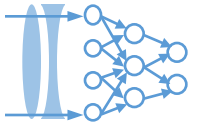


Lecture 15: Introduction to Physical Layers in Machine Learning

Machine Learning and Imaging

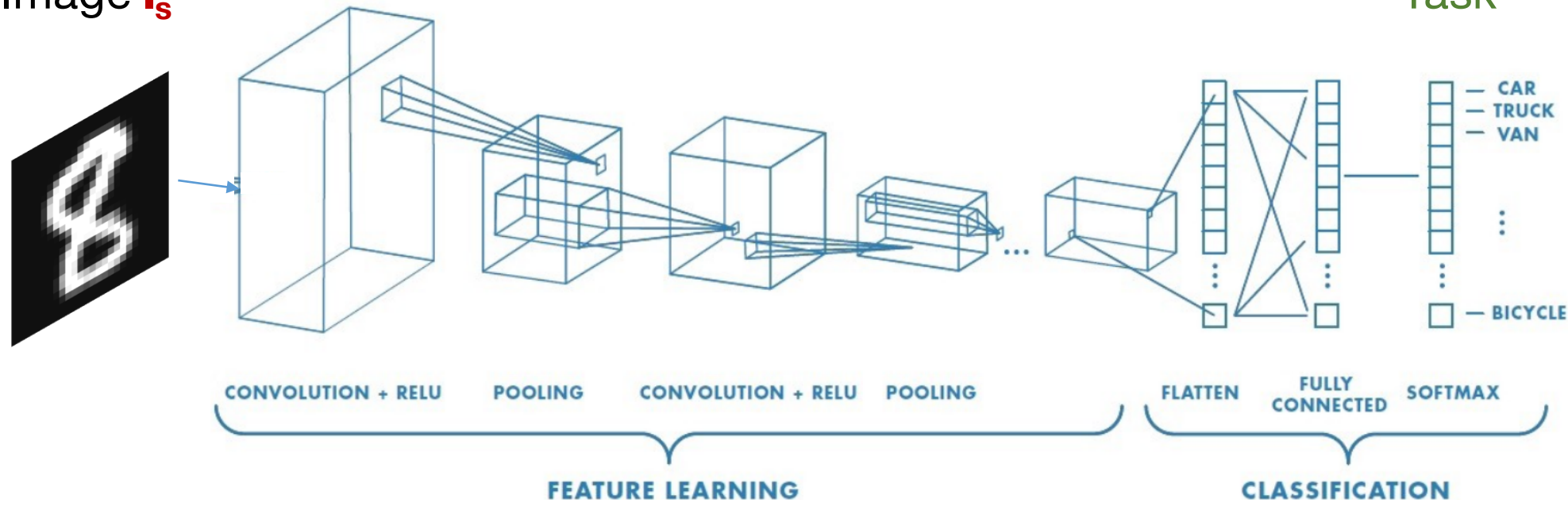
BME 590L
Roarke Horstmeyer

Bringing together physical and digital image representations

deep imaging

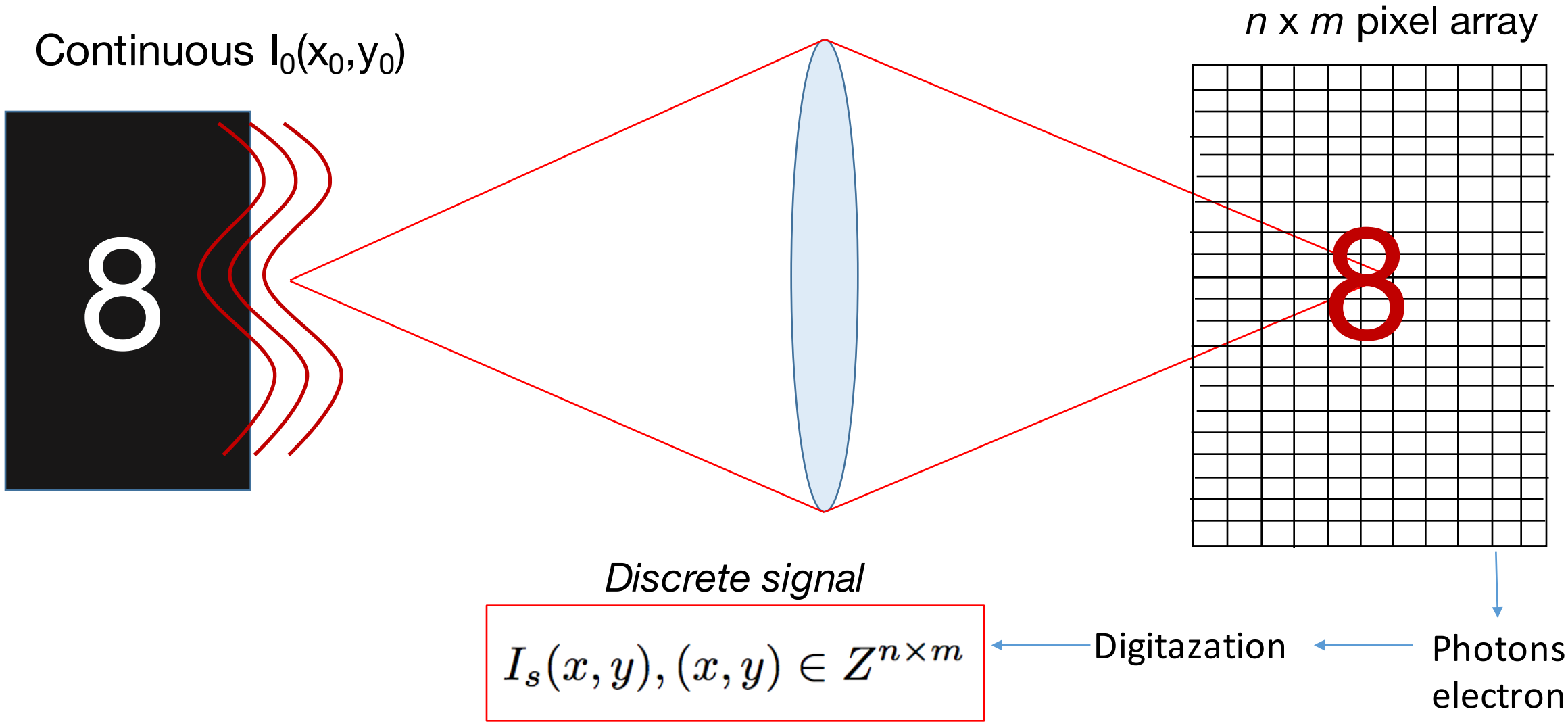
Image I_s

Task

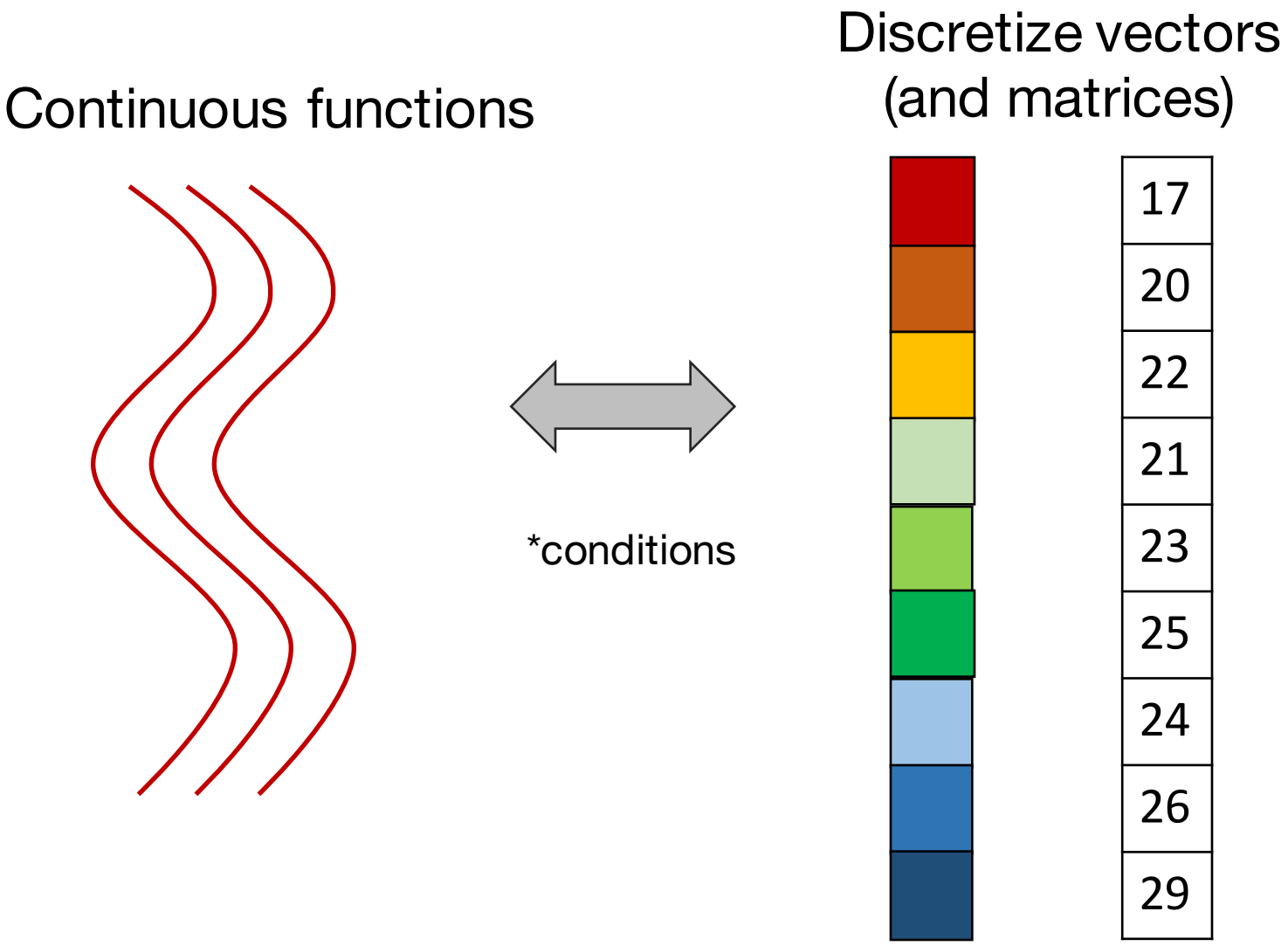


$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, I_s] \ldots]$$

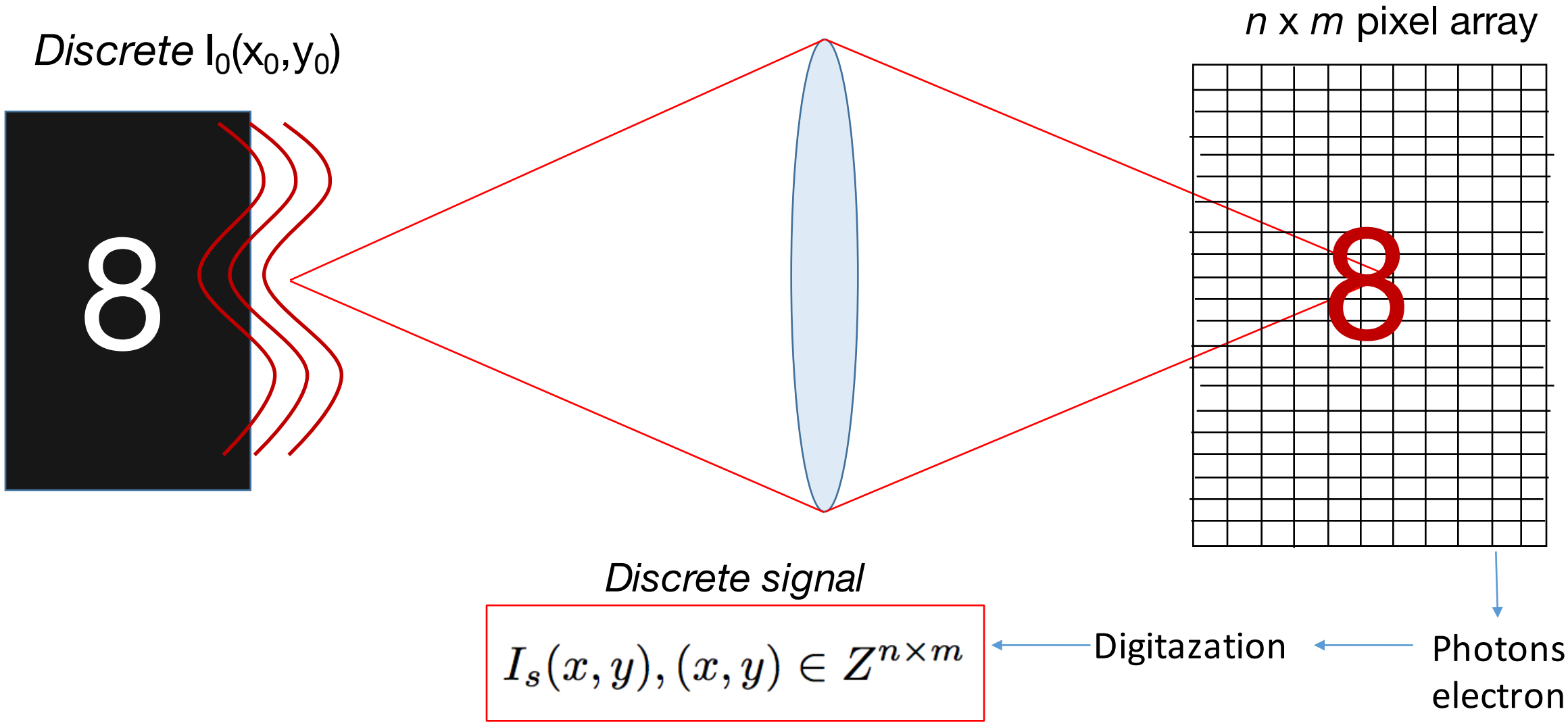
Simple model of image formation



What does the Sampling Theorem mean for us?



Simple model of image formation



Bringing together physical and digital image representations

deep imaging

Physical Layers

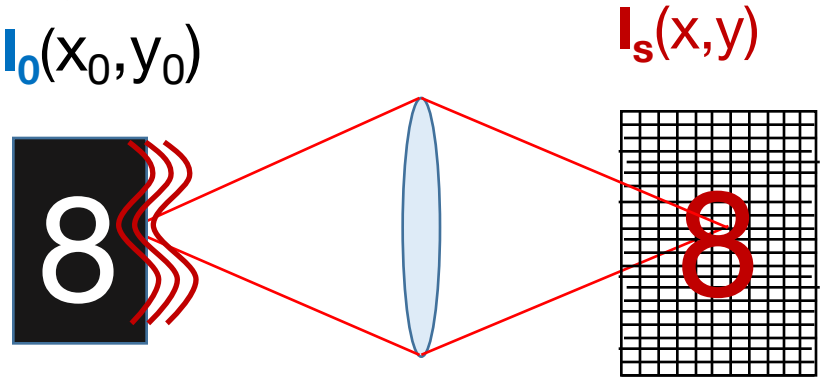
Digital Layers

Physical world

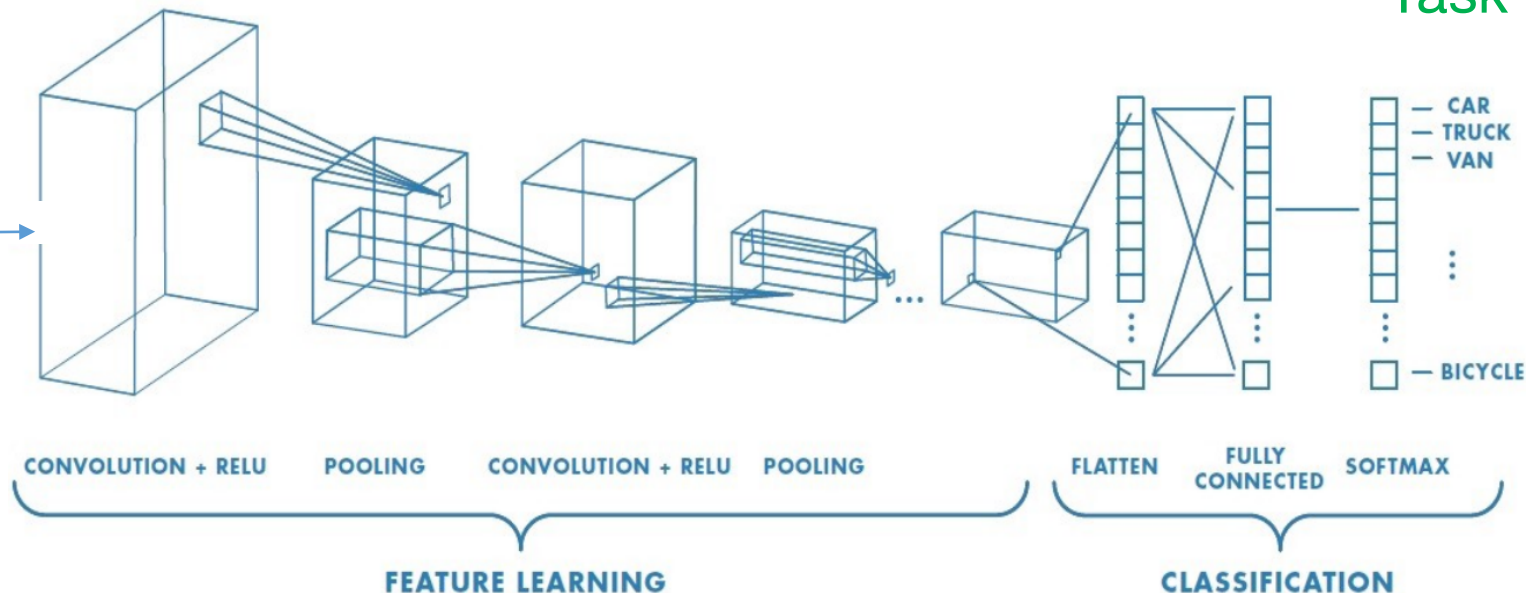
$I_0(x_0,y_0)$

Digital Image

$I_s(x,y)$



Task



$$I_s = f[I_0]$$

Digital layers

Physical layers

$$\text{Task} = W_n \ldots \text{ReLU}[W_1 \, \text{ReLU}[W_0 \, f[I_0]] \ldots]$$

Bringing together physical and digital image representations

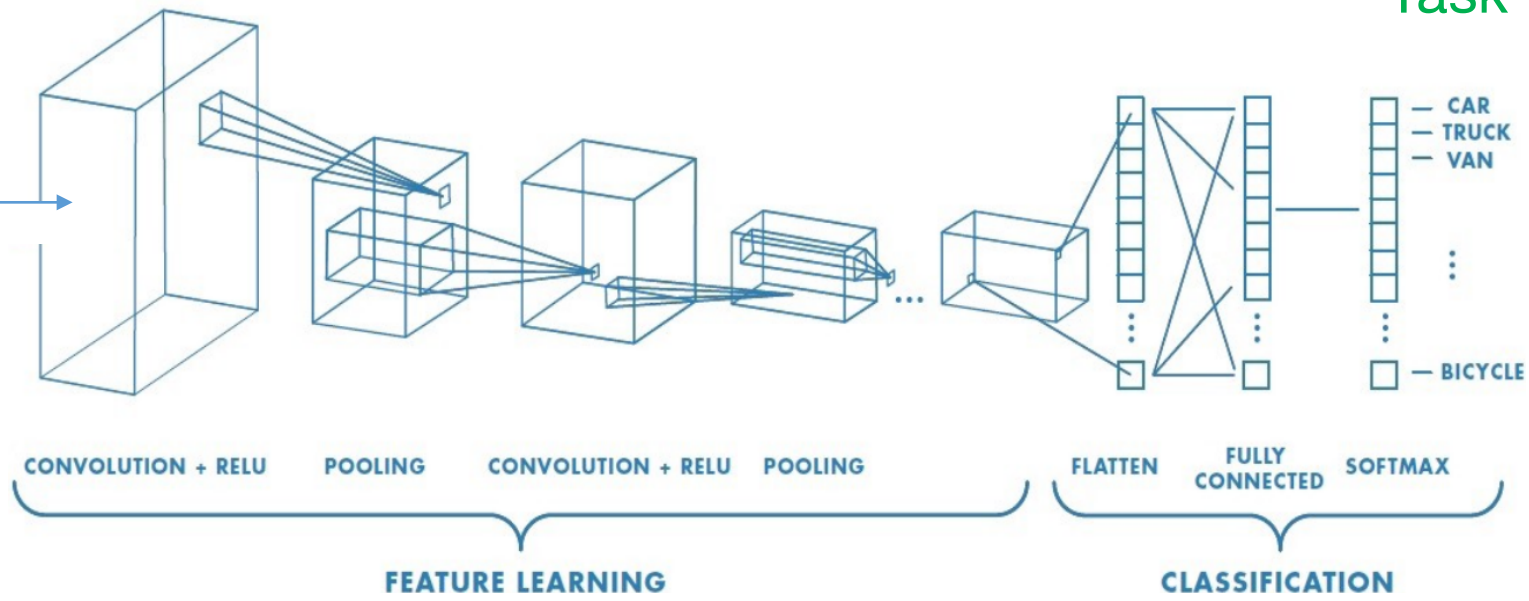
deep imaging

Physical Layers

Digital Layers

Task

Physical Function I_0 → f[ ] → Digitized I_s →



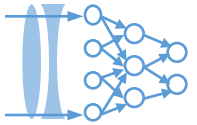
$$I_s = f[I_0]$$

Digital layers

Physical layers

$$\text{Task} = W_n \ldots \text{ReLU}[W_1 \, \text{ReLU}[W_0 \, f[I_0]] \ldots]$$

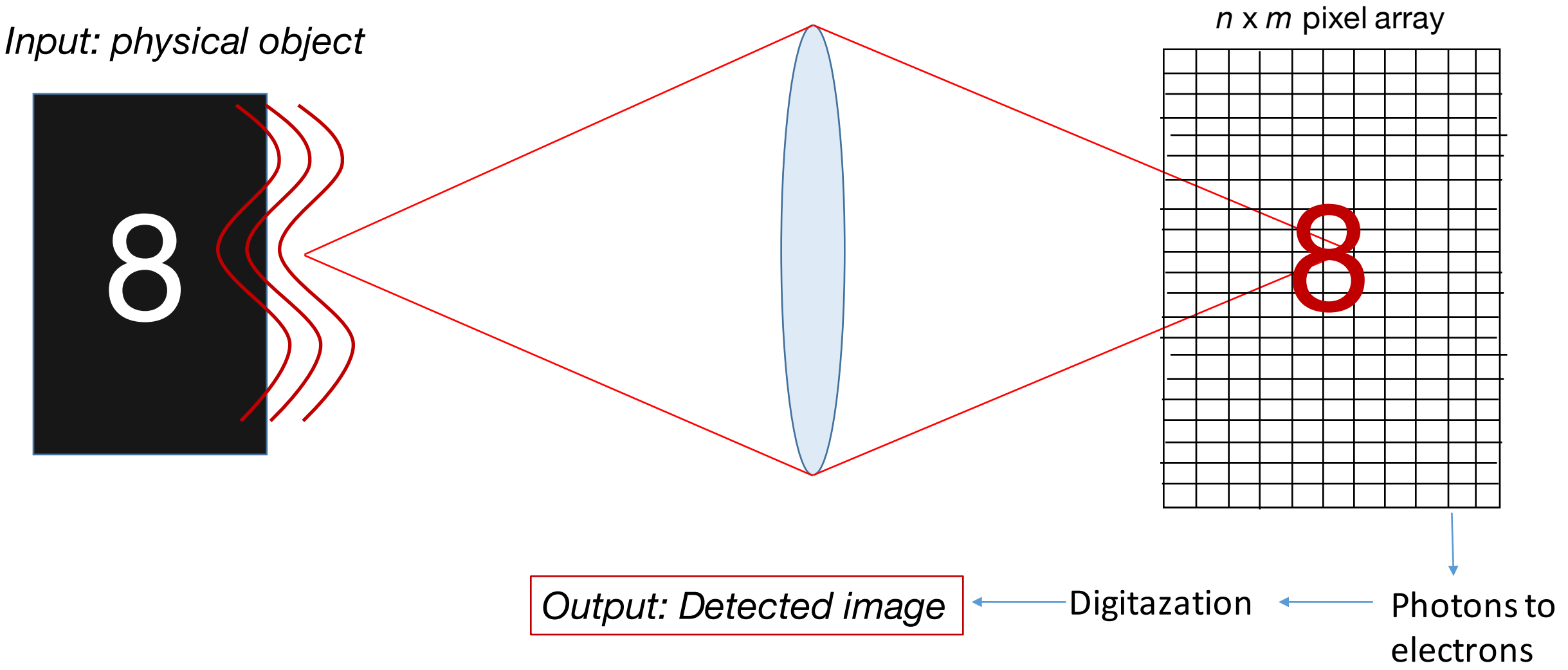
Required properties of physical mapping f[ ] for DNN optimization?



- Finite
- Non-zero gradients
- Differentiable*
- Known structure (for now…)
- Anything else?

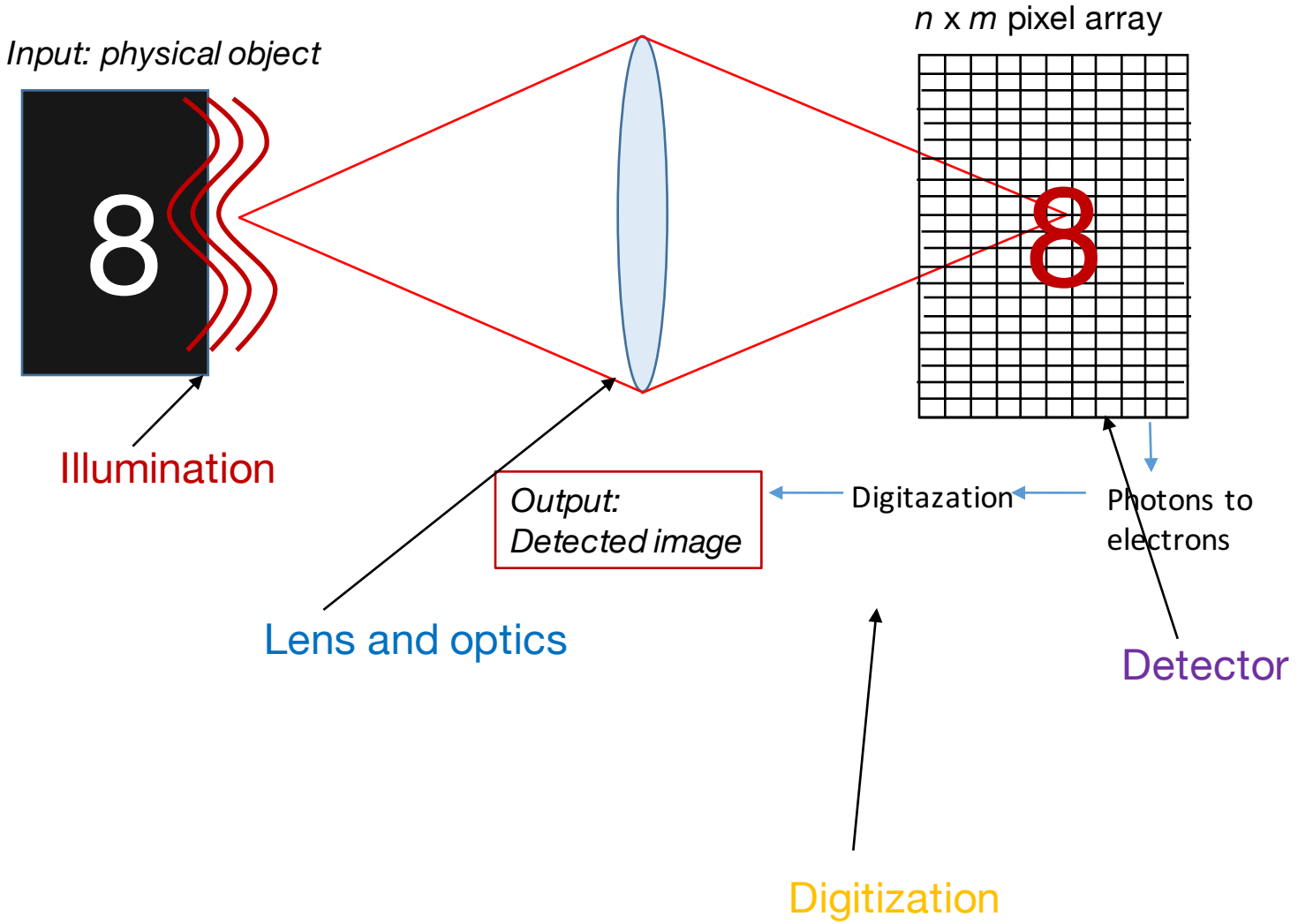
deep imaging

What physical parameters effect image formation?

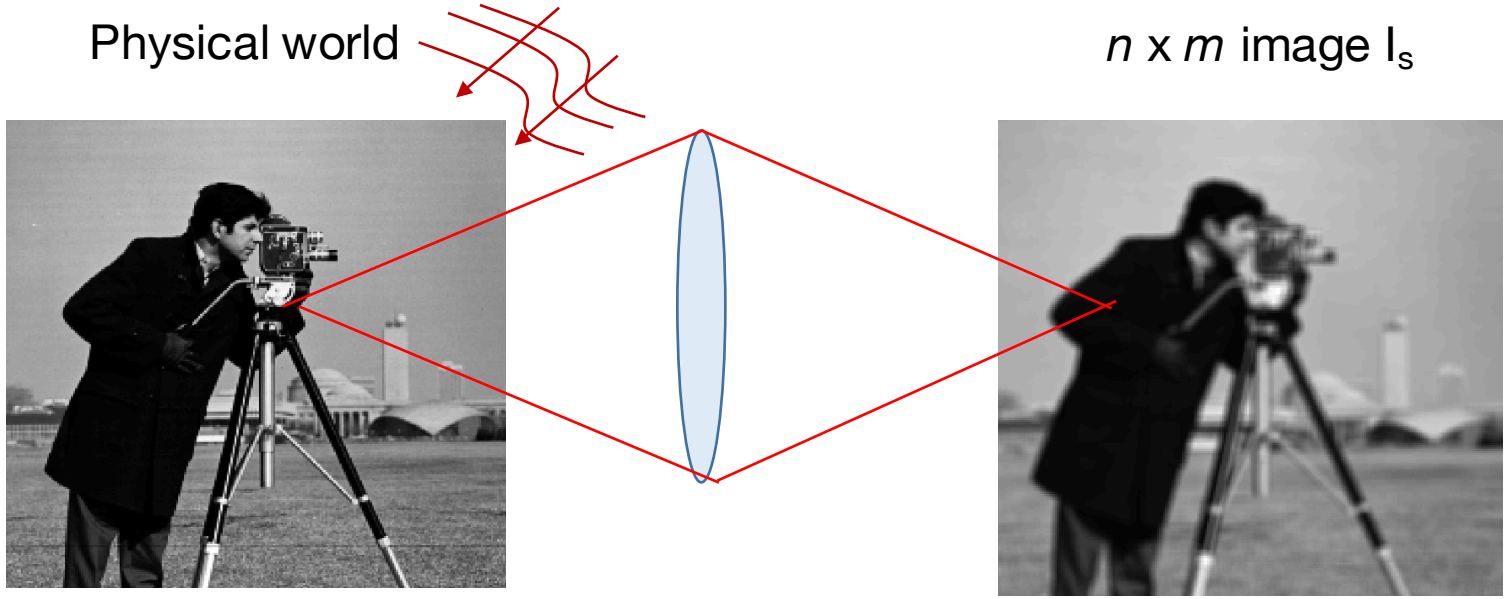


What physical parameters effect image formation?

- Illumination
  - Spatial pattern
  - Angle of incidence
  - Color, polarization
- Lens and optics
  - Position/orientation
  - Shape
  - Focus
  - Transparency
- Detector
  - Pixel size
  - Pixel shape & fill factor
  - Color filters
  - Other filters
- Digitization
  - E to P curves
  - Digitization schemes/thresholds
  - Data transmission, multiplexing
- Physical object

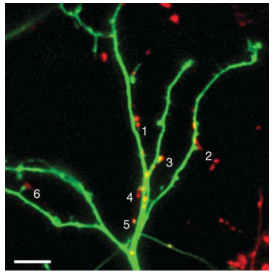


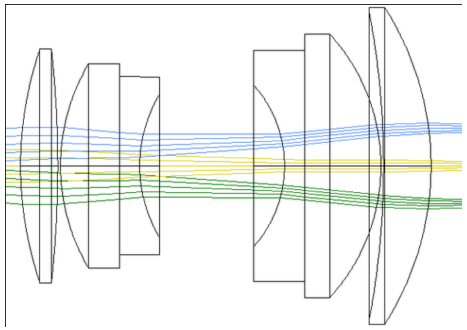
Simple mathematical model of image formation



First - what is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays



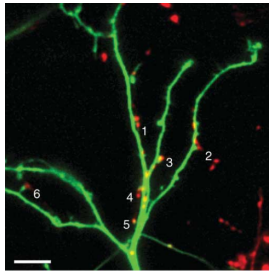


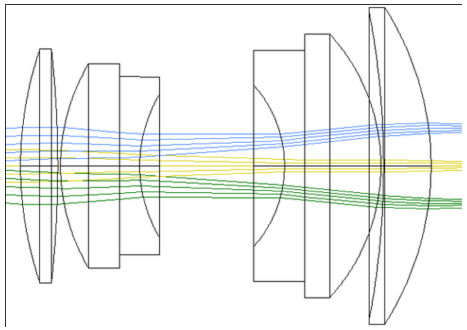
- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

First - what is light and how can we model it?

- Interpretation #1: Radiation (*Incoherent*)
- Model: Rays

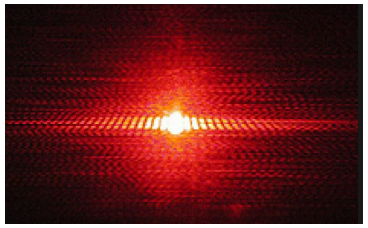


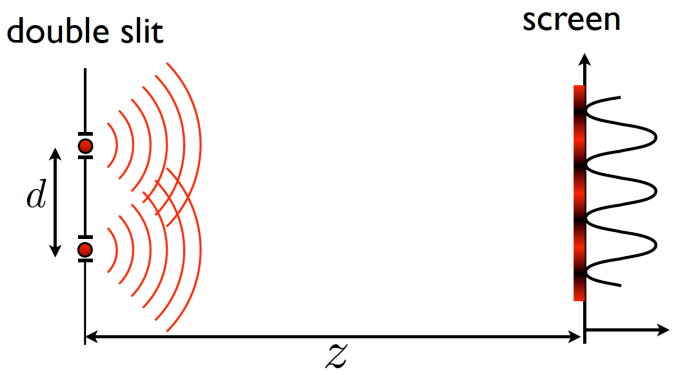


- Real, non-negative
- Models absorption and brightness

$$I_{tot} = I_1 + I_2$$

- Interpretation #2: Electromagnetic wave (*Coherent*)
- Model: Waves



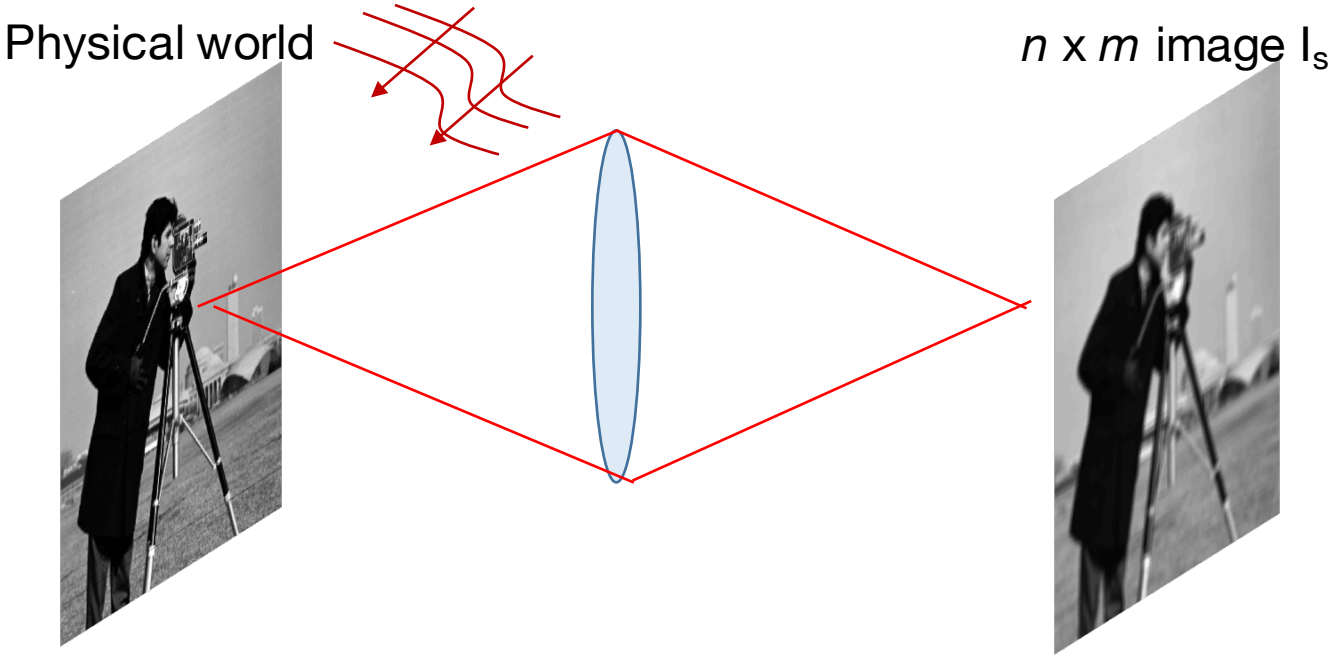


- Complex field
- Models Interference

$$E_{tot} = E_1 + E_2$$

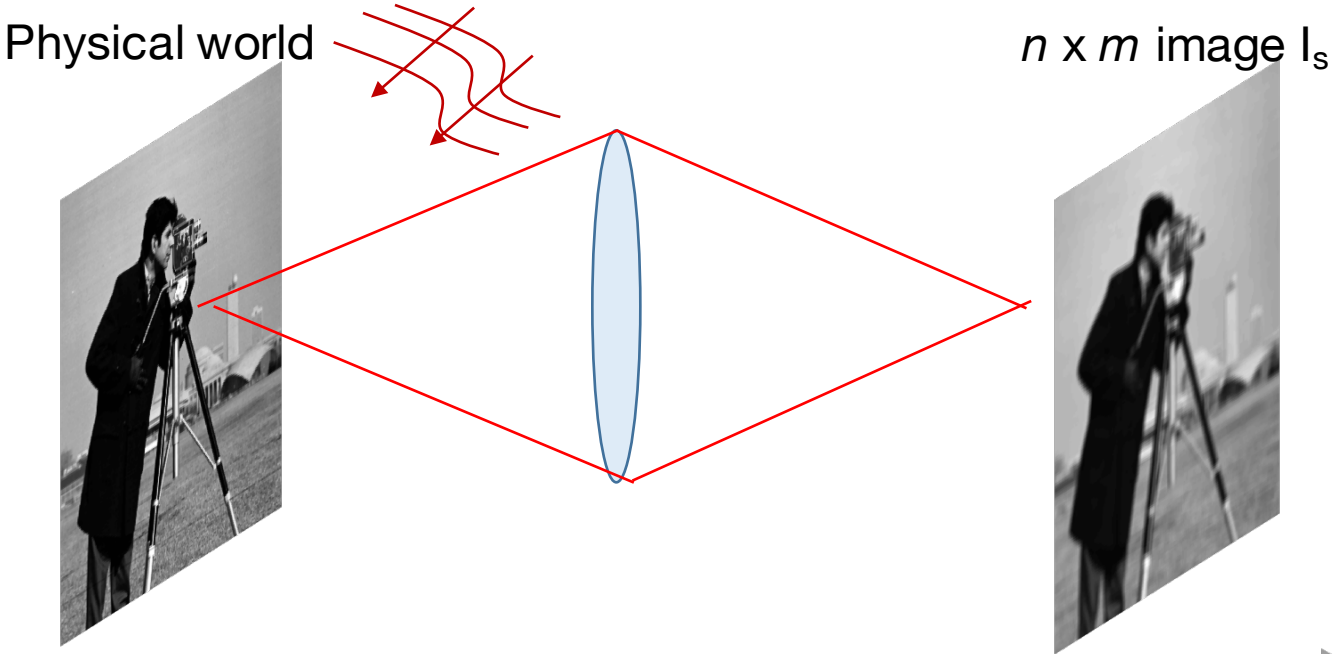
- Interpretation #3: Particle
- Model: Photons

Simple mathematical model of image formation



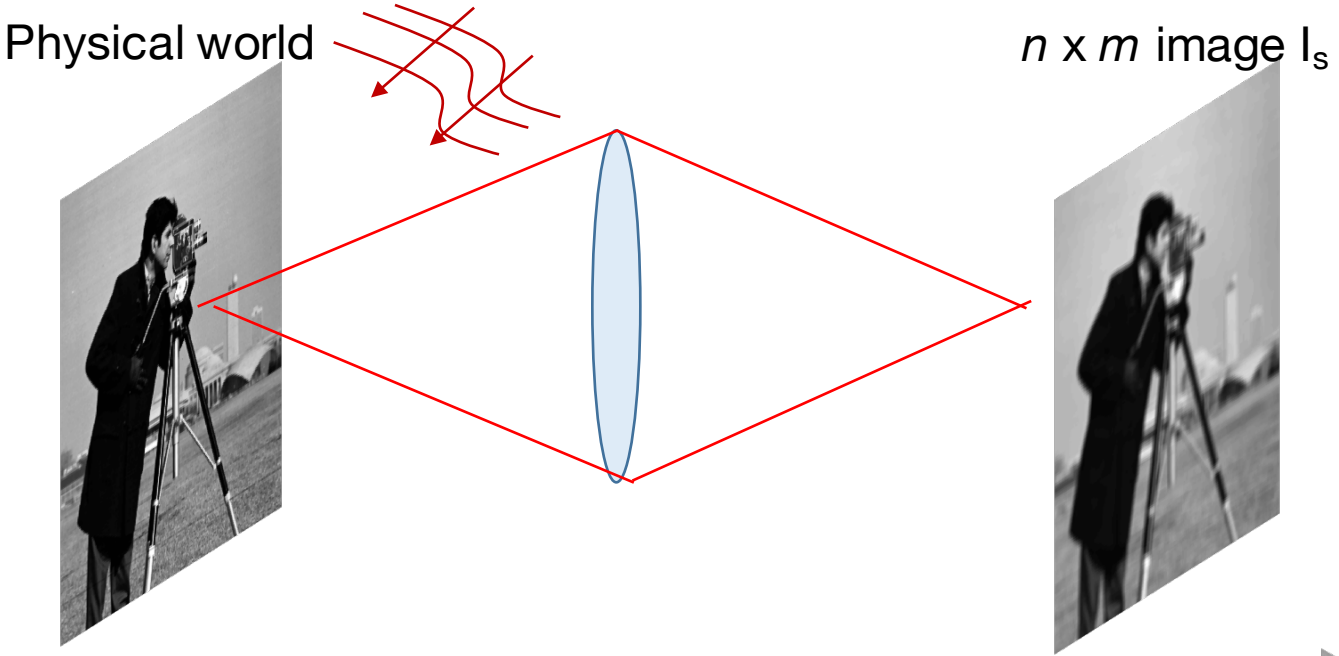
- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Simple mathematical model of image formation

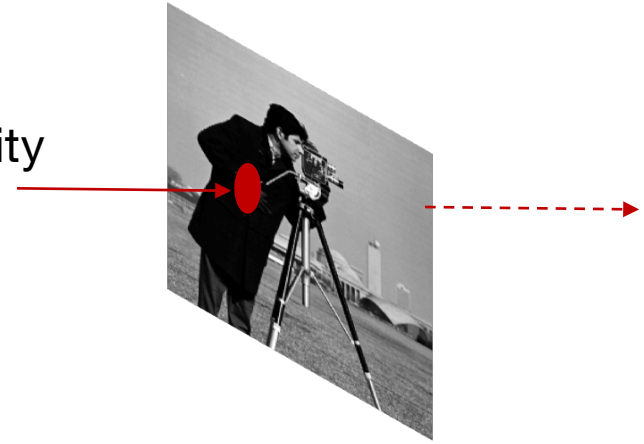


- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

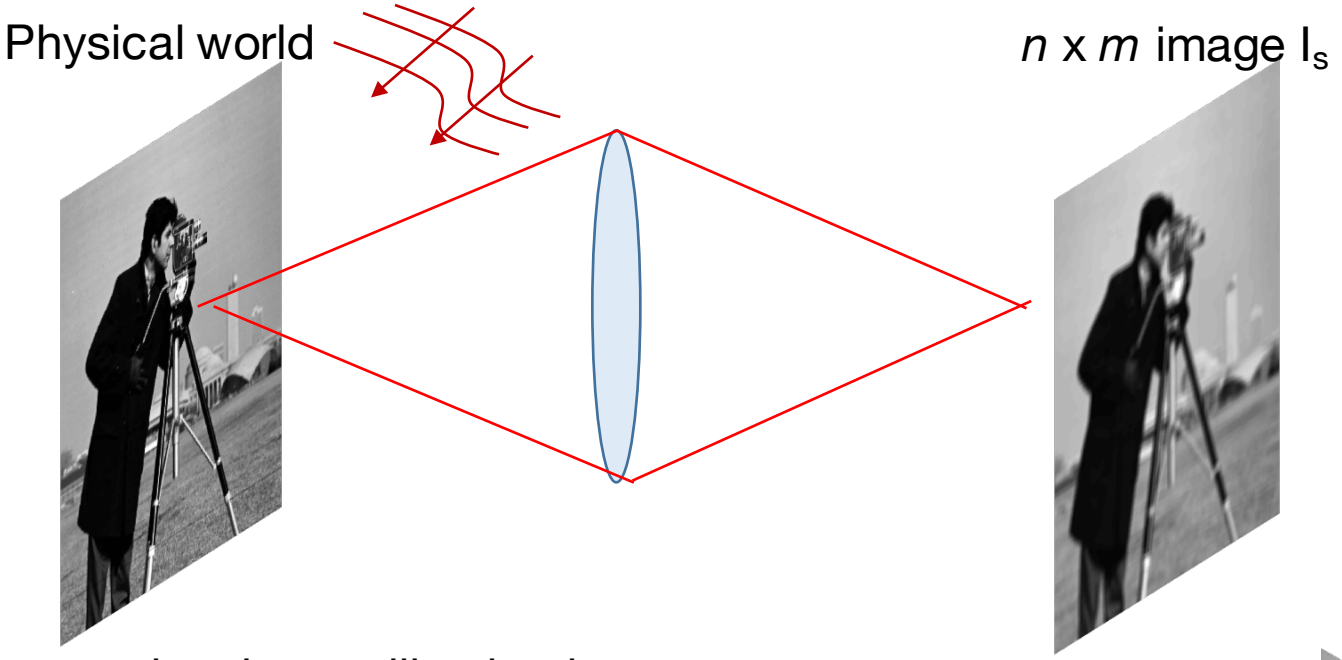
Simple mathematical model of image formation



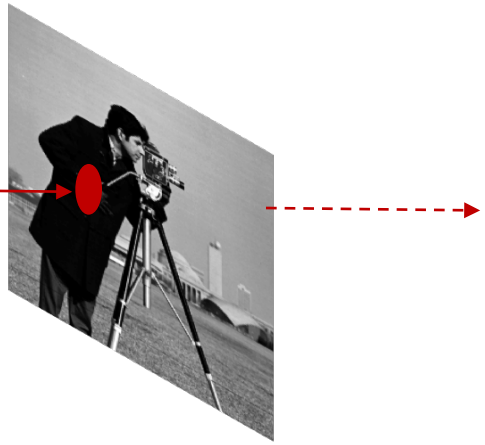
- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity



Simple mathematical model of image formation

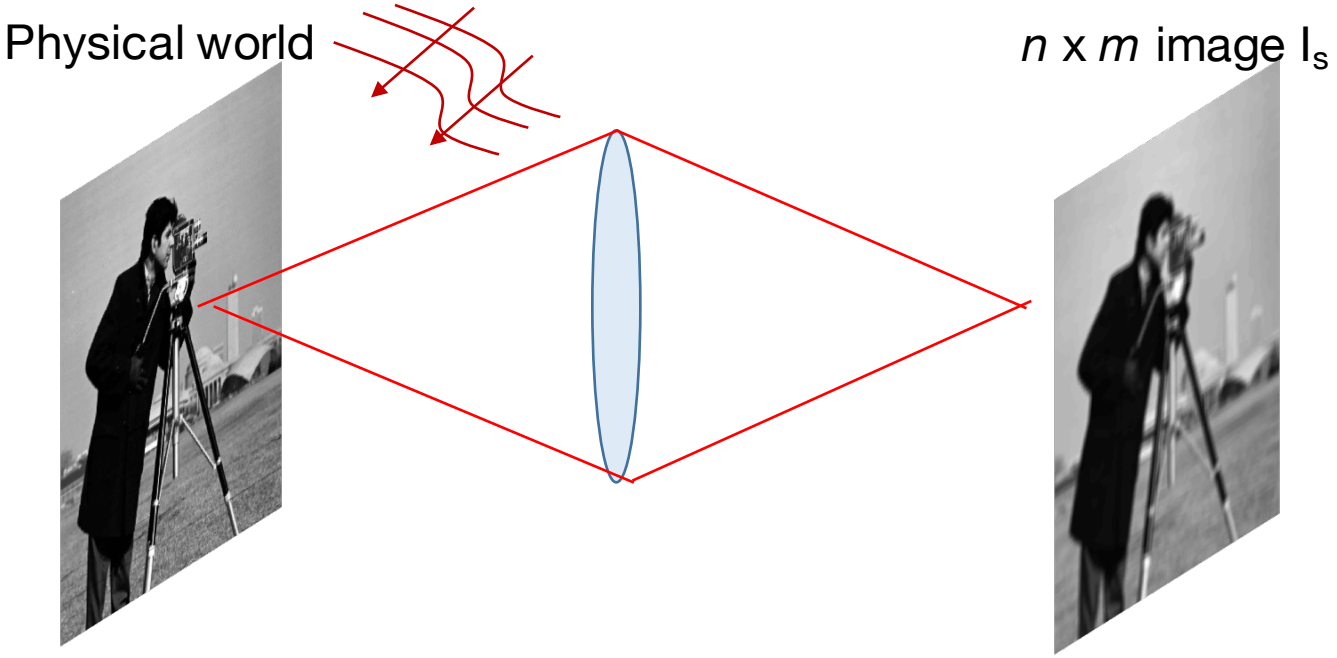


- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity



Object absorption: $\mathbf{I_0}(x,y)$
Illumination pattern: $\mathbf{s}(x,y)$
Light exiting object surface: $\mathbf{I_e}(x,y) = \mathbf{I_0}(x,y) \circ \mathbf{s}(x,y)$

Simple mathematical model of image formation



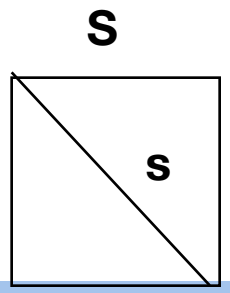
- Assume incoherent illumination
- Assume thin 2D object
- Object is real, non-negative map of absorption/reflectivity

Modeling incoherent illumination

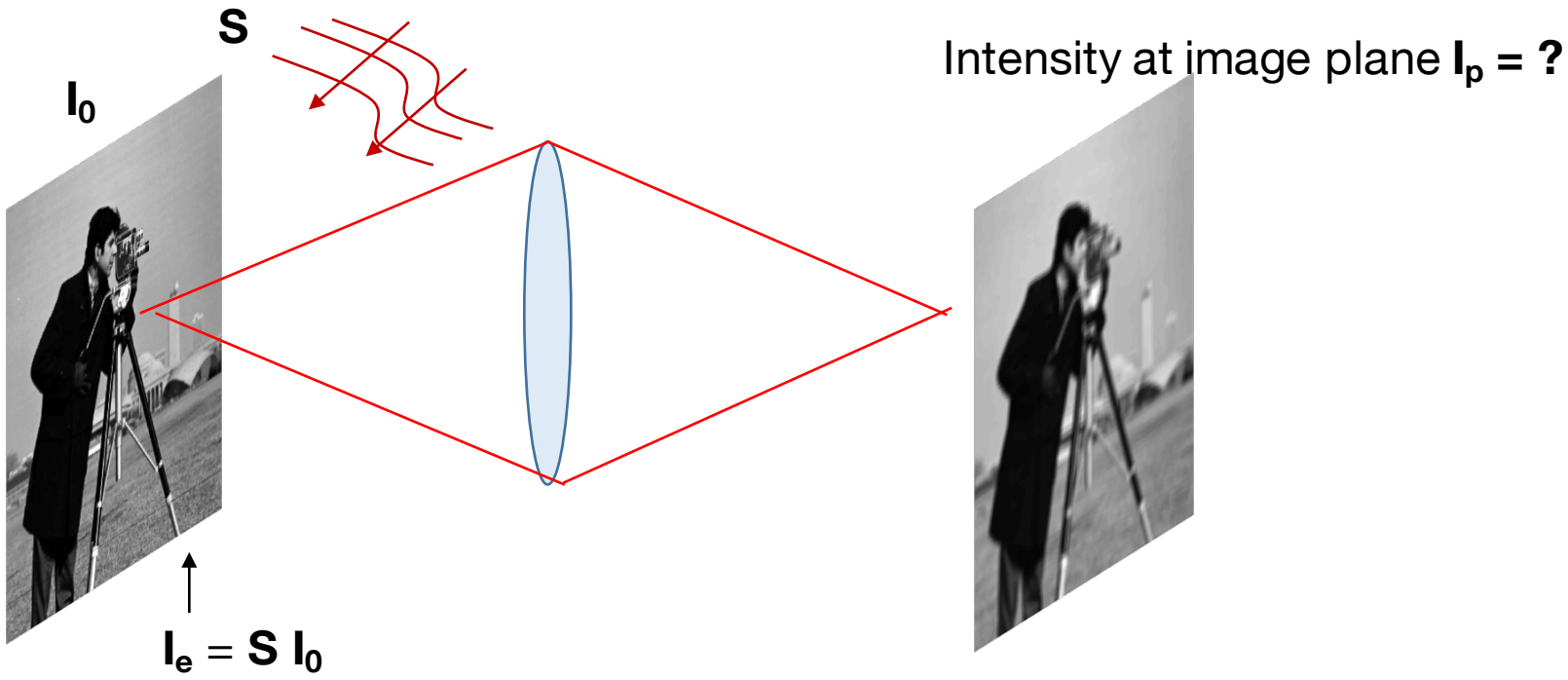
$$\mathbf{I}_e(x,y) = \mathbf{I}_0(x,y) \circ \mathbf{s}(x,y)$$

$$\mathbf{I}_e = \mathbf{S}\,\mathbf{I}_0$$

$\mathrm{diag}(\mathbf{S}) = \mathbf{s}$



Simple mathematical model of image formation



Simple mathematical model of image formation

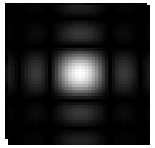
deep imaging

Lenses blur and rescale images:
(We'll learn how exactly next few weeks)

Input intensity

Convolution filter h

$*$



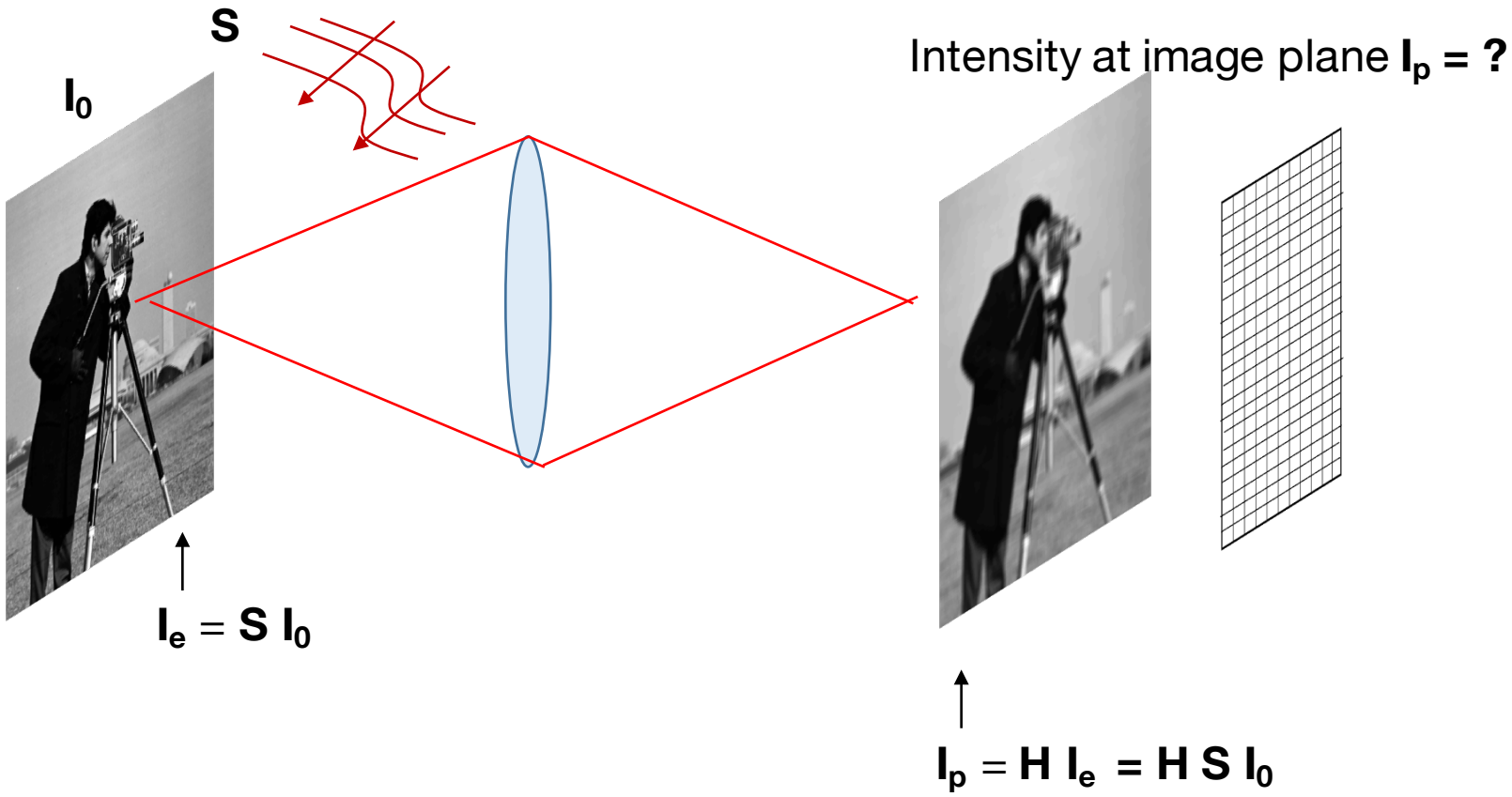
$=$

Output intensity

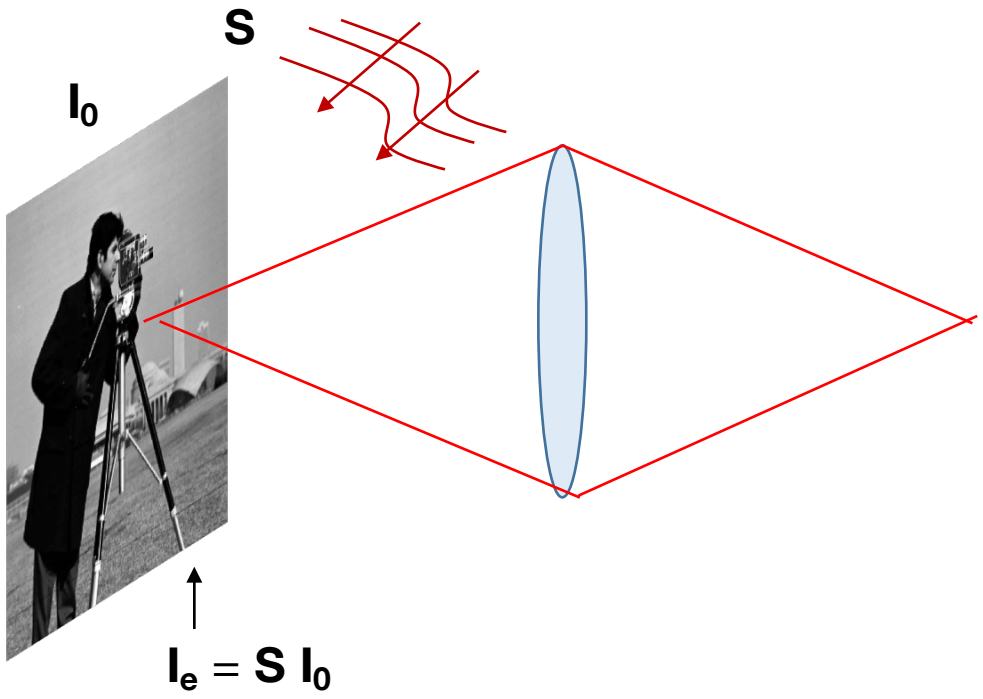
$\mathbf{I_e}(x/M, y/M) \quad * \quad \mathbf{h}(x,y) \quad = \quad \mathbf{I_p}(x,y)$

Assuming we've resized by M, $\mathbf{I_p = I_e * h = H I_e}$

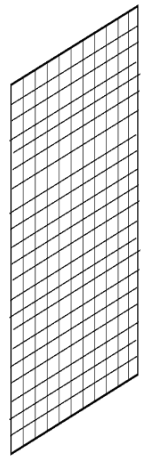
Simple mathematical model of image formation

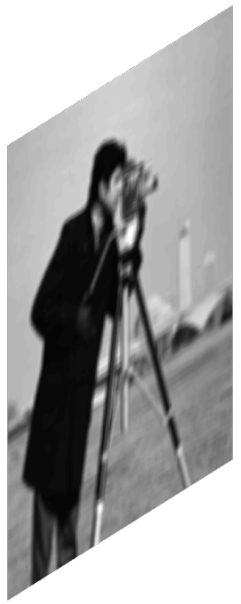


Simple mathematical model of image formation



Discretization by detector ?





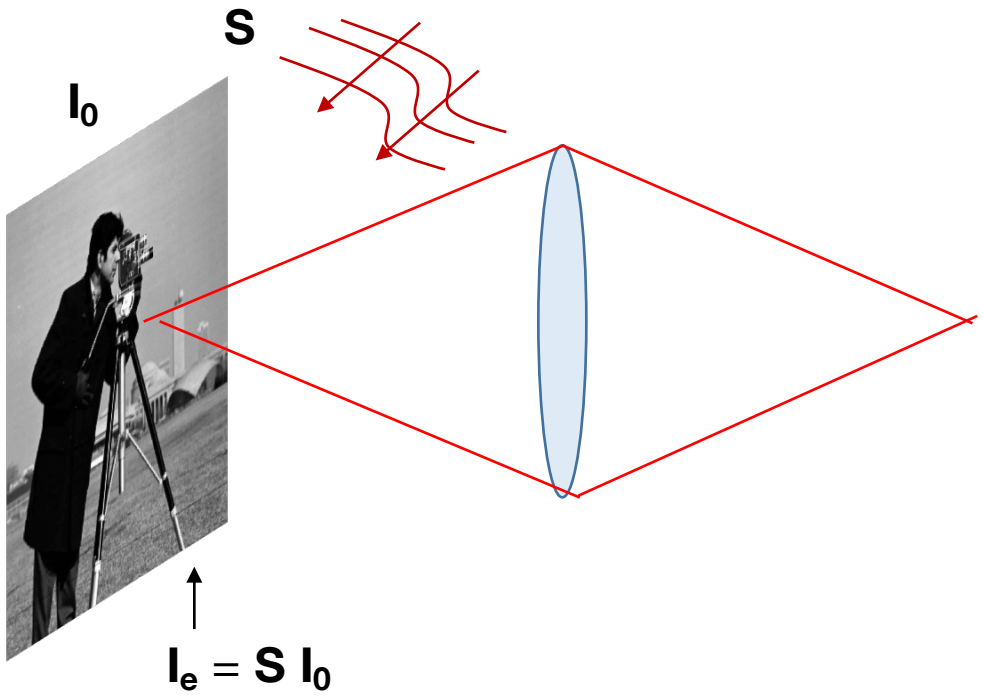
$I_p = H\ I_e = H\ S\ I_0$

Use downsampling matrix (sum-pooling)

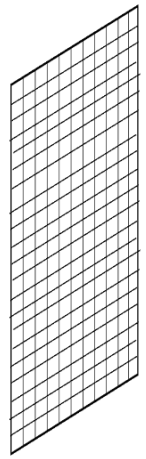
$D =$

$$\begin{bmatrix} 0.5\ 0.5\ 0\ 0\ 0\ \ldots. & & \\ & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. & \\ & & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. \\ \vdots & & \vdots \end{bmatrix}$$

Simple mathematical model of image formation



Discretization by detector?



$$I_s = D\ I_p = D\ H\ S\ I_0$$

Use downsampling matrix (sum-pooling)

$D =$

$$\begin{bmatrix} 0.5\ 0.5\ 0\ 0\ 0\ \ldots. & & \\ & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. & \\ & & 0.5\ 0.5\ \ 0\ 0\ 0\ \ldots. \\ \vdots & & \vdots \end{bmatrix}$$

deep imaging

Physical Layers

Digital Layers

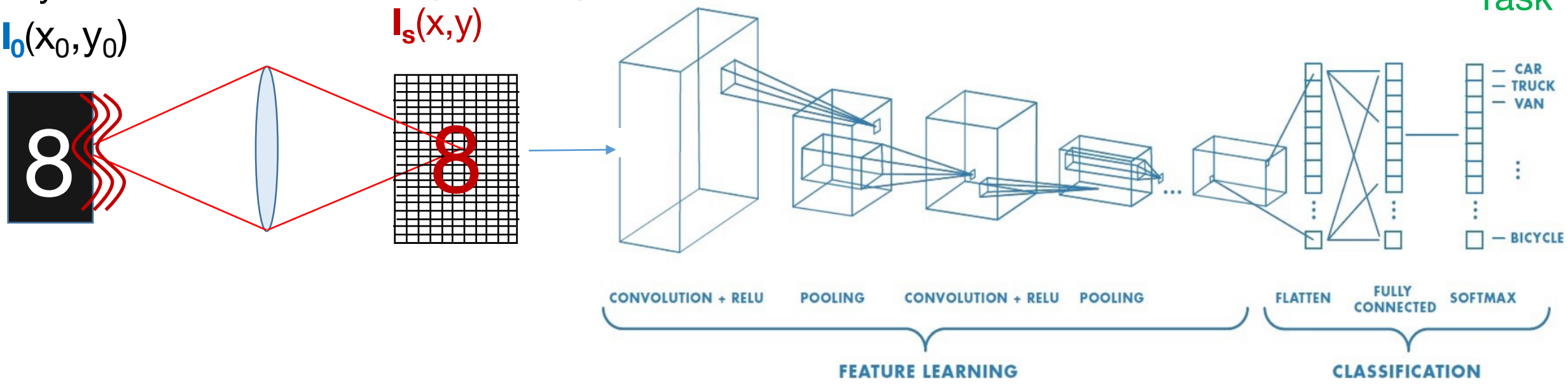
Physical world

$I_0(x_0,y_0)$

Digital Image

$I_s(x,y)$

Task



Digital layers

Physical layers

$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, f[I_0]]\ldots]$$

$$\text{Task} = \mathbf{W}_n \ldots \text{ReLU}[\mathbf{W}_1 \, \text{ReLU}[\mathbf{W}_0 \, \mathbf{D} \, \mathbf{H} \, \mathbf{S} \, I_0]\ldots]$$