nabla \times \vec{\mathcal{E}} = -\mu \frac{\partial \vec{\mathcal{H}}}{\partial t}$$

$$\nabla \times \vec{\mathcal{H}} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t}$$

$$\nabla \cdot \epsilon \vec{\mathcal{E}} = 0$$

$$\nabla \cdot \mu \vec{\mathcal{H}} = 0.$$

1. Take the curl of both sides of first equation
2. Substitute 2nd and 3rd equation
3. Arrive at the wave equation:

$$\nabla^2 \vec{\mathcal{E}} - \frac{n^2}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0 \qquad n = \left(\frac{\epsilon}{\epsilon_0}\right)^{1/2} \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

Plane-to-plane light propagation via the "paraxial approximation"

We are usually concerned about propagation between two planes (almost always in an optical system):

Paraxial approximation:

$$\nabla_\perp^2 U + 2ik\frac{dU}{dz} = 0$$

Substitute in $U(P) = E(x, y, z)e^{ikz}$ and crank the wheel,

$$\nabla_\perp^2 E + 2ik\frac{dE}{dz} + 2k^2 E = 0$$

Paraxial Helmholtz Equation. This has an exact integral solution:

$$\boxed{E(x, y, z) = \frac{e^{ikz}}{i\lambda z}\iint_{-\infty}^{+\infty} E(x', y', 0)e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]}dx'dy'}$$

Fresnel diffraction integral

This is how light propagates from one plane to the next. It's a convolution!

From the Fresnel approximation to the Fraunhofer approximation

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Fresnel Approximation:

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint_{-\infty}^{+\infty} E(x',y',0) e^{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]} dx' dy'$$

Lets assume that the second plane is "pretty far away" from the first plane. Then,

$$\boxed{z > \frac{2D^2}{\lambda}}$$

1. Expand the squaring

$$E(x,y,z) = \frac{e^{ikz}}{i\lambda z} \iint E(x',y',0) e^{\frac{ik}{2z}(x^2+y^2)} e^{\frac{ik}{2z}(x'^2+y'^2)} e^{\frac{ik}{2z}(xx'+yy')} dx' dy'$$

2. Front term comes out, assume second term goes away, then,

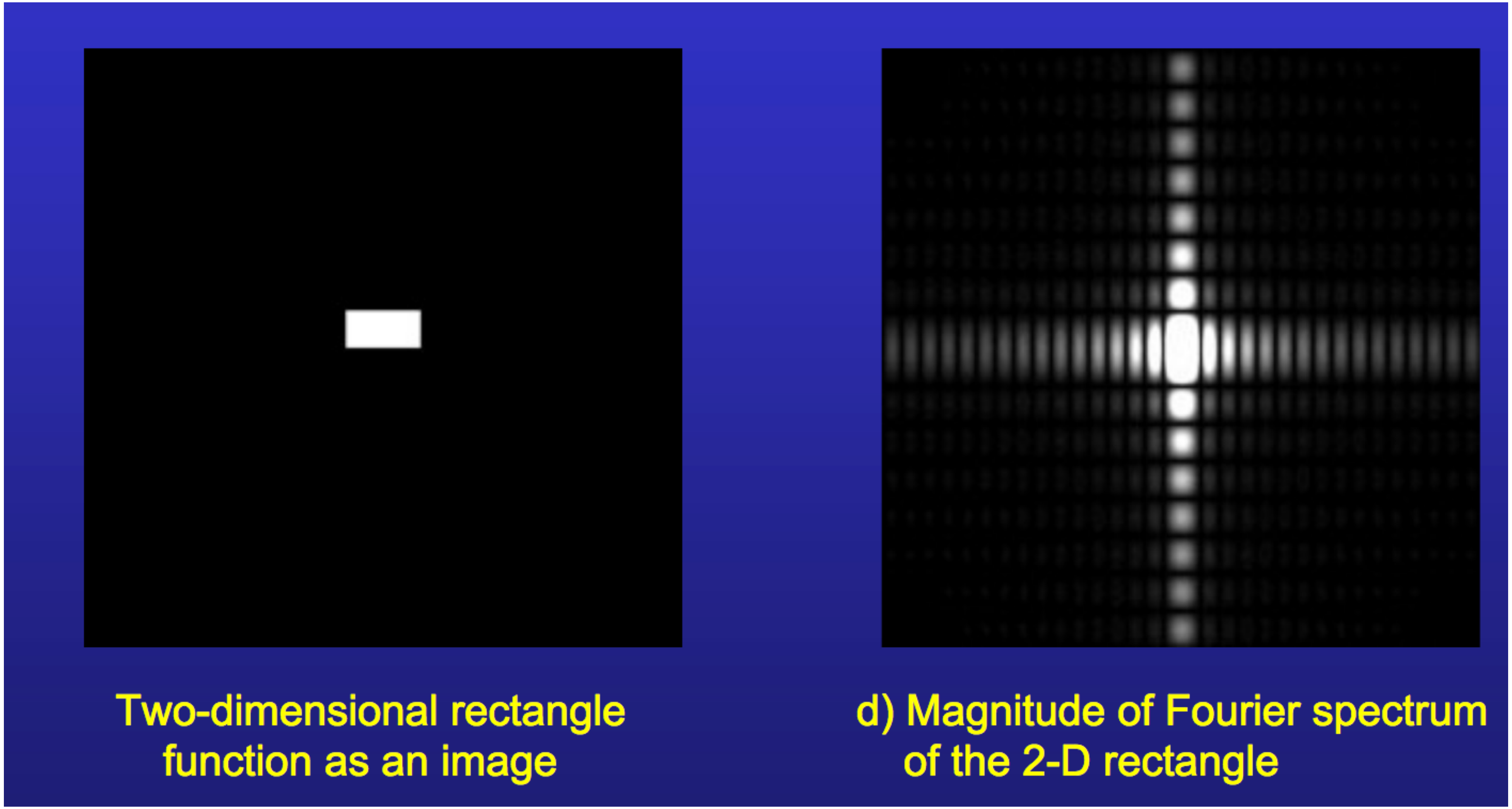
$$\boxed{E(x,y,z) = C \iint E(x',y',0) e^{\frac{ik}{2z}(xx'+yy')} dx' dy'}$$

$$C = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)}$$

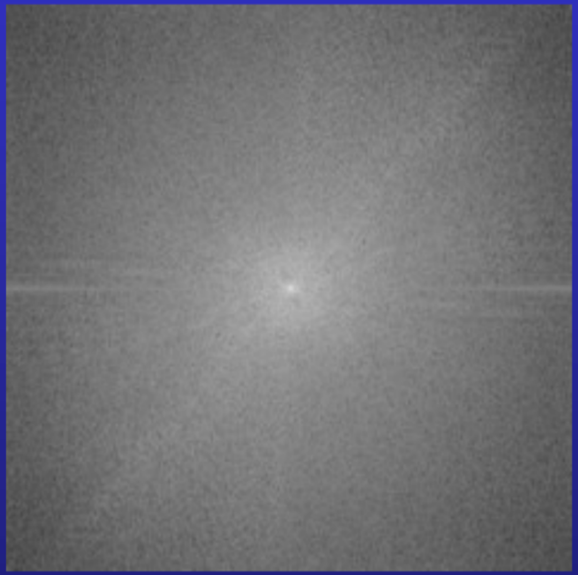
Fraunhofer diffraction is a Fourier transform!!!!!!!

This is the aperture

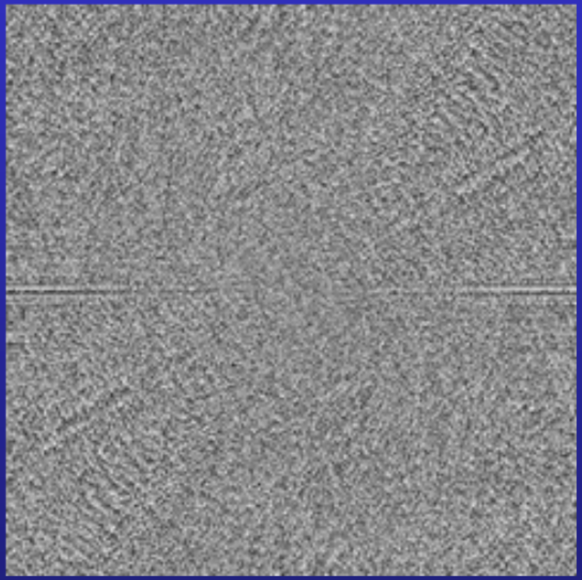
This is what you see far away



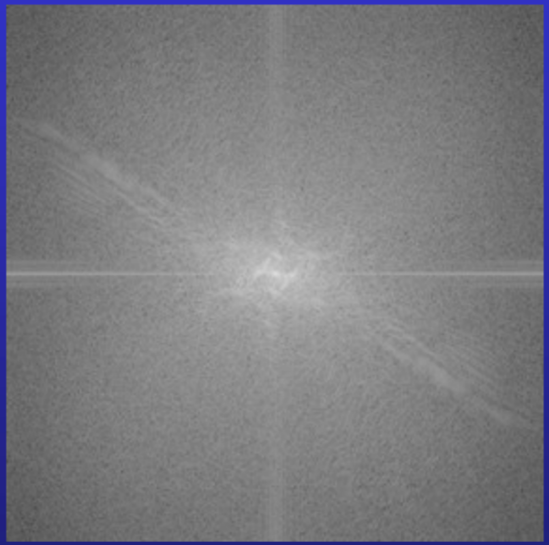

Cheetah
Zebra



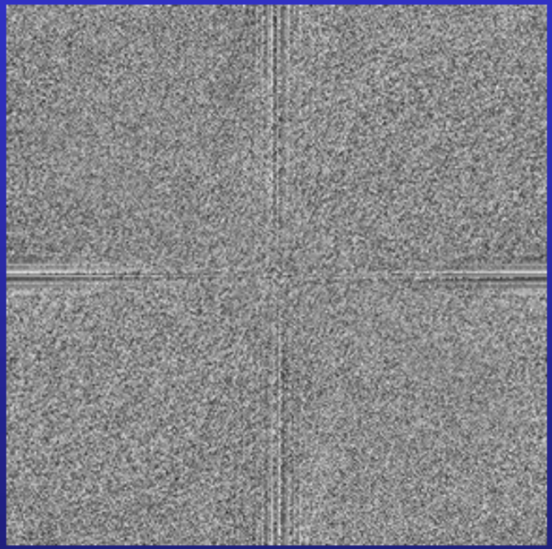
magnitude of cheetah



phase of cheetah



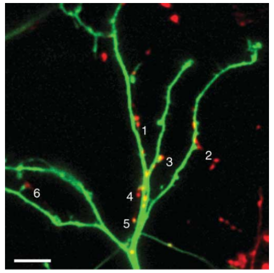
magnitude of zebra

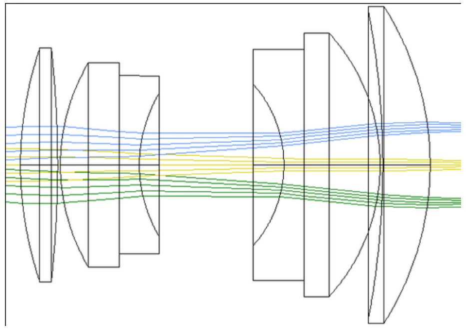


phase of zebra

Summary of two models for image formation

- **Interpretation #1: Radiation (*Incoherent*)**
- Model: Rays





- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

Mathematical model of for incoherent image formation

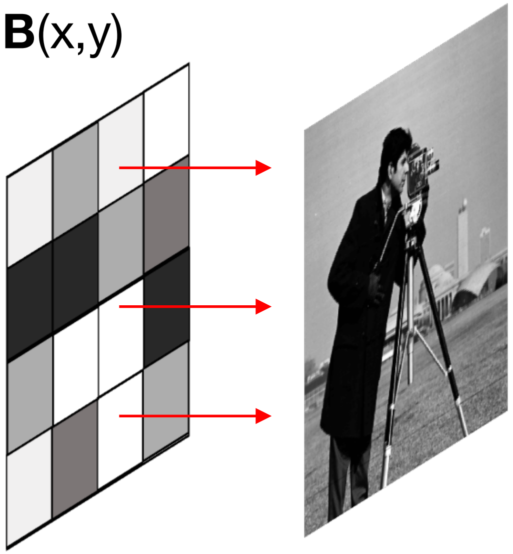
- All quantities are real, and non-negative

Object absorption:

S_0(x,y)

Illumination brightness:

B(x,y)



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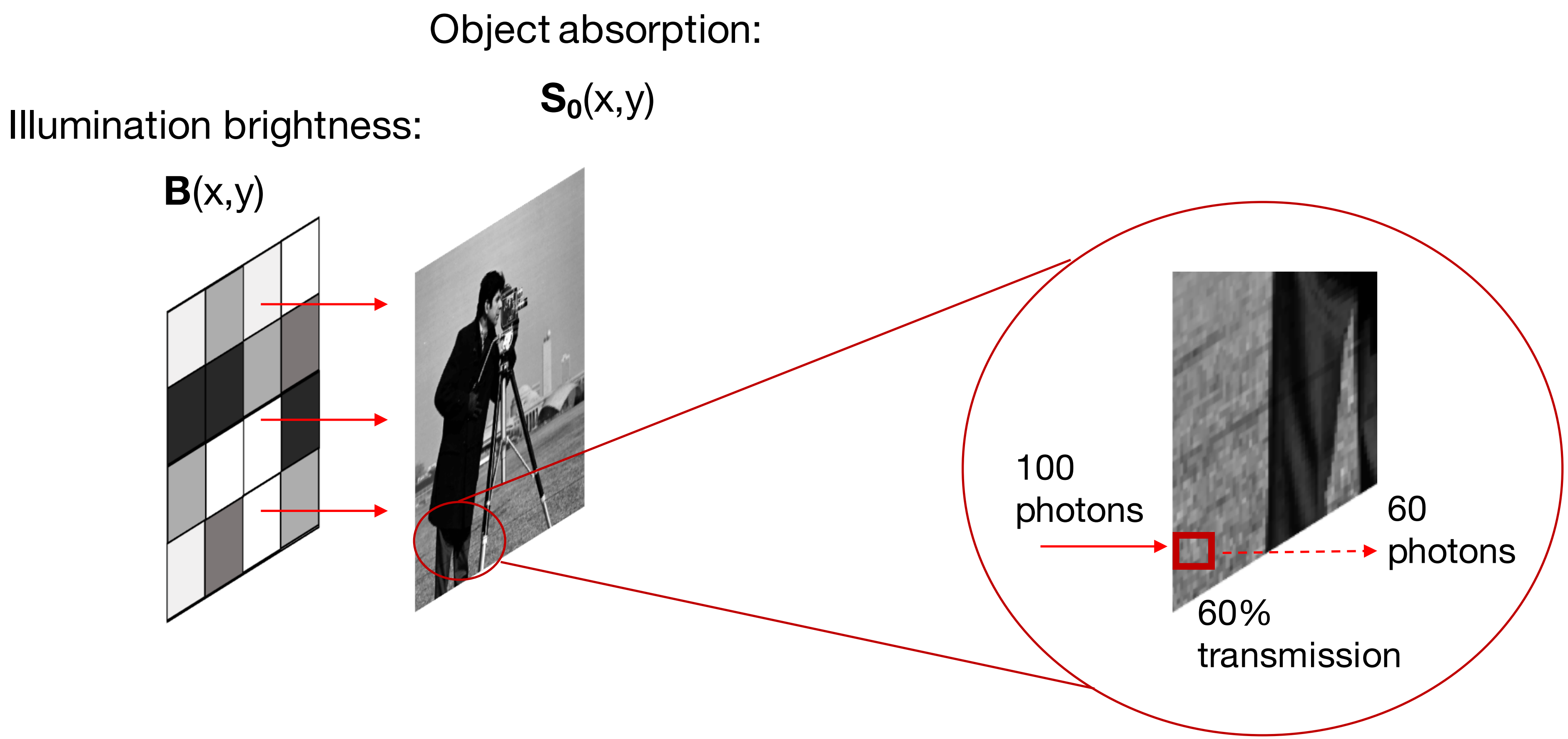

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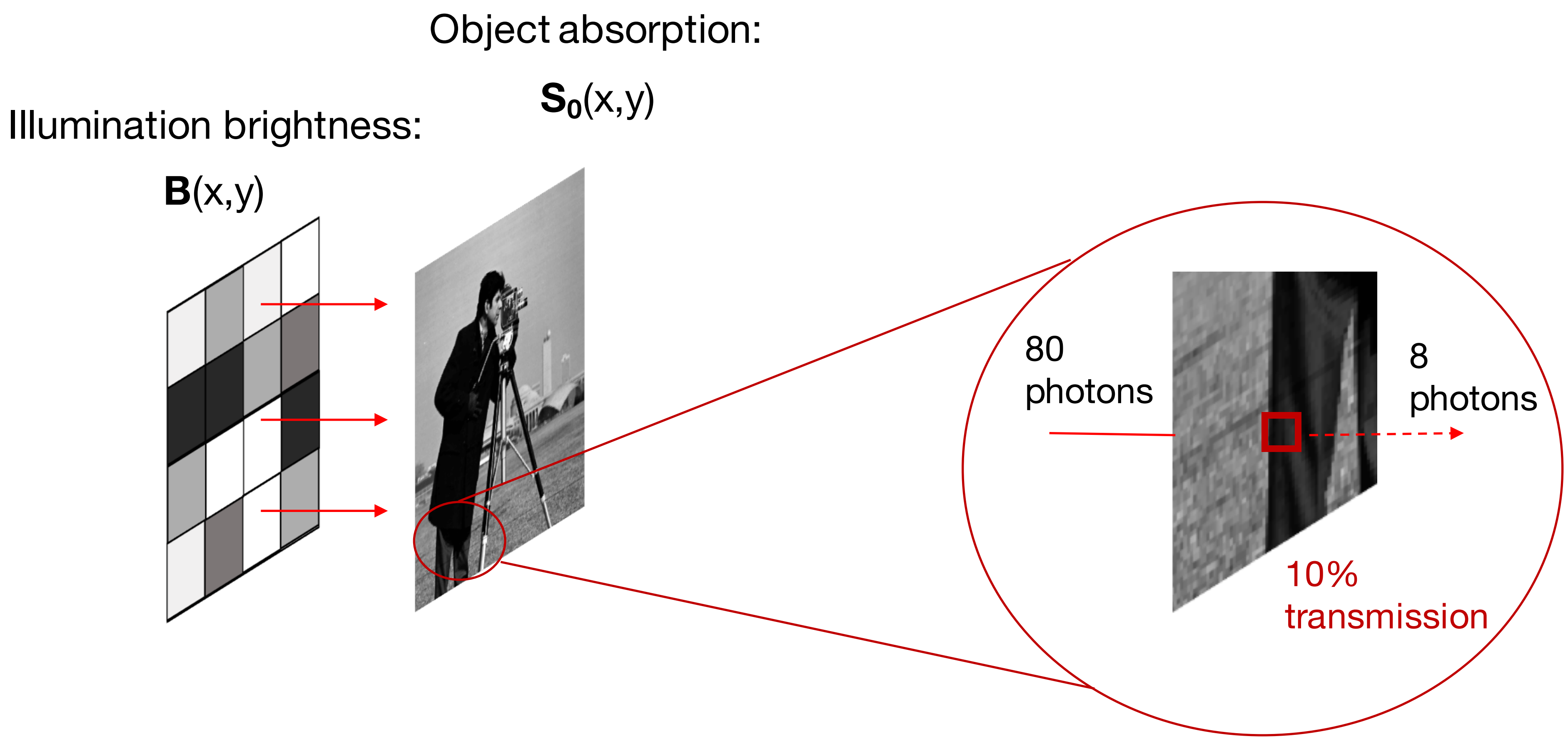

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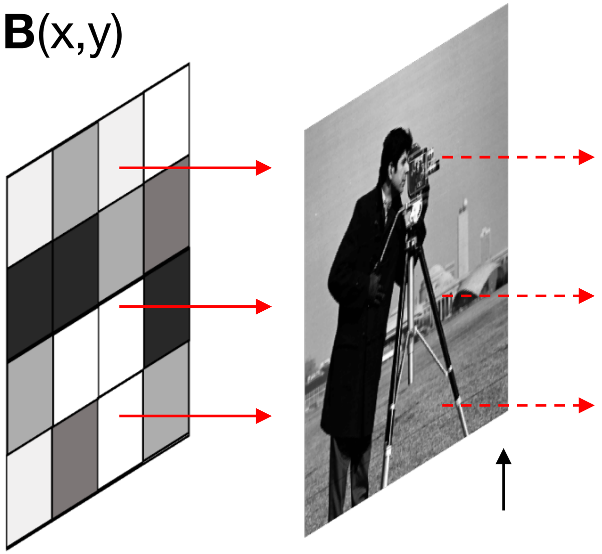

$\mathbf{B\ S_0}$

multiplication

Mathematical model of for incoherent image formation

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Illumination brightness:

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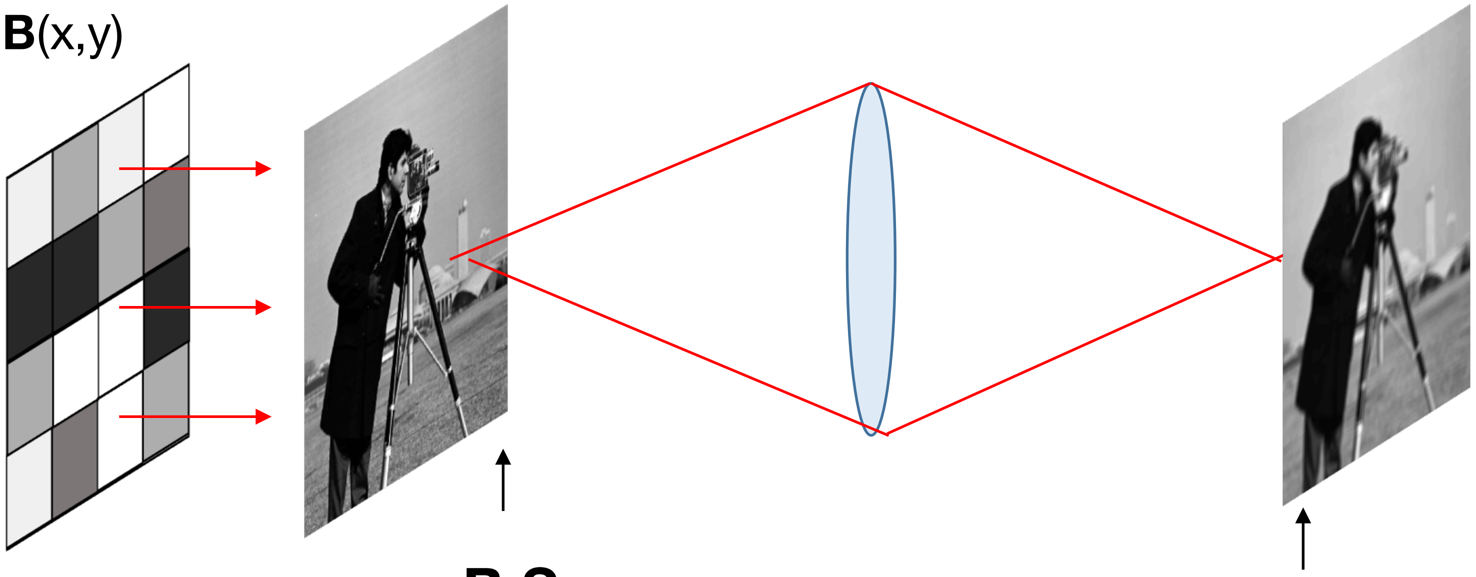


$\mathbf{B\ S_0}$

multiplication

$(\mathbf{B\ S_0}) * \mathbf{h}$

convolution

Mathematical model of for incoherent image formation

- All quantities are real, and non-negative

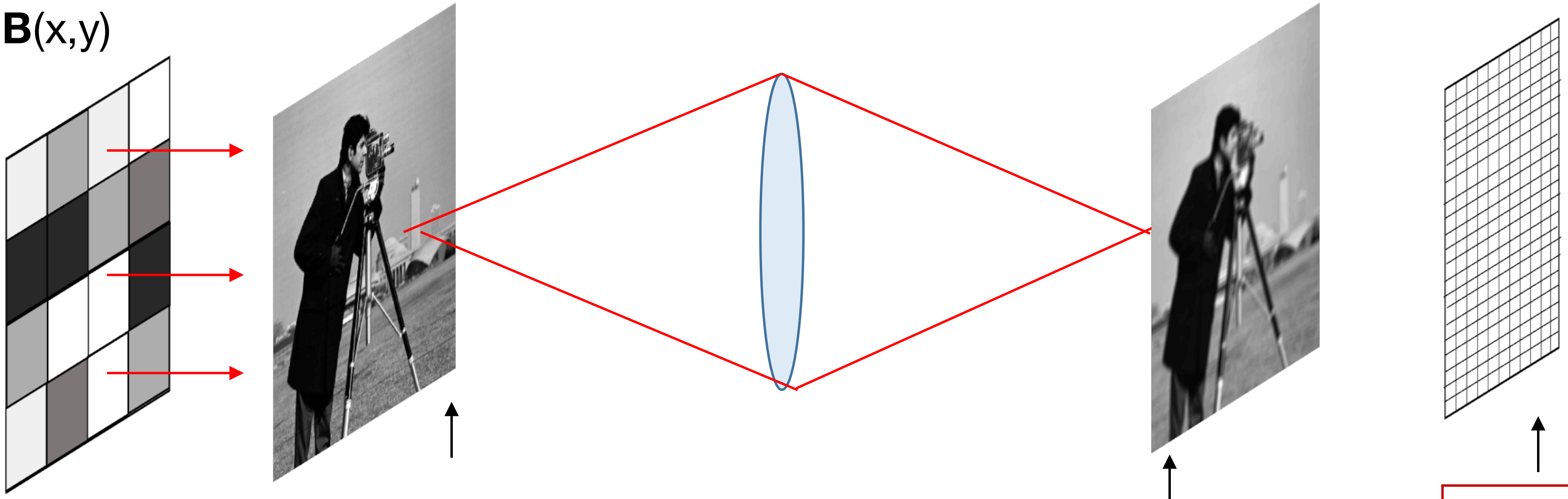
Object absorption:

$\mathbf{S_0}(x,y)$

Illumination brightness:

$\mathbf{B}(x,y)$

Photons (intensity) hits\
detector



$\mathbf{B\ S_0}$

multiplication

$(\mathbf{B\ S_0}) * \mathbf{h}$

convolution

$$\boxed{\mathbf{I_s} = \mathbf{H\ B\ S_0}}$$

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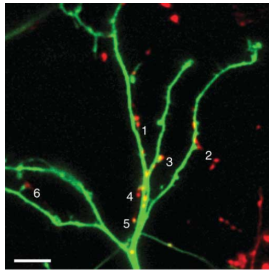

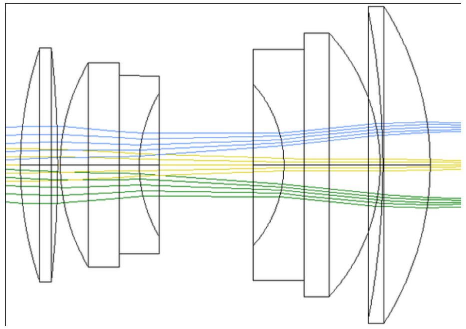

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$$I_{tot} = I_1 + I_2$$

$$\mathbf{I_s = H\,B\,S_0}$$

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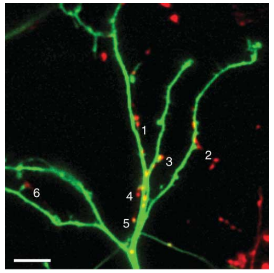

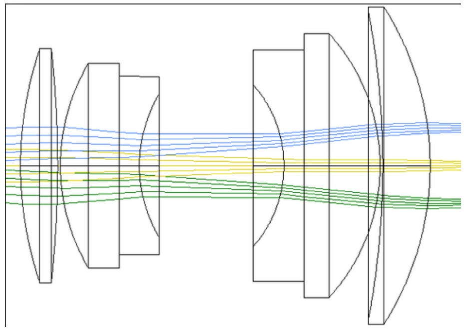

- Real, non-negative
- Models absorption and brightness

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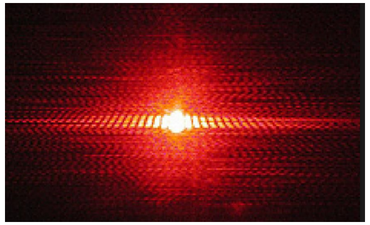
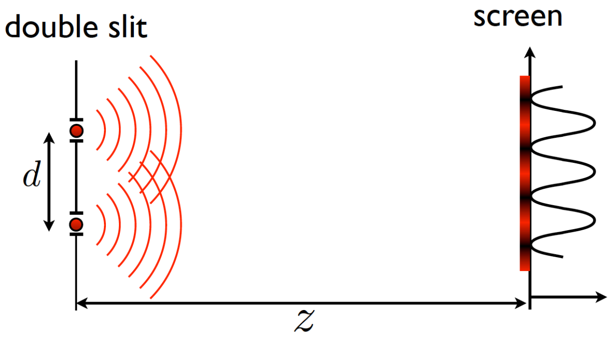
- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex field
- Models Interference

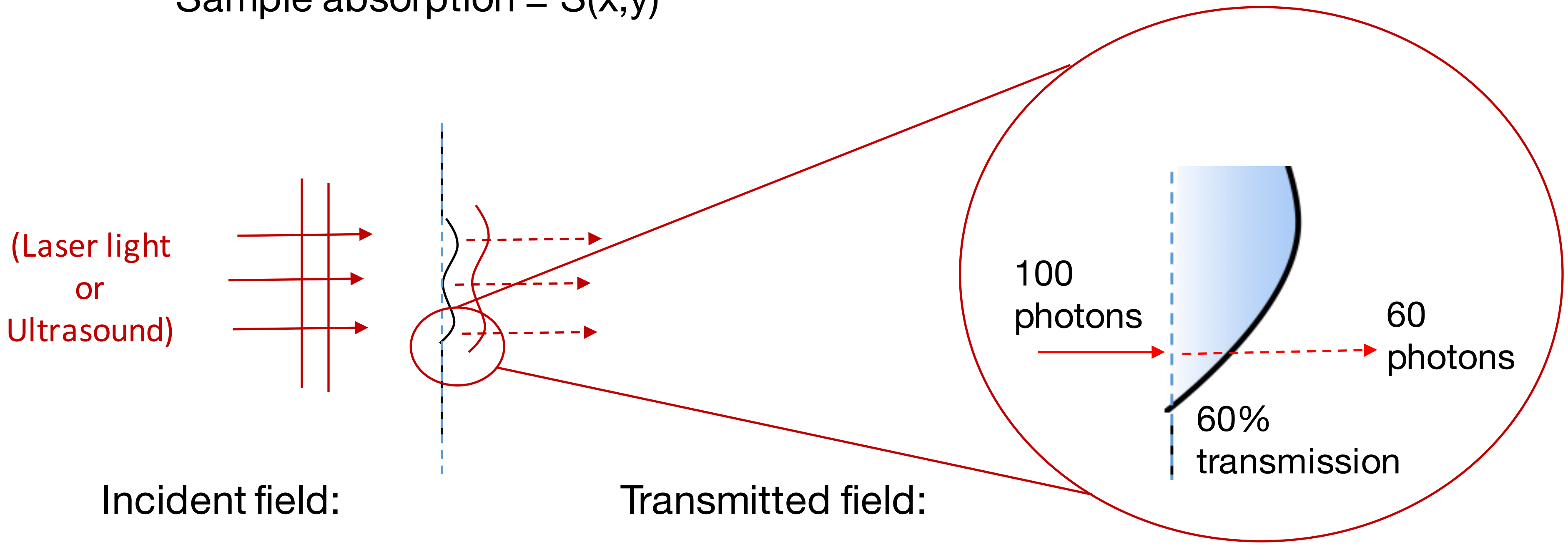
$$E_{tot} = E_1 + E_2$$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Point #1: Amplitudes behave just like before



$A_t(x,y) = A_i(x,y)\ S(x,y)$

Incident field:

$C(x,y) = A_i(x,y)$

Transmitted field:

$U(x,y) = A_t(x,y) = ?$

Mathematical model of for coherent image formation

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Sample absorption = $S(x,y)$

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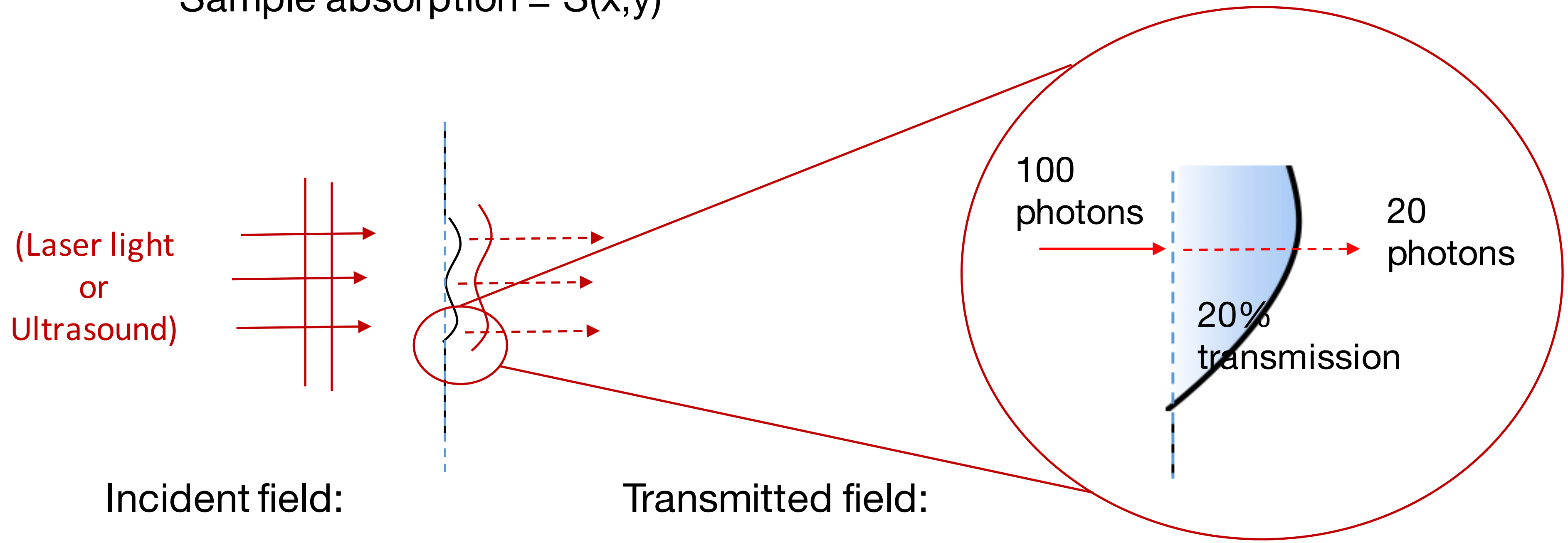

$$\boxed{A_t(x,y) = A_i(x,y)\, S(x,y)}$$

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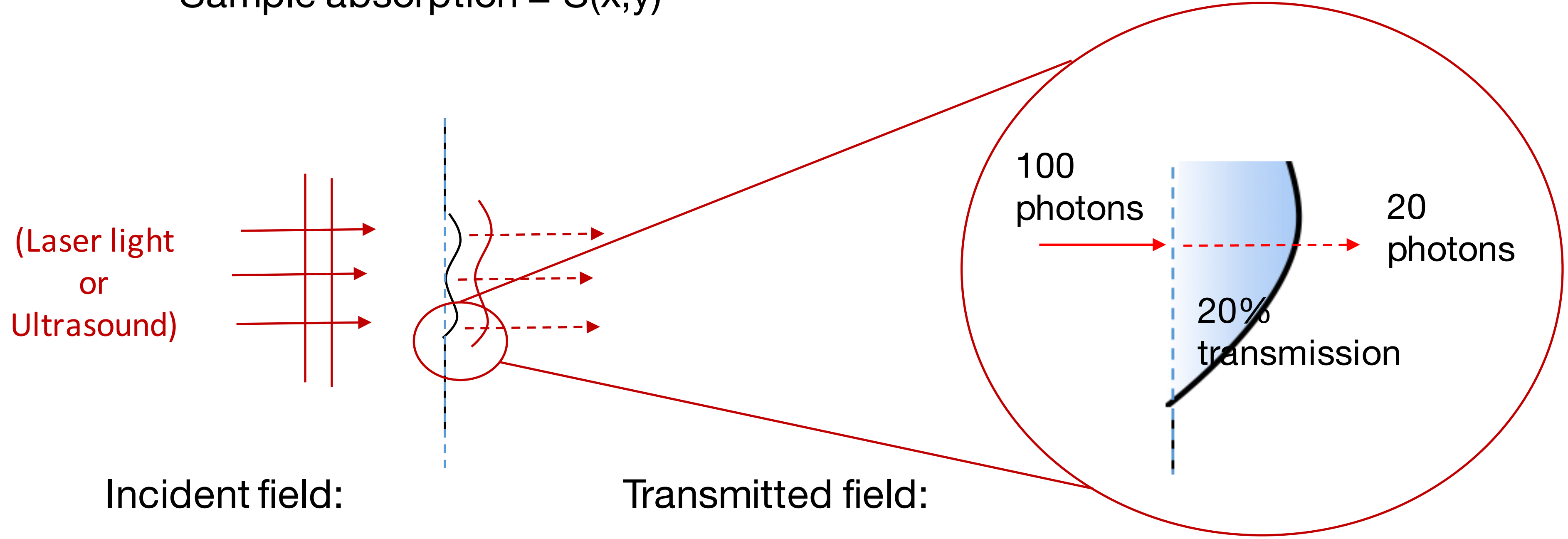

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$U(x,y) = A_t(x,y) = A_i(x,y)\,S(x,y)$

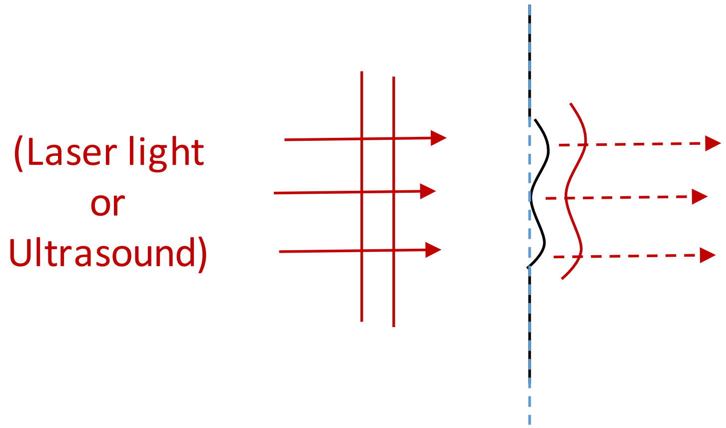
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = $\exp[ik\varphi(x,y)]$

New: complex phase delay
- Needed to represent wave
- Represents wave delay across space



Incident field:

$C(x,y) = A_i(x,y)\exp[ik\varphi_i(x,y)]$

Transmitted field:

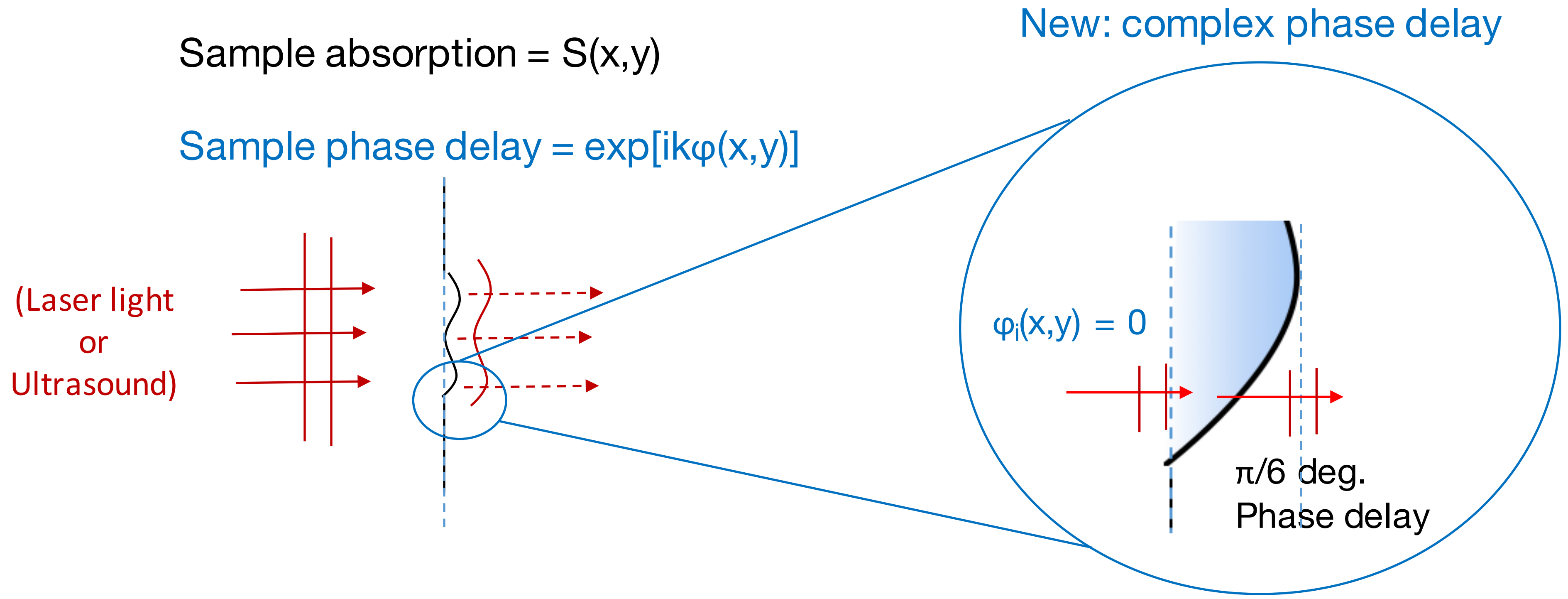
$U(x,y) = A_i(x,y)\,S(x,y)\exp[ik\varphi_t(x,y)]$

Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = exp[ikφ(x,y)]



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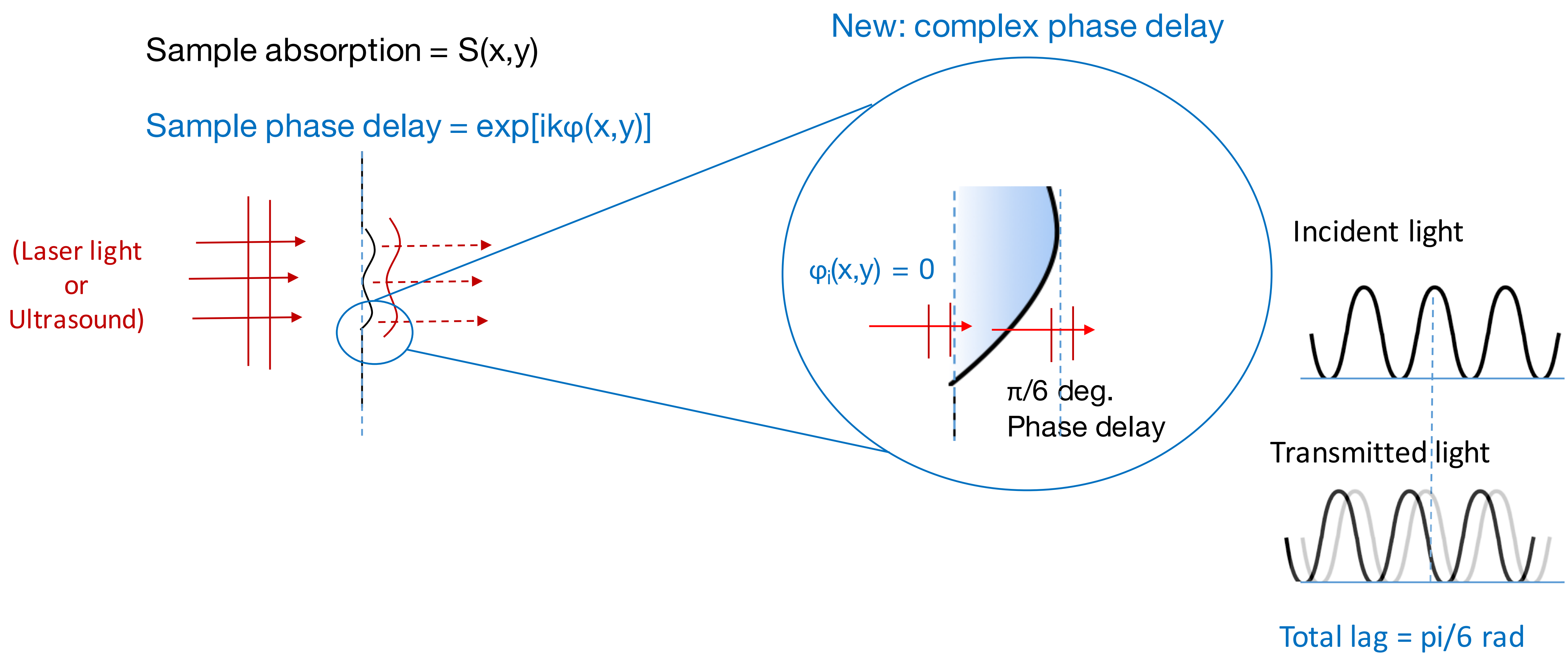

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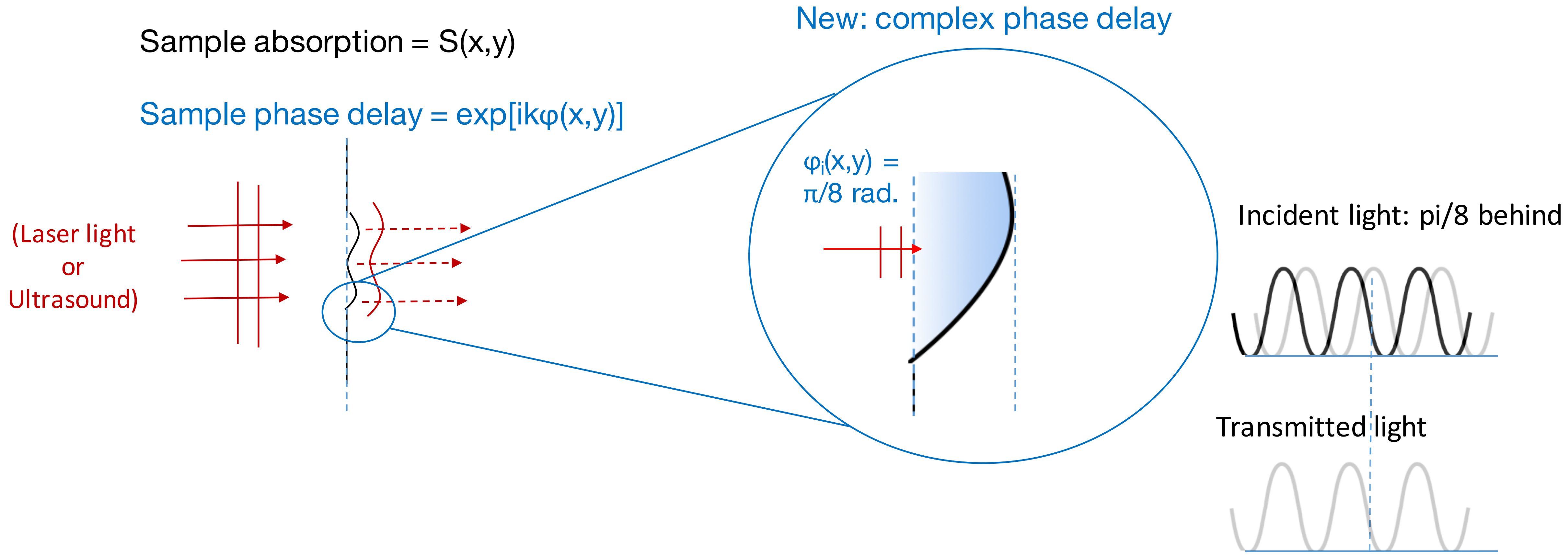


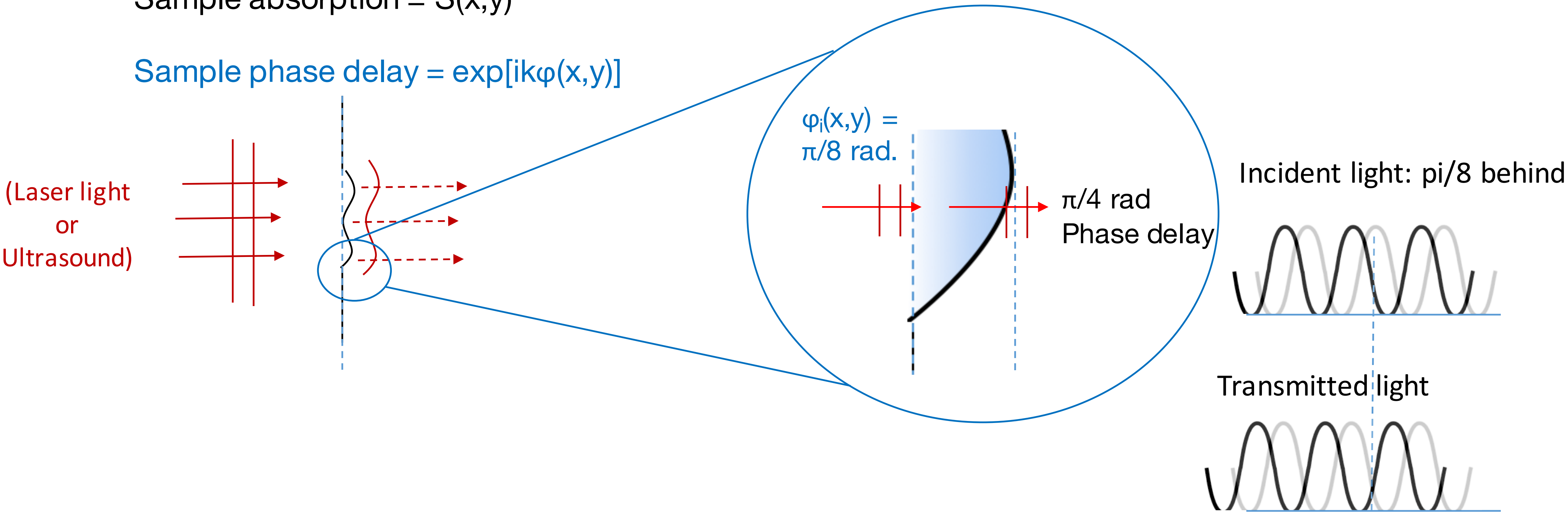
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = S(x,y)

Sample phase delay = exp[ikφ(x,y)]

New: complex phase delay



Total lag = π/8+ π/4= 3π/8 rad

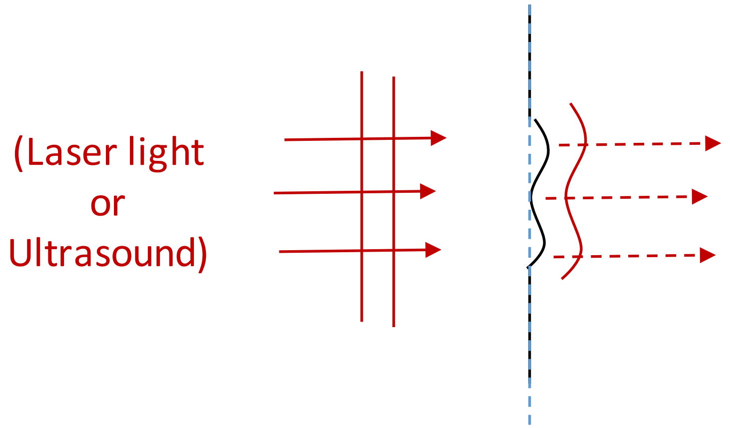
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = $S(x,y)$

Sample phase delay = $\exp[ik\varphi(x,y)]$

Output phase is sum of phase delays, product of phasors



$$\varphi_t(x,y) = \varphi(x,y) + \varphi_i(x,y)$$

$$\exp[ik\varphi_t(x,y)] = \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$$

Incident field:

$C(x,y) = A_i(x,y)\ \exp[ik\varphi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\ S(x,y)\ \exp[ik\varphi_i(x,y)]\ \exp[ik\varphi(x,y)]$

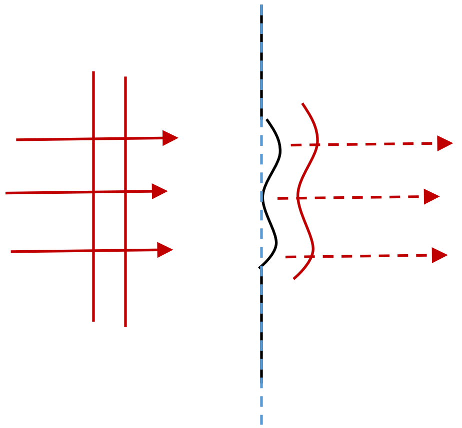
Mathematical model of for coherent image formation

- Pretty much the same thing, but now we have an amplitude and a complex phase

Sample absorption = $S(x,y)$

Sample phase delay = $\exp[ik\varphi(x,y)]$

(Laser light
or
Ultrasound)



Incident field:

$C(x,y) = A_i(x,y)\exp[ik\varphi_i(x,y)]$

Transmitted field:

$U(x,y) = A_i(x,y)\, S(x,y)\exp[ik\varphi_i(x,y)]\exp[ik\varphi(x,y)]$

Conclusion:

Transmitted field = incident field x complex sample :

$U(x,y) = C(x,y)\, S(x,y)\exp[ik\varphi(x,y)]$

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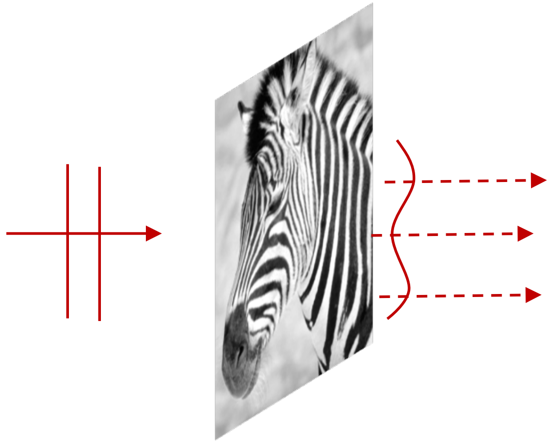
Model of image formation for wave optics (coherent light):

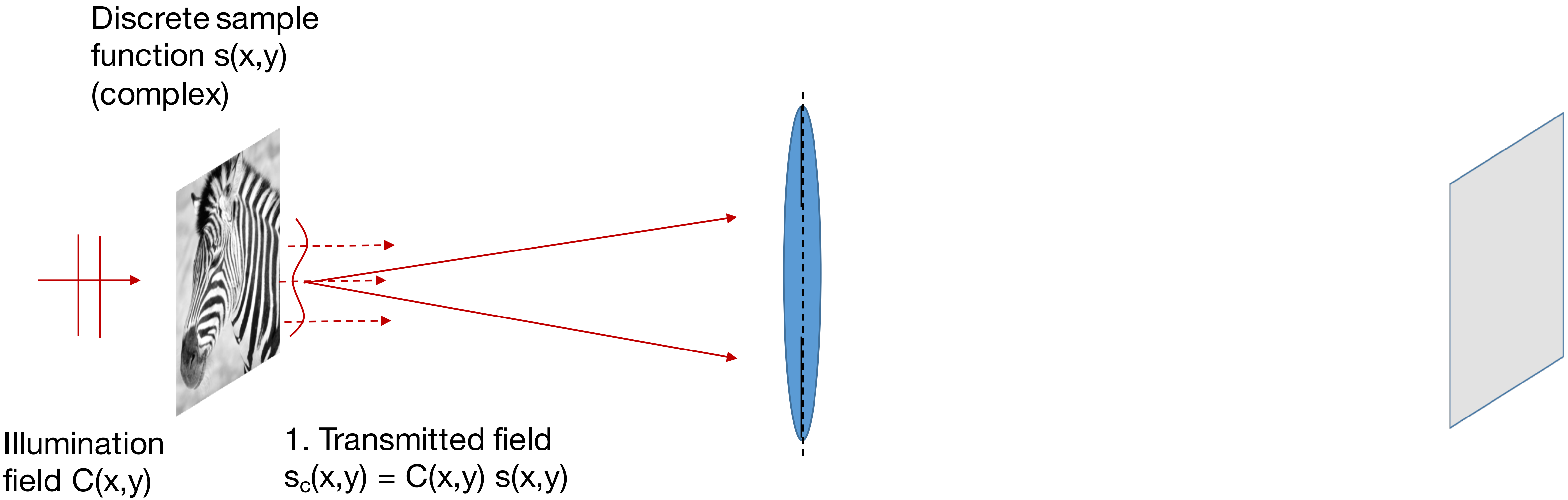
Discrete sample function s(x,y) (complex)



Illumination field C(x,y)

Transmitted field $s_c(x,y) = C(x,y)\ s(x,y)$

Model of image formation for wave optics (coherent light):



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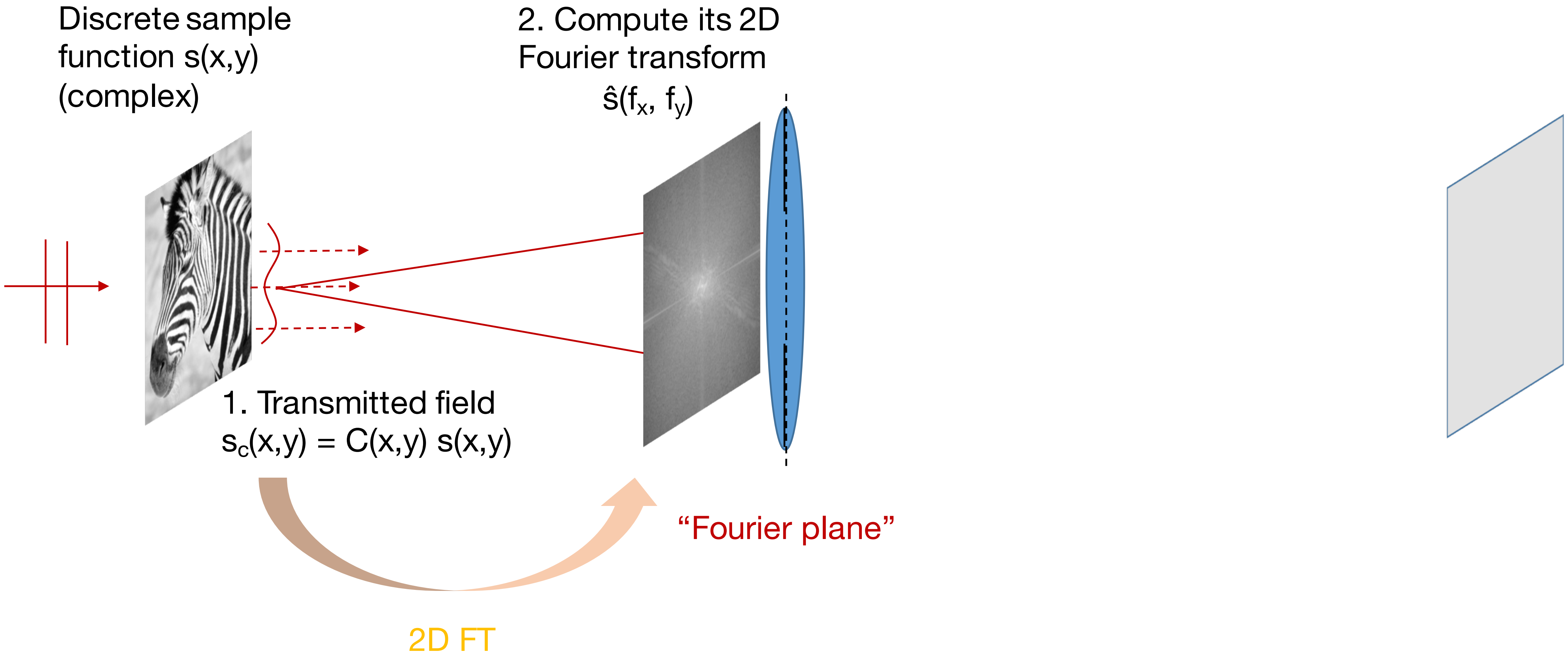

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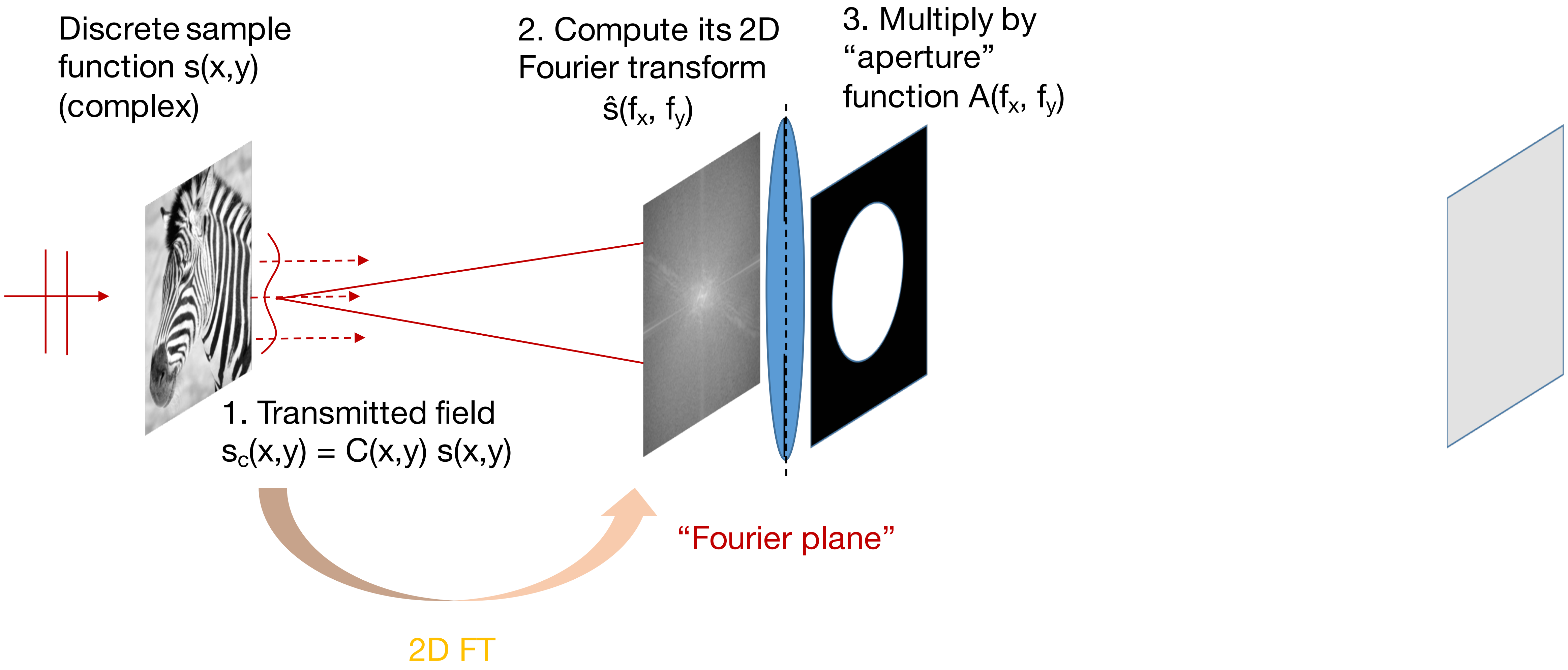

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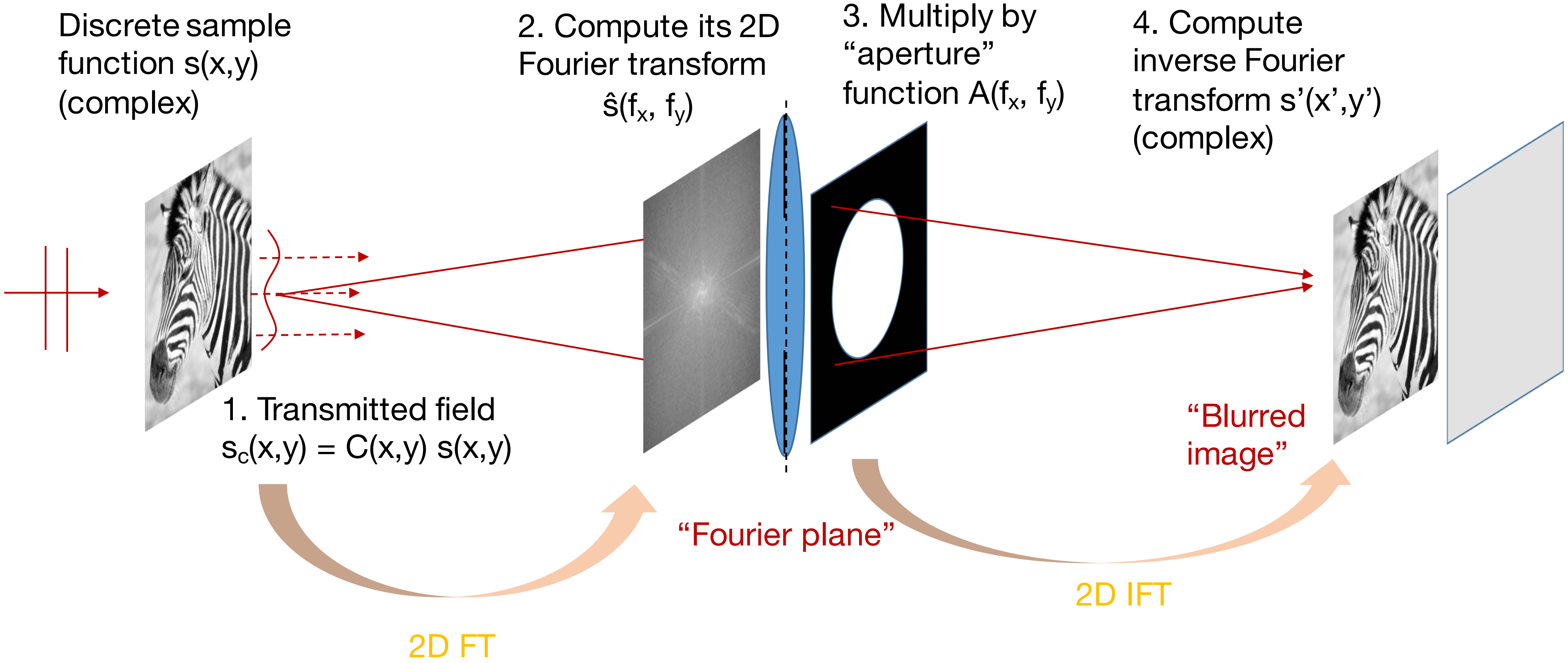

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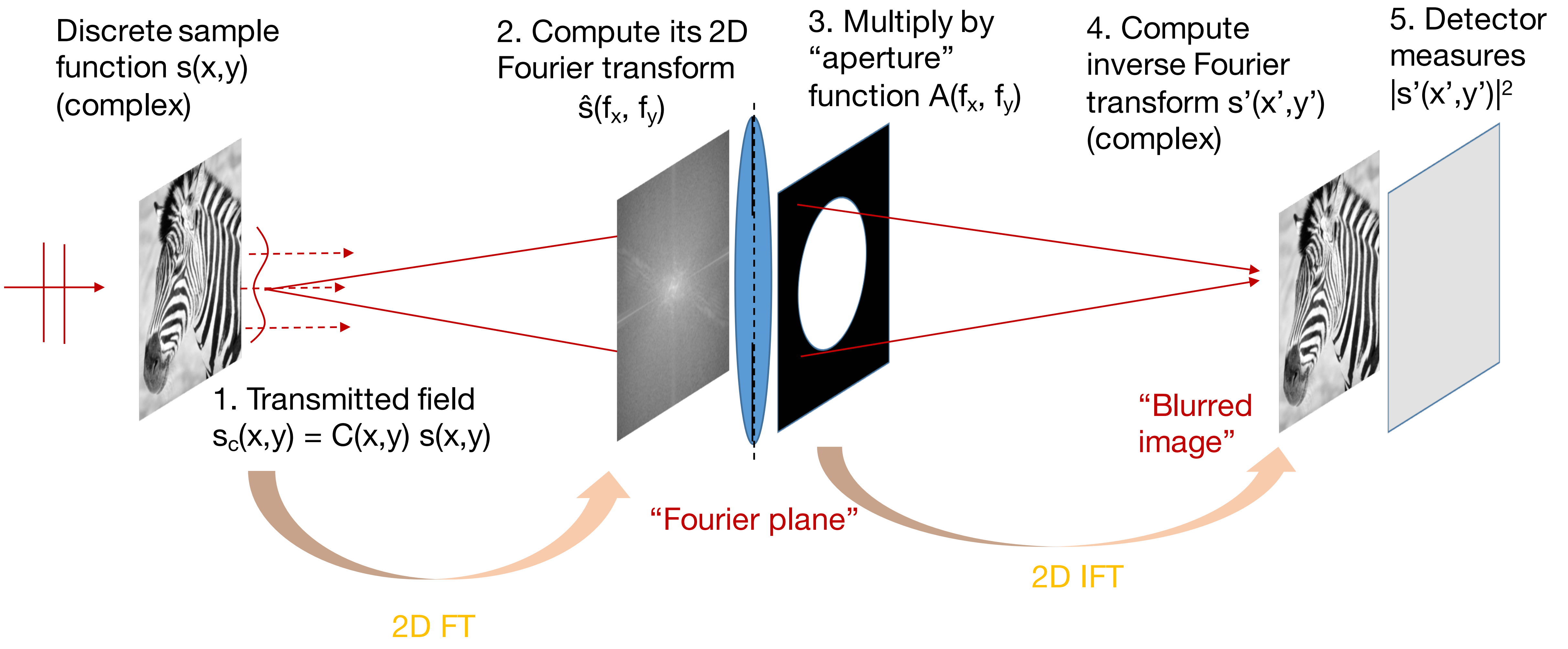

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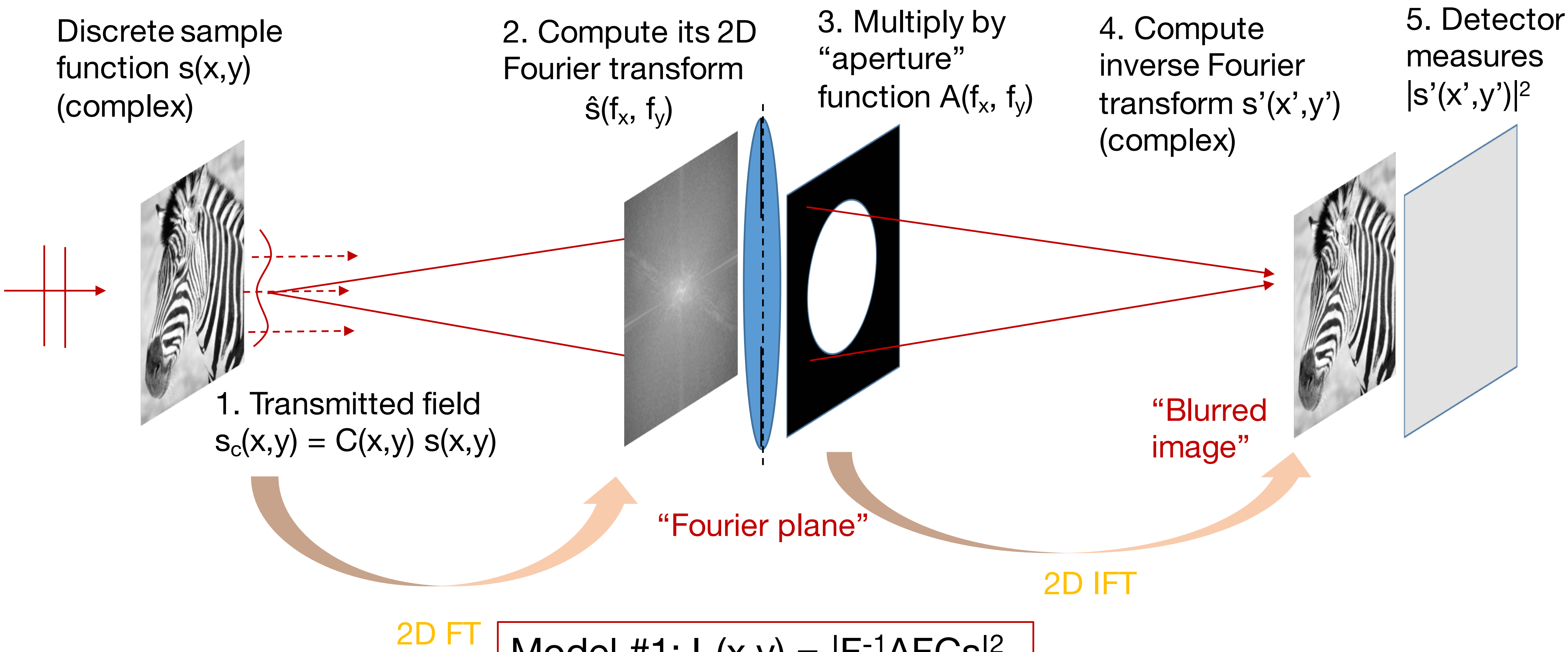

$$\boxed{\text{Model \#1: } I_c(x,y) = |F^{-1}AFCs|^2}$$

Can also model this using the Convolution Theorem

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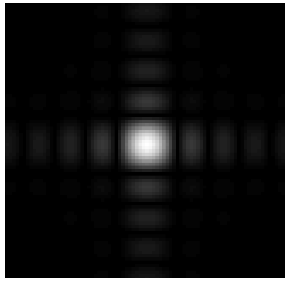
Aperture function (lens shape)

Camera blur function (IFT of lens shape)

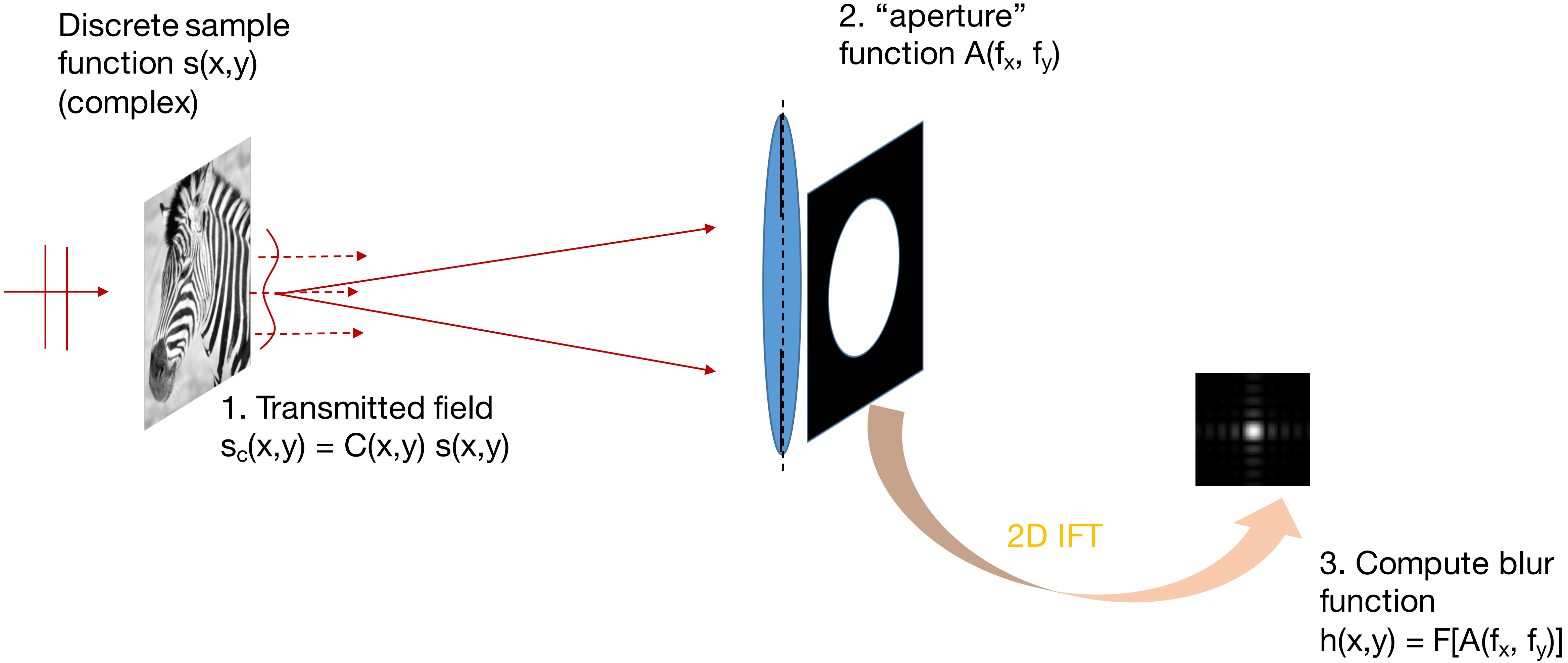
$A(f_x, f_y)$

$h(x, y)$



2D IFT

Model of image formation for wave optics (coherent light):



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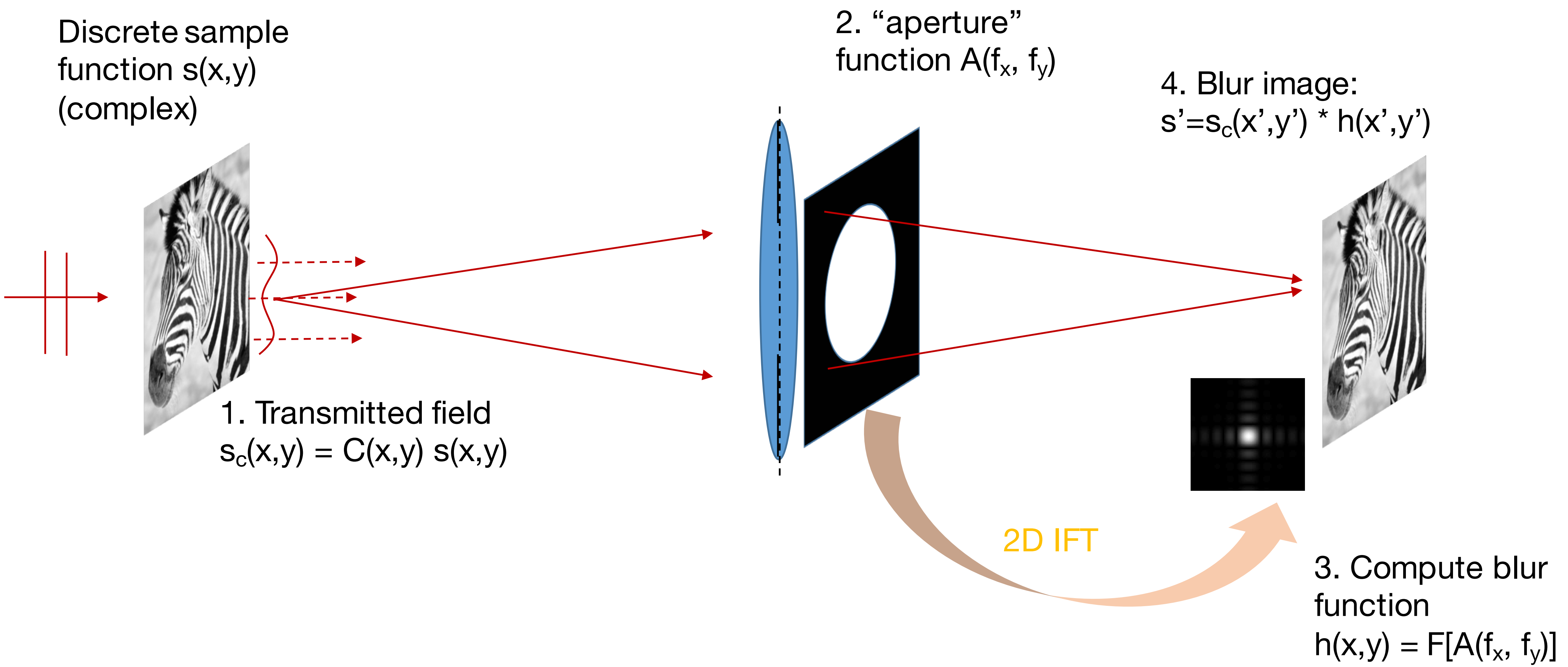

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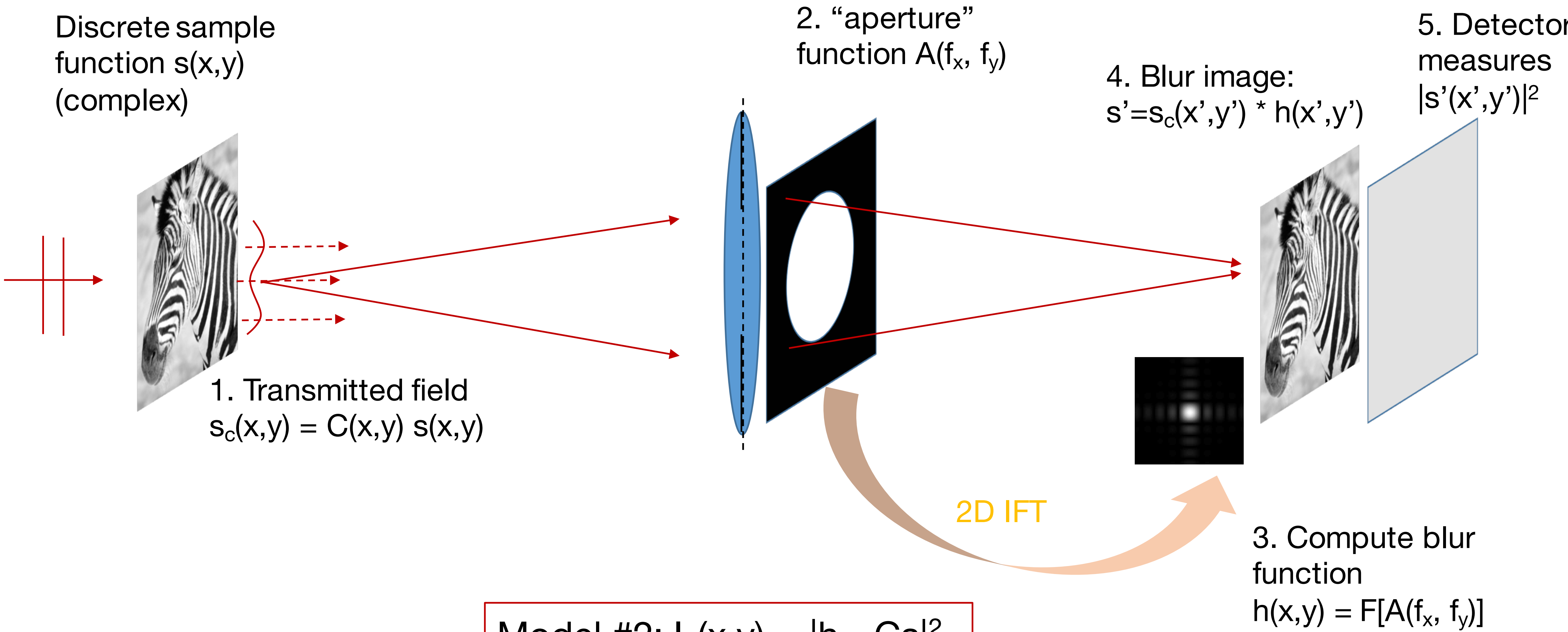

$$\boxed{\text{Model \#2: } I_c(x,y) = |h * Cs|^2}$$

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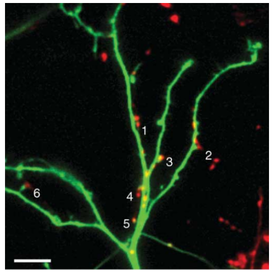
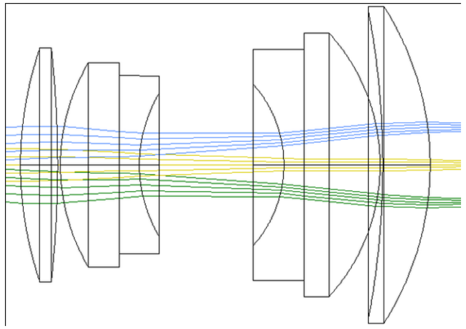
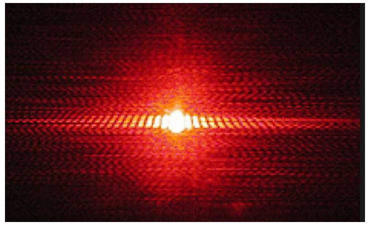



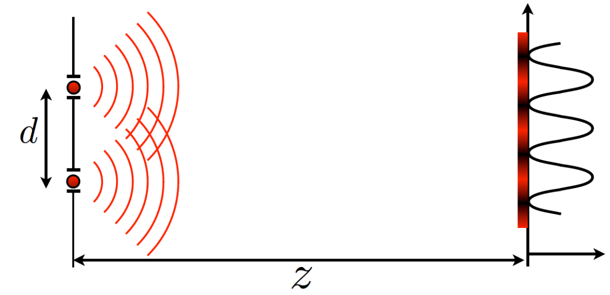
- Real, non-negative

$$I_s = H\,B\,S_0$$

- Sample absorption **S**
- Illumination brightness **B**
- Blur in **H**

- **Interpretation #2: Electromagnetic wave (*Coherent*)**
- Model: Waves





- Complex field

$$I_C = |\,H\,C\,S_C|^2$$

- Sample abs./phase **S**
- Illumination wave **B**
- Blur in **H**

Coherent image formation equation as CNN operations

$$\boxed{I_C = D\,|\,H\,C\,S_C\,|^2}$$

CNN layer

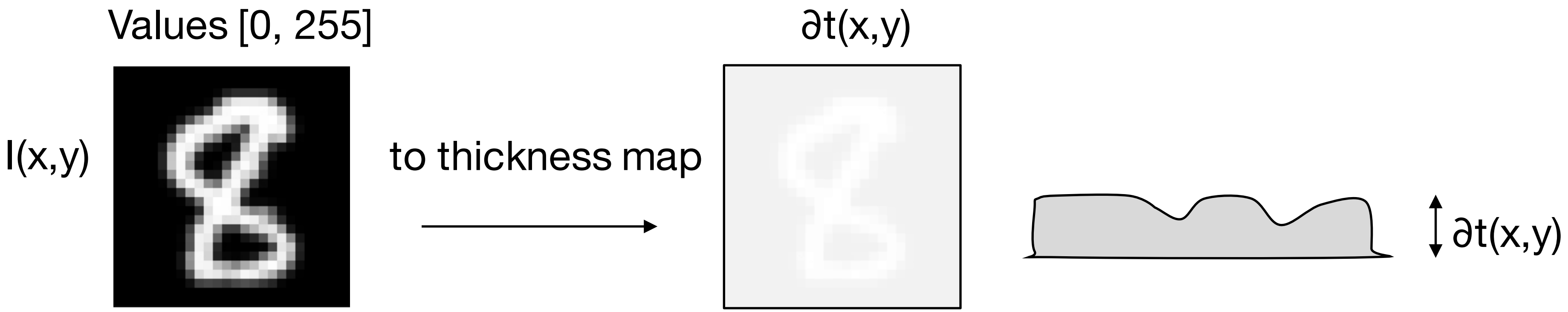
Step 1: Multiply with weights	(Step 1: Normalization)
Step 2: Convolution	Step 2: Convolution
Step 3: Absolute value square (non-linearity)	Step 3: Non-linearity
Step 4: Down-sampling by detector	Step 4: Down-sampling by max pooling

Example #1: Optimizing coherent illumination pattern for improved classification

Example future situation: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

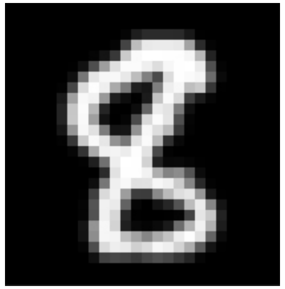
Question: What type of illumination should you use to maximize the classification accuracy of the numbers on the check?

Step 1: Transform MNIST image data set into transparent plastic sheets with varying thickness



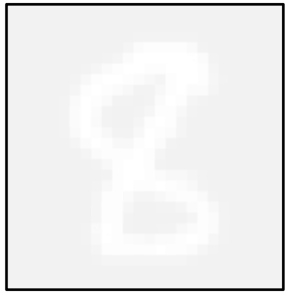
Values [0, 255]

I(x,y)



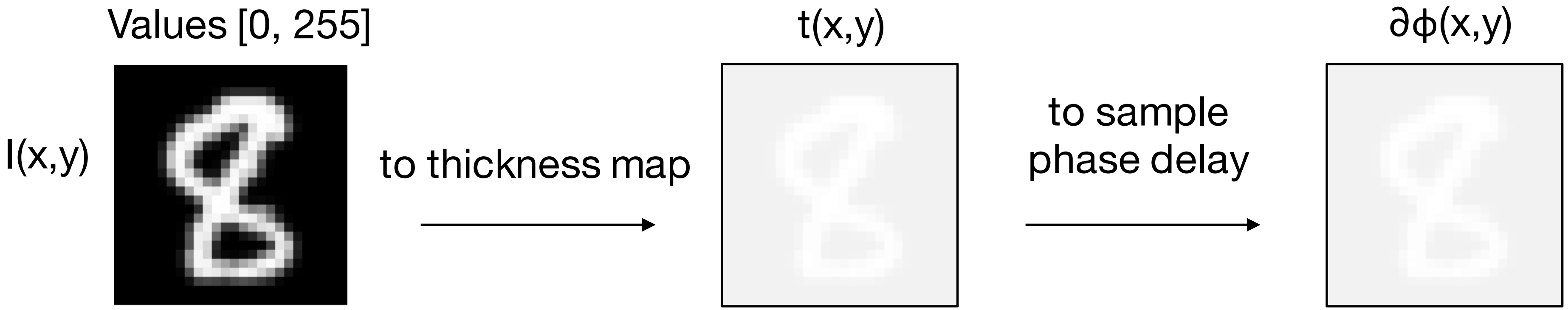
to thickness map

t(x,y)

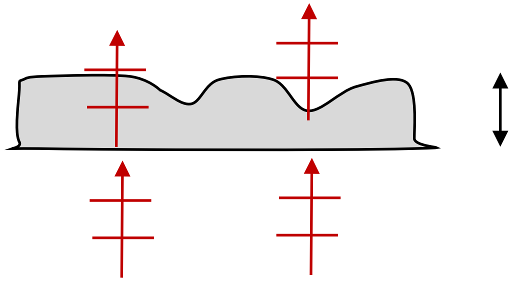


1. Normalize intensity map to 1

2. Define thickness map at some reasonable amount (100 μm max change)



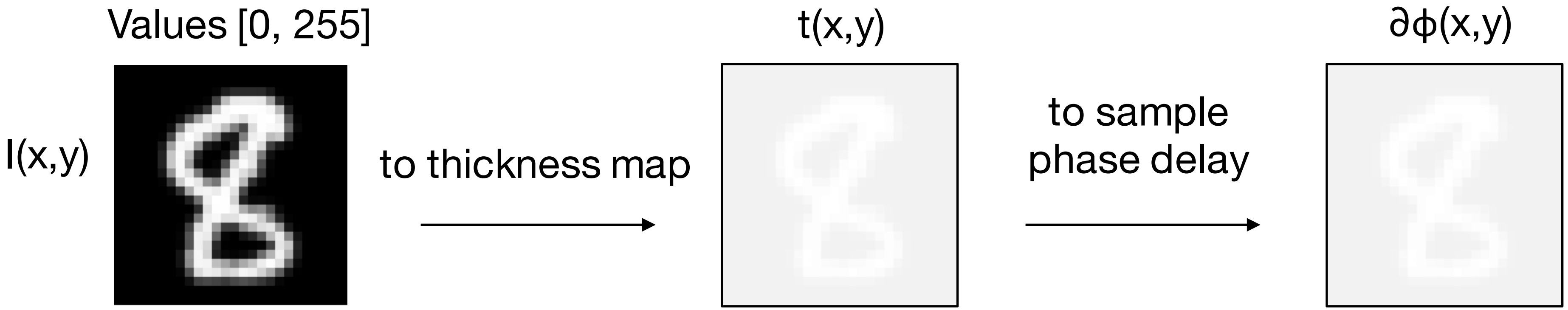
1. Normalize intensity map to 1
2. Define thickness map at some reasonable amount (100 μm max change)
3. Convert thickness map into optical phase delay:



$$\boxed{\partial\phi(x,y) = \exp[\, j\, n\, t(x,y) / \lambda \,]}$$

nt = Optical path length

deep imaging



1. Normalize intensity map to 1

2. Define thickness map at some reasonable amount (100 μm max change)

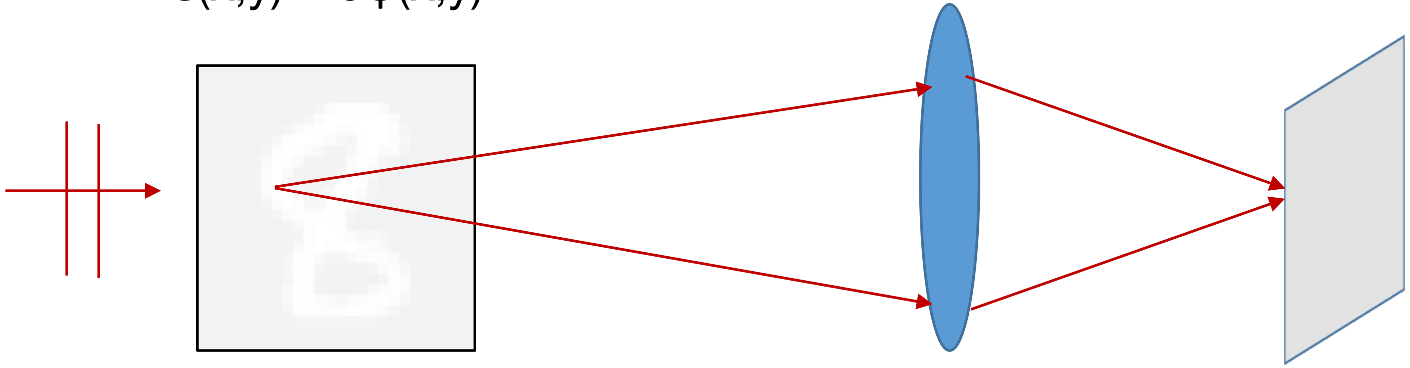
3. Convert thickness map into optical phase delay:

```
n = 1
wavelength = 0.5e-3
mnist_raw_images = tf.placeholder(tf.float32, [image_size, None])
thickness_map = mnist_raw_images/np.amax(mnist_raw_images)
mnist_phase_delay_real = cos(thickness_map * n/wavelength)
mnist_phase_delay_imag = sin(thickness_map * n/wavelength)
mnist_phase_delay = tf.complex(mnist_phase_delay_real,mnist_phase_delay_imag)
```

Example #1: Optimizing coherent illumination pattern for improved classification

$\boxed{\text{Coherent image Model: } I_c(x,y) = |h * Cs|^2}$

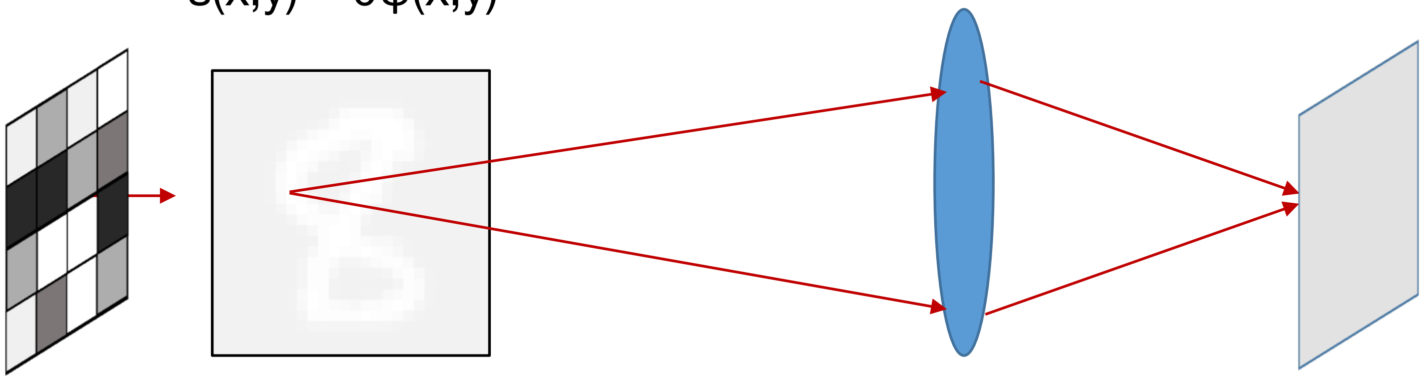
$s(x,y) = \partial\phi(x,y)$



Example #1: Optimizing coherent illumination pattern for improved classification

deep imaging

Coherent image Model: $I_c(x,y) = |h * Cs|^2$



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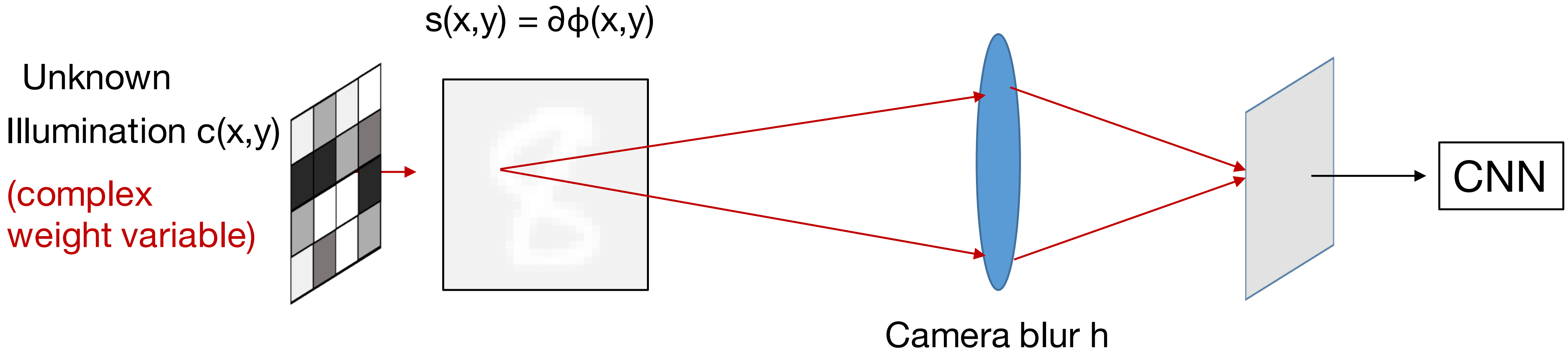
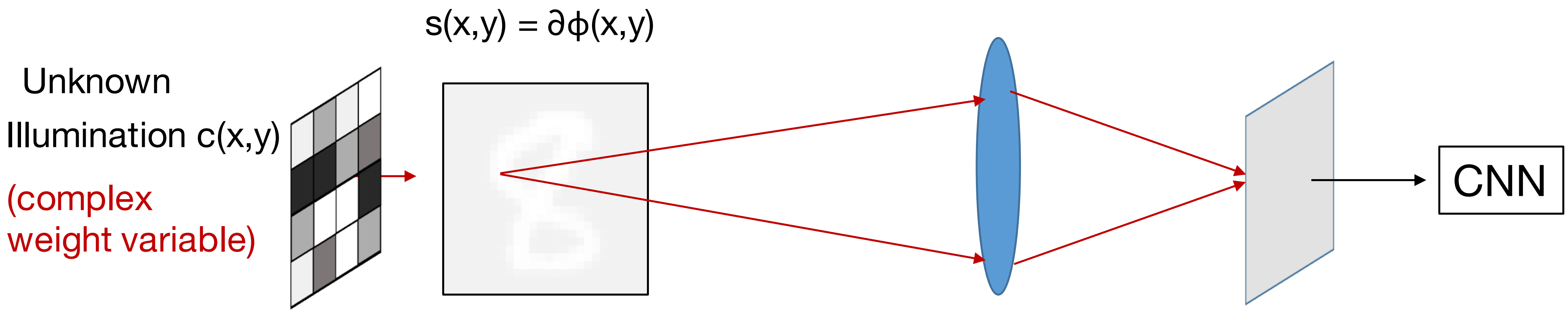


Example #1: Optimizing coherent illumination pattern for improved classification

Coherent image Model: $I_c(x,y) = |h * Cs|^2$




```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0_real = tf.Variable([image_size, image_size])
C0_imag = tf.Variable([image_size, image_size])
C0_complex = tf.complex(C0_real, C0_imag)
x_C_complex = tf.mul(mnist_phase_delay, C0_complex)
image_complex = conv2d(x_C_complex, camera_blur)
detected_image = tf.complex_abs(image_complex)
```

detected_image then enters standard CNN classification pipeline

Example #2: Optimizing aperture shape for improved digit classification

Example future situation: Hacking has brought online banking to a halt. We now rely on a special form of physical check that is made of visibly transparent plastic. To write the amount in, you press down with a pen-like instrument, and then the check is read out by shining a particular pattern of laser light onto it, and then imaging it with a lens.

Question #2: What type of aperture shape should you use to maximize classification accuracy?

Example #2: Optimizing aperture shape for improved digit classification

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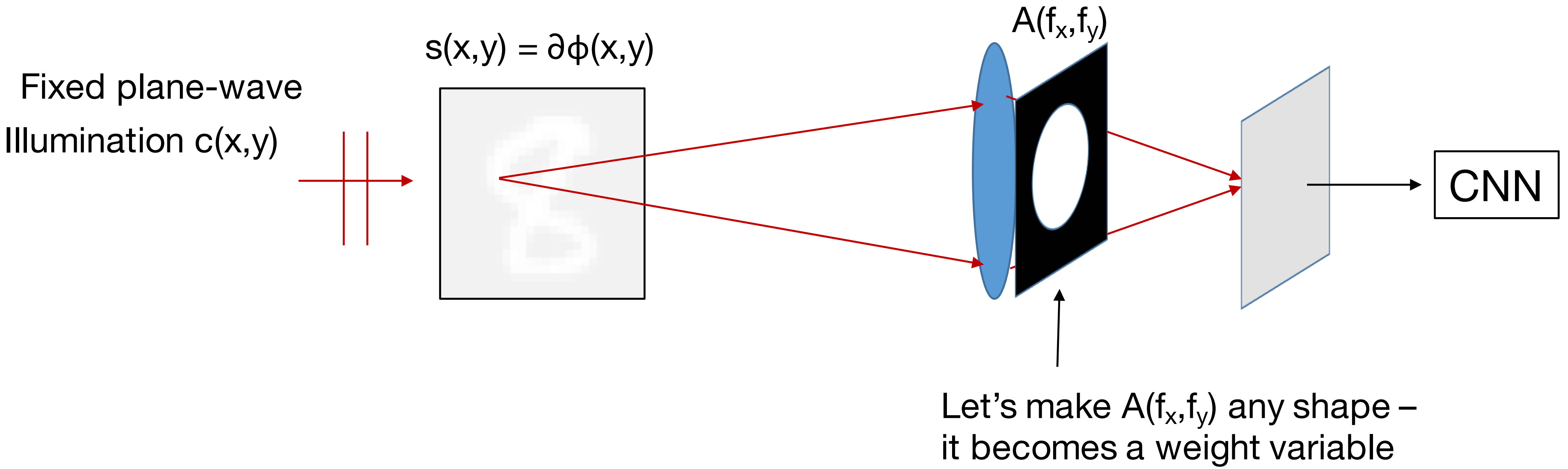
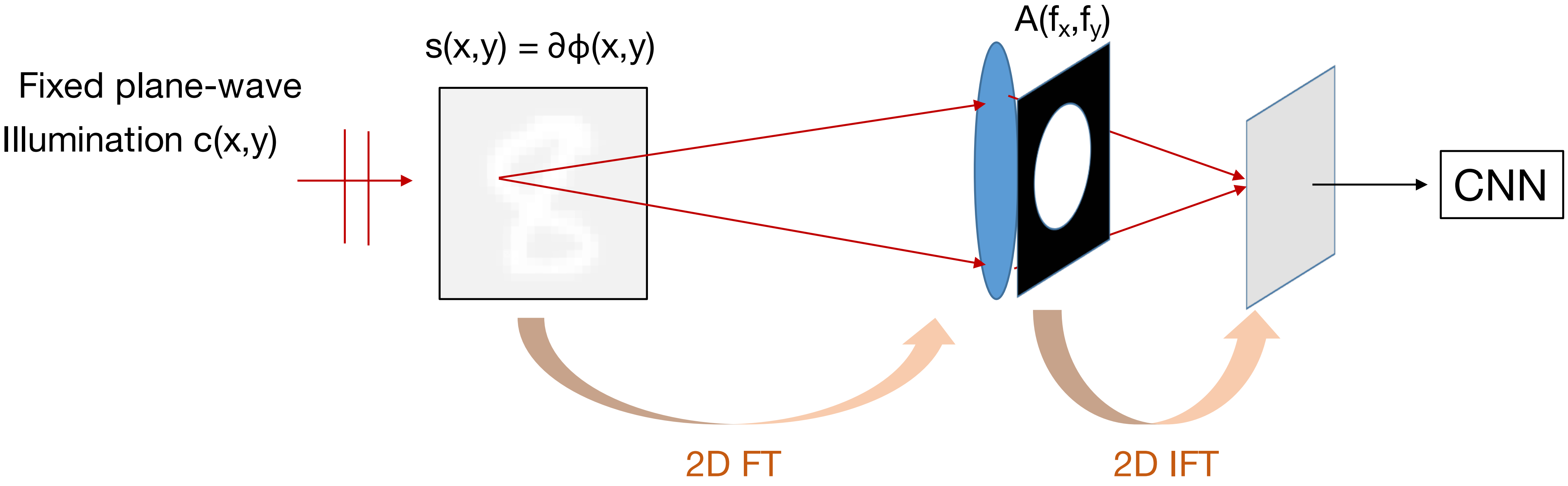
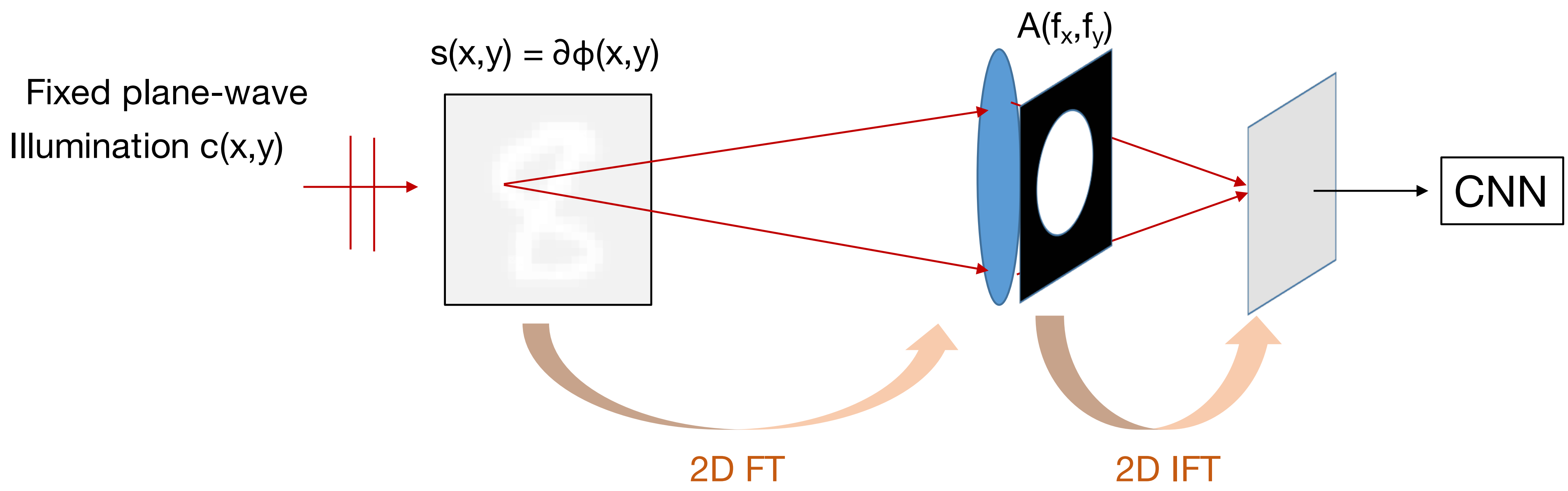


Example #2: Optimizing aperture shape for improved digit classification



Example #2: Optimizing aperture shape for improved digit classification




```
mnist_phase_delay = tf.reshape(mnist_phase_delay, [-1, image_size, image_size])
C0 = np.ones(image_size, image_size)
C0 = tf.constant(C0)
x_C_complex = tf.mul(mnist_phase_delay, C0)
fx_C_complex = tf.fft2d(x_C_complex)
ap_filter = tf.Variable([image_size, image_size])
filtered_x_C =  tf.mul(fx_C_complex, ap_filter)
image_complex = tf.ifft2d(filtered_x_C)
detected_image = tf.complex_abs(image_complex)
```